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by

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COMPUTER PROGRAMMES TO CALCULATE THE RESPONSE OF FLEXIBLE AIRCRAFT TO GUSTS AND CONTROL MOVEMENTS

by

C. G. B. Mitchell

SUMMARY

The equations of motion for flexible aircraft with or without automatic control systems exposed to excitation by harmonic gusts or control movements are given. The response to transient excitation of step form is found by a Fourier transforn process.

Computer programmes to solve these equations are described, together with supporting data preparation and analysis programmes. Manual data preparation is minimised. Detailed programme specifications are given in Appendices.

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CONTENTS

1 INTRODUCTION

The development of large digital computers has enabled far more rigorous and rapid analyses of the dynamic response characteristics of aircraft to be made than were previously possible[1,2]. More degrees-of-freedom can be included, allowing a better representation of the flexibility of the aircraft structure. The aerodynamic forces on the aircraft can be calculated by various lifting surface theories that cover a range of planforms, Mach numbers and frequency parameters. The equations of motion can include elements of the automatic control system if such a system is fitted to the aircraft.

Despite these advances many problems do still exist, of which the most serious are non-linear effects (for example, of aerodynamic forces at large incidences, of control systems, or of undercarriages); transonic aerodynamics and aerodynamic interference; lack of understanding of the damping in such complex structures as modern aircraft; effects due to pilot-induced control movements; and, for excitation by random processes, fundamental theoretical problems when the process is non-stationary or the aircraft characteristics are non-linear.

However, the more rigorous analyses now possible do allow calculation of the dynamic responses of aircraft to turbulent air that show good quantitative agreement with experiments (at least for cases where non-linear system behaviour and pilot control movements are negligible), to such an extent that the largest unknown is often the description of the excitation.

To undertake such analyses involves handling large amounts of data in digital form. To enable this to be done quickly and accurately it is quite essential that computer programmes should require a minimum of data to be prepared by hand, and that where a series of programmes is used the output from one should be compatible mith the input to the next.

This Report describes a series of Mercury Autocode computer programmes written at the Royal Aircraft Establishment, Farnborough, for the calculation of the dynamic response of flexible aircraft to atmospheric turbulence and to control movements. Section 2 outlines the equations of motion for the system and Section 3 describes the computer programmes developed to solve the equations. These programmes calculate either the harmonic response to harmonic excitation or the transient response to transient excitation, the time history of which is a step function. The flexibility of the structure is represented by a number of nodes (which need not be normal) and aerodynamic data from any source can be used.

The response programmes are not restrioted to use on aircraft. They could be used for any linear multi-degree-of-freedom second order system acted on by non-conservative forces that can be represented as linear functions of the system response.

2 THE EQUATIONS OF MOTION

2.1 General

The aircraft is assumed to be represented by a linear system. Axes are taken with the origin moving along the undisturbed straight flight path at the constant flight speed, as shown in Fig.?. The origin coincides with a fixed point on the aircraft in steady flight, but not when the aircraft is disturbed from this condition. The x axis is along the steady flight direction, positive towards the rear: the y axis ishorieontal, normal to the flight path, positive to the right; the z axis is normal to the x and y axes, positive upwards. Forces on the aircraft needed to maintain steady flight (including all forces in the x direction) are ignored, and the equations of motion written in terms of the forces tending to perturb the aircraft from this state. The response parameters are those describing the perturbation movements of the aircraft about the steady flight state,

Motion of the aircraft is represented as the linear combination of a finite number of degrees-of-freedom. Some of these (heave, pitch, sideslip, yaw and roll) represent rigid body motions and are shown in Fig.1. Structural flexibility is usually represented by a number of calculated normal modes or measured natural vibration modes. Finally, degrees-of-freedom are required to represent control movements and oontrol systems. The modes are described by their shapes and their associated generalised masses, dampings and stiffnesses.

Unless there is asymmetry of the aircraft the motions may be separated into symmetric and antisymmetric, which in practice are usually analysed separately. The programmes described here are written to allow both symmetric and antisymmetric degrees-of-freedom to be included simultaneously. Furthermore, as structural terms coupling degrees-of-freedom can be included the programmes are not limited to freedoms represented by normal modes.

The equations of motion are written in matrix form and are derived from Lagrange's equation, displacements in rigid and flexible modes being used as generalised coordinates[3]. Associated with the modes are generalised inertia, damping and stiffness forces due to the structure, and also generalised aerodynamic forces. The structural forces are assumed to be linear functions of the generalised coordinates representing perturbations of the aircraft from its steady flight path.

The aerodynamic forces are of two types. The first are the forces which are linear functions of the generalised coordinates. These only exist when the aircraft is disturbed from its steady state and so are called the response aerodynamic forces. They do not depend directly on the excitation, but only on the response to it. The second set of aerodynamic forces are those acting on the aircraft to cause the disturbance. These are not in any way related to the response of the aircraft, and are called the excitation forces. They can be considered to be the forces on the aircraft if it were exposed to the excitation while being restrained against responding. The two sets of forces can only be separated for linear systems,

2.2 Harmonic motion with fixed controls

Consider the aircraft exposed to harmonio excitation at a circular frequency ω rad/sec which started long enough ago for any starting transient responses to have decayed to a negligible level. The harmonic excitation force could be due to harmonic motion of the control system or due to flight at velocity V through an array of harmonic gusts of wavelength $\lambda = 2\pi V/\omega$. The aircraft response is a sustained harmonic motion, also at a circular frequency ω, but normally the various generalised coordinates will not be in phase with each other, with the excitation force or with the excitation parameter (gust velocity w or control angle).

At time t the j^{th} generalised coordinate q_j has a value

$$q_j = \mathrm{Re}\,(q_{j0}\, e^{i\omega t}) \tag{1}$$

where q_j is the response generalised coordinate representing freedom j and q_{j0} is in general complex. With each generalised coordinate q_j is associated a mode shape f_{rj} such that when the displacement at the reference point for the mode is ℓ the displacement at some point r is ℓf_{rj}, where ℓ is a reference length.

Let the aircraft have n degrees-of-freedom represented by n modes. Each pair of modes has associated with it a generalised mass A_{jk} defined by

$$A_{jk} = \sum_{\text{all values of } r} \ell^2 m_r f_{rj} f_{rk} \tag{2}$$

where m_r is the mass of the element of the aircraft that is collected at the point r. The $n \times n$ matrix [A] of which A_{jk} is an element is symmetric and is positive definite. If the modes represented by f_{rj} and f_{rk} are orthogonal with respect to mass then $A_{jk} = 0$ unless $j = k$, end the matrix reduces to one of diagonal form. The column-matrix of the n generalised inertia forces is given by $[A]\{\ddot{q}\}$. (The notation here is that [ ] is a general matrix, { } is a column matrix and $\overline{\{\ \}}$ is a row matrix.)

Similarly the matrices of generalised damping and stiffness forces are given by $[B]\{\dot{q}\}$ and $[C]\{q\}$. The coefficients B_{jk} and C_{jk} can be evaluated from the work done by the dissipative and stiffness forces, but in general these are found by an approximate method that is only strictly valid for negligibly damped normal modes. If this method is used, and the system has a natural circular frequency ω_j in mode j then

$$C_{jj} = \omega_j^2 A_{jj} \quad . \tag{3}$$

If the damping in mode j is a fraction γ_j of critical then

$$B_{jj} = 2\gamma_j A_{jj} \omega_j \quad . \tag{4}$$

The aerodynamic oscillatory generalised forces in mode j due to harmonic motion of the aircraft in mode k are, in the notation of Ref.4,

$$Q_{jk}(t) = \rho V^2 S \ell \; q_{ko} \; e^{i\omega t} \; (Q'_{jk}(\nu) + i\nu \; Q''_{jk}(\nu)) \tag{5}$$

where ρ is the atmospheric density, V is the flight speed, S is a reference area, $Q'_{jk}(\nu)$ and $\nu Q''_{jk}(\nu)$ are the in phase and quadrature components of the non-dimensional oscillatory aerodynamic generalised force in mode j due to harmonic motion in mode k, and $\nu = \omega\ell/V$ is the frequency parameter.

Let the excitation oscillatory aerodynamic generalised forces in mode j be given by

$$\Phi_j(t) = \rho V^2 S \ell \; e_o \; e^{i\omega t} \; (\Phi'_j(\nu) + i \; \Phi''_j(\nu)) \tag{6}$$

where e_o is a non-dimensional excitation amplitude (control angle or gust velocity w/V) and $\Phi'_j(\nu)$ and $\Phi''_j(\nu)$ are the in phase and. quadrature components of the non-dimensional oscillatory aerodynamic generalised force in mode j due to harmonic excitation of the type represented by e_o.

For the case of excitation by harmonic gusts the excitation forces, represented by the matrix $\{\Phi(\nu)\} = \{\Phi'(\nu)\} + i\{\Phi''(\nu)\}$ of which $\Phi'_j(\nu)$ and $\Phi''_j(\nu)$ are typical elements, are caused by the passage over the aircraft of waves of harmonio downwash. Calculation of these forces cannot be done directly by existing lifting surface aerodynamic foroe programmes since these are written to accept only modes that are looated on the aircraft, while the gust mode convects back along it at the flight speed, This problem can be overcome by the use of two synthetic modes that the programme can accept, which when combined generate a travelling downwash wave[5]. The-forces due to the synthetic modes can then be combined to give the force In the gust mode. The method by which this can be done, and the shapes of the synthetic modes (which are funotions of frequency), are given in Appendix A of this Report (page 23).

For harmonio motion at circular frequenoy ω

$$\left.\begin{aligned} \ddot{q}_{jo}\, e^{i\omega t} &= -\omega^2\, q_{jo}\, e^{i\omega t} \\ \dot{q}_{jo}\, e^{i\omega t} &= i\,\omega\, q_{jo}\, e^{i\omega t} \end{aligned}\right\} \quad . \qquad (7)$$

The equation of motion for the harmonio response of the aircraft to harmonio excitation of amplitude e_o and circular frequenoy ω can be written

$$(-\omega^2[A] + i\omega[B] + [C])\,\{q_o(\nu)\}\, e^{i\omega t}$$
$$= \rho V^2 S\ell\, e^{i\omega t}\,([Q'(\nu)] + i\nu\,[Q''(\nu)])\,\{q_o(\nu)\} + \rho V^2 S\ell\, e_o\, e^{i\omega t}\,(\{\Phi'(\nu)\} + i\{\Phi''(\nu)\}) \qquad \ldots(8)$$

where $\{q_o(\nu)\}$ is a matrix of which a typical element is q_{jo}. It is convenient to reduce this to non-dimensional form by the substitutions

$$[a] = [A]/\rho S\ell^3 \;;\; [b] = [B]/\rho S\ell^2 V \;;\; [c] = [C]/\rho S\ell V^2 \quad . \qquad (9)$$

This gives

$$(-\nu^2[a] + i\nu[b] + [c])\,\{q_o(\nu)\}$$
$$= ([Q'(\nu)] + i\nu\,[Q''(\nu)])\,\{q_o(\nu)\} + e_o\,(\{\Phi'(\nu)\} + i\,\{\Phi''(\nu)\})$$

or

$$(-\nu^2[a] + i\nu\,[b] + [c] - [Q'(\nu)] - i\nu\,[Q''(\nu)])\,\{q_o(\nu)\}$$
$$= e_o\,(\{\Phi'(\nu)\} + i\,\{\Phi''(\nu)\}) \qquad (10)$$

and the column matrix of transfer functions representing the harmonic responses of the generalised coordinates to unit excitation of the type considered is

$$\{T_q(\nu)\} = \frac{\{q_o(\nu)\}}{e_o} = (-\nu^2\,[a] + i\nu\,[b] + [c] - [Q'(\nu)] - i\nu\,[Q''(\nu)])^{-1} \times$$
$$\times\,(\{\Phi'(\nu)\} + i\,\{\Phi''(\nu)\})\,. \qquad (11)$$

$\{T_q(\nu)\}$ is a column matrix with elements $T_{q_j}(\nu)$ which are in general complex numbers.

In addition to evaluating equation (11) for the transfer function it is prudent to calculate the latent roots of the matrix $(-\nu^2\,[a] + i\nu\,[b] + [c] - [Q'(\nu)] - i\nu\,[Q''(\nu)])$. This is to ensure that the modal system representing the aircraft is stable at the flight condition oonsidered. Instability can be detected from a vector plot of the transfer function but this requires some experience and could possibly be missed.

To obtain the dimensional response at a point r the modal displacements f_{rj} must be multiplied by the generalised coordinates q_j and a summation over all the modes taken.

$$z_r = \mathrm{Re}\,(z_{ro}\,e^{i\omega t}) = \mathrm{Re}\,(e_o\,\ell\,e^{i\omega t}\sum_{j=\ell}^{n} f_{rj}\;T_{q_j}(\nu))\,. \qquad (12)$$

If a number of points are involved the relation becomes

$$\{z_r\} = \mathrm{Re}\,(e_o\,\ell\,e^{i\omega t}\,[f]\,\{T_q(\nu)\}) \qquad (13)$$

where $[f]$ is a matrix of modal displaoements, of which f_{rj} is a typioal element. Similarly, the relation for acceleration is

$$\{\ddot{z}_r\} = \mathrm{Re}\,(-e_o\,\frac{V^2}{\ell}\,e^{i\omega t}\,\nu^2\,[f]\;\{T_q(\nu)\})\,. \qquad (14)$$

The transfer functions for other quantities of interest can be deduced from the modal transfer function matrix. These quantities might be pressures at points on a wing, forces on elements of the wing, or shear forces, bending momenta ad stresses at points in the structure.

Take for example the sum of the aerodynamic and inertia foroes on the elements of a wing. The aerodynamic forces, which could oome from theory or measurements, are given by

$$\{P_{aero}(t)\} = \rho V^2 S\ e_o\ e^{i\omega t}\ (\{\bar{p}'(\nu)\} + i\ \{\bar{p}''(\nu)\}) + \rho V^2 S\ e_o\ e^{i\omega t}\ ([p'(\nu)] + i\ [p''(\nu)]) . \{T_q(\nu)\} \quad (15)$$

where $\{\bar{p}'(\nu)\}$ and $\{\bar{p}''(\nu)\}$ are matrices of the in-phase and quadrature components of the non-dimensional osoillatory aerodynamic forces on the elements of the wing, due to harmonio exoitation, of which typical elements are $\bar{p}'_r$ and $\bar{p}''_r$; $[p'(\nu)]$ and $[p''(\nu)]$ are matrioea of the in phase end quadrature components of the non-dimensional oaoillatory aerodynamic forces on the elements of the wing due to harmonio motion in mode k, of which typical elements are p'_{rk} and p''_{rk}; r identifies the element..

The inertia foroea on the elements are given by

$$\{P_{inertia}(t)\} = -\omega^2 \frac{V^2}{\ell}\ e_o\ e^{i\omega t}\ [m]\ [f]\ \{T_q(\nu)\} \quad (16)$$

where $[m]$ is a diagonal matrix of element masses. Thus the total harmonio foroea on the elements due to the response of the aircraft to *harmonic* excitation is

$$\{P(t)\} = \rho V^2 S\ e_o\ e^{i\omega t}\ (\{\bar{p}'(\nu)\} + i\ \{\bar{p}''(\nu)\} + ([p'(\nu)] + i\ [p''(\nu)])\ \{T_q(\nu)\}) - \omega^2 \frac{V^2}{\ell}\ e_o\ e^{i\omega t}\ [m]\ [f]\ \{T_q(\nu)\} . \quad (17)$$

The only significant *difference* between caloulating the local response from the modal response (equation (13)) ad calculating the response forces from the modal response is that in the latter oaae the weighting matrix'is oomplex and frequency-dependent.

2.3 Harmonic controlled motion

Modern aircraft are often fitted with automatic flight control systems which modify the response and stability of the basic aircraft. Consider a system that moves a control surface when a response parameter at some point on the aircraft is disturbed from the value it would have in steady flight, the amount of control movement being proportional to the size of the disturbance (Fig.2).

Including the control surface in the dynamic system that represents the aircraft has two effects on the characteristics of the aircraft. Firstly, increasing the number of degrees-of-freedom used to represent the aircraft will allow new forms of response or modify the previous response characteristics. For example, control surface flutter will now be possible. Secondly, closing the feedback loop from the aircraft response to the control surface allows the response characteristics to be tailored by a suitable choice of feedback gain and phase, both of these being in general frequency-dependent.

The power control systems generally used are designed so that a signal from the control column or autopilot represents a control surface position. The power control unit jacks, which have sufficient force capability to overcome aerodynamic loads in the normal flight regime, move the control surface to the required position and then stop.

Consider initially the aircraft with a control surface held in position by a flexible Jack, but with no control movement signals applied to the jack. The equation of motion is still that given as equation (10)

$$(-\nu^2 [a] + i\nu [b] + [c] - [Q'(\nu)] - i\nu [Q''(\nu)]) \{q_o(\nu)\} = e_o (\{\Phi'(\nu)\} + i \{\Phi''(\nu)\}) \qquad (10)$$

where the degrees-of-freedom include control surface rotation against the Jack (treated as a linear spring and damper) and, if necessary, control surface elastio distortion modes.

Next consider a control system that can detect linear and angular motion at some point r on the structure ad generate a demand for a control movement η_o. Let

$$\eta_o = (C_1 + i\nu C_2) \sum_{k=1}^{n} f_{rk} q_{ko} + (C_3 + i\nu C_4) \sum_{k=1}^{n} F_{rk} q_{ko} \qquad (18)$$

where F_{rk} is the slope in mode k at point r and the gains C_1 to C_4 are real numbers that are functions of frequency. For example, in a-system that made the demand signal equal to the linear acceleration at point r and had the same gain at all frequencies C_1 would be $-\nu^2$ and C_2 to C_4 would be eero.

Equation (18) can be simplified to

$$\eta_o = \sum_{k=1}^{n} (G'_k(\nu) + i\nu\, G''_k(\nu))\, q_{ko} \qquad (19)$$

where

$$\left.\begin{aligned} G'_k(\nu) &= C_1 f_{rk} + C_3 F_{rk} \\ G''_k(\nu) &= C_2 f_{rk} + C_4 F_{rk} \end{aligned}\right\} \qquad (20)$$

This demanded oontrol angle can be superimposed on the natural osoillatory control rotation in equation (10) and the flexibility already in the equation will allow blow-back of the surface against jaok flexibility and, if the neoessary degreea-of-freedom are included, elastio distortion of the surface. The non-dimensional oscillatory aerodynamic generalised foroe in mode j due to a harmonic control surface rotation of amplitude η_o is $(Q'_{jo}(\nu) + i\nu\, Q''_{jo}(\nu))\eta_o$, where $Q'_{jo}(\nu)$ and $\nu\, Q''_{jo}(\nu)$ are the in phase and quadrature oomponents of the non-dimensional oscillatory aerodynamic generalised force in mode j due to harmonic motion of the control. The equation of motion beoomes

$$(-\nu^2 [a] + i\nu [b] + [c] - [Q'(\nu)] - i\nu [Q''(\nu)]) \{q_o(\nu)\}$$
$$= e_o (\{\Phi'(\nu)\} + i \{\Phi''(\nu)\}) + (\{Q'_c(\nu)\} + i\nu \{Q''_c(\nu)\})\, \eta_o \qquad (21)$$

where $\{Q'_c(\nu)\}$ and $\{Q''_c(\nu)\}$ are matrices of whioh typical elements are $Q'_{jo}(\nu)$ and $Q''_{jo}(\nu)$. Now substitute for η_o in terms of the response coordinates from equation (19).

$$(-\nu^2 [a] + i\nu [b] + [c] - [Q'(\nu)] - i\nu [Q''(\nu)]) \{q_o(\nu)\}$$
$$= e_o (\{\Phi'(\nu)\} + i \{\Phi''(\nu)\}) + (\{Q'_c(\nu)\} + i\nu \{Q''_c(\nu)\})(\{\overline{G'(\nu)}\} + i\nu \{\overline{G''(\nu)}\}) \{q_o(\nu)\}$$
$$\ldots (22)$$

where the elements of $\{\overline{G'(\nu)}\} + i\nu \{\overline{G''(\nu)}\}$ are $G'_k(\nu)$ and $G''_k(\nu)$. Collect terms in $\{q_o(\nu)\}$ to give

$$(-\nu^2[a] + i\nu[b] + [c] - [Q'(\nu)] - i\nu[Q''(\nu)] - (\{Q_c'(\nu)\} + i\nu\{Q_c''(\nu)\})$$
$$\times (\{\overline{G'(\nu)}\} + i\nu\{\overline{G''(\nu)}\})) \{q_o(\nu)\} = e_o (\{\Phi'(\nu)\} + i\{\Phi''(\nu)\}) \quad . \quad (23)$$

The transfer functions for the harmonic responses of the generalised coordinates representing the aircraft with an active control system to harmonic excitation are obtained from equation (23)

$$\{T_q(\nu)\} = (-\nu^2[a] + i\nu[b] + [c] - [Q'(\nu)] - i\nu[Q''(\nu)]$$
$$- (\{Q_c'(\nu)\} + i\nu\{Q_c''(\nu)\}) (\{\overline{G'(\nu)}\} + i\nu\{\overline{G''(\nu)}\}))^{-1} (\{\Phi'(\nu)\} + i\{\Phi''(\nu)\}) \quad . \quad ...(24)$$

Thus the automatic control system appears in this form of the equation of motion as an additional set of aerodynamic response forces. These will be frequenoy-dependent, as are the normal response forces. However, as well as depending on the-geometry and Mach number of the aircraft the control system forces depend on the complex gain of the system and so can be changed relatively easily to provide the desired aircraft response.

All the analysis of this and the preceding section has been based on a number of assumptions. These are that the aircraft can be represented by a linear dynamic system consisting of a finite number of degrees-of-freedom; that perturbations from the steady flight path are small, and that the flight speed is constant. If the disturbance is sufficiently large for the controls to reach their limits (automatic control systems have limited authority for safety reasons) or the aerodynamic forces can no longer be represented by linear functions of the response coordinates, then the whole set of equations representing the motion of the aircraft become non-linear and a different form of analysis is required. Indeed, the concept of the transfer function only exists for linear systems.

2.4 Transient response to step excitation

For a linear system the response to transient excitation by some process can be obtained from the harmonic response to harmonic excitation by the same process, and vice versa. . Consider a system for which the transfer function for the response to exoitation by' harmonic gusts is $\{T(\omega)\}$. Apply to the system transient exoitation'by a gust with velocity history w(t), which has a Fourier transform [3]

$$\bar{w}(\omega) = \int_{-\infty}^{\infty} w(t)\, e^{-i\omega t}\, dt \,. \tag{25}$$

Then the Fourier transform of the transient response is[6]

$$\{\bar{q}(\omega)\} = \{T(\omega)\}\ \bar{w}(\omega) \,. \tag{26}$$

$\{T(\omega)\}$ is a general transfer function matrix, not necessarily that for modal displacements. The transient response at time t is given by the inverse transform

$$\{q(t)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \{\bar{q}(\omega)\}\ e^{i\omega t}\ d\omega \,. \tag{27}$$

This is a powerful. method of calculating transient responses, in that it allows for lags of the excitation and response aerodynamic forces in the time domain if the frequency dependence of the corresponding foroes is included in the frequency domain.

A specific applioation of equation (26) is to the transient response of an aircraft entering a step gust. This is not a practical &sign case, as for these the gust always has some finite gradient length, but the response to a step gust can be considered to be the fundamental transient solution for the aircraft (at the specified flight conditions). From this fundamental solution, denoted $\{q_{step}(t)\}$, the response to a gust of velooity history w(t) ($w(t) = 0$, $t < 0$) can be found by the use of Duhamel's superposition integral[3]

$$\{q(t)\} = w(o)\ \{q_{step}(t)\} + \int_{0}^{t} \{q_{step}(t-\tau)\}\ \frac{dw}{dt}(\tau)\ d\tau \,. \tag{28}$$

Thus the response to transient excitation of step form is important. Consider a gust with a velocity history

$$w(t) = 0 \,, \quad t < 0 \,; \quad = 1 \,, \quad t > 0 \,. \tag{29}$$

The Fourier transform of this is

$$\bar{w}(\omega) = \pi\delta(\omega) + \frac{1}{i\omega} \tag{30}$$

The Dirac delta function does not pose a problem in practice, as the transfer function $\{T(\omega)\}$ must tend to zero as ω tends to zero if the transform response $\{\bar{q}(\omega)\}$ is not to become infinite at $\omega = 0$. Thus the integral

$$\int_{-\infty}^{\infty} \{T(\omega)\}\ \delta(\omega)\ d\omega \tag{31}$$

for this class of transfer function is zero and the transform of the transient response to a step gust is

$$\{\bar{q}(\omega)\} = \frac{\{T(\omega)\}}{i\omega} \quad . \tag{32}$$

For the response of aircraft to harmonic gusts it is found that in the translation modes (heave and sideslip) the transfer functions for displacement and velocity have non-zero values at zero frequency, as do those for displacement in the rotational modes (pitch, yaw and roll). The physical explanation for this is that the response *parameters* concerned do not return to their undisturbed values (zero) a long time after the aircraft has entered the step gust. The translation velocities and angular displacements settle at some finite value (for a stable aircraft) while the translation displacements continue to increase with increasing time after entering the gust.

For this reason the programmes described in this report are written in terms of the response transient accelerations. If other response parameters (velocity or displacement) are required they can be recovered by integrating the acceleration solution in the time domain. Transforms of the responses of structural loads to excitation by a step gust do exist, so that these loadings can be evaluated directly. For the case of excitation by a step elevator movement all acceleration response transforms except that for heave exist. This is because, after a step elevator movement, the aircraft pitch velocity stabilises at some finite value and the incidence increases with increasing time. The lift, and hence the heave acceleration, is proportional to this incidence. The transform of the rate of change of heaving acceleration exists and is identically equal to the transfer function of heave acceleration.

2.5 Random response

It is often desirable to calculate the statistical properties of the response of a system to excitation by a process that can only be described in statistical terms. Provided the system is linear and the excitation has the properties of ergodicity, stationarity, isotropy and homogeneity the theory is well established[6].

Let the system have a transfer function T(o) when excited by some process occurring harmonically, and let the same system be exposed to the same process occurring randomly. The distribution of the power of the exoitation process as a function of frequency is called the power spectrum and $\Phi_E(\omega)$, the value of the spectrum at a given frequency, is the power spectral density at that frequency. At a circular frequency ω the power spectral density of the response is given by

$$\Phi_R(\omega) = |T(\omega)|^2 \Phi_E(\omega) \quad . \tag{33}$$

Other statistical quantities that can be obtained from the spectral density are the root mean square value ad, if the probability density distribution of the prooess is Gaussian, the frequency of zero crossings and frequenoy with which specified response levels are exoeeded.

The rms of a process hating a power spectral density $\Phi(\omega)$ is given by the integral.

$$\sigma^2 = \int_0^\infty \Phi(\omega) \, d\omega \quad . \tag{34}$$

If the process has a Guassian probability density distribution then the frequency of zero crossings (in one direotion only) is

$$N_o = \frac{1}{2\pi\sigma} \left(\int_0^\infty \omega^2 \, \Phi(\omega) \, d\omega \right)^{1/2} \tag{35}$$

and the frequency with which a level R of the process (which has an rms σ) is exceeded is then

$$N(R) = N_o \, e^{-R^2/2\sigma^2} \quad . \tag{36}$$

3 COMPUTER PROGRAMMES

3.1 General

Seotion 2 of this Report has given the equations of motion for the response of a flexible aircraft, represented by the sum of a finite number of modes, to harmonic and step excitation. To solve these equations the following digital computer programmes have been written.

RAE 299A Harmonic response to harmonic excitation (equation (11)).

RAE 299B Harmonic response to harmonic excitation with a feedback loop connecting one control to the response (equation (24)).

RAE 299C Transient response to step excitation (equations (11), (32) and (27)).

RAE 299D Transient response to step excitation with a control feedback loop (equations (24), (32) and (27)).

These programmes are written in Mercury Autocode for use on the Ferranti Mercury and ICT Atlas computers. In addition to the response programmes that solve the equations listed above, supporting programmes have been written. These prepare data for the programmes that calculate the generalised aerodynamic forces, sort the output from these programmes into a form compatible with the input to the response programmes, transform the modal output from the response programmes into response parameters of more practical interest (accelerations of points on the structure, for example), and obtain the responses of these parameters when the aircraft is exposed to excitation either of a known transient form or of a random nature with a known power spectrum.

These latter are relatively trivial tasks, but they must be automated if the whole calculation is to be done quickly, accurately and with a minimum of effort.

3.2 Calculation chain

The response programmes described in this Report can be used on their own, but to achieve the maximum ease of calculation they should be regarded as a part of a calculation chain. This chain uses a number of programmes, some not written by the author, to calculate the aerodynamic forces on the aircraft, the modal responses and the responses of derived parameters to specified excitation. Minor programmes are used to link the major ones together. The rest of this section will describe the chain used when the oscillatory generalised forces are calculated by Davies's lifting surface theory[4] programmed as RAE 16iA. This is the chain of which the author has most experience, but it is in no way the only possible one. Aerodynamic data could come from experiments or from other programmes. The various linking programmes would need modification, but the principle of a semi-automatic chain monitored at a number of stages would remain.

Any calculation starts with the data defining the system. In this case the data required is the external geometry of the aircraft (camber, twist and relative incidences of the various lifting surfaces are not necessary).

Also needed is sufficient structural stiffness and mass data to allow the aircraft's normal vibration modes to be calculated. Since standard programmes exist for the caloulation of normal modes it will be assumed that the shapes and frequencies of these are known.

With the geometry and modal data as the starting point the calculation flow chart is shown in Fig.3. The caloulation can be broken into the following steps;

Preparation of data for the calculation of the aerodynamic forces.

Calculation of the aerodynamic forces.

Preparation of data, including the aerodynamic forces, for the response programme.

Calculation of the response in modal form.

Calculation of the responses of the derived parameters to specific excitation,

Use of programme RAE 161A is described in Ref.7. It employs a collocation point method and the number of collocation points must be chosen. A guide to this is given in Ref.8; the author commonly uses 25 or 36 points on a half-wing. The positions of the collocation points must be calculated and the slopes and displacements of the mode shapes found at these points. If control surface modes are used equivalent smooth modes should be calculated to remove disoontinuities at the control hinge-line. This can be done by another programme, RAE 213A[9].

Finding the slopes and displacements of the elastic modes at the collocation points can be done in a number of ways. A curve fitting interpolation programme can be used with both calculated and measured modes (the fit being 'exact' in the first case and 'least squares' in the latter). However, in the latter oase it is often preferable to draw out the mode shapes by hand and interpolate graphically. This makes it possible to allow for the effect of the structural layout on the mode shapes and avoids trouble with waviness of the fitted mode.

Finally the mode shapes for the synthetic gust modes must be calculated for each frequency at which RAE 161A is to be run. The programme to do this, RAE 300A, is described in Appendix A (page 23).

The data tape for RAE 161A is prepared. This consists of the geometry of the lifting surface, the Mach number and frequency parameter, and the slopes and displacements of the rigid body, elastic, control and gust modes at the collocation points. This has to be done for each Mach number and frequency parameter combination for which results are required. However, only the gust modes change with frequency so that the data tapes can be oopied from a master tape, inserting the gust mode data from the output tape of programme RAE 300A.

Output from RAE 161A is in the form of the square matrix of oscillatory aerodynamic generslised forces for one frequency, eaoh complex element $Q'_{jk} + i\nu Q''_{jk}$ being presented as the two numbers Q'_{jk} and Q''_{jk}. These matrioes are repeated for each frequency at which the forces are oaloulated.

The response programme takes in the aerodynamic excitation and response force coefficients in a different order. The order is that of increasing frequency for each coefficient in turn. Thus, if the aerodynamic forces are oaloulated for three frequencies the input to the response programme is in the order . . . . $Q''_{j(k-1)}(\nu 3)$: $Q'_{jk}(\nu 1)$, $Q'_{jk}(\nu 2)$, $Q'_{jk}(\nu 3)$; $Q''_{jk}(\nu 1)$, $Q''_{jk}(\nu 2)$, $Q''_{jk}(\nu 3)$: $Q'_{j(k+1)}(\nu 1)$, ... The reason for this order is that it makes easy the deteotion of a numerioal error, as this stands out from the smooth variation of values with frequency.

Two programmes, RAE 300B and RAE 300C, have been written to read the output from RAE 161A and sort this into the order required for the response programmes. These aocept forces on a wing and on a wing and a tailplane respectively, combine the forces due to the synthetic modes into those due to the travelling gust, combine the forces on wing and tailplane into a single set (scaling some for downwash effects, if required), ard omit males that are *not* required for the response calculation. These programmes are described in Appendix B (page 26).

Structural data for the response programmes consists, for degrees-of-freedom represented by normal modes, of diagonal matrices of generalised inertia, damping and stiffness coefficients. These, plus the aerodynamic exoitation forces (due to harmonio control movements or gusts), the aerodynamic. response foroe coefficients (which are independent of the nature of the excitation), and the complex gains of the feedback loop to the control surfaces, are the data needed for the response progrsmmes. The detailed speoifioations of the programmes are given in Appendix C (harmonio response, page 30) and Appendix D (transient response, page 35).

Output of results form the response programmes is in modal form. The harmonic response programmes output the response at the frequencies for which it has been demanded, while the transient response programmes output both the harmonio response at speoified frequencies and the transient response at the times specified by the user of the programme. These outputs can be on tape and are compatible with the input to subsequent programmes.

The final stags of the calculation is to obtain the responses of parameters of practical interest that can be derived from the modal response by a linear transformation. One programme, RAE 300D, does this for harmonic response. The elements of the transformation matrix can be complex and frequency-dependent. When the power spectrum of a random excitation process is given RAE 300D calculates the power spectra, rms and frequencies of zero crossings for the responses of the derived parameters to this excitation. This programme is described in Appendix E (page 39).

A similar programme, RAE 300E, calculates the responses of parameters that can be derived from the transient modal response, and their responses when the excitation has a particular transient form. In this case the elements of the transformation matrix cannot be functions of time. This effectively limits RAE 300E to calculating local transient accelerations. If the transient response of other parameters (structural loads, for example) are required, RAE 300D can be used to calculate the harmonic response of the parameter and the transient response found by a Fourier transformation as outlined in Section 2.4. Programme RAE 300E is described in Appendix E.

It will be seen that the chain of programmes described in this section are intended to enable the forced response of aircraft to be calculated quickly. However, the programmes are not so automatic that all understanding of the problem is lost. At each important intermediate stage the results to that stage are output for inspection. This allows, for example, aerodynamic forces to be adjusted to agree with tunnel or flight measurements, or numerical errors to be detected before they contaminate later parts of the calculation.

3.3 Details of the response programmes

Of the programmes forming the complete chain the only one written by the author that may have some intrinsic interest is that to calculate the transient response to step excitation by using equations (11), (32) and (27). This section describes the procedure which was adopted for this.

The problem is to set up the matrix equation (11) (in which all the matrices have elements that are complex and frequency-dependent), evaluate the transfer function matrix, divide the result by $i\omega$, and carry out a number of Fourier inverse transforms on this new matrix of solutions.

Equation (11) can be rewritten in terms of the real and imaginary parts of the transfer functions. This makes the equation, given below, a real algebraic one.

$$\begin{Bmatrix} Re(T_q(\nu)) \\ Im(T_q(\nu)) \end{Bmatrix} = \begin{bmatrix} -\nu^2 a + o - Q'(\nu) & -vb + \nu Q'' \\ vb - \nu Q'' & -\nu^2 a + o - Q'(v) \end{bmatrix}^{-1} \begin{Bmatrix} \Phi'(\nu) \\ \Phi''(\nu) \end{Bmatrix} \qquad (37)$$

The elements of equation (37)are still frequency-dependent.

The flow diagram for the programme is shown in Fig.4. The aerodynamic forces, which are frequenoy-dependent, are read at a small number of frequencies and fitted by simple polynomial functions of frequency. The real and imaginary parts of the transfer function matrix are then calculated at a large number of frequencies using values of the aerodynamio forces obtained from the'fitted curves. These transfer functions are output as they are being calculated. They are also stored and later are divided by the frequency parameter to give the response transform, the relation between the transfer function and response transform being

$$\begin{Bmatrix} Re(\bar{q}) \\ Im(\bar{q}) \end{Bmatrix} = \begin{Bmatrix} Im(T_q(\nu))/\nu \\ -Re(T_q(\nu))/\nu \end{Bmatrix} \qquad (38)$$

The transform is found at intermediate frequencies by linear interpolation.

The transformation is then a numerical integration carried out separately on the real and imaginary parts, This should yield the same answer from each part, the difference being a measure of the numerical and truncation errors.' The process is described in detail in Ref.10. Cheoks on the acouracy of this part of the programme suggest that for smooth functions this is about ±2%, and that for very erratic functions not more than ±12%.

4 DISCUSSION

The set of programmes described in this Report illustrates the way in whioh large and complicated programmes written by different authors can be combined into a single oaloulation chain. When this is done by means of small programmes linking the larger ones these offer the additional advantage of allowing the csloulationto be monitored at intermediate stages.

The specific set of programmes described are designed to oalculate the response of flexible airoraft to aerodynamic exoitation, but there appears to be no reason why they should not be used for other dynamio systems. In the airoraft application *the* part of the oaloulation that the transient response programmes treat more rigorously than other methods (for example, step-by-step integration of the equations) is the inolusion of unsteady aerodynamic effects in the response aerodynamic foroes.

To allow for these effects the variation of the aerodynamic foroes with frequenoy must be known. If the aerodynamic forces are calculated by a lifting surface theory this means that the lifting surface programme must be run a number of times at different frequencies. Since these prcgrammes are usually longer to run than the response programmes it is often found that the calculation of the aerodynamic forces accounts for over three-quarters of the total oomputing time in a given problem.

It might be felt that this *is a good reason for ignoring* the variation of aerodynamio forces with frequency. A number of recent oaloulations have shown that for the aircraft oonsidered (all of low aspeot ratio swept or delta planform) this variation of the response aerodynamio forces has a small effect on the aircraft response, but that the variation with frequency of the excitation forces has a large effect. Since to know the latter variation the lifting surface programme has to be run at a number of frequencies it is no more expensive to include variation of the response aerodynamic forces at the same time. This provides a safeguard against the aircraft for which the response is sensitive to variations with frequency of the response forces.

A further general result that has been shown by specific caloulations is that a very high standard of both struotural and aerodynamio data is required for the accurate calculation of the sub-critical response of the aircraft.

Finally, prcgrammes of the type described here can produce a large number of results in a short time. This is one reason for the need to automate data handling between prcgrammes. In addition, care is needed in the presentation of the final results. From this point of view there is considerable merit in the oaloulation of the rms of the random response of the aircraft to random exoitation, or the peak value of the response to a discrete gust. In these oases the final result is a single number for each response parameter. This is an advantage if variation with flight conditions or with some design variable (control system gain, for example) is being studied.

5 CONCLUSION2

A set of Mercury Autoocde programmes has been developed to calculate the harmonic and transient responses of aircraft to harmonic and step excitation by gusts and control movements. Automatic control systems can be included in the aircraft. The programmes can be applied to dynamic systems other than airoraft.

The. programmes are semi-automatio and keep the preparation of data by hand to a minimum while allowing the calculation to be monitored at intermediate stages.

The set of programmes described show how large and complicated programmes written by a number of authors can be integrated into a single caloulation ohain.

Appendix A

SYNTHETIC MODES TO REPRESENT HARMONIC GUSTS

Introduction

An array of harmonio gusts oonveoting pest the airoraft at flight speed induoes a downwash at a point x that is given by

$$w = w_0 e^{i\omega\left(t - \frac{x}{V}\right)} \qquad (A1)$$

where $\omega = 2\pi V/\lambda$, λ is the gust wavelength and w_0 is the gust velocity amplitude. On a wing the mode shape that, when oscillated at a oiroular frequenoy ω, will generate this downwash distribution is

$$\frac{z}{\ell} = -\frac{x}{\ell} e^{i\omega t}\left(\cos\frac{\omega x}{V} - i \sin\frac{\omega x}{V}\right) \qquad (A2)$$

This mode shape is complex and so cannot be used directly as input data for a lifting surface programme. However, if it is split into its real and imaginary oomponents these can be used separately as mode shapes for suoh a programme. These are

$$\left.\begin{aligned} \left(\frac{z}{\ell}\right)_{\cos} &= -\frac{x}{\ell} e^{i\omega t} \cos\frac{\omega x}{V} \\ \left(\frac{z}{\ell}\right)_{\sin} &= -\frac{x}{\ell} e^{i\omega t} \sin\frac{\omega x}{V} \end{aligned}\right\} \qquad (A3)$$

The non-dimensional osoillatory aerodynamio generalised forces due to these modes are $\{Q'_{\cos}(\nu) + i\nu Q''_{\cos}(\nu)\}$ and $\{Q'_{\sin}(\nu) + i\nu Q''_{\sin}(\nu)\}$. The generelised forces due to the array of harmonio gusts oan be obtained from these by combining them as

$$\{\Phi'(\nu)\} + i\{\Phi''(\nu)\} = \{Q'_{\cos}(\nu) + \nu Q''_{\sin}(\nu)\} + i\{\nu Q''_{\cos}(\nu) - Q'_{\sin}(\nu)\} . \qquad (A4)$$

In order to use the lifting surface programme RAE 161A to oaloulate the generalised foroes in the modes of a lifting surfaoe passing through harmonic gusts the programme must be prodded with the modal slope and displacement data for the modes of the surfaoe. In addition the slopes and displacements at the downwash points in the two synthetic gust modes must be included, but the displacements at the loading points may be entered as zeros. The reason for this is that the generalised forces required are those in the surface modes due to the synthetic modes, and not vioe versa.

A programme, RAE 300A GUST MWE SHAPES, has been written to calculate the slopes and displacements at the downwash points for the modes

$$\left.\begin{aligned} \frac{z}{\ell} &= -\frac{x}{\ell}\cos\frac{\nu x}{\ell} \\ \frac{z}{\ell} &= -\frac{x}{\ell}\sin\frac{\nu x}{\ell} \end{aligned}\right\} \qquad (A5)$$

which are the same as those of equation (A3), but written in terms of the frequenoy parameter ν, and with the tern $e^{i\omega t}$ omitted. The output from this programme is suitable for use direotly as part of the data tape for programme RAE 161A.

Programme specification

Title RAE 300A GUST MWE SHAPES

Purpose

To caloulate the modal slopes and displacements at the collocation downwash points for the modes

$$\frac{z}{\ell} = -\frac{x}{\ell}\cos\frac{\nu x}{\ell}$$

$$\frac{z}{\ell} = -\frac{x}{\ell}\sin\frac{\nu x}{\ell}$$

Input

k number of frequencies at which calculation is to be performed

m number of spanwise stations

n number of ohordwise points

$\nu_1, \nu_2, \ldots \nu_k$ values of the frequency parameter at which the oaloulation is to be performed

x/ℓ (1,1), x/ℓ (1,2), ... x/ℓ (1,n)
x/ℓ (2,1), x/ℓ (2,2), ... x/ℓ (2,n)
......
x/ℓ (m,1), x/ℓ (m,2), ..., x/ℓ (m,n)

coordinates of the downwash points. x/ℓ (r,s) is a number equal to the value of x/ℓ at spanwise station r and ohordwise location s.

output

ν_1 frequency parameter
an n × m matrix of modal slopes in the cosine mode
an n × m matrix of modal deflections in the cosine mode
an n × m matrix of modal slopes in the sine mode
an n × m matrix of modal deflections in the sine mode

This output is repeated for eaoh of the k values of the frequency parameter in turn. .

Notes

(1) Limitations $k \leq 20$; $mn \leq 90$.

(2) The programme will read data for a further case (starting with k, m, n) on completion of one oaloulation.

Appendix B

AERODYNAMIC DATA PREPARATION PROGRAMMES

Introduction

The lifting surface programmes are expensive, in terns of computing time, to use. Most of the running time is spent in forming the influence coefficients and so the run time is virtually independent of the number of modes for which generalised forces are calculated. Thus it is efficient to run a lifting surface programme once for one vehicle/flight condition combination, including all the modes that are ever likely to be needed. Then for any particular response calculation the forces for the modes concerned can be selected.

The programmes described here do this selection, combine the generalised foroes due to the two synthetio modes into a single generalised force due to harmonic gusts, ad sort the generalised forces calculated at a number of values of the frequency parameter into the order needed for input to the response programmes. Two versions of the data preparation programmes exist. One of these, RAE 300B, handles the forces on one lifting surface only. The other, RAE 300C, accepts forces due to two separate surfaces (wing and tail) and combines them into a single set of foroes, scaling those from one surface if this is speoified.

If these programmes are to be used the order in whioh modes are speoified in RAE 161A must be:-

Rigid modes; heave, pitch or slip, yaw, roll.
Flexible modes.
Control rotation modes.
Synthetic cosine mode.
Synthetic sine mode.

Programme specification

(i) Programme for one wing only.

Title RAE 300B AERODYNAMIC DATA PREPARATION

Purpose

To rearrange the generalised forces on one wing caloulated by RAE 161A for a number of frequencies into the order required for input to the RAE 299 series of response programmes.

Input

k number of frequencies for which RAE 161 has been run

i number of rigid plus elastio modes for which RAE 161A has been run

j number of oontrol rotation modes for which RAE 161A has been run

n number of modes required for response programme

p number of controls used as exoitation in the response programme

$\nu_1, \nu_2, \ldots \nu_k$ values of the frequenoy parameter for which RAE 161A has been run

Output from RAE 161A (omitting geometric data, Mach number and frequency parameter) in the order

$$Q'_{11}(\nu_1) \qquad Q''_{11}(\nu_1)$$
$$Q'_{12}(\nu_1) \qquad Q''_{12}(\nu_1)$$
$$\ldots\ldots \qquad \ldots\ldots$$
$$Q'_{sinsin}(\nu_1) \qquad Q''_{sinsin}(\nu_1)$$
$$Q'_{11}(\nu_2) \qquad Q''_{11}(\nu_2)$$
$$\ldots\ldots \qquad \ldots\ldots$$
$$\ldots\ldots \qquad \ldots\ldots$$
$$Q'_{sinsin}(\nu_k) \qquad Q''_{sinsin}(\nu_k).$$

Modes must be in the order rigid, elastio, oontrol, cosine and sine modes.

output

The gust aerodynamio forces are printed first, in the order

$$\Phi'_1(\nu_1), \quad \Phi'_1(\nu_2), \quad \ldots, \quad \Phi'_1(\nu_k); \quad \Phi''_1(\nu_1); \quad \Phi''_1(\nu_2), \quad \ldots, \quad \Phi''_1(\nu_k)$$
$$\Phi'_2(\nu_1), \quad \Phi'_2(\nu_2), \quad \ldots, \quad \Phi'_2(\nu_k); \quad \Phi''_2(\nu_1), \quad \Phi''_2(\nu_2), \quad \ldots, \quad \Phi''_2(\nu_k)$$
$$\ldots\ldots$$
$$\Phi'_n(\nu_1), \quad \Phi'_n(\nu_2), \quad \ldots, \quad \Phi'_n(\nu_k); \quad \Phi''_n(\nu_1), \quad \Phi''_n(\nu_2), \quad \ldots, \quad \Phi''_n(\nu_k).$$

These are followed by the control surface exoitation forces, in the order above, for p sets of oontrols. Finally, the response aerodynamic forces are printed in the order

$$Q'_{11}(\nu_1), Q'_{11}(\nu_2), \ldots, Q'_{11}(\nu_k); \quad Q''_{11}(\nu_1), Q''_{11}(\nu_2), \ldots, Q''_{11}(\nu_k)$$

$$Q'_{12}(\nu_1), Q'_{12}(\nu_2), \ldots, Q'_{12}(\nu_k); \quad Q''_{12}(\nu_1), Q''_{12}(\nu_2), \ldots, Q''_{12}(\nu_k)$$

....

$$Q'_{nn}(\nu_1), Q'_{nn}(\nu_2), \ldots, Q'_{nn}(\nu_k); \quad Q''_{nn}(\nu_1), Q''_{nn}(\nu_2), \ldots, Q''_{nn}(\nu_k)$$

Notes

(I) Limitations $i \leqslant 50$; $k \leqslant 20$; $i^2k < 10000$

(2) When n modes are seleoted for output these will be the first n in the order of the input (i.e. rigid, elastio, control). Thus with the present programme it is not possible to obtain the aerodynamic forces for a control surface that is included in the response freedoms without obtaining the forces for all the rigid and elastic modes as well. However, once the coefficients Q_{jk} for all frequencies have been grouped into a single blook on the output tape any subsequent editing of this tape is not a time oonsuming process.

(ii) Programme for two lifting surfaces.

Title RAE 300C AERODYNAMIC DATA PREPARATION

Purpose

To oombine the generalised forces on two lifting surfaces, calculated separately by RAE 161A at a number of frequencies, and to rearrange these in the order required for input to the RAE 299 series of response programmes. The forces for the second surface can be scaled, so that the total is typically

$$Q'_{jk}(\nu)_{total} = Q'_{jk}(\nu)_{\substack{wing\\one}} + F_k\, Q'_{jk}(\nu)_{\substack{wing\\two}}$$

where the scaling factor F_k depends on the mode in which motion is occurring.

Input

k number of frequencies for which RAE 161A has been run

i number of rigid plus elastic modes for which RAE 161A has been run

j number of control rotation modes for wing one j' number of oontrol rotation modes for wing two

n number of modes required for the response programme

p number of oontrols on wing one used as exoitation in the response programme

p' number of oontrols on wing two used as excitation in the response programme

$F_1, F_2, \ldots F_{(n+p')}$ scale factors

$\nu_1, \nu_2, \ldots \nu_k$ values of the frequenoy parameter for which RAE 161 A has been run.

Output from RAE 161A (omitting geometric data, Mach number and frequenoy parameter) for wing one in the order given in the specification of programme RAE 300B.

similar output from RAE 161A for wing two.

Modes must be in the order rigid, elastic, control, cosine and sine.

output

As for programme RAE 300B.

Notes

. . As for programme RAE 300B, exoept that $i^2k < 5000$.

Appendix C

PROGRAMMES TO CALCULATE THE RESPONSE OF AIRCRAFT TO HARMONIC EXCITATION

Introduction

Once the aerodynamic and struck-al data are available for the aircraft its response can be calculated. The programmes specified in this Appendix read the structural data (which is independent of frequency) and the frequency dependent aerodynamic data at a relatively small number of frequencies. If the aircraft has an automatic control system then the system coefficients are read at the same values of the frequency parameter as the aerodynamic data. The programmes then set up and evaluate equation (11) (for the aircraft with controls fixed) or equation (24) (for the aircraft with an automatic control system). This is done for a number of frequencies, at which the aerodynamic and system ooeffioients are evaluated by polynomial interpolation.

Programme specifications

(i) Controls fixed.

Title RAF 299A AIRCRAFT TRANSFER FUNCTION

Purpose

This programme sets up and solves equation (11) for selected frequencies.

Input

n number of modes

k number of frequencies for the aerodynamic data

ℓ number of frequencies for which response is required

$\nu_1, \nu_2, \ldots, \nu_k$ values of the frequency parameter at which aerodynamic data is Given

$\mu_1, \mu_2, \ldots, \mu_\ell$ values of the frequency parameter at which response is required.

Structural generalised mass, damping and stiffness data is then input in the order

$$
\begin{array}{lll}
a_{11} & b_{11} & c_{11} \\
a_{12} & b_{12} & c_{12} \\
\cdots\cdots & & \\
a_{1n} & b_{1n} & c_{1n} \\
a_{21} & b_{21} & c_{21} \\
\cdots\cdots & & \\
a_{nn} & b_{nn} & c_{nn}
\end{array}
$$

Next are the excitation generalised forces (either due to gusts or controls) in the order

$$
\begin{array}{l}
\Phi_1'(\nu_1)\ ,\ \Phi_1'(\nu_2)\ ,\ \ldots\ ,\ \Phi_1'(\nu_k)\ ;\quad \Phi_1''(\nu_1)\ ,\ \Phi_1''(\nu_2)\ ,\ \ldots\ ,\ \Phi_1''(\nu_k) \\
\Phi_2'(\nu_1)\ ,\ \Phi_2'(\nu_2)\ ,\ \ldots\ ,\ \Phi_2'(\nu_k)\ ;\quad \Phi_2''(\nu_1)\ ,\ \Phi_2''(\nu_2)\ ,\ \ldots\ ,\ \Phi_2''(\nu_k) \\
\cdots\cdots \\
\Phi_n'(\nu_1)\ ,\ \Phi_n'(\nu_2)\ ,\ \ldots\ ,\ \Phi_n'(\nu_k)\ ;\quad \Phi_n''(\nu_1)\ ,\ \Phi_n''(\nu_2)\ ,\ \ldots\ ,\ \Phi_n''(\nu_k)
\end{array}
$$

Finally the response aerodynamic generalised forces in the order

$$
\begin{array}{l}
Q_{11}'(\nu_1)\ ,\ Q_{11}'(\nu_2)\ ,\ \ldots\ ,\ Q_{11}'(\nu_k)\ ;\ Q_{11}''(\nu_1)\ ,\ Q_{11}''(\nu_2)\ ,\ \ldots\ ,\ Q_{11}''(\nu_k) \\
Q_{12}'(\nu_1)\ ,\ Q_{12}'(\nu_2)\ ,\ \ldots\ ,\ Q_{12}'(\nu_k)\ ;\ Q_{12}''(\nu_1)\ ,\ Q_{12}''(\nu_2)\ ,\ \ldots\ ,\ Q_{12}''(\nu_k) \\
\cdots\cdots\cdot \\
Q_{nn}'(\nu_1)\ ,\ Q_{nn}'(\nu_2)\ ,\ \ldots\ ,\ Q_{nn}'(\nu_k)\ ;\ Q_{nn}''(\nu_1)\ ,\ Q_{nn}''(\nu_2)\ ,\ \ldots\ ,\ Q_{nn}''(\nu_k)
\end{array}
$$

It mill be noticed that the order in which the aerodynamic generalised forces are input to RAE 299A is the same as that in which they are output from RAE 300B and RAE 300C.

<u>output</u>

Output is on two channels; on Mercury these are 1 and 2, on Atlas 0 and 1.

<u>Channel 1 (Mercury) or 0 (Atlas)</u>

This consists of, for each response frequency in turn, the value of the frequency parameter, the real and imaginary parts of the response accelerations, and the moduli of the displacements and accelerations in the response freedoms. Thus for the first frequency is printed

$$\begin{array}{llll} \mu_1 & & & \\ Re(\ddot{q}_1) & Im(\ddot{q}_1) & |q_1| & |\ddot{q}_1| \\ Re(\ddot{q}_2) & Im(\ddot{q}_2) & |q_2| & |\ddot{q}_2| \\ \cdots\cdots & & & \\ Re(\ddot{q}_n) & Im(\ddot{q}_n) & |q_n| & |\ddot{q}_n| \end{array}$$

and this output is repeated for each frequency in turn.

Channel 2 (Mercury) or 1 .(Atlas

This consists of the real and imaginary parts of the displacements of the response freedoms. For the first frequenoy'this is

$$\begin{array}{ll} \mu_1 & \\ Re(q_1) & Im(q_1) \\ Re(q_2) & Im(q_2) \\ \cdots\cdots & \\ Re(q_n) & Im(q_n) \end{array}$$

and this is repeated for each frequency in turn. The output tape from this channel is used as input to programmes RAE 300D and RAE 300E.

Notes

(I) Limitations $k \geqslant 2$, $n \leqslant 8$ (Mercury) or 20 (Atlas)

(2) To avoid spurious results $\nu_k \geqslant \mu_\ell$

(3) The programme will read data for further oases, one at a time, on oompletion of the oalculation on the first oase. This data starts with n, k, ℓ. There is no limit to the number of oases that can be run oonseoutively.

(4) The Mercury Autocode oontains an optional print faoility known as query printing. If this is seleoted when the programme is run this results in the output on channel 1 (Mercury) of the aerodynamio generalised force

coefficients at each response frequency in addition to the output specified above.

(5) Programme definition (for Atlas). The title by whichthe programme is called in is RAE 299A.

For n = 8, k = 8, ℓ = 50 the store required is 25 blockswith a computing time equivalent to 5000 instruction interrupts.

(ii) With automatic flight control system.

Title RAE 299B AIRCRAFT TRANSFER FUNCTION WITH CONTROL MOVEMENTS

Purpose

This programme solves equation (24) for selected frequencies.

Input

Frequency, structural and aerodynamic data is as for programme RAE 299A. Control system gain data follows the response aerodynamic data in the order

$$G_1'(\nu_1)\ ,\ G_1'(\nu_2)\ ,\ \dots\ ,\ G_1'(\nu_k)$$
$$G_1''(\nu_1)\ ,\ G_1''(\nu_2)\ ,\ \dots\ ,\ G_1''(\nu_k)$$
$$G_2'(\nu_1)\ ,\ G_2'(\nu_2)\ ,\ \dots\ ,\ G_2'(\nu_k)$$
$$\dots\dots$$
$$\dots\dots$$
$$G_n''(\nu_1)\ ,\ G_n''(\nu_2)\ ,\ \dots\ ,\ G_n''(\nu_k)$$

output

This is identical with that from RAE 299A.

Notes

(1) Limitations are as for RAE 299A

(2) The programme can only treat one active control system. Rotation of this control surface must be inserted as the last of the n response modes in the calculation.

(3) The programme will read data for further cases following completion of the first case.

(4) Programme definition (for Atlas). The title by whioh the programme is called in is RAE 2VYB.

Storage and computing time requirements are as for RAE 299A.

Appendix D

PROGRAMMES TO CALCULATE THE RESPONSE OF AIRCRAFT TO EXCITATION BY A STEP GUST CR STEP CONTROL MOVEMENT

Introduction

In Seotion 2.4 of this Report is described a method by whioh the response of an aircraft to excitation by a step gust or oontrol movement can be caloulated onoe the transfer function giving the harmonio response of the aircraft to exoitation by harmonio gusts or harmonic control movements is known. The programmes desoribed in this Appendix calculate the response of the aircraft (with oontrol surfaces either fixed or moved by an automatic oontrol system) to harmonio excitation in exaotly the same wsy as those described in Appendix C. They then go on to use the relations given in equations (32) and. (27) to caloulate the response of the airoraft to step exoitation. The results of the transformations of both the real and imaginary parts of the transfer funotion are given separately, together with their average. Laok of agreement between the transforms of the real and imaginary parts indicates lack of reliability of the results, This can be due to the transfer motion not being caloulated at enough frequencies to describe the system adequately. If the aircraft is unstable either as a rigid body or as a flexible structure then the results will be completely spurious.

Programme specification

(i) Controls fixed.

Title RAE 299C TRANSIENT RESPONSE TO STEP EXCITATION

Purpose

To set-up and solve equation (11) for the harmonio response of the aircraft to harmonic excitation, and then to use equations (32) and (27) to calculate its transient response to step excitation.

Input

- n number of modes
- k number of frequencies for the aerodynamic data
- ℓ number of frequencies for which the response is required
- m number of times for whioh the transient response is required
- g integration parameter (see notes)

$\nu_1, \nu_2, \ldots \nu_k$ values of the frequency parameter at which aerodynamic data is given

$\mu_1, \mu_2, \ldots, \mu_\ell$ values of the frequency parameter at which harmonic response is required

$s_1, s_2, \ldots, s_m$ values of the distance parameter ($s = Vt/\ell$) at which transient response is required.

Input of structural and-aerodynamic data is as for programme RAE 299A.

Output

Channel 1 (Mercury) or 0 (Atlas)

The first part of the output on this channel is identical to that from programme RAE 299A. After the output of the harmonic response at frequency μ_ℓ is printed the transient response at each value of the distance parameter in turn. This takes the form

$$
\begin{array}{lll}
s_1 & & \\
\ddot{q}_{1r} & \ddot{q}_{1i} & \frac{1}{2}(\ddot{q}_{1r} + \ddot{q}_{1i}) \\
\ddot{q}_{2r} & \ddot{q}_{2i} & \frac{1}{2}(\ddot{q}_{2r} + \ddot{q}_{2i}) \\
\ldots\ldots & & \\
\ddot{q}_{nr} & \ddot{q}_{ni} & \frac{1}{2}(\ddot{q}_{nr} + \ddot{q}_{ni})
\end{array}
$$

where $\ddot{q}_{nr}$ is the transient acceleration in mode n obtained by the transformation of the real part of the transformed response, and $\ddot{q}_{ni}$ is that from the imaginary part.

This output is repeated for each specified value of the distance parameter in turn.

Channel 2 (Mercury) or 1 (Atlas)

Output on this channel is identical with that from RAE 299A.

Notes

(1) Limitations $k \geq 2$, $\nu_1 < \pi/(50g)$, $\nu_1 < \pi/(8\, s_m)$.
Mercury: $n \leq 8$, $n\ell \leq 150$
Atlas: $n \leq 20$, $n\ell \leq 2000$

g is the integration parameter which sets the integration step length and limits. It should be chosen to satisfy the limitation given in notes 1 and 2.

(2) To avoid spurious results the following limitations should be observed. If the highest frequenoy parameter at which the transfer function is varying rapidly (say that for resonanoa in the highest frequency mode) is ν_o, then

$$\nu_o < 12/g \ , \qquad \nu_o < 75/s_m \ .$$

$$\nu_k \geq \mu_\ell \ .$$

These limitations arise from two causes. The first two ensure that the transformation integration is oarried to a frequency beyond the highest at whioh significant information exists. The last reoognises that the polynomial interpolation method used to evaluate the aerodynamic data at intermediate frequenoies will give wildly inaoourate results if it is used to extrapolate the data to frequencies beyond the highest for whioh the data is provided.

(3) For the transformation integration the transfer funotion is determined by linear interpolation at frequencies between those at which it is calculated. Tine frequencies for whioh the response is calculated must be chosen to give a good desoription of the transfer funotion. Thus response frequencies should be close together in regions where the transfer function is expeoted to vary rapidly, and may be more widely spaoed where little change with frequency is expected.

(4) The programme will read data for further oases, starting with n, k, ℓ, m, g, on oompletion of the first oase. There is no limit to the number of oases that can be run oonseoutively.

(5) Seleotion of query printing (see Appendix C, note 4) results in the output on ohannel 1 (Mercury) of the aerodynamic generalised foroe coefficients at each response frequenoy in addition to the output specified above.

(6) Programme definition (for Atlas). The title by which the programme is oalled in is RAE 299C.

Forn = 7, k = 9, ℓ = 50, m = 30 the storage required is 24 blooks and the oomputing time is equivalent to 7000 instruction interrupts.

(ii) With automatio flight oontrol system.

Title RAE299DTRANSIENT 'RESPONSE TO STEP EXCITATION WITH CONTROL MOVEMENT

Purpose

This programme sets up and solves equation (24) for the harmonic response to harmonic excitation, and then uses equations (32) and (27) to calculate the transient response to step excitation.

Input

Frequency, distance, structural and aerodynamic data is as for RAE 2VVC. Control system gain data follows the response aerodynamic data in the order given for RAE 299B.

output

This is identical to that from RAE 299C.

Notes

(I) Limitations are as for RAE 299C.

(2) The programme can only treat one active control system. Rotation of this control surface must be inserted as the last of the n response modes in the calculation.

(3) The programme will read data for further cases following completion of the first case.

(4) Programme definition (for Atlas). The title by which the programme is called is RAE 299D.

Storage and computing time requirements are as for RAE 299D.

Appendix E

SUPPLEMENTARY PROGRAMMES

Introduction

The programmes desoribed in Appendices C and D caloulate the modal responses of airoraft exposed to excitation of unit amplitude harmonio or step form. However, for design purposes the parameters of interest are likely to be ones that can be derived from the modal responses. Examples are the displacements, velooities and aooelerations at points on the struoture; pressures at points on the wing; stresses or loads in the structure. With. the assumptions of this analysis all these quantities are linear funotions of the modal responses, and for the oase of harmonio motion are given by the general expression

$$\{T_r(\nu)\} = b\,[H'(\nu) + i\nu\, H''(\nu)]\,\{T_q(\nu)\} \qquad (E1)$$

where $\{T_r(\nu)\}$ is the transfer function of the derived responses, $\{T_q(\nu)\}$ is the transfer function for the modal displaoements, [H(v)] is a transformation matrix, the elements of which may be complex and frequency dependent, and b is a soaling faotor.

In the case of the indioial responses of the derived parameters (that is, the responses to transient excitation by a prooess having a history of unit step form) the equation oonneoting the modal and derived responses can be written

$$\{r(s)\} = b\,[H]\,\{\ddot{q}(s)\} \qquad (E2)$$

where $\{r(s)\}$ is the matrix of derived responses, $\{\ddot{q}(s)\}$ is the matrix of modal accelerations, [H] is a transformation matrix and b is a scale faotor. s is a measure of time and oan be either dimensional or take the non-dimensional form $s = Vt/\ell$. When the equation is written in the form of (E2) the elements of [H] cannot be time dependent.

For some derived parameters (any involving aerodynamic pressures or forces, for example) the elements of [H] are time dependent. In this case the transfer function of the derived parameter can be found using equation (El) and the indioial response evaluated by the method described in Seotion 2.4.

Once the transfer function for some derived parameter is known the statistical characteristics of the random response of that parameter to random excitation can be calculated, as was shown in Section 2.5. Similarly, if the aircraft is exposed to some general transient excitation e(t) having e(t) = 0, t ≤ 0, then the response history of the derived parameters is given by Duhamel's superposition integral

$$r_e(t) = \int_0^t r(t - \tau) \quad e(\tau) \quad d\tau \tag{E3}$$

where $r_e(t)$ is the response to excitation e(t).

To obtain the derived responses is thus straightforward mathematically but tedious in practice, and has been automated. For this purpose two further programmes have been written. These are:

(i) RAE 300D DERIVED CONTINUOUS RESPONSE. This calculates the transfer functions for derived parameters using the output from the RAE 299 programmes as input data. The transformation matrix [H] can be complex and frequency dependent. If a description of an exoitation power spectrum is given the programme oaloulates the spectral density, rms and frequency of zero orossings for the derived responses that result from exposure of the aircraft to excitation by a stationary random prooess having the given power spectrum and a Gaussian probability density distribution,

(ii) RAE 300E DERIVED TRANSIENT RESPONSE. This oaloulates the indicial response of the derived parameters using the output from programmes RAE 2VYC and D as input data. The elements of the transformation matrix [H] cannot be time dependent. If a history of a transient exoitation prooess e(t) is given the programme calculates the transient derived responses to this exoitation at speoified times.

Programme specifications

Title RAE 300D DERIVED CONTINUOUS RESPONSE

Purpose

To evaluate equations (El), (33), (34) and (3.5) using the output from the RAE 299 programmes as input data.

Input

n number of modes used. in modal response programme

m number of derived response parameters

j number of values of the frequency (c/s) at which the excitation power spectrum is defined. j can be 0; otherwise $j \geq 2$

k number of values of the frequency parameter at which the transformation matrix $[H'(\nu) + i\nu H''(\nu)']$ is defined. k can be 0; otherwise $k \geq 2$

a scale factor for frequency. $f = a\nu$ o/a, $a = V/(2\pi\ell)$

b scale factor for response

If $j \neq 0$, the excitation power spectral density is given next as a function of the frequency (c/s).

$$f_1 \quad \Phi(f_1)$$
$$f_2 \quad \Phi(f_2)$$
$$\cdots\cdots$$
$$f_j \quad \Phi(f_j)$$

If $j = 0$ this data is omitted.

Next is given the values of the frequency parameter at which the transformation matrix is defined.

$$\nu_1, \nu_2, \ldots \nu_k$$

if $k = 0$ this data is omitted.

This is followed by the elements of the transformation matrix, in the order

$H'_{11}(\nu_1)$, $H'_{11}(\nu_2)$, ... , $H'_{11}(\nu_k)$; $H''_{11}(\nu_1)$, $H''_{11}(\nu_2)$, ... , $H''_{11}(\nu_k)$.

$H'_{12}(\nu_1)$, $H'_{12}(\nu_2)$, ... , $H'_{12}(\nu_k)$; $H''_{12}(\nu_1)$, $H''_{12}(\nu_2)$, ... , $H''_{12}(\nu_k)$.

....

$H'_{1n}(\nu_1)$, $H'_{1n}(\nu_2)$, ... , $H'_{1n}(\nu_k)$; $H''_{1n}(\nu_1)$, $H''_{1n}(\nu_2)$, ... , $H''_{1n}(\nu_k)$.

$H'_{21}(\nu_1)$, $H'_{21}(\nu_2)$, ... , $H'_{21}(\nu_k)$; $H''_{21}(\nu_1)$, $H''_{21}(\nu_2)$, ... , $H''_{21}(\nu_k)$.

....

....

$H'_{mn}(\nu_1)$, $H'_{mn}(\nu_2)$, ... , $H'_{mn}(\nu_k)$; $H''_{mn}(\nu_1)$, $H''_{mn}(\nu_2)$, ... , $H''_{mn}(\nu_k)$.

If k = 0 the above data is replaced by the following

H_{11}, H_{12}, ..., H_{1n}.

H_{21}, H_{22}, ..., H_{2n}.

....

H_{m1}, H_{m2}, ..., H_{mn}.

The modal displacement transfer function is then red in, directly from the channel 2 (Mercury) or channel 1 -(Atlas) output from the RAE 299 programmes. This is in the order

μ_1 — *frequency* parameter for transfer function real and imaginary parts of the modal transfer function

$Re(q_1)$ $Im(q_1)$

$Re(q_2)$ $Im(q_2)$

......

$Re(q_n)$ $Im(q_n)$

This is repeated for each response frequency parameter in turn. The order must be that of ascending frequency. The data must be terminated by the character * . This will cause the programme to print the caption

rms NO

followed by the values of the rms and frequency of zero crossings (o/s) for each of the derived parameters in turn. The programme then reads data for a new case, starting with n, m, j, k, . . .

<u>output</u>

The output takes the following form. First are printed. the frequency (c/s), real ad imaginary part of the transfer function of the derived parameters in turn, the modulus of the transfer function and the spectral density. These are in the order .

f_1

$Re(T_{r1}(f_1))$ $Im(T_{r1}(f_1))$ $|T_{r1}(f_1)|$ $\Phi_{r1r1}(f_1)$

$Re(T_{r2}(f_1))$ $Im(T_{r2}(f_1))$ $|T_{r2}(f_1)|$ $\Phi_{r2r2}(f_1)$

....

$Re(T_{rm}(f_1))$ $Im(T_{rm}(f_1))$ $|T_{rm}(f_1)|$ $\Phi_{rmrm}(f_1)$

If no excitation spectrum is given the last column is omitted.

This output is repeated for each frequency in turn until the oharaoter * is read, when values of the rms and No of the derived parameters is output.

rms	No
σ_{r1}	No_{r1}
σ_{r2}	No_{r2}
....	
σ_{rm}	No_{rm}

Notes

(I) Limitations $n \leqslant 20$, m 6 20, $nm \leqslant 179$, $j \leqslant 10$, k 6 10.

(2) The programme will read data for further oases, one at a time, starting with n, m, j, k. This data is oalled in by the oharaoter * .

(3) The excitation power speotrum is defined dimensionally as this then applies to a number of different aircraft having different values of the typical length ℓ. In the programme the spectrum is fitted by a k^{th} order polynomial in logarithmio ooordinates. Selection of query prints (see Appendix C, note 4) results in the printing of the value of the dimensional power spectral density following the frequency.

Title RAE 300E DERIVED TRANSIENT RESPONSE

Purpose

To evaluate equations (E2) and (E3) using the output from programmes RAE 299C and D as input data.

Input

n number of modes used in the modal response programme

m number of derived response parameters

a scale factor for time. $t = a s$, $a = \ell/V$

b scale faotor for response

The transformation matrix is given in the order

$$H_{11}, \ H_{12}, \ \ldots, \ H_{1n}$$
$$H_{21}, \ H_{22}, \ \ldots, \ H_{2n}$$
....
$$H_{m1}, \ H_{m2}, \ \ldots \ H_{mn}$$

The modal indicial response is then read in, directly from the channel 1 (Mercury) or channel 0 (Atlas) output from RAE 299C or D. This is in the order

$$s_1$$
$$\ddot{q}_{1r} \quad \ddot{q}_{1i} \quad \tfrac{1}{2}(\ddot{q}_{1r} + \ddot{q}_{1i})$$
$$\ddot{q}_{2r} \quad \ddot{q}_{2i} \quad \tfrac{1}{2}(\ddot{q}_{2r} + \ddot{q}_{2i})$$
....
$$\ddot{q}_{nr} \quad \ddot{q}_{ni} \quad \tfrac{1}{2}(\ddot{q}_{nr} + \ddot{q}_{ni})$$

This is repeated for each specified value of s in turn, in order of increasing s, for the f values of s at which response data is read in.

The modal indicial response data must terminate with the character * if details of some specific transient excitation follows, or the character ‡ if a new case is to be read.

If the character * has ended the indicial data the details of the specific transient excitation history must be provided. These take the form

- j number of times at which the transient excitation history is defined-.
- k an integer (see below)
- o the integration step length (in seconds) for the evaluation of Duhamel's integral

If k = 0 the excitation is defined by the slope of the 'gust velocity profile at specified times. In this case the input is of the form

t_1 $dw/dt(t_1)$

t_2 $dw/dt(t_2)$

....

t_j $dw/dt(t_j)$

t is in seconds and dw/dt in the appropriate acceleration units. dw/dt at intermediate times is determined by linear interpolation

If $k = 1$ the excitation is speoified by the gust velocity at specified times. The input is then

t_1	$w(t_1)$	The gust velooity a times intermediate between t_p and t_{p+1} is found by polynomial interpolation. A polynomial is fitted through $w(t_{p-1})$, $w(t_p)$, $w(t_{p+1})$, and $w(t_{p+2})$. The slope $dw/dt(t)$ is the slope of the polynomial at t.
t_2	$w(t_2)$	
....		
t_j	$w(t_j)$	

Finally times are read at which the transient response to the specified disorete gust is to be calculated.

t_{r1}, t_{r2}, . . . These times in seconds.

Output

The output takes the following form. First are printed the responses to a step gust of the m derived response parameters at eaoh time for which the modal transient responses are given.

$t(s = s_1)$	If there are five or fewer derived response parameters these are printed in a row instead of a column.
$r_1(s_1)$	
$r_2(s_1)$	
......	
$r_m(s_1)$	

$t(s = s_2)$

$r_1(s_2)$

......

......

......

$r_m(s_f)$

Following the responses to a step gust are printed the responses to the specified gust, at the speoified times.

t_{r1}

$r_1(t_{r1})$

$r_2(t_{r1})$

....

$r_m(t_{r1})$

This output is repeated for each response time tr specified.

Notes

(1) Limitations $n \leqslant 20$, $m \leqslant 10$, $f \leqslant 30$, $mf \leqslant 300$, $nm \leqslant 100$.

(2) The programme will read data for further specific gusts starting with J, k, c, . . . This data is called in by the character * . The character ‡ calls in data for a new case, beginning n, m, a, b.

(3) If the modal response is zero at $s = s_f$ then calculation of the response to a specified gust may be performed for times greater than the maximum for which indicial response data has been provided.

SYMBOLS (Main text only)

[al	non-dimensional generalised inertia matrix
$[A]$	generalised inertia matrix
$[b]$	non-dimensional generalised damping matrix
$[B]$	generalised damping matrix
[o]	non-dimensional generalised stiffness matrix
$[C]$	generalised stiffness matrix
e_o	amplitude of harmonio excitation parameter
$[f]$	modal displaoement matrix
$[F]$	modal slope matrix
$\{G' + i\,G''\}$	oontrol system gains
i	$\sqrt{-1}$
ℓ	referenoe length
$[m]$	mass distribution matrix
n	number of degrees-of-freedom
NO	frequency of zero orossings
$[p' + i\,p'']$	non-dimensional oscillatory aerodynamio load distribution matrix
$\{p\}$	transfer function for load
$\{q_o\}$	matrix of complex generalised coordinates during harmonio motion
q_j	instantaneous value of the j^{th} generalised ooordinate
[Q' + iv Q"]	non-dimensional oscillatory aercdynsmio response generalised forces
Q_{jk}	instantaneous value of the aerodynamic generalised force in mode j due to harmonio motion in mode k.
R	response level
s	non-dimensional time $s = \frac{Vt}{\ell}$
S	reference area
t	time
$\{T\}$	general transfer funotion
$\{T_q\}$	transfer funotion for the displaoements of the generalised ooordinatea
V	flight velocity
w	gust velocity
x, y, z	ooordinates (see Fig.1)
γ	fraction of oritical damping
$\delta(\omega)$	Dirao delta
η	oontrol angle

SYMBOLS (Contd)

λ	harmonic gust wavelength
ν	frequency parameter $\nu = \frac{\omega \ell}{V}$
ρ	atmospheric density
σ	rms value
τ	time
Φ	instantaneous value of the excitation aerodynamic force
Φ_E	power spectral density of excitation
Φ_R	power spectral density of response
$\{\Phi' + i\ \Phi''\}$	non-dimensional aerodynamic excitation oscillatory generalised force
ω	circular frequency
suffices	
o	complex value of harmonically varying quantity
j,k	mode identification
r	location identification
other symbols	
$\bar{\ }$	denotes the Fourier transform of a quantity
[ ]	general matrix
{ }	column matrix
$\overline{\{\ \}}$	row matrix

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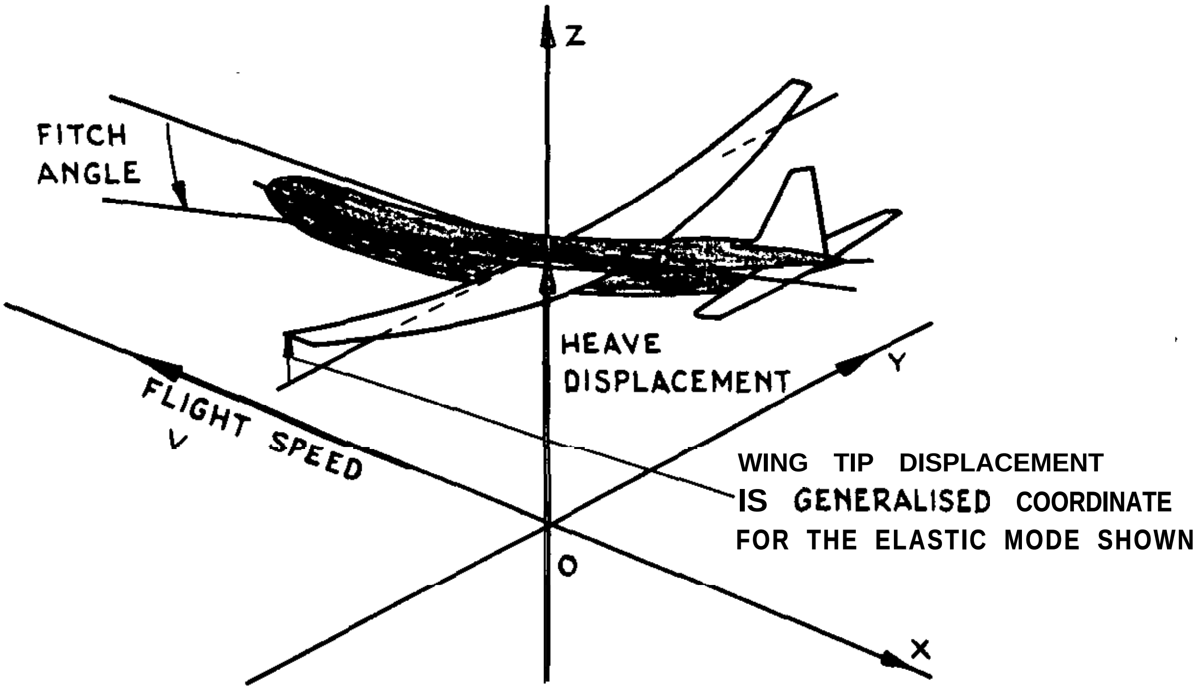
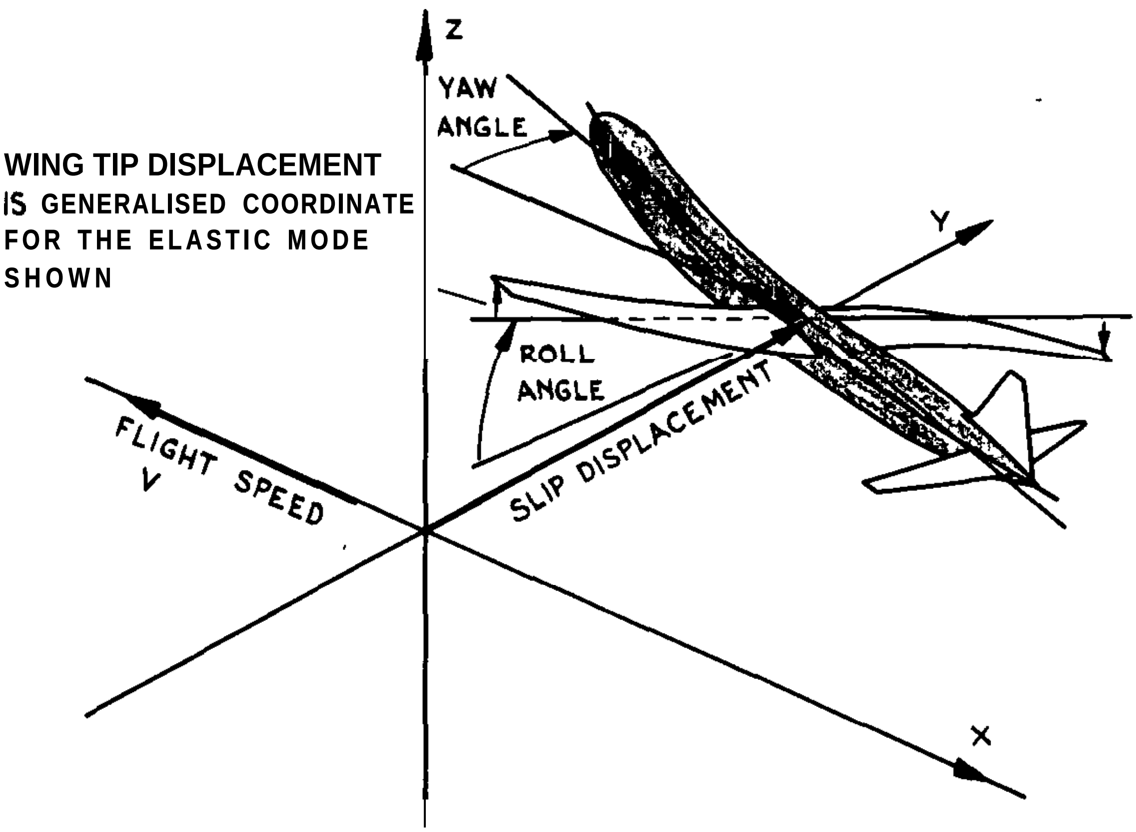


SYMMETRIC MOTION



ANTI SYMMETRIC MOTION

FIG. I COORDINATE SYSTEMS REPRESENTING THE PERTURBATION MOTIONS OF A FLEXIBLE AIRCRAFT

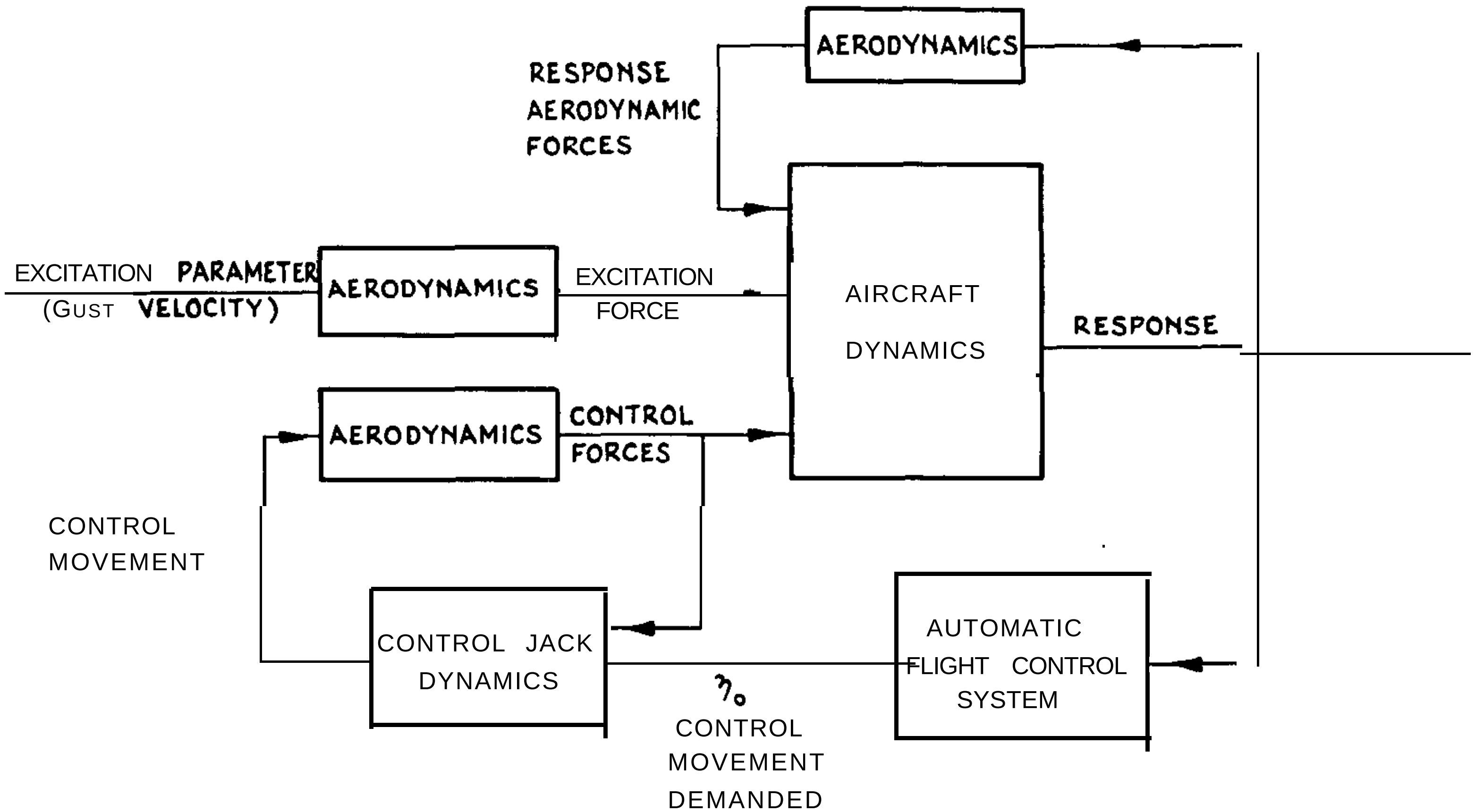


FIG. 2 THE DYNAMIC MODEL OF AN AIRCRAFT WITH AN AUTOMATIC FLIGHT CONTROL SYSTEM

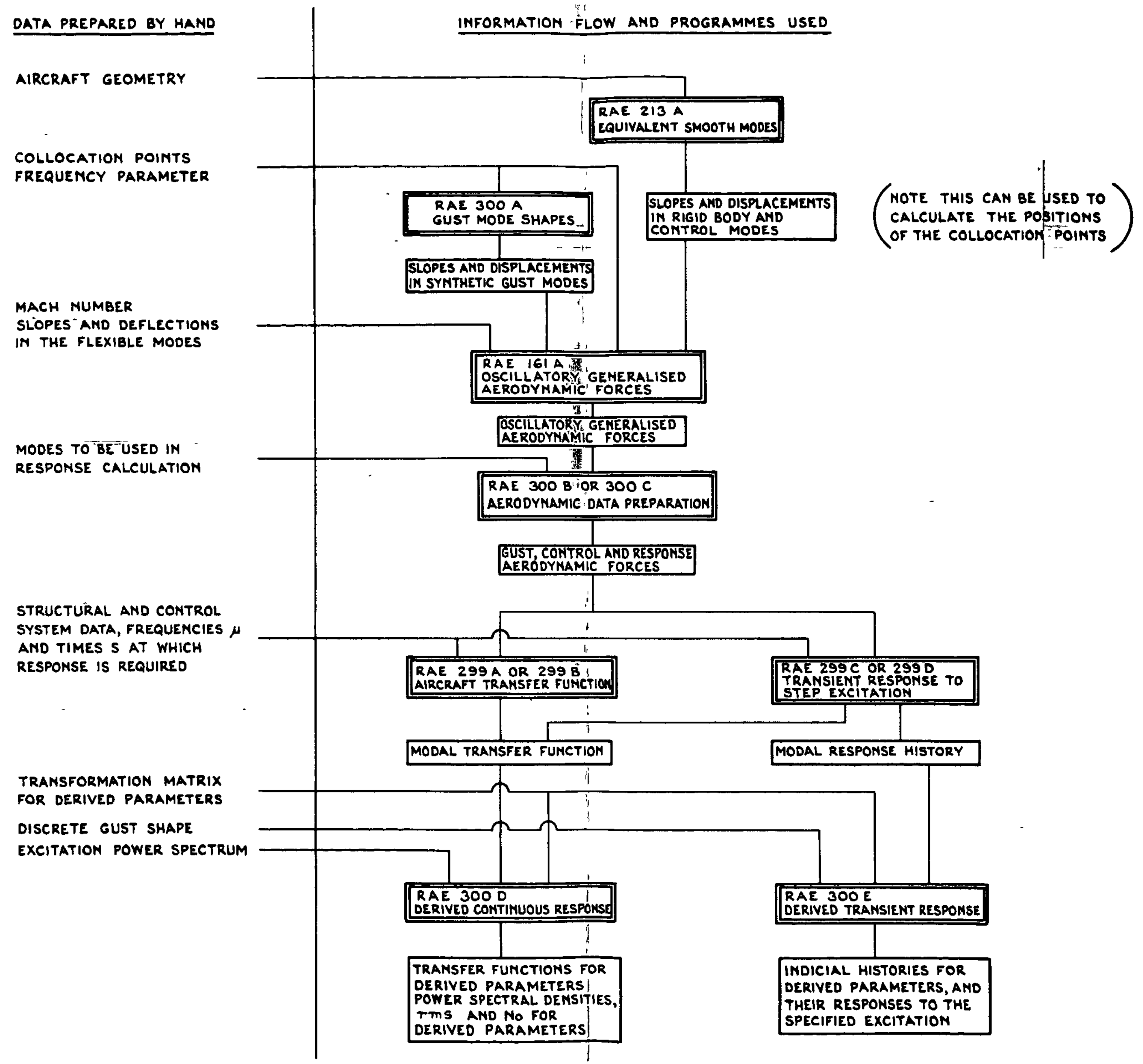
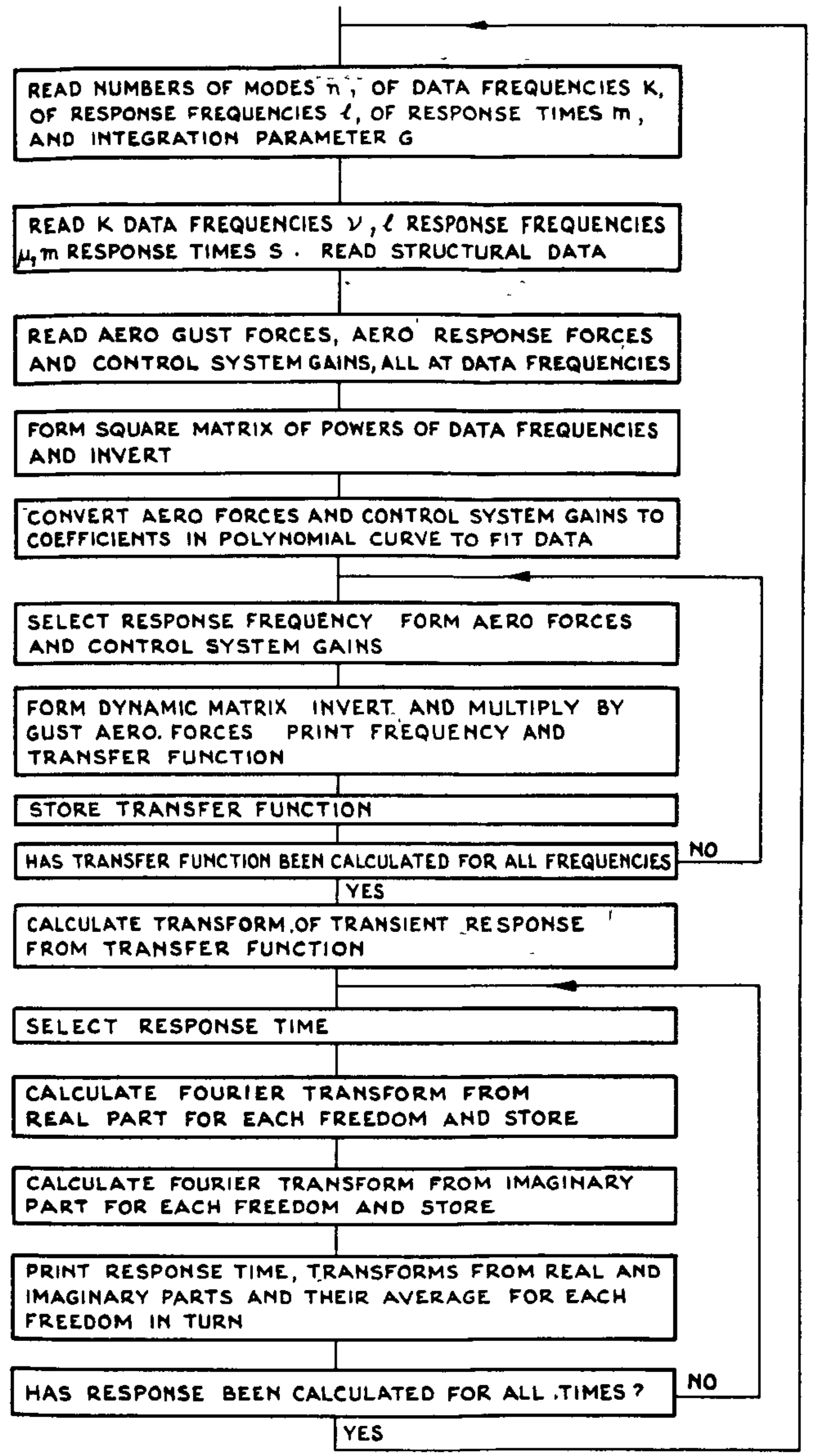


FIG. 3 RESPONSE CALCULATION FLOW DIAGRAM



NOTE DETAILS OF THE TRANSFORM ROUTINES ARE GIVEN IN REFERENCE 10

FIG. 4 "CALCULATION FLOW FOR PROGRAMME R.A.E. 299D "TRANSIENT RESPONSE TO STEP EXCITATION WITH CONTROL MOVEMENTS" (SEE APPENDIX D)

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COMPUTER PROGRAMMES TO CALCULATE THE RESPONSE OF FLEXIBLE AIRCRAFT TO GUSTS AND CONTROL MOVEMENTS

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Computer programmes to solve these equations are described, together with supporting data preparation and analysis programmes. Manual data preparation is minimised. Detailed programme specifications are given in Appendices.

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