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# A Use of Arbitrary Modes in Control-Surface Flutter Investigations

*by*

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# A Use of Arbitrary Modes in Control-Surface Flutter Investigations

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SUMMARY

A method of determining the kind of distortion modes of an aircraft that are likely to lead to control-surface flutter is described. In it arbitrary modes of flexure and torsion are selected and, for given critical flutter speeds and frequencies, the flutter equations are solved for the amplitude ratio between the arbitrary modes and the frequency of the resulting composite mode. The method has been used in an accident investigation to indicate the type of flutter that might ensue from a structural weakening of the wing. It is thought that the method may also have useful applications in design.

* Replaces R.A.E. Technical Note No.Structures 148 - A.R.C. 17826

This paper was written originally in 1955, when it had limited circulation. Because it is felt to be of general interest even today, it is being reissued with a limited amount of revision with a view to wider circulation.

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## 1 INTRODUCTION

Some time ago an aircraft suffered a structural failure in flight which quickly resulted in its disintegration. Eye-witness accounts and ciné films taken from the ground established that the wing tips appeared as blurs just before they broke away, pointing to flutter as a possible cause of the accident. A theoretical investigation of the flutter possibilities was then made at the R.A.E.

The most likely form of flutter was wing-aileron flutter and conventional calculations were made using the measured ground resonance modes of the aircraft, but the only flutter speeds obtained were well beyond the capabilities of the aircraft. It was thought, however, that the wing might have been weakened. The conventional type of calculation could easily allow for weakening in so far as it might have affected the modal frequencies (i.e. the overall stiffnesses), but consideration of the effect of possible changes in the model shapes, which could be important, required a different approach. This Note describes the investigation that was made of the latter aspect.

A direct approach to the investigation would have been to specify the structural changes considered likely due to weakening, calculate the corresponding changes in the normal modes, and then calculate the flutter characteristics appropriate to the modified modes. The approach actually adopted was to calculate the types of mode that would give flutter at a specified speed and frequency, and then consider which of these types of mode, if any, might have resulted from a structural weakening. This inverse method is considered to have useful applications in design as well as in accident investigations of the kind described here.

## 2 BASIS OF THE INVESTIGATION

Attention is confined to wing-aileron flutter since, at the time of the investigation, this was deemed more likely. The extension of the method to other forms of flutter, such as wing flexure-torsion, is straightforward.

The wing of the aircraft is allowed to distort in a mode composed of simple arbitrary modes of flexure and torsion, the amplitude ratio of these components being left variable. Binary flutter equations with the composite mode and aileron rotation as degrees of freedom are derived. These equations are solved for the amplitude ratio of the component modes and the frequency of the composite mode for an assumed critical flutter speed and frequency (and therefore frequency parameter). Whether it is an upper or lower critical speed can be determined by substituting the values of the amplitude ratio and modal stiffness in the original equations and applying the stability tests given by Templeton[1].

The procedure is repeated for different flutter frequencies and modal stiffnesses so that a family of modes giving flutter at a specified speed is obtained. These modes are then compared with the ground resonance modes of the aircraft. If any of the ground resonance modes are similar to the critical modes the pertinent flutter coefficients can be compared. From this comparison it is comparatively easy, with binary coefficients, to determine the amount the ground resonance mode (or aerodynamic derivatives if these are in doubt) needs to be in error to promote flutter.

There follows a more detailed account of the calculations that were made as part of the accident investigation.

## 3 DETAILS OF CALCULATION

For simplicity the distortion in the arbitrary modes is confined to the outboard parts of the wings. This limitation is considered justified in view of the minor importance of the rest of the aircraft in aileron flutter and the qualitative nature of the investigation. The arbitrary modes are parabolic flexure and linear torsion about the wing leading edge. The deflection of any point on the wing is therefore given by

$$z = z_\ell + \alpha x =$$

$$= fc_r + Fx = (\eta - 0.4)^2 c_r 1(\eta - 0.4) + P(\eta + R/P - 0.5) x 1(\eta - 0.5)$$

where $z_\ell$ is the local displacement at the wing leading edge,

$\alpha$ is the local incidence of the wing,

x is the distance downwind of the leading edge,

f and F are flexure and torsion functions respectively,

$c_r$ is a reference chord,

$\eta$ is y/s i.e. the distance from the centre line as a fraction of the semi-span,

1(u), the Heaviside step function, is zero when u is negative and unity when u is positive and

P and R are variables.

The variable R is included to take account of twist of the inboard wing which, although not important in itself, influences the incidence of the outboard wing. The deflection of the leading edge is zero until $\eta$ is larger than 0.4 and this will be representative of the deflections in most ground resonance modes for they are generally small near the middle of the wing. The point $\eta = 0.5$ for the torsion mode is chosen for a similar reason.

The aerodynamic force and moment coefficients are estimated on the basis of strip theory and equivalent constant strip derivatives, the spanwise integrations being carried out analytically. The derivatives are evaluated according to the recommendations for low-aspect-ratio wings by Minhinnick[2]. Steady motion values are assumed for the stiffness derivatives. The damping derivatives are assumed the same as the equivalent stiffness derivatives where possible (e.g. $\ell_{\dot{z}}$ assumed to be $\ell_\alpha$); otherwise they are obtained by a comparison of the three-dimensional steady-motion derivatives and the turning-point values of the two-dimensional damping and stiffness derivatives. The inertia derivatives are given their two-dimensional values. All these derivatives are independent of frequency parameter.

For the evaluation of the inertia coefficients the continuous wing structure is replaced by a number of discrete masses and the aileron divided into chordwise strips. The aileron circuit incorporates a power unit with no feedback. Although the unit is positioned unsymmetrically in the circuit there is no detectable different between the natural frequencies of rotation of the two ailerons, which are about 10 c.p.s.

The equations for binary flutter between the composite wing mode and aileron rotation are derived in the Appendix. The flutter determinant is a function of the modal variables (P and R), the frequency parameter ($\nu$), the airspeed (V) and the stiffness of the aircraft in the composite mode (E). Since the determinant is complex its equation to zero can be solved for two variables. R is given definite values which are likely to result in reasonable modes. The values of frequency parameter chosen are those usually encountered in control surface flutter - chiefly a value of unity. The airspeed at which the accident occurred is known with a fair degree of accuracy, being estimated at about 1000 ft/sec. The combination of a flutter speed of

1000 ft/sec and a frequency parameter of unity gives a flutter frequency of 17.3 c.p.s., which is above the aileron frequency, as it should be, and is not at variance with the order of frequency indicated by the ciné film record.

## 4 RESULTS OF CALCULATION

The modes obtained from the calculation are presented in Figs.1 and 2. For most of them the flutter frequency parameter is unity which corresponds to a flutter frequency of 17.3 c.p.s. at the assumed flutter speed of 1000 ft/sec at sea level. All the modes have a large amount of torsion and their nodal lines run in spanwise directions. The number of modes obtained is limited but it is clear that any mode in which the nodal line intersects the tip between the aileron mass-balance and the trailing edge, or just in front of the leading edge, is likely to promote flutter.

As part of the accident investigation, these critical modes were compared with the ground resonance modes of the aircraft. The latter included an antisymmetric mode at 18.8 c.p.s. whose nodal line ran in a spanwise direction and intersected the tip just in front of the trailing edge (see Fig.3(a)). Attention was then transferred to this and no attempt was made to find any further critical modes. It is unlikely that a predominantly flexural mode would give flutter as the aileron mass-balance arrangement is probably quite effective in such a mode. A binary calculation with the ground resonance mode at 18.8 c.p.s. confined to the outboard parts of the wing gave no flutter. However, increasing the torsion in this mode by 32%, the leading edge displacement and the modal frequency being unchanged (see Fig.3(b)), flutter occurred at a speed of 930 ft/sec and a frequency parameter of 1.2.

Whilst a probable explanation of the primary failure of the aircraft has been found on strength grounds, films of the accident suggest that the aircraft vibrated before breaking up. If this is so, it seems likely that the outer parts of the wing broke away as a result of aileron flutter in a mode similar to the mode at 18.8 c.p.s., the modification of the mode necessary to make it critical arising from the primary strength failure in the main structure. There is no suggestion that the aircraft lacked a satisfactory margin of safety in its original condition.

## 5 USES OF THE TECHNIQUE

The technique described in this Note proved useful in an accident investigation and should be useful in other accident investigations[3] where the flutter mode is not known or where none of the known modes give flutter.

The technique might also be useful in proving the mass-balance systems of new aircraft. It is sometimes the practice that control-surface flutter calculations are not made until the ground resonance modes are available; that is until the aircraft is ready to fly. The ground resonance modes are then examined and possibly dangerous modes selected for flutter calculations; and full flight clearance of the aircraft must await a satisfactory outcome of these calculations. The time this process takes is often a serious matter, and the position might be eased if simple calculations were made in the design stage to indicate what types of mode are likely to prove dangerous.

## 6 CONCLUSIONS

A method of calculating types of mode of an aircraft that give control-surface flutter has been described. The method has been used in an accident investigation and has provided useful results. It is thought that the method has other useful applications.

## REFERENCES


| No. | Author | Title, etc. |
| --- | --- | --- |
| 1 | H. Templeton | The technioue of flutter calculations. A.R.C. C.P.172. April 1953 |
| 2 | I.T. Minhinnick | Aerodynamic derivatives - Paper No.4 of a symposium on the flutter problem in aircraft design (edited by H. Templeton and G.R. Brooke) A.R.C. 16,081. May 1953 |
| 3 | H.F.L. Pinkney | Analysis of the incidence of a tailplane flutter arising from modified stiffness and elastic coupling. N.A.E. Report LR-320. December 1961 |

## APPENDIX I

### Derivation of binary flutter equations and form of solution

The flutter equations are derived, as is usual[1], from the Lagrangian equations of motion. Let $q_1$ and $q_2$ be the generalised coordinates and the motion be simple harmonic so that $\ddot{q}_i = -\omega^2 q_i$.

The deflection of any point on the wing is

$$z = (fc_r + Fx)q_1 \tag{A.1}$$

where

$$\left.\begin{aligned} f &= (\eta - 0.4)^2 \, 1(\eta - 0.4) \\ \text{and} \quad F &= P(\eta + R/P - 0.5) \, 1(\eta - 0.5) \end{aligned}\right\} \tag{A.2}$$

A point on the aileron has a further deflection

$$z_a = x_a \, q_2 \tag{A.3}$$

where $x_a$ is the distance downwind of the aileron hinge line.

The structural kinetic energy is

$$T = \tfrac{1}{2} \sum \delta \, m(\dot{z} + \dot{z}_a)^2 = \tfrac{1}{2} \sum \delta m \, \{(fc_r + Fx)\dot{q}_1 + x_a \, \dot{q}_2\}^2 \tag{A.4}$$

Putting

$$\frac{d}{dt} \cdot \frac{\partial T}{\partial \dot{q}_i} = \hat{A}_{ii} \, \ddot{q}_i + \hat{A}_{ij} \, \ddot{q}_j \tag{A.5}$$

gives

$$\left.\begin{aligned} \hat{A}_{11} &= \int \{mc_r^2 \, f^2 + 2 \, \overline{mx} \, c_r \, fF + \overline{mk}^2 \, F^2\} \, s \, d\eta \; , \\ \hat{A}_{12} &= A_{21} = \int \{\overline{mx}_a \, c_r \, f + (\overline{mk}_a^2 + d \cdot \overline{mx}_a)F\} \, s \, d\eta \\ \text{and} \quad \hat{A}_{22} &= \int \overline{mk}_a^2 \, s \, d\eta \end{aligned}\right\} \tag{A.6}$$

where $m$ is the mass of the wing per unit span,

$\overline{mx}$ is the first moment of wing about its leading edge per unit span,

$\overline{mk}^2$ is the second moment of the wing about its leading edge per unit span,

$\overline{mx}_a$ is the first moment of the aileron about its hinge line per unit span,

$\overline{mk}_a^2$ is the second moment of the aileron about its hinge line per unit span,

and $d$ is the distance from the aileron hinge line to the wing leading edge.

The structural stiffness in the composite mode is given by the final solution. The structural stiffness in the aileron degree-of-freedom is given by its natural frequency in still-air and its inertia coefficient, for

$$\left.\begin{aligned} \frac{d}{dt} \cdot \frac{\partial T}{\partial \dot{q}_2} + \frac{\partial V_e}{\partial q_2} &= 0 \\ \text{i.e.} \qquad -\omega_2^2 A_{22} + E_{22} &= 0 \end{aligned}\right\} \qquad \text{A.7}$$

where

$$\frac{\partial V_e}{\partial q_2} = E_{22}\, q_2$$

and $\omega_2$ is the natural frequency of the aileron.

The work done by the aerodynamic forces in a small displacement,

$$\left.\begin{aligned} \delta W &= -L\delta z + M\delta\alpha + H\delta\beta = Q_1\,\delta q_1 + Q_2\,\delta q_2 = \\ &= -\rho V^2 s \int_0^1 c\left(L_z \frac{c_r f q_1}{c} + L_\alpha F q_1 + L_\beta q_2\right) c_r f\, \delta q_1\, d\eta \\ &\quad + \rho V^2 s \int_0^1 c^2 \left(M_z \frac{c_r f q_1}{c} + M_\alpha F q_1 + M_\beta q_2\right) F \delta q_1\, d\eta \\ &\quad + \rho V^2 s \int_{ail} c^2 \left(H_z \frac{c_r f q_1}{c} + H_\alpha F q_1 + H_\beta q_2\right) \delta q_2\, d\eta \end{aligned}\right\} \qquad \text{A.8}$$

where

$$L_z = -\nu^2 \ell_{\ddot{z}} + i\nu\, \ell_{\dot{z}} + \ell_z\,, \quad \text{etc.}$$

$$\nu = \frac{\omega c}{V}$$

Putting

$$Q_i = -\rho V^2 s c_r^2 \left(\delta_{ii}\, q_i + \delta_{ij}\, q_j\right) \qquad \text{A.9}$$

where

$$\delta_{ij} = -\nu_r^2\, \bar{a}_{ij} + i\nu_r\, b_{ij} + c_{ij}\,, \quad \nu_r = \frac{\omega c_r}{V}$$

gives

$$\left.\begin{aligned}
c_{11} &= \int \left\{ f^2 \ell_z + \frac{c}{c_r} fF(\ell_\alpha - m_z) + \left(\frac{c}{c_r}\right)^2 F^2(-m_\alpha) \right\} d\eta \\
c_{12} &= \int \left\{ \frac{c}{c_r} f\ell_\beta + \left(\frac{c}{c_r}\right)^2 F(-m_\beta) \right\} d\eta \\
c_{21} &= \int \left\{ \frac{c}{c_r} f(-h_z) + \left(\frac{c}{c_r}\right)^2 F(-h_\alpha) \right\} d\eta \\
c_{22} &= \int \left(\frac{c}{c_r}\right)^2 (-h_\beta)\, d\eta
\end{aligned}\right\} \qquad \text{A.10}$$

and b and $\bar{a}$ coefficients which can be obtained from the corresponding c coefficients by including additional factors $\frac{c}{c_r}$ and $\left(\frac{c}{c_r}\right)^2$ within the integral and using the appropriate derivatives.

The Lagrangian equations of motion are

$$\frac{d}{dt} \cdot \frac{\partial T}{\partial \dot{q}_i} + \frac{\partial V_e}{\partial q_i} = Q_i \qquad \text{A.11}$$

Dividing them by $\rho V^2 s c_r^2$ to make them non-dimensional they become

$$\left.\begin{aligned}
(-\nu^2 a_{11} + i\nu b_{11} + c_{11} + e_{11} y) q_1 + (-\nu^2 a_{12} + i\nu b_{12} + c_{12}) q_2 &= 0 \\
(-\nu^2 a_{21} + i\nu b_{21} + c_{21}) q + (-\nu^2 a_{22} + i\nu b_{22} + c_{22} + e_{22} y) q_2 &= 0
\end{aligned}\right\} \qquad \text{A.12}$$

where

$$a_{ij} = \frac{\hat{A}_{ij}}{\rho c_r^4 s} + \bar{a}_{ij}$$

$$e_{ij} = \frac{E_{ij}}{\rho c_r^2 s} \quad \text{and} \quad y = \frac{1}{V^2}$$

When numerical values are substituted for $\nu_r$, R and y the direct wing term is quadratic in P and linear in $e_{11}$ with complex coefficients, the cross terms are linear in P with complex coefficients and the direct aileron term is a complex number. The real and imaginary parts of the determinantal equation give two equations which are quadratics in P and linear in $e_{11}$. $e_{11}$ is eliminated from them and the resulting quadratic is solved for P.

The roots P will not necessarily be real but only real roots have a meaning since it is assumed in the derivation of the equations that all points on the wing are either in or out of phase when it is oscillating in the composite mode. If P is real $e_{11}$ will be real but not necessarily positive.

From physical considerations the roots P only have meaning if the associated $e_{11}$ the direct stiffness of the composite mode, is positive. The still-air frequency of the isolated composite mode is given by the equation $\omega_1^2 = e_{11}/c_r^2 a_{11}$. If P and $e_{11}$ are real the coefficients in equations A.12 will be real and the solution of these equations by equating the real and imaginary parts of the determinantal equation separately to zero is equivalent to equating the penultimate Routh Test Function to zero. The question of whether the system flutters below or above the assumed critical speed can be resolved by finding the sign of the function for a speed close to that assumed, using the derived values of P and $e_{11}$.

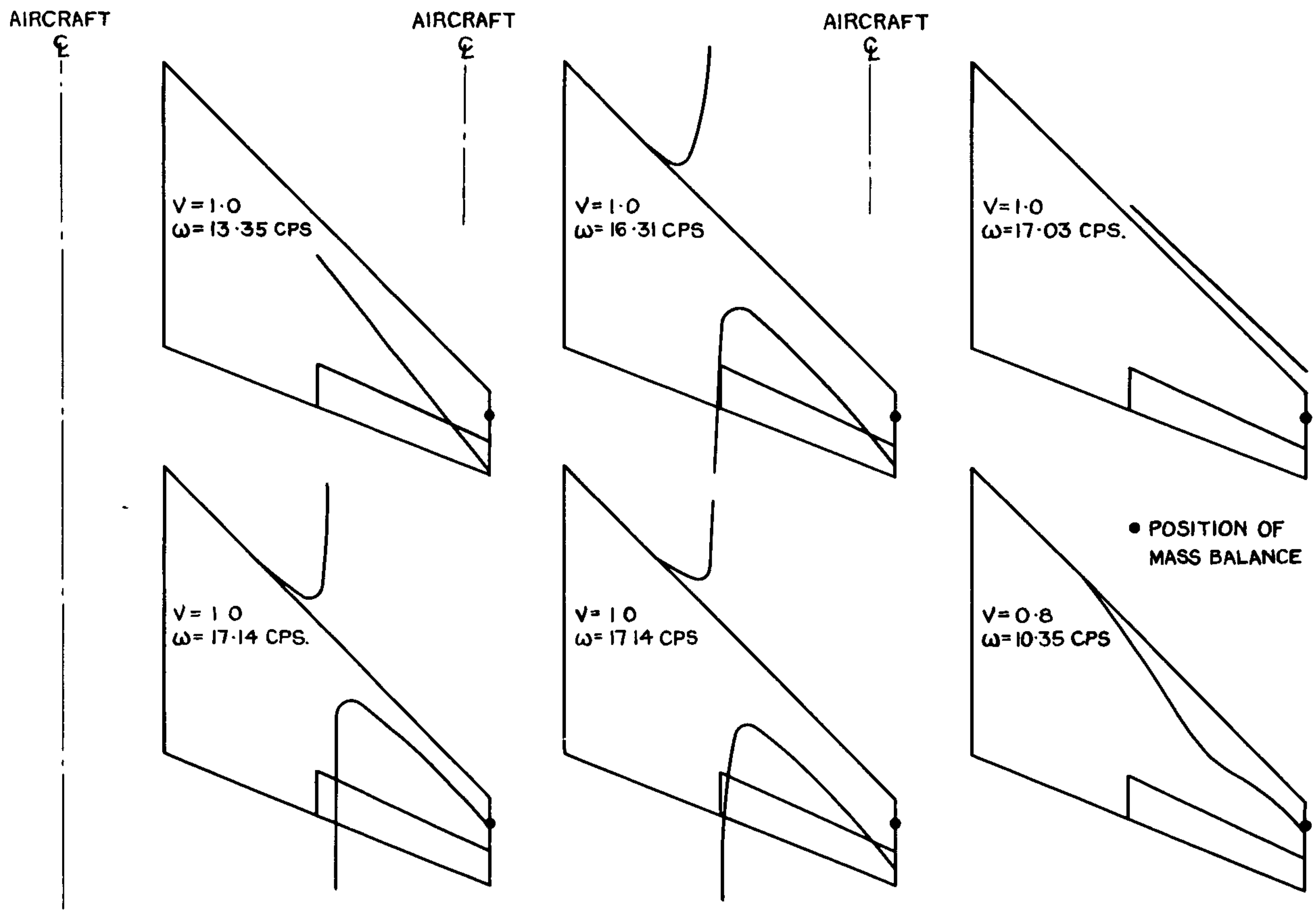



FIG. 1 NODAL LINES OF THE A/C MODES WHICH GIVE AILERON FLUTTER AT 1,000 ft./sec.

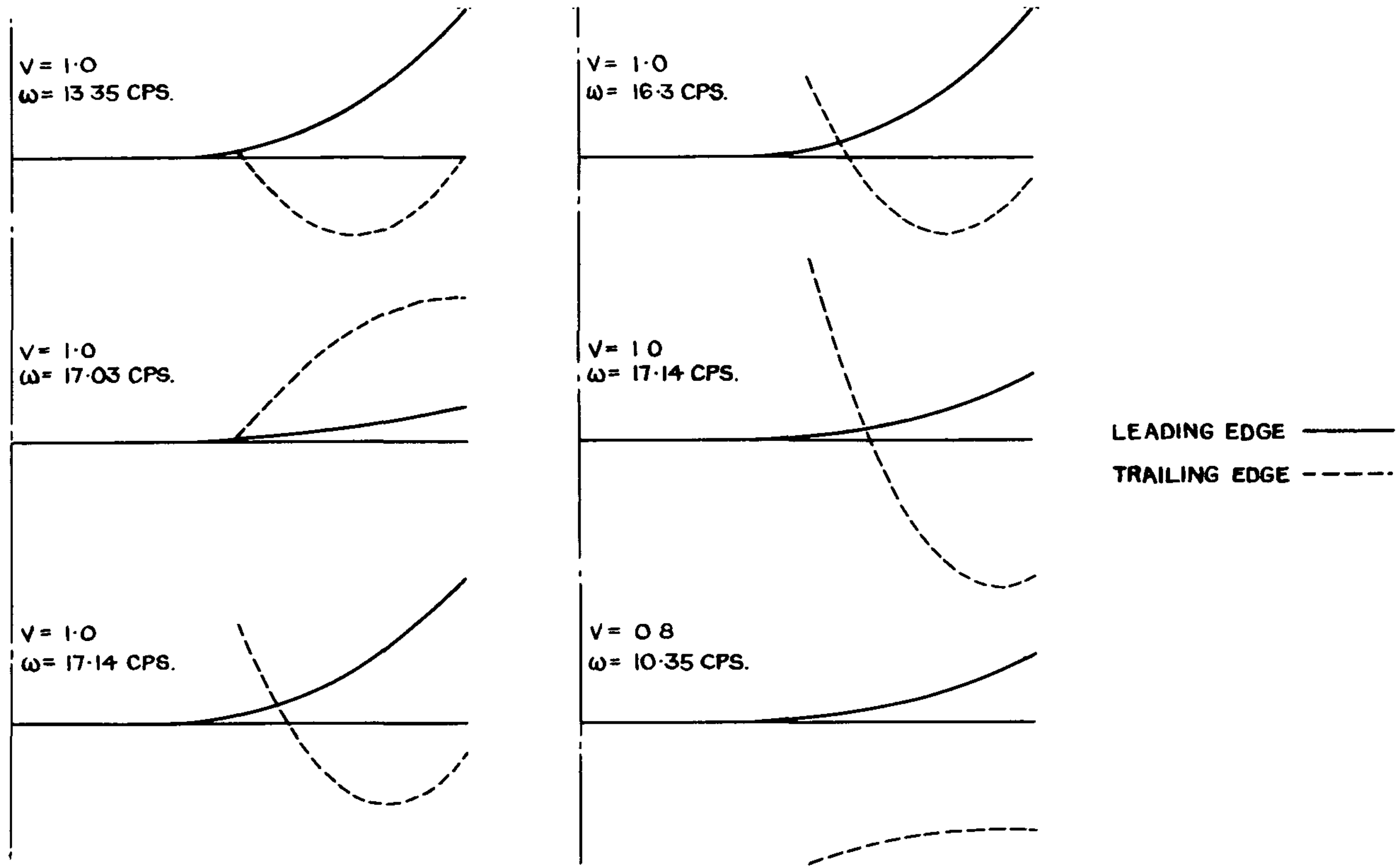
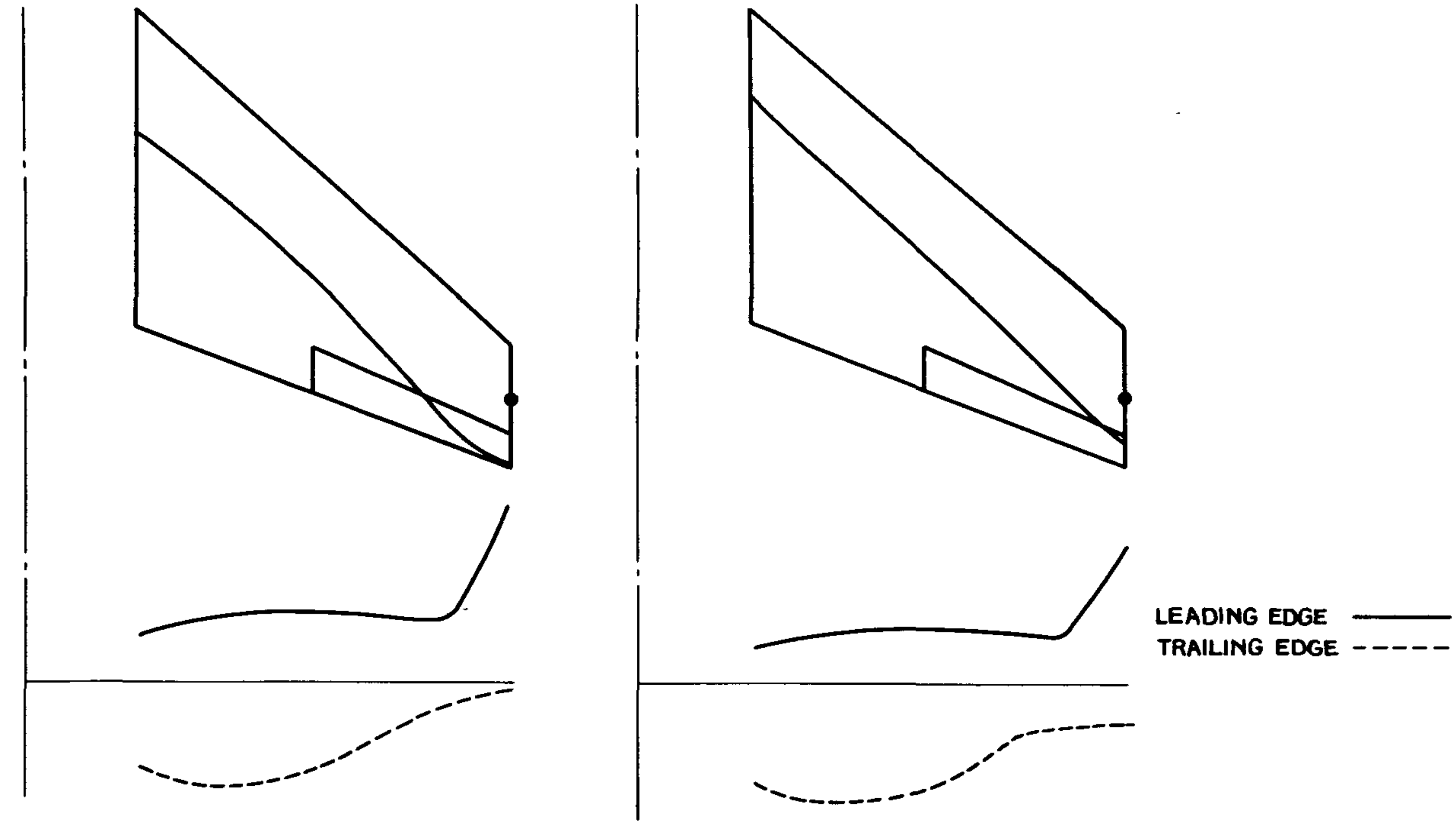



FIG. 2 DISPLACEMENTS IN AIRCRAFT MODES WHICH GIVE AILERON FLUTTER AT 1,000 ft./sec.




(a) ORIGINAL MODE

(b) MODE WITH TWIST INCREASED

FIG.3 ANTISYMMETRIC GROUND RESONANCE MODE AT 18·8 CPS.— ORIGINAL MODE & MODE WITH TWIST INCREASED BY 32%

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