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Supersonic Flutter Derivatives for a Series of Swept and Cropped Delta Wings

by

G. Z. Harris

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SUPERSONIC FLUTTER DERIVATIVES FOR A SERIES OF SWEPT AND CROPPED DELTA WINGS

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SUMMARY

An investigation of the experimental and theoretical determination of flutter derivatives for a series of swept and cropped delta wings was initiated in 1954. This report gives values of the pitching and heaving derivatives for a number of these planforms in the intermediate supersonic speed range. The derivatives have been found using supersonic lifting surface theory.

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CONTENTS

1. INTRODUCTION

An investigation of flutter derivatives was initiated in 1954 to assess the adequacy of the methods available for calculating derivatives by making comparisons between theoretical and experimental derivative values for a family of cropped delta and swept planforms at subsonic, transonic and supersonic speeds. The research programme used the six planforms shown in Fig.1. Derivatives were to be found for various displacement modes, including that of control surface rotation. This report describes calculations undertaken to find derivatives for the intermediate supersonic speed range and gives results for pitching and heaving motion.

The method of calculation used is based on the Multhopp-Richardson lifting surface theory[1], and is described in a separate report[2]. This relies on replacing the integral equation which connects the lift and downwash by a matrix equation connecting the values of the lift and the downwash at two sets of points on the wing. This equation can be solved to give the lift values and expressions can then be derived for the generalised aerodynamic forces when the wing oscillates in a mode of any shape. Standard programmes[3] have been written to carry out computations using the method of Ref.2 and modified versions of these were used on the Ferranti Atlas computer to find the values described in this report.

Most of the theoretical work which was envisaged in the derivative research programme is now complete. For example, at subsonic speeds derivatives for low frequency parameters have been given by Hornsby[4] while at high values of the frequency parameter values have been found by Woodcock[5]; in the purely supersonic speed range, when both leading and trailing edges are supersonic, derivatives have been calculated by Barnes[6]. The experimental part of the programme, covering subsonic, transonic and supersonic speeds, is being undertaken by Hawker Siddeley Dynamics Ltd. and derivatives for rolling, pitching and heaving motion have been reported[7]. A series of measurements of pitching and heaving derivatives, using rocket models, has been undertaken at the Weapons Research Establishment, Salisbury, South Australia[8], while a series of wind tunnel tests at supersonic speeds is being undertaken at the National Aeronautical Establishment, Ottawa, Canada.

On completion of the theoretical and experimental work it is proposed to issue a report making a full comparison between experiment and theory throughout the speed range. Consequently, this report is restricted to presenting and discussing the results calculated using supersonic lifting surface theory. No experimental comparisons are made and the theoretical comparisons which are carried out are restricted to what is necessary to establish the probable limits of accuracy of the results.

2 NOTATION

The notation used is as follows:

A	wing aspect ratio
$\bar{c}$	wing mean chord
M	Mach number
S	wing area
V	airspeed
z	Cartesian co-ordinate, measured downwards
z_o	heaving amplitude of wing apex
β	$= (M^2 - 1)^{\frac{1}{2}}$
θ	angle of pitch
θ_o	pitching amplitude
ν	frequency parameter $= \omega\bar{c}/V$
ρ	air density
ω	circular frequency

The flutter derivatives on a wing oscillating in a heaving mode $z = z_o e^{i\omega t}$ and a pitching mode $\theta = \theta_o e^{i\omega t}$ are then given by the expressions

$$\text{Lift} = \rho V^2 S \left\{ (\ell_z + i\nu\, \ell_{\dot{z}}) \frac{z_o}{\bar{c}} + (\ell_\theta + i\nu\, \ell_{\dot{\theta}})\, \theta_o \right\} e^{i\omega t}$$

and

$$\text{Moment} = \rho V^2 S\bar{c} \left\{ (m_z + i\nu\, m_{\dot{z}}) \frac{z_o}{\bar{c}} + (m_\theta + i\nu\, m_{\dot{\theta}})\, \theta_o \right\} e^{i\omega t} \quad .$$

The moment, about the wing apex, is positive in a nose-up direction.

3 THE CALCULATION

3.1 Method

The method of calculation used, based on Richardson's extension[1] of the Multhopp lifting surface theory[9], is described in Ref.2 while the mechanisation of the method is described in Ref.3. Although the method was originally mechanised for the Ferranti Mercury computer, the amount of computing time needed for the work described in this report (of the order of 250 hours on Mercury) made it necessary at an early stage to modify the programmes and take advantage of the increased speed of the Ferranti Atlas computer.

In lifting surface theory the assumption is made that the lift and downwash distributions on the wing can each be represented by double series of lift and downwash functions in the chordwise and spanwise co-ordinates, the coefficients in these double series being related to the values of the lift and downwash at sets of points on the wing. The lift and downwash functions are chosen on the basis of two-dimensional flow for the chordwise functions and on slender wing theory for the spanwise functions. The sets of points at which the lift and downwash values are taken are known as the lift points and the downwash points respectively. With these approximations made, the integral equation which gives the downwash at any point as an integral containing the lift can be replaced by a matrix equation; in this the downwash at the point is given as the product of an influence matrix with another matrix containing the values of the lift at the lift points. Given a set of unknown downwash values over the wing, this matrix equation can then be solved for the corresponding lift values merely by inverting this aerodynamic influence matrix and the generalised aerodynamic forces and flutter derivatives follow. The crux of the calculation is the formulation of the aerodynamic influence matrix, the elements of which depend in a complicated manner on a number of surface integrals taken over that area of the wing cut off by the forward Mach lines through the downwash point in question. To evaluate these surface integrals a set of integration points is taken over the area being considered, these points lying at the intersections of a series of chordwise sections and a series of spanwise sections which depend on the exact area being considered.

At the outset of the calculations, parameters m (number of chordwise lift and downwash points), n (number of spanwise lift and downwash points), p (number of chordwise integration points) and q (number of spanwise integration points) must be chosen. The considerations which weigh in arriving at a suitable choice of these are discussed in Ref.3; following trial calculations at zero frequency the choice (m, n, p, q) = (5, 10, 5, 11) was made.

3.2 Cases considered

The investigation used wings A, B, C, D, E and F, of which details are given in Table 1 and which are shown in Fig.1. It will be seen that wings A, B and C belong to a family of cropped delta wings, while wings D, E and F belong to a family of swept wings. Each family has a constant taper ratio and each contains one wing of aspect ratio 3, 2 and 1.25 with the same leading edge sweep as the corresponding wing in the other family. The values of the Mach number were chosen to yield certain prescribed values of the quantity $A\beta$, and are summarised below.

		$A\beta$ = 1.0	1.5	2.0	2.5	3.0
Wings A, D (A=3)	M =	1.054	1.12	1.20	1.30	1.41
Wings B, E (A=2)	M =	1.12	1.25	1.41	1.60	1.80
Wings C, F (A=1.25)	M =	1.28	1.56	1.89	-	-

Flutter derivatives were calculated for the rigid-body modes of pitch and heave at frequency parameters, based on mean chord, of 0.1, 0.25 and 0.5; in addition, the in-phase lift and moment derivatives due to pitch were calculated at zero frequency.

Methods from other sources were also used to calculate derivatives in order to establish the probable limits of accuracy in the computations. These are discussed in detail in section 4.1.

4 RESULTS

4.1 Derivatives used for comparison

For the wings considered, theoretical derivatives are available from other sources and these enable an independent check to be made of the results of the calculations of this report.

The method of Allen and Sadler[10] has been used by Garvey[11] to find derivatives for wing C and wing F in the same modes as are considered here and these are given in Table 2. The method of Ref.10 is a supersonic theory valid for general frequency, planform and Mach number, and is based on the integral equation which gives the downwash in terms of the velocity potential. This equation is solved to give the velocity potential at the vertices of a fine mesh. For wings A, B, D and E Barnes[6] has calculated derivatives using the method of Hunt[12]. In this method, a mesh is placed over the wing; for sufficiently high Mach number the velocity potential can be expressed directly in terms of the downwash and this enables the velocity potential to be calculated at the vertices of the mesh. The derivatives then follow directly. Results taken from Ref.6 are also given in Table 2. The theories of Refs.10 and 12, and hence the results given in Refs.6 and 11, are approximate.

Derivatives have been calculated for a delta wing of aspect ratio 3.46 and are given in Table 2. In the method used[13] the velocity potential is expressed as a series in the frequency parameter; for modes of simple shape and for Mach number not too close to unity retention of the first few terms of the series is sufficient and derivatives obtained by the method may be regarded as exact.

For the swept wings D, E and F in steady flow, closed form approximations to the aerodynamic lift and moment derivatives are available and expressions from which ℓ_θ may be calculated are given by Jones and Cohen[14]. With the assumption used in Ref.14, that the loading is uniform along each section downstream from the tip Mach line, the lift distributions given by Cohen[15] may be used to give values for m_θ. Values of ℓ_θ and m_θ calculated by these methods are given in Table 3. Tables which provide lift and moment derivatives for cropped delta wings in steady flow have been given by Smith, Beasley and Stevens[16] and these have been used in finding the derivatives for wings A, B and C which are given in Table 3. The derivatives of Table 3 will be regarded as being exact when making comparisons with lifting surface theory results in section 4.2.

4.2 Discussion

The values of the derivatives obtained from the lifting surface theory calculations are given in Tables 4 to 9. To present all the results which have been found in graphical form would require an excessive number of figures, so representative plots are given in Figs.2 to 18 of the variation of the main derivatives with Mach number and frequency parameter. In order to obtain an assessment of the accuracy of the results obtained, comparison is made in these figures with the derivatives obtained from other sources, given in Tables 2 and 3, which were discussed in section 4.1.

From Tables 4 to 9 it is seen that the greatest variation of derivatives with frequency occurs for the highest aspect ratio wings A and D, while the least variation occurs for the lowest aspect ratio wings C and F. For wings C and F, the derivatives ℓ_z and m_z apart, the frequency variation of derivatives is only about 1% for the Mach numbers considered. For wings A and D there is a greater variation with frequency; this is more marked at the lower end of the Mach number range and the greatest variation is that in the pitching damping derivatives $\ell_{\dot\theta}$ and $m_{\dot\theta}$. The derivatives ℓ_z and m_z are in all cases small; their variation is usually of no significance in practice and it has not been thought worth-while to plot them here.

Figs.2 to 7 show the variation with Mach number of the derivatives ℓ_θ, m_θ, $\ell_{\dot\theta}$, $m_{\dot\theta}$, $\ell_{\dot z}$ and $m_{\dot z}$ for the two highest aspect ratio wings A and D. To illustrate the variation of these derivatives with frequency they have been plotted for the sample frequency parameters $\nu = 0.1$ and 0.5. It will be seen from Figs.2 and 3 that frequency has a perceptible effect on the in-phase lift and moment derivatives ℓ_θ and m_θ and this effect becomes less with increasing Mach number. Figs.4 and 5 show the considerable effect which it is predicted that frequency will have at the lower Mach numbers on the pitching damping derivatives $\ell_{\dot{\theta}}$ and $m_{\dot{\theta}}$. Figs.6 and 7 show the lesser effect of frequency on the heaving damping derivatives $\ell_{\dot{z}}$ and $m_{\dot{z}}$. The effect of frequency on these derivatives decreases with increasing Mach number, but the effect at high Mach number on the heaving damping derivatives is slightly greater than that on the pitching damping derivatives.

Included in Figs.2 to 7 are the derivatives for wings A and D at $M = 1.53$ given by Barnes[6]. These lie beyond the Mach number range considered here and are, like the derivatives of this report, approximations to the true values. It can nevertheless be seen that qualitative agreement between the results of this report and those of Ref.6 is good although, since extrapolation is involved, no quantitative assessment can be made.

The variation with Mach number of the main derivatives for wings B, C, E and F is shown in Figs.8 to 15. Since there is little variation with frequency of the derivatives for these wings, particularly in the case of the lower aspect ratio wings C and F, attention has been restricted in these figures to the single frequency parameter $\nu = 0.1$. Ref.6 gives derivative values for wings B and E at $M = 2.0$ which again fall beyond the Mach number range considered here. These values are included in the figures and it can be seen that qualitative agreement with the results of this report is again good. Garvey[11] has given derivatives for wings C and F at Mach numbers of 1.077 and 1.2806; these are repeated in Table 2 and are shown in Figs.8, 9 and 12 to 15.

In Ref.11 frequency parameters $\nu = 0.087$ and 0.433 were chosen for wing C and $\nu = 0.1$ and 0.5 for wing F. Thus, only for wing F at $M = 1.2806$ can a direct quantitative comparison between the two sets of derivatives be made; Tables 2 and 9 show the maximum disagreement to be one of 3.9% in the derivative m_{θ} when $\nu = 0.5$. This agreement is very close since, as was remarked in section 4.1, both the results of Ref.11 and those of this report are based on approximate theories and this would allow for each set of derivative values being within at most 2% of the unknown exact value. It can be seen that the derivatives ℓ_z and m_z derived by the two different methods disagree; however they, together with the $\ell_{\dot{z}}$ and $m_{\dot{z}}$ derivatives, arise from the lift distribution due to heaving motion and this has only a very small in-phase component. The disagreement is thus of no practical significance. The wing C derivatives of this report have been evaluated at different frequency parameters from those of Ref.11 and it can be seen from Tables 2 and 6 that

Ref.11 predicts a slightly greater frequency effect than does this report. The differences are small, however. Assuming, which is not strictly true, that the direct comparison can be made between the results of Ref.11 at $\nu = 0.087$ and $\nu = 0.433$ and the results of this report at $\nu = 0.1$ and $\nu = 0.5$ respectively, the maximum disagreements shown are of 5.5% in $m_{\dot{z}}$ at the lower frequency parameter and 6.5% in m_θ at the higher frequency parameter. This is well within the limits to be expected for comparisons between results from two approximate theories.

The variation of derivatives with frequency for wings A and D at the particular Mach number $M = 1.054$ is shown in Figs.16 to 18. This is the Mach number at which the greatest frequency variation of the damping derivatives $\ell_{\dot{\theta}}$ and $m_{\dot{\theta}}$ occurs and this variation is shown in Fig.17. Figs.16 to 18 also show the variation with frequency of the main derivatives for a delta wing of aspect ratio 3.46. This has the same leading edge sweep as the cropped delta wing A so that, in addition to showing derivative variation with frequency, Figs.16 to 18 illustrate the quantitative effect on the derivatives of cropping the tip. Fig.16 also shows the exact values of the derivatives ℓ_θ and m_θ when $\nu = 0$.

A further check on the results of this report is afforded by direct comparison of the exact values of the derivatives ℓ_θ and m_θ when $\nu = 0$, given in Table 3, and the derivative values at zero frequency which are given in Tables 4 to 9. The greatest discrepancy is one of 3.6% between the two values of m_θ for wing F at $M = 1.28$, which is also plotted in Fig.16. In view of this and the comparisons with other approximate theories made above it is suggested that an upper limit of 4% may be taken for the error in the derivatives presented in this report.

Table 1

DETAILS OF PLANFORMS

Wing	Aspect ratio	Leading edge sweep	Trailing edge sweep	$\frac{\text{Tip chord}}{\text{Root chord}}$
A	3	49.1°	0°	0.0718
B	2	60°	0°	0.0718
C	1.25	70.13°	0°	0.0718
D	3	49.1°	18.43°	0.238
E	2	60°	26.57°	0.238
F	1.25	70.13°	38.67°	0.238

Table 2

DERIVATIVES USED FOR COMPARISON

Source	Wing	M	ν	ℓ_z	ℓ_θ	$-m_z$	$-m_\theta$	$\ell_{\dot z}$	$\ell_{\dot\theta}$	$-m_{\dot z}$	$-m_{\dot\theta}$
Quoted in Ref.11; calculated by the method of Ref.10	C	1.077	0.087	0.0002	1.0730	-0.0002	1.3198	1.0629	1.8598	1.2939	2.6152
			0.433	-0.0318	1.0750	-0.0497	1.3179	1.0317	1.9608	1.2610	2.7361
		1.2806	0.087	0.0003	1.0037	-0.0009	1.2160	1.0302	1.7776	1.2489	2.4963
			0.433	-0.0193	1.0372	-0.0373	1.2518	1.0120	1.7473	1.2241	2.4551
	F	1.077	0.1	-0.0004	0.9738	-0.0011	1.1409	0.9641	1.6412	1.1184	2.2279
			0.5	-0.0464	0.9550	-0.0719	1.1010	0.9430	1.7413	1.1038	2.3470
		1.2806	0.1	-0.0002	0.9538	-0.0017	1.1260	0.9762	1.6948	1.1528	2.3548
			0.5	-0.0348	0.9772	-0.0633	1.1451	0.9609	1.6759	1.1368	2.3305
Quoted in Ref.6; calculated by the method of Ref.12	A	1.53	0.1	0.0282	1.6450	0.0094	1.9690	1.6383	1.3077	1.954	1.7982
			0.25	0.0414	1.6345	0.0545	1.9538	1.5946	1.3173	1.8967	1.8128
			0.5	-	1.6080	-	1.9265	1.515	1.335	1.7900	1.8415
	B	2.0	0.1	0.0018	1.0800	0.0022	1.2800	1.0781	1.1431	1.2776	1.5639
			0.25	0.0108	1.0796	0.0132	1.2793	1.0691	1.1429	1.2651	1.5637
			0.5	0.0375	1.0786	0.1021	1.2766	1.0402	1.1425	1.2460	1.5634
	D	1.53	0.1	0.0076	1.6862	0.104	2.0524	1.6900	1.3218	2.0616	1.8456
			0.25	0.0446	1.6865	0.0688	2.0569	1.6438	1.3308	2.056	1.8592
			0.5	-	1.6870	-	2.0600	1.571	1.3455	2.039	1.8800
	E	2.0	0.1	0.0020	1.1011	0.0027	1.3438	1.0991	1.1682	1.3410	1.6393
			0.25	0.0121	1.1012	0.0162	1.3439	1.0892	1.1674	1.3264	1.6379
			0.5	0.0424	1.1010	0.1245	1.3433	1.0573	1.1650	1.2990	1.6334
Calculated for this report by the method of Ref.13	Delta wing A=3.46	1.054	0.00005	0	2.4948	0.0001	3.3264	2.4948	0.2823	3.3264	0.4234
			0.025	0.0025	2.4867	0.0658	3.3134	2.4815	0.3309	3.3051	0.5044
			0.05	0.0096	2.4643	0.1244	3.2780	2.4443	0.4669	3.2460	0.7301
			0.1	0.0325	2.4084	0.2061	3.1940	2.3377	0.8641	3.0819	1.3763

Table 3

DERIVATIVES FOR WINGS A TO F AT ZERO FREQUENCY USING AERODYNAMIC THEORY FOR STEADY FLOW

Wing	M	ℓ_θ	$-m_\theta$
A	1.054	2.36	2.86
	1.12	2.20	2.67
	1.20	2.05	2.49
	1.30	1.90	2.31
	1.41	1.76	2.15
B	1.12	1.58	1.91
	1.25	1.47	1.78
	1.41	1.37	1.66
	1.6	1.26	1.54
	1.8	1.17	1.43
C	1.28	0.99	1.19
	1.56	0.92	1.12
	1.89	0.85	1.03
D	1.054	2.31	2.76
	1.12	2.18	2.63
	1.20	2.05	2.49
	1.30	1.91	2.33
	1.41	1.79	2.18
E	1.12	1.54	1.84
	1.25	1.46	1.76
	1.41	1.37	1.66
	1.6	1.27	1.55
	1.8	1.19	1.45
F	1.28	0.96	1.15
	1.56	0.91	1.10
	1.89	0.85	1.03

Table 4

LIFTING SURFACE THEORY DERIVATIVES FOR WING A

M	ν	ℓ_z	ℓ_θ	$-m_z$	$-m_\theta$	$\ell_{\dot z}$	$\ell_{\dot\theta}$	$-m_{\dot z}$	$-m_{\dot\theta}$
1.054	0		2.37		2.83				
	0.1	0.03	2.29	0.03	2.72	2.24	1.13	2.64	1.73
	0.25	0.08	2.19	0.10	2.61	1.99	1.82	2.33	2.67
	0.5	0.14	2.16	0.14	2.58	1.78	2.08	2.08	2.97
1.12	0		2.23		2.67				
	0.1	0.02	2.20	0.03	2.63	2.17	1.33	2.58	1.96
	0.25	0.08	2.11	0.10	2.52	1.95	1.67	2.29	2.44
	0.5	0.15	2.07	0.15	2.49	1.73	1.95	2.03	2.80
1.20	0		2.08		2.51				
	0.1	0.02	2.07	0.02	2.49	2.05	1.45	2.46	2.08
	0.25	0.08	2.02	0.09	2.42	1.90	1.57	2.25	2.25
	0.5	0.15	1.95	0.17	2.34	1.65	1.78	1.93	2.55
1.30	0		1.94		2.33				
	0.1	0.01	1.93	0.01	2.33	1.92	1.46	2.31	2.08
	0.25	0.06	1.90	0.08	2.29	1.82	1.51	2.17	2.15
	0.5	0.16	1.84	0.18	2.21	1.60	1.62	1.87	2.31
1.41	0		1.82		2.18				
	0.1	0.01	1.81	0.01	2.18	1.80	1.38	2.16	1.96
	0.25	0.05	1.79	0.07	2.15	1.73	1.40	2.07	2.00
	0.5	0.15	1.74	0.19	2.08	1.55	1.48	1.82	2.11

Table 5

LIFTING SURFACE THEORY DERIVATIVES FOR WING B

M	ν	ℓ_z	ℓ_θ	$-m_z$	$-m_\theta$	$\ell_{\dot z}$	$\ell_{\dot\theta}$	$-m_{\dot z}$	$-m_{\dot\theta}$
1.12	0		1.58		1.89				
	0.1	0.00	1.57	0.01	1.88	1.56	2.09	1.86	2.92
	0.25	0.02	1.56	0.02	1.87	1.50	2.15	1.78	3.00
	0.5	0.02	1.59	0.00	1.91	1.43	2.16	1.70	3.01
1.25	0		1.49		1.78				
	0.1	0.00	1.49	0.00	1.78	1.48	1.92	1.77	2.68
	0.25	0.02	1.49	0.02	1.78	1.44	1.92	1.72	2.69
	0.5	0.03	1.49	0.02	1.80	1.37	1.93	1.63	2.70
1.41	0		1.39		1.67				
	0.1	0.00	1.39	0.00	1.67	1.38	1.70	1.66	2.38
	0.25	0.02	1.39	0.02	1.67	1.36	1.70	1.63	2.38
	0.5	0.04	1.40	0.04	1.68	1.30	1.70	1.55	2.37
1.60	0		1.29		1.56				
	0.1	0.00	1.29	0.00	1.56	1.29	1.50	1.55	2.10
	0.25	0.01	1.29	0.02	1.56	1.27	1.50	1.53	2.10
	0.5	0.04	1.30	0.05	1.57	1.22	1.50	1.46	2.09
1.80	0		1.21		1.46				
	0.1	0.00	1.21	0.00	1.46	1.21	1.31	1.45	1.84
	0.25	0.01	1.21	0.02	1.46	1.19	1.31	1.43	1.84
	0.5	0.05	1.21	0.05	1.46	1.15	1.31	1.38	1.84

Table 6

LIFTING SURFACE THEORY DERIVATIVES FOR WING C

M	ν	ℓ_z	ℓ_θ	$-m_z$	$-m_\theta$	$\ell_{\dot z}$	$\ell_{\dot\theta}$	$-m_{\dot z}$	$-m_{\dot\theta}$
1.28	0		0.99		1.18				
	0.1	0.00	0.98	0.00	1.18	0.98	1.78	1.18	2.47
	0.25	-0.01	0.98	-0.02	1.17	0.98	1.78	1.17	2.47
	0.5	-0.05	0.98	-0.08	1.17	0.99	1.78	1.18	2.46
1.56	0		0.93		1.11				
	0.1	0.00	0.93	0.00	1.11	0.93	1.53	1.11	2.12
	0.25	-0.01	0.93	-0.01	1.11	0.93	1.53	1.11	2.12
	0.5	-0.03	0.93	-0.05	1.12	0.93	1.52	1.12	2.11
1.89	0		0.87		1.04				
	0.1	0.00	0.87	0.00	1.05	0.87	1.29	1.04	1.80
	0.25	0.00	0.87	-0.01	1.05	0.87	1.29	1.04	1.80
	0.5	-0.01	0.87	-0.03	1.05	0.87	1.29	1.05	1.79

Table 7

LIFTING SURFACE THEORY DERIVATIVES FOR WING D

M	ν	ℓ_z	ℓ_θ	$-m_z$	$-m_\theta$	$\ell_{\dot z}$	$\ell_{\dot\theta}$	$-m_{\dot z}$	$-m_{\dot\theta}$
1.054	0		2.28		2.67				
	0.1	0.02	2.21	0.03	2.58	2.16	1.18	2.51	1.86
	0.25	0.08	2.12	0.08	2.48	1.94	1.82	2.24	2.70
	0.5	0.13	2.10	0.11	2.47	1.75	2.06	2.04	2.96
1.12	0		2.19		2.58				
	0.1	0.02	2.16	0.02	2.54	2.12	1.27	2.50	1.91
	0.25	0.08	2.07	0.09	2.43	1.91	1.61	2.22	2.39
	0.5	0.14	2.02	0.14	2.39	1.69	1.90	1.98	2.75
1.20	0		2.06		2.47				
	0.1	0.02	2.05	0.02	2.45	2.03	1.36	2.42	1.96
	0.25	0.08	1.99	0.10	2.37	1.88	1.49	2.21	2.14
	0.5	0.15	1.91	0.17	2.27	1.62	1.71	1.89	2.46
1.30	0		1.92		2.32				
	0.1	0.01	1.92	0.02	2.31	1.90	1.41	2.29	2.00
	0.25	0.06	1.89	0.08	2.27	1.81	1.45	2.15	2.07
	0.5	0.16	1.82	0.19	2.18	1.58	1.57	1.85	2.23
1.41	0		1.81		2.18				
	0.1	0.01	1.81	0.01	2.18	1.80	1.34	2.16	1.91
	0.25	0.06	1.79	0.07	2.15	1.73	1.36	2.06	1.94
	0.5	0.16	1.74	0.19	2.08	1.55	1.43	1.81	2.04

Table 8

LIFTING SURFACE THEORY DERIVATIVES FOR WING E

M	ν	ℓ_z	ℓ_θ	$-m_z$	$-m_\theta$	$\ell_{\dot z}$	$\ell_{\dot\theta}$	$-m_{\dot z}$	$-m_{\dot\theta}$
1.12	0		1.52		1.78				
	0.1	0.00	1.51	0.00	1.77	1.50	2.02	1.76	2.81
	0.25	0.02	1.51	0.01	1.76	1.45	2.08	1.69	2.89
	0.5	0.01	1.52	0.01	1.79	1.39	2.10	1.63	2.91
1.25	0		1.46		1.72				
	0.1	0.00	1.46	0.00	1.72	1.45	1.84	1.71	2.57
	0.25	0.02	1.45	0.02	1.72	1.41	1.85	1.66	2.59
	0.5	0.03	1.46	0.02	1.72	1.33	1.86	1.57	2.60
1.41	0		1.37		1.65				
	0.1	0.00	1.37	0.00	1.65	1.37	1.64	1.64	2.28
	0.25	0.02	1.37	0.02	1.65	1.34	1.64	1.60	2.28
	0.5	0.05	1.38	0.05	1.65	1.27	1.64	1.51	2.28
1.60	0		1.28		1.54				
	0.1	0.00	1.28	0.00	1.54	1.28	1.46	1.54	2.04
	0.25	0.02	1.28	0.02	1.55	1.26	1.46	1.52	2.03
	0.5	0.05	1.29	0.05	1.55	1.21	1.45	1.45	2.03
1.80	0		1.21		1.45				
	0.1	0.00	1.21	0.00	1.45	1.21	1.28	1.45	1.80
	0.25	0.02	1.21	0.02	1.45	1.19	1.28	1.43	1.80
	0.5	0.05	1.21	0.06	1.45	1.15	1.28	1.37	1.79

Table 9

LIFTING SURFACE THEORY DERIVATIVES FOR WING F

M	ν	ℓ_z	ℓ_θ	$-m_z$	$-m_\theta$	$\ell_{\dot z}$	$\ell_{\dot\theta}$	$-m_{\dot z}$	$-m_{\dot\theta}$
1.28	0		0.95		1.11				
	0.1	0.00	0.95	0.00	1.11	0.95	1.71	1.11	2.34
	0.25	-0.01	0.95	-0.02	1.11	0.95	1.71	1.11	2.34
	0.5	-0.05	0.94	-0.08	1.10	0.95	1.70	1.12	2.34
1.56	0		0.91		1.08				
	0.1	0.00	0.91	0.00	1.08	0.91	1.47	1.08	2.03
	0.25	-0.01	0.91	-0.01	1.08	0.91	1.47	1.08	2.03
	0.5	-0.03	0.91	-0.05	1.07	0.91	1.47	1.08	2.02
1.89	0		0.86		1.03				
	0.1	0.00	0.86	0.00	1.03	0.86	1.25	1.03	1.73
	0.25	0.00	0.86	0.00	1.03	0.86	1.25	1.03	1.73
	0.5	-0.01	0.86	-0.02	1.03	0.85	1.24	1.02	1.72

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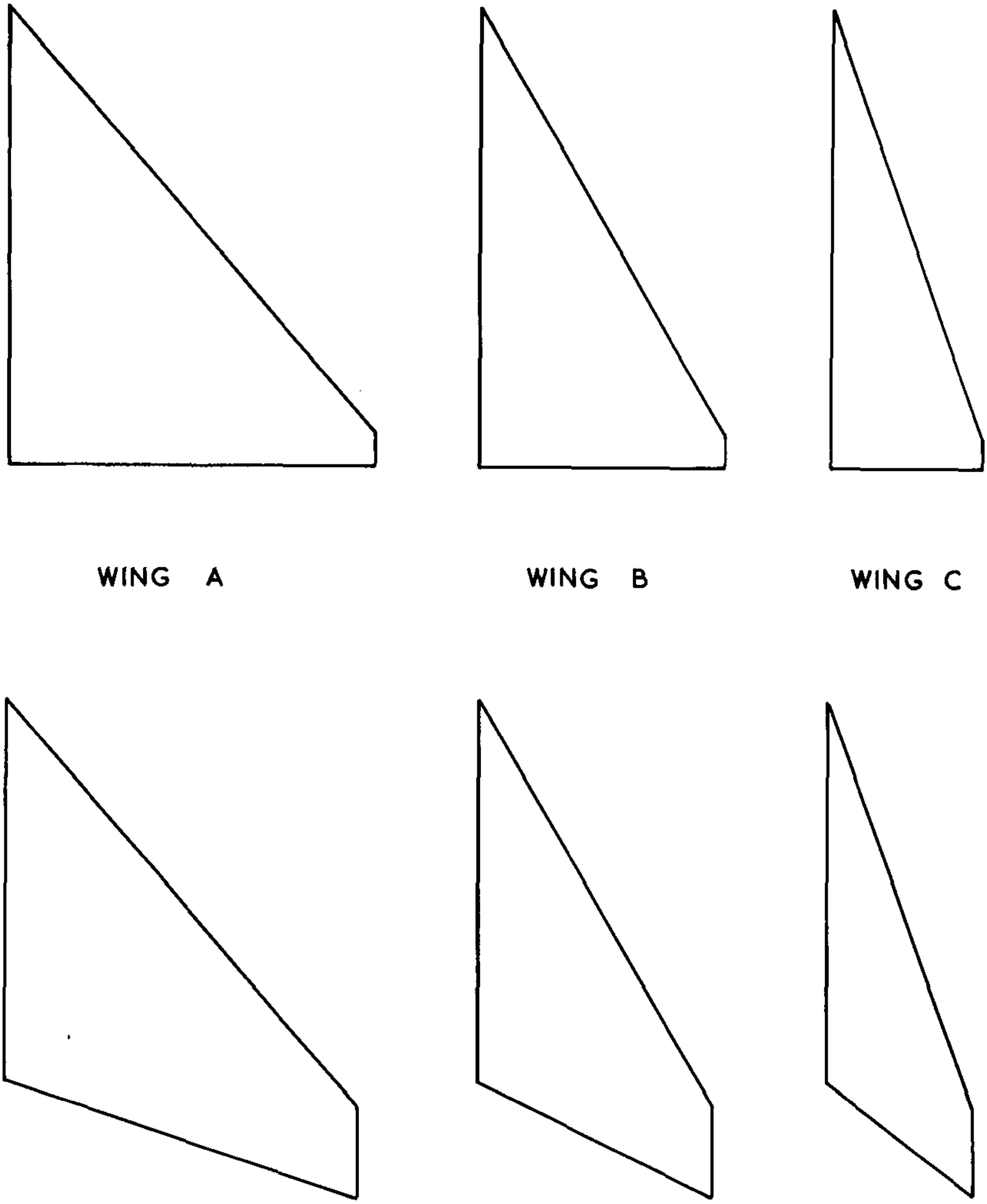


WING A WING B WING C

WING D WING E WING F

FIG.1 PLANFORMS A TO F OF THE M.O.A. DERIVATIVE PROGRAMME

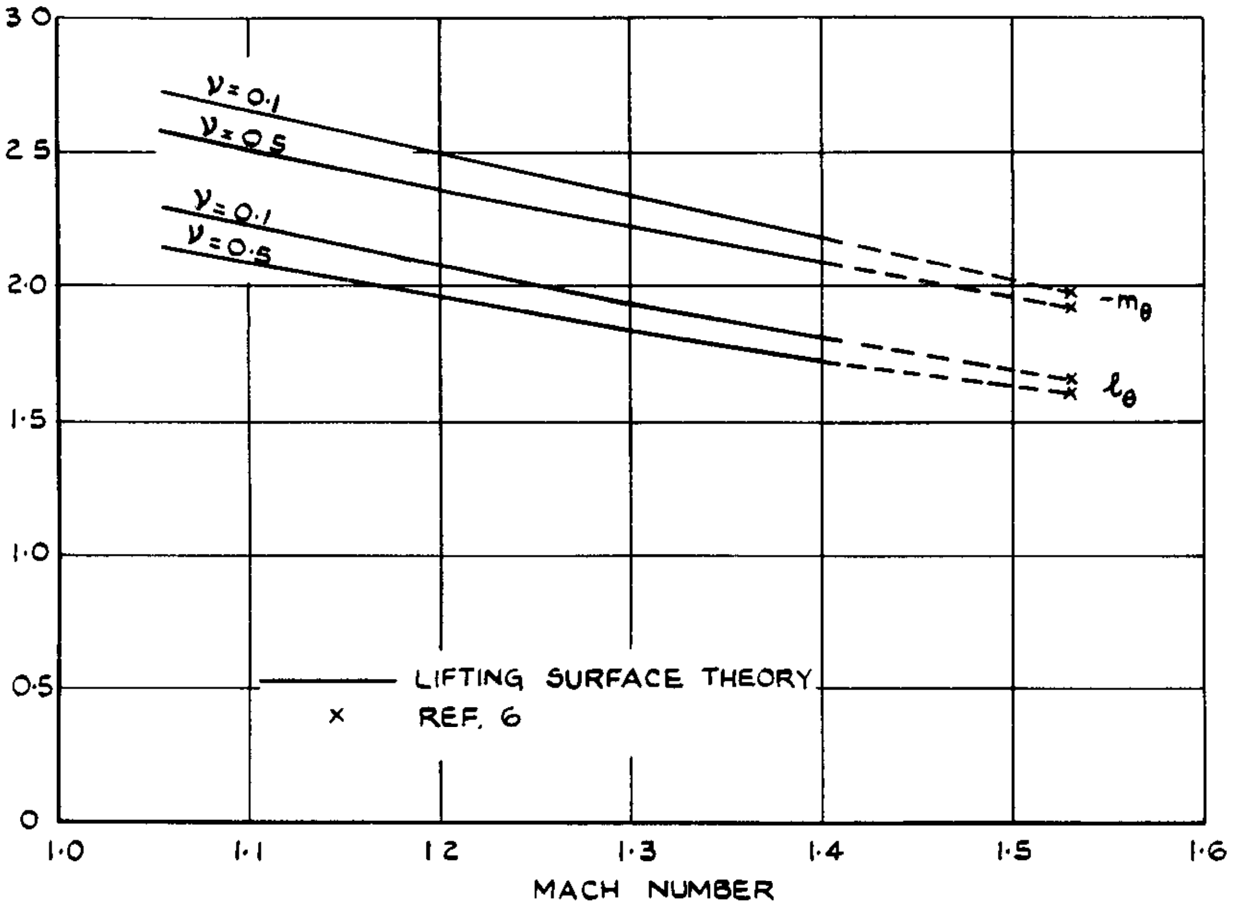


FIG. 2 VARIATION OF l_θ & $-m_\theta$ WITH M FOR WING A

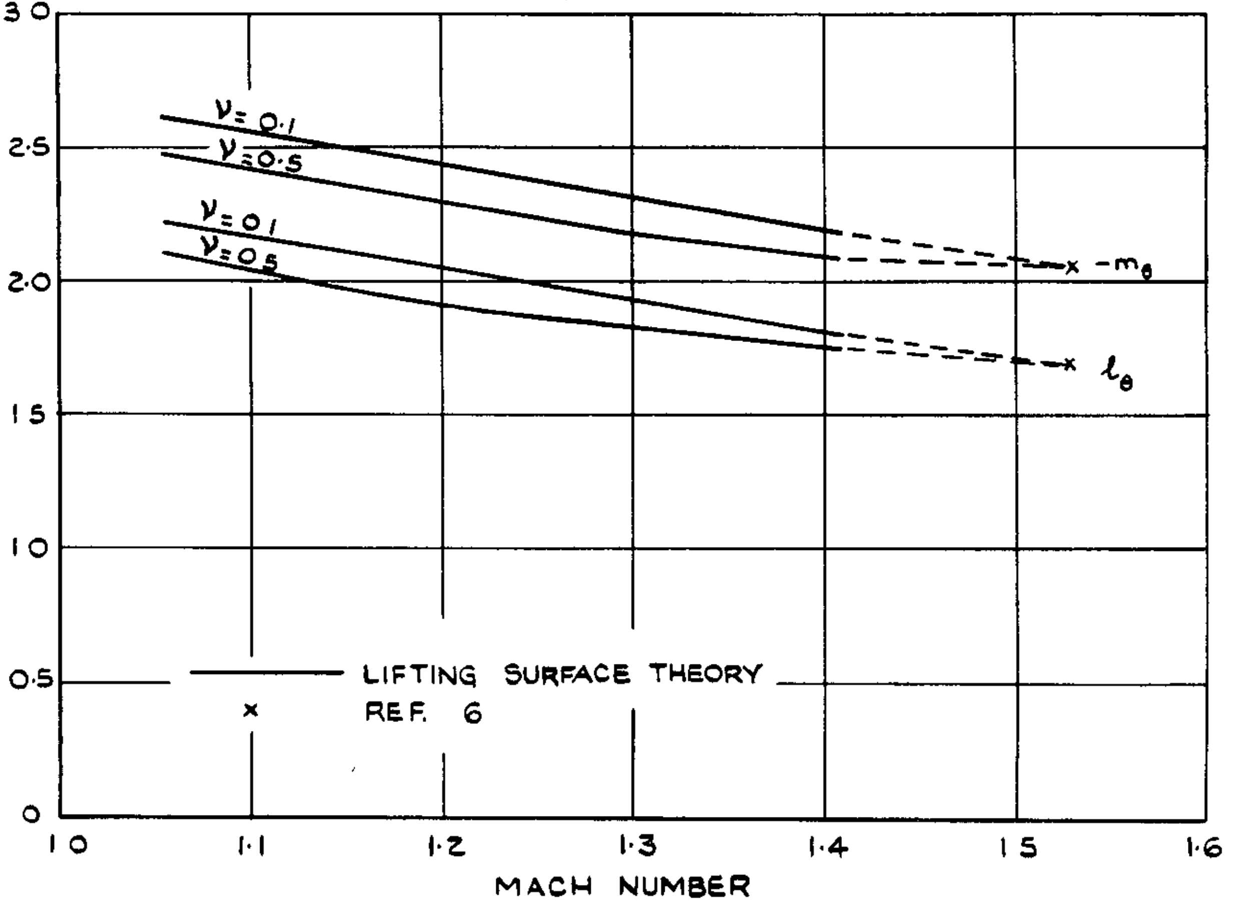


FIG. 3 VARIATION OF l_θ & $-m_\theta$ WITH M FOR WING D

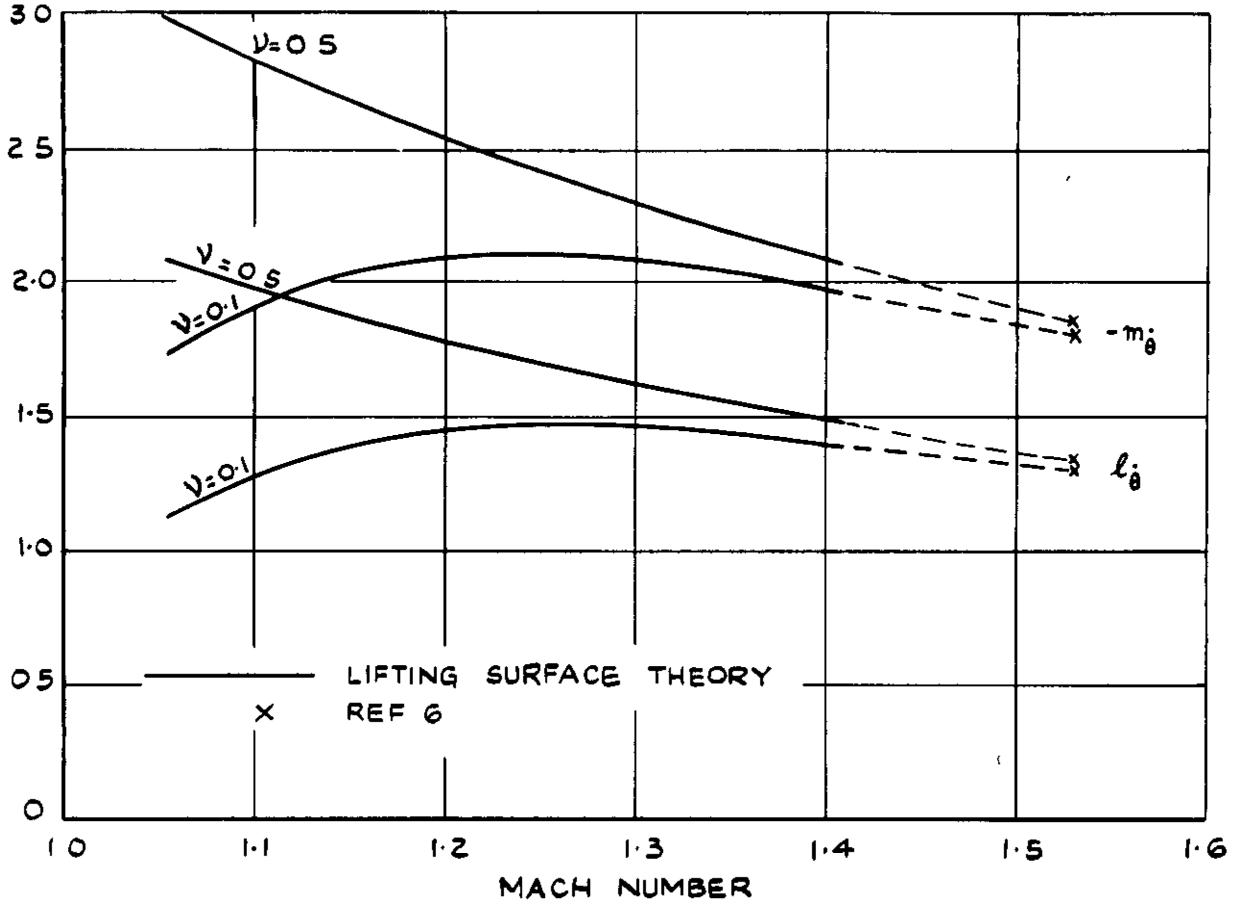


FIG. 4 VARIATION OF $\ell_{\dot{\theta}}$ & $-m_{\dot{\theta}}$ WITH M FOR WING A

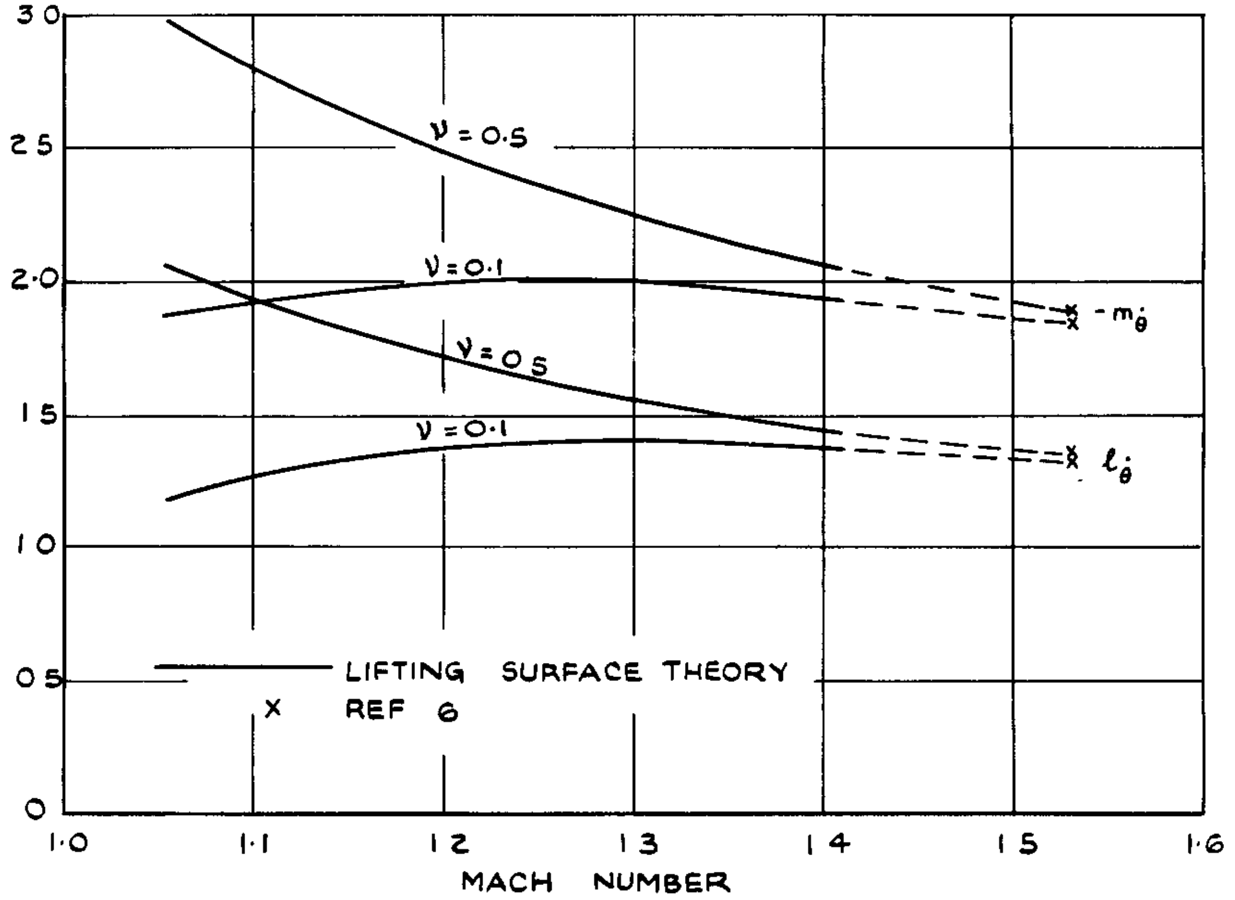


FIG. 5 VARIATION OF $\ell_{\dot{\theta}}$ & $-m_{\dot{\theta}}$ WITH M FOR WING D

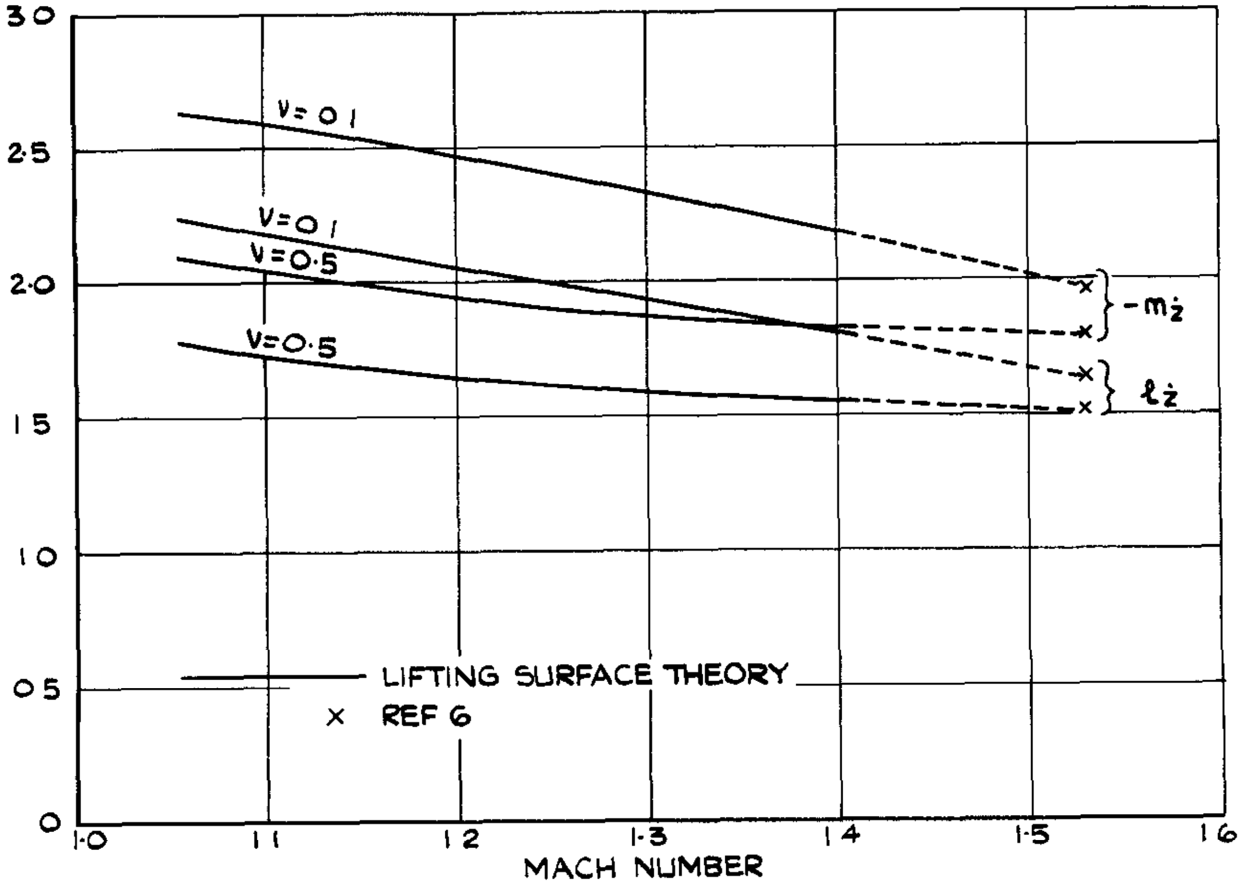


FIG. 6 VARIATION OF $\ell_{\dot z}$ AND $-m_{\dot z}$ WITH M FOR WING A

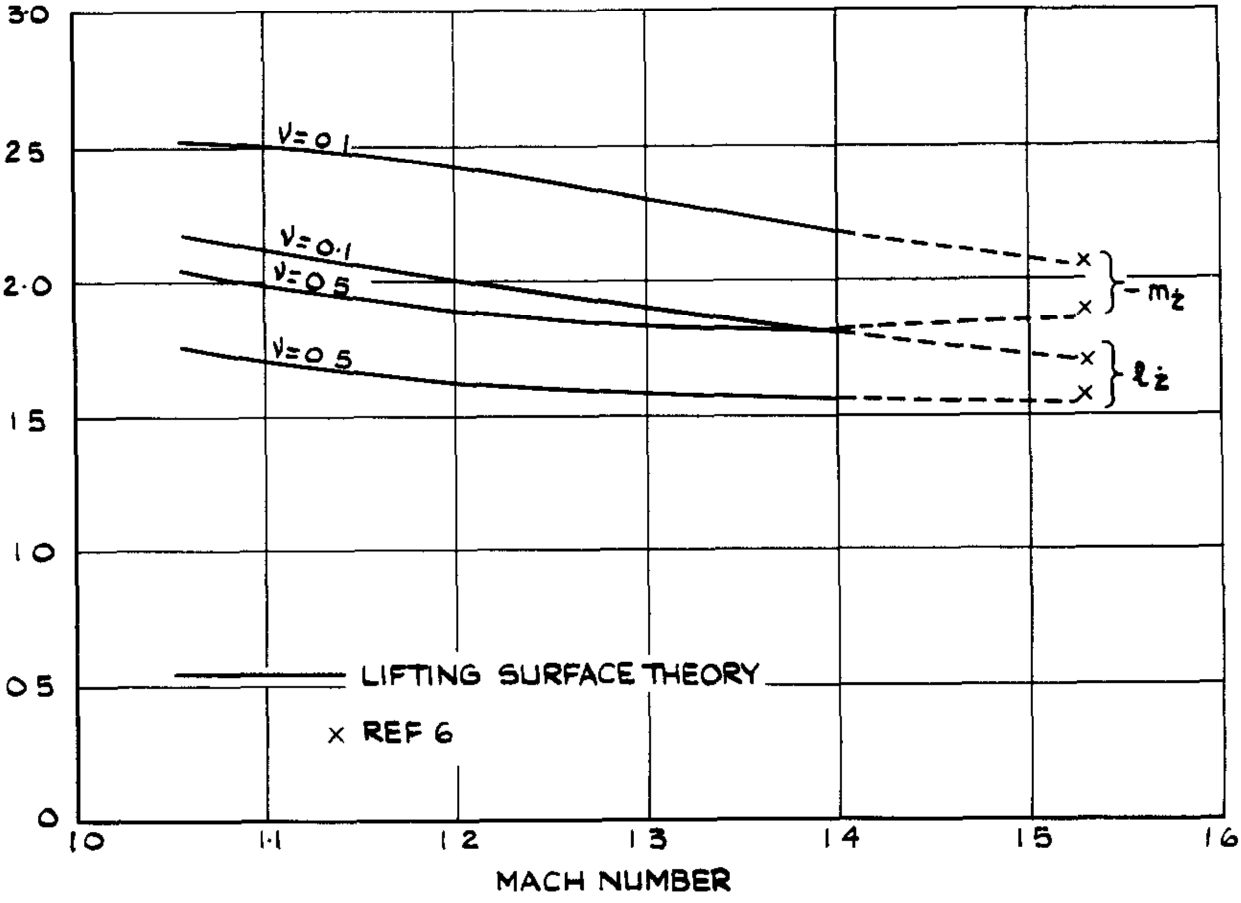


FIG. 7 VARIATION OF ℓ_z AND $-m_{\dot z}$ WITH M FOR WING D

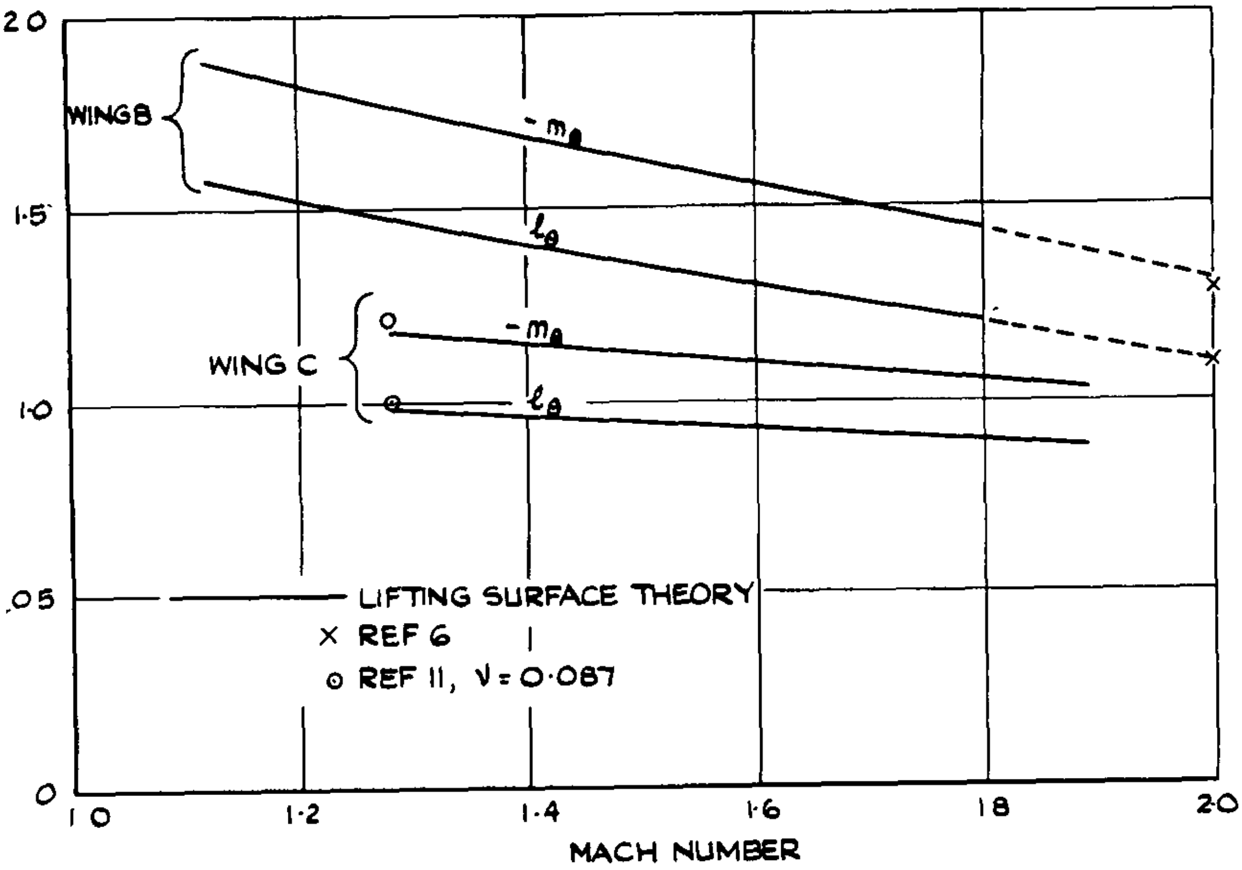


FIG. 8 VARIATION OF ℓ_θ AND $-m_\theta$ WITH M FOR WINGS B AND C AT $\nu = 0.1$

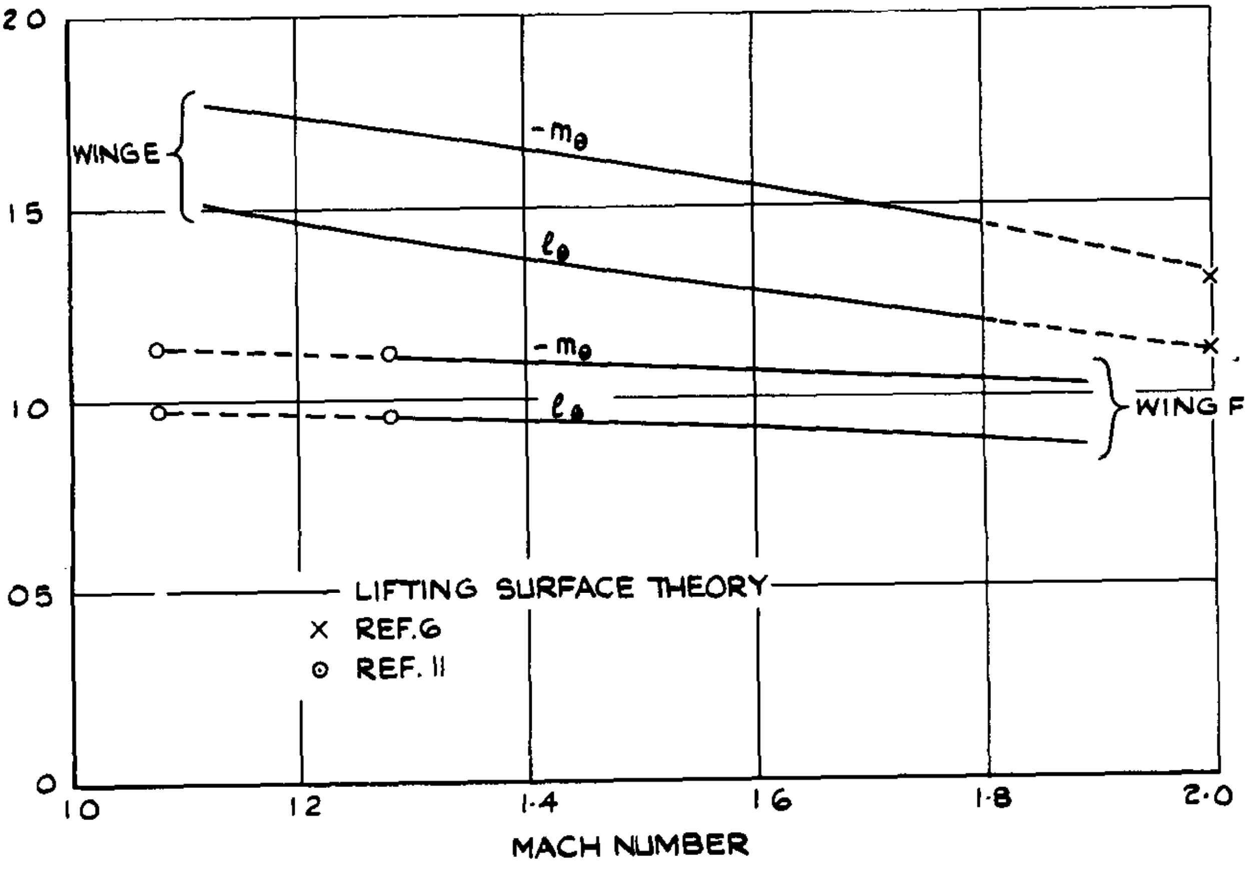


FIG 9 VARIATION OF ℓ_θ AND $-m_\theta$ WITH M FOR WINGS E AND F AT $\nu = 0.1$

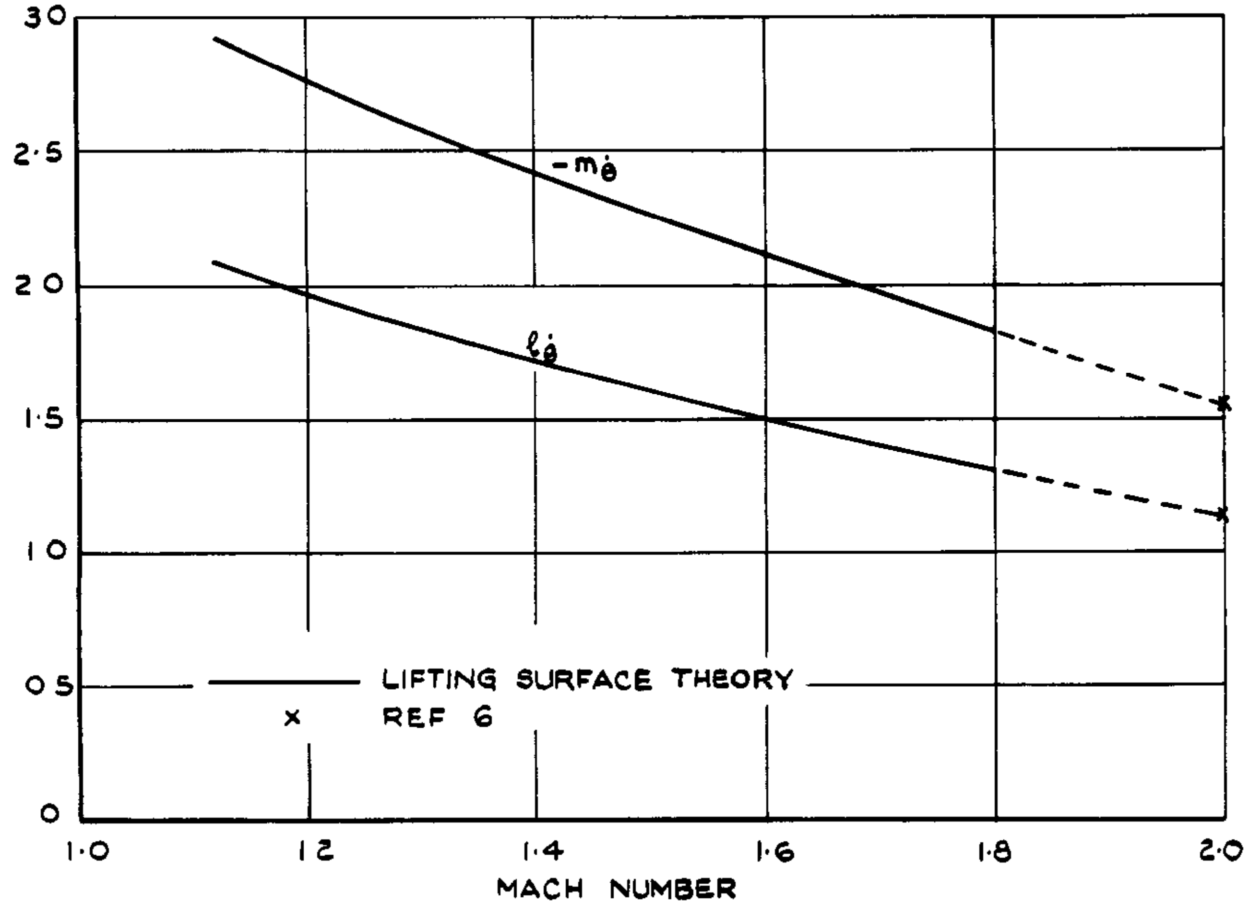


FIG. 10 VARIATION OF $l_{\dot{\theta}}$ AND $-m_{\dot{\theta}}$ WITH M FOR WING B AT $\nu = 0{\cdot}1$

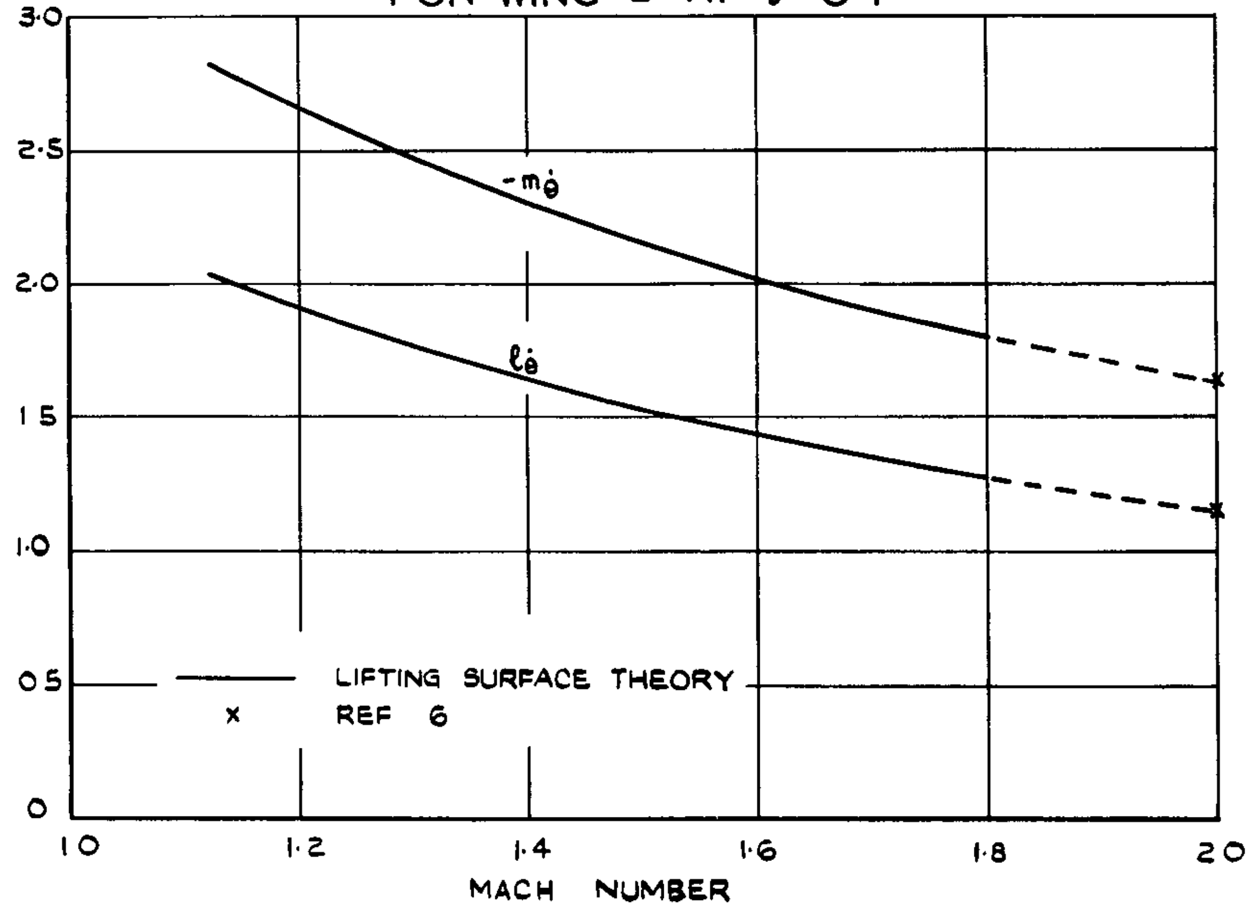


FIG. 11 VARIATION OF $l_{\dot{\theta}}$ AND $-m_{\dot{\theta}}$ WITH M FOR WING E AT $\nu = 0{\cdot}1$

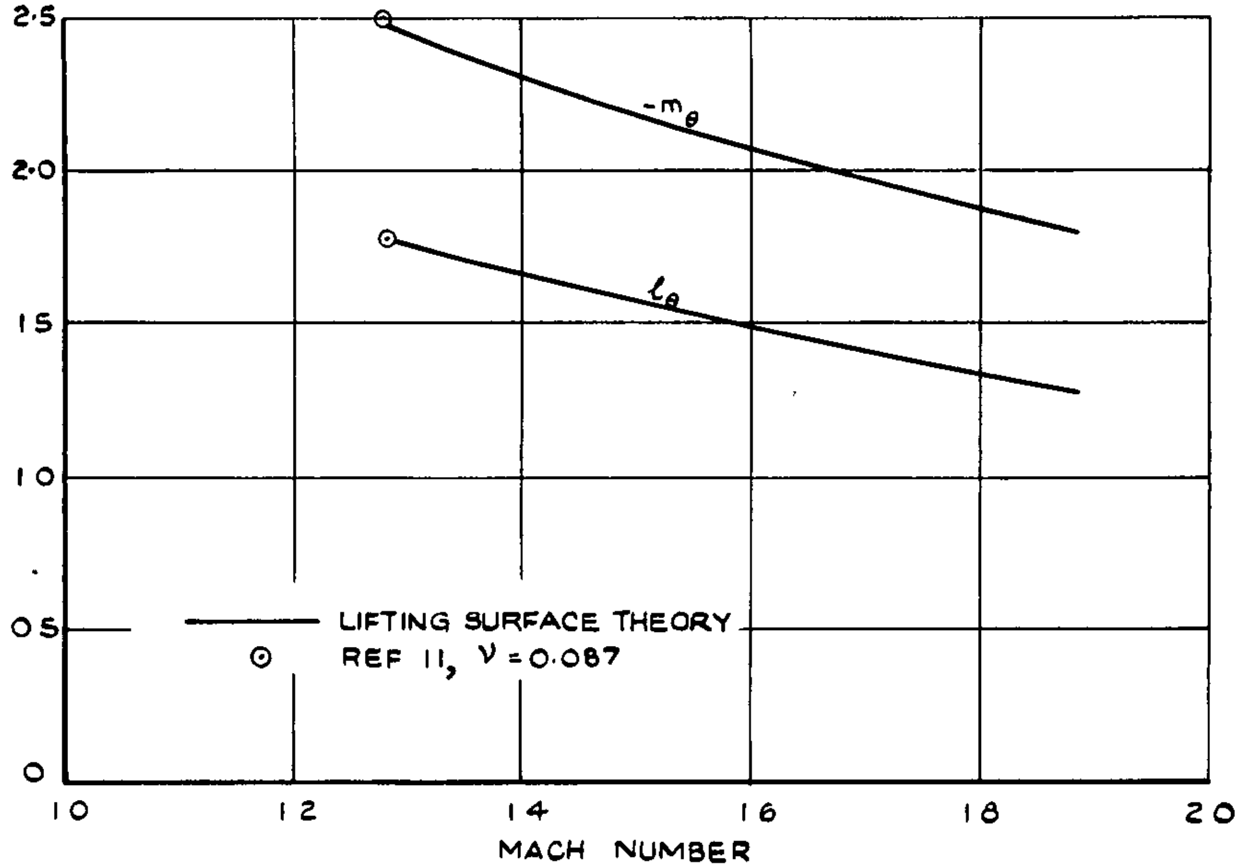


FIG.12 VARIATION OF $l_{\dot\theta}$ AND $-m_{\dot\theta}$ WITH M FOR WING C AT $\nu = 0{\cdot}1$

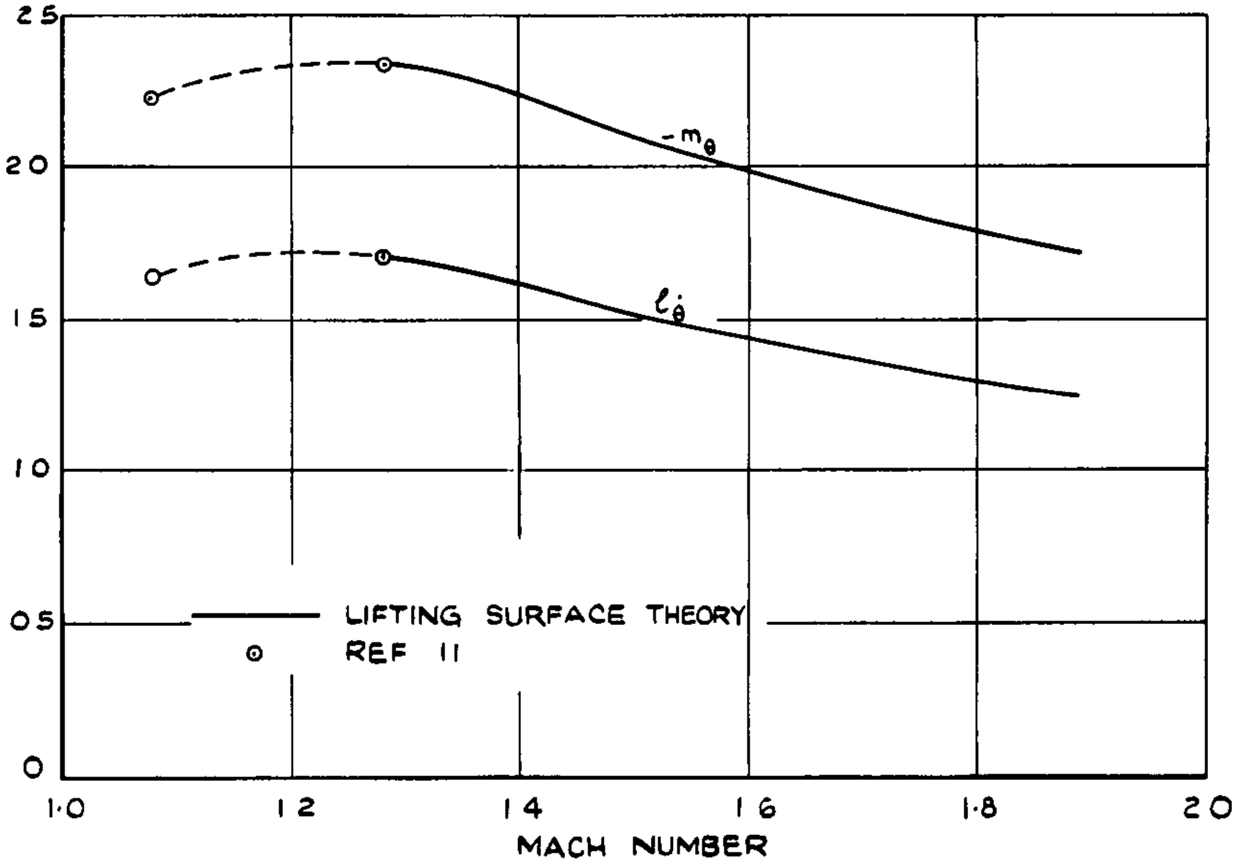


FIG.13 VARIATION OF $l_{\dot\theta}$ AND $-m_{\dot\theta}$ WITH M FOR WING F AT $\nu = 0{\cdot}1$

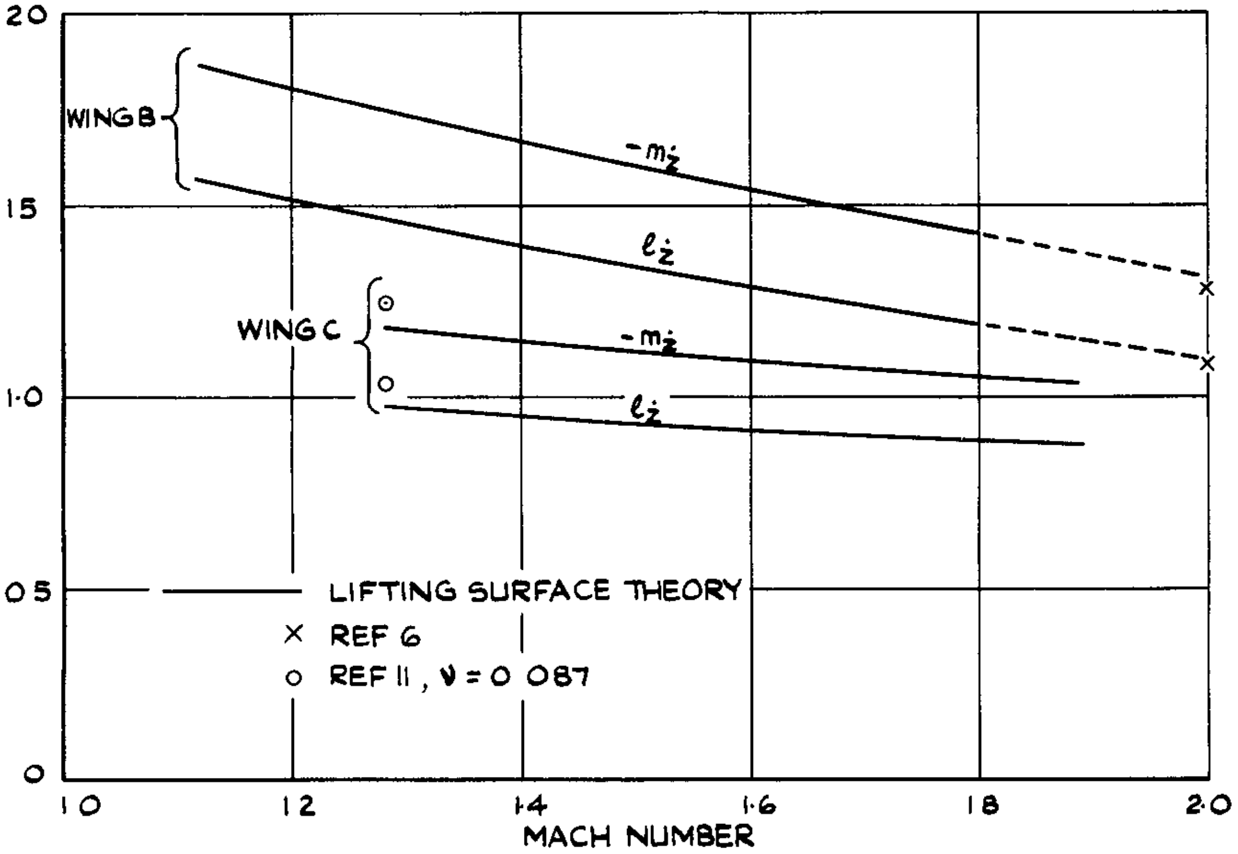


FIG. 14 VARIATION OF $\ell_{\dot{z}}$ AND $-m_{\dot{z}}$ WITH M FOR WINGS B AND C AT $\nu = 0{\cdot}1$

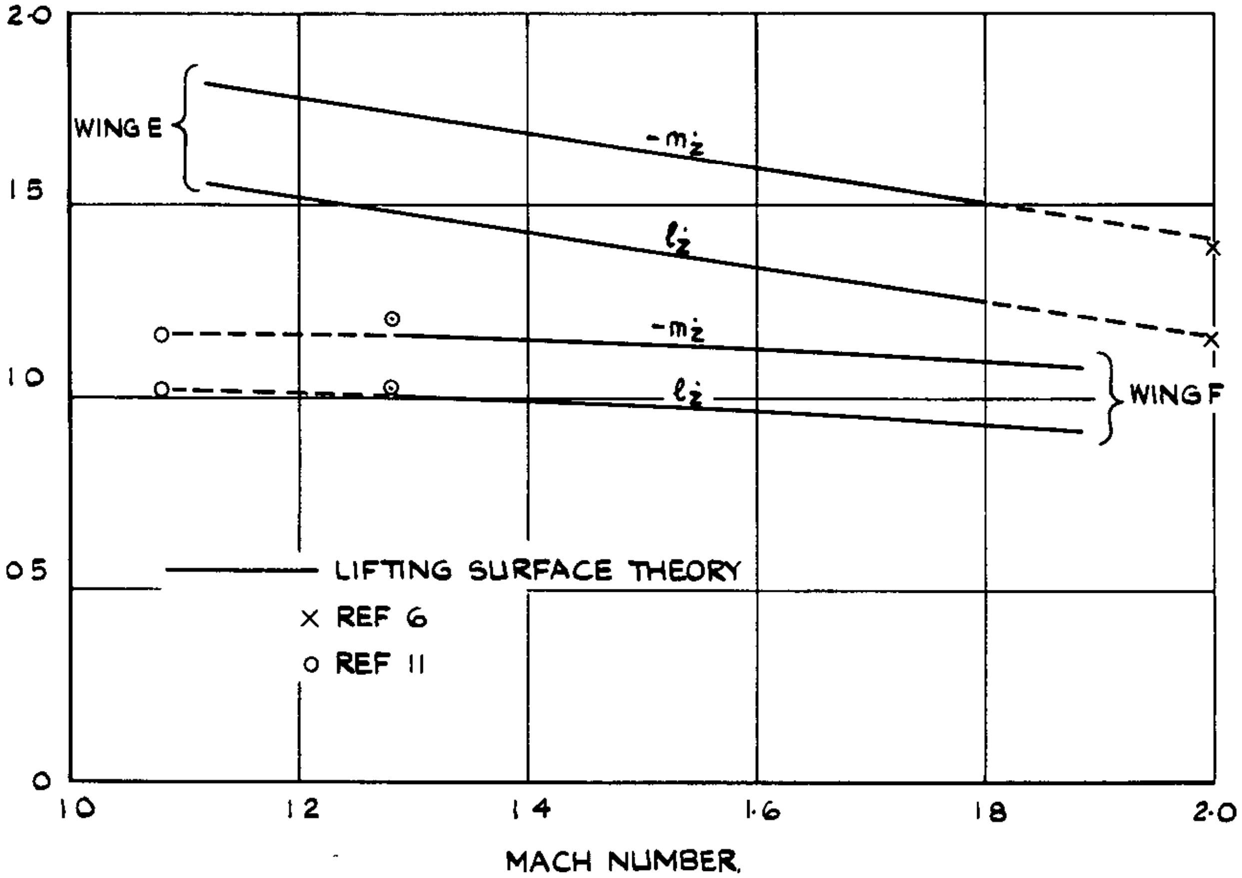


FIG 15 VARIATION OF $\ell_{\dot{z}}$ AND $-m_{\dot{z}}$ WITH M FOR WINGS E AND F AT $\nu = 0{\cdot}1$

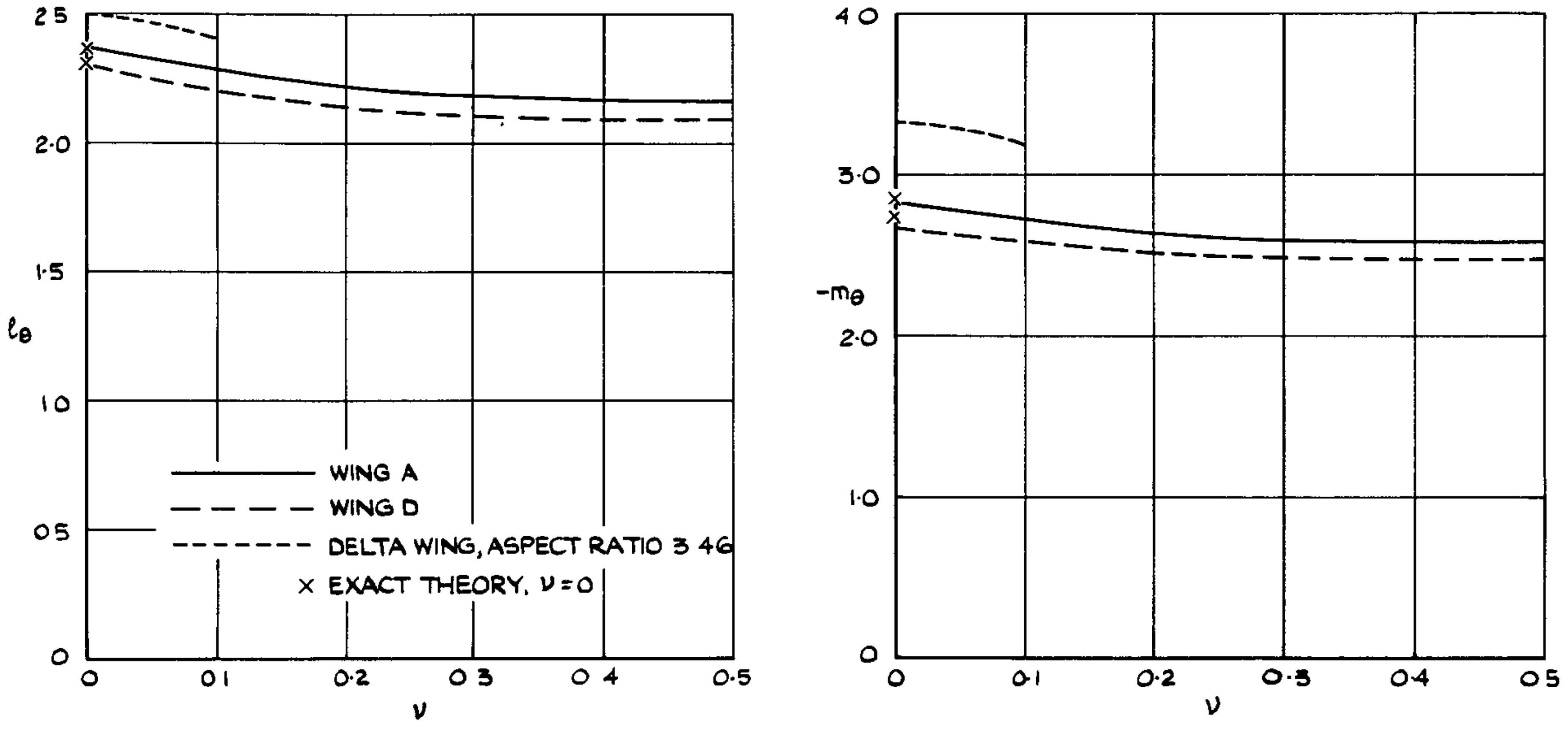
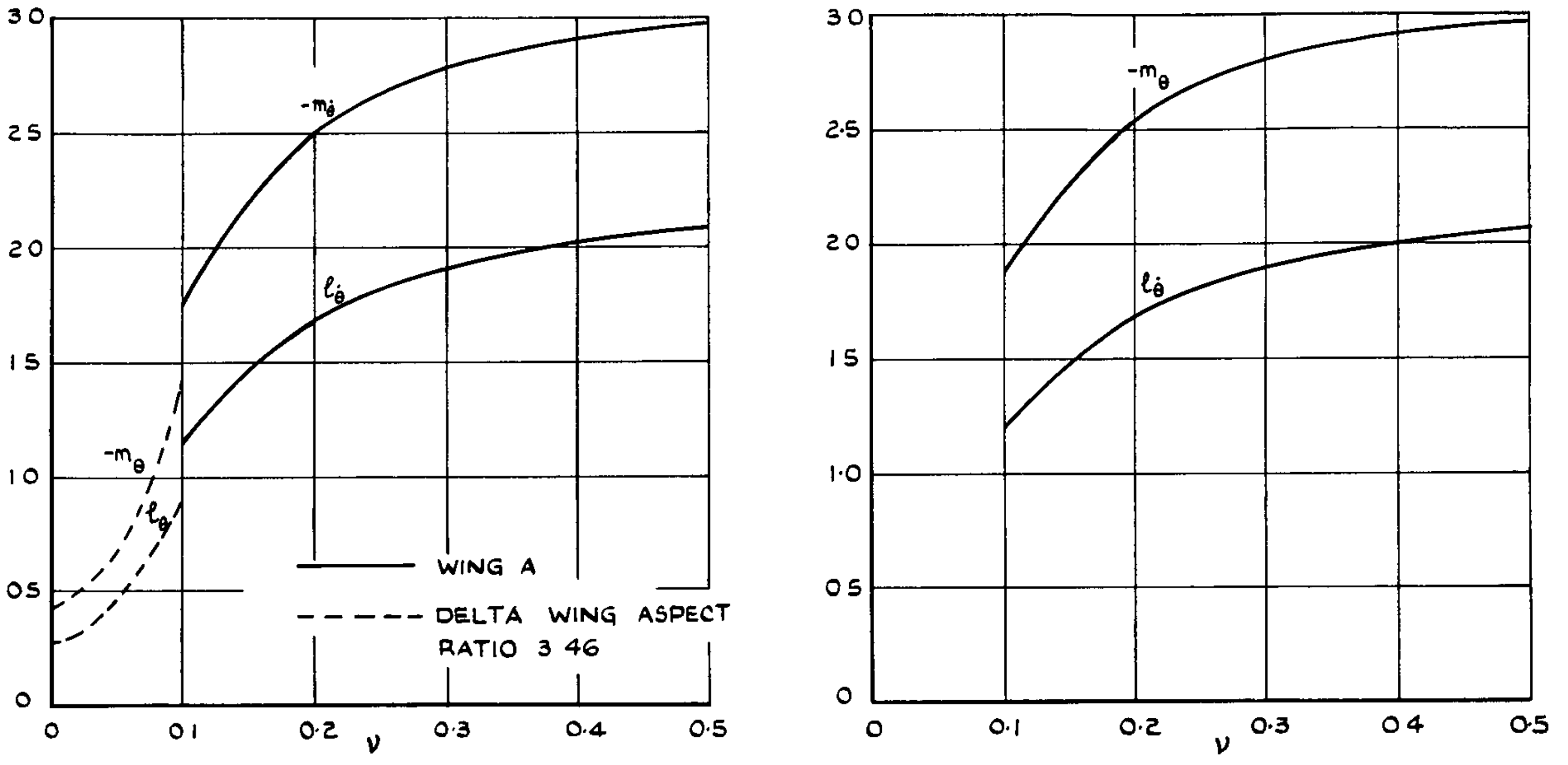


FIG. 16 VARIATION OF l_θ AND $-m_\theta$ WITH ν FOR WINGS A & D AT $M=1.054$



(a) WING A

(b) WING D

FIG. 17 (a & b) VARIATION OF $\ell_{\dot\theta}$ AND $-m_{\dot\theta}$ WITH ν FOR WINGS A & D AT M = 1·054

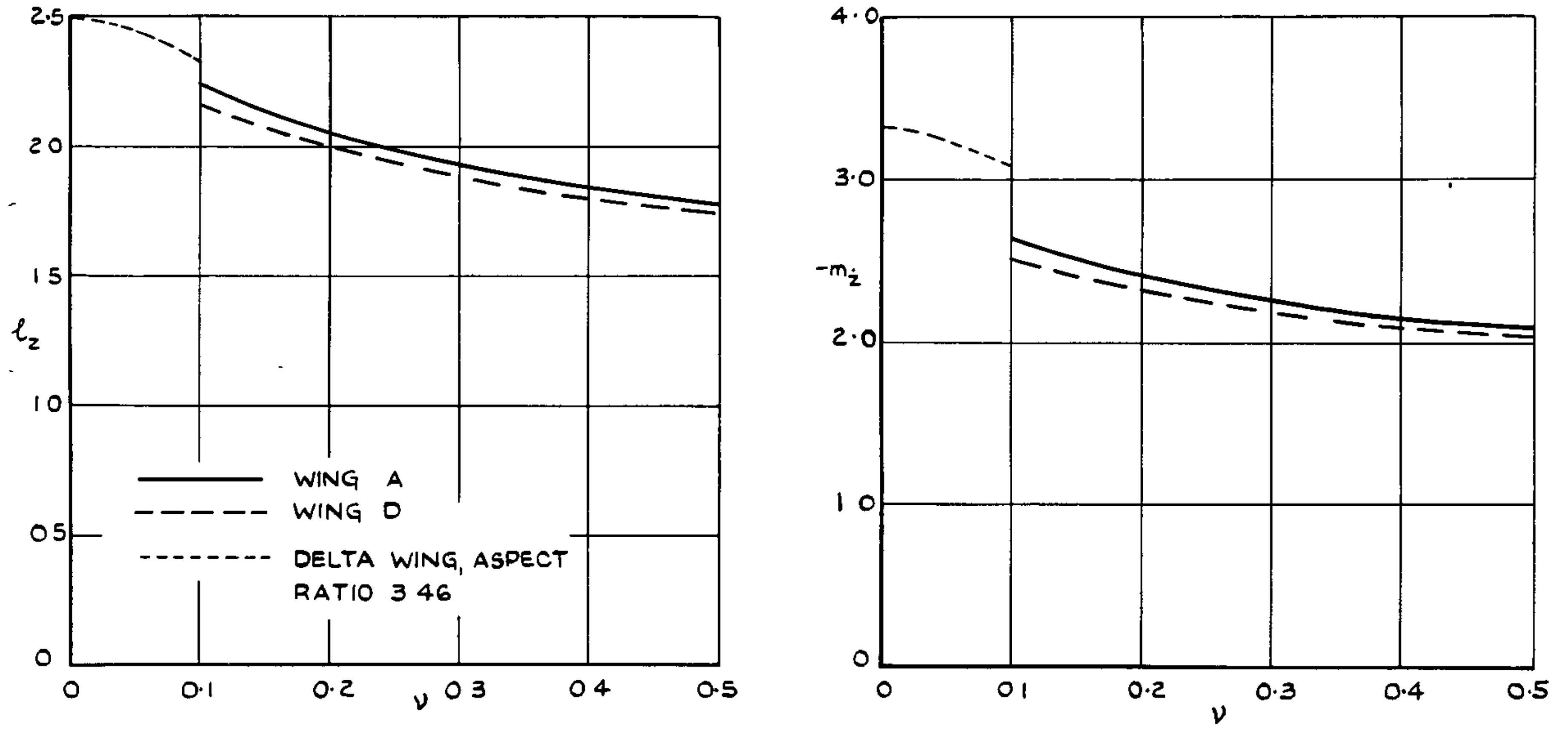


FIG. 18 VARIATION OF $\ell_{\dot z}$ AND $-m_{\dot z}$ WITH ν FOR WINGS A & D AT M = 1.054

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