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# Solution of the Catapult Take-off Performance Equations by an Analogue Method

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# SOLUTION OF THE CATAPULT TAKE-OFF PERFORMANCE EQUATIONS BY AN ANALOGUE METHOD

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## SUMMARY

An analogue computer programme was derived, making as few approximations as possible, for the calculation of the flight path of an aircraft leaving the end of a ship-borne catapult. Using this 'complete' calculation, it was shown that, for most aircraft, other approximations could be made without significantly impairing the accuracy of the result, and greatly simplifying the programme.

A description of both programmes is given here, together with their derivation and method of use.

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* Replaces R.A.E. Tech. Report 66026 - A.R.C. 28521

CONTENTS

## 1 INTRODUCTION

One aspect of assessing the performance of Naval aircraft is the estimation of take-off characteristics. This may arise during design appraisal or when studying the effects of proposed modifications or changes in handling technique.

The equations governing the aircraft flight path off the catapult are such that no general solution is possible, and each individual case must be integrated either step-by-step, by hand, or by a digital computer such as Mercury, which has been used in the past for this purpose, or by an analogue computation. As will be seen later, the problem is extremely well suited to solution by analogue methods, and programmes were designed for the Solartron S.C.30 computer, though it is clearly a simple matter to adapt them for use on any other analogue computer.

The purpose of this paper is to put on record the work that has been done on this problem in order to avoid duplication at a later date, and to assist other workers in this field when it may be helpful to have available a standard procedure.

## 2 GENERAL PRINCIPLES

The analysis used is fairly standard. The problem is essentially concerned with quite large changes of speed and incidence, and thus the small perturbation method is not applicable. However, in the interests of being able to use as large percentage changes in each variable as possible, a datum speed and datum incidence were chosen at about the mean values of these parameters expected during the first few seconds of flight, and incremental speed and incidence variables considered about these datum points. Thus, although the usual small perturbation approximations cannot be made, fullest use is made of the computer scaling accuracy, and approximations for $\sin(\alpha' + \alpha)$ and $\cos(\alpha' + \alpha)$ are facilitated; these approximations being an order better than the simple $\sin \alpha \simeq \alpha$, and considerably better than the use of a servo resolver, whose accuracy would be very poor for the angular changes we are considering.

## 3 ANALYSIS

The vertical component of velocity, $\dot{h}$, is given by

$$\dot{h} = (V_0 + u) \sin \gamma .$$

For all practical flight paths, $\gamma$ is less than $4^\circ$ (the catapult being only 50 feet above sea level) so with less than 0·05% error we may write

$$\dot{h} = P\gamma + Qu\gamma \qquad (1)$$

where P and Q are constants.

At any point on the flight path, the forces on the aircraft are as shown in Fig.1 and may be resolved to produce equations of motion along and perpendicular to the flight path as given below.

3.1 Motion along the flight path

Resolving:-

$$m\dot{u} = \left(T + \frac{\partial T}{\partial u} u\right) \cos(\alpha' + \alpha) + m g \sin\gamma - D_m - \frac{\partial D_m}{\partial u} u - D. \qquad (2)$$

We make the following approximations:-

(a) As in equation (1), $\sin\gamma \approx \gamma$

(b) $\cos(\alpha' + \alpha) = \cos\alpha' \cos\alpha - \sin\alpha' \sin\alpha$

$$\approx \cos\alpha' - \alpha \sin\alpha'$$

with only a small error $\delta c_1$, which depends on $\alpha'$ and $\alpha$ as shown in Fig.2

(c) $D = C_D \tfrac{1}{2}\rho (V_o + u)^2 S$

where $C_D = C_{D_{oV}} + k C_L^2$; $C_{D_{oV}}$ being a function of $C_\mu$ only, for fixed configuration, and k being constant - the induced drag factor.

At constant throttle setting, $C_\mu$ depends mainly on airspeed, and we may assume the linearity

$$C_{D_{o_V}} = C_{D_o} + \frac{\partial C_{D_o}}{\partial u} u$$

where $C_{D_o}$ is the value of $C_{D_{o_V}}$ at speed $V_o$.

Also, in fixed configuration, $C_L$ is very nearly a linear function of $\alpha$ for fixed $C_\mu$, and, over the range of $C_\mu$ we need to consider, a linear function of $C_\mu$ for fixed $\alpha$. Thus we may write

$$C_L = C_{L_{\alpha'}} + \frac{\partial C_L}{\partial \alpha}\alpha + \frac{\partial C_L}{\partial u}u$$

where $C_{L_{\alpha'}}$ is the value of $C_L$ at incidence $\alpha'$ and speed $V_o$. Thus

$$C_D = C_{D_o} + \frac{\partial C_{D_o}}{\partial u}u + k\left(C_{L_{\alpha'}} + \frac{\partial C_L}{\partial \alpha}\alpha + \frac{\partial C_L}{\partial u}u\right)^2 .$$

Now $(C_{D_o} + k\,C_{L_{\alpha'}}^2)$ is the value of $C_D$ at $\alpha'$ and $V_o$; i.e. $C_{D_{\alpha'}}$.

Therefore

$$C_D = C_{D_{\alpha'}} + \frac{\partial C_{D_o}}{\partial u}u + k\left(2C_{L_{\alpha'}}\frac{\partial C_L}{\partial \alpha}\alpha + 2C_{L_{\alpha'}}\frac{\partial C_L}{\partial u}u + 2\frac{\partial C_L}{\partial \alpha}\frac{\partial C_L}{\partial u}u\alpha + \left(\frac{\partial C_L}{\partial \alpha}\right)^2\alpha^2 + \left(\frac{\partial C_L}{\partial u}\right)^2 u^2\right).$$

The linearities assumed in (c) above should be extremely good for most aircraft, but in any particular case, the error involved should be checked from the available aerodynamic data. In most cases, the terms affected are small enough to accept quite large percentage errors without significantly impairing the overall accuracy of the calculation.

Using these expressions therefore, equation (2) becomes:-

$$m\dot{u} = T\cos\alpha' - T\sin\alpha'\,\alpha + \frac{\partial T}{\partial u}\cos\alpha'\,u - \frac{\partial T}{\partial u}\sin\alpha'\,u\,\alpha + mg\gamma - D_m - \frac{\partial D_m}{\partial u}u$$

$$- \frac{1}{2}\rho S\Big\{V_o^2 C_{D_{\alpha'}} + V_o^2\frac{\partial C_{D_o}}{\partial u}u + V_o^2\,2k\,C_{L_{\alpha'}}\frac{\partial C_L}{\partial \alpha}\alpha + V_o^2\,2k\,C_{L_{\alpha'}}\frac{\partial C_L}{\partial u}u$$

$$+ V_o^2\,2k\frac{\partial C_L}{\partial \alpha}\frac{\partial C_L}{\partial u}u\,\alpha + k V_o^2\left(\frac{\partial C_L}{\partial \alpha}\right)^2\alpha^2 + k V_o^2\left(\frac{\partial C_L}{\partial u}\right)^2 u^2$$

$$+ 2V_o\,u\,C_{D_{\alpha'}} + 2V_o\frac{\partial C_{D_o}}{\partial u}u^2 + 4k\,C_{L_{\alpha'}}\frac{\partial C_L}{\partial \alpha}V_o\,u\,\alpha + 4k\,C_{L_{\alpha'}}\frac{\partial C_L}{\partial u}V_o\,u^2$$

$$+ 4k\frac{\partial C_L}{\partial \alpha}\frac{\partial C_L}{\partial u}V_o\,u^2\alpha + 2k\,V_o\left(\frac{\partial C_L}{\partial \alpha}\right)^2 u\,\alpha^2 + 2k\,V_o\left(\frac{\partial C_L}{\partial u}\right)^2 u^3$$

$$+ u^2 C_{D_{\alpha'}} + \frac{\partial C_{D_o}}{\partial u}u^3 + 2k\,C_{L_{\alpha'}}\frac{\partial C_L}{\partial \alpha}u^2\alpha + 2k\,C_{L_{\alpha'}}\frac{\partial C_L}{\partial u}u^3 + 2k\frac{\partial C_L}{\partial \alpha}\frac{\partial C_L}{\partial u}u^3\alpha$$

$$+ k\left(\frac{\partial C_L}{\partial \alpha}\right)^2 u^2\alpha^2 + k\left(\frac{\partial C_L}{\partial u}\right)^2 u^4\Big\}.$$

Rearranging, we get:-

$$\underline{10\dot{u} = C_o + C_1\gamma - C_2\alpha - C_3 u - C_4 u\alpha - C_5\alpha^2 - C_6 u^2 - C_7 u^2\alpha}$$

$$\underline{- C_8 u\alpha^2 - C_9 u^3 - C_{10}u^2\alpha^2 - C_{11}u^3\alpha - C_{12}u^4} \qquad (3)$$

where

$$C_o = \frac{10}{m}\left(T\cos\alpha' - D_m - \frac{1}{2}\rho V_o^2 S\,C_{D_{\alpha'}}\right)$$

$$C_1 = 10g\,\frac{1}{57\cdot 3}$$

$$C_2 = \frac{10}{m}\left(\frac{T\sin\alpha'}{57\cdot 3} + k\,C_{L_{\alpha'}}\rho V_o^2 S\frac{\partial C_L}{\partial \alpha}\right)$$

$$C_3 = \frac{10}{m}\left(\frac{\partial D_m}{\partial u} + \frac{1}{2}\rho V_o^2 S\frac{\partial C_{D_o}}{\partial u} + k\,C_{L_{\alpha'}}\rho V_o^2 S\frac{\partial C_L}{\partial u} + C_{D_{\alpha'}}\rho V_o S - \frac{\partial T}{\partial u}\cos\alpha'\right)$$

$$C_4 = \frac{10}{m} k \rho V_o S \frac{\partial C_L}{\partial \alpha}\left(V_o \frac{\partial C_L}{\partial u} + 2 C_{L_{\alpha'}}\right) + \frac{10}{m} \frac{\partial T}{\partial u} \frac{\sin \alpha'}{57 \cdot 3}$$

$$C_5 = \frac{10}{m} k \left(\frac{\partial C_L}{\partial \alpha}\right)^2 \frac{1}{2} \rho V_o^2 S$$

$$C_6 = \frac{10}{m}\left(\frac{1}{2} \rho S C_{D_{\alpha'}} + \rho V_o S \left\{\frac{1}{2} k V_o \left(\frac{\partial C_L}{\partial u}\right)^2 + \frac{\partial C_{D_o}}{\partial u} + 2k C_{L_{\alpha'}} \frac{\partial C_L}{\partial u}\right\}\right)$$

$$C_7 = \frac{10}{m}\left(2k \frac{\partial C_L}{\partial \alpha} \frac{\partial C_L}{\partial u} \rho V_o S + k C_{L_{\alpha'}} \frac{\partial C_L}{\partial \alpha} \rho S\right)$$

$$C_8 = \frac{10}{m} k \left(\frac{\partial C_L}{\partial \alpha}\right)^2 \rho V_o S$$

$$C_9 = \frac{10}{m} \rho S \left(k V_o \left(\frac{\partial C_L}{\partial u}\right)^2 + \frac{1}{2} \frac{\partial C_{D_o}}{\partial u} + k C_{L_{\alpha'}} \frac{\partial C_L}{\partial u}\right)$$

$$C_{10} = \frac{10}{m} \frac{1}{2} \rho S k \left(\frac{\partial C_L}{\partial \alpha}\right)^2$$

$$C_{11} = \frac{10}{m} \rho S k \frac{\partial C_L}{\partial \alpha} \frac{\partial C_L}{\partial u}$$

$$C_{12} = \frac{10}{m} \frac{1}{2} \rho S k \left(\frac{\partial C_L}{\partial u}\right)^2$$

$\gamma$ and $\alpha$ are measured in degrees; $\frac{\partial C_L}{\partial \alpha}$ is measured per degree.

3.2 Forces normal to the flight path

Resolving:-

$$L + m V_o \dot{\gamma} + m u \dot{\gamma} + \left(T + \frac{\partial T}{\partial u} u\right) \sin(\alpha' + \alpha) = m g \cos \gamma. \qquad (4)$$

We make the following approximations:-

(a) As in equation (1), $\gamma < 4°$ so with less than 0·001% error we may write

$$\cos \gamma \approx 1 - \frac{\gamma^2}{2}.$$

(b) $\sin(\alpha' + \alpha) = \sin \alpha' \cos \alpha + \cos \alpha' \sin \alpha$

$$\approx \sin \alpha' + \alpha \cos \alpha'$$

with only a small error, $\delta e_2$, which depends on $\alpha'$ and $\alpha$ as shown in Fig.3.

(c) $L = C_L \frac{1}{2} \rho (V_o + u)^2 S - Z_\eta \eta$

where $C_L = C_{L_{\alpha'}} + \frac{\partial C_L}{\partial \alpha} \alpha + \frac{\partial C_L}{\partial u} u$ (see para. 3.1 note (c))

and $Z_\eta$ is the tail download per unit elevator deflection.

Thus:-

$$L = \left(C_{L_{\alpha'}} + \frac{\partial C_L}{\partial \alpha} \alpha + \frac{\partial C_L}{\partial u} u\right)\left(V_o^2 + 2V_o u + u^2\right) \frac{1}{2} \rho S - Z_\eta \eta$$

$$= \frac{1}{2} \rho S \left\{V_o^2 C_{L_{\alpha'}} + V_o^2 \frac{\partial C_L}{\partial \alpha} \alpha + \left(2V_o C_{L_{\alpha'}} + V_o^2 \frac{\partial C_L}{\partial u}\right) u + \left(C_{L_{\alpha'}} + 2V_o \frac{\partial C_L}{\partial u}\right) u^2 + \frac{\partial C_L}{\partial u} u^3 + 2V_o \frac{\partial C_L}{\partial \alpha} u \alpha + \frac{\partial C_L}{\partial \alpha} u^2 \alpha\right\} - Z_\eta \eta .$$

Using these expressions therefore, equation (4) becomes:-

$$m V_o \dot{\gamma} = m g \left(1 - \frac{\gamma^2}{2}\right) - T \sin \alpha' - T \cos \alpha' \alpha - \frac{\partial T}{\partial u} \sin \alpha' u - \frac{\partial T}{\partial u} \cos \alpha' u \alpha$$

$$- m u \dot{\gamma} - \frac{1}{2} \rho V_o^2 S C_{L_{\alpha'}} - \frac{1}{2} \rho V_o^2 S \frac{\partial C_L}{\partial \alpha} \alpha - \frac{1}{2} \rho S \left(2V_o C_{L_{\alpha'}} + V_o^2 \frac{\partial C_L}{\partial u}\right) u$$

$$- \frac{1}{2} \rho S \left(C_{L_{\alpha'}} + 2V_o \frac{\partial C_L}{\partial u}\right) u^2 - \frac{1}{2} \rho S \frac{\partial C_L}{\partial u} u^3 - \frac{1}{2} \rho S \, 2V_o \frac{\partial C_L}{\partial \alpha} u \alpha$$

$$- \frac{1}{2} \rho S \frac{\partial C_L}{\partial \alpha} u^2 \alpha + Z_\eta \eta.$$

Rearranging, we get:-

$$\underline{- 10\dot{\gamma} = K_o + K_1 \alpha + K_2 u + K_3 u \alpha + K_4 u^2 + K_5 u^2 \alpha + K_6 u^3 + K_7 u \dot{\gamma} + K_8 \gamma^2 + K_9 \eta} \qquad \ldots. (5)$$

where

$$K_o = \frac{573}{m V_o} \left(T \sin \alpha' + \frac{1}{2} \rho V_o^2 S C_{L_{\alpha'}} - m g\right)$$

$$K_1 = \frac{573}{m\,V_o}\left(\frac{T\cos\alpha'}{57{\cdot}3} + \frac{1}{2}\,\rho\,V_o^2\,S\,\frac{\partial C_L}{\partial\alpha}\right)$$

$$K_2 = \frac{573}{m\,V_o}\,\rho\,V_o\,S\left(C_{L_{\alpha'}} + \frac{1}{2}\,V_o\,\frac{\partial C_L}{\partial u}\right) + 573\,\frac{\partial T}{\partial u}\,\frac{\sin\alpha'}{m\,V_o}$$

$$K_3 = \frac{573}{m\,V_o}\,\rho\,V_o\,S\,\frac{\partial C_L}{\partial\alpha} + \frac{10}{m\,V_o}\,\frac{\partial T}{\partial u}\cos\alpha'$$

$$K_4 = \frac{573}{m\,V_o}\left(\frac{1}{2}\,\rho\,S\,C_{L_{\alpha'}} + \rho\,S\,V_o\,\frac{\partial C_L}{\partial u}\right)$$

$$K_5 = \frac{573}{m\,V_o}\left(\frac{1}{2}\,\rho\,S\,\frac{\partial C_L}{\partial\alpha}\right)$$

$$K_6 = \frac{573}{m\,V_o}\,\frac{1}{2}\,\rho\,S\,\frac{\partial C_L}{\partial u}$$

$$K_7 = \frac{10}{V_o}$$

$$K_8 = -\frac{5g}{57{\cdot}3\,V_o}$$

$$K_9 = \frac{573}{m\,V_o}\,Z_\eta$$

$\eta$, $\gamma$ and $\alpha$ are measured in degrees; $Z_\eta$ and $\frac{\partial C_L}{\partial\alpha}$ are measured per degree and $\dot{\gamma}$ is measured in degrees/sec.

The solution of equations (1), (3) and (5) forms the basis of the 'complete' programme (Programme I) shown in Fig.4 (see also Fig.5). The programme plots out the flight path of the aircraft, from which the maximum height dropped below deck level may be read off.

The scaling coefficients are all unity, i.e. for one volt we have

$$u = 1\ \text{ft/sec}$$
$$\dot{u} = 1\ \text{ft/sec}^2$$
$$\theta = \eta = \gamma = \alpha = 1^\circ$$
$$\dot{\theta} = \dot{\gamma} = \dot{\alpha} = 1^\circ/\text{sec.}$$

Integration takes placed in 'real' time, however the time-scale in volts/sec is purely arbitrary and may be altered at will by altering the coefficient R.

## 4 ERRORS

### 4.1 Linearity assumptions and basic data errors

The errors introduced by the assumptions of linearity in the assessment of thrust, lift and drag are considerably smaller than those due to the tolerances to which these parameters are measured in aircraft flight tests. These measurement errors are separate from, and additional to, the calculation errors, hence the effects of the linearity assumptions are not included here in the assessment of overall calculation errors, and the values of thrust, lift and drag are assumed to be known with perfect accuracy. Since this is not normally the case, the catapult performance of the aircraft should be determined for a range of values of these parameters to cover the expected range of error in the basic data.

### 4.2 Other sources of error

The percentage errors introduced by the angle approximations made in the calculation are as follows:-

In equation (2)

(a) $\cos(\alpha' + \alpha)$ is given to $^{+0}_{-}\delta e_1\%$ by the approximation $(\cos\alpha' - \alpha\sin\alpha')$. If we impose a limitation that $(\alpha' + \alpha) \not> 80^\circ$, $\delta e_1 \leqslant 2{\cdot}5$ - see Fig.2 and para. 6.8.

(b) $\sin\gamma$ is given to $^{+\delta s\%}_{-0}$ by the approximation $\sin\gamma = \gamma$, $\delta s$ being less than 0·05% - see equation (1), para.3.

Simplifying equation (2), we have:-

$$m\dot{u} = T\cos(\alpha' + \alpha) + m\,g\sin\gamma - D'$$

where D' is aerodynamic drag + momentum drag, and the errors in T and D' are ignored as discussed above in para.4.1. Thus the error in $\dot{u}$ is given by:-

$$-\frac{T}{m}\,\delta e_1 \leqslant \delta\dot{u} \leqslant g\sin\gamma\,\delta s.$$

If $t_D$ sec is the time to the bottom of the drop, the error in u is given by:-

$$-\frac{T}{m} t_D \, \delta e_1 \leq \delta u \leq g \, t_D \sin\gamma \, \delta s \, .$$

In equation (4)

(c) u, as described above, is given to $\left.\begin{matrix} + g \, t_D \sin\gamma \, \delta s \\ - T/m \, t_D \, \delta e_1 \end{matrix}\right\}$ ft/sec

(d) $\sin(\alpha' + \alpha)$ is given to $\begin{smallmatrix}+0\\-\end{smallmatrix} \delta e_2$% by the approximation $(\sin\alpha' + \alpha\cos\alpha')$, see Fig.3. By careful choice of $\alpha'$, $\delta e_2 \leq 2{\cdot}5$%.

(e) $\cos\gamma$ is given to $\begin{smallmatrix}+0\\-\end{smallmatrix} \delta c$% by the approximation $\cos\gamma = 1 - \frac{\gamma^2}{2}$, $\delta c$ being less than 0·001% - see note (a), para. 3.2.

Simplifying equation (4), we have:-

$$\frac{m\dot{\gamma}}{57{\cdot}3} = \frac{m \, g \cos\gamma - T\sin(\alpha' + \alpha) - L}{(V_o + u)}$$

the errors in T and L being ignored, as discussed in para. 4.1 above.

Thus, the error in $\dot{\gamma}$ is given by:-

$$-\frac{57{\cdot}3}{(V_o + u)}\left\{g\cos\gamma\,\delta c + \frac{g\,t_D\sin\gamma\,\delta s}{57{\cdot}3}\dot{\gamma}\right\} \leq \delta\dot{\gamma} \leq \frac{57{\cdot}3}{(V_o + u)}\,\frac{T}{m} \times\left\{\delta e_2\sin(\alpha' + \alpha) + \frac{t_D\,\delta e_1}{57{\cdot}3}\dot{\gamma}\right\}.$$

Integrating,

$$-\frac{57{\cdot}3}{(V_o + u)}\left\{g\,t_D\cos\gamma\,\delta c + \frac{g\,t_D\sin\gamma\,\delta s}{57{\cdot}3}\gamma\right\} \leq \delta\gamma \leq \frac{57{\cdot}3}{(V_o + u)}\,\frac{T}{m} \times\left\{t_D\,\delta e_2\sin(\alpha' + \alpha) + \frac{t_D\,\delta e_1}{57{\cdot}3}\gamma\right\}.$$

Substituting typical values, T = 18000 lb, m = 1400 slugs
$(V_o + u)$ = 200 ft/sec, $t_D$ = 2 sec.

Approximately, $-18(\delta c + 0{\cdot}001\,\delta s\,\gamma) \leq \delta\gamma \leq 7{\cdot}4\,(\delta e_2 + 0{\cdot}013\,\delta e_1\,\gamma)$.

Now $\gamma \not> 4^\circ = 0{\cdot}07$ rad, therefore we may neglect the $\delta s$ and $\delta e_1$ terms. Thus we may write:

$$-\frac{57{\cdot}3}{(V_o + u)} g \cos\gamma \, \delta c \, t_D \leqslant \delta \dot{\gamma} \leqslant \frac{57{\cdot}3}{(V_o + u)} \frac{T}{m} \delta e_2 \sin(\alpha' + \alpha) \, t_D .$$

This results might have been expected from consideration of the equations of motion; $\dot{\gamma}$ depends on the balance of forces normal to the flight path, and these forces are modified only slightly by the longitudinal acceleration of the aircraft during the first few seconds of flight. Thus one would expect that the errors in u as calculated by equation (3) will produce only a small error in $\gamma$ compared with the 'direct' assumption errors $\delta c$ and $\delta e_2$.

Now from equation (1),

$$\dot{h} = \frac{(V_o + u)\,\gamma}{57{\cdot}3} .$$

Thus percentage error in $\dot{h}$ is given by

$$\frac{\delta\dot{h}}{\dot{h}} = \frac{1}{57{\cdot}3}\left\{\frac{\delta u}{(V_o + u)} + \frac{\delta\gamma}{\gamma}\right\}$$

therefore

$$-\frac{\dot{h}}{57{\cdot}3}\left\{\frac{T}{m}\,\frac{t_D\,\delta e_1}{(V_o + u)} + \frac{57{\cdot}3}{(V_o + u)}\,\frac{g\cos\gamma\,\delta c\,t_D}{\gamma}\right\} \leqslant \delta\dot{h} \leqslant \frac{\dot{h}}{57{\cdot}3}$$

$$\times \left\{\frac{g\,t_D\sin\gamma\,\delta s}{(V_o + u)} + \frac{57{\cdot}3}{(V_o + u)}\,\frac{T}{m}\,\frac{\delta e_2\sin(\alpha' + \alpha)\,t_D}{\gamma}\right\} .$$

Substituting typical values again,

$$-\dot{h}\left(0{\cdot}0025\,\delta e_1 + 0{\cdot}32\frac{\delta c}{\gamma}\right) \leqslant \delta\dot{h} \leqslant \dot{h}\left(0{\cdot}0004\,\delta s + 0{\cdot}13\frac{\delta e_2}{\gamma}\right) .$$

Now $\gamma \not> 4^\circ = 0{\cdot}07$ rad, thus once more we may neglect the $\delta e_1$ and $\delta s$ terms.

Substituting from equation (1)

$$\frac{\dot{h}}{\gamma} = \left(\frac{V_o + u}{57{\cdot}3}\right) .$$

Thus

$$-\frac{g \cos \gamma \ \delta c \ t_D}{57 \cdot 3} \leqslant \delta \dot{h} \leqslant \frac{T}{m} \ \frac{\delta e_2 \ \sin(\alpha' + \alpha) \ t_D}{57 \cdot 3} .$$

On integrating, error in h is given by

$$-\frac{g \cos \gamma \ \delta c \ t_D^2}{57 \cdot 3} \leqslant \delta h \leqslant \frac{T}{m} \ \frac{\delta e_2 \ \sin(\alpha' + \alpha) \ t_D^2}{57 \cdot 3} .$$

Typical values give $-0 \cdot 00003$ ft $\leqslant \delta h \leqslant 0 \cdot 9 \ \delta e_2 \ \sin(\alpha' + \alpha)$ ft.

The lower limit of error is negligible in consideration of height drops of the order of 15 feet.

On the right-hand side, $\sin(\alpha' + \alpha)$ for a conventional aircraft will be of the order of 0·3, but for V/STOL aircraft with deflected thrust, may be almost 1. Examining Fig.3, it will be seen that we can always choose $\alpha'$ such that $\delta e_2 < 2 \cdot 5\%$ (see para. 6.2). Thus we may say that in the worst case, the height drop calculated by the computer may be optimistic by up to 0·025 feet, although for a conventional aircraft and a good choice of parameter datums, this error is probably less than 0·01 feet.

It should be noted that this error in h is directly proportional to the error introduced by the approximation for $\sin(\alpha' + \alpha)$ in equation (4) i.e. $\delta e_2$. Thus the choice of incidence datum is of paramount importance.

This analysis takes no account of the computer system errors (of the order of 0·1%) which should have little effect, or of the accuracy to which the output trace can be read - this being completely under the control of the computer operator.

## 5 SIMPLIFICATION OF THE EQUATIONS OF MOTION

The above assessment of errors is necessarily simplified, but is sufficiently pessimistic to make the final figure more than adequate.

An error of this size is obviously swamped completely by the variations which can occur in basic data, even from one aircraft to another of the same type.

The size of these errors, which are beyond the control of the computer, leads to the idea that a very much simpler programme might be adequate for most purposes.

The simplification of the programme necessitates a comparison of the relative magnitude of the terms in the calculation as follows:-

The terms of equation (3) for a typical modern Naval aircraft have the following approximate numerical values and represent the stated percentage of 10 $\dot{u}$.

| Term | | Approx. value | Percentage of 10 $\dot{u}$ |
|---|---|---|---|
| 10 $\dot{u}$ = | | 70 | 100 |
| $C_o$ | (+ve) | 22 | 31·5 |
| $C_1\ \alpha$ | (+ve) | 15 | 21·5 |
| $C_2\ \alpha$ | (−ve) | 20 | 29 |
| $C_3\ u$ | (−ve) | 9 | 13 |
| $C_4\ u\ \alpha$ | (−ve) | 1·5 | 2 |
| $C_5\ \alpha^2$ | (−ve) | 2 | 3 |
| $C_6\ u^2$ | (−ve) | 0·3 | Negligible |
| $C_7\ u^2\ \alpha$ | (−ve) | 0·04 | " |
| $C_8\ u\ \alpha^2$ | (−ve) | 0·12 | " |
| $C_9\ u^3$ | (−ve) | 0·006 | " |
| $C_{10}\ u^2\ \alpha^2$ | (−ve) | 0·003 | " |
| $C_{11}\ u^3\ \alpha$ | (−ve) | 0·0003 | " |
| $C_{12}\ u^4$ | (−ve) | $5 \times 10^{-6}$ | " |

Clearly, ignoring terms of suffix 6 and above introduces less than 1% error in $\dot{u}$. Hence we will reduce equation (3) to:-

$$10\dot{u} = C_o + C_1 \gamma - C_2 \alpha - C_3 u - C_4 u \alpha - C_5 \alpha^2 \qquad (3b)$$

Equation (5) for the same aircraft has terms with the following approximate values:-

| Term | | Approx. value | Percentage of $(-10\,\dot{\gamma})$ |
|---|---|---|---|
| $-10\,\dot{\gamma}$ = | | 30·6 | 100 |
| $K_0$ | (+ve) | 2 | 6·5 |
| $K_1\,\alpha$ | (+ve) | 20 | 65·5 |
| $K_2\,u$ | (+ve) | 5 | 16·5 |
| $K_3\,u\,\alpha$ | (+ve) | 0·6 | 2 |
| $K_4\,u^2$ | (+ve) | 0·1 | Negligible |
| $K_5\,u^2\,\alpha$ | (+ve) | 0·02 | " |
| $K_6\,u^3$ | (+ve) | 0·001 | " |
| $K_7\,u\,\dot{\gamma}$ | (+ve) | 1·2 | 4 |
| $K_8\,\gamma^2$ | (+ve) | 0·09 | Negligible |
| $K_9\,\eta$ | (+ve) | 1·6 | $5\frac{1}{2}$ |

Ignoring terms of suffix 4,5,6 and 8 only, introduces a total error of less than 1% in $\dot{\gamma}$. Hence we will reduce equation (5) to

$$- 10\dot{\gamma} = K_o + K_1\,\alpha + K_2\,u + K_3\,u\,\alpha + K_7\,u\,\dot{\gamma} + K_9\,\eta\,. \qquad (5b)$$

Equations (1), (3b) and (5b) form the basis of the simplified programme; programme II, shown in Fig.6 (see also Fig.7).

## 6 USE OF THE TWO PROGRAMMES

### 6.1 Calculation of the coefficient values

The tables given in Figs.10 and 11 aid the calculation of the coefficient values for both programmes. Initially the first column should be completed, since this consists entirely of information derived from the basic aerodynamic data. Column two contains the initial calculations required to put this data into a usable form, and the other columns are simply the calculation of the required constants.

### 6.2 Choice of speed and incidence datums, $V_o$ and $\alpha'$

The choice of $V_o$ and $\alpha'$ is to some extent arbitrary. However, it is a good guide to choose $V_o$ as being about 10 ft/sec above the expected minimum launch speed. This ensures that in the most important of the series of calculations (the minimum launch condition) the value of u is kept small, and its variation is in the right direction for greatest accuracy.

The choice of $\alpha'$ is somewhat simpler than this, since the incidence time-history is fixed beforehand, and a good datum value can be chosen. It will be noted, by reference to Fig.3, that if $(\alpha' + \alpha)$ is to take any values less than $10^\circ$, it is best to choose $\alpha'$ as small as possible consistent with keeping $\alpha < 10^\circ$, so as to stay as far away as possible from the discontinuity at $(\alpha' + \alpha) = 0$ when $\alpha$ takes negative values. Similarly, by reference to Fig.2, if $(\alpha' + \alpha)$ is very large, it is best to choose $\alpha'$ as large as possible, consistent with $\alpha > -10^\circ$, to stay as far away as possible from the second discontinuity at $(\alpha' + \alpha) = 90^\circ$ when $\alpha$ takes positive values.

## 6.3 Generation of the incidence programme

The incidence programme has to be set up on a function generator. It may be taken directly from incidence time-history records of flight tests, or as a simple "ramp" type rotation between two incidences at a given rate. Experiments have shown that the results of the height drop calculation are very sensitive to changes in the $\alpha$-programme, and for the purpose of comparing results, a standard $\alpha$-programme must be chosen and adhered to throughout. If the programme is used in conjunction with a flight simulator the $\alpha$-term may be computed within the simulator itself and fed into the appropriate terminal.

## 6.4 The tail-plane download term (See footnote on list of Symbols)

Although having quite large extreme values the tail-plane download term often almost cancels out when integrated over the initial part of the flight path, since, if a large tail-plane download is required to initiate a nose-up rotation after launch, a correspondingly large upload will be required to stop it, hence it may often be neglected. When used, the value of $\eta$ may either be set up on a function generator, using flight test time-histories; be put in from the stick output of a flight simulator, or be computed, for a given incidence programme, using a longitudinal stability programme such as that shown in Fig.8 (see also Fig.9). In the latter case, it will be noted that the straight line ramp incidence programme used for simplicity in comparison calculations (see above) cannot be reproduced.

## 6.5 Alternative use of the two programmes

For some aircraft, the approximations made in simplifying the programme may be inadequate, and a check should always be made before using the second programme, that this is not the case; otherwise programme I must be used.

### 6.6 Computing procedure

Having "patched-up" the programme required on the computer, and set up the relevant potentiometer values, operation is simplicity itself. The α-programme is fed into the appropriate terminal, and the catapult end (air) speed required is set up on pot. C32 on programme I and B21 on programme II (values of initial u, both positive and negative, may be set up using Key I to change the sign) and the flight path, speed variation or α time-history may be plotted out for the initial part of the flight path, depending on the position of Key II.

Sophistications to the basic calculation can be made such as those listed below.

#### 6.6.1 Deck run

Allowance can be made for the few feet of deck beyond the end of the catapult by calculating the new airspeed at the end of the deck using rectilinear kinematics. This is a simple calculation by hand, or if required, it could easily be programmed into the computer circuit.

#### 6.6.2 Initial condition of γ

Pot. E42 in programme I and pot. B51 in programme II, provide a facility for varying the initial condition of the flight path angle, γ. This would be one way of estimating the effect of energy recovery from undercarriage oleos and tyres, or the effect of launching from a catapult on a pitching and/or heaving deck, etc.

### 6.7 Aircraft with boundary layer control

Since most modern naval aircraft are equipped with blown flaps and/or blown leading edge, the variation of the effects of "blow" was written into the programmes. Should the programmes be required to deal with the case of an aircraft with surface suction boundary layer control, $\frac{\partial C_L}{\partial u}$ and $\frac{\partial C_{D_0}}{\partial u}$ may be determined from the effects of variation of suction mass flow rate and the rate of change of this quantity with airspeed. It is suspected that, as with the case of "blow" boundary layer control, this variation will have little effect on the overall result of the calculation.

### 6.8 V/STOL aircraft with deflected thrust

This paper does not deal with the case of a varying thrust deflection angle during the period of the height drop. However, it is doubtful if such variation would be a feasible technique during the first few seconds after a catapult launch.

The present calculation does cover the case of a large but constant thrust deflection angle, which means, in effect, a large value of $\alpha'$. Study of Figs.2 and 3 shows that this requirement is quite compatible with the assumptions made, provided that $\alpha$ still lies within the range $\pm 10^\circ$ (which is almost certain since the chosen $\alpha$-time-history will be dictated by the requirements of a more or less conventional wing) and that the total angle $(\alpha' + \alpha)$ does not exceed a value of about $80^\circ$ (which again is very likely since the purpose of the catapult launch of such an aircraft is to give the aircraft forward speed, and in order to maintain this speed or accelerate, there must be at least some horizontal component of thrust).

To sum up, then; the voctored thrust aircraft may be dealt with by the programmes herein, provided that the total inclination of the thrust line above the flight path does not exceed about $80^\circ$, and the inclination of the thrust line to the wing datum does not vary during the first few seconds of flight.

Clearly, if either of the above requirements are not fulfilled, the present programmes are inadequate and modification must be made to cover the case.

## 7 CONCLUDING REMARKS

The calculating accuracy of both programmes is considerably better than the accuracy to which the basic aircraft data can be obtained. Consequently, for most aircraft, there is little to choose between the programmes. However, for some unusual cases (e.g. large excess thrust; small initial lift) the "complete" programme is necessary to include all relevant terms.

The main advantage of the 2nd programme (apart from its inherent simplicity) is that it leaves more than half the SC30 computer free for other purposes, such as the longitudinal stability programme shown in Fig.8.

Fig.12 shows a comparison between results for both programmes for an aircraft in two very different configurations with results for the same cases calculated independently on a Mercury digital computer and by hand. The main reason for disagreement with the hand calculation results is a discrepancy in basic aerodynamic data. Fig.13 shows a plot of the height dropped against time as produced by programme I and as calculated by hand for identical basic data.

## SYMBOLS

| | |
|---|---|
| $\alpha$ | incremental angle of incidence above datum |
| $\alpha'$ | datum incidence of thrust line relative to the flight path |
| $C_D$ | drag coefficient $= \dfrac{D}{\frac{1}{2}\rho V^2 S}$ |
| $C_{D_o}$ | drag coefficient at zero lift and speed $V_o$ |
| $C_{D_{o_V}}$ | drag coefficient at zero lift and speed V |
| $C_{D_{\alpha'}}$ | drag coefficient at datum incidence and speed |
| $C_L$ * | lift coefficient $= \dfrac{L}{\frac{1}{2}\rho V^2 S}$ |
| $C_{L_{\alpha'}}$ | lift coefficient at datum incidence and speed |
| $C_\mu$ | boundary layer control momentum coefficient |
| D | aerodynamic drag force |
| $D_m$ | momentum drag force (total for aircraft) |
| $\gamma$ | angle between flight path and horizontal (positive on descent) |
| g | acceleration due to gravity |
| h | height dropped below deck level |
| k | induced drag factor $= \dfrac{\partial C_D}{\partial (C_L^2)}$ |
| L * | aerodynamic lift force |
| m | mass of aircraft at take-off |
| $\eta$ * | elevator angle |
| $\rho$ | air density at take-off conditions |
| S | total wing area of aircraft |
| T | gross thrust of aircraft measured at speed $V_o$ and ambient air density |
| u | incremental velocity of aircraft above datum |
| $V_o$ | datum speed |
| V | total speed $= V_o + u$ |
| $Z_\eta$ | vertical force/degree elevator deflection |
| | A dot implies differentiation with respect to time. |

* If trimmed values of lift and $C_L$ are used, $\eta$ is the incremental elevator angle over the trimmed elevator position. This is clearly the method to be adopted if the $Z_\eta$ term is to be neglected (see later). If stick fixed (untrimmed) values of lift and $C_L$ are used, $\eta$ has the usual meaning of elevator displacement from the datum position.

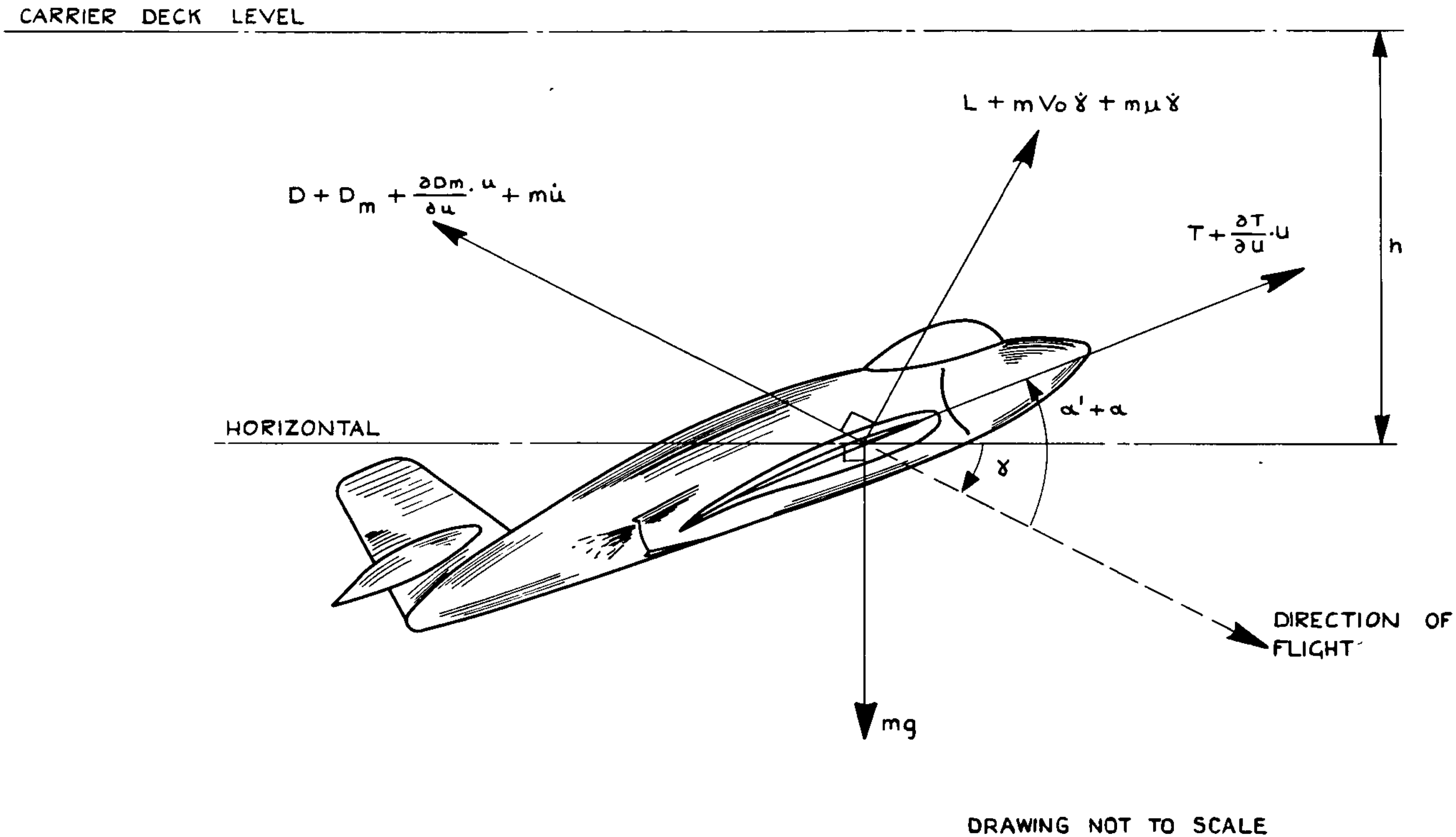



FIG.1 FORCES ON AN AIRCRAFT AFTER A CATAPULT LAUNCH

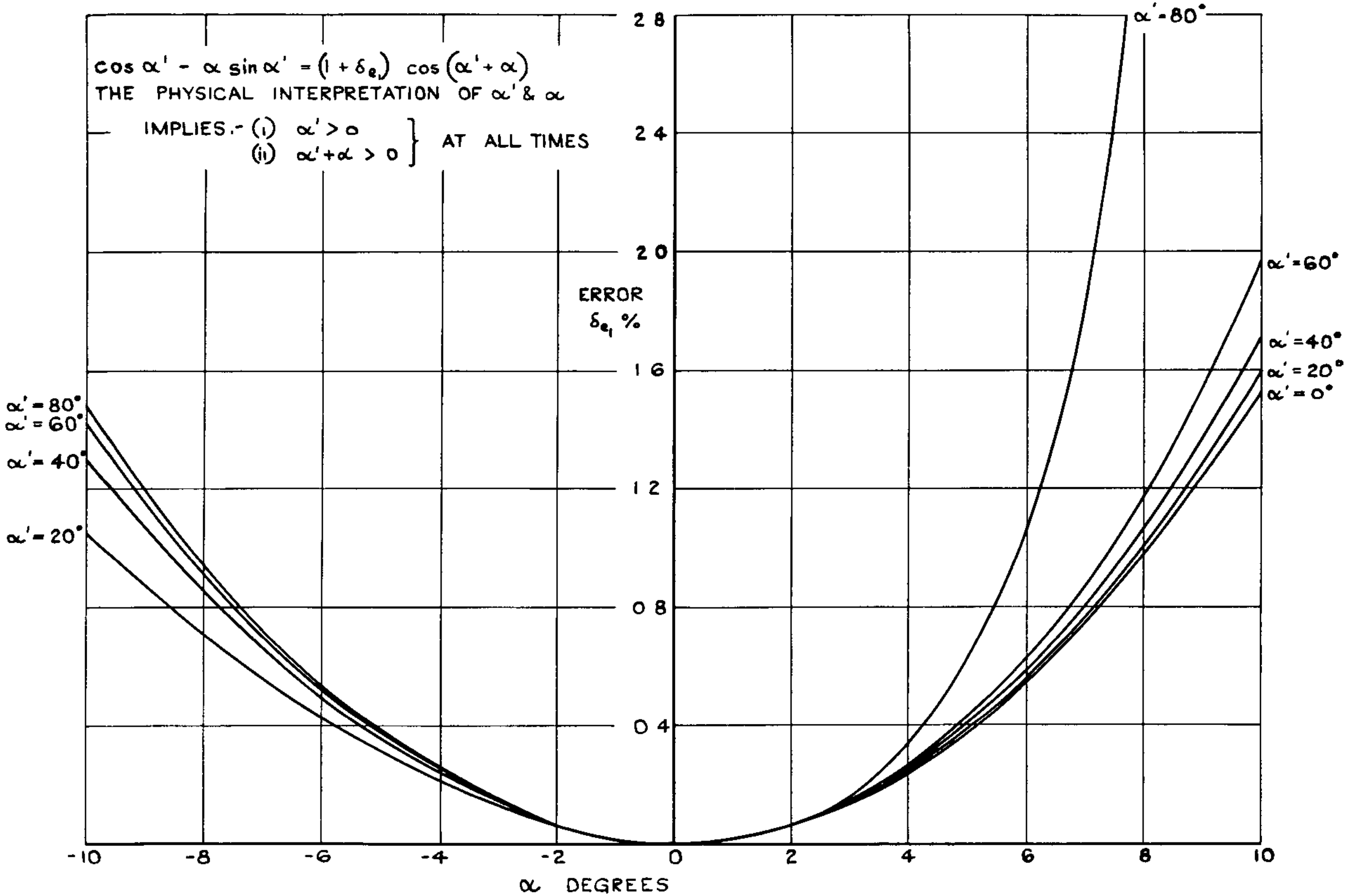



FIG. 2 PLOT OF THE ERRORS IN THE APPROXIMATION $\cos(\alpha'+\alpha) \approx \cos\alpha' - \alpha\sin\alpha'$

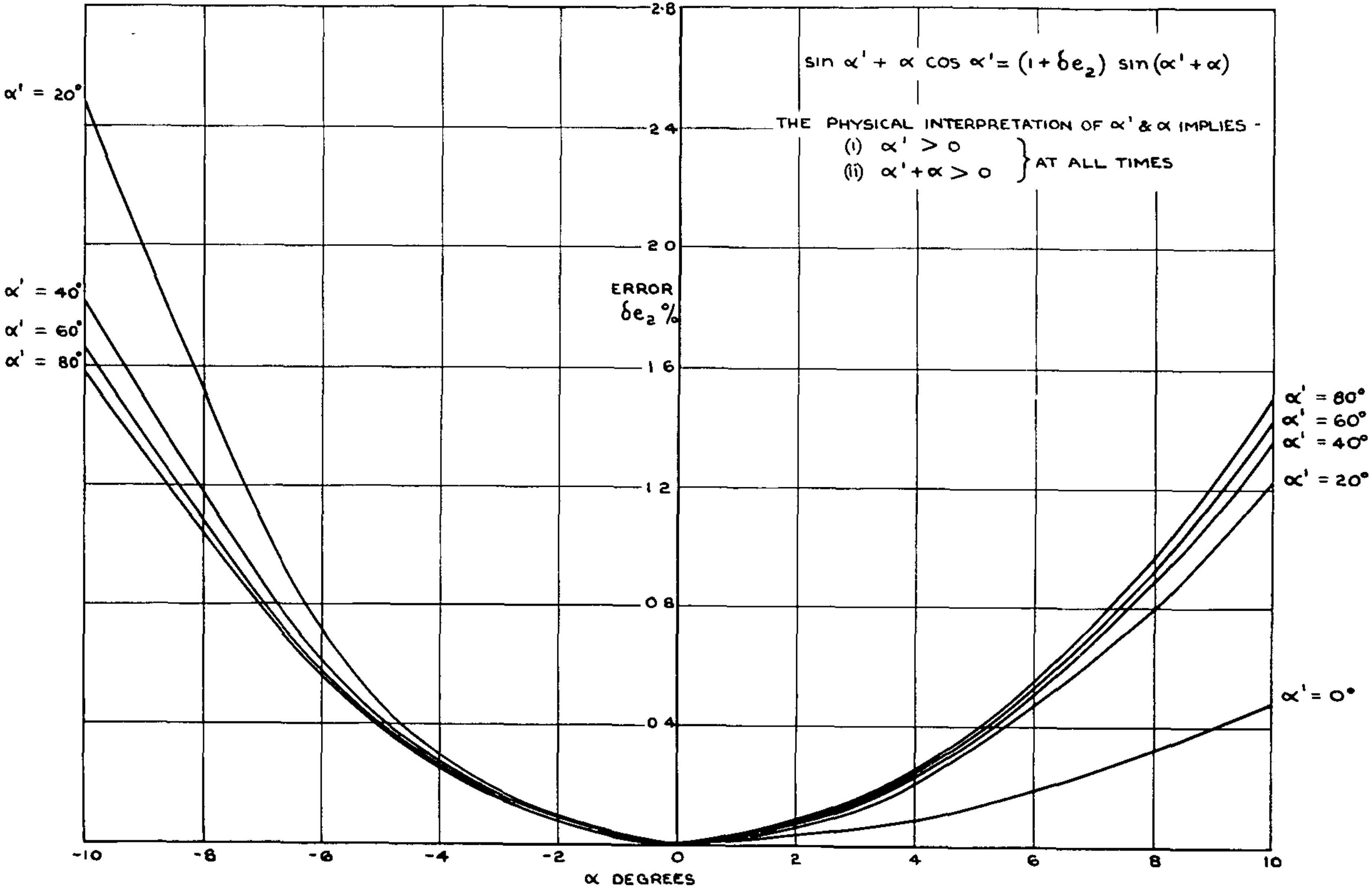
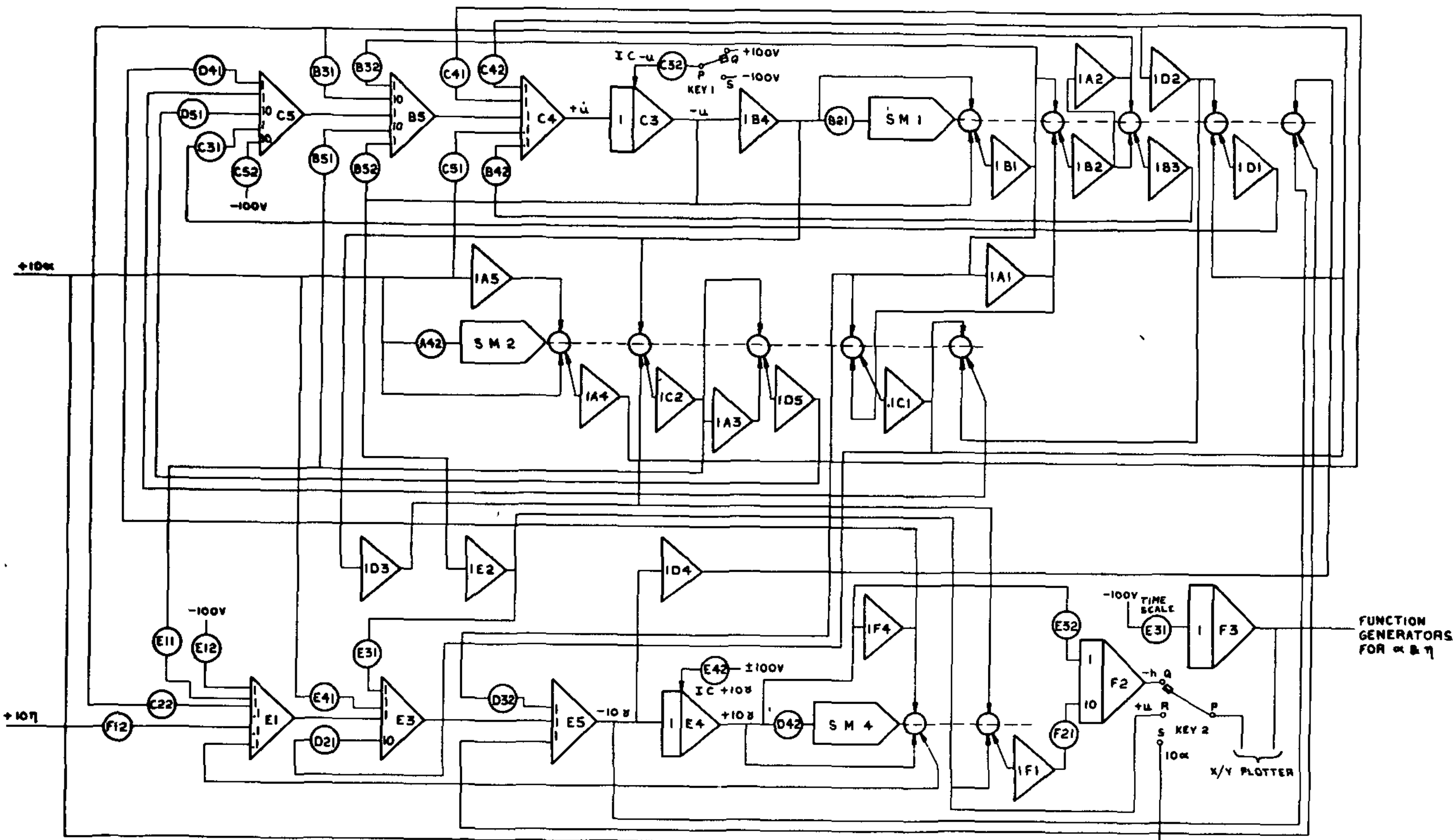



FIG. 3 PLOT OF THE ERRORS IN THE APPROXIMATION $\sin(\alpha' + \alpha) \approx \sin\alpha' + \alpha\cos\alpha'$




(1) $h = P\gamma + Qu\gamma$

(2) $10u = C_0 + C_1\gamma - C_2\alpha - C_3u - C_4u\alpha - C_5\alpha^2 - C_6u^2 - C_7u^2\alpha - C_8u\alpha^2 - C_9u^3 - C_{10}u^2\alpha^2 - C_{11}u^3\alpha - C_{12}u^4$

(3) $-10\gamma = K_0 + K_1\alpha + K_2u + K_3u\alpha + K_4u^2 + K_5u^2\alpha + K_6u^3 + K_7u\gamma + K_8\gamma^2 + K_9\eta$

FIG. 4 PROGRAMME I

| POT | INPUT | COEFF | VALUE | AMP GAIN | AMP No | AMP OUTPUT |
|---|---|---|---|---|---|---|
| A11 | | | | | | |
| A12 | | | | | | |
| A21 | | | | | | |
| A22 | | | | | | |
| A31 | | | | | | |
| A32 | | | | | | |
| A41 | | | | | | |
| A42 | $10\alpha$ | $C$ | | SHAFT OF SM 2 | | |
| A51 | | | | | | |
| A52 | | | | | | |
| B11 | | | | | | |
| B12 | | | | | | |
| B21 | $u$ | $10\,K_7$ | | SHAFT OF SM 1 | | |
| B22 | | | | | | |
| B31 | $-\frac{K_7^3u^3}{100}$ | $\frac{10\,C_8}{K_7^3}$ | | 10 | B5 | $+C_8u^3$ |
| B32 | $-\frac{K_7u^2}{10}$ | $\frac{10\,C_6}{K_7}$ | | 1 | B5 | $+C_6u^2$ |
| B41 | | | | | | |
| B42 | $+\frac{K_7^4u^4}{10^4}$ | $\frac{10^4C_{12}}{K_7^4}$ | | 1 | C4 | $-\frac{C_{12}u^4}{10}$ |
| B51 | $-\frac{C}{10}u\alpha$ | $\frac{C_4}{C}$ | | 10 | B5 | $+C_4u\alpha$ |
| B52 | $-u$ | $C_3$ | | 1 | B5 | $+C_3u$ |
| C11 | | | | | | |
| C12 | | | | | | |
| C21 | | | | | | |
| C22 | $-\frac{K_7^3u^3}{100}$ | $\frac{10^3K_6}{K_7^3}$ | | 1 | E1 | $K_6u^3$ |
| C31 | $\frac{K_7^3C}{10^3}u^3\alpha$ | $\frac{10^3C_{11}}{K_7^3C}$ | | 1 | C5 | $-C_{11}u^3\alpha$ |
| C32 | $IC - u$ | | | | | |
| C41 | $\frac{C}{10}\alpha^2$ | $\frac{C_5}{C}$ | | 1 | C4 | $-\frac{C_5\alpha^2}{10}$ |
| C42 | $\frac{K_7C}{100}u^2\alpha$ | $\frac{10\,C_7}{K_7C}$ | | 1 | C4 | $-\frac{C_7u^2\alpha}{10}$ |
| C51 | $10\alpha$ | $\frac{C_2}{10}$ | | 1 | C4 | $-\frac{C_2\alpha}{10}$ |
| C52 | $-100$ | $\frac{C_0}{1000}$ | | 10 | C5 | $+C_0$ |

KEY 1 SELECTS SIGN OF I C − u (C32)

KEY 2 SELECTS O/P TO X/Y PLOTTER

| POT | INPUT | COEFF | VALUE | AMP GAIN | AMP No | AMP OUTPUT |
|---|---|---|---|---|---|---|
| D11 | | | | | | |
| D12 | | | | | | |
| D21 | $\frac{K_7Cu^2\alpha}{100}$ | $\frac{10\,K_5}{K_7C}$ | | 10 | E3 | $-K_5u^2\alpha$ |
| D22 | | | | | | |
| D31 | | | | | | |
| D32 | $-\frac{K_7u^2}{10}$ | $\frac{10\,K_4}{K_7}$ | | 1 | E5 | $+K_4u^2$ |
| D41 | $-10\,\gamma$ | $\frac{C_1}{10}$ | | 1 | C5 | $+C_1\gamma$ |
| D42 | $10\,\gamma$ | $10\,K_8$ | | SHAFT OF SM 4 | | |
| D51 | $\frac{C^2u\alpha^2}{100}$ | $\frac{10\,C_9}{C^2}$ | | 10 | C5 | $-C_9u\alpha^2$ |
| D52 | | | | | | |
| E11 | $-\frac{Cu\alpha}{10}$ | $10\frac{K_3}{C}$ | | 1 | E1 | $+K_3u\alpha$ |
| E12 | $-100$ | $\frac{K_0}{100}$ | | 1 | E1 | $K_0$ |
| E21 | | | | | | |
| E22 | | | | | | |
| E31 | $u$ | $K_2$ | | 1 | E3 | $-K_2u$ |
| E32 | $10\,\gamma$ | $\frac{P}{10}$ | | 1 | F2 | $-P\gamma\,dt$ |
| E41 | $10\alpha$ | $\frac{K_1}{10}$ | | 1 | E3 | $-K_1\alpha$ |
| E42 | $IC + 10\,\gamma$ | | | | | |
| E51 | | | | | | |
| E52 | | | | | | |
| F11 | | | | | | |
| F12 | $+10\,\eta$ | $K_9$ | | -1 | E1 | $-K_9\eta$ |
| F21 | $+K_8u\dot{\gamma}$ | $\frac{Q}{10\,K_8}$ | | 10 | F2 | $-Qu\dot{\gamma}\,dt$ |
| F22 | | | | | | |
| F31 | $-100$ | $\frac{R}{100}$ | | 1 | F3 | $R\,dt$ |
| F32 | | | | | | |
| F41 | | | | | | |
| F42 | | | | | | |
| F51 | | | | | | |
| F52 | | | | | | |

FIG. 5 LIST OF COEFFICIENT POTENTIOMETERS FOR PROGRAMME I

(1) $h = P\gamma + Qu\gamma$

(2) $10u = C_0 + C_1\gamma - C_2\alpha - C_3u - C_4u\alpha - C_5\alpha^2$

(3) $-10\gamma = K_0 + K_1\alpha + K_2u + K_3u\alpha + K_7u\gamma + K_8\eta$

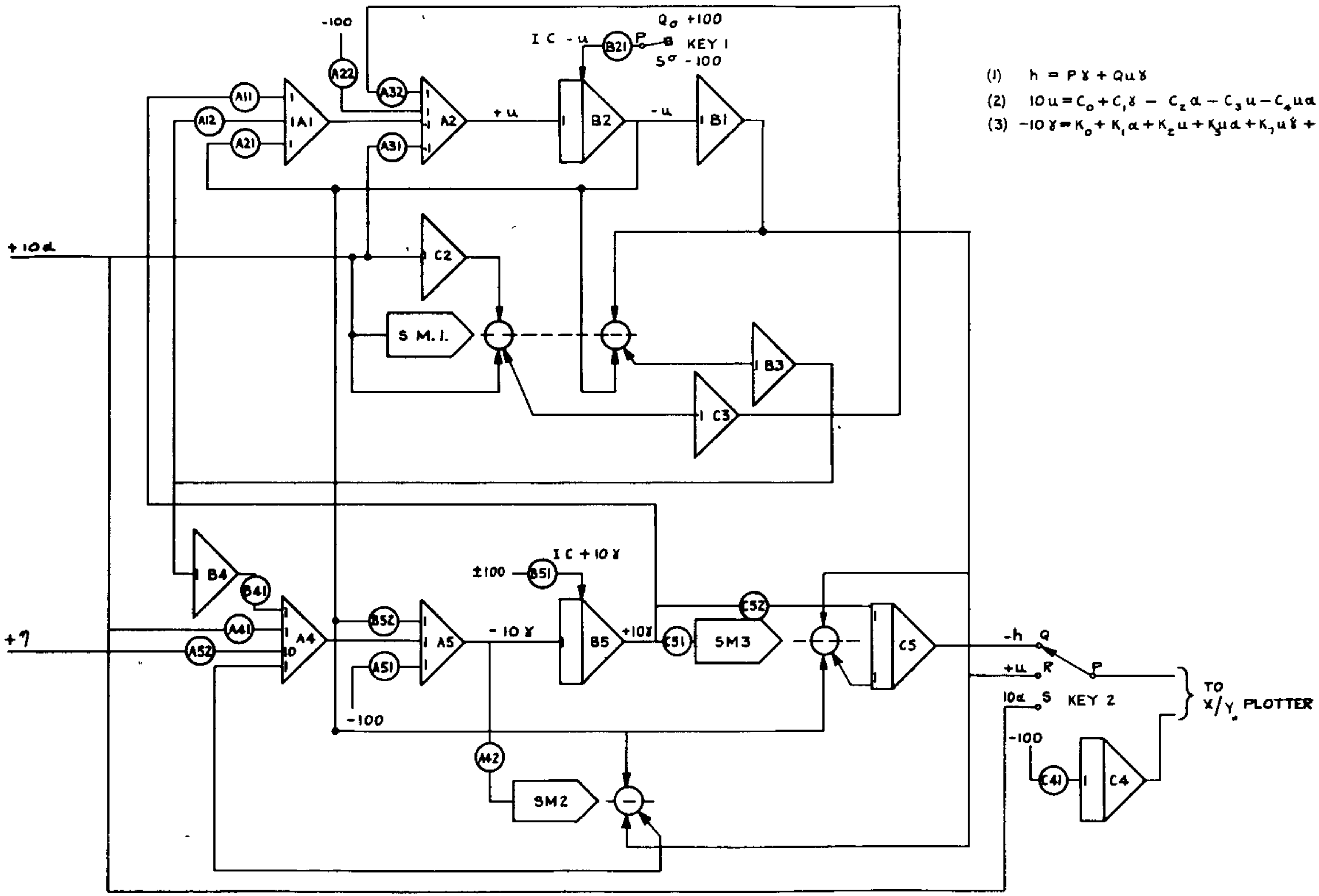


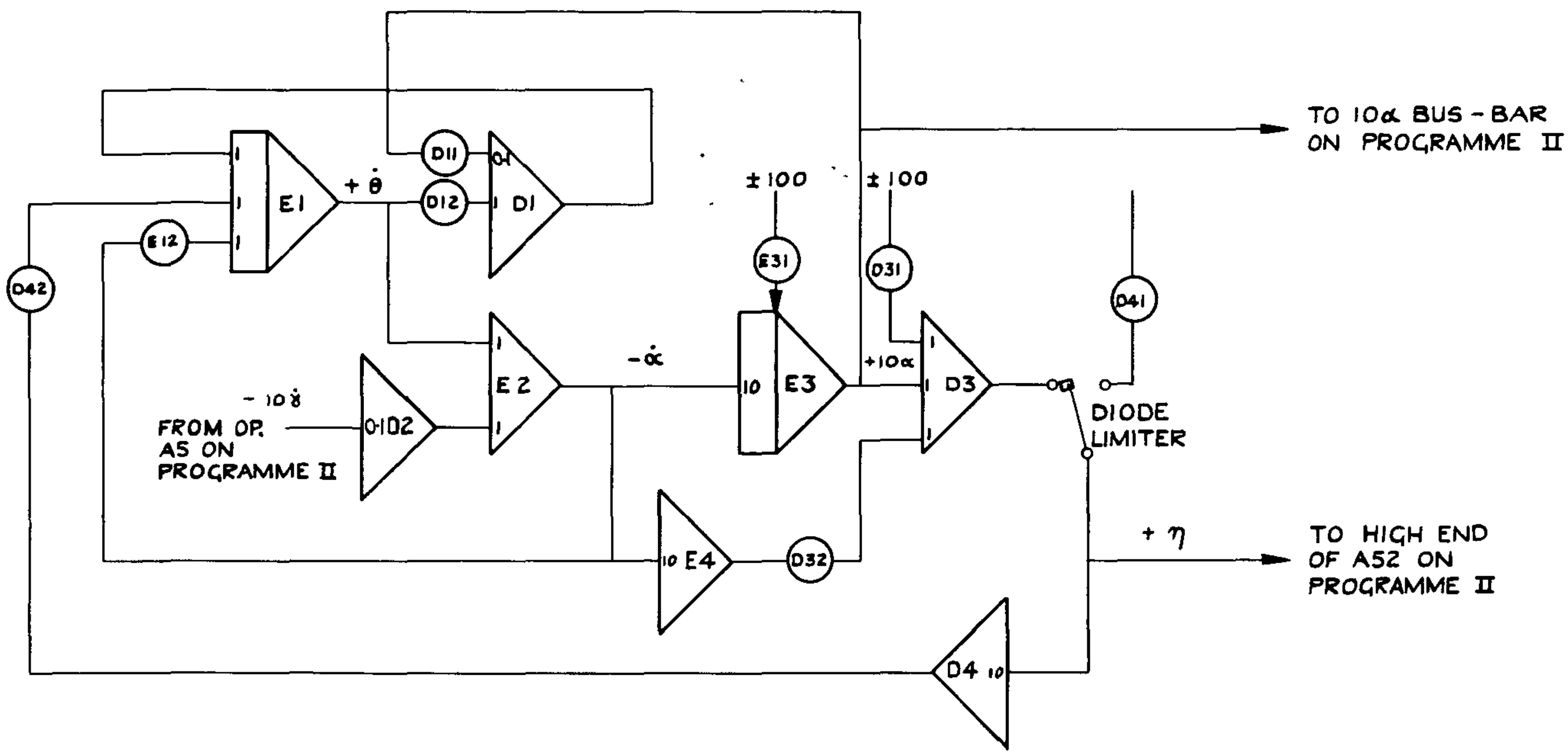

FIG.6 PROGRAMME II

| POT | INPUT | COEFF | VALUE | AMP GAIN | AMP No. | AMP OUTPUT |
|---|---|---|---|---|---|---|
| A11 | $+10\gamma$ | $\frac{C_1}{10}$ | | 1 | A1 | $-C_1\gamma$ |
| A12 | $-\frac{u\alpha}{10}$ | $10C_4$ | | 1 | A1 | $C_4u\alpha$ |
| A21 | $-u$ | $C_3$ | | 1 | A1 | $C_3u$ |
| A22 | $-100$ | $\frac{C_0}{1000}$ | | 1 | A2 | $\frac{C_0}{10}$ |
| A31 | $+10\alpha$ | $\frac{C_2}{10}$ | | ·1 | A2 | $-\frac{C_2\alpha}{10}$ |
| A32 | $\frac{\alpha^2}{10}$ | $C_5$ | | 1 | A2 | $-\frac{C_5\alpha^2}{10}$ |
| A41 | $+10\alpha$ | $\frac{K_1}{10}$ | | 1 | A4 | $-K_1\alpha$ |
| A42 | $-10\dot{\gamma}$ | $10K_7$ | | SHAFT OF | | SM 2 |
| A51 | $-100$ | $\frac{K_0}{100}$ | | 1 | A5 | $+K_0$ |
| A52 | $+\eta$ | $\frac{K_g}{10}$ | | 10 | A4 | $-K_g\eta$ |
| B11 | | | | | | |
| B12 | | | | | | |
| B21 | I.C. $-u$ | | | | | |
| B22 | | | | | | |
| B31 | | | | | | |
| B32 | | | | | | |
| B41 | $\frac{u\alpha}{10}$ | $10K_3$ | | 1 | A4 | $-K_3u\alpha$ |
| B42 | | | | | | |
| B51 | I.C. $+10\gamma$ | | | | | |
| B52 | $-u$ | $K_2$ | | 1 | A5 | $K_2u$ |
| C11 | | | | | | |
| C12 | | | | | | |
| C21 | | | | | | |
| C22 | | | | | | |
| C31 | | | | | | |
| C32 | | | | | | |
| C41 | $-100$ | $\frac{R}{100}$ | | 1 | C4 | $R\,dt$ |
| C42 | | | | | | |
| C51 | $+10\gamma$ | $10Q$ | | SHAFT OF | | SM 3 |
| C52 | $+10\gamma$ | $\frac{P}{10}$ | | 1 | C5 | $-P\gamma\,dt$ |

KEY 1 SELECTS SIGN OF I.C. $-u$ (row B21)

KEY 2 SELECTS O/P TO X/Y PLOTTER

FIG.7 LIST OF COEFFICIENT POTENTIOMETERS FOR PROGRAMME II




$$\ddot{\theta} = M_q \dot{\theta} + M_\eta \eta + M_{\dot{\alpha}} \dot{\alpha} + M_\alpha \alpha$$

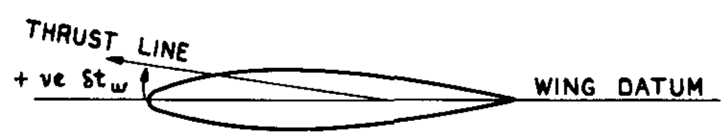
FIG. 8 LONGITUDINAL STABILITY PROGRAMME FOR GENERATION OF $\eta$ FOR A GIVE $\alpha$ TIME-HISTORY

| POT | INPUT | COEFF | VALUE | AMP GAIN | AMP NO | AMP OUTPUT |
|---|---|---|---|---|---|---|
| D11 | $+10\alpha$ | $M_\alpha$ | | .1 | D1 | $-M_\alpha \alpha$ |
| D12 | $+\dot{\theta}$ | $M_q$ | | 1 | D1 | $-M_q \dot{\theta}$ |
| D21 | | | | | | |
| D22 | | | | | | |
| D31 | ±100 | REQUIRED FINAL $\alpha$ | | 1 | D3 | + REQ° FINAL $\alpha$ |
| D32 | $+10\dot{\alpha}$ | DAMPING COEFF | | 1 | D3 | $-10K\dot{\alpha}$ |
| D41 | $\eta$–LIMIT | | | | | |
| D42 | $-10\eta$ | $\frac{M_\eta}{10}$ | | 1 | E1 | $+M_\eta \eta dt$ |
| D51 | | | | | | |
| D52 | | | | | | |
| E11 | | | | | | |
| E12 | $-\dot{\alpha}$ | $M_{\dot{\alpha}}$ | | 1 | E1 | $+M_{\dot{\alpha}} \dot{\alpha} dt$ |
| E21 | | | | | | |
| E22 | | | | | | |
| E31 | I.C.$+10\alpha$ | | | | | |
| E32 | | | | | | |
| E41 | | | | | | |
| E42 | | | | | | |
| E51 | | | | | | |
| E52 | | | | | | |
| F11 | | | | | | |
| F12 | | | | | | |
| F21 | | | | | | |
| F22 | | | | | | |
| F31 | | | | | | |
| F32 | | | | | | |
| F41 | | | | | | |
| F42 | | | | | | |
| F51 | | | | | | |
| F52 | | | | | | |

FIG. 9 LIST OF COEFFICIENT POTENTIOMETERS FOR LONGITUDINAL STABILITY PROGRAMME

| CONSTANT | RAW DATA | CALCULATE | VALUE |
|---|---|---|---|
| ACCELERATION DUE TO GRAVITY (ft/sec²) | $g$ | | |
| ALL-UP WEIGHT OF a/c AT t/o (lb) | $W$ | | |
| MASS OF a/c AT t/o (SLUGS) | | $m = W/g$ | |
| RATE OF CHANGE OF GROSS THRUST WITH SPEED | $\delta T/\delta u$ | | |
| GROSS THRUST AT SPEED $V_0$ (lb) | $T$ | | |
| GROSS THRUST AT SPEED $V_0$ (lb) | | $T = T_S + \partial T/\partial u^2 \; V_0^2$ | |
| RATE OF CHANGE OF MOMENTUM DRAG WITH SPEED | $\partial D_m/\partial u$ | | |
| MOMENTUM DRAG AT SPEED $V_0$ (lb) | | $D_m = \partial D_m/\partial u \cdot V_0$ | |
| 1 ANGLE BETWEEN WING DATUM AND THRUST LINE | $\delta t_w$ | | |
| 2 DATUM WING INCIDENCE | $\alpha'_w$ | | |
| DATUM INCIDENCE REFERRED TO THRUST LINE | | $\alpha' = \alpha'_w + \delta t_w$ | |
| | | $\cos \alpha'$ | |
| | | $\sin \alpha'$ | |
| 3 LIFT COEFFICIENT AT DATUM INCIDENCE | $C_{L\alpha'}$ | | |
| 3 DRAG COEFFICIENT AT DATUM INCIDENCE | $C_{D\alpha'}$ | | |
| LIFT CURVE SLOPE AT DATUM INCIDENCE | $\partial C_L/\partial \alpha$ | | |
| LIFT-DEPENDANT DRAG FACTOR ($\partial C_D/\partial C_L^2$) | | | |
| RATE OF CHANGE OF $C_\mu$ WITH SPEED | $\partial C_\mu/\partial u$ | | |
| RATE OF CHANGE OF $C_{D_0}$ WITH $C_\mu$ | $\partial C_{D_0}/\partial C_\mu$ | | |
| RATE OF CHANGE OF $C_L$ WITH $C_\mu$ | $\partial C_L/\partial C_\mu$ | | |
| 4 RATE OF CHANGE OF $C_{D_0}$ WITH SPEED | | $\partial C_{D_0}/\partial u = \partial C_{D_0}/\partial C_\mu \; \partial C_\mu/\partial u$ | |
| 4 RATE OF CHANGE OF $C_L$ WITH SPEED | | $\partial C_L/\partial u = \partial C_L/\partial C_\mu \; \partial C_\mu/\partial u$ | |
| AIR DENSITY AT t/o CONDITIONS | $\rho$ | | |
| OVERALL WING AREA | $S$ | | |
| RATE OF CHANGE OF TAIL DOWNLOAD WITH ELEVATOR POSITION (lb/°) | $Z_\eta$ | | |
| 2 DATUM SPEED (ft/sec) | $V_0$ | | |
| USEFUL CONSTANTS | | $\frac{1}{2}\rho S$ | |
| | | $\rho V_0 S$ | |
| | | $\frac{1}{2}\rho V_0^2 S$ | |
| | | $10/m$ | |
| | | $573/m V_0$ | |

1 SIGN CONVENTION FOR $\delta t_w$ IS AS FOLLOWS :—




2 DATUMS CHOSEN TO BE IN THE CENTRE OF THE WORKING RANGE OF INCIDENCE AND SPEED

3 $C_L$ AND $C_D$ AT DATUM WING INCIDENCE

4 DERIVATIVES TAKEN AT DATUM INCIDENCE AND SPEED

# FIG.10 TABLE FOR CALCULATION OF POTENTIOMETER VALUES

| | CONSTANT | CALCULATE | VALUE |
|---|---|---|---|
| $C_0$ | $\frac{10}{m}\left(T\cos\alpha' - D_m - \frac{1}{2}\rho V_0^2 S C_{D\alpha'}\right)$ | | |
| $C_1$ | $\frac{10g}{57.3}$ | | |
| $C_2$ | $\frac{10}{m}\left(\frac{T\sin\alpha'}{57.3} + \frac{1}{2}\rho V_0^2 S . 2KC_{L\alpha'}\frac{\partial C_L}{\partial\alpha}\right)$ | | |
| $C_3$ | $\frac{10}{m}\left(\frac{1}{2}\rho V_0^2 S\frac{\partial C_{D0}}{\partial u} + \frac{1}{2}\rho V_0^2 S\, 2KC_{L\alpha'}\frac{\partial C_L}{\partial u} + \rho S V_0 C_{D\alpha'} + \frac{\partial D_m}{\partial u} - \frac{\partial T}{\partial u}\cos\alpha'\right)$ | | |
| $C_4$ | $\frac{10}{m}K\rho V_0 S\frac{\partial C_L}{\partial\alpha}\left(V_0\frac{\partial C_L}{\partial u} + 2C_{L\alpha'}\right) + \frac{10}{m}\frac{\partial T}{\partial u}\frac{\sin\alpha'}{57.3}$ | | |
| $C_5$ | $\frac{10}{m}\frac{1}{2}\rho V_0^2 S K\left(\frac{\partial C_L}{\partial\alpha}\right)^2$ | | |
| $C_6$ | $\frac{10}{m}\left(\frac{1}{2}\rho S C_{D\alpha'} + \rho V_0 S\left\{\frac{1}{2}KV_0\left(\frac{\partial C_L}{\partial u}\right)^2 + \frac{\partial C_{D0}}{\partial u} + 2KC_{L\alpha'}\frac{\partial C_L}{\partial u}\right\}\right)$ | | |
| $C_7$ | $\frac{10}{m}\left(2K\frac{\partial C_L}{\partial\alpha}\frac{\partial C_L}{\partial u}\rho V_0 S + KC_{L\alpha'}\frac{\partial C_L}{\partial\alpha}\rho S\right)$ | | |
| $C_8$ | $\frac{10}{m}K\left(\frac{\partial C_L}{\partial\alpha}\right)^2\rho V_0 S$ | | |
| $C_9$ | $\frac{10}{m}\cdot\frac{1}{2}\rho S\left(2KV_0\left(\frac{\partial C_L}{\partial u}\right)^2 + \frac{\partial C_{D0}}{\partial u} + 2KC_{L\alpha'}\cdot\frac{\partial C_L}{\partial u}\right)$ | | |
| $C_{10}$ | $\frac{10}{m}\frac{1}{2}\rho S K\left(\frac{\partial C_L}{\partial\alpha}\right)^2$ | | |
| $C_{11}$ | $\frac{10}{m}\frac{1}{2}\rho S . 2K\frac{\partial C_L}{\partial\alpha}\cdot\frac{\partial C_L}{\partial u}$ | | |
| $C_{12}$ | $\frac{10}{m}\cdot\frac{1}{2}\rho S K\left(\frac{\partial C_L}{\partial u}\right)^2$ | | |
| $K_0$ | $\frac{573}{mV_0}\left(\frac{1}{2}\rho V_0^2 S C_{L\alpha'} + T\sin\alpha' - mg\right)$ | | |
| $K_1$ | $\frac{573}{mV_0}\left(\frac{1}{2}\rho V_0^2 S\frac{\partial C_L}{\partial\alpha} + \frac{T\cos\alpha'}{57.3}\right)$ | | |
| $K_2$ | $\frac{573}{mV_0}\rho V_0 S\left(C_{L\alpha'} + \frac{1}{2}V_0\frac{\partial C_L}{\partial u}\right) + \frac{573}{mV_0}\frac{\partial T}{\partial u}\sin\alpha'$ | | |
| $K_3$ | $\frac{573}{mV_0}\rho S V_0\frac{\partial C_L}{\partial\alpha} + \frac{10}{mV_0}\frac{\partial T}{\partial u}\cos\alpha'$ | | |
| $K_4$ | $\frac{573}{mV_0}\left\{\frac{1}{2}\rho S\left(C_{L\alpha'} + 2V_0\frac{\partial C_L}{\partial u}\right)\right\}$ | | |
| $K_5$ | $\frac{573}{mV_0}\left(\frac{1}{2}\rho S\frac{\partial C_L}{\partial\alpha}\right)$ | | |
| $K_6$ | $\frac{573}{mV_0}\cdot\frac{1}{2}\rho S\frac{\partial C_L}{\partial u}$ | | |
| $K_7$ | $\frac{10}{V_0}$ | | |
| $K_8$ | $\frac{5g}{57.3V_0}$ | | |
| $K_9$ | $\frac{573}{mV_0}Z_\eta$ | | |
| P | $\frac{V_0}{57.3}$ | | |
| Q | $\frac{1}{57.3}$ | | |
| C | $100\sqrt{\frac{C_{10}}{10K_7}}$ | | |

EQUATIONS

DRAG: $10u = C_0 + C_1\delta - C_2\alpha - C_3 u - C_4 u\alpha - C_5\alpha^2 - C_6 u^2 - C_7 u^2\alpha - C_8 u^3$

LIFT: $-10\dot{\delta} = K_0 + K_1\alpha + K_2 u + K_3 u\alpha + K_4 u^2 + K_5 u^2\alpha + K_6 u^2 + K_7 u\delta + K_8\delta^2 + K_9\eta$

HEIGHT DROP: $h = P\delta + Qu\delta$

**FIG. 11 CONSTANTS FOR EQUATIONS OF MOTION**

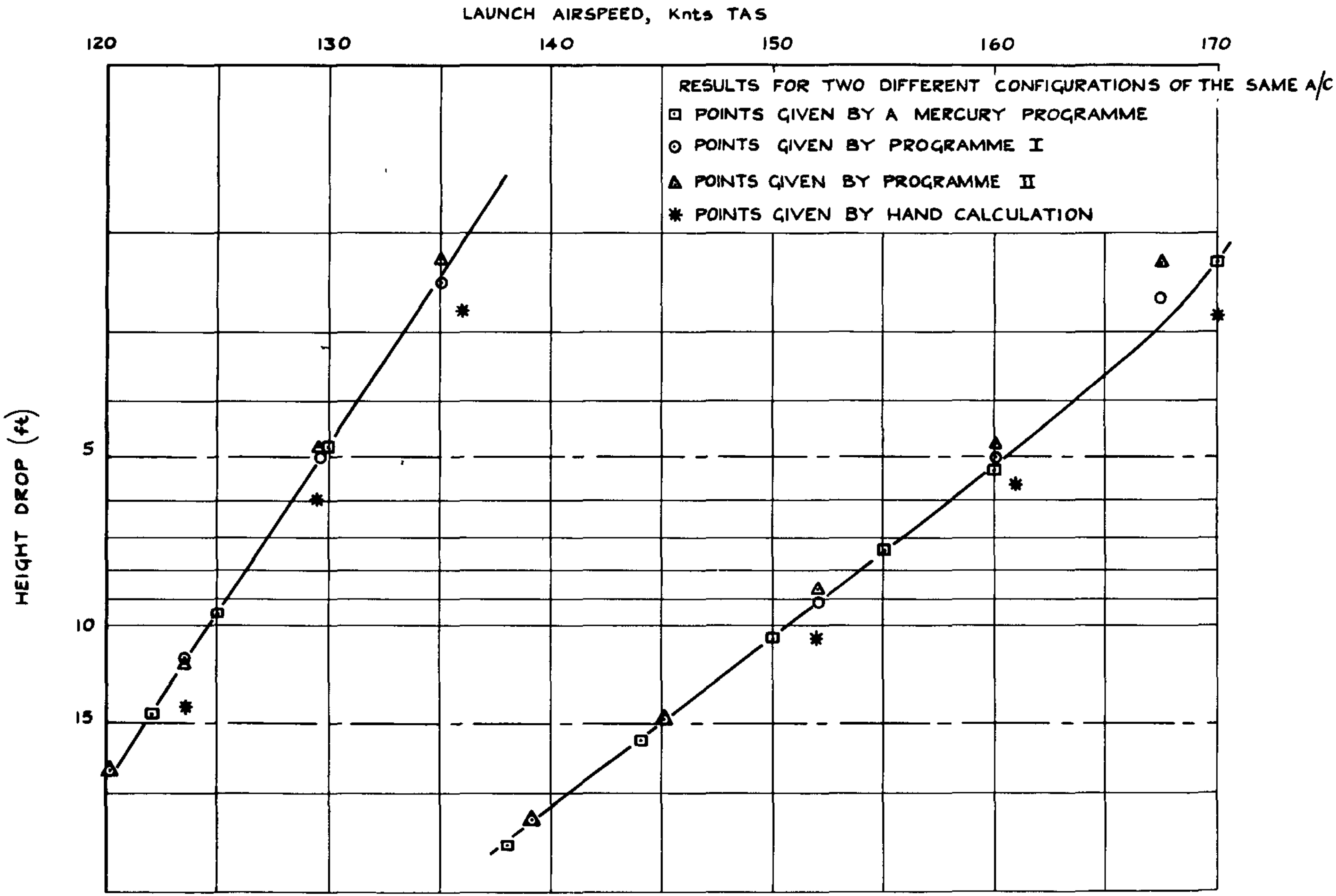



FIG. 12 COMPARISON OF RESULTS OBTAINED BY VARIOUS METHODS

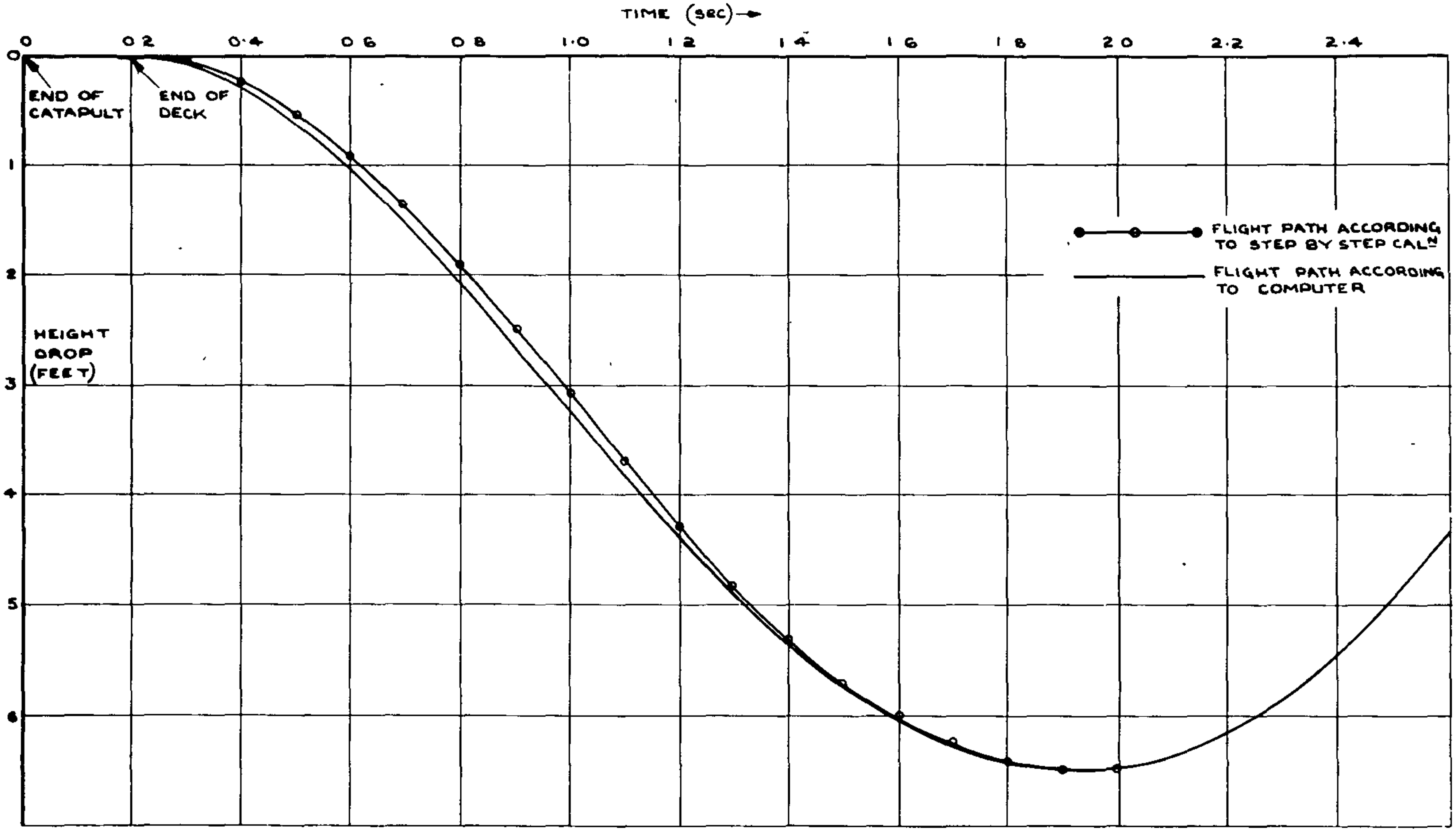



FIG. 13 COMPARISON BETWEEN COMPUTER PLOT & HAND CALN. FOR IDENTICAL BASIC DATA

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Jones, R. W.

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518.5:
621.317.79:
629.13.015.612.2:
533.6.015.1

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An analogue computer programme was derived, making as few approximations as possible, for the calculation of the flight path of an aircraft leaving the end of a ship-borne catapult. Using this 'complete' calculation, it was shown that, for most aircraft, other approximations could be made without significantly impairing the accuracy of the result, and greatly simplifying the programme.

A description of both programmes is given here, together with their derivation and method of use.



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