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Project Topcat

Power Spectral Measurements of Clear Air Turbulence Associated with Jet Streams

by

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Power Spectral Measurements of Clear Air Turbulence Associated with Jet Streams

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Anne Burns, B.Sc.

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SUMMARY

The note presents preliminary information on the power spectral data obtained during Project TOPCAT in Australia. Measurements of moderately severe clear air turbulence were obtained from a Canberra aircraft flying at altitudes of 26000 to 33000 ft in the vicinity of the sub-tropical jet stream. In some respects, particularly at long wavelengths, the spectra are dissimilar to those found for low level turbulence and there are indications that the turbulence is anisotropic. At short wavelengths the power spectra agree well with a -5/3rd power law, and the energy densities of the three components (u_g, v_g, w_g) are similar.

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* Replaces R.A.E. Tech. Report No. 65210 - A.R.C. 27611.

** Of the Aeronautical Research Laboratories, Melbourne.

CONTENTS

1 INTRODUCTION

Although a great deal of information has been obtained on the power spectra of atmospheric turbulence at low altitudes where the turbulence occurs in the earth's boundary layer due to mechanical or convective causes[1,2], there is still little information on turbulence at higher altitudes. The picture is now being successfully filled in as regards turbulence in severe storms and in cumulus cloud[3], but published data on clear air turbulence at the higher altitudes is still sparse. The only known published data are those of Shur[4] obtained in the U.S.S.R. These are in reasonable agreement with the spectra of the present report at wavelengths shorter than 2000 ft, but there is a discrepancy at longer wavelengths. This has yet to be resolved; it may well arise from basic differences in the methods of measurement as discussed by Reiter[5]. Whatever method is used the problem of measuring long wavelength motions of the atmosphere with a jet aircraft, which itself becomes increasingly unstable at the higher altitudes, is a difficult one.

The present report presents power spectral data obtained during Project TOPCAT in Australia from August to October 1963. In this investigation, atmospheric turbulence associated with the sub-tropical jet stream was measured by means of a specially instrumented Canberra aircraft. The aircraft measurements were strongly supported by meteorological ground and upper air observations. Details of the meteorological aspects of the project are given in Ref. 6. The sub-tropical jet stream, which flows from west to east, is usually located across the southern half of Australia during the winter months, with the jet axis at an average height of 35000 to 42000 ft. Wind velocities greater than 150 kt are not uncommon although maximum turbulence does not necessarily occur in the region of strongest wind. The power spectra relate to traverses made on 4 different days through discrete patches of clear air turbulence. In most cases only one traverse through any one patch has been analysed, although recordings were usually made of a number of traverses at different heights and in different directions. Because of the extensive data reduction and computation involved, these other traverses have not yet been analysed. The turbulence was judged subjectively to be moderate in severity. Vertical incremental accelerations of the order of 0.5g to 0.8g were recorded but there was some evidence that the horizontal components of turbulence were more severe than the vertical. Power spectra were obtained for all three components of turbulence. The patches studied occurred mostly at heights of 26000 to 29000 ft on the polar side of the jet stream (the search for turbulence was mainly confined to this side). The patch of turbulence encountered in flight 33 was an exception to this, in that it occurred at 32000 ft near the height of the jet axis, which was unusually low that day.

2 OUTLINE OF METHOD

The aircraft, a Canberra B6, instrumented with gust measuring probe, was the same as that used for previous investigations of low level turbulence. Slight modifications were made to the instrumentation, described in its original form in Ref. 7, to adapt it to the higher altitude environment. Additional items fitted included Doppler for wind measurements and a smoke generator for visual assessment of air flow patterns. This smoke generating system proved invaluable for marking patches of turbulence and allowing subsequent traverses through the turbulence to be positioned relative to each other.

In the earlier investigations of low altitude turbulence, power spectra were not derived for the lateral component of turbulence, although recordings were made, since it was suspected that such spectra would be contaminated by the amount of Dutch roll which occurred when the aircraft encountered turbulence. An attempt was made, however, to tackle this problem for the high altitude measurements. The response of the aircraft in the Dutch roll was decreased to some extent by the addition of wing tip tanks, which were kept three-quarter full of fuel, since this condition proved optimum for minimising the motion; (various effects, including increased moment of inertia and increased damping from fuel sloshing, contributed to this reduction). Best corrections for eliminating the effect of the remaining Dutch roll from the gust sensing probe signals were determined empirically from an analysis of pilot-induced Dutch roll in smooth air. It was found that correction could be made from angle of yaw only, the contribution from rate of yaw being insignificant.

The standard equation developed by Chilton[8], Crane[9] and others was used for determining the vertical velocity of turbulence from the air flow sensed by the probe and from the aircraft motions. Since the measurement of the vertical gust spectra at low frequencies depends strongly on the correction for changes in aircraft attitude, this quantity was estimated both from a free gyro signal and from an integrated pitch rate gyro signal. The vertical gust spectra obtained using the alternative methods were in good agreement over the frequency range 0.1 to 10 cycles/sec. Somewhat simplified equations were used for the horizontal components. In the case of fore-and-aft turbulence the aircraft ground speed was assumed sensibly constant, i.e. all variation in air speed was assumed due to turbulence. A comparison of power spectra obtained from synchronised measurements on a radio tower at Darmstadt and on the aircraft flown in the neighbourhood of the tower, suggests that this assumption is justified, at least at low altitudes. In the case of the lateral turbulence, corrections were made for the lateral movements of the aircraft in the Dutch roll only, as described above. Comparison of the tower and aircraft power spectra suggests that in this case the aircraft spectra tend to over-estimate the energy content at long wavelengths. This could be due to incomplete elimination of the Dutch roll or to lateral motions of which no account has been taken*.

The power spectra were obtained from the Fourier transforms of the auto-correlation functions of the time histories of the vertical, lateral and longitudinal components of the true air velocities encountered by the aircraft. The original data were digitised at discrete intervals of 1/20th sec. Processes used to refine the accuracy of the spectra included pre-whitening and "hanning"[10]. The computer programme was written to obtain estimates of the spectra over two ranges of the frequency. In the first, estimates based on unsmoothed data were obtained from 0 to 10 c/s at an interval of 0.2 c/s. In the second, non-overlapping groups of four were pre-whitened and estimates of the spectrum from 0.05 to 1 c/s obtained, the resolution being 0.05 c/s. The final spectra were composed from the two sets of estimates. Spectra were transformed to a wave-number plane by removing the effect of the aircraft true airspeed. It is believed that the processes used for obtaining the spectra give reliable estimates over a wavelength range of the order 10000 to 100 ft.

3 RESULTS

3.1 Set-out of results

Estimates of the individual spectra are plotted in Figs.1-7. The estimates show a certain amount of scatter and smooth curves have been faired in for comparative purposes. These are used in Fig.8 where the three

*Full correction for the aircraft motion would have required inclusion of lateral acceleration and angle of roll terms in the equations for the lateral gust component. Lateral accelerations were, however, small - except for the Dutch roll component - and contained an unwanted fuselage flexural component. It was, therefore, considered that the inclusion of the lateral acceleration would probably do more harm then good. Angle of roll is not required if the lateral acceleration term is omitted.

components of turbulence for each traverse are grouped together. Fig.9 compares spectra obtained for the whole of a traverse of $4\frac{1}{2}$ minutes with spectra for the last $1\frac{2}{3}$ minutes of the same traverse.

Table 1 gives a summary of the traverses, flight conditions and root mean square gust velocities. These velocities are truncated values obtained by measuring the square root of the area under curves of best fit for the wavelength range 10000 to 100 ft. Cubic polynomials in log f were fitted to log P for this purpose.

The spectra of flight 44 relate to a low-level flight made during the TOPCAT trials in which turbulence measurements were taken at 300 ft. Turbulence was light and mainly due to convective causes. The spectra for this flight are included for comparison with the high altitude spectra.

3.2 Accuracy of results

Some of the spectra included in this note have a bearing on the accuracy of the results. The spectra of Fig.8(c), relating to the low-level flight, indicate that the equipment was working reasonably well and giving results comparable with those obtained previously from measurements on the same aircraft and by other workers in this field. The spectra of Fig.9 indicate that there is no significant loss of energy at the long wavelengths due to the shortage of the sample - at least down to samples of length $1\frac{2}{3}$ minutes. The slightly lower values obtained in the shorter sample are due to the end of the run being less turbulent; this results in a decrease of energy at all wavelengths, rather than at very long wavelengths as would be the case if the decrease was due to shortage of sample. The traverses 18.02 and 18.04 were, however, shorter than this, and the validity of their spectra at the longer wavelengths has yet to be established.

As noted in the outline of method, it is likely that the spectra for the lateral component of turbulence slightly over-estimate the energy content, particularly at the longer wavelengths. In the case of the traverse 27.06, a very large value of v_g was obtained at a wavelength of 15000 ft, and this value has been omitted as it is suspect. It should be noted that the reliability of all estimates in the region of 15000 ft is somewhat uncertain because of the difficulty of extending measurements to such long wavelengths.

3.3 Discussion of results

A study of Fig.8(a) to (d) reveals some interesting features which are discussed briefly below:-

(i) The spectral density curve of the vertical component of turbulence w_g tends to lie below that of the horizontal components, particularly at the longer wavelengths. At shorter wavelengths, less than about 500 ft, the curves come together again with a slope approximating to a -5/3rd power law. At these short wavelengths the spectra are similar to those found for other types of atmospheric turbulence. The feed-in of vertical energy at wavelengths of the order 2000 to 4000 ft (denoted by the flattening of the spectra), is perhaps not surprising in view of the shallowness of the layers in which high altitude clear air turbulence occurs. Such layers are usually of the order of 3000 ft in depth.

(ii) A study of Fig.8 shows that the curve for u_g tends to lie above or very close to, that for v_g when flying in the direction of wind, and below it when flying cross-wind. This suggests that the horizontal component of turbulence is greater in the direction of the jet stream than across it.

(iii) The curves of spectral density, particularly those of the vertical component, often exhibit a cubic curvature (or S-bend) when plotted on a log-log scale. Figs.8(a) and (b) suggest that this is a characteristic of spectra obtained when the flight path is across the wind, but this does not hold true for the traverses of Fig.8(d) unless the upper and more stronger southerly wind is considered to be the dominant factor. On this day the situation was complex with a sharp change in wind direction, as well as in speed, at about the level of turbulence.

(iv) The value of the cross-correlation coefficient between the longitudinal component of turbulence, i.e. the component along the aircraft flight path, and the vertical component varies according to the direction of the flight path relative to the wind. Fig.10 shows a plot of this cross-correlation coefficient against direction of flight relative to the wind. The sign of the cross correlation coefficient indicates an association of upwind and upward turbulent air flow, and vice versa. One exception is found to this - that of the traverse 33.05 where the effect is reversed. It is perhaps significant that this is the only traverse made at the level of the jet stream axis, where the wind shear would be opposite to that of the other traverses, i.e. the wind would decrease rather than increase with height. The cross-correlations are those of the pre-whitened values and, therefore, relate to accelerations of the air flow rather than velocities.

4 CONCLUDING REMARKS

Analysis of further recorded data would help to determine how the features noted above, such as the variation in the ratio u_g/v_g, the input of spectral energy in the middle range of wavelengths and the variation in the cross-correlation between w_g and u_g, are related to parameters such as wind direction and shear. These answers should contribute to a theoretical understanding of the nature of clear air turbulence associated with jet streams. In the meantime some general conclusions can be drawn. In particular, there is considerable evidence that such clear air turbulence is anisotropic, at least at wavelengths greater than about 500 ft. The anisotropic region is characterised by the horizontal components of the turbulence being greater than the vertical. At short wavelengths, less than 500 ft, this type of turbulence tends to be more nearly isotropic with a slope approximating to a -5/3rd power law. Thus, at the shorter wavelengths it is similar to that already measured at lower altitudes and in cumulus and storm clouds.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the assistance given by the following Australian authorities in carrying out the flight trials and in the data reduction:- the Weapons Research Establishment, Salisbury who were responsible for the co-ordination and overall conduct of the trials; the Royal Australian Air Force, Edinburgh who provided servicing and maintenance facilities; and the Bureau of Meteorology, Adelaide and Melbourne who made special forecasts and diagnoses of the meteorological situations. Particular thanks are due to the Mathematics Services Group of W.R.E. for trace reading and data tabulation, and to the University of Melbourne for the use of the IBM 7044 for computing the correlations. The assistance given by Dr. U. Rakok of the University of Melbourne in planning the trials, and the guidance given by Dr. E. R. Reiter of Colorado State University, U.S.A. during the second half of the trials contributed largely to their success.

Table 1

SUMMARY OF TRAVERSES

Date	Flight and run No.	Average barometric height - feet	Heading	Average airspeed ft/sec, true	Duration of run seconds	Wind deg/kt	Root mean square gust velocity ft/sec			Coefficient of cross correlation between w_g and u_g
							σ_u^*	σ_v^*	σ_w^*	
21.8.63	18.02 (Run B)	28200	155°	734.1	37.5	270°/90	2.76	3.86	2.61	-0.188
21.8.63	18.04 (Run D)	29100	260°	737.9	50	261°/90	3.86	3.90	2.91	0.289
4.9.63	27.06 (Run F)	26690	280°	730.5	150	226°/94	2.74	3.10	2.42	0.069
12.9.63	33.05 (Run E)	32040	255°	742.2	150	260°/90	3.70	3.34	2.38	-0.263
1.10.63	44.04 (Run D)	300	-	507.0	150	-	1.81	2.10	1.98	-0.022
3.10.63	45.08 (Run H)	28450	086°	741.8	270	256°/43	3.71	2.46	2.18	-0.198
3.10.63	46.05 (Run E)	29260	216°	744.4	100	278°/45	4.59	4.90	3.53	0.160

*Truncated values for wavelengths rangirg from 100 ft to 10000 ft.

SYMBOLS

w_g	Vertical component of velocity of turbulence	ft/sec
v_g	Lateral component of velocity of turbulence relative to aircraft	ft/sec
u_g	Longitudinal component of velocity of turbulence relative to aircraft	ft/sec
σ_w, σ_v, σ_u	Root mean-square values of vertical, lateral and longitudinal components of velocities of turbulence	ft/sec
P	Spectral density of turbulence	$(\text{ft/sec})^2$/cycle/ft
f	Frequency	c.p.s.

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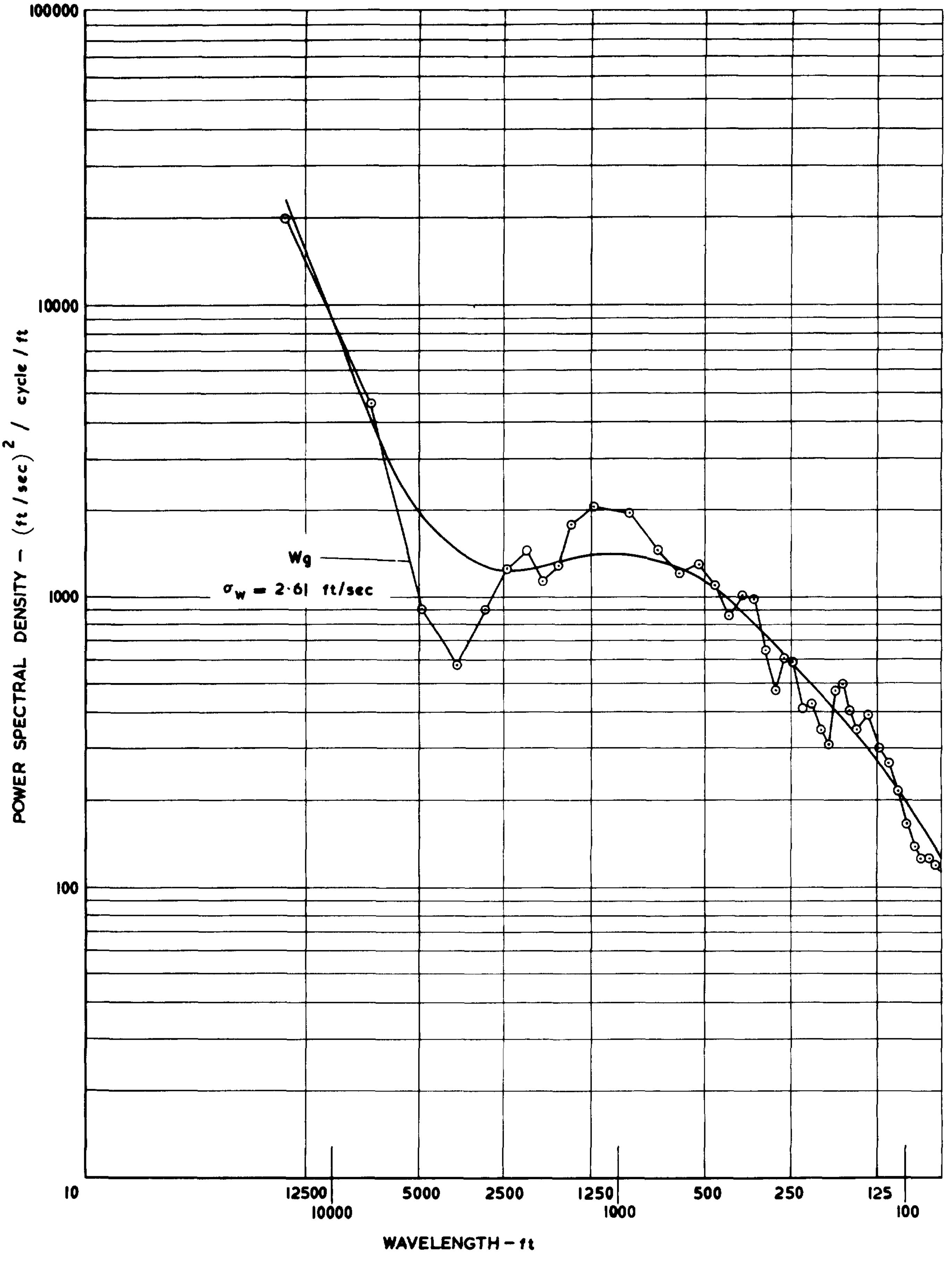

FIG I (a) POWER SPECTRA OF VERTICAL COMPONENT OF TURBULENCE
FLIGHT No 18 RUN B

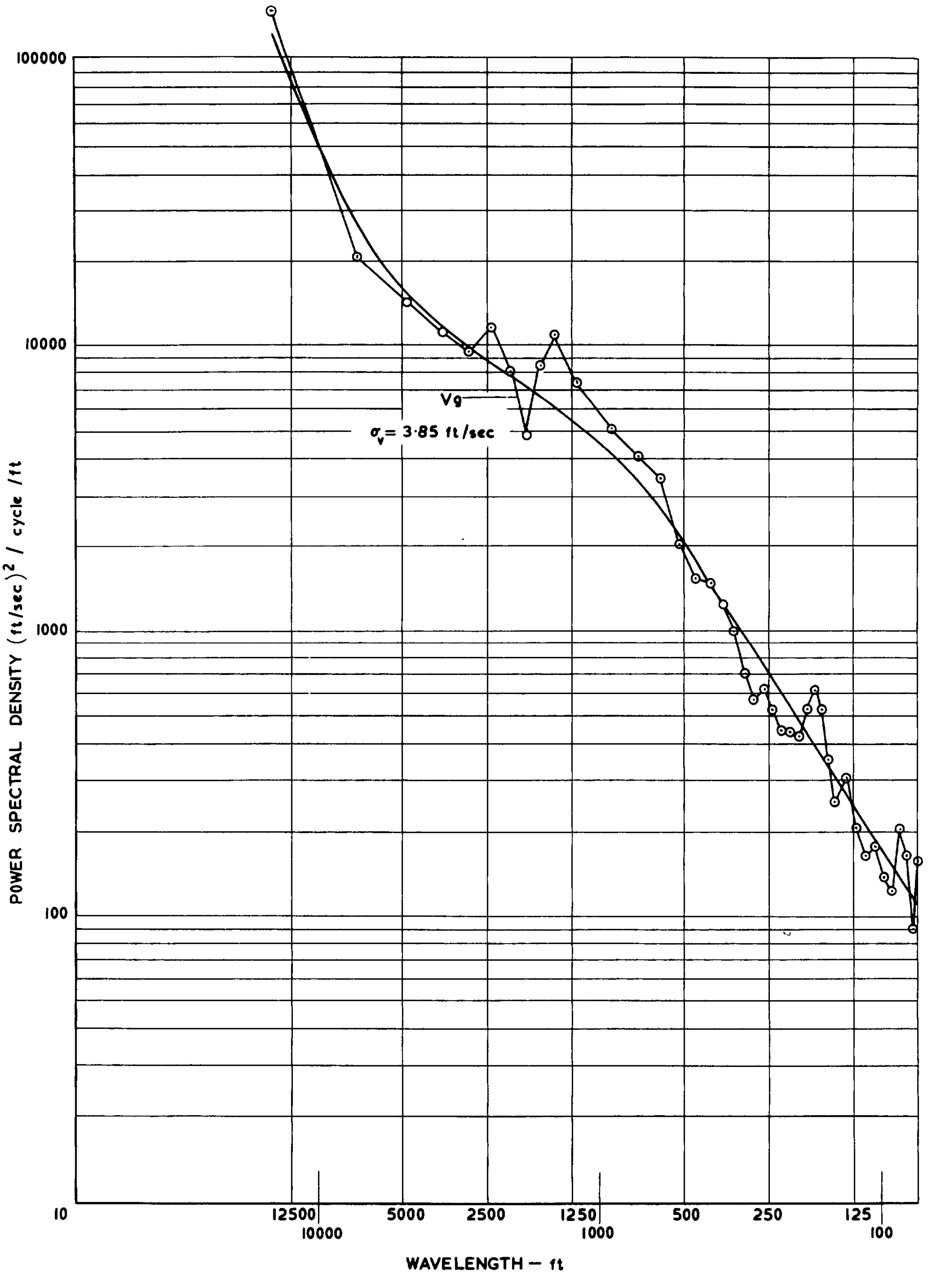

FIG 1(b) POWER SPECTRA OF LATERAL COMPONENT OF TURBULENCE

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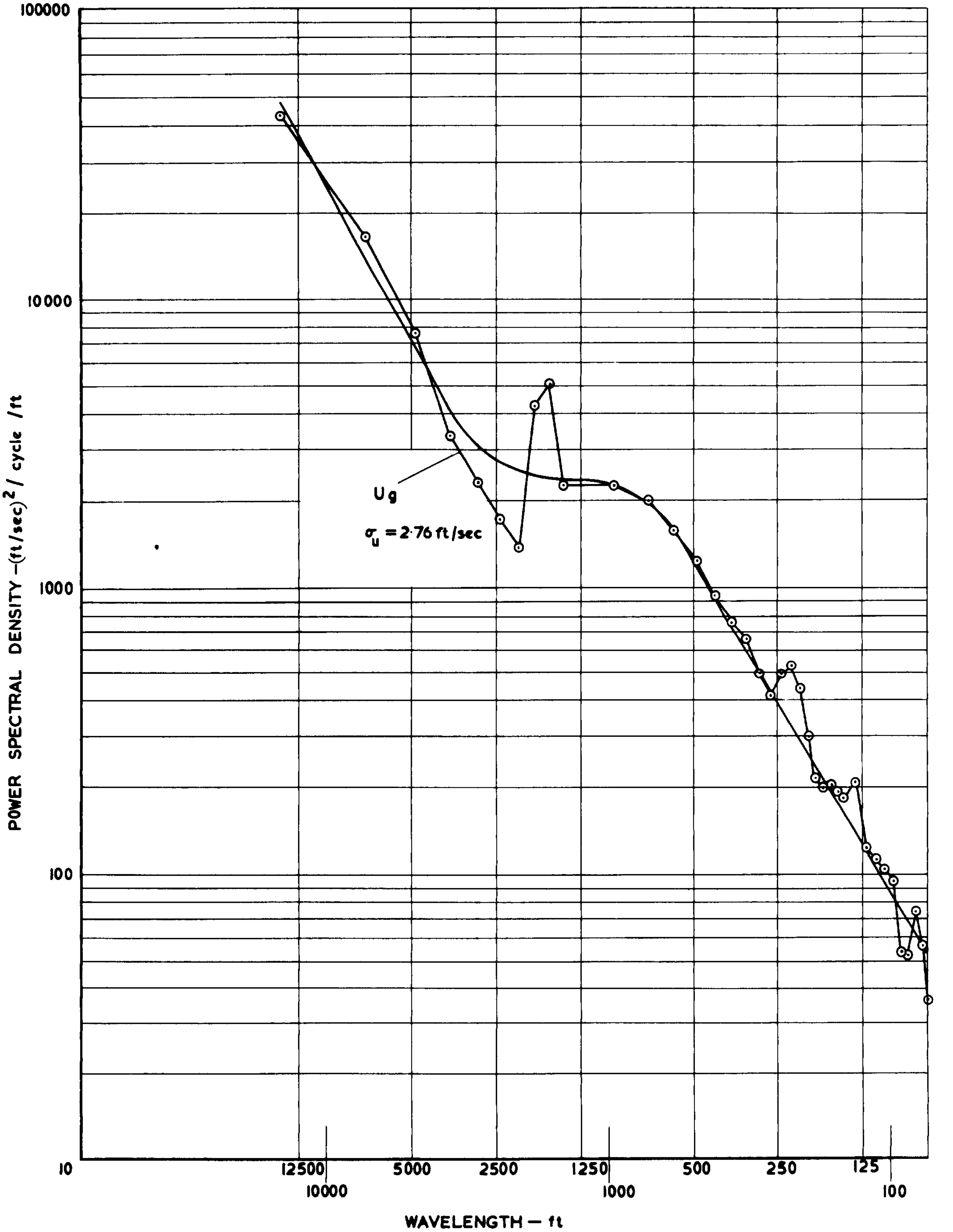

FIG I(c) POWER SPECTRA OF LONGITUDINAL COMPONENT OF TURBULENCE
FLIGHT No 18 RUN B

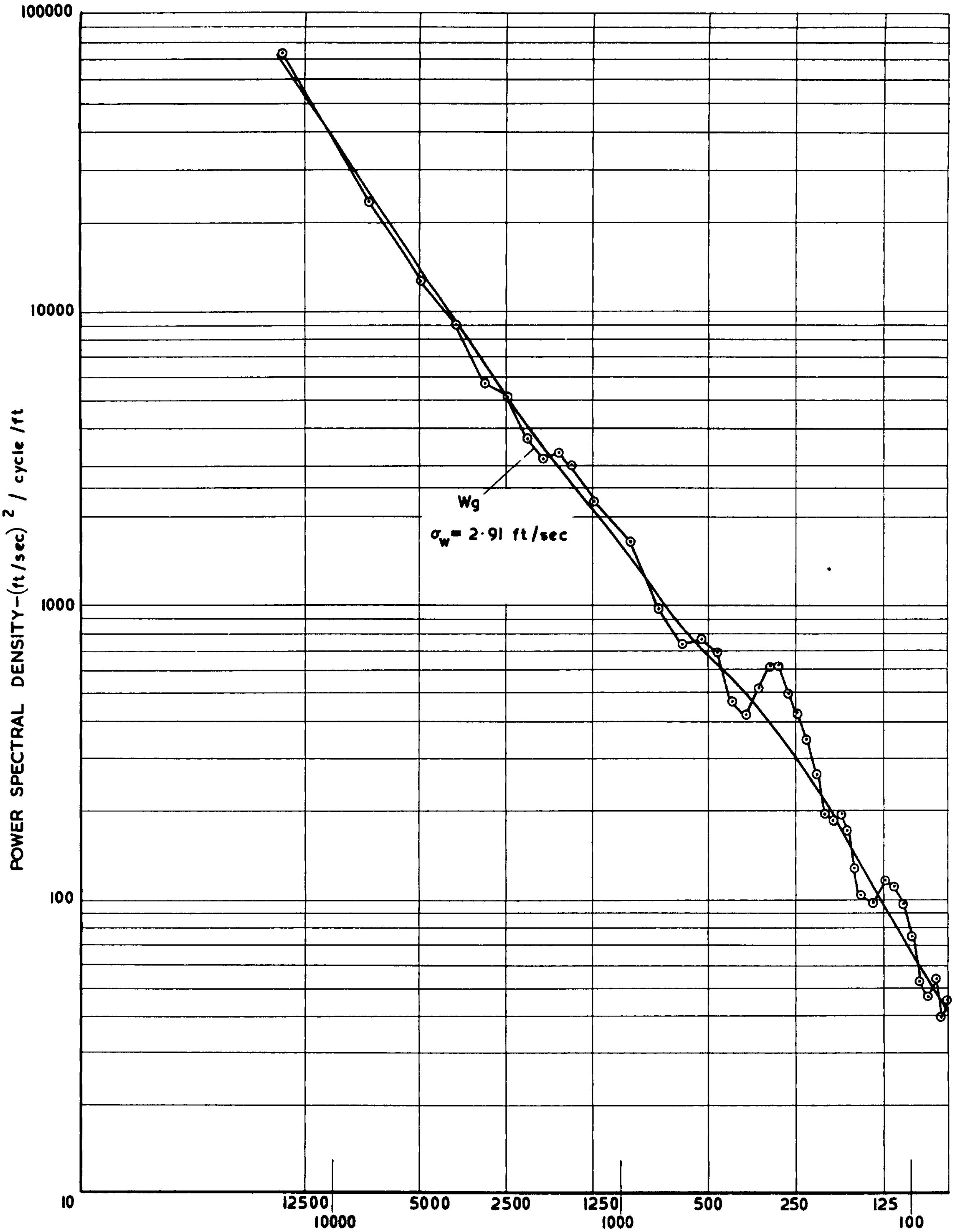

FIG 2(a) POWER SPECTRA OF VERTICAL COMPONENT OF TURBULENCE
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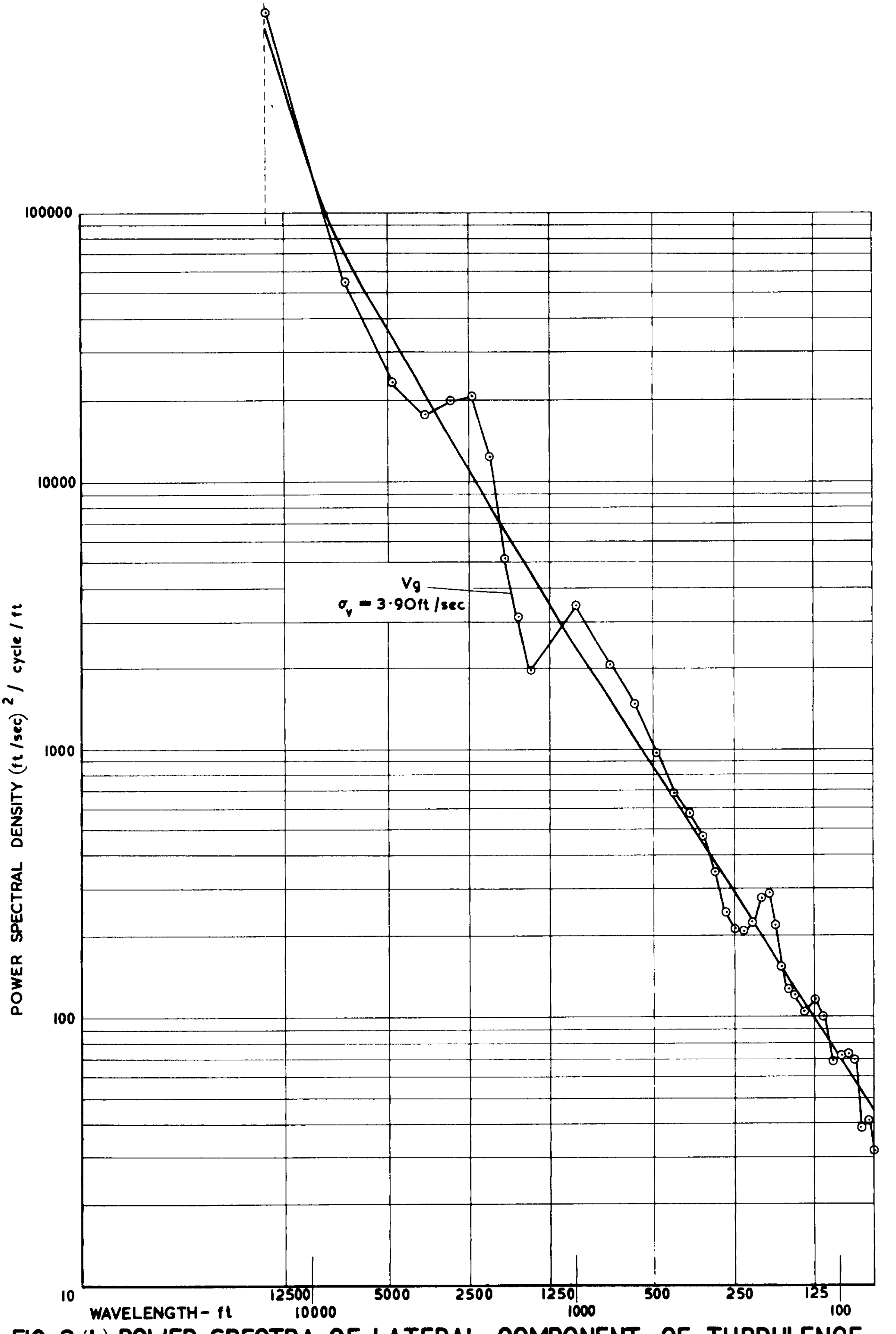

FIG 2 (b) POWER SPECTRA OF LATERAL COMPONENT OF TURBULENCE
FLIGHT No 18 RUN D

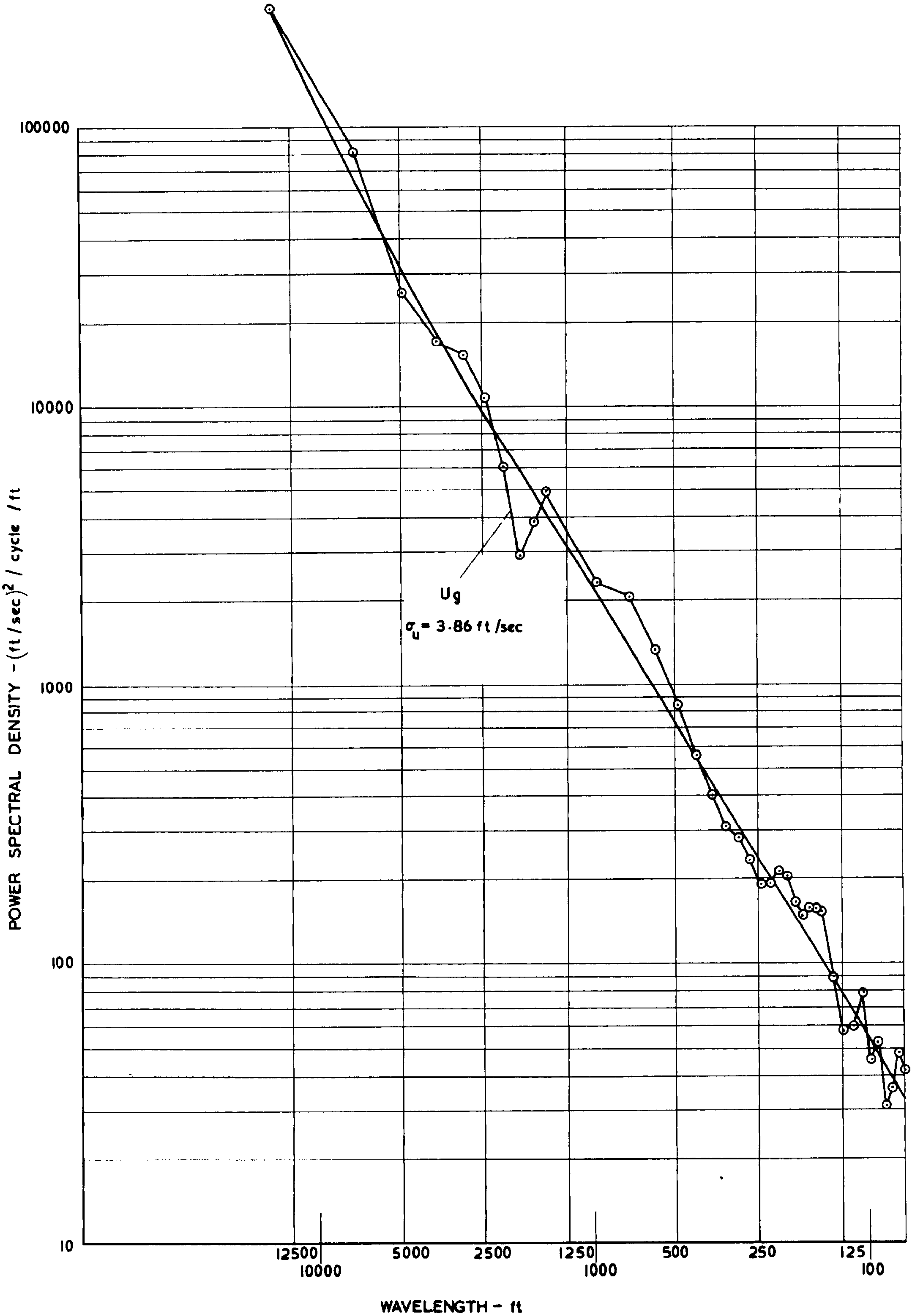

FIG 2(c) POWER SPECTRA OF LONGITUDINAL COMPONENT OF TURBULENCE
FLIGHT No 18 RUN D

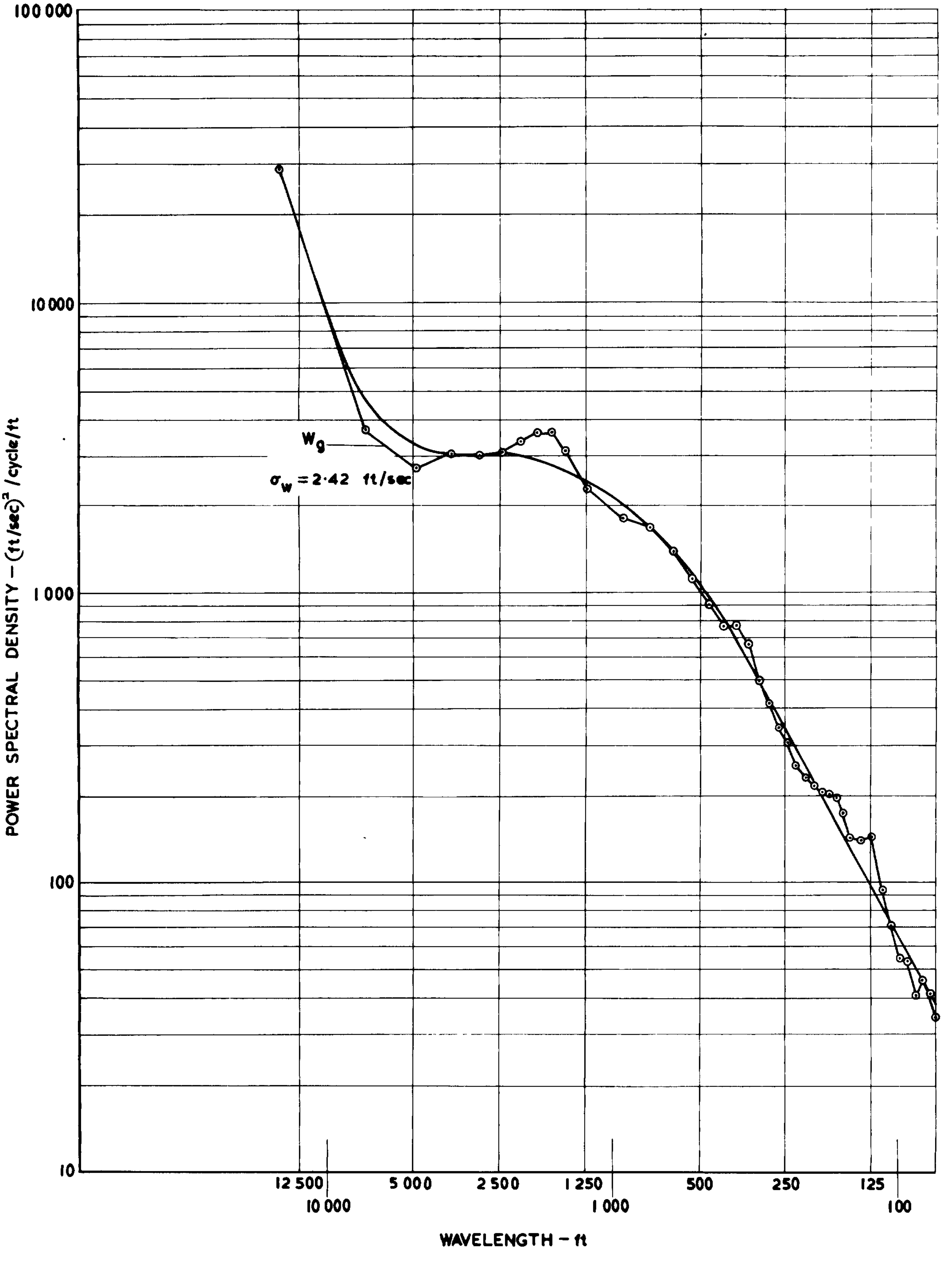

FIG. 3 (a) POWER SPECTRA OF VERTICAL COMPONENT OF TURBULENCE
FLIGHT No 27 RUN F

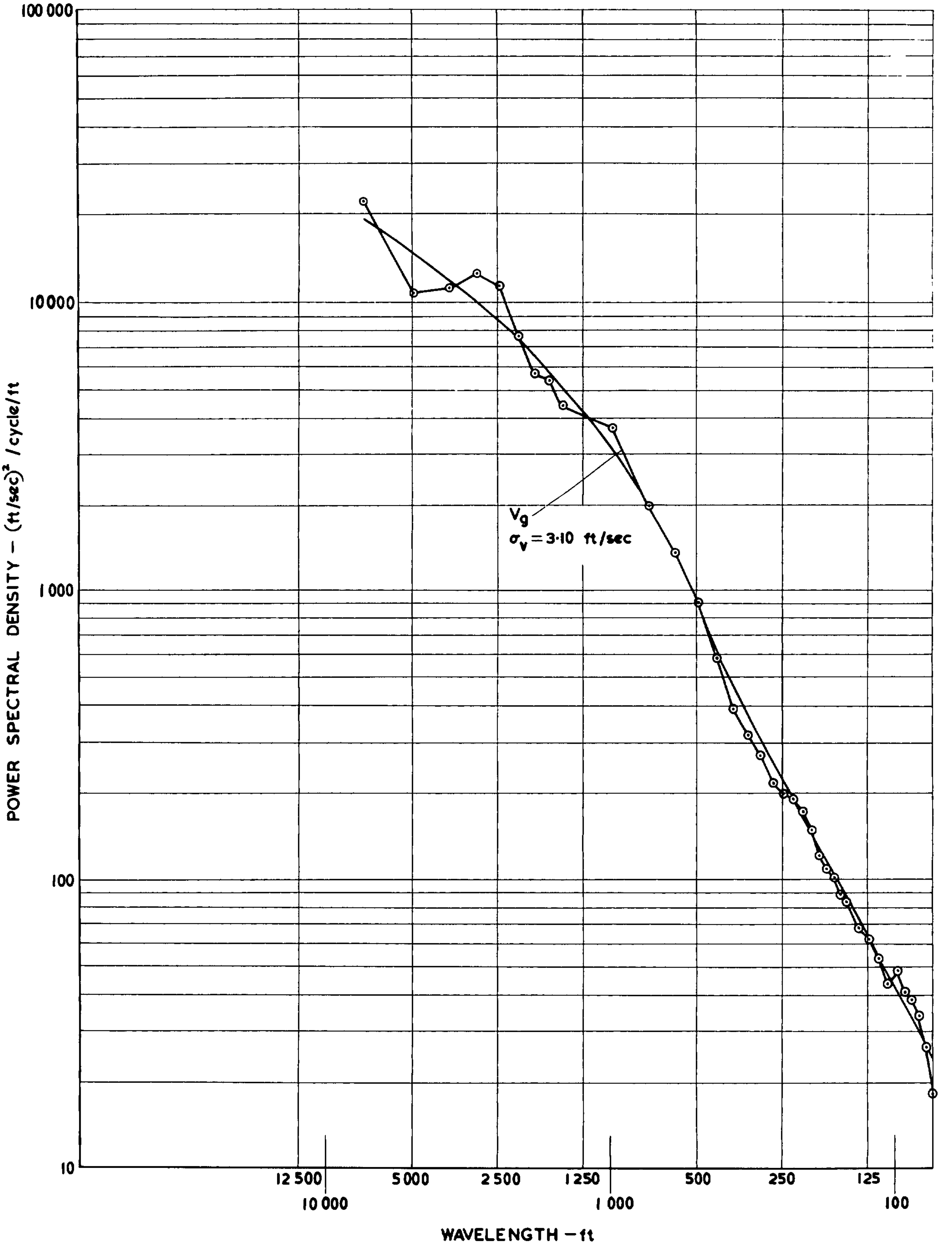

FIG. 3 (b) POWER SPECTRA OF LATERAL COMPONENT OF TURBULENCE
FLIGHT No 27 RUN F

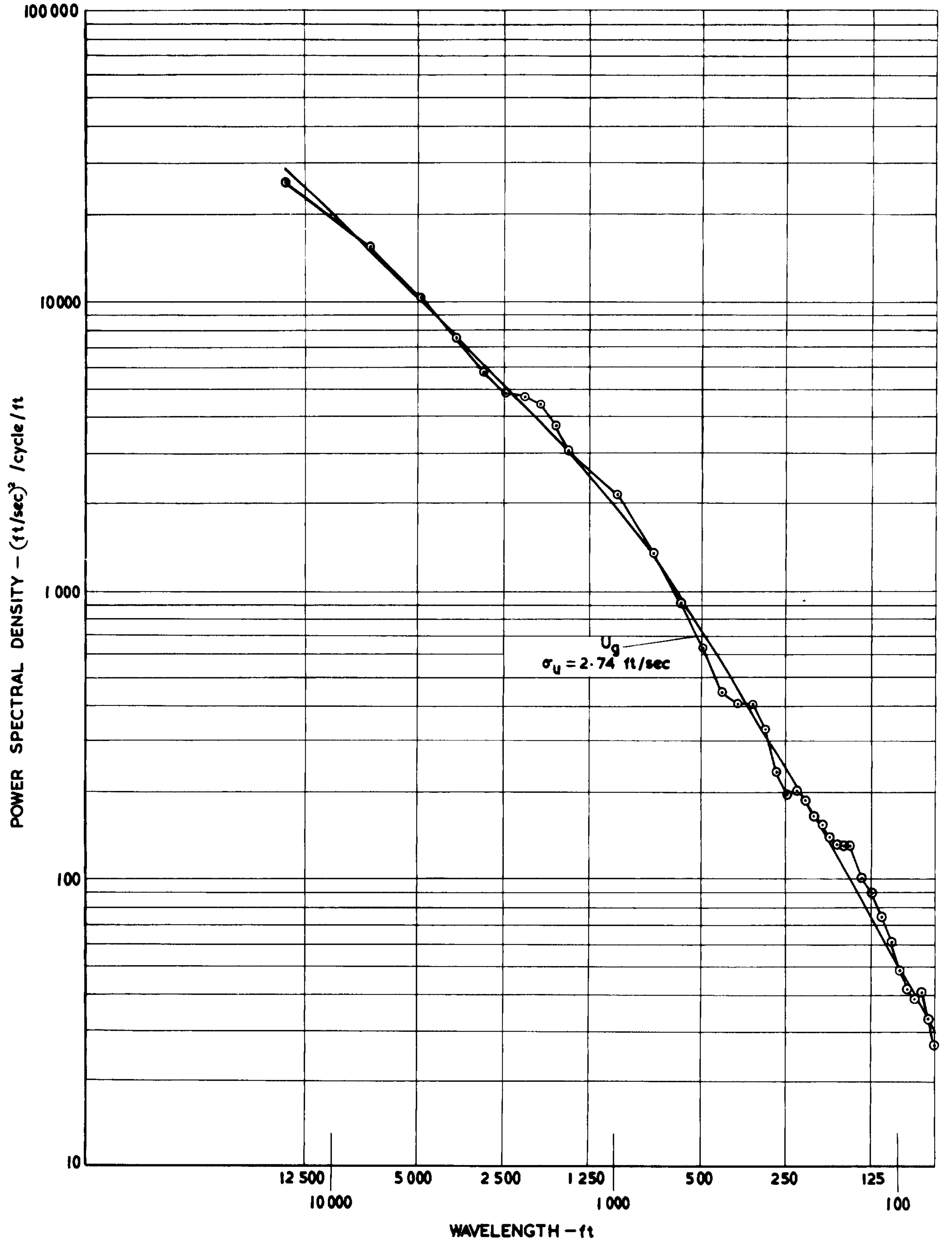

FIG. 3(c) POWER SPECTRA OF LONGITUDINAL COMPONENT OF TURBULENCE FLIGHT No 27 RUN F

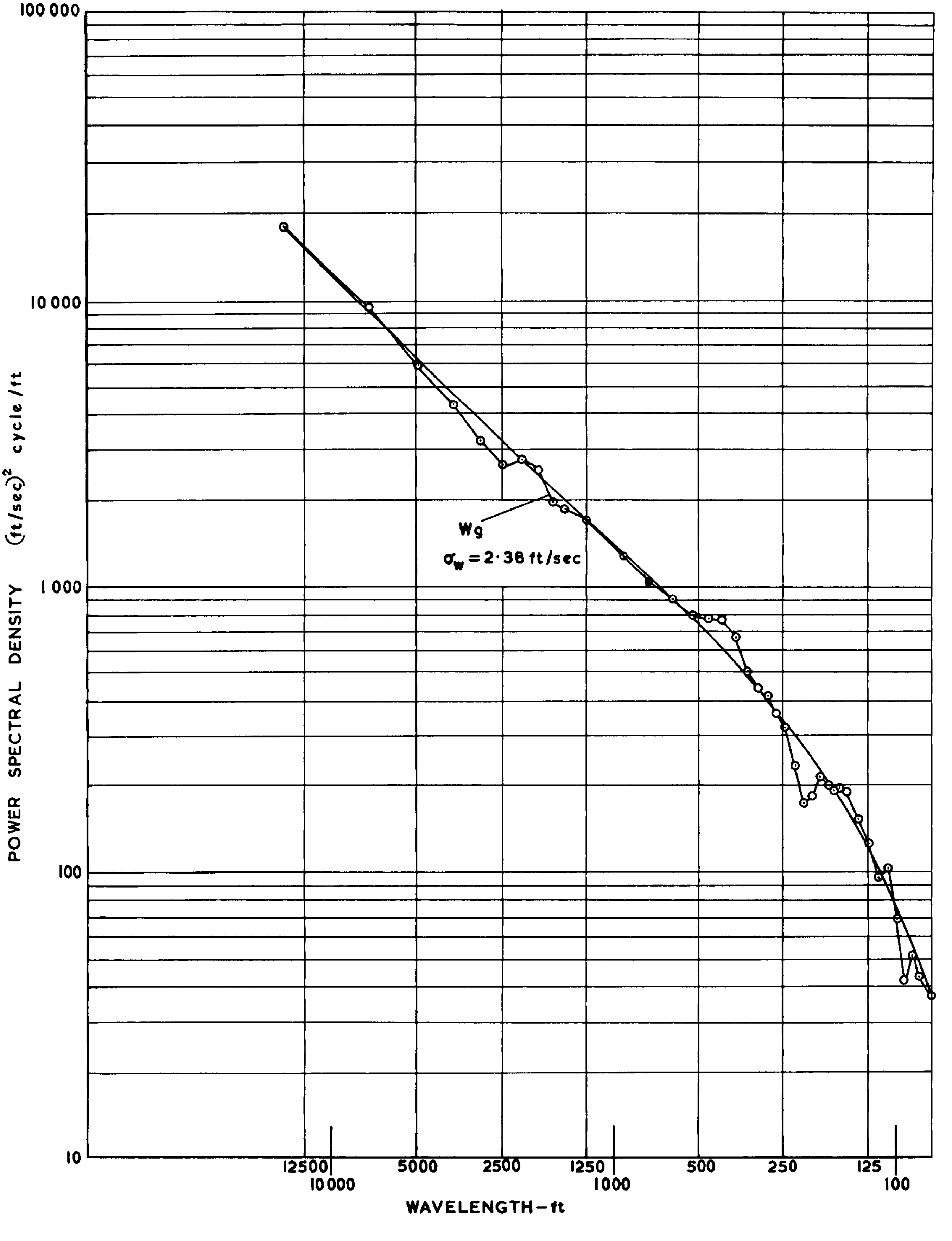

FIG 4 (a) POWER SPECTRA OF VERTICAL COMPONENT OF TURBULENCE
FLIGHT No 33 RUN E

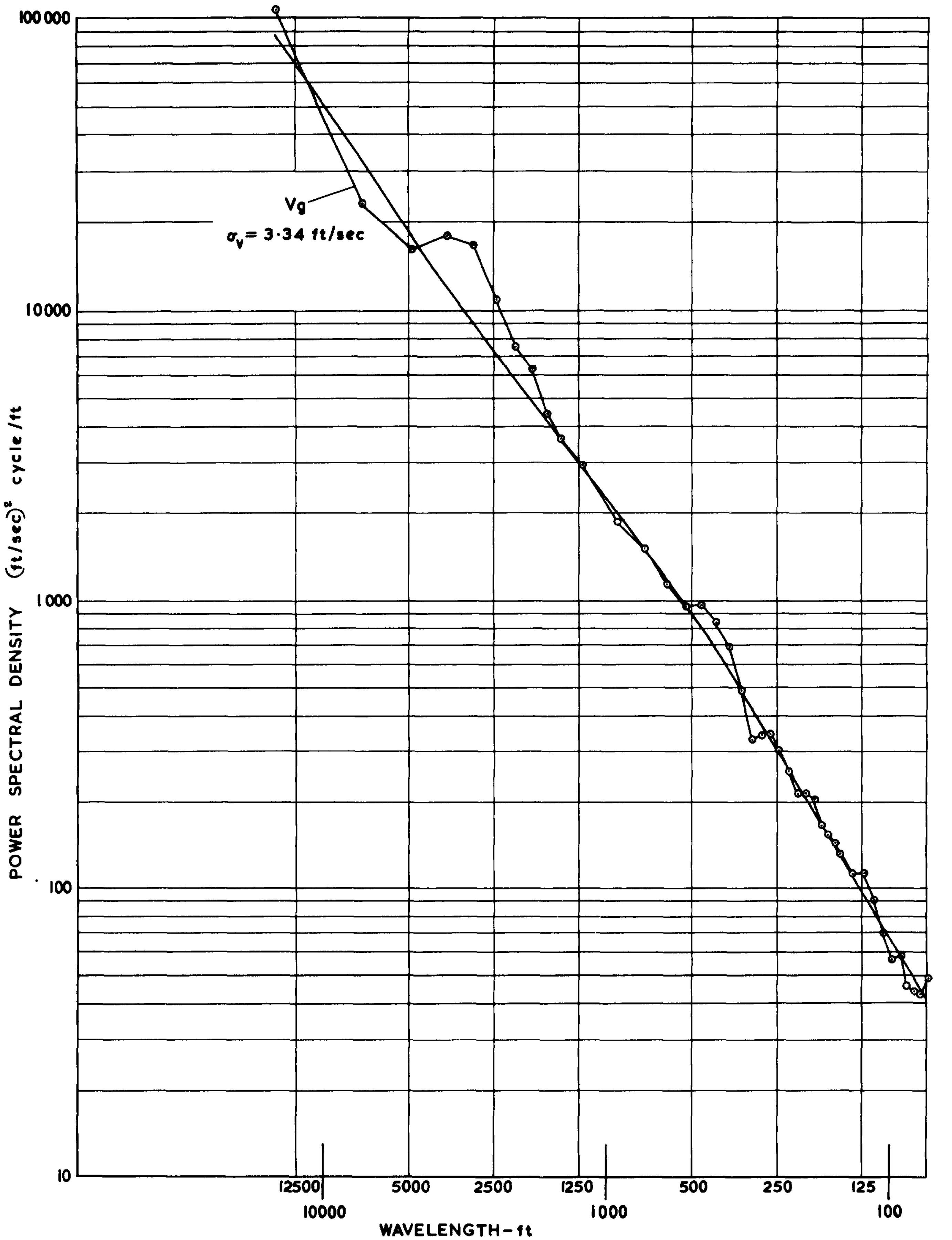

FIG 4 (b) POWER SPECTRA OF LATERAL COMPONENT OF TURBULENCE
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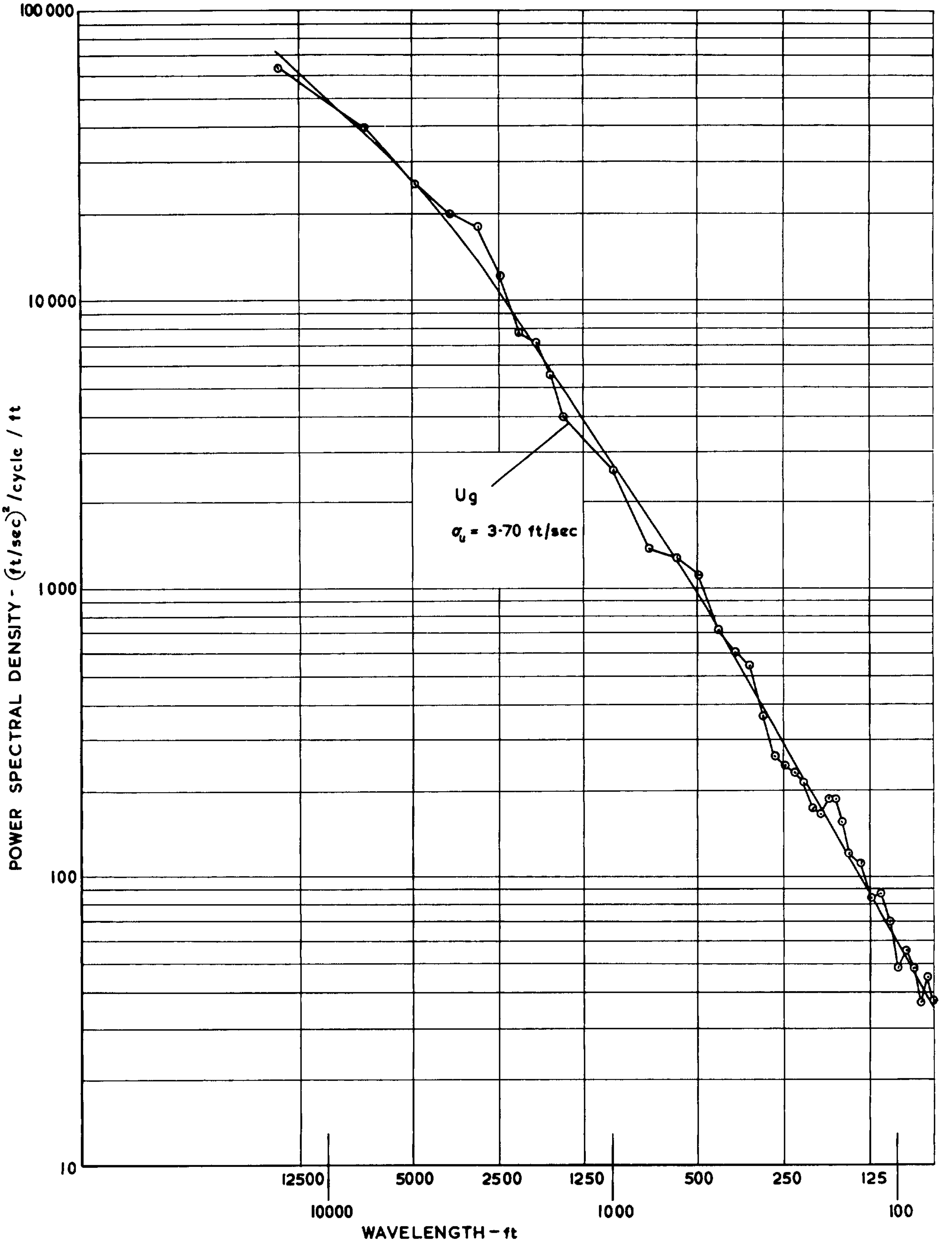

FIG.4(c) POWER SPECTRA OF LONGITUDINAL COMPONENT OF TURBULENCE
FLIGHT No 33 RUN E

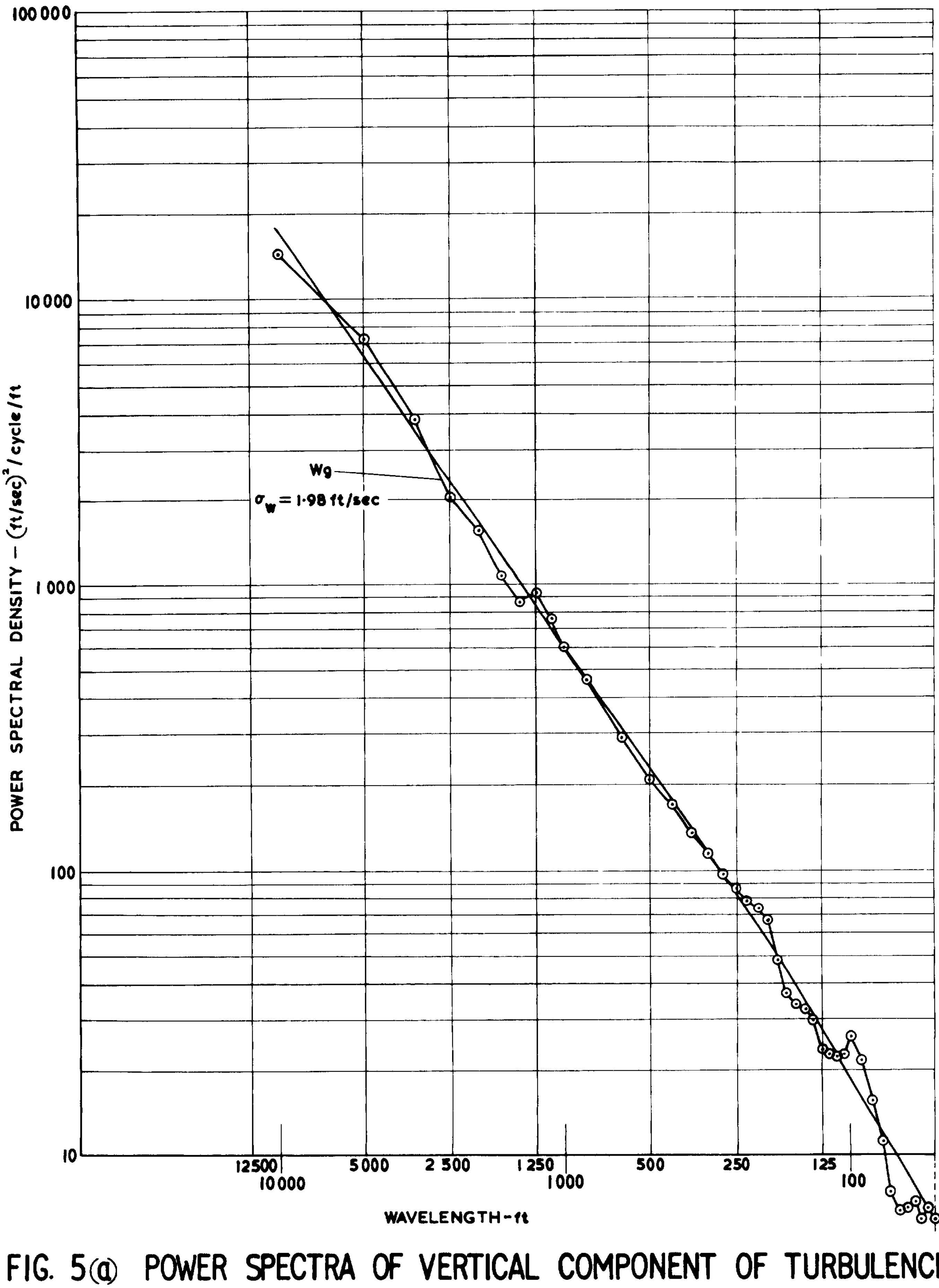

FIG. 5(a) POWER SPECTRA OF VERTICAL COMPONENT OF TURBULENCE
FLIGHT No 44 RUN D-LOW LEVEL

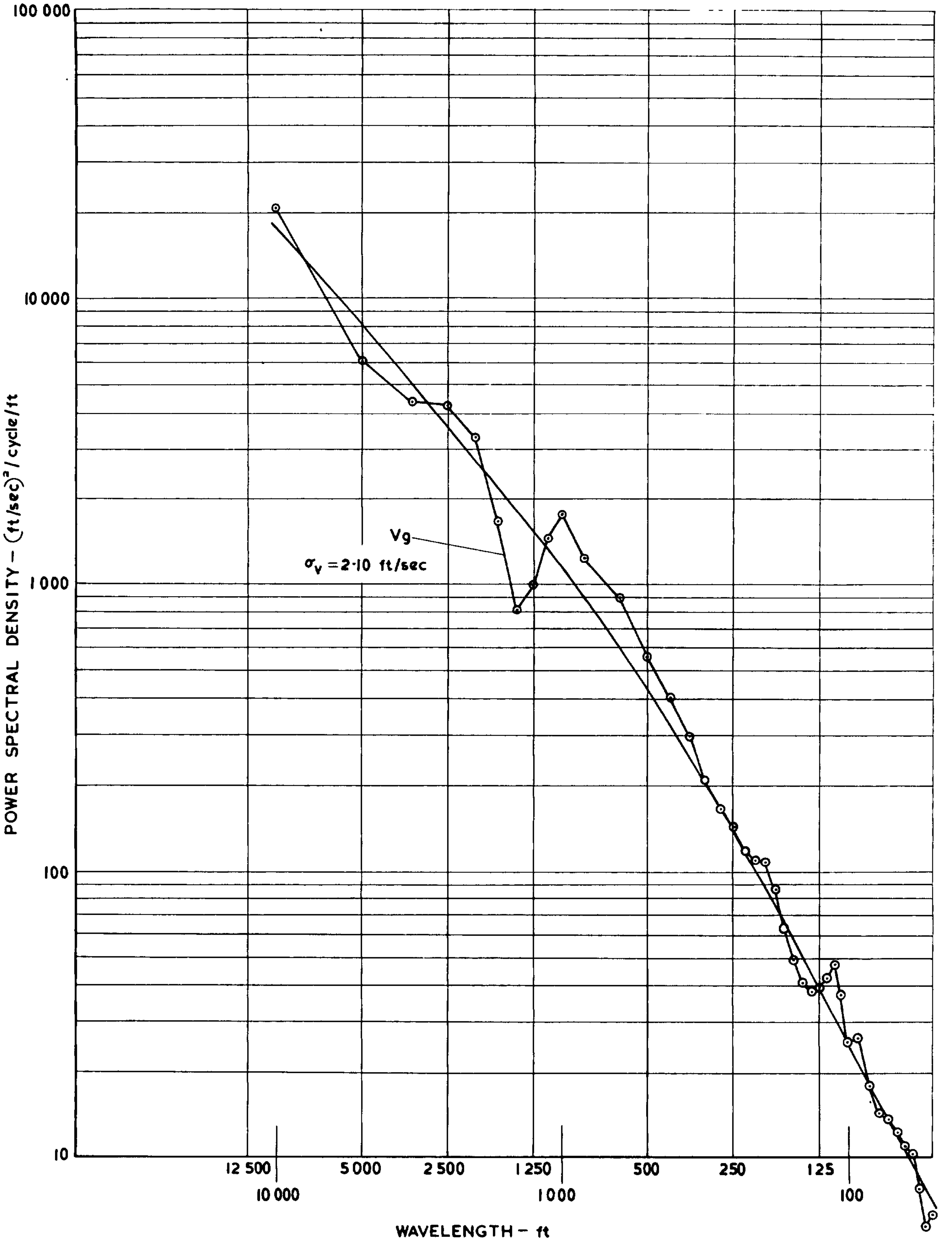

FIG. 5(b) POWER SPECTRA OF LATERAL COMPONENT OF TURBULENCE
FLIGHT No 44 RUN D-LOW LEVEL

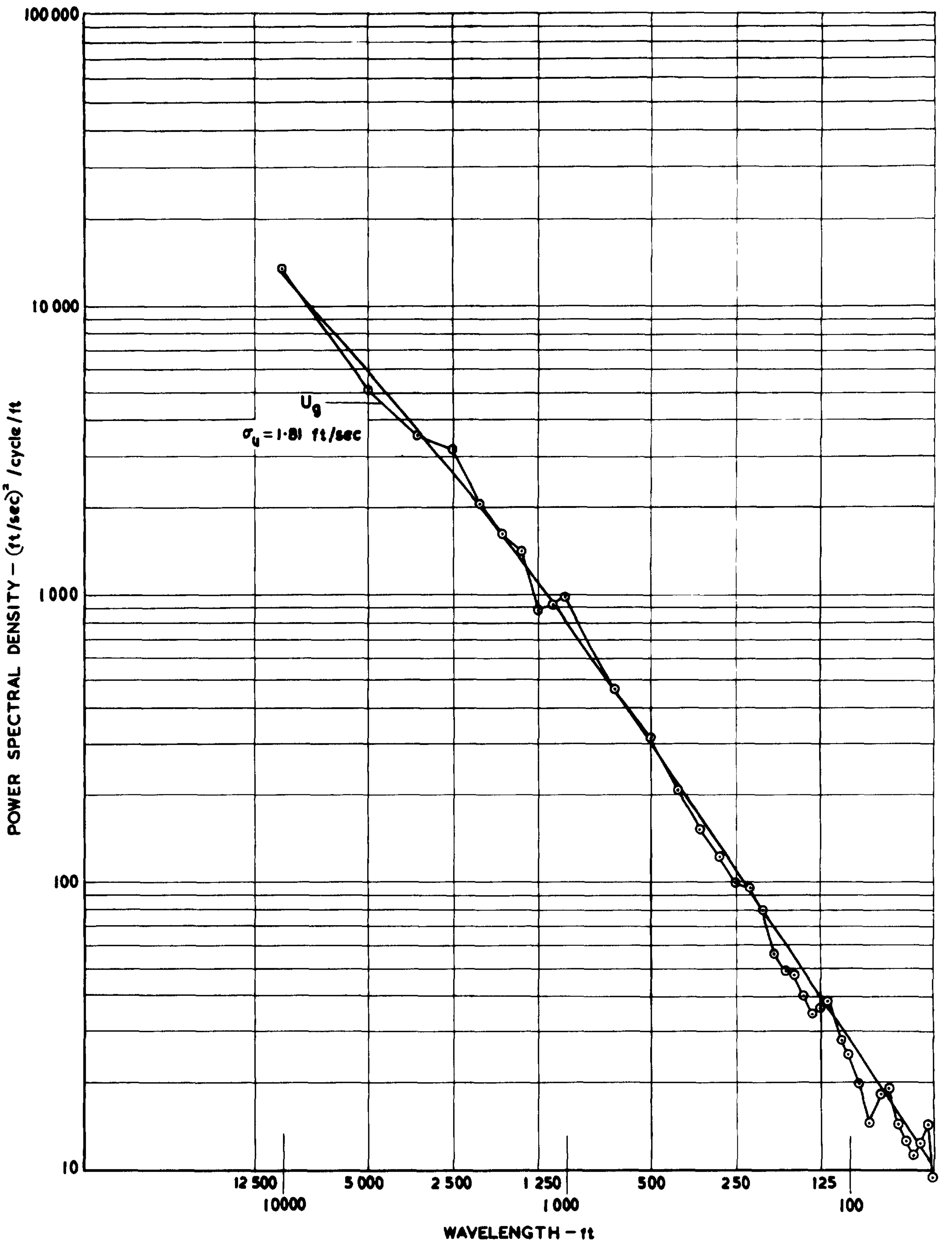

FIG. 5 (c) POWER SPECTRA OF LONGITUDINAL COMPONENT OF TURBULENCE
FLIGHT No 44 RUN D-LOW LEVEL

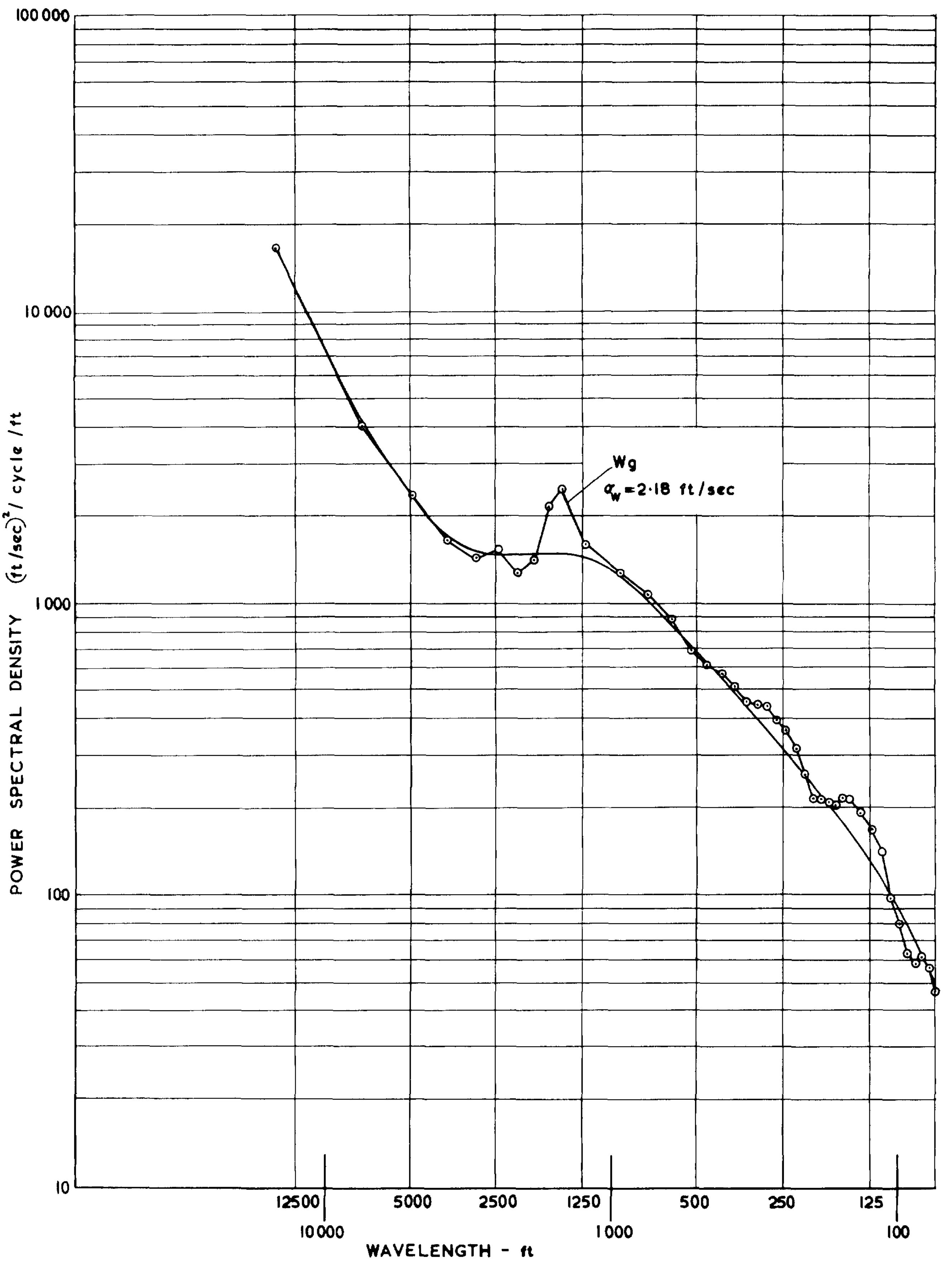

FIG. 6(a) POWER SPECTRA OF VERTICAL COMPONENT OF TURBULENCE
FLIGHT No. 45 RUN H

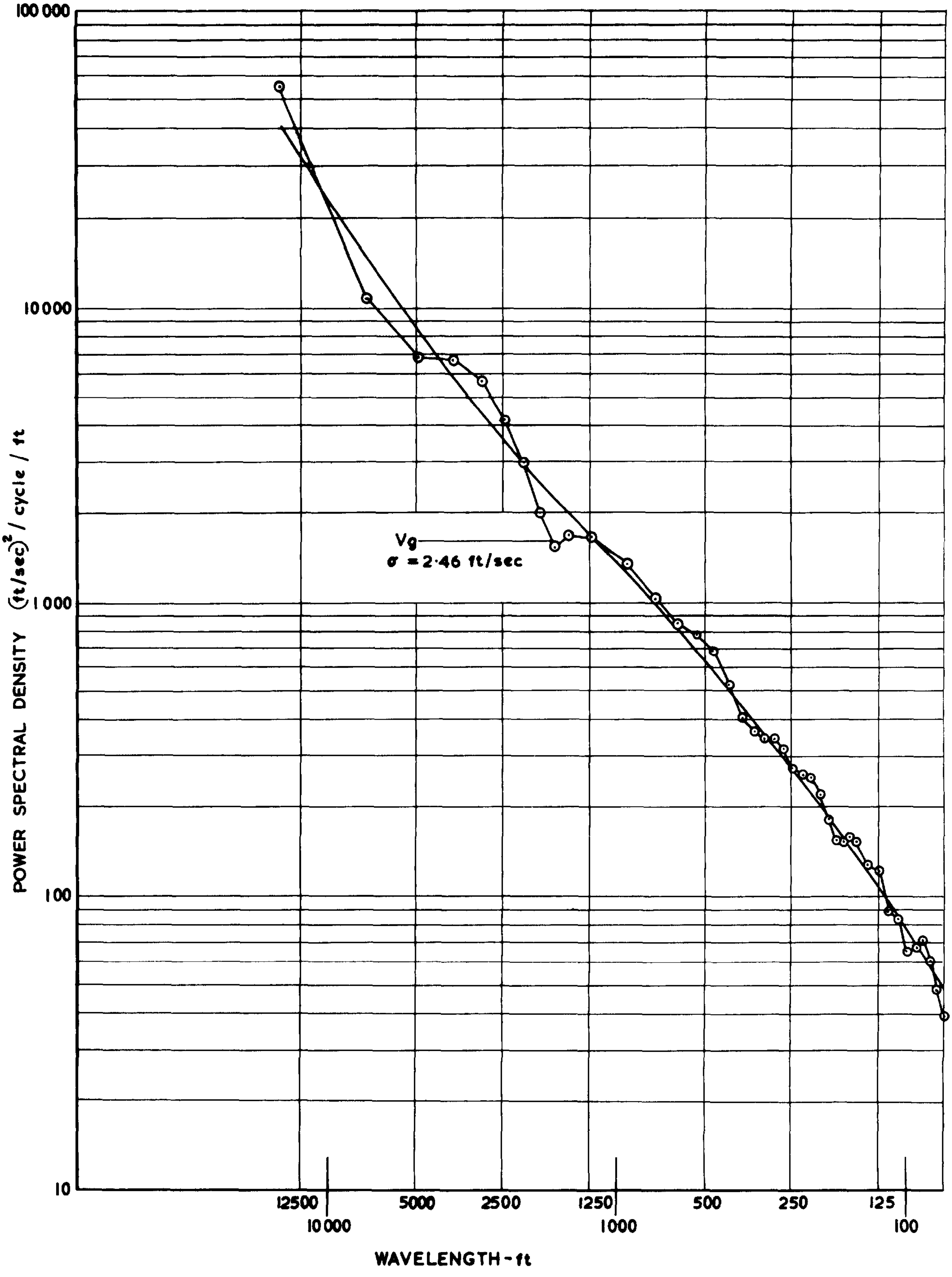

FIG 6 (b) POWER SPECTRA OF LATERAL COMPONENT OF TURBULENCE
FLIGHT No 45 RUN H

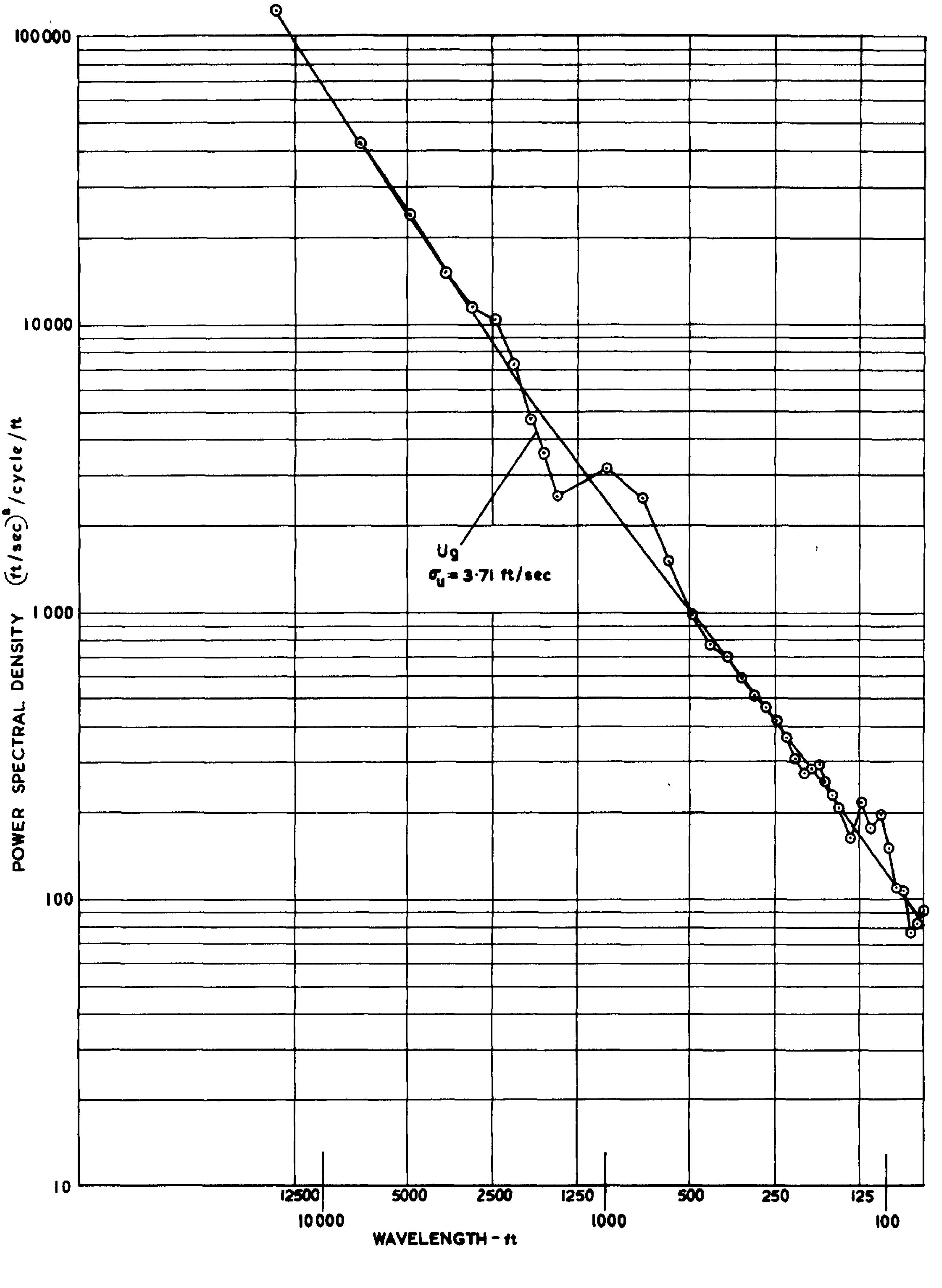

FIG 6 (c) POWER SPECTRA OF LONGITUDINAL COMPONENT OF TURBULENCE
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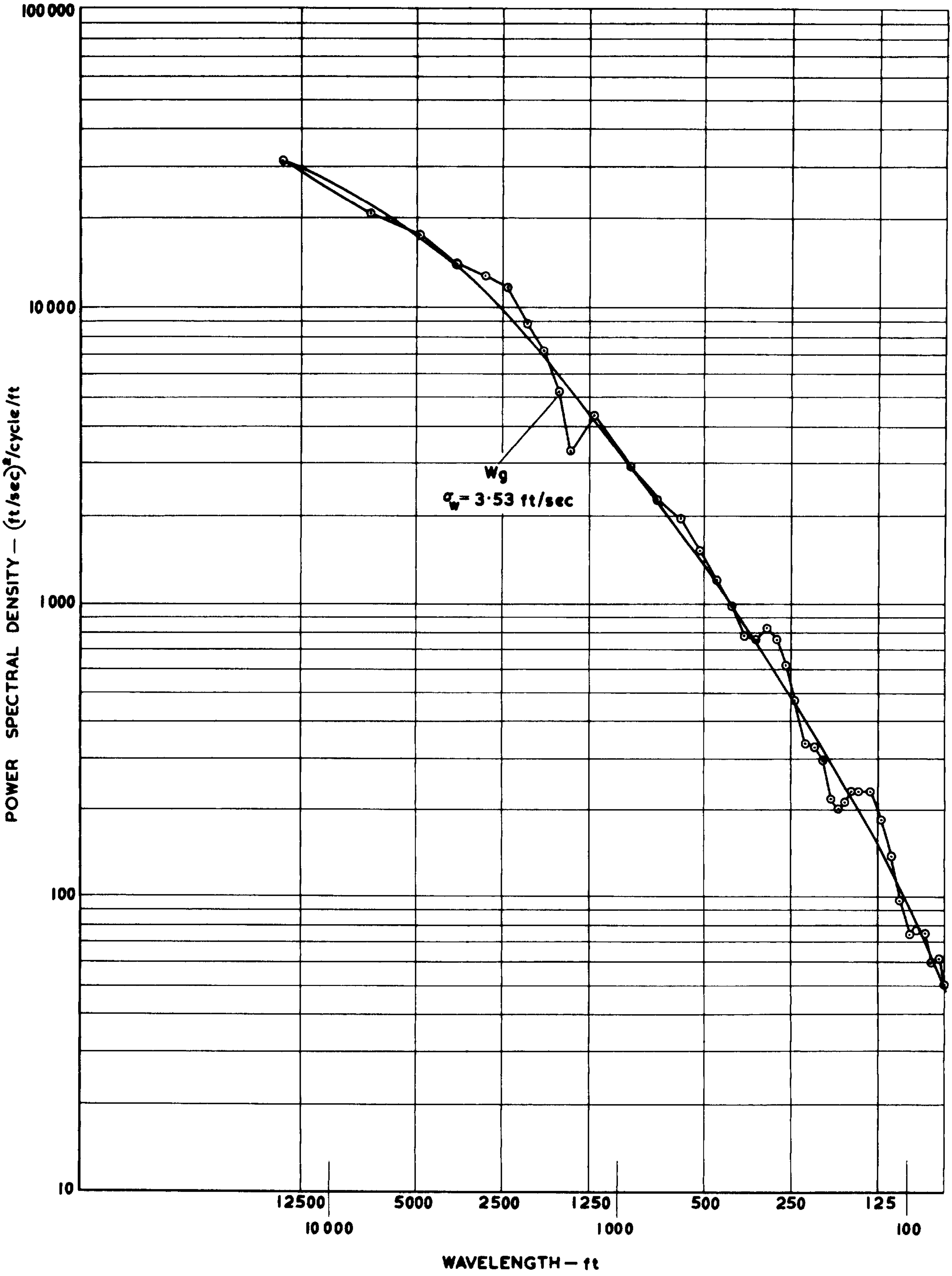

FIG 7(a) POWER SPECTRA OF VERTICAL COMPONENT OF TURBULENCE
FLIGHT No 46 RUN E

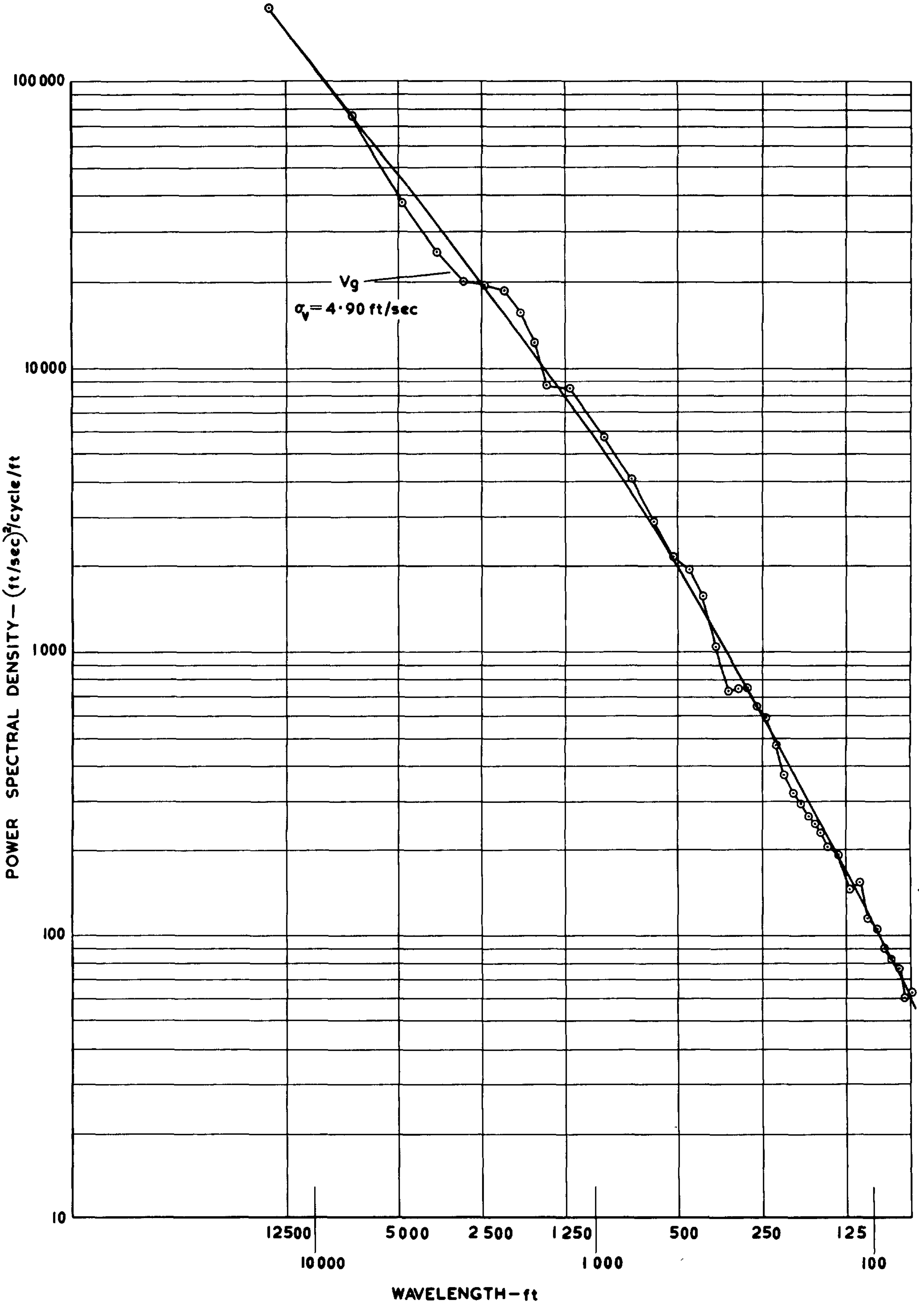

FIG 7 (b) POWER SPECTRA OF LATERAL COMPONENT OF TURBULENCE

FLIGHT No 46 RUN E

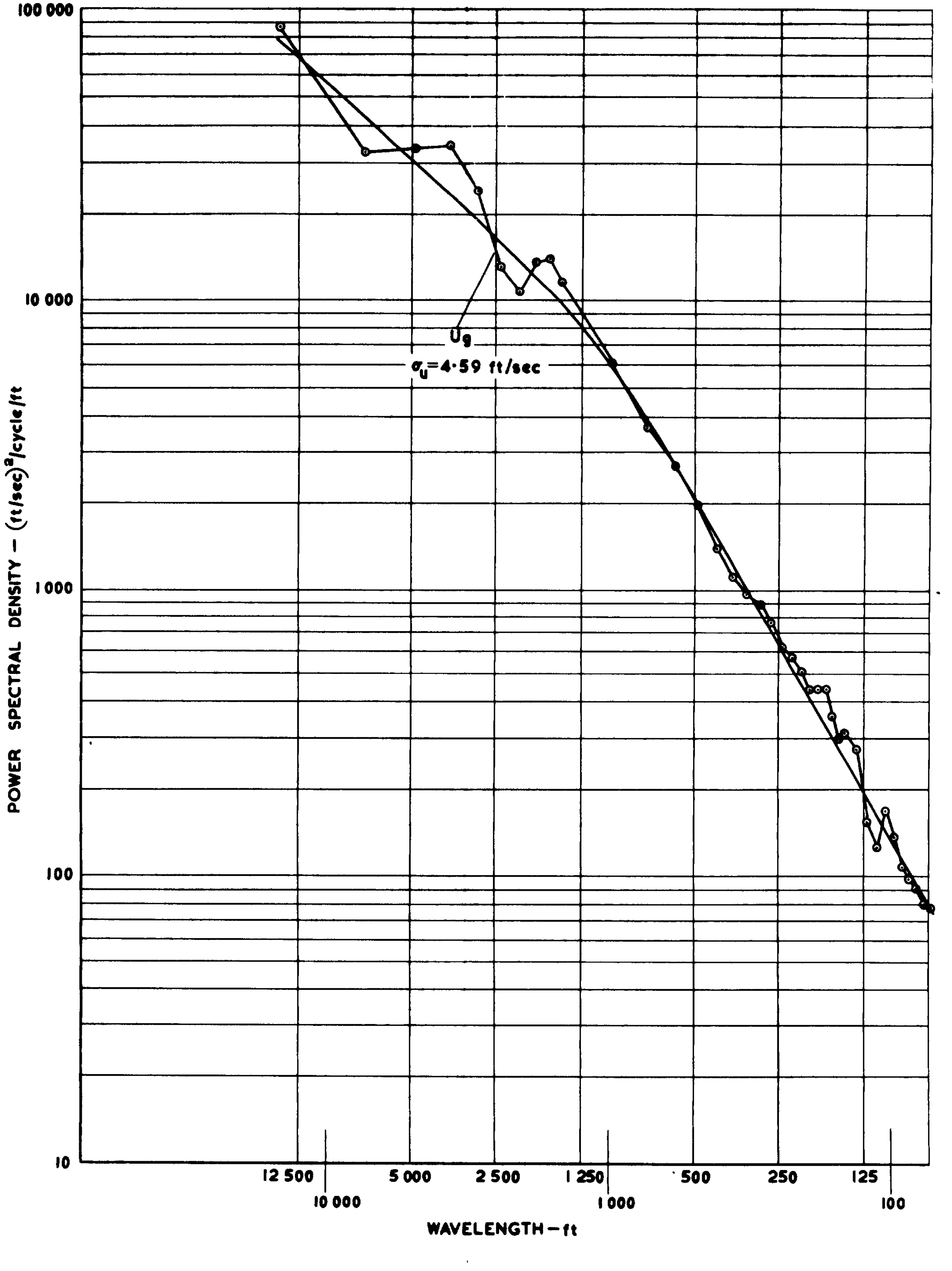

FIG 7(c) POWER SPECTRA OF LONGITUDINAL COMPONENT OF TURBULENCE

FLIGHT No 46 RUN E

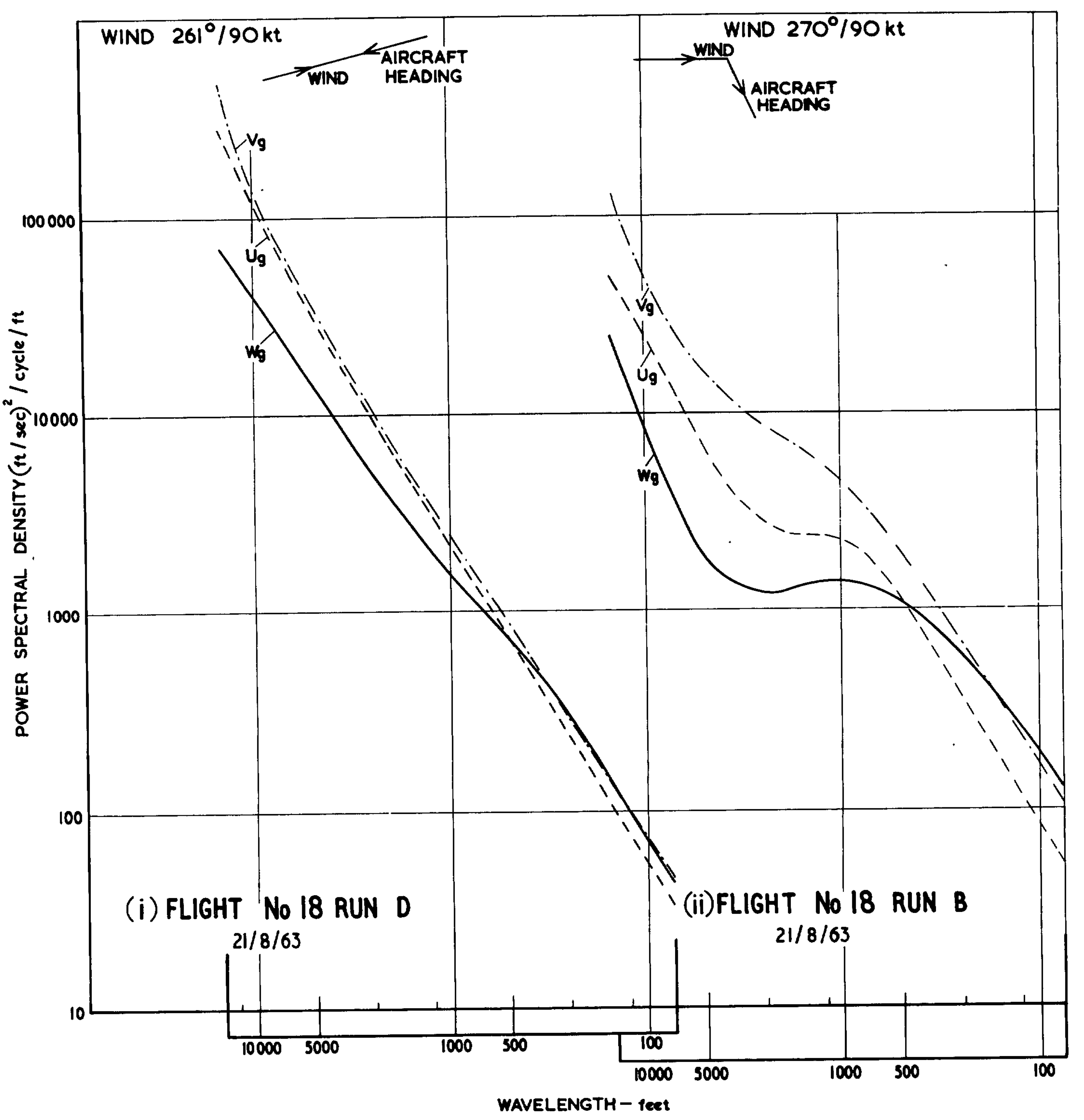

FIG 8(a) POWER SPECTRA OF THREE COMPONENTS OF TURBULENCE

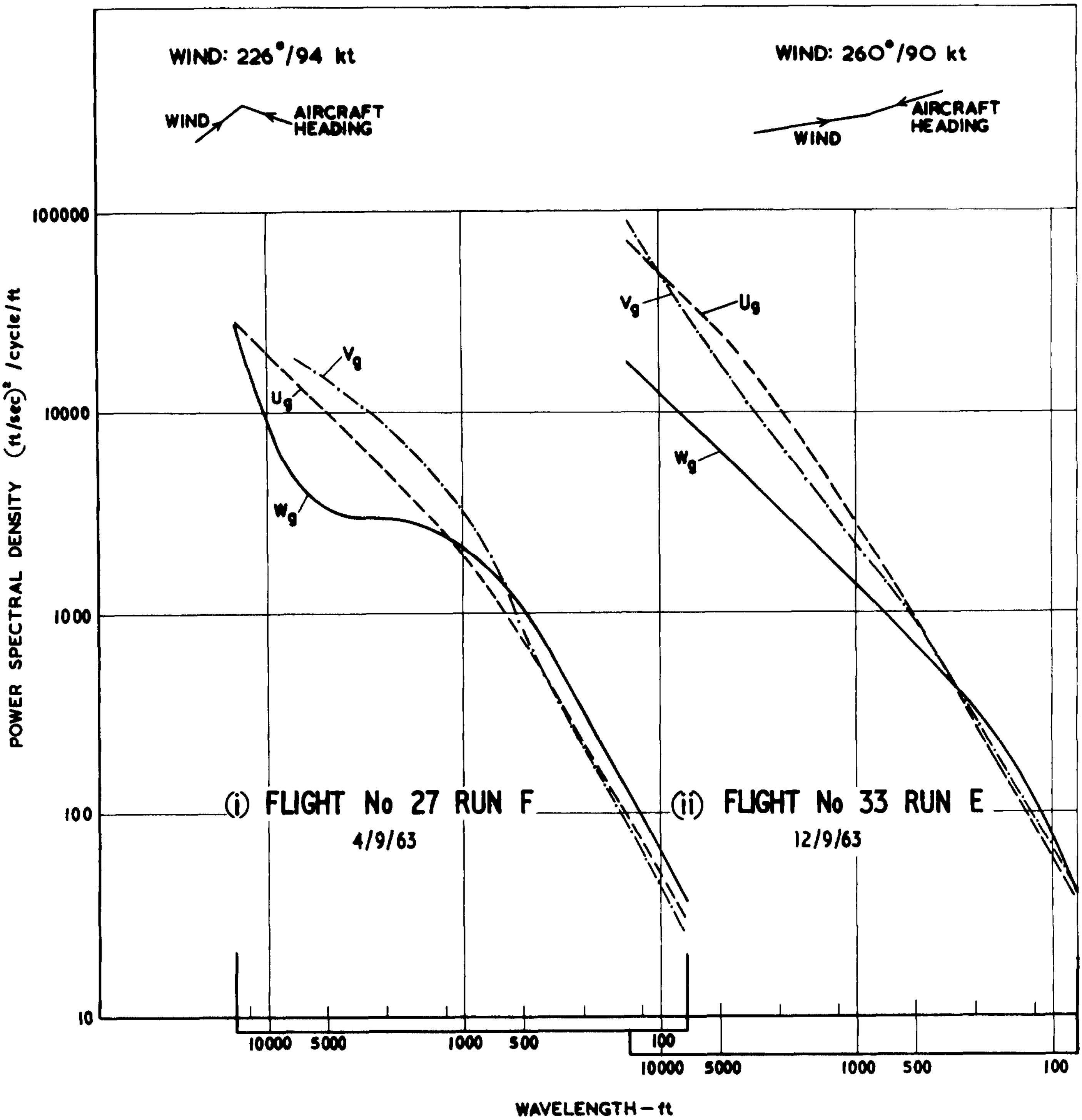

FIG 8(b) POWER SPECTRA OF THREE COMPONENTS OF TURBULENCE

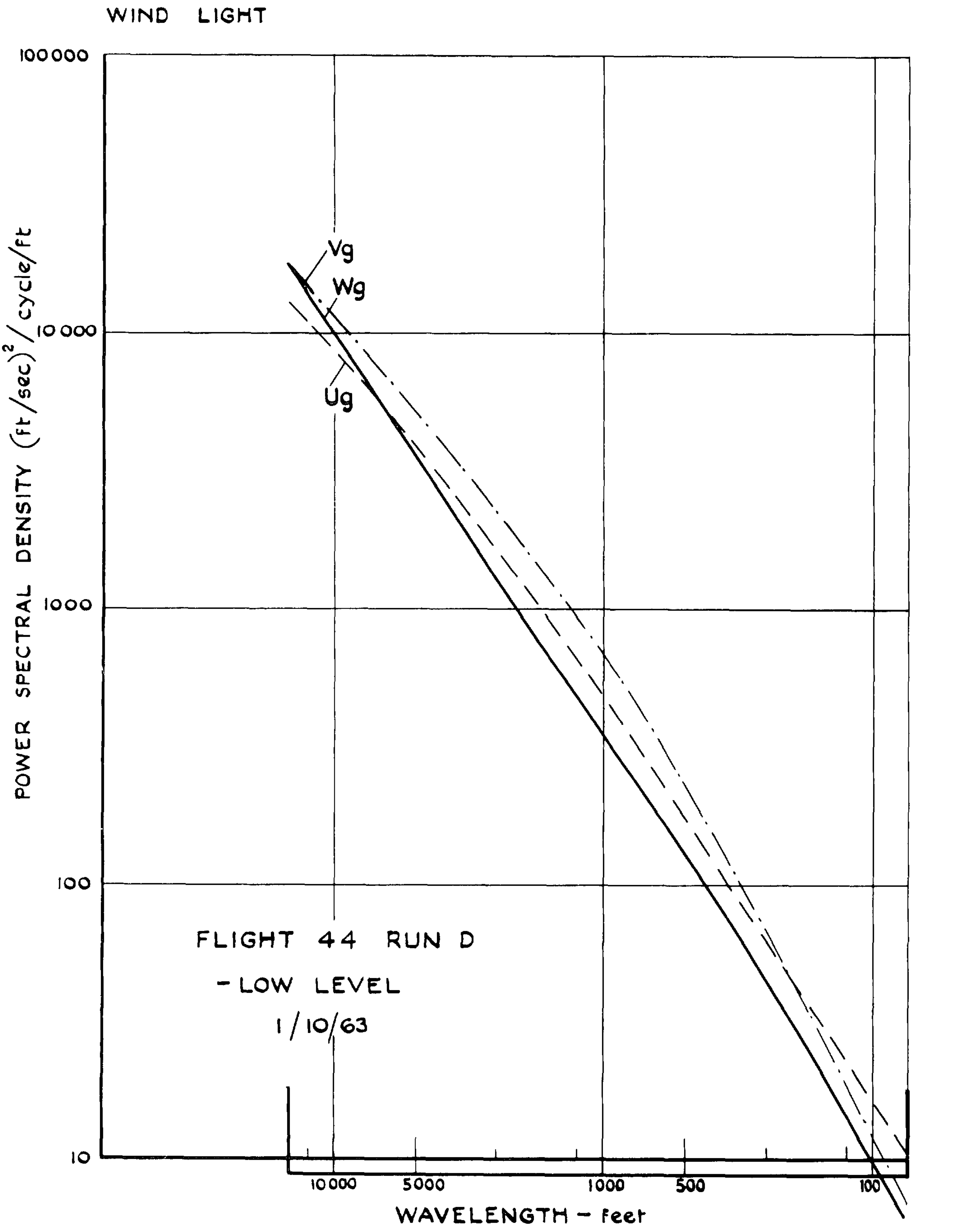

FIG 8 (c) POWER SPECTRA OF THREE COMPONENTS OF TURBULENCE

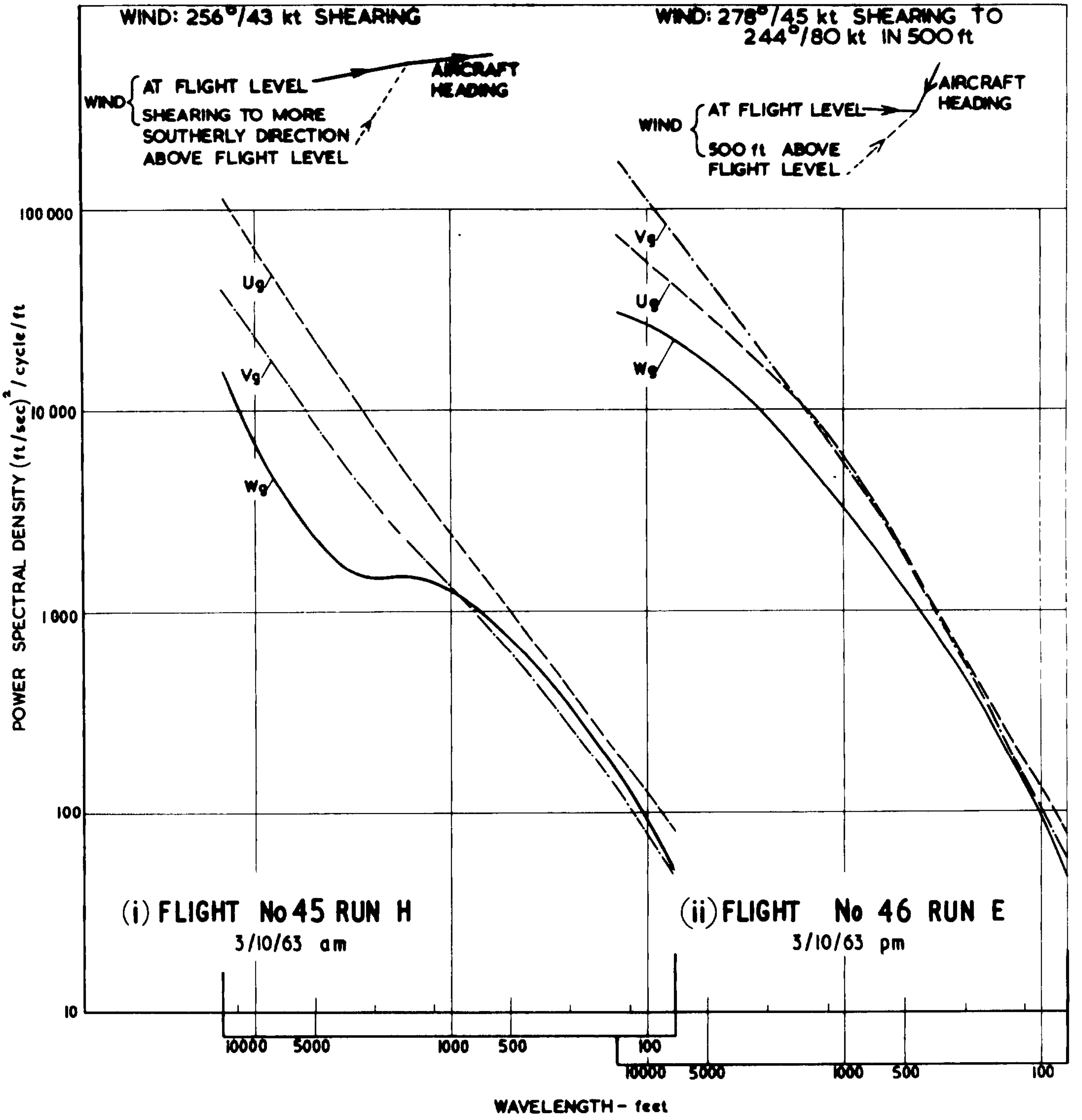

FIG 8(d) POWER SPECTRA OF THREE COMPONENTS OF TURBULENCE

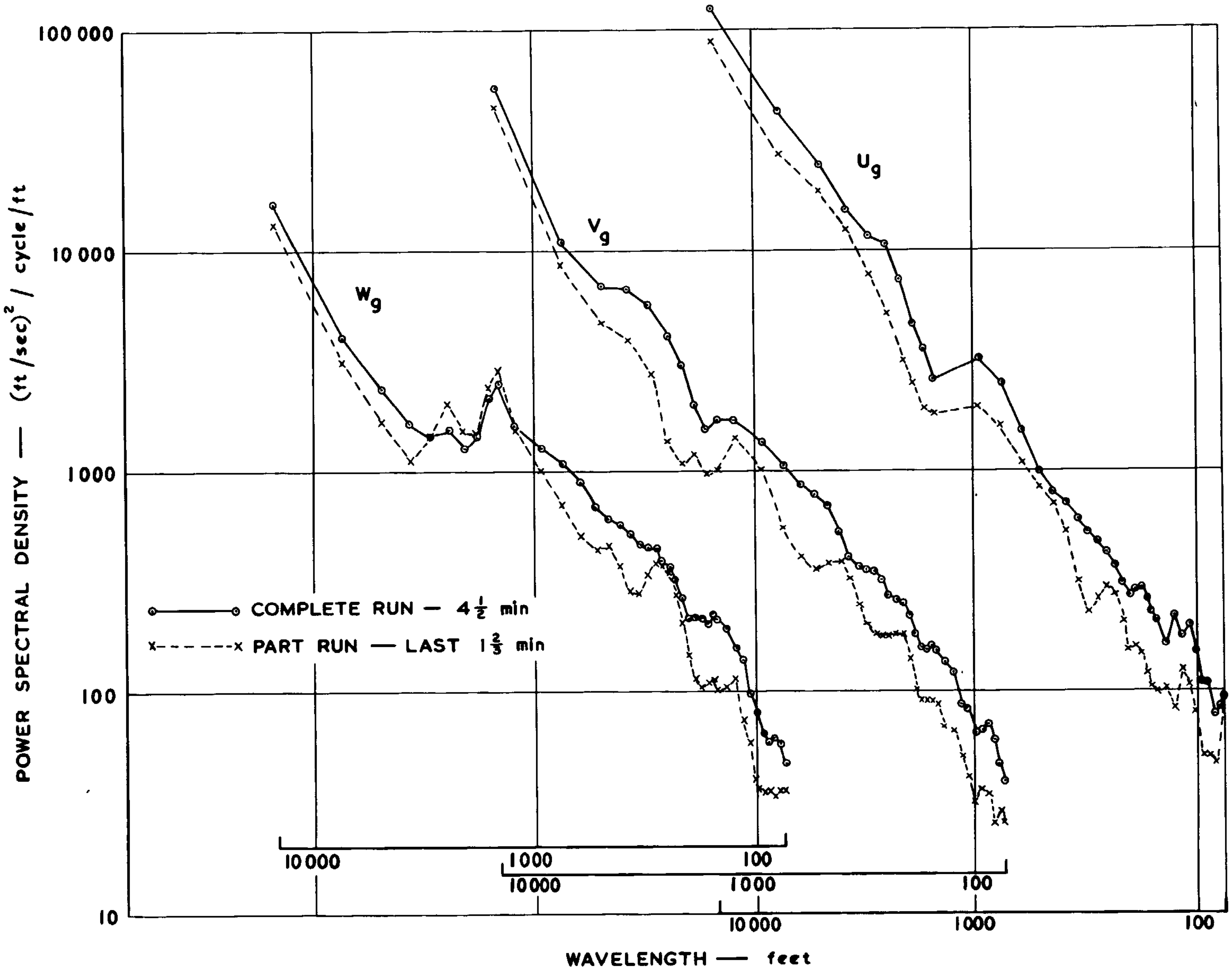

FIG 9 COMPARISON OF SPECTRA FOR COMPLETE RUN AND PART OF RUN
FLIGHT No 45 RUN H

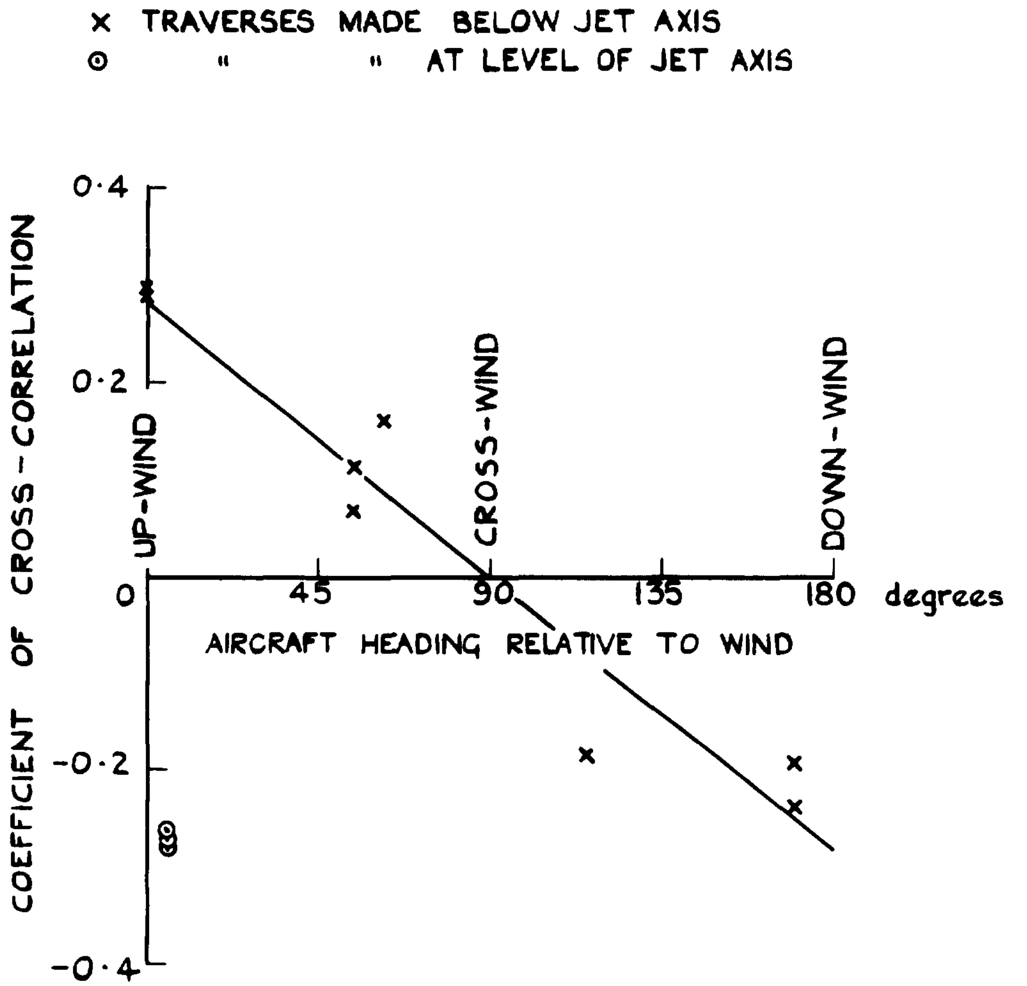

FIG. 10 VARIATION IN CROSS CORRELATION OF ϖ_g AND u_g (PREWHITENED) WITH RESPECT TO AIRCRAFT HEADING RELATIVE TO WIND

N.B. ϖ_g AND u_g ARE MEASURED RELATIVE TO AIRCRAFT AXES

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