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The Effect of Initial Conditions on the Development of Turbulent Boundary Layers

By

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SUMMARY

Calculations by a method that is believed to be reasonably accurate have been made to demonstrate how perturbations decay, in zero pressure gradient, until all that remains is a change in boundary layer thickness. Initial perturbations in typical aerofoil pressure gradients can usually be represented by an equivalent change in initial boundary layer thickness, which produces percentage changes of the same order of magnitude in distance to separation.

1. Introduction

Because of the widely differing responses of boundary-layer calculation methods to changes in the initial conditions[1], there is uncertainty and controversy about the sensitivity of real boundary layers to initial conditions. The question is of some practical importance apart from its bearing on the accuracy of calculation methods, because conditions existing just after transition are not known with any accuracy and it is often difficult to estimate the location of transition on an aerofoil: if boundary layers are really as sensitive to initial conditions as some of the calculation methods indicate, the determination of aerofoil drag is practically an ill-posed problem. It is clear, however, that the older calculation methods[2], which implicitly relate the local shear stress profile to the local mean velocity profile, cannot reproduce the behaviour of a real boundary layer: the sensitivity to a change of the velocity profile parameter H depends on the particular empirical relation between H and the shear-stress profile or entrainment function. The attempt by McDonald and Stoddart[1] to introduce a separate parameter describing the shear-stress integral is not different in principle from the above methods, because their empirical relation for this parameter implies that it is a function of H , the exact function depending (permanently) on the initial conditions, and in practice this may result in a gross overestimate of the influence of initial conditions.

More realistic attempts to represent the shear stress profile independently of the mean velocity profile are now being made [3,4,10] but the resounding failure of the older methods, which neglected "history" effects on the turbulent shear stress, and which McDonald therefore calls the "pre-historic" methods, has led to the feeling that, since history is obviously important, the initial conditions must exercise a large influence on the development of the boundary layer. It has been suggested that a definitive set of measurements should

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* Replaces NPL Aero Report 1223 - A.R.C.28 771

be made in a typical pressure gradient with a range of initial conditions, but we do not know which parameters we should attempt to vary, nor indeed how to vary one without the others: certainly, the traditional parameters such as δ_2 , c_f and H do not uniquely specify a turbulent boundary layer. One is therefore reduced to studying the response of boundary layers to real perturbations, such as might be caused by a region of transpiration or roughness, rather than to attempt to generate boundary layers artificially, for instance by vortex generators or graded grids.

As a first contribution to this study, and as a guide to any future experiments, some calculations of this sort have been done by the method of ref. 5, which is believed to be reliable over a wide range of different boundary layers and which represents the physical situation rather more closely than the integral methods. This method satisfies the mean-motion equation, the continuity equation and an empirically-modified version of the turbulent kinetic energy equation at each point in the boundary layer, enabling complete velocity and shear stress profiles to be calculated. Evidence of the accuracy of the calculations is given in ref. 5 and in fig. 13 below: in the present paper it will be assumed, unless otherwise stated, that the calculations are accurate enough for the comparative purposes for which they will be used. They should be useful as test cases for simpler methods, and full details are available from the authors. This paper does not treat the equally important problem of what initial conditions occur in practice - say, just after transition - because that problem must be studied experimentally.

In Section 2 the types of perturbation to be used are outlined, and their qualitative effect on the velocity and shear stress profiles is discussed . In Section 3 the distance required for recovery from typical perturbations in zero pressure gradient is discussed: it is concluded, not unexpectedly, that there is no universal decay law, and it appears that considerable disturbances may remain within the layer even after the surface shear stress has returned nearly to the unperturbed value for the appropriate Reynolds number. However, even large disturbances decay to negligible level in, say, 80 boundary-layer thicknesses, and the greater part of the disturbance disappears in less than half this distance. In Section 4, details are given of the response to a sudden small change in surface roughness, which is academically interesting because the disturbance is self-preserving[6]: it is also of great practical interest in meteorology. A good measure of the flexibility of a calculation method would be how soon after the change in roughness it could be used successfully.

In Section 5, the response to perturbations in arbitrary pressure gradient is discussed. It is pointed out that in many practical cases of aerofoil design the boundary layer will have largely recovered from a perturbation at the transition point (say) before being markedly affected by the adverse pressure gradient, so that the final effect of the perturbation is the same as that of an increase in initial boundary layer thickness, just as in zero pressure gradient. It is found that, usually, a given (small) percentage change in initial boundary layer thickness produces percentage changes of no greater order of magnitude in the properties at the trailing edge or in the pressure rise to separation.

The conclusion of this work is that turbulent boundary layers are less sensitive to initial conditions than has sometimes been supposed. This is quite compatible with the argument that the past history of the boundary layer must be taken into account in calculations - to use the language of linear systems, boundary layers have a very long periodic time, but they are heavily damped.

2. Choice of Perturbations

In the context of aerofoil design, the only real perturbing influences that can usefully be studied are changes of surface conditions or of pressure gradient. The response to the introduction of large obstacles into the flow is not relevant and cannot be calculated by any known method. We have chosen three methods of introducing types of disturbances, whose region of application extends over a streamwise distance of not more than a few boundary layer thicknesses: they can thus be treated as fairly weak point disturbances followed by a long region of response. By "perturbation" we mean the difference between the boundary layer immediately after the disturbance and the undisturbed boundary layer at the same Reynolds number (taken as $U_1 \delta_2 / \nu$).

In the discussion we shall make frequent use of the concept that the turbulent boundary layer is approximately hyperbolic in behaviour, as shown by the model equations of ref. 5 and assumed implicitly by several authors, so that the effect of a disturbance in velocity or shear stress introduced near the surface (say) propagates outward at a finite rate (at an angle of the order of 2 deg.). Also, we shall base the discussion on the behaviour of the shear stress rather than the mean velocity. Since the dimensionless eddy viscosity is of the same order of magnitude in all boundary layers, the shear stress mirrors the behaviour of the velocity gradient and is therefore a more sensitive parameter than the velocity itself.

Figs. 1 and 2 show the variation of c_f and H in the perturbed boundary layers. The "region of disturbance" in which the perturbation is applied is shown by dotted lines and the "region of recovery" by full lines. The bold line without points shows the behaviour of the unperturbed boundary layer. The perturbations all start at the point P , except for the case of roughness change. Before discussing the region of recovery, we discuss the way in which each type of disturbance affects the boundary layer.

(i) Short region of suction or injection. Here, V_w/U_1 rises linearly to 0.005 in a distance of two boundary layer thicknesses and remains at that value for 4 boundary layer thicknesses before returning quickly to zero. This produces a change of shear stress and mean velocity only near the surface: the change then decays, spreading into the outer layer. The outer layer, although not directly affected by the application of the disturbance, is perturbed from its equilibrium state later in the region of response, and gradually returns to equilibrium as the perturbation dies away altogether. The effect is thus similar to that of a band of roughness, but the perturbation can conveniently be made larger.

(ii) Change of surface roughness. For convenience, we simulated this by running a boundary layer with a decreased value of 1.5 for the additive constant A in the logarithmic law for a smooth surface

$$\frac{U}{\sqrt{\tau_w/\rho}} = \frac{1}{K}\left[\log \frac{(\tau_w/\rho)^{\frac{1}{2}} y}{\nu} + A\right] :$$

and then, after the layer reached equilibrium, restoring the additive constant to its proper value of 2. Therefore, the boundary layer does not start from the same point as the others in figs. 1 and 2. The shear stress in the initial "rough wall" boundary layer is everywhere higher than in the equilibrium boundary layer on a smooth surface but τ/τ_w is about the same at given y/δ :

/a restoring

a restoring change in shear stress and mean velocity propagates outward from the surface and finally reduces the shear stress to the equilibrium value everywhere.

(iii) Short region of adverse pressure gradient. The free-stream velocity varies as $c+(1-c)\cos x/2\delta$ from $x = o$ to $x = 2\pi\delta$, for $c = 0.975$ and 0.95 (5 percent and 10 percent reduction in free stream velocity respectively). The velocity profile changes as if the flow were inviscid, except close to the surface; the absolute shear stress is unaltered, except again near the surface, so that $\tau/\rho U_1^2$ is either 10 or 20 percent higher at the end of the disturbance region than in an equilibrium boundary layer with the same free stream velocity. Near the surface the shear stress falls considerably, as is the usual response to adverse pressure gradient: it rises again rapidly after the region of disturbance. It is interesting to see, from figs. 3 and 4, that in the early stages the surface shear stress and the shape parameter H are functions of the pressure rise, nearly the same for both pressure gradients. In the case of the shape parameter this follows from the concept of quasi-inviscid response. In the case of the surface shear stress it follows if the logarithmic law is satisfied at the inner boundary of the quasi-inviscid region: this will be a fair approximation if the shear stress responds much more quickly than the mean velocity, which it will do at points well within the inner layer; therefore the approximation works only for x/δ not too large.

The only disturbances to produce unexpected results were the short regions of suction or injection, for which the shear stress and velocity profiles are plotted in figs. 5 and 6 against $y-\delta_1$: this scale was chosen as a rough approximation to the stream function, so that the undisturbed outer layer can be distinguished from the perturbed inner layer. On the application of injection, the shear stress at the wall decreases rapidly as expected (fig. 5(a)), but the shear stress in the inner layer rises to about twice the wall value. The reason is that in the calculation method it is assumed that the dissipation length parameter (a typical eddy size) is proportional to the distance from the surface, so that eddies which are suddenly displaced away from the surface by injection of fluid at the surface are supposed to undergo a sudden increase in length scale and a corresponding decrease in dissipation: since the velocity gradient, shear stress and (therefore) turbulence production on a given streamline are initially unaltered, the turbulent intensity and shear stress start to grow rapidly. The same phenomenon occurs in practice in the inner layer of a boundary layer just after separation from a sharp edge[7], so that the present calculations should be qualitatively correct. Both the dissipation length parameter and the diffusion parameter G used in the method of ref. 5 will be altered by such a large perturbation so the results are certainly not accurate - the "shock wave" at the outer edge of the perturbation would be diffused in real life - and for present purposes the perturbations resulting at the end of the region of transpiration should be regarded merely as perturbations of unspecified origin.

3. Decay of Perturbations in Zero Pressure Gradient

We now proceed to discuss the full-line portions of the curves in figs. 1 and 2, representing the region of recovery from the disturbances. Some of the curves from fig. 1(a) are reproduced in figs. 1(b) and 1(c) for clarity. The first thing to notice from the miscellany of curves in fig. 1(a) and fig. 2 is that there is no simple decay law for c_f or H, even when the perturbation is small. This is contrary to the assumptions

/basic

basic to the older integral methods[4], which use a shape parameter equation $\delta_2 \frac{dH}{dx} = f_1\left(H, \frac{U_1 \delta_2}{\nu}\right)$ in zero pressure gradient together with a "Ludwieg-Tillman" type of skin function law, $c_f = f_2\left(H, \frac{U_1 \delta_2}{\nu}\right) = 2\, d\delta_2/dx$,

so that $\delta_2 \frac{dc_f}{dx} = f_3\left(H, \frac{U_1 \delta_2}{\nu}\right)$: in figs. 1 and 2 we see that at a given Reynolds number there is no unique relation between c_f (or H) and its gradient, and therefore no equation like $\delta_2\, dc_f/dx = f\left(c_f - c_{f_{eqm}}\right)$ which would yield a generalized "time constant" for the boundary layer. As a demonstration of the non-linearity of the problem, it may be noted that the response to a short region of suction (fig. 1(b)) is much less violent and decays much more quickly than the response to a similar region of injection, and that the response to a short region of adverse pressure gradient (fig. 1(c)) is more than doubled when the change of pressure is doubled. The effect of a 10 percent reduction in free stream velocity over a distance of six boundary layer thicknesses takes about 80 boundary layer thicknesses to die away, so that some perturbations, at least, are very slow to decay completely. However, all the perturbations have decayed to a low level after 40 boundary layer thicknesses.

The surface shear stress or H alone is not an adequate measure of the decay of a perturbation. In figs. 7(a) - (f) are shown the shear stress profiles for each disturbed boundary layer at a point where the perturbation in c_f has decreased to approximately 1 percent of the undisturbed c_f at the same Reynolds number.

The profiles vary widely. The same behaviour was noted in ref. 8, where it was shown that the surface shear stress in a strongly-perturbed boundary layer approached the unperturbed value while the shear stress profile as a whole was quite unlike the unperturbed profile. The properties of these profiles could not be represented by any function of H and c_f alone.

4. Self-preserving Response of the Inner Layer to a Change in Wall Roughness

A numerically exact solution for this problem, discussed by Townsend[6], could be obtained by substituting self-preserving forms for the velocity and shear stress profiles in the three equations of ref. 5 to give two simultaneous ordinary differential equations, simulating a change in roughness by changing the roughness length scale z_0 in the logarithmic law

$$U = \frac{(\tau_w/\rho)^{\frac{1}{2}}}{K} \log y/z_0$$

which is used as the boundary condition in the calculations. We have not yet done this, and the results shown in figs. 8-11 were obtained by solving the partial differential equations for a boundary layer with a sudden change in the additive constant A in the logarithmic law for a smooth surface,

$$U = \frac{(\tau_w/\rho)^{\frac{1}{2}}}{K}\left[\log\, (\tau_w/\rho)^{\frac{1}{2}}\, y/\nu + A\right].$$

The "roughness length" is $\nu/(\tau_w/\rho)^{\frac{1}{2}} \exp A$ and therefore changes slightly

/with

with τ_w. For the calculation shown here, A was maintained at 1.5 for long enough for the boundary layer to reach equilibrium, and then returned to the usual value of 2, giving a decrease of 40 percent in z_0.

The length scale y_c, plotted in fig. 10 as $\frac{u_\tau y_c}{\nu}$, where u_τ is an average value of $(\tau_w/\rho)^{\frac{1}{2}}$, is the distance from the surface of the outgoing characteristic starting from the point at which the "roughness" changes, calculated from

$$\frac{dy_c}{dx} = \tan \gamma = \sqrt{2a_1 \tau_w}/U$$

The shear-stress scale $\Delta \tau_w$(fig.11) is the difference between the surface shear stress in the perturbed and unperturbed boundary layers, assumed small. According to Townsend's analysis $\Delta \tau_w$ should be proportional to $1/\log y_c$ for not too small x. Very roughly, therefore, $\Delta \tau_w \sim 1/\log x$ which is an extremely slow rate of decay. (Note that this expression is not valid for very large x because it is conditional on the disturbance being confined to the inner layer). ΔU is the total difference in velocity: the part of ΔU attributable to streamline displacement rather than stress gradients has not been subtracted: this alters the self-preserving velocity function (slightly) but not the fact of self-preservation. The departures from self-preservation of the profiles are partly the result of failure to represent a fully-rough wall, partly the fault of the numerical method (the first profile in figs. 8 and 9 extends over only 8 mesh points) but chiefly the fact that y_c/δ is as large as 0.5 at the largest value of x whereas the inner-layer extends only to $y/\delta \simeq 0.2$.

The results of figs. 8-11 are directly applicable to changes of terrain under the Earth's boundary layer.

5. Response to Perturbations in an Adverse Pressure Gradient

The chief conclusion to be drawn from the miscellaneity of the responses to various perturbations in zero pressure gradient is that there is no hope at all of deriving any general rules for arbitrary pressure gradient: in particular, it is not realistic to correlate the responses or even the initial perturbations in terms of H and $U_1 \delta_2/\nu$ alone so that no judgement can be made of the accuracy of the integral methods.

Since the effect of a simple increase in thickness of the initial boundary layer on the subsequent development in a pressure gradient is easy to understand, we can also understand the cases in which a perturbed boundary layer (or a given part of it) recovers from the perturbation before being greatly altered by the pressure gradient: the parenthesis is useful because the effects of pressure gradient are first felt near the surface, where the effects of a perturbation die out most quickly. If the pressure gradient or the perturbation are so strong that this sort of superposition is not applicable, the effect of the perturbation may be greater than the effect of the equivalent increase in thickness, but it is difficult to think of circumstances in which it could be an order of magnitude greater unless it provokes almost immediate separation.

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In fig. 12 are shown calculations for a free-stream velocity proportional to $(x+24)^{-0.255}$ [9], the initial boundary layer approximating to zero pressure gradient. For an initial value of δ_2 of 0.28 in. the boundary layer just avoids separation and reaches equilibrium in the power-law pressure gradient far downstream, but for larger values it separates. Figs. 12(a)-(c) show the results of calculations with initial momentum thicknesses of 1.11 and 1.18 times this critical value (but with the same Reynolds number for computational convenience): the pressure rise to separation is respectively 0.59 and 0.55 times the initial dynamic pressure. Therefore, even for a boundary layer which only just separates, the percentage change in pressure rise or distance to separation is of the same order of magnitude as the percentage change in initial boundary layer thickness, and the ratio of momentum thickness to the initial momentum thickness is virtually unaltered. This last conclusion can be reached by ignoring the c_f term in the momentum integral equation and setting H = constant, but it is surprising to see it confirmed in boundary layers that only just separate.

It is also interesting to note the effect of small changes in the initial velocity profile (representing two attempts at reading the very small published graphs) in a boundary layer which just avoids separation,[10] because this is also about the most sensitive case one could imagine.* A 3 percent increase in momentum thickness at almost constant displacement thickness (giving a 3 percent increase in H) causes practically no change in momentum thickness at the end of the run and only about a 2 percent increase in the final displacement thickness. However, the minimum value of c_f decreases by 30 percent, an order of magnitude greater than the percentage change in initial conditions.

It seems fair to conclude from these two particularly sensitive examples that the effect of a change in initial conditions is rarely of greater order of magnitude than the change itself: the only obvious exception is a boundary layer which just separates (or nearly separates) in the unperturbed state.

6. Conclusions

(1) The response of a boundary layer to simple perturbations is too complicated for any general rules about decay rate to be formulated, even in zero pressure gradient. Calculation methods employing only one or two integral parameters of the velocity profile cannot represent these processes accurately, even in principle, so that it is not meaningful to compare their sensitivity to perturbations with that of a real boundary layer.

(2) Since most of the effect of a perturbation in zero pressure gradient disappears in 30 or 40 boundary-layer thicknesses, perturbations arising in the transition region on an aerofoil may die out before the pressure gradient becomes strong: if the perturbation does not interact with the pressure gradient, its final effect is the same as a simple change in boundary layer thickness.

(3) In an adverse pressure gradient, changes in initial conditions usually produce changes of no greater order of magnitude at the end of the run. The only obvious exception is a boundary layer which just separates, or just avoids separation, in the unperturbed state.

/Notation

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* It is also a very severe test of the calculation method.

Notation

A	additive constant in logarithmic law for a smooth surface, Section 2
c_f	$\tau_w / \frac{1}{2}\rho U_1^2$
H	velocity-profile shape parameter, δ_1/δ_2
K	factor in logarithmic law, Sections 2,4
U,V	mean velocities in x,y directions
u_τ	average value of $(\tau_w/\rho)^{\frac{1}{2}}$, Section 4
x	streamwise distance from start of perturbation
y	distance normal to surface
y_0	length scale of perturbation, Section 4
z_0	roughness length scale, Section 4
δ_0	total boundary layer thickness at start of perturbation, $1.10\ \delta_{995}$ at that position
δ_1	displacement thickness
δ_2	momentum thickness
ν	kinematic viscosity
ρ	density
τ	shear stress

Suffixes:

o	start of perturbation
1	free stream
w	wall

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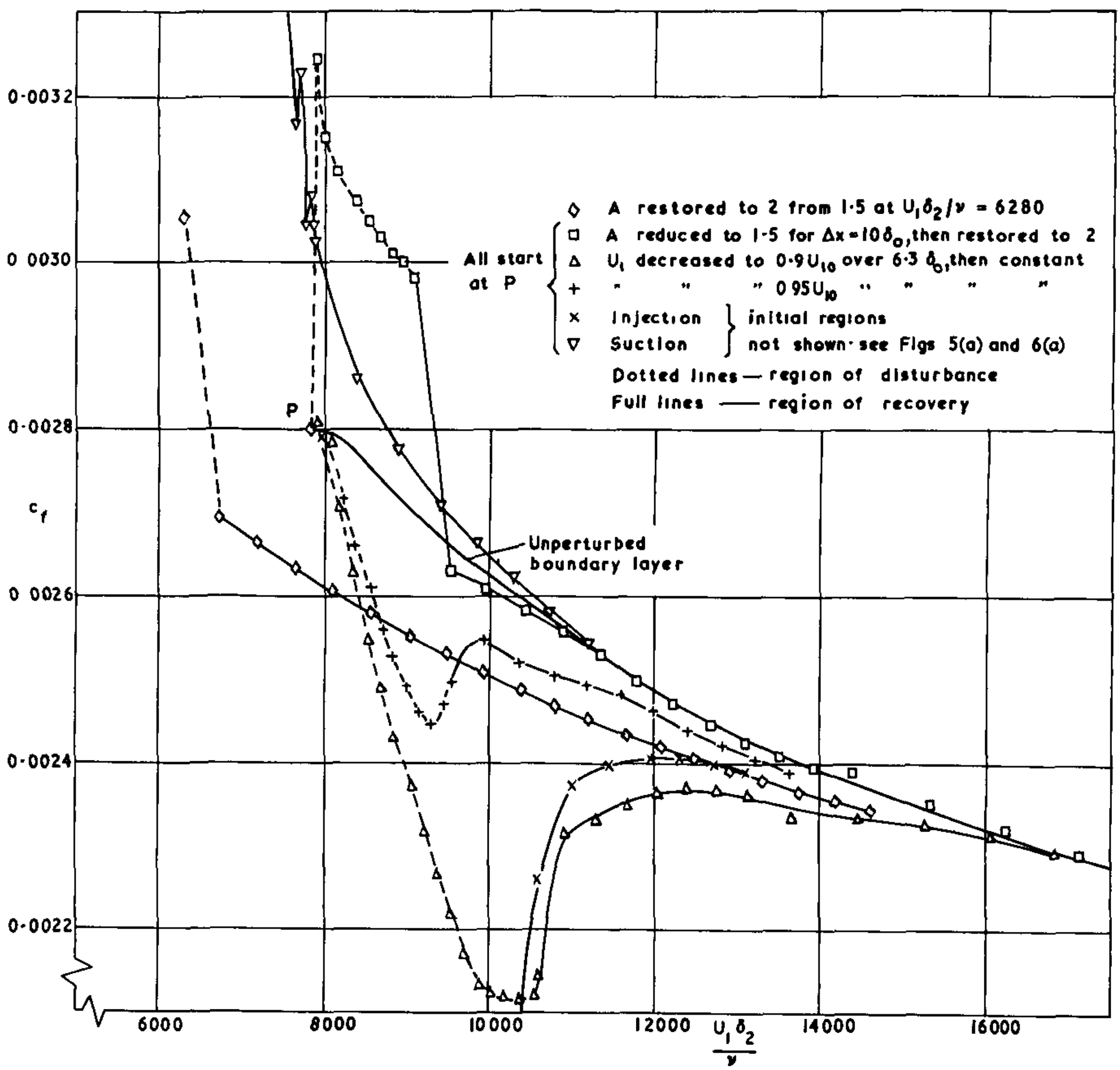
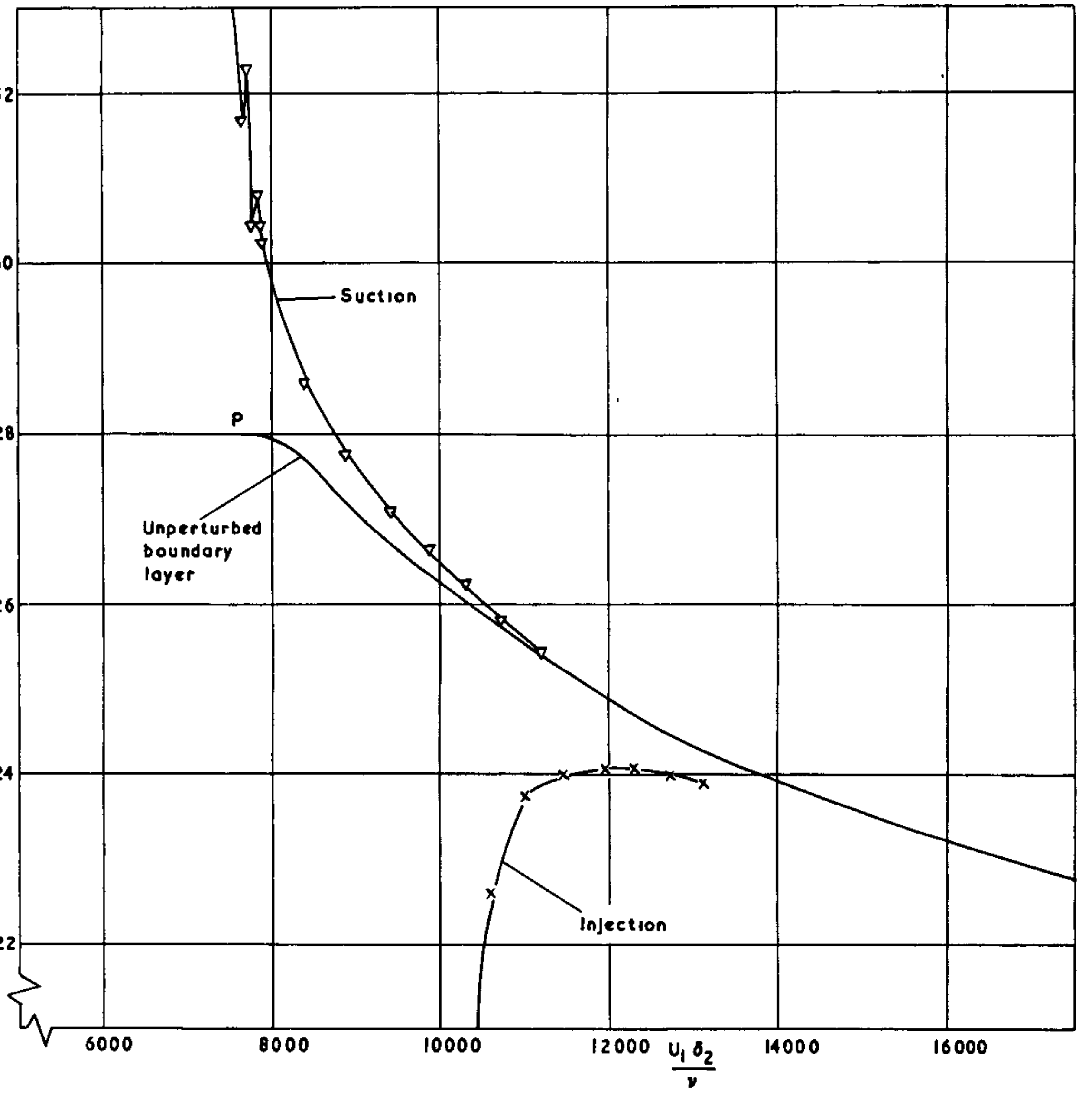
FIG.1 (a)



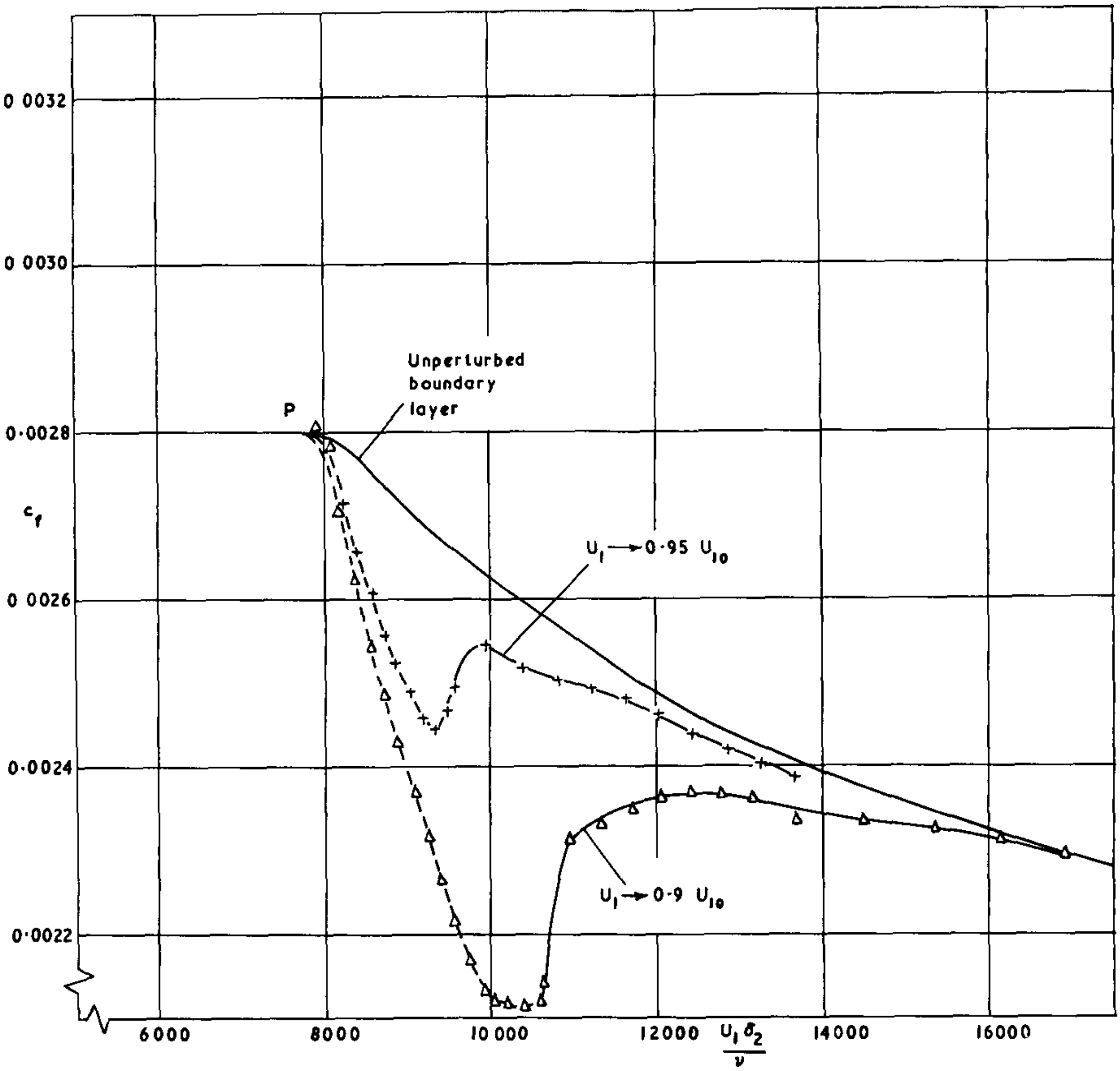
Response of surface shear-stress coefficient in zero pressure gradient

FIG 1(b)

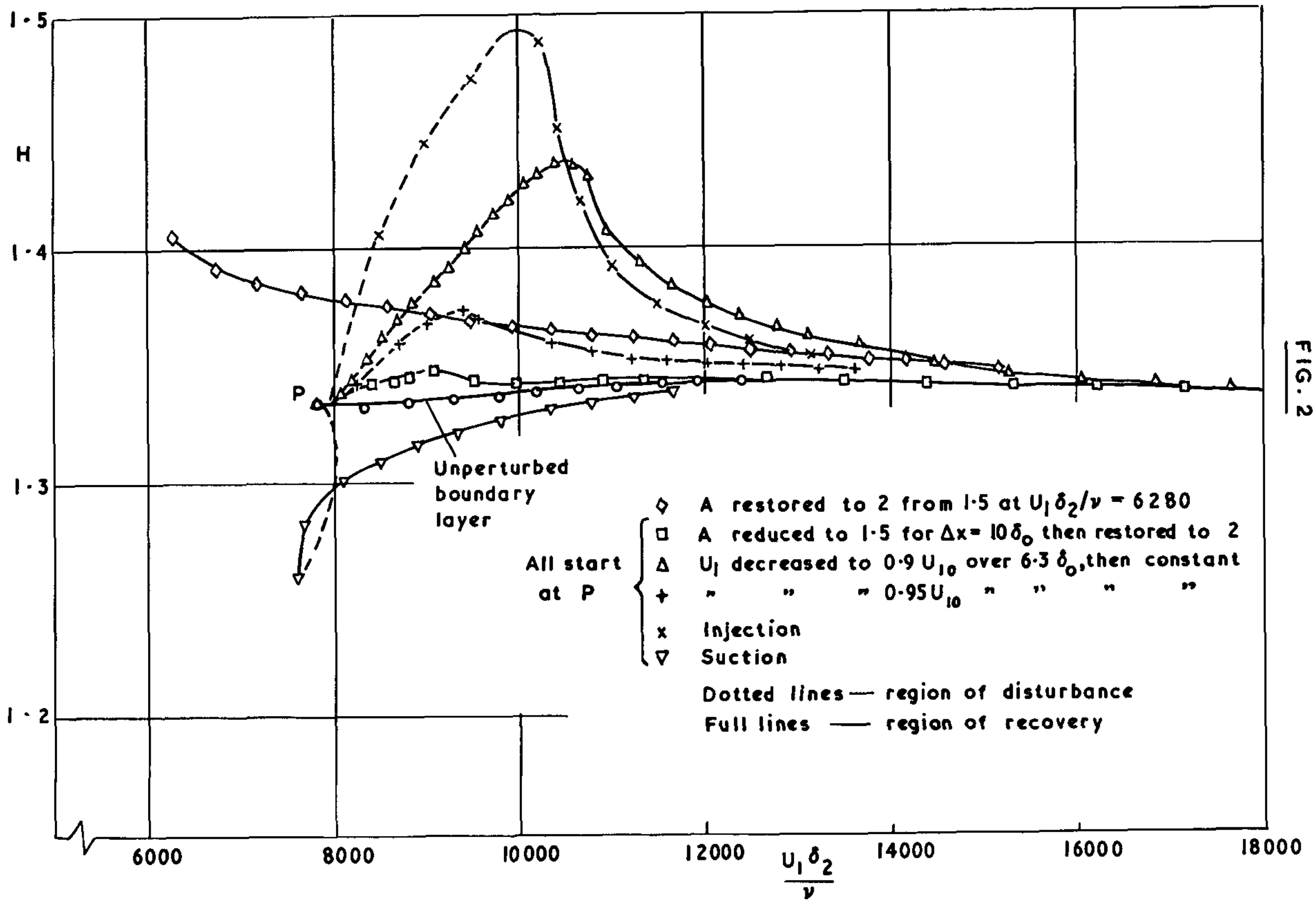


Response of c_f to short regions of suction or injection

FIG 1(c)

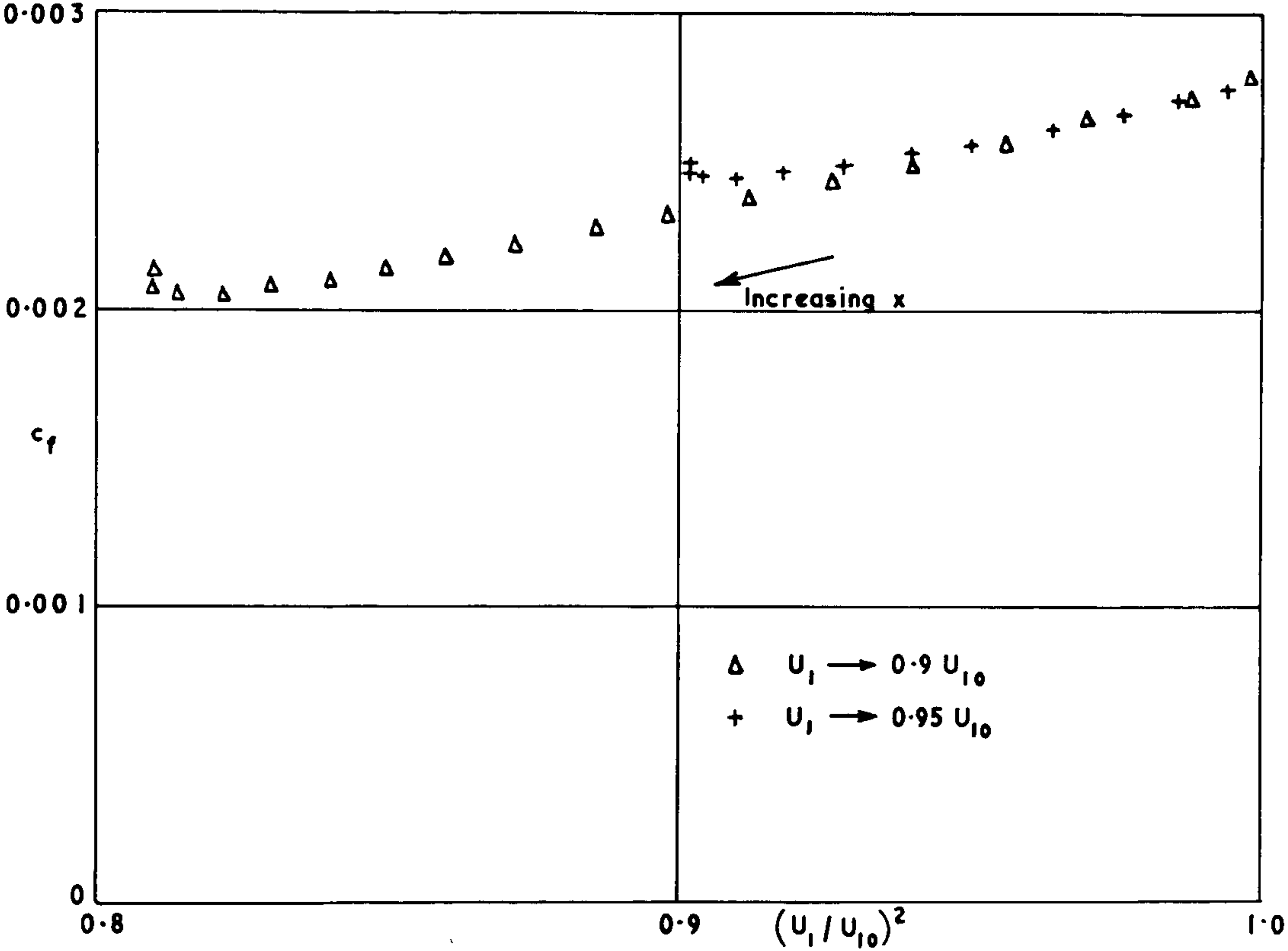


Response of c_f to rapid reduction in free-stream velocity



Response of shape parameter in zero pressure gradient

FIG. 2



Connection between c_f and U_1 in strong adverse pressure gradient

FIG. 3

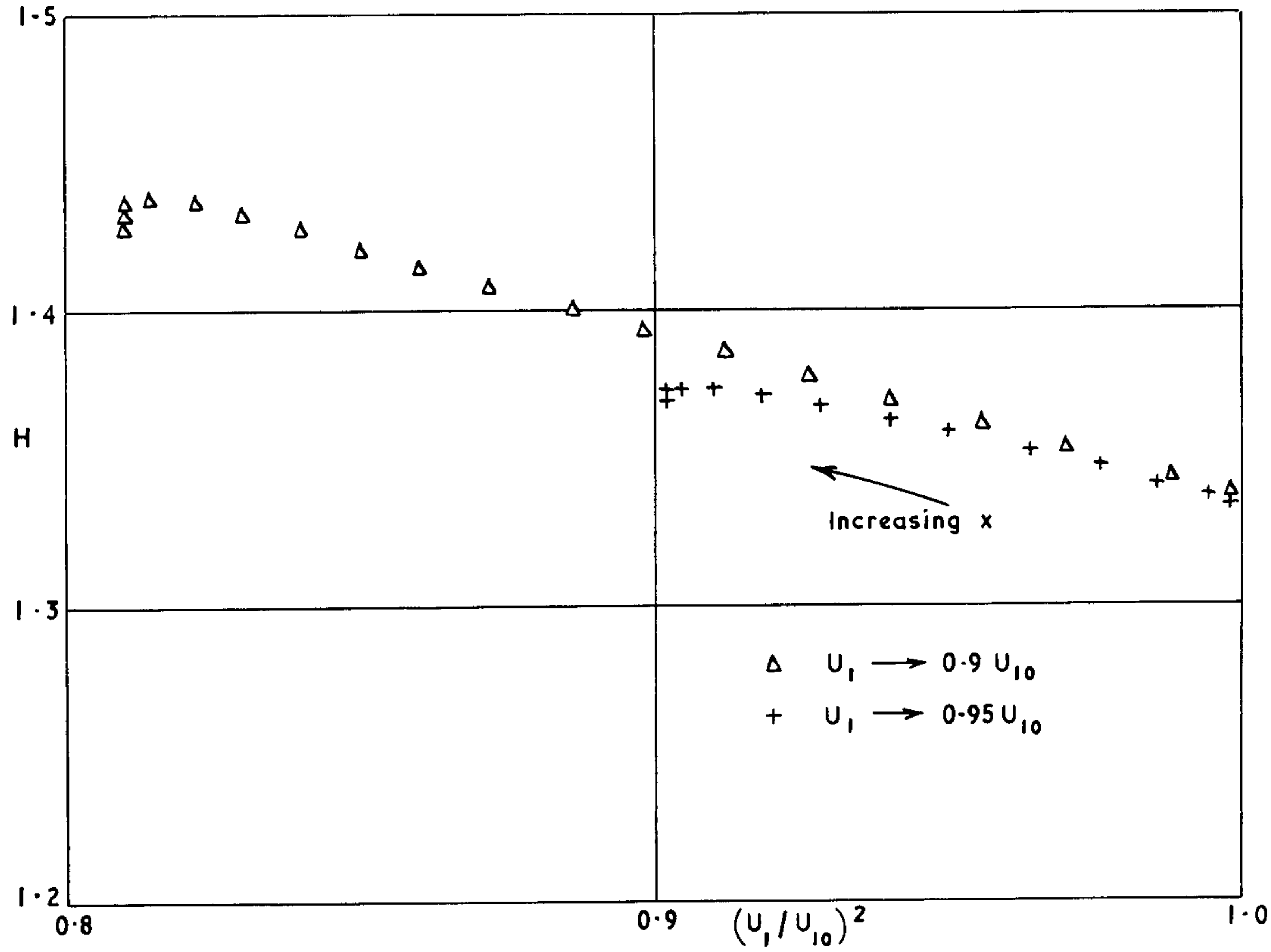


FIG. 4

Connection between H and U_1 in strong adverse pressure gradient

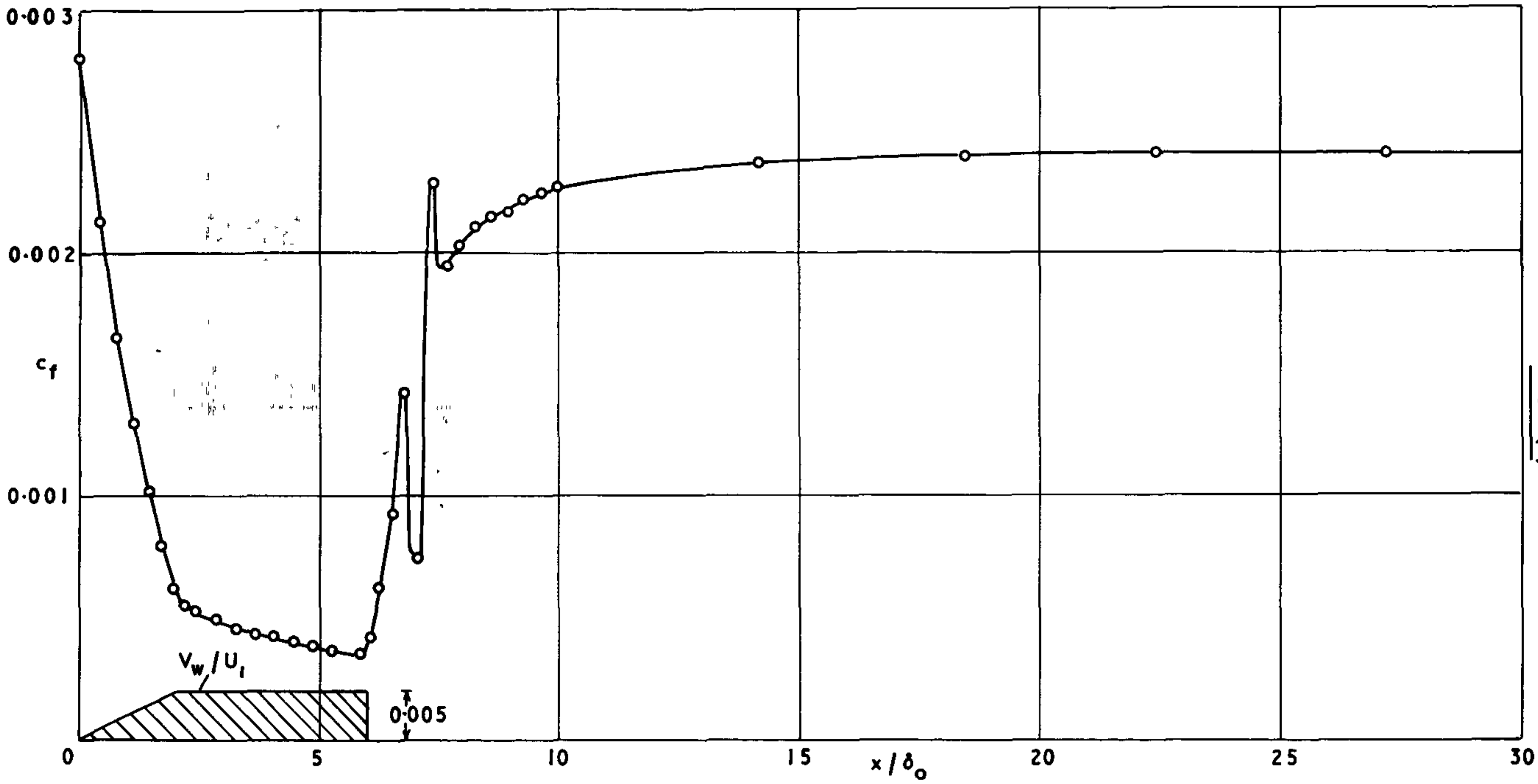
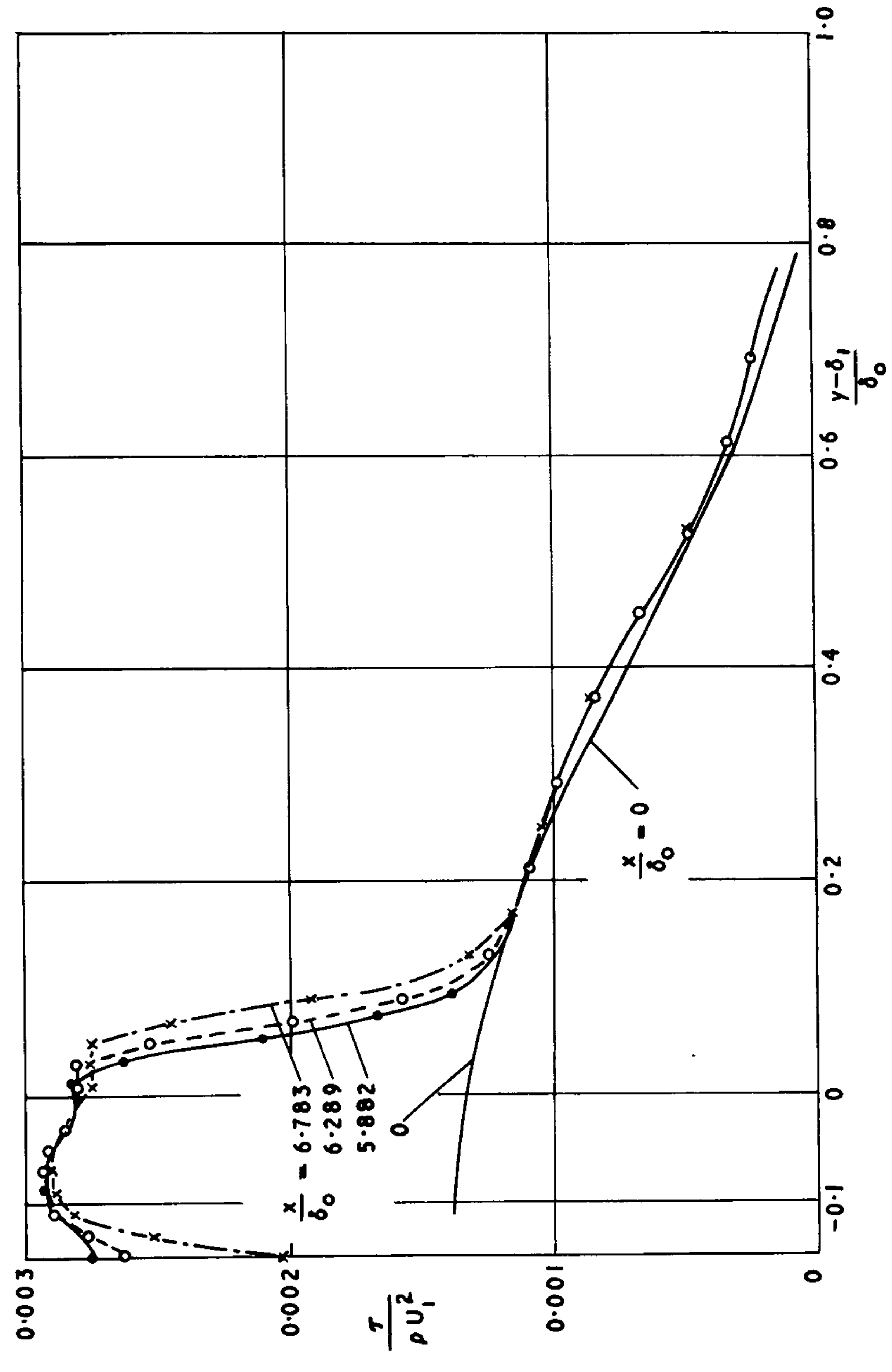


FIG.5(a)

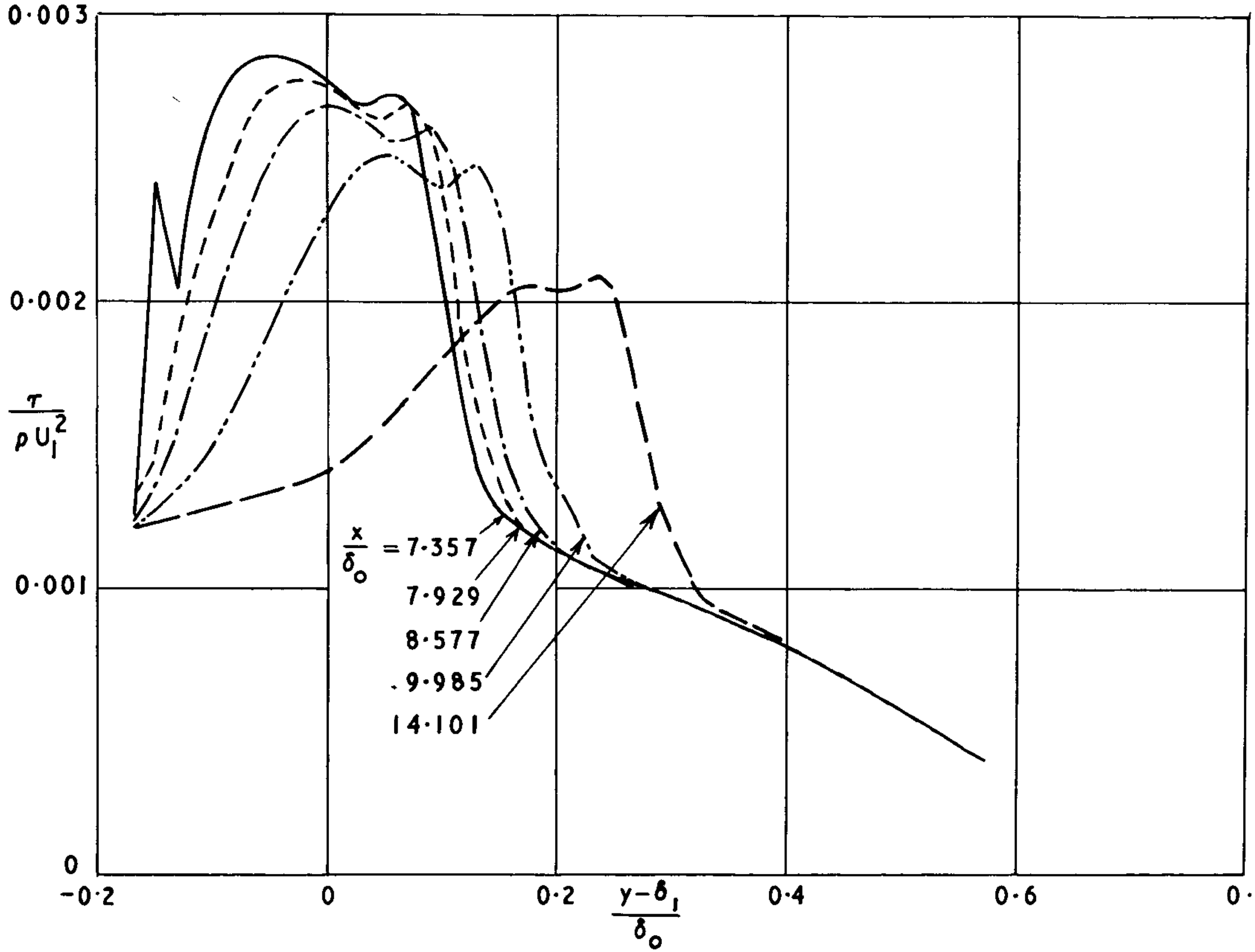
Surface shear stress with short region of injection

FIG 5(b)



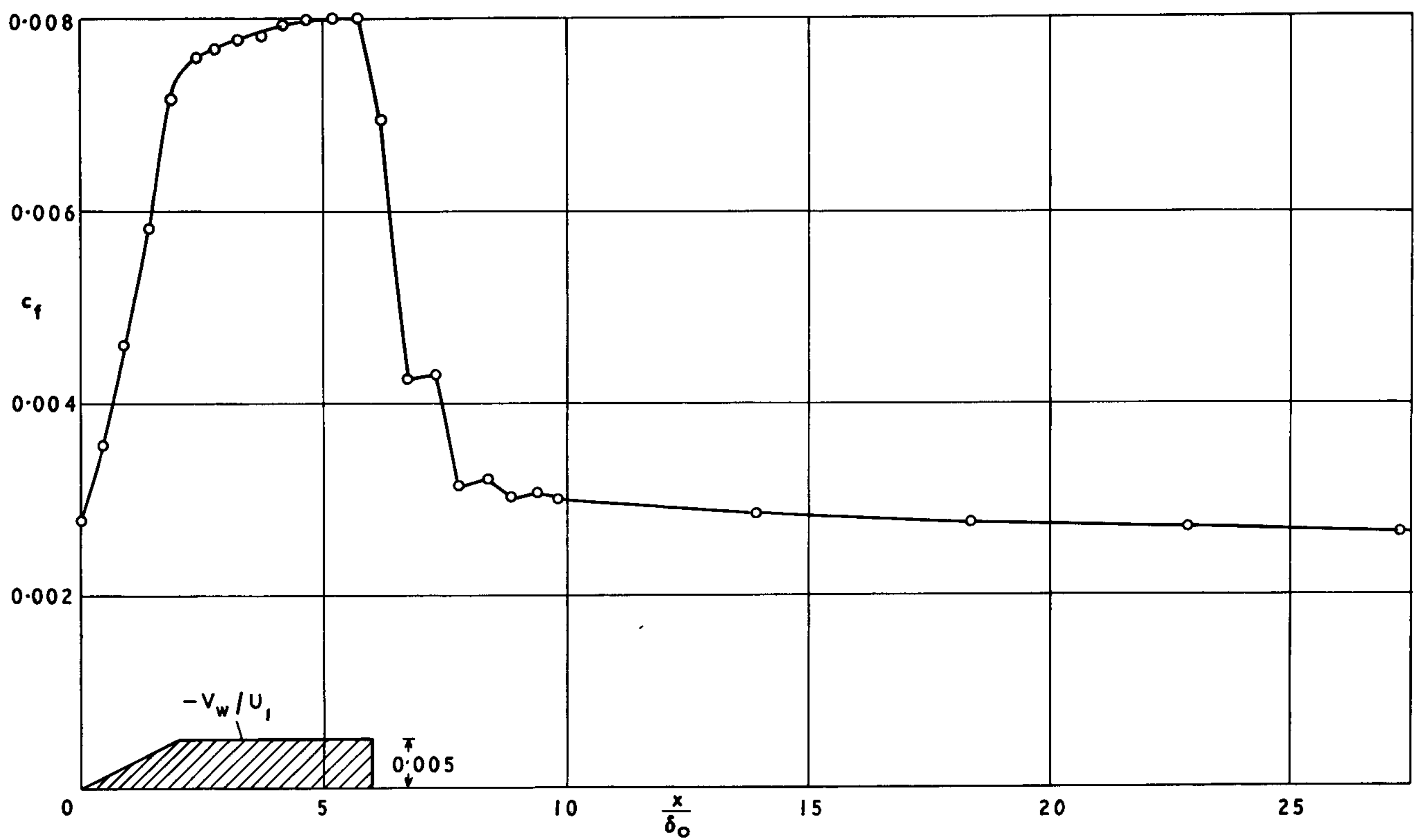
Shear-stress profile after short region of injection

FIG. 5(c)

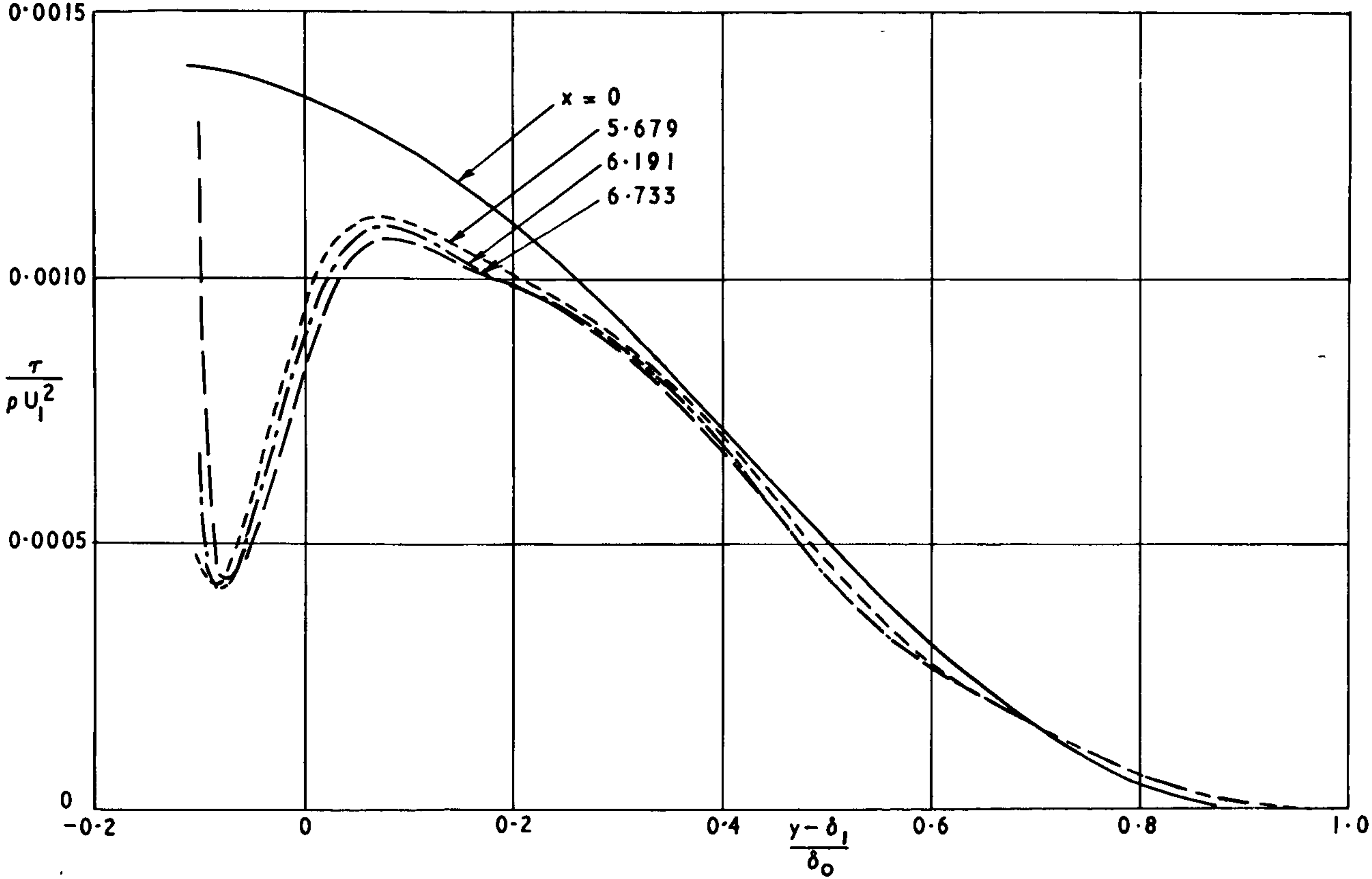


Shear stress profiles after short region of injection — continued. See also Fig 7(e)

FIG. 6(a)



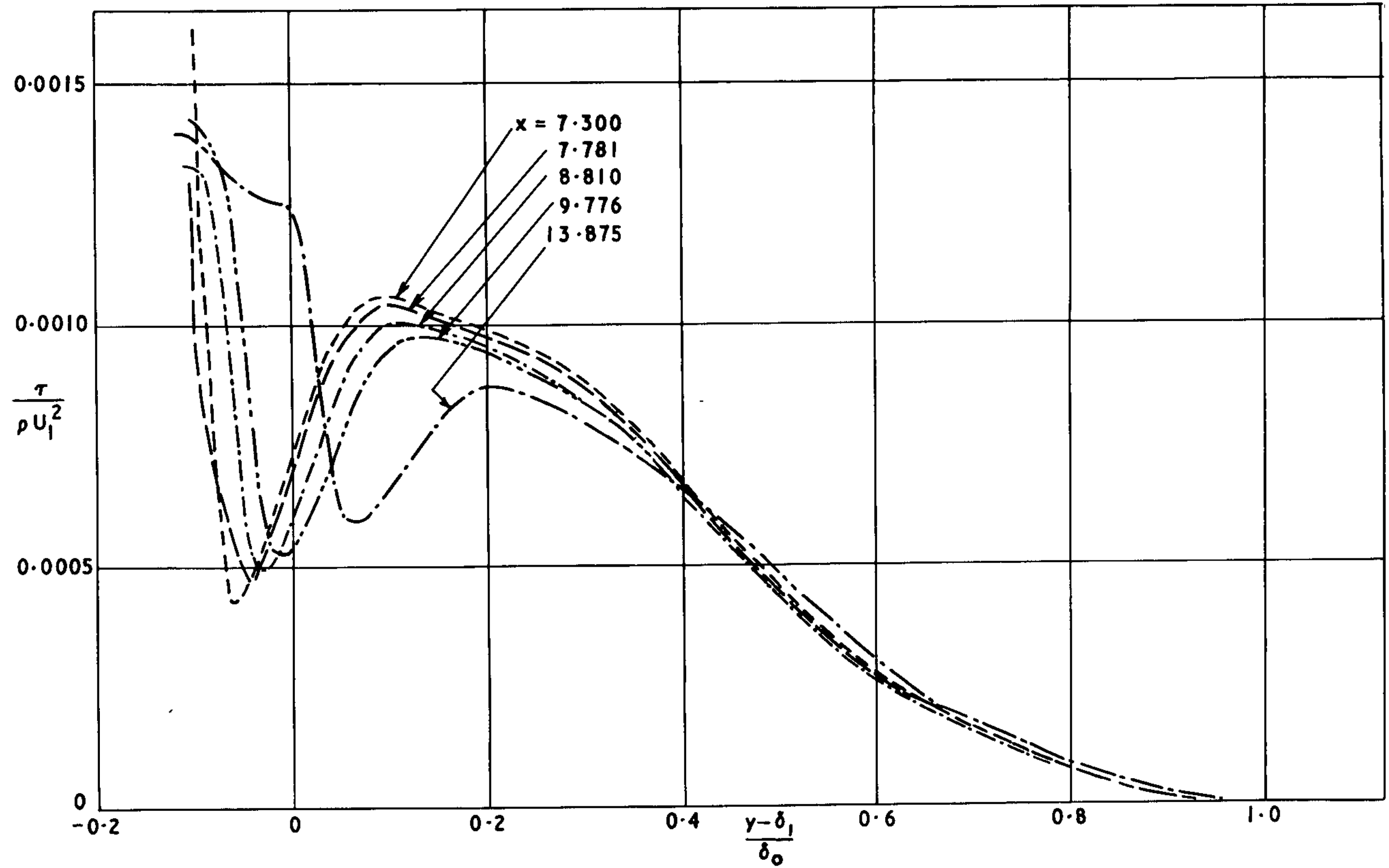
Surface shear stress with short region of suction



Shear stress profiles after short region of suction

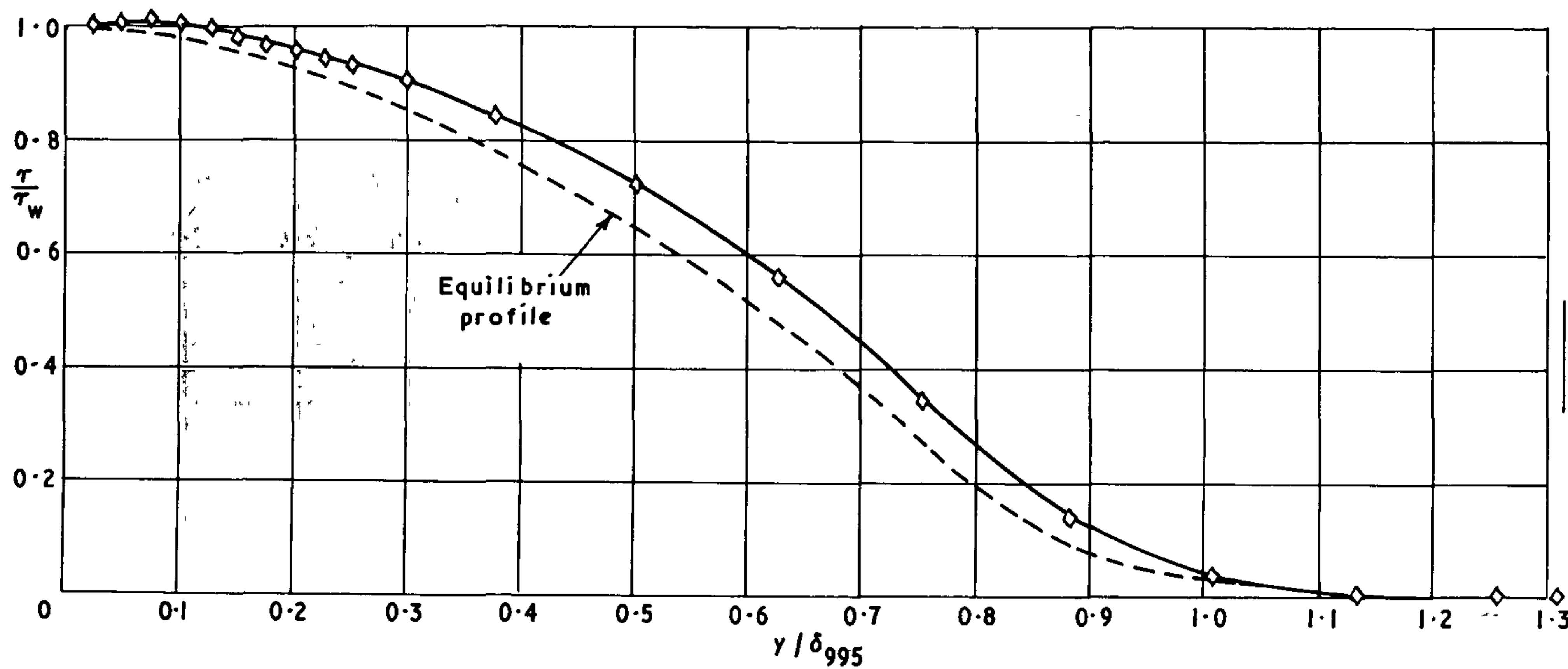
FIG. 6(b)

FIG 6(c)



Shear stress profiles after short region of suction — continued. See also Fig. 7(f)

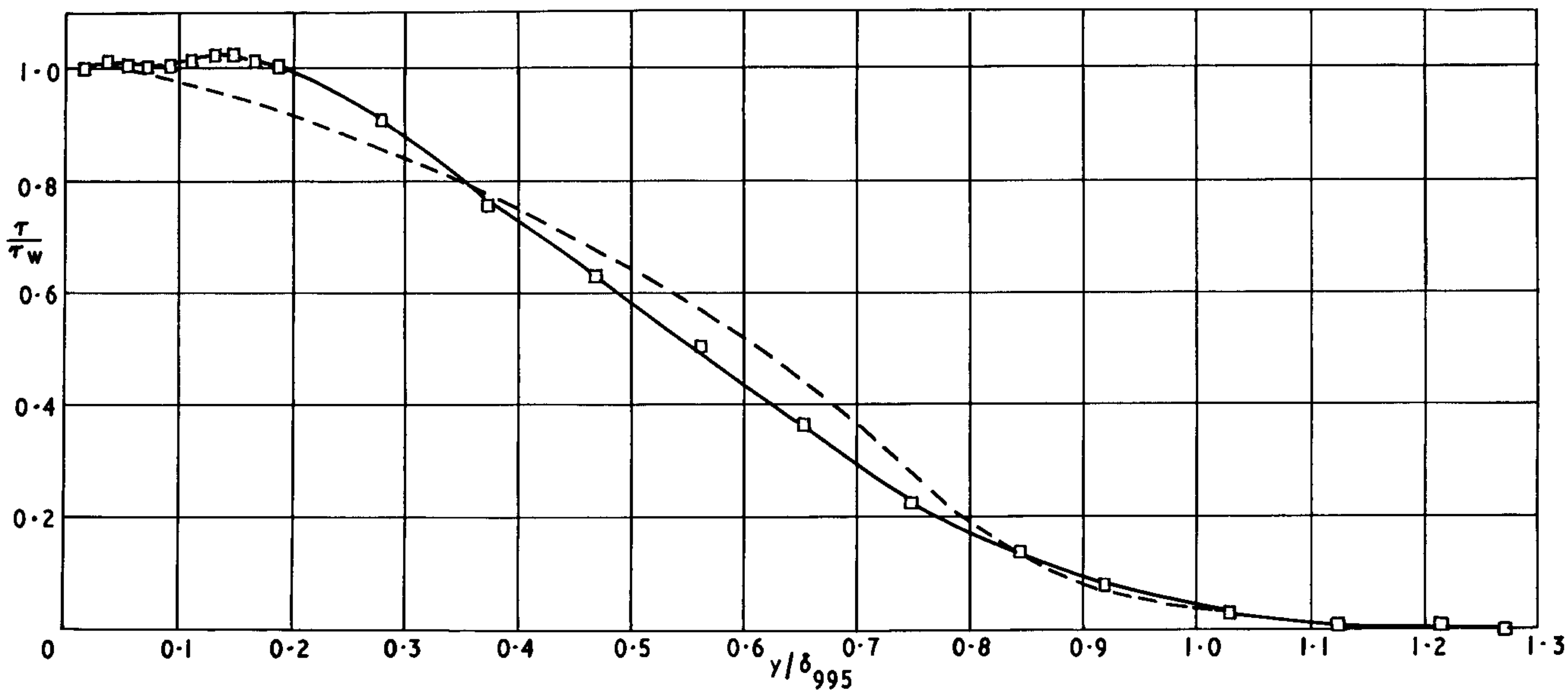
FIG.7(a)



(a) A restored to 2 from 1·5 : $\frac{x}{\delta_o} = 102$

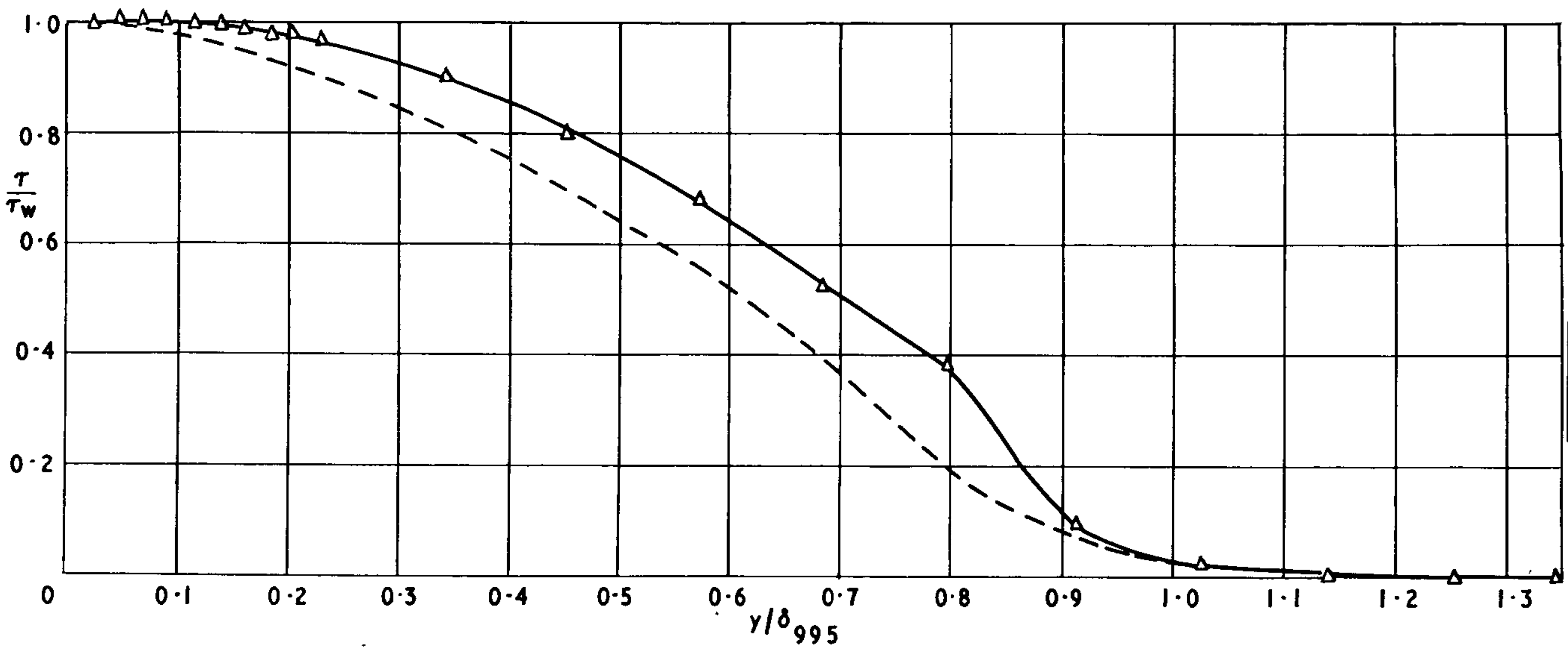
Shear stress profiles when $\Delta\tau_w/\tau_w \simeq 0{\cdot}01$: symbols as in Fig. 1(a)

FIG. 7(b)



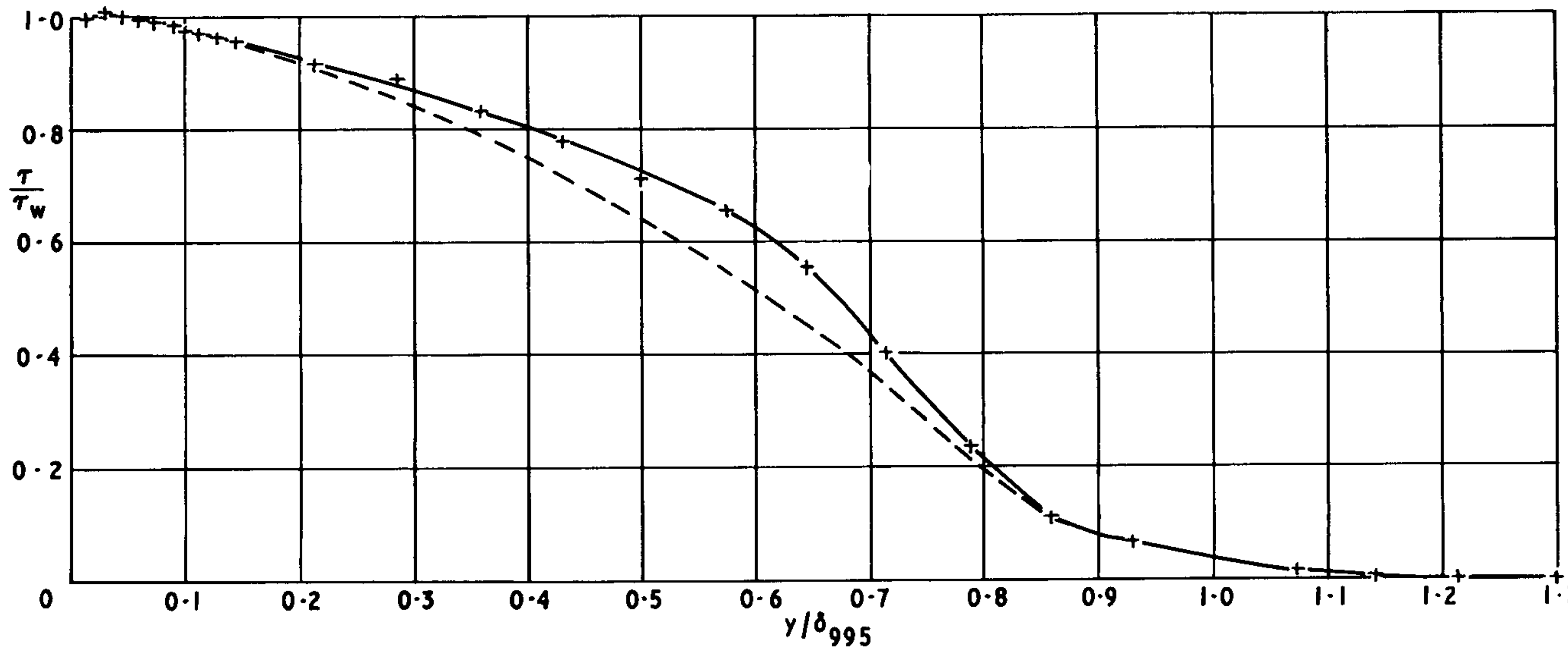
(b) Short region of A change : $\frac{x}{\delta_o} = 13$

FIG. 7(c)



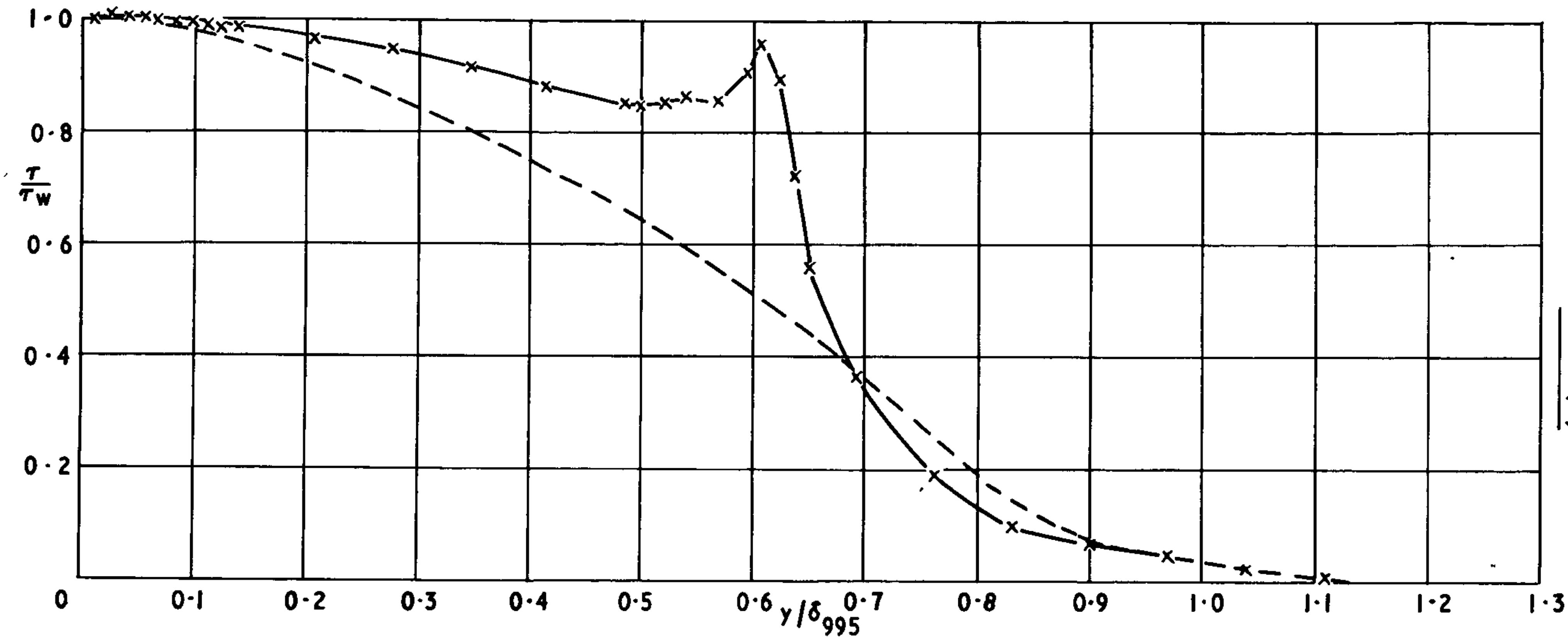
(c) $U_1 \rightarrow 0{\cdot}9 : \frac{x}{\delta_o} = 40$

FIG. 7 (d)



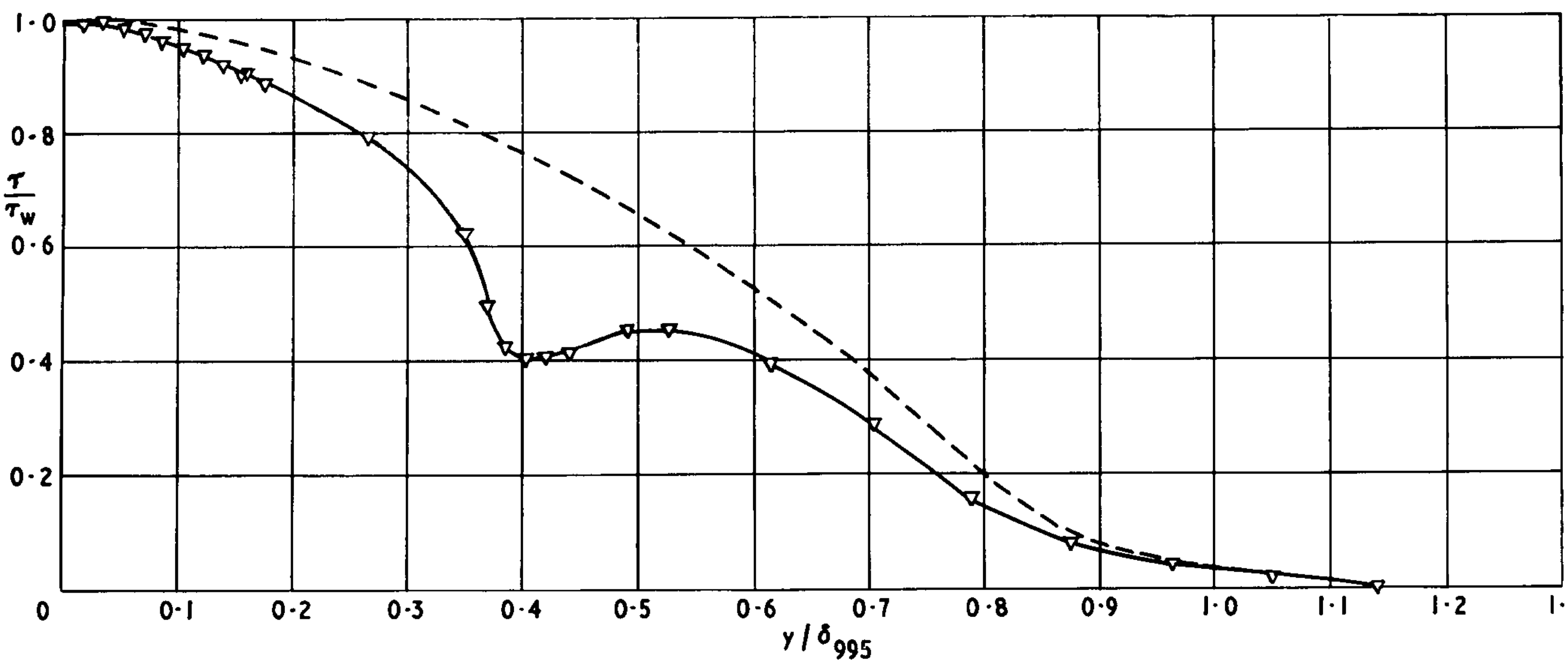
(d) $U_1 \rightarrow 0 \cdot 95$: $\frac{x}{\delta_o} = 23$

FIG. 7 (e)



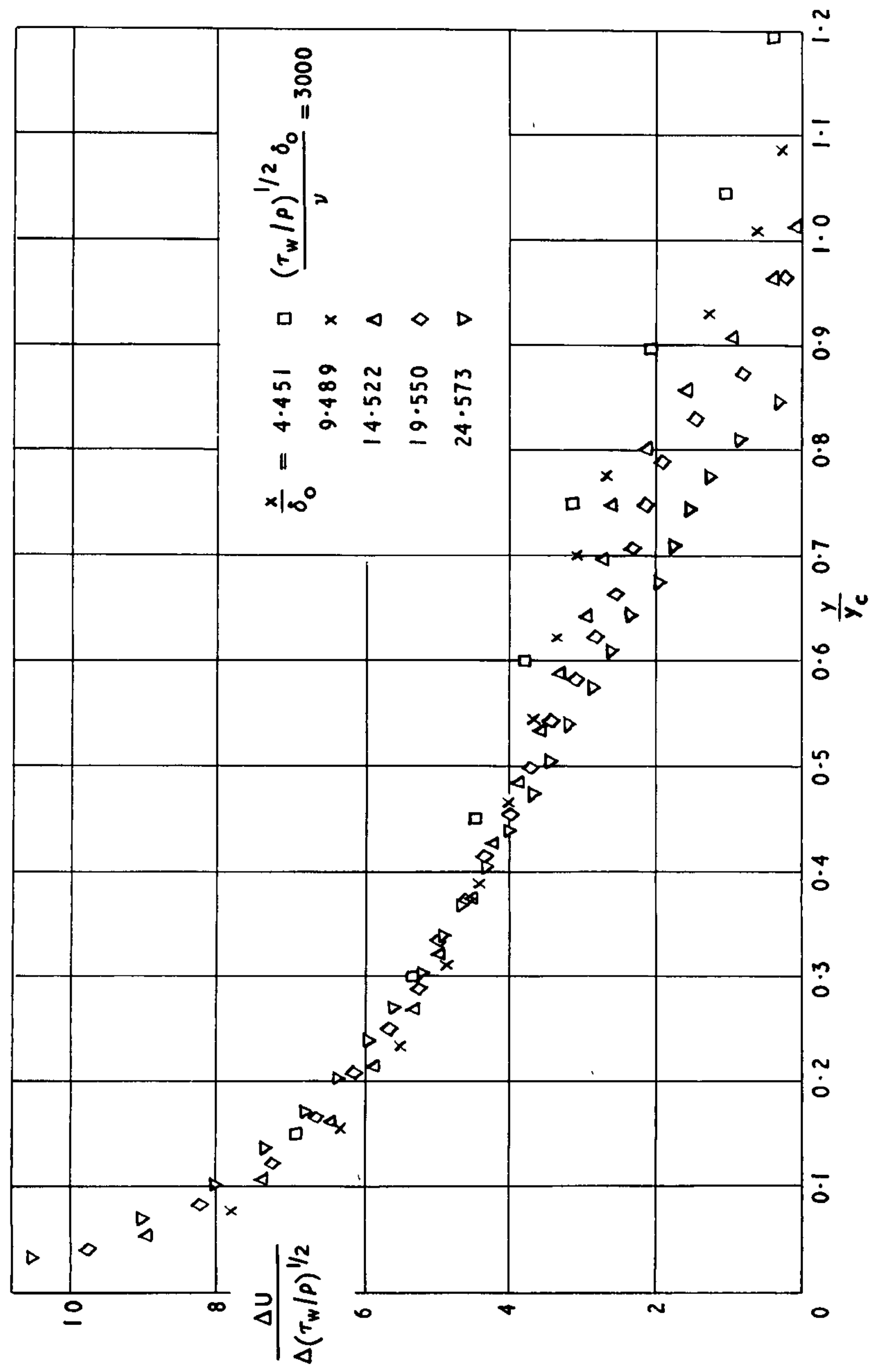
(e) Short region of injection : $\frac{x}{\delta_o} = 36$

FIG. 7(f)



(f) Short region of suction: $\frac{x}{\delta_o} = 27$

FIG. 8



Self-preserving response of velocity profile to change in A

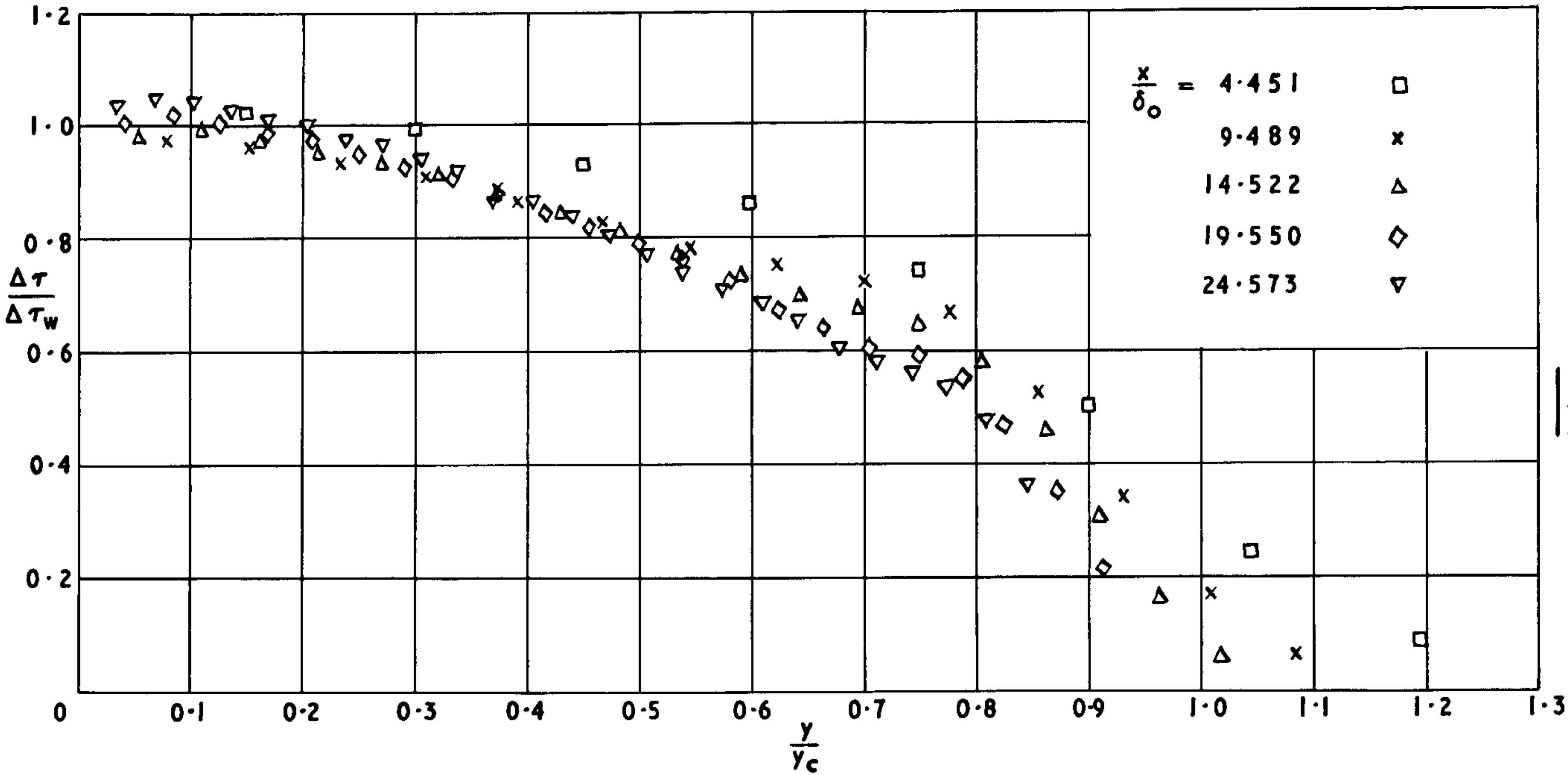
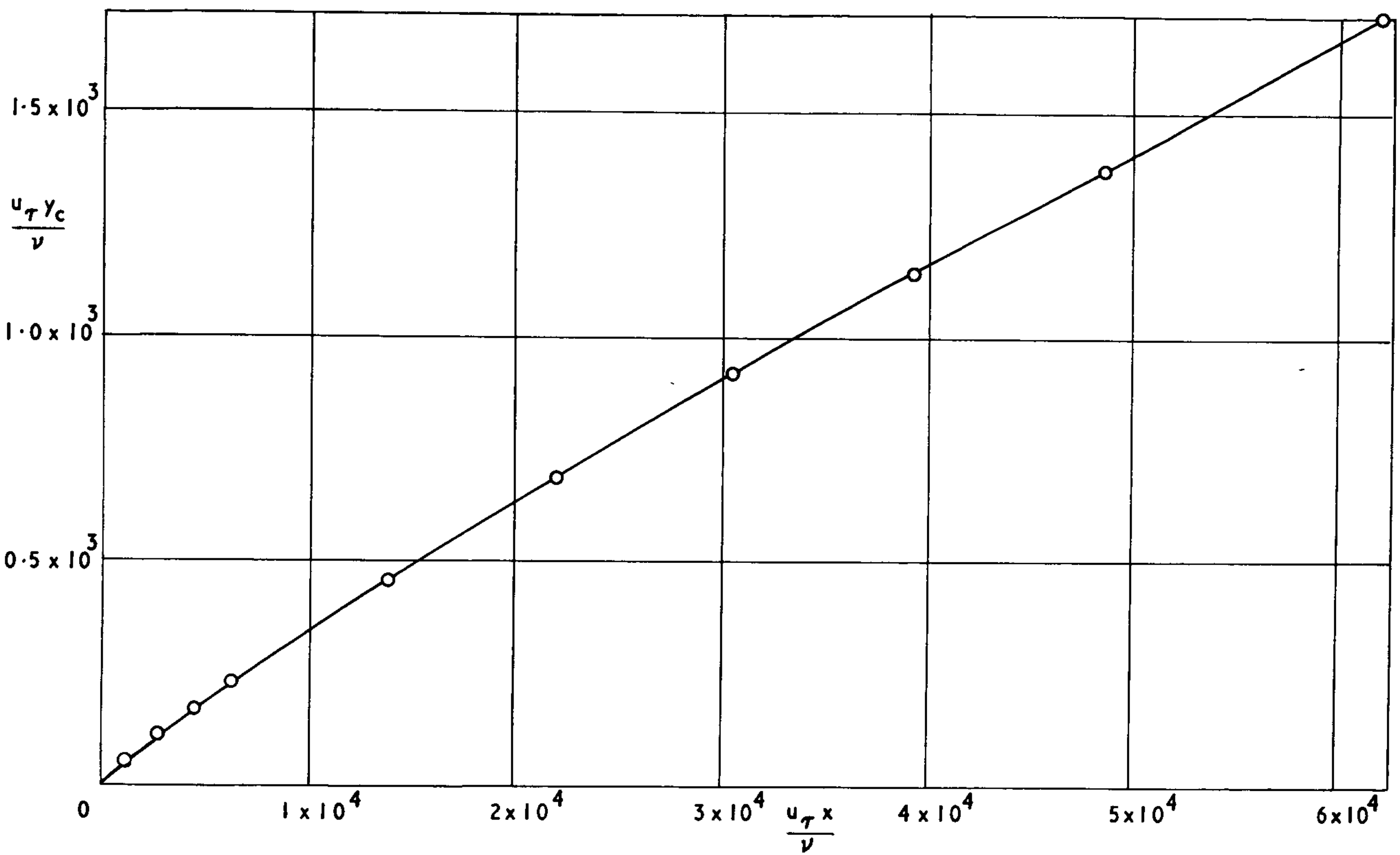


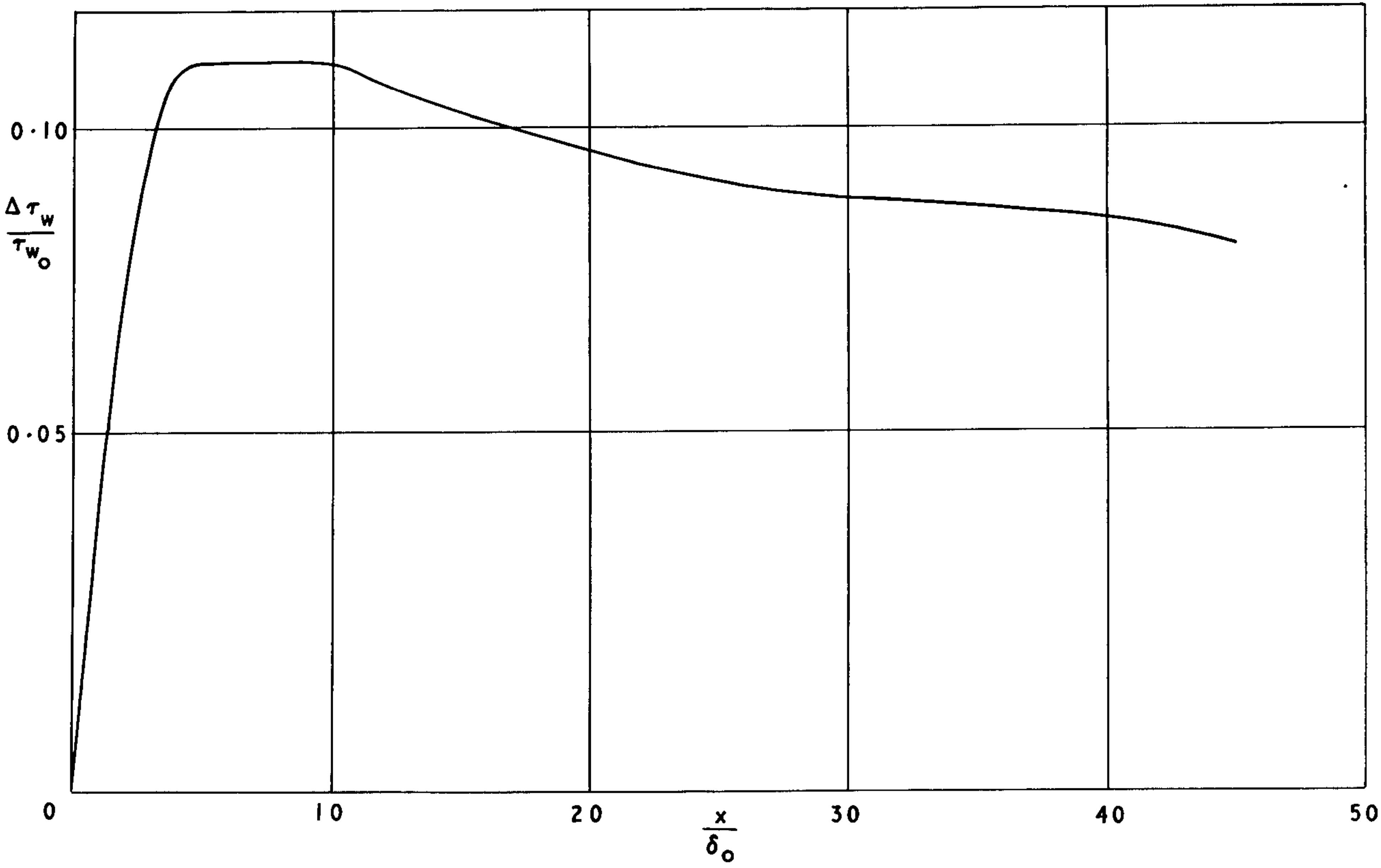
FIG. 9

Self-preserving response of shear stress profile to change in A

FIG. 10



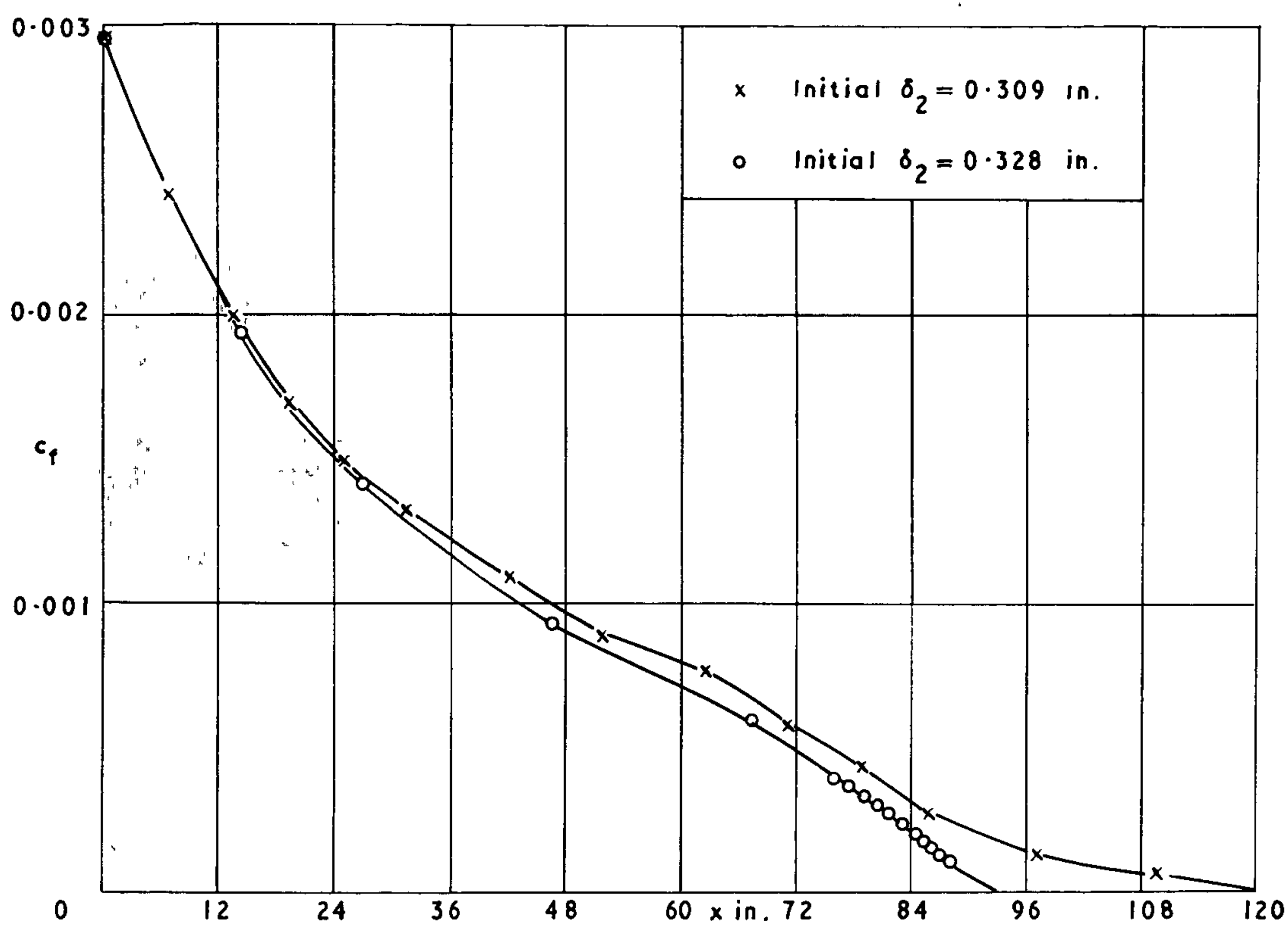
Length scale y_c for self-preserving profiles : $u_\tau \equiv (\tau_{w_0}/\rho)^{1/2}$



Shear stress scale $\Delta \tau_w$ for self-preserving profiles

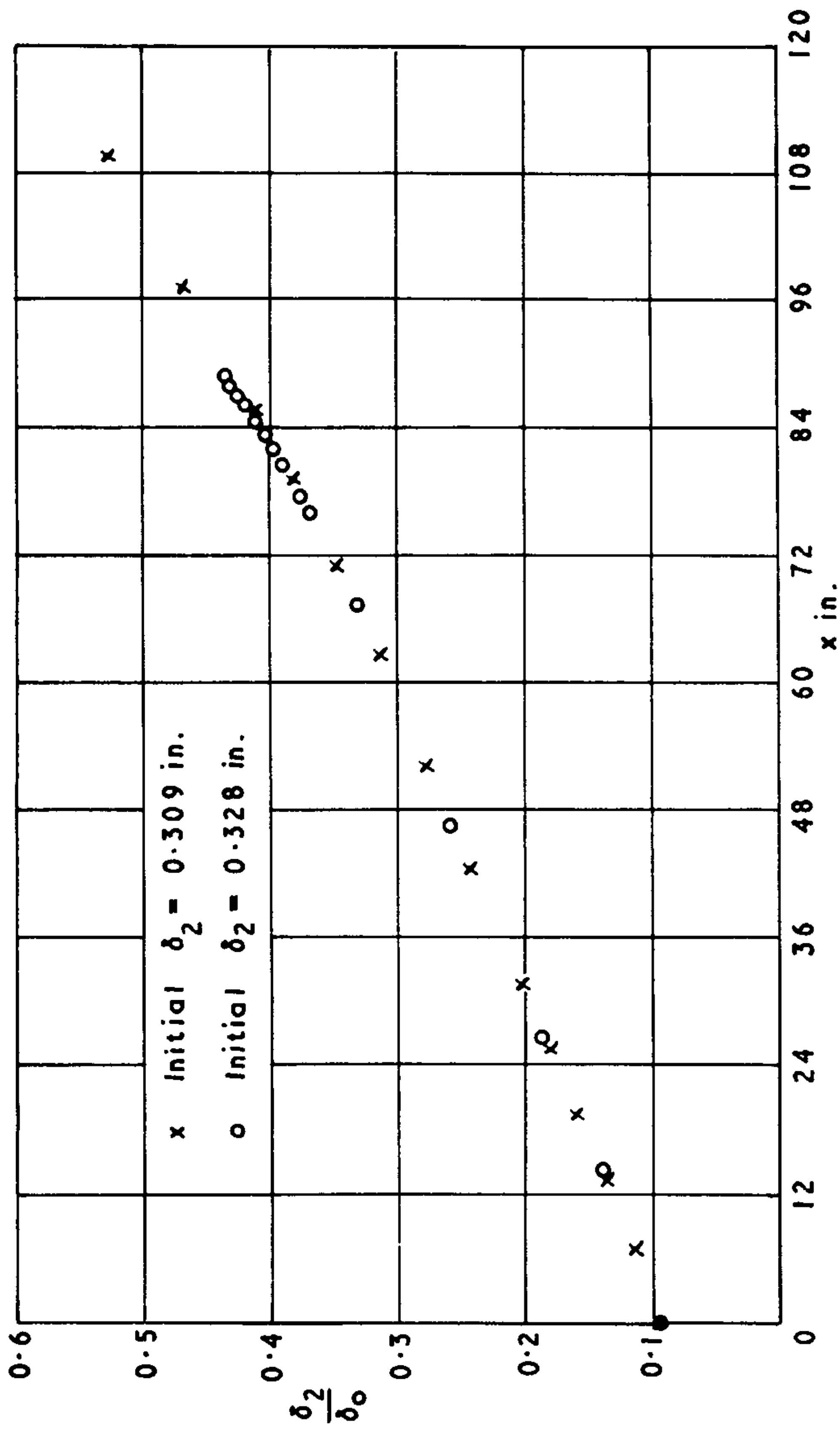
FIG. 11

FIG 12(a)



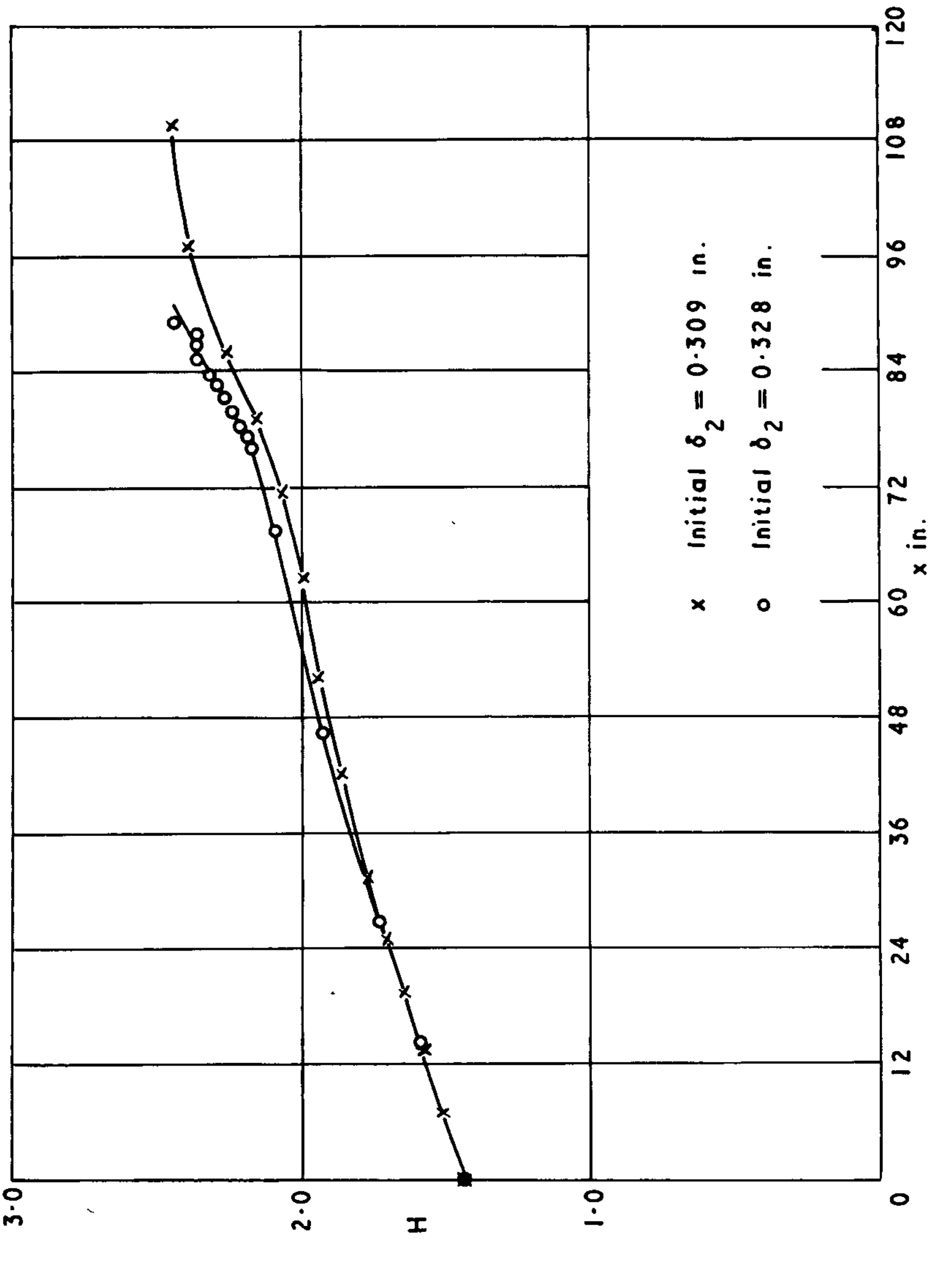
Calculations for two initial boundary-layer thicknesses: $U_1 \propto (x + 24)^{-0 \cdot 255}$ from x=0 onward

FIG. 12 (b)



x Initial δ_2 = 0·309 in.
o Initial δ_2 = 0·328 in.
$\frac{\delta_2}{\delta_0}$
0·6
0·5
0·4
0·3
0·2
0·1
x in.
0
12
24
36
48
60
72
84
96
108
120

FIG. 12 (c)



3·0
2·0
H
1·0
0
12
24
36
48
60
72
84
96
108
120
x in.
x Initial $\delta_2 = 0·309$ in.
o Initial $\delta_2 = 0·328$ in.

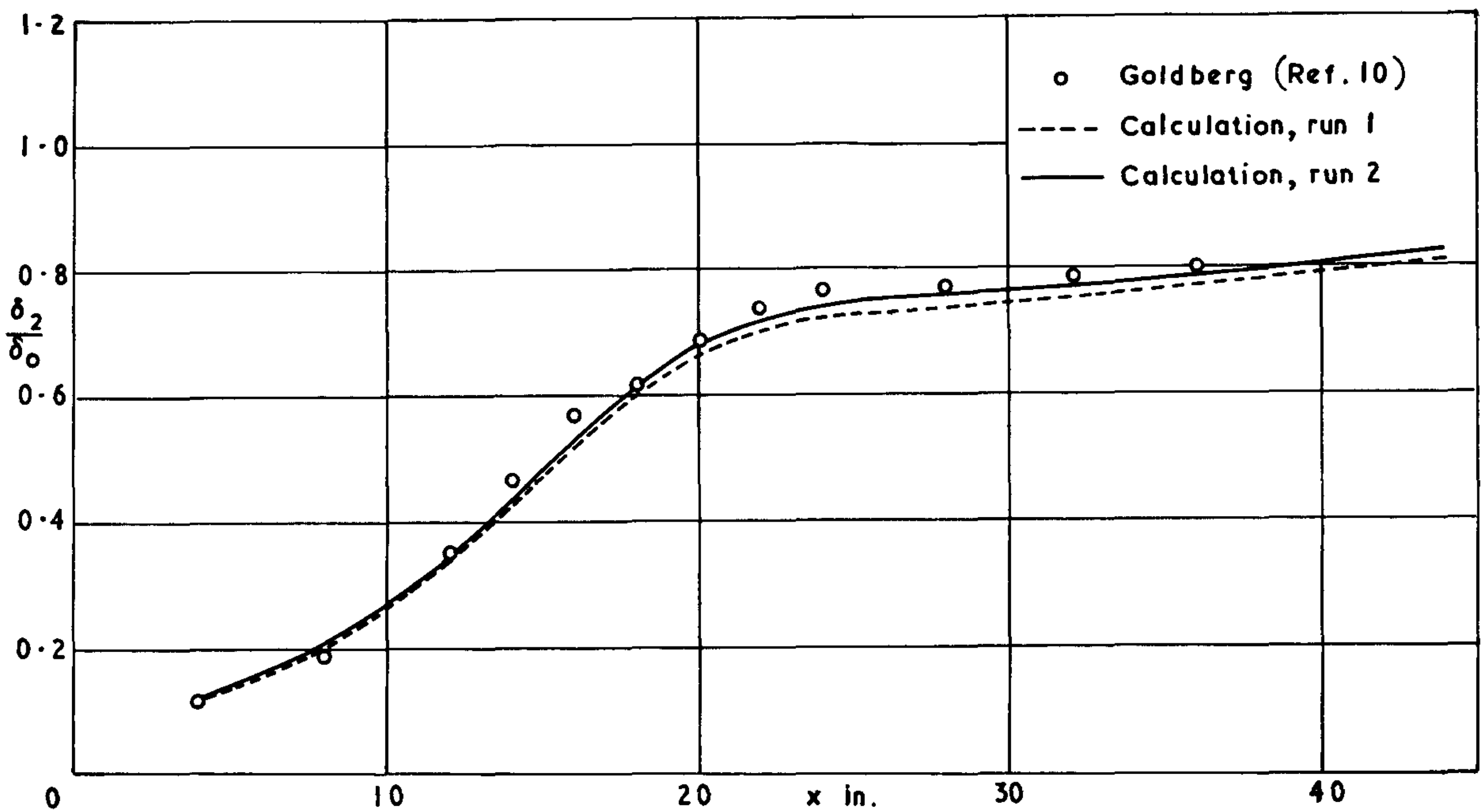
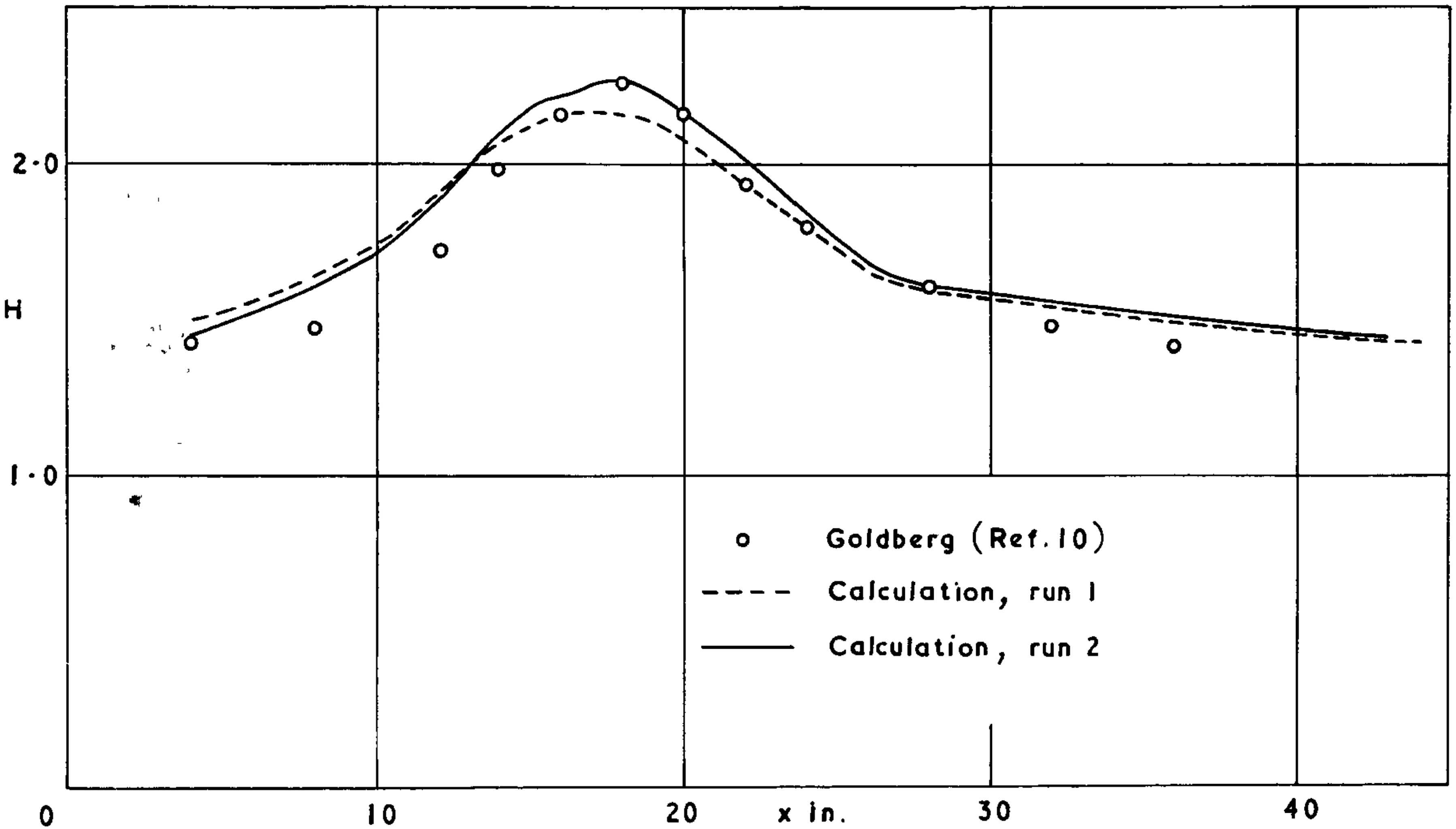


FIG. 13(a)

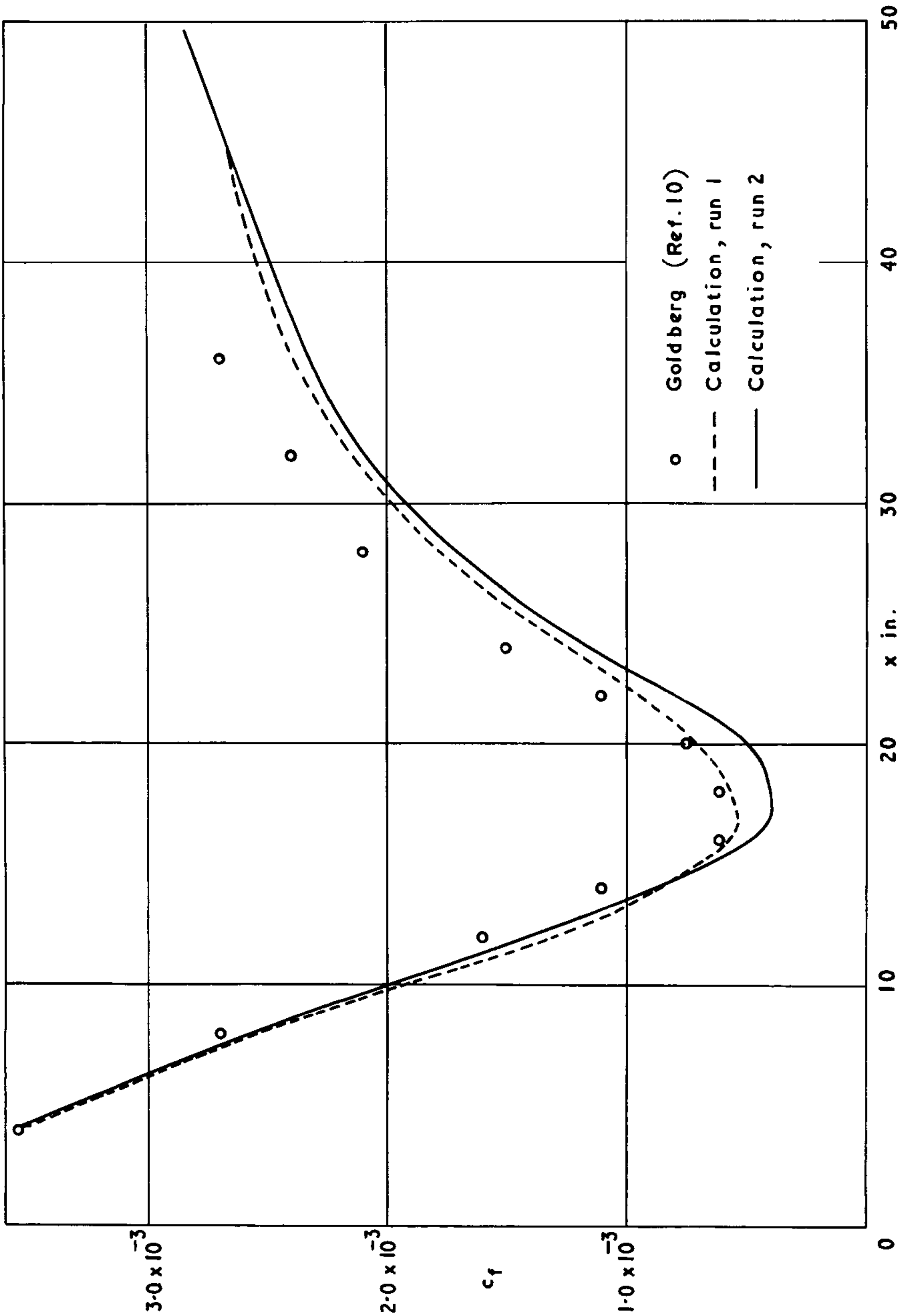
Effect of initial conditions on a nearly-separating boundary layer



2·0
1·0
H
0
10
20
x in.
30
40
Goldberg (Ref. 10)
Calculation, run 1
Calculation, run 2

FIG. 13(b)

FIG 13(c)



Goldberg (Ref. 10)
Calculation, run 1
Calculation, run 2
x in.
0
10
20
30
40
50
c_f
$3\cdot0\times10^{-3}$
$2\cdot0\times10^{-3}$
$1\cdot0\times10^{-3}$

A.R.C. C.P. No. 986
February, 1967.
Bradshaw, P. and Ferriss, D. H.

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