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Studies of the Flow Fields Created by Single Vertical Jets Directed Downwards upon a Horizontal Surface

By

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SUMMARY

The velocity and temperature conditions in the space surrounding a single nozzle discharging hot gas vertically downwards on to a horizontal surface have been studied. Except at very low exit velocities, the amount of recirculation to an intake situated on the axis of the jet is small. Within limits, the dynamic head at ground level at a point away from the axis is independent of the height of the nozzle above the ground. These limits are decided by the spread of the jet before it reaches the ground, and by the lateral extent of the jet before it separates and rises from the ground.

A parameter including the initial velocity of the jet and its temperature is used to define both the vertical penetration of the jet and its lateral extent if it strikes the ground. The rate of decay of dynamic head in the jet along its axis and across the ground has been studied. In transient experiments, the rate of progress of the initial spread of the jet across the ground has been determined.

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1.0 Introduction

The use of direct jet thrust for assisting aircraft to take off vertically raises a number of problems in operation, one of which is the possibility that the temperature of the air into the engine intake may be increased owing to recirculation of hot gases from the lifting jets. In the region near the jet exits, where the gases still have an appreciable velocity, the flow of the gases is dependent on geometrical factors, that is, on the relative disposition of the jets and adjacent surfaces. Thus recourse is often made to model tests[1] to investigate heating effects on surfaces of the aircraft, and it is usual to simulate the full-scale velocity and temperature of the jet gases.

Temperatures at the engine intake of a few degrees above ambient can seriously affect engine performance; for example, a rise of 10°C may cause a loss of up to 5 per cent in thrust. Thus we are concerned in this respect also with gases which may have recirculated from more distant parts of the flow field, where velocities are low, and where forces on the gases due to their buoyancy become an important factor in establishing the flow patterns. In this situation it is not clear what the relationships between the various parameters are which have to be observed when simulating full-scale conditions in model tests. The object of the present experiments was to study the flow from a single jet directed towards the ground, particularly in regard to laws of scaling appropriate to the outer regions of the flow field, and to investigate whether recirculation from this region, both under steady-state and transient starting conditions, was affected by the presence of an intake situated just above the jet discharging hot gas.

2.0 Apparatus

The set up for the experiments is illustrated in Figure 1. A supply of hot gas was provided by an aircraft combustion chamber burning kerosine; a simple air-driven ejector was used as a source of sub-atmospheric pressure. Both these supplies were ducted through concentric pipes to a point well away from surrounding obstructions where the hot gas was led to a jet pointing vertically downwards, and the suction pipe terminated in a rounded inlet located above the jet. The height of the jet above the floor could be altered by interposing suitable distance pieces in the pipes. Both supplies could be turned on or off suddenly by operating plate valves in the supply pipes.

The temperature of the air around the jet was measured by a rake of thermocouples. The couples themselves were made of fine wires about 0.005 in. diameter, and a suction of about 5 in. of water gauge was applied to draw air over the couples to ensure rapid response. The outputs from the couples and from a thermocouple located in the gas just upstream of the jet exit were recorded on a sensitive recording galvanometer having twelve separate channels.

In many cases, it was necessary to warm the pipe leading from the combustion chamber to the jet before starting the experiment proper. To do this, an exhaust pipe was positioned under the jet nozzle, so that the hot gas was led away from the test area. Having warmed the pipe, the valves were moved to by-pass the hot gas and the exhaust pipe was removed.

The floor was smooth cement, but no special preparation of the surface was made. Usually, the hot gas was not allowed to flow for more than 30 seconds during a test, and there was no significant erosion of the floor.

3.0 Transient temperatures in the field around the jet

The thermocouple rake was erected at various radial distances from the jet axis and recordings of the temperatures were made during a 30 second period after the jet and intake flows were turned on suddenly. A thermocouple located just inside the intake was monitored to determine the temperature of the air at this point.

So far as could be determined, the temperatures in the region around the jet and in the vicinity of the intake were the same whether or not suction was applied to the intake. There appeared to be a small amount of recirculation of hot gas to the intake region, the temperature rise being about 1 to 2 per cent of the temperature above ambient of the jet at exit. Observations of temperature close to the nozzle were made difficult because there was always a small leakage of hot gas from the nozzle when the valve was closed, and this gas convected upwards and caused the intake thermocouple readings to fluctuate. When the jet was turned on, the readings at the intake steadied very considerably, but all the thermocouple readings during the test runs showed fluctuations in temperature, except when the couples were directly in the faster moving parts of the jet near the floor.

The fact that the temperature fluctuations in the outer regions of the jet field were of a similar magnitude to the temperature differences being measured, meant that it was difficult to get precise quantitative measurements. An indication of the steady-state temperature pattern is given in Figure 2. Approximately, the maximum value of excess temperature close to the ground, θ, varied inversely with radial distance from the axis of the jet and it was noted that these indications were similar to the measurements of Reeves[1].

4.0 Initial rate of spread of the jet

A transient phenomenon which could clearly be determined from the temperature records was the rate at which the initial boundary of the jet spread across the floor when the jet was first turned on. The sudden change in temperature was clearly defined on the record, and by placing the rake in a horizontal position just above the floor surface the time at which the jet reached a given radius could be found. Figure 3 shows how these measurements fall on curves in which distance is proportional to $(\text{time})^{\frac{1}{2}}$. The curves for the same jet exit velocity, but with the height of the jet above the ground varying in the range 1 to 36 diameters, are the same. Results for nozzles with diameters of 2 in. and 1 in. are shown.

The curves of Figure 3 give the rate of progress of the jet boundary, u, as the slope of the curve at any instant, t. The curves are of the form

$$R = ct^{\frac{1}{2}} \quad \text{(where c is a constant)}$$

$$u = \frac{dR}{dt}$$

$$= \frac{ct^{-\frac{1}{2}}}{2}$$

$$= \frac{R}{2t}$$

Values of the velocity u have been obtained from the results in this way and are plotted on Figure 4. The velocity is shown relative to V_J, the velocity of the jet at the nozzle and the distance is related to the diameter of the nozzle. It is seen that the measurements may be correlated quite well by this form of non-dimensional plotting.

It was not immediately apparent that the rate of spreading of the jet should be the same for widely different heights of the nozzle above the ground, since it was clear that the pressures in the jet at the region of impingement with the ground were very much reduced as the nozzle height was increased. Therefore, measurements of the steady state velocity field were made.

5.0 Steady-state velocity measurements

The dynamic head of the jet flow at ground level was measured on a sensitive water manometer using a total head probe which was held at a small distance above the ground (usually about $\frac{1}{2}$ to 1 in.) at the height which was found to give the maximum pressure reading. The pitot pressure at the exit plane from the nozzle and the pressure close to the ground at the point of jet impingement were also measured with the same pitot tube. Generally, the jet air was at ambient temperature, and the jet velocities ranged from 350 ft/s to 1100 ft/second.

5.1 Velocity field at ground level

A non-dimensional plot of the measurements is shown on Figure 4; the square root of the ratio of the dynamic pressure reading to the dynamic pressure at the jet exit, $(q/q_J)^{\frac{1}{2}}$, is equivalent to the velocity ratio V/V_J for isothermal, incompressible conditions. The readings lie close to a characteristic

$$\left(\frac{q}{q_J}\right)^{\frac{1}{2}} = \frac{1}{(R/D_J)}$$

for all heights in the range $z_o/D_J = 1$ to 36, all jet velocities and two nozzle diameters. That is, within limits the velocity at a point near the ground away from the jet axis is a fixed proportion of the jet exit velocity, irrespective of the height of the nozzle above the ground. Near the axis of the jet, the velocity does vary with the height of the nozzle above the ground.

If, alternatively, the dynamic pressure q is related to q_o, the pressure at ground level on the axis of the jet, and plotted against radial distance as shown on Figure 5, the proportionality is different for each nozzle height so that the straight-line characteristics intercept the axis $(q/q_o)^{\frac{1}{2}} = 1$ at values of R/D_J which increase with height. These distances might be termed "effective" diameters of the jet at impingement; that is

$$\left(\frac{q}{q_o}\right)^{\frac{1}{2}} = \frac{1}{R/D_e}$$

A few tests were made with the jet heated to about 300°C, and these confirmed that the relationship $(q/q_J)^{\frac{1}{2}}$ to (R/D_J) was the same as when the jet was at ambient temperature.

Referring again to Figure 4 it is seen that the rate of advance of the initial wave, u, is about one quarter of the final steady velocity appropriate to each radial station.

5.2 Velocity along axis of jet

The decay of dynamic pressure along the axis of the jet was measured with the jet exit at several heights above the ground, Figure 6. After an initial distance over which there is little change, the velocity decreases linearly with distance.

$$\left(\frac{q_z}{q_J}\right)^{\frac{1}{2}} = \frac{k}{z/D_J}$$

The value of k is between 6 and 6.5 which is similar to that reported by Squire[2,3] and others. It also appears that the dynamic pressure at the point of impingement is the same as the value at that axial distance in absence of ground effect.

On the same graph, the "effective" diameters of the jet D_e/D_J at the various nozzle heights determined from Figure 5 have been plotted to show that D_e has the same but inverse relationship with nozzle height as the velocity on the jet axis. That is

$$\left(\frac{q_o}{q_J}\right)^{\frac{1}{2}} = \frac{D_J}{D_e}$$

which establishes the compatability of the two equations relating the velocity of the spreading jet with radial distance given above.

6.0 Horizontal extent of jet flow

As the jet spreads across the ground, the maximum dynamic pressure of the flow decreases in inverse proportion to the square of the radial distance; however, the temperature of the gases near the ground decreases at a much slower rate, the excess temperature varying in inverse proportion to approximately the first power of the distance. Whilst at small radial distances the dynamic pressure of the flow is very great compared with the forces due to buoyancy of the gases, in the outer regions the magnitudes of these quantities become comparable, and the jet flow will tend to lift from the ground.

6.1 Choice of correlating parameter

The non-dimensional parameter usually associated with natural convective processes is the Grashof number, $\frac{\ell^3 g\beta\theta\rho^2}{\mu^2}$. It is formed from the product $\frac{V\rho\ell}{\mu} \times \frac{\ell^2 g\beta\theta\rho}{V\mu}$; the first term is Reynolds number, the second is the non-dimensional parameter which appears in the generalised analysis of velocity and temperature fields in a fluid. This latter parameter, rewritten as $\ell^3 g\beta\theta\rho \div \frac{V\ell^2\mu}{\ell}$ is seen to be the ratio between the buoyancy forces on an element and the viscous shearing forces. In the present instance we are primarily concerned with the relative effects of buoyancy and the kinetic forces in the flow so that a more suitable parameter would be

$$\ell^3 g\beta\theta\rho \div \rho V^2 \ell^2 = \frac{\ell g\beta\theta}{V^2}$$

Since velocity and temperature are related to the initial values V_J and θ_J, and D_J is a characteristic dimension of the system, a suitable correlating parameter for a first attempt at analysing the experimental results is

$$\frac{V_J^2}{\beta g\theta_J D_J}$$

6.2 Experimental results

The point at which the jet flow leaves the ground was determined by advancing a sensitive suction thermocouple held close to the floor inwards towards the jet axis until a point was reached where fluctuations in the galvanometer indication showed the presence of some hotter gas. Although the point at which fluctuations commenced was well defined, its radial distance from the jet axis varied considerably in a random way with time, so that the radial extent of the jet, R^*, is an estimate based on a series of observations made over the space of a minute or so.

The observations are plotted on Figure 7, with the extent of the jet as the ratio R^*/D_J, in terms of the parameter derived in the previous paragraph. Results are shown for two sizes of nozzle 1 in. and 2 in. diameter, for jet velocities from 50 to 500 ft/s and temperature at the nozzle exit from 63°C to 265°C above ambient; the height of the nozzle exit above the ground ranged from 6 diameters to 23 D_J.

From the upper graph of Figure 7, where log/log scales are used it is seen that a good approximation to the course of the points is R^*/D_J proportional to $(V_J^2/\beta g\theta_J D_J)^{\frac{1}{2}}$; therefore, the horizontal scale has been chosen accordingly in the lower graph.

6.3 Effect of temperature ratio T_J/T_a

Although Figure 7 shows a fair correlation between the parameters chosen, the range of temperature used was too small, and the boundary of the flow too unsteady for the results to show whether there was a factor involving the temperature ratio (T_J/T_a) which should be taken into account. As will be indicated later, this term appears in the theoretical analysis and accordingly a further test was made to attempt to evaluate the temperature (or density) ratio effect.

The apparatus was re-arranged to bring the combustion chamber nearer to the jet exit, and three suction thermocouples at distances 40, 51 and 59 D_J were supported just above ground level. Records of the temperature at these points were taken over periods of about 30 seconds while the gas conditions at the nozzle were held constant. Several tests of this sort were made at different combinations of jet pressure and temperature. Later, the temperature records were examined to see whether the trace was steady or fluctuating denoting that the boundary of the exhaust gases fell short of or extended beyond the fixed point. The results are shown on Figure 8; the jet dynamic pressure has been corrected in each case to apply to a radial distance $R^* = 50\ D_J$.

If for a given horizontal extent of the jet the parameter

$$\frac{V_J^2}{\beta_g \theta_J D_J}\left(\frac{T_a}{T_J}\right)^n = \text{constant}$$

then we can draw characteristics on Figure 8 corresponding to various constant values of n. Those for $n = 0$ and $n = \frac{1}{2}$ are shown; it seems unlikely that n should lie much outside these limits.

7.0 Vertical extent of the jet

Having determined the horizontal extent of spread of the jet after it touched the ground, it seemed worthwhile to find how high above the ground the nozzle had to be before the vertical part of the jet just failed to reach the ground. Similar experiments were, therefore, made by searching beneath the nozzle, which was raised well from the floor, with the suction thermocouple so that the lowest point reached by the hot gases might be determined.

Again the results correlate quite well using the parameter $V_J^2/\beta g\theta_J D_J$, as shown on Figure 9.

8.0 Discussion of choice of parameters

From the data presented we can gain an assessment of the factors on which the velocity and temperature patterns in the field around a single jet depend. The dynamic pressure of the spreading flow is proportional to the exit dynamic pressure at the nozzle and varies inversely with the square of the radius; similar dynamic pressure ratios occur at points having the same ratio of radial distance to nozzle diameter. Near the axis of the jet, the dynamic pressures depend on the height of the jet nozzle above the ground; the pressures in this region and their extent can be decided from considerations of the decay of dynamic pressure and the growth of a jet discharging axially from a nozzle.

The rate at which the boundary of the jet spreads along the ground is given approximately as one quarter of the steady-state velocity at each radial position.

The extent of the radial spread of the jet across the ground is found to correlate with a non-dimensional parameter $V_J^2/\beta g\theta_J D_J$, which relates the jet dynamic pressure to the initial buoyancy forces on the jet.

The several parameters used in these relationships will now be discussed, with reference particularly to extending the application of the results to higher velocities and temperatures than those used in the tests.

8.1 Velocity and temperature ratios

In most of the tests, the Mach number of the jet flow was too low for there to be a great difference between the quantities dynamic pressure (total pressure - static pressure) and kinetic pressure ($\frac{1}{2}\rho V^2$) or between total and static temperature. In any case, in regions of flow more than a few jet diameters away from the point of impingement, these differences will always be negligible. However, in many practical cases, where hot gases at choking pressure ratios are involved, the choice whether the operative parameter is $(P_t - P_s)/(P_t - P_s)_J$ or $\frac{1}{2}\rho V^2/(\frac{1}{2}\rho V^2)_J$ can alter estimates of velocity in the spreading jet by an important fraction. In cases where the velocity decay curves for the same system have been compared over a range of jet pressure ratios, we have noted that the correlation between tests at different pressures was better when dynamic pressures were used. An examination of Kuhn's[3] data shows a similar tendency for the correlation to be worse if kinetic pressure ratios are used instead of ratios of dynamic pressure. Anderson and Johns[4] studied jets at Mach numbers up to 3.5, and concluded that linearity of the axial decay parameters when plotted to logarithmic scales was obtained when the pressure ratio was formed from pitot tube readings, related to a pitot pressure measurement made immediately downstream of the nozzle exit. Therefore, in the present tests, the quantity q is taken to be the difference between the pressure measured on a pitot tube aligned with the flow, and the surrounding ambient static pressure, and it is proposed that this relationship should be used when extending present data to pressure levels beyond those actually tested.

The same authors[4] report measurements of the temperature decay along the axis of high velocity jets from which it is concluded that total temperatures should be used in forming non-dimensional ratios between temperature differences in order to correlate measurements.

8.2 The buoyancy parameter, horizontal extent of flow

In an initial correlation of the experimental data the parameter $V^2/\beta g\theta D$ was used to express the interaction between buoyancy and kinetic forces in the flow field. While it is clearly possible that the interaction of these forces can decide the path of a freely-moving buoyant element of fluid, it is not so clear that they are relevant when the flow is bounded on one side by a surface. Dr. B. S. Stratford suggested that the separation of the buoyant flow from the ground might be analogous to the separation of isothermal flows which occurred in many cases when the rise in static pressure at the surface exceeded a fraction (usually between 0.3 and 0.55) of the dynamic head of the main stream. In the present instance the pressure rise at separation may be equated to a defect in pressure of the jet flow at the surface due to the buoyancy of the heated gas above (Figure 10a). It might be anticipated that separation will occur when the pressure rise exceeds a certain fraction of the local maximum dynamic head in the spreading jet, $\frac{1}{2}\rho V^2$.

A theoretical analysis of the buoyancy effect has been made in Appendix II. It has been assumed that the distributions of velocity and temperature transverse to the direction of flow follow a normal error law, and that the ratio between the rates at which temperature and velocity effects spread across the flow is characterised by a factor 'b', the value of which is not much different from 1. It is shown that the pressure defect at the surface due to buoyancy is

$$\Delta p = \beta g \cdot \frac{D^2{}_J V_J \rho_J \theta_J}{RV} \cdot \frac{\sqrt{(1 + b^2)}}{8}$$

Since, approximately, V/V_J and R/D_J are in inverse relationship, Δp is constant within the region of the spreading flow and, for a given jet temperature, is proportional to nozzle diameter. It is further shown that the ratio of this pressure defect to the local maximum dynamic pressure of the flow is

$$\frac{\Delta p}{q} = \frac{\sqrt{(1 + b^2)}}{8} \cdot \frac{\beta g \theta_J D_J}{q_J} \cdot \rho_a \left(\frac{T_a}{T_J}\right)^{\frac{1}{2}} \left(\frac{R}{D_J}\right)^2$$

Where compressibility effects in the jet at the nozzle exit are small, this relationship may be written

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{\sqrt{(1 + b^2)}}{V_J^2} \cdot \left(\frac{T_J}{T_a}\right)^{\frac{1}{2}} \left(\frac{R}{D_J}\right)^2$$

Since this expression contains a temperature ratio term, and the results shown on Figure 8 are compatible also with a factor $(T_J/T_a)^{\frac{1}{2}}$, the results of horizontal extent have been replotted to include this on Figure 11. The straight line which best fits the observations is given by

$$\frac{R}{D_J} = 0.62\left[\frac{V^2{}_J}{\beta g\theta_J D_J}\cdot\left(\frac{T_a}{T_J}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

So that when separation occurs

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = (0.62)^2\,\frac{\sqrt{(1+b^2)}}{4}$$

$$= 0.14 \ (b = 1)$$

$$= 0.17 \ (b = 1.4)$$

The value of b is probably within the limits indicated; if b = 1, then temperature effects are spreading across the flow at the same rate as momentum. Usually in mixing systems, temperature effects spread slightly the faster, so that b is probably rather greater than unity. It may be said that qualitatively the extent of the jet to the point of separation follows the present analysis, but that the pressure coefficient $\Delta p/\frac{1}{2}\rho V^2$ is about $\frac{1}{2}$ to $\frac{1}{3}$ of the value found in other types of separating flow.

Another situation where separation of the spreading flow occurs is shown in Figure 10b, where a wind of velocity U opposes the jet flow along the ground. The pressure rise which the jet flow is seeking to overcome will be near to $\frac{1}{2}\rho U^2$. Therefore, the pressure coefficient is

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{\frac{1}{2}\rho U^2}{\frac{1}{2}\rho V^2}$$

For the values 0.14 to 0.17 found previously, we have

$$\frac{U}{V} = 0.37 \text{ to } 0.41$$

In the present tests we have the velocity of advance of the initial wave, u, is 0.25V. However, the two cases are not strictly comparable as the initial wave is a transient phenomenon. If we assume that the pressure rise to be accomplished in the transient case is about twice that in the equivalent steady-state where a cross-wind of velocity u is opposing the flow, then

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{\rho u^2}{\frac{1}{2}\rho V^2}$$

$$\frac{u}{V} = 0.26 \text{ to } 0.29$$

which is not far from the experimental value.

8.3 The buoyancy parameter, vertical penetration of jet

A theoretical approach to the interaction of kinetic and buoyancy forces in a downwardly-directed hot jet is given in Appendix III. This leads to a buoyancy parameter of the form used initially to correlate the experimental results, but a temperature ratio term is now included. Thus

$$\left(\frac{z^*}{D_J}\right)^2 = \frac{k^2}{k'} \times \frac{V_J^2}{\beta g \theta_J D_J} \times \frac{T_a}{T_J}$$

Again there is reasonable qualitative agreement between theory and experiment, although the theoretical line shown on Figure 13 overestimates the penetration by a factor of about 1.4, indicating some shortcomings in the theoretical analysis. There are certain similarities between this problem and that of the penetration of a column of heated air upwards into the atmosphere which has been studied by Taylor and others[5], but in their case the buoyancy forces at the starting condition were of the same magnitude as the dynamic forces in the plume. It may be that the analysis could be improved by considering the momentum of the system, starting from a point on the axis of the jet where mixing has reduced the dynamic pressures to the same order of magnitude as the buoyancy forces. However, a satisfactory result along these lines has not been found.

9.0 Conclusions

The velocity and temperature conditions in the space surrounding a single nozzle discharging hot gas vertically downwards have been studied.

When the jet strikes the ground it spreads radially outwards and the amount of recirculation of hot gas to the region of the nozzle appears to be small.

The dynamic head of the flow at ground level decreases inversely as the square of the distance from the jet axis and is proportional to the dynamic head at the nozzle exit. To a close approximation, the dynamic head at ground level at any point outside the region of direct impingement of the jet is independent of the height of the nozzle above the ground.

The lateral extent of the flow at ground level has been correlated in terms of a parameter relating the jet dynamic head to its buoyancy due to excess temperature above ambient. A similar parameter has been used to correlate the vertical extent of a jet discharging well above ground level.

The evidence of the tests supports the view that in model experiments where the flow field around jets directed towards the ground is of importance, the value of the parameter $\frac{V_J^2}{\beta g \theta_J D_J} \times \left(\frac{T_a}{T_J}\right)^n$ should be the same in model and full-scale cases. The value of the index n should be 0.5 where the jet strikes the ground, and 1.0 if the jet does not reach the ground. These values for n are based on a theoretical approach, and while the experiments do not establish absolutely that they are correct, the experimental results and theoretical values are nevertheless compatible. A slightly altered form of the parameter is recommended for cases where compressibility effects in the jet flow at the nozzle exit must be taken into account.

In tests where the jet was suddenly switched on, the rate at which the initial boundary of the jet advances across the ground has been found to be about one quarter of the final steady velocity of the gases at each radius.

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APPENDIX I

List of symbols

b	factor, h_2/h_1, defining relative rates at which temperature and momentum effects penetrate transversely to the flow
c	constant
Cp	specific heat
D	diameter of jet
D_e	equivalent diameter of jet at ground level
g	acceleration due to gravity
h	height above ground in field away from jet axis
h_1, h_2	height, moving away from surface, at which v and $(T - T_a)$ have decreased to 1/e of the maximum local values, V and θ
L	$\frac{z}{kD_J}$
L^*	L at limit y = 0
ℓ	a characteristic length
Δp	defect in pressure at surface
q	dynamic head in jet flow
R	radial distance from jet axis
R^*	limit of radial extent of jet across ground
T	absolute temperature, T_J = jet total temperature
t	time
u	velocity of advance of initial jet wave
U	velocity of cross-wind
v	velocity of flow above ground level
V	maximum velocity in jet flow at given radial distance
x	$(1 + b^2)^{\frac{1}{2}} \cdot \frac{h}{h_2}$
y	$\frac{q}{q_J}$ along axis of jet
z	vertical distance downwards from nozzle exit

APPENDIX I (cont'd)

z^*	limit of vertical penetration of hot jet
β	expansion coefficient of gas $\frac{1}{T_a}$
ρ	density
θ	temperature difference $(T - T_a)$, maximum value in jet flow at given radial distance

Suffices

a	ambient conditions
J	conditions at exit plane of nozzle
o	conditions at ground on axis of jet

APPENDIX II

The buoyancy effect in flow spreading from an impinging, heated jet

The flow outwards from the point of impingement is assumed to be symmetric about the vertical axis of the jet. Let us consider the conditions where the flow crosses a surface defined by a vertical cylinder radius R, and suppose that the variations of temperature and horizontal velocity with height h follow normal error distributions; that is that

$$v = Ve^{-\left(\frac{h}{h_1}\right)^2}$$

$$T - T_a = \theta e^{-\left(\frac{h}{h_2}\right)^2}$$

V and θ are the values of velocity and temperature difference at ground level, that is they are maximum values, and θ is small compared with T_a.

The buoyancy force acting on an element of the flowing gas height dh and unit cross-section due to its density ρ being different from ambient is

$$(\rho_a - \rho) \cdot g \cdot dh = \rho\left(\frac{\rho_a}{\rho} - 1\right) g\, dh$$

$$= \rho\beta\ (T - T_a)\ g\, dh$$

Therefore, the defect in pressure at ground level due to the buoyancy of the gas above is

$$\Delta p = \int_0^\infty \rho\beta\ (T - T_a)\ g\, dh$$

$$= \rho\beta g\theta \int_0^\infty e^{-\left(\frac{h}{h_2}\right)^2} dh \qquad \ldots.(1)$$

Because the change in absolute value of ρ is small, it is regarded as constant in this integration.

The total quantity heat passing the cylindrical surface in unit time is

$$\Sigma\ 2\pi R\rho v\ (T - T_a)\ Cp \cdot dh$$

$$= 2\pi R\rho Cp \int_0^\infty v\ (T - T_a)\ dh$$

$$= 2\pi R\rho V\theta Cp \int_0^\infty e^{-\left(\frac{h}{h_1}\right)^2 - \left(\frac{h}{h_2}\right)^2} dh$$

$$= 2\pi R\rho V\theta Cp \int_0^\infty e^{-(1+b^2)\left(\frac{h}{h_2}\right)^2} dh$$

where

$$b = \frac{h_2}{h_1}$$

Put

$$(1 + b^2)\left(\frac{h}{h_2}\right)^2 = x^2$$

$$dh = \frac{h_2}{\sqrt{(1 + b^2)}} \cdot dx$$

$\therefore$ Total quantity of heat $= \dfrac{2\pi R\rho V\theta Cph_2}{\sqrt{(1 + b^2)}} \displaystyle\int_0^\infty e^{-x^2}\ dx$

This must equal the total heat discharged from the nozzle

$$= \frac{\pi}{4} D_J^2 V_J \rho_J \theta_J Cp$$

$$\therefore \frac{2\pi R\rho V\theta Cph_2}{\sqrt{(1 + b^2)}} \int_0^\infty e^{-x^2}\ dx = \frac{\pi}{4} D_J^2 V_J \rho_J \theta_J Cp$$

$$\beta g \rho \theta \cdot h_2 \int_0^\infty e^{-x^2}\, dx = \beta g \cdot \frac{D_J^2 V_J \rho_J \theta_J}{RV} \cdot \frac{\sqrt{(1+b^2)}}{8} \qquad \ldots.(2)$$

But we have from Equation (1) that

$$\Delta p = \beta g \rho \theta \int_0^\infty e^{-\left(\frac{h}{h_2}\right)^2} dh$$

$$= \beta g \rho \theta \cdot h_2 \int_0^\infty e^{-\left(\frac{h}{h_2}\right)^2} d\left(\frac{h}{h_2}\right)$$

and since
$$\int_0^\infty e^{-x^2} dx = \int_0^\infty e^{-\left(\frac{h}{h_2}\right)^2} d\left(\frac{h}{h_2}\right)$$

it follows, by substitution in Equation (2), that

$$\Delta p = \beta g \cdot \frac{D_J^2 V_J \rho_J \theta_J}{RV} \cdot \frac{\sqrt{(1+b^2)}}{8} \qquad \ldots.(3)$$

It has been observed (Figure 4) that

$$\frac{q}{q_J} = \left(\frac{D_J}{R}\right)^2 \qquad \ldots.(4)$$

If we consider first flow conditions where compressibility effects are absent then

$$\frac{D_J}{R} = \left(\frac{\frac{1}{2}\rho V^2}{\frac{1}{2}\rho_J V^2{}_J}\right)^{\frac{1}{2}}$$

$$= \frac{V}{V_J} \cdot \left(\frac{\rho}{\rho_J}\right)^{\frac{1}{2}}$$

Substituting this in Equation (3) we have

$$\Delta p = \beta g\, \theta_J\, D_J\, \rho_J\, \theta_J \cdot \left(\frac{\rho}{\rho_J}\right)^{\frac{1}{2}} \frac{\sqrt{(1 + b^2)}}{8} \qquad \ldots.(5)$$

Thus the pressure defect due to buoyancy varies very little with radius and is independent of jet velocity. At the radius of separation, R*, the maximum dynamic head of the flow, is

$$\tfrac{1}{2}\,\rho V^2 = \tfrac{1}{2}\,\rho_J\, V^2{}_J \left(\frac{D_J}{R^*}\right)^2$$

and the ratio of the buoyancy pressure defect to this is

$$\frac{\Delta p}{\frac{1}{2}\,\rho V^2} = \frac{\beta g\, \theta_J\, D_J}{V^2{}_J}\left(\frac{\rho}{\rho_J}\right)^{\frac{1}{2}} \frac{\sqrt{(1 + b^2)}}{4} \cdot \left(\frac{R^*}{D_J}\right)^2 \qquad \ldots.(6)$$

In the present experiments, (ρ/ρ_J) can be replaced by (T_J/T_a) so that

$$\frac{\Delta p}{\frac{1}{2}\,\rho V^2} = \frac{(R^*/D_J)^2}{\dfrac{V^2{}_J}{\beta g\, \theta_J\, D_J}\left(\dfrac{T_a}{T_J}\right)^{\frac{1}{2}}} \cdot \frac{\sqrt{(1 + b^2)}}{4}$$

The proportionality shown by the straight line drawn through the results on Figure 11 is that

$$\frac{R^*}{D_J} = 0.62\left[\frac{V^2{}_J}{\beta g\, \theta_J\, D_J}\left(\frac{T_a}{T_J}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

So that, at separation

$$\frac{\Delta p}{\frac{1}{2}\,\rho V^2} = (0.62)^2\, \frac{\sqrt{(1 + b^2)}}{4}$$

$$= 0.14 \qquad (b = 1)$$

$$= 0.17 \qquad (b = 1.4)$$

This analysis would seem to indicate, therefore, that separation occurs when the difference between ambient pressure and local surface static pressure has a value of about 15 per cent of the local maximum dynamic pressure of the flow. This critical condition for separation might be compared with the critical pressure rise coefficient found in other types of separating flow. For example, when detachment of a semi-infinite flow from a surface occurs, $\Delta p/\frac{1}{2}\rho V^2$ is often found to be between 0.3 and 0.55, V in this case being the flow velocity of the main stream outside the boundary layer. Clearly, however, it is invalid to expect a direct quantitative comparison between a semi-infinite flow on the one hand and a wall-jet flow on the other, and it might be expected that the latter is incapable of overcoming a static pressure rise of as large a proportion of the maximum local dynamic pressure as the former.

If the approximation $q_J \approx \frac{1}{2}\rho_J V^2{}_J$ is not valid, Equations (4) and (3) can be combined to give

$$\frac{\Delta p}{q} = \frac{\sqrt{(1+b^2)}}{8}\left[\frac{\beta g\ \theta_J\ D_J}{q_J}\cdot\rho\cdot\left(\frac{\rho_J}{\rho}\right)^{\frac{1}{2}}\cdot\left(\frac{\frac{1}{2}\rho_J\ V^2{}_J}{q_J}\right)^{\frac{1}{2}}\right]\cdot\left(\frac{R}{D_J}\right)^2$$

$$\text{or}\ \frac{\sqrt{(1+b^2)}}{8}\left[\frac{\beta g\ \theta_J\ D_J}{q_J}\cdot\rho_J\cdot\left(\frac{\rho}{\rho_J}\right)^{\frac{1}{2}}\cdot\left(\frac{\frac{1}{2}\rho_J\ V^2{}_J}{q_J}\right)^{\frac{1}{2}}\right]\cdot\left(\frac{R}{D_J}\right)^2$$

Thus we see that if we wish to preserve similarity in those parts of the flow field where buoyancy effects are significant, then we have to ensure similarity between values of the parameter represented by the expressions in the square brackets.

In most practical cases where the gas is air and for nozzle pressure ratios up to choking value, the substitution

$$\left(\frac{\rho_J}{\rho}\right)^{\frac{1}{2}}\left(\frac{\frac{1}{2}\rho_J\ V^2{}_J}{q_J}\right)^{\frac{1}{2}} \approx \left(\frac{T}{T_J}\right)^{\frac{1}{2}}$$

may be made, where T_J is the total temperature of the jet at the nozzle exit; the error is less than 4 per cent. Furthermore, temperatures in the spreading flow at points where buoyancy is becoming relatively important are not far from ambient, so the buoyancy parameter becomes

$$\frac{q_J}{\beta g\ \theta_J\ D_J}\cdot\frac{2}{\rho_a}\cdot\left(\frac{T_J}{T_a}\right)^{\frac{1}{2}}$$

For cases where compressibility effects at the nozzle exit are small, this parameter has the form discussed in Section 6.3 viz:-

$$\frac{V_J^2}{\beta g\ \theta_J\ D_J}\cdot\left(\frac{T_a}{T_J}\right)^{\frac{1}{2}}$$

APPENDIX III

The buoyancy effect in a hot jet directed vertically downwards

One line of approach to the problem of calculating the point of arrest of a jet of heated air directed downwards into quiescent surroundings is to consider the rate at which the initial energy of the jet is dissipated by mixing and by the buoyancy forces on the heated gases.

If we consider the conditions on the axis of the jet, we have from measurements in the absence of buoyancy effects that

$$\left(\frac{q}{q_J}\right)^{\frac{1}{2}} = \frac{k}{z/D_J} \quad \text{(where k is a constant)}$$

$$q^{\frac{1}{2}} = q_J^{\frac{1}{2}} \frac{kD_J}{z}$$

$$-\frac{d_q}{d_z} = q_J \cdot \frac{2k^2 D_J^{\,2}}{z^3}$$

$$= \left(\frac{q}{q_J}\right)^{\frac{3}{2}} \cdot \frac{2q_J}{kD_J}$$

This term is taken to represent the rate of decrease of energy per unit volume of the flow at the axis of the jet. From Squire[2] we have the temperature decay along the axis as

$$\frac{\theta}{\theta_J} = \frac{k'}{z/D_J} \quad \text{(where k' is a constant)}$$

We may now relate the temperature and dynamic pressure in the core of the jet by

$$\frac{\theta}{\theta_J} = \frac{k'}{k} \cdot \left(\frac{q}{q_J}\right)^{\frac{1}{2}}$$

$$\frac{d\theta}{dz} = \frac{1}{2}\,\frac{k'}{k} \cdot \frac{\theta_J}{(q_J)^{\frac{1}{2}}} \cdot q^{-\frac{1}{2}}\,\frac{dq}{dz}$$

$$= -\frac{k'}{k^2} \cdot \frac{\theta_J}{D_J} \cdot \frac{q}{q_J}$$

For temperature, the initial condition is that $\theta = \theta_J$ at $z = k'D_J$ so that

$$\theta = \theta_J - \int_{k'D_J}^{z} \frac{k'}{k^2} \cdot \frac{\theta_J}{D_J} \frac{q}{q_J}\, dz$$

We have seen that the upthrust, or buoyancy per unit volume of gas is $\theta\beta g\rho$ so that over a distance dz, the loss of energy of the jet is $\theta\beta g\rho dz$.

Therefore, the total decrease in energy of the jet over a distance dz is

$$-\frac{2q_J}{kD_J} \cdot \left(\frac{q}{q_J}\right)^{\frac{3}{2}} dz - \beta g\rho \left(\theta_J - \int_{k'D_J}^{z} \frac{k'}{k^2} \frac{\theta_J}{D_J} \frac{q}{q_J} \cdot dz\right) dz$$

For energy at the axis, the initial condition is that $q = q_J$ when $z = kD_J$ so that

$$q = q_J - \frac{2q_J}{kD_J} \int_{kD_J}^{z} \left(\frac{q}{q_J}\right)^{\frac{3}{2}} dz - \int_{kD_J}^{z} g\beta\rho\theta_J \left(1 - \int_{k'D_J}^{z} \frac{q}{q_J} \cdot \frac{k'}{k^2} \cdot \frac{dz}{d_J}\right) dz$$

Put

$$\frac{q}{q_J} = y \text{ and } L = \frac{z}{kD_J}, \quad dz = kD_J \cdot dL$$

$$y = 1 - 2\int_{1}^{L} y^{\frac{3}{2}}\, dL - \int_{1}^{L} \frac{g\beta\rho\theta_J kD_J}{q_J} \left(1 - \int_{\frac{k'}{k}}^{L} \cdot \frac{k'}{k} \cdot y dL\right) dL$$

$$= 1 - 2\int_{1}^{L} y^{\frac{3}{2}}\, dL - \int_{1}^{L} \frac{g\beta\rho\theta_J k'D_J}{q_J} \left(1 - \int_{1}^{L} y dL\right) dL$$

A way has not been found of integrating this expression to obtain the value of L when $y = 0$, but it can be done in a step-by-step fashion. A few hand calculations were made and it is clear that in the region of greatest interest, θ is small when buoyancy begins to have an effect, so that we may take the density ρ to be the ambient value. The buoyancy parameter is then

$$\frac{g\beta\rho_a\theta_J D_J}{q_J} \cdot k'$$

A computer programme† was used to find values of L when y = 0 for several values of the buoyancy parameter. These values are shown on Figure 12, and the relationship indicated between the variables is

$$(L^*)^2 = 2.0 \cdot \frac{q_J}{k' g\beta\rho_a\theta_J D_J}$$

The distance L*, the point where no kinetic energy remains in the gases at the jet axis is taken to be the point of arrest of the jet, so

$$\left(\frac{z^*}{kD_J}\right)^2 = \frac{2.0}{k'} \cdot \frac{q_J}{g\beta\theta_J D_J\rho_a}$$

$$\left(\frac{z^*}{D_J}\right)^2 = 2.0\,\frac{k^2}{k'} \cdot \frac{q_J}{g\beta\theta_J D_J\rho_a}$$

$$\simeq \frac{k^2}{k'} \cdot \frac{V_J^2}{g\beta\theta_J D_J} \cdot \frac{T_a}{T_J}$$

in the present tests where the jet velocity was low.

On Figure 13 the experimental observations have been plotted against the parameter given above, and the calculated line has been added, taking k = 6.5 and k' = 4.8, which are the values quoted by Squire[2] for velocity and temperature decay along the axis of a jet.

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†P. W. H. Howe programmed this operation and obtained the results quoted.

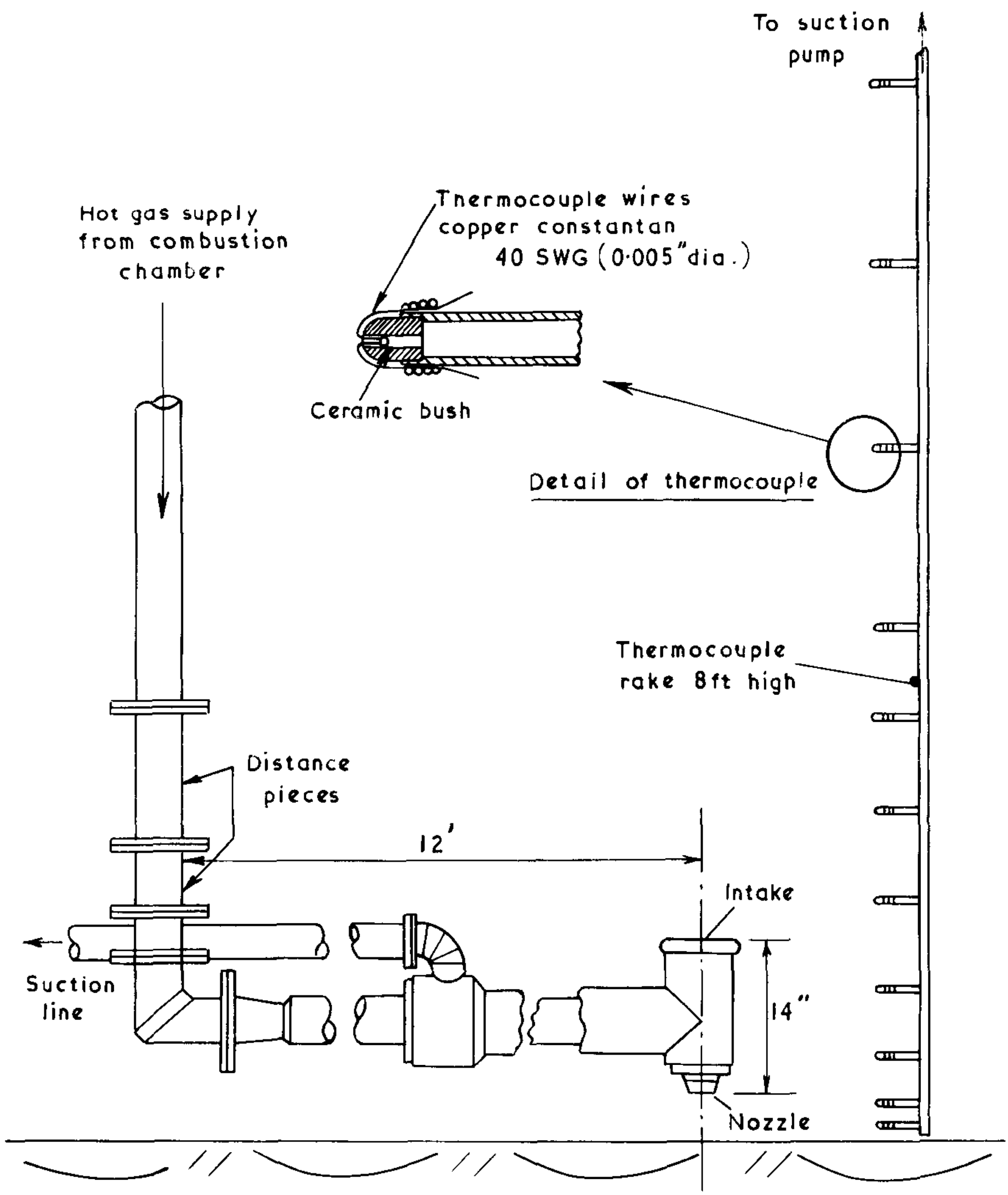

FIG. I Test apparatus

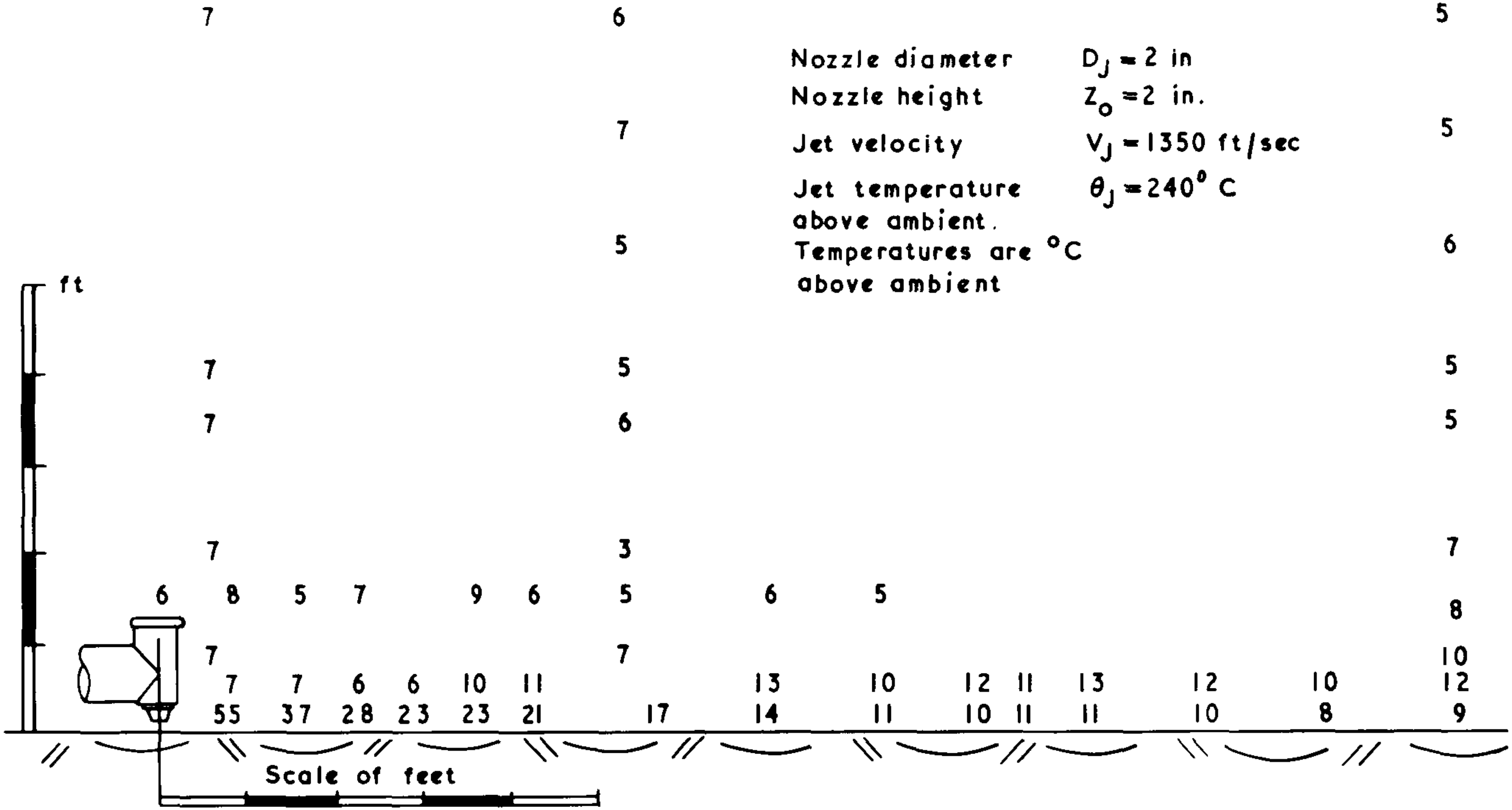

FIG.2 Temperature field

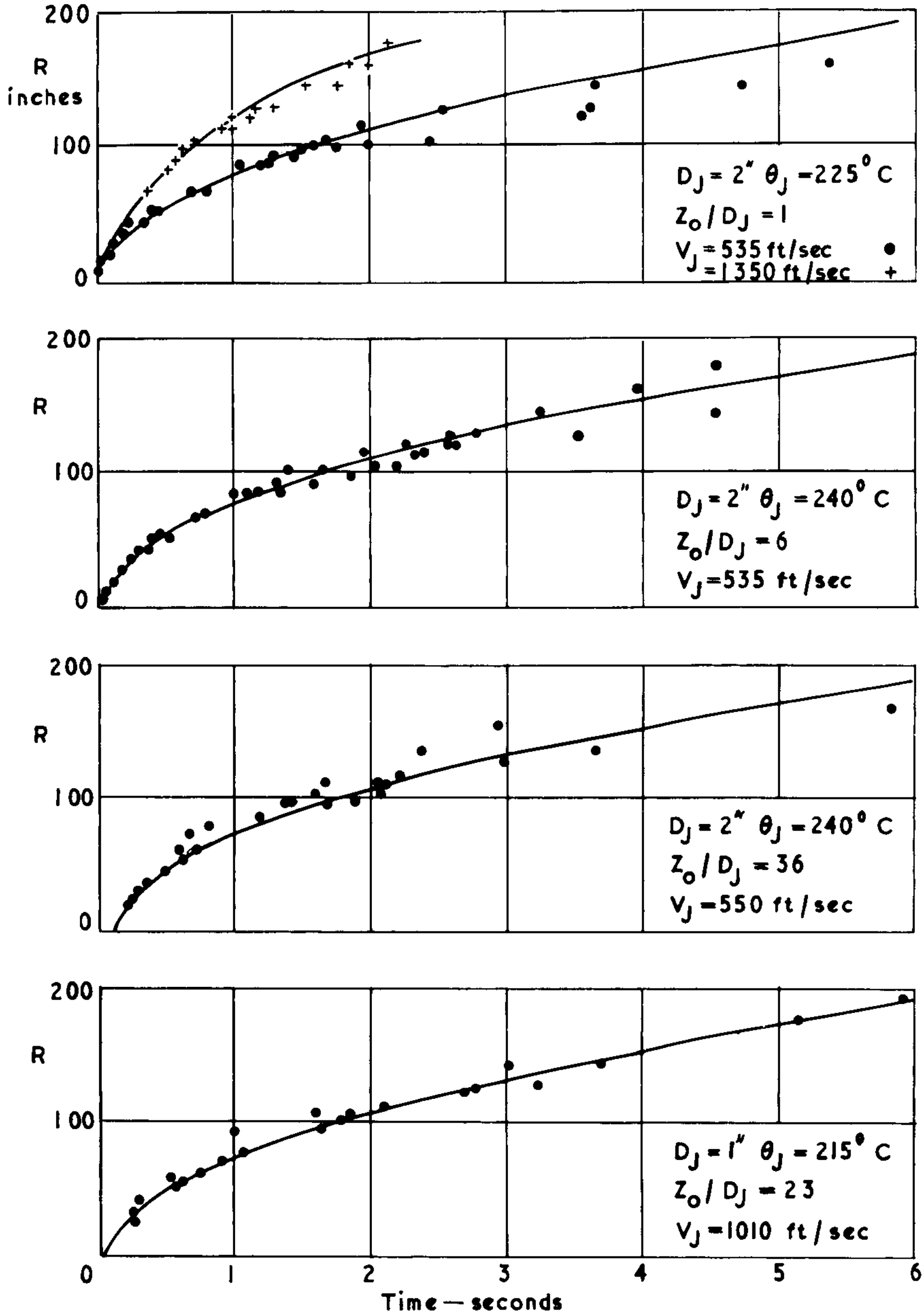

FIG. 3 Initial progress of jet across ground

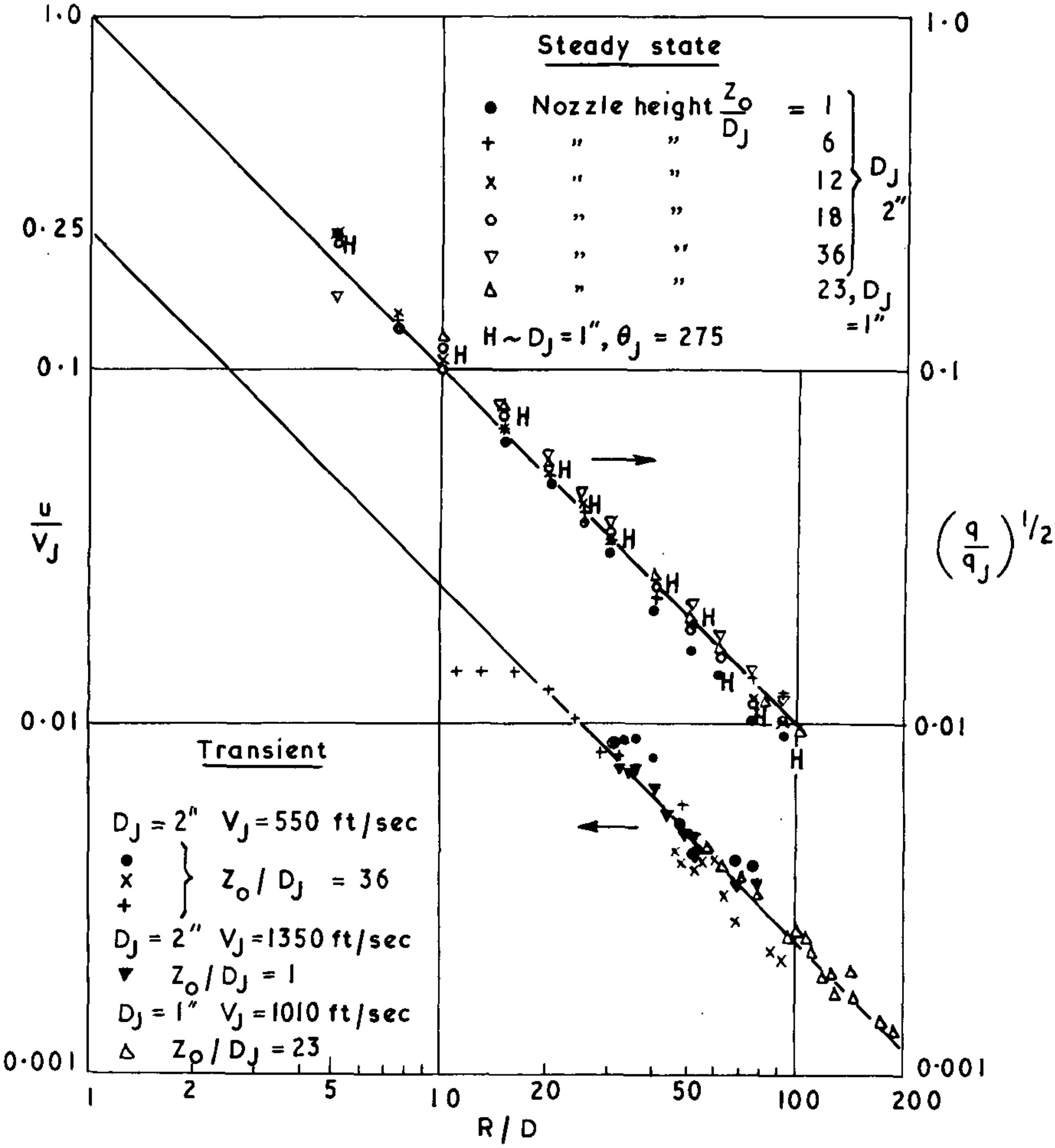

FIG.4 Relation between velocity at ground level and distance from axis of vertical jet in transient and steady state conditions

that we may take the density ρ to be the ambient value. The buoyancy parameter is then

$$\frac{g\beta\rho_a\theta_J D_J}{q_J} \cdot k'$$

A computer programme† was used to find values of L when $y = 0$ for several values of the buoyancy parameter. These values are shown on Figure 12, and the relationship indicated between the variables is

$$(L^*)^2 \quad = \quad 2.0 \cdot \frac{q_J}{k' g\beta\rho_a\theta_J D_J}$$

The distance L*, the point where no kinetic energy remains in the gases at the jet axis is taken to be the point of arrest of the jet, so

$$\left(\frac{z^*}{kD_J}\right)^2 \quad = \quad \frac{2.0}{k'} \cdot \frac{q_J}{g\beta\theta_J D_J \rho_a}$$

$$\left(\frac{z^*}{D_J}\right)^2 \quad = \quad 2.0\,\frac{k^2}{k'} \cdot \frac{q_J}{g\beta\theta_J D_J \rho_a}$$

$$= \quad \frac{k^2}{k'} \cdot \frac{V_J^2}{g\beta\theta_J D_J} \cdot \frac{T_a}{T_J}$$

in the present tests where the jet velocity was low.

On Figure 13 the experimental observations have been plotted against the parameter given above, and the calculated line has been added, taking $k = 6.5$ and $k' = 4.8$, which are the values quoted by Squire[2] for velocity and temperature decay along the axis of a jet.

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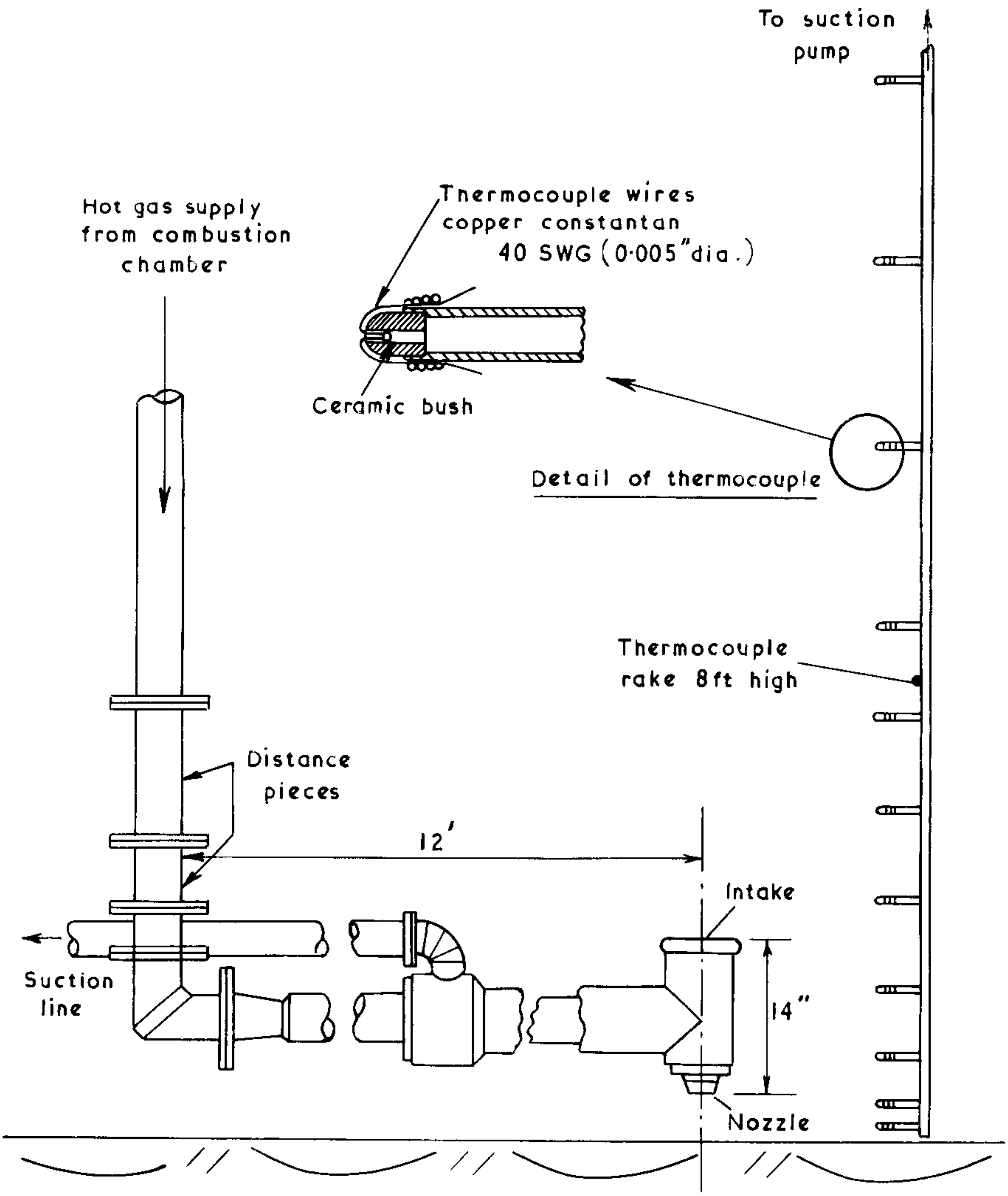


FIG. 1 Test apparatus

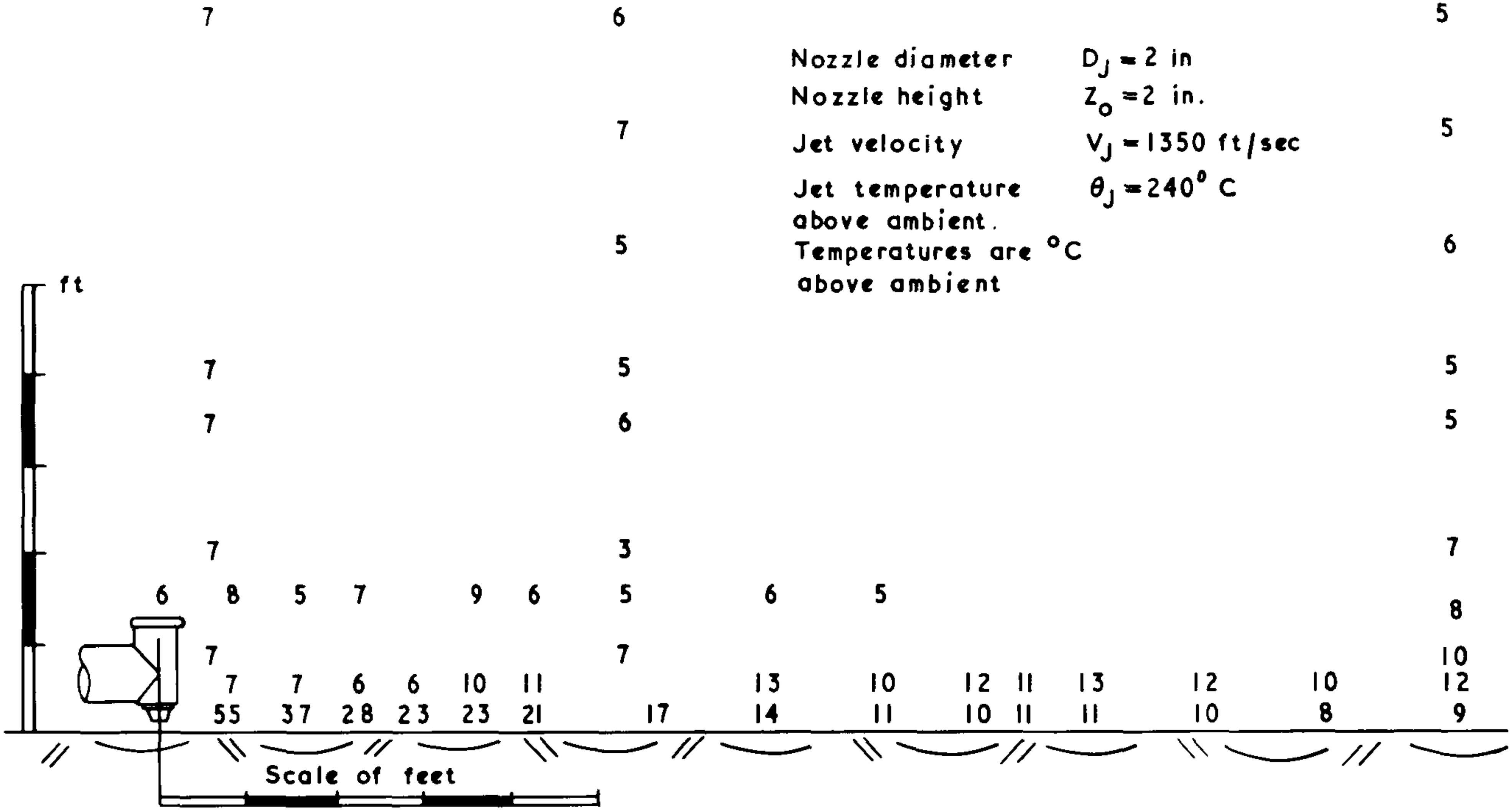


FIG.2 Temperature field

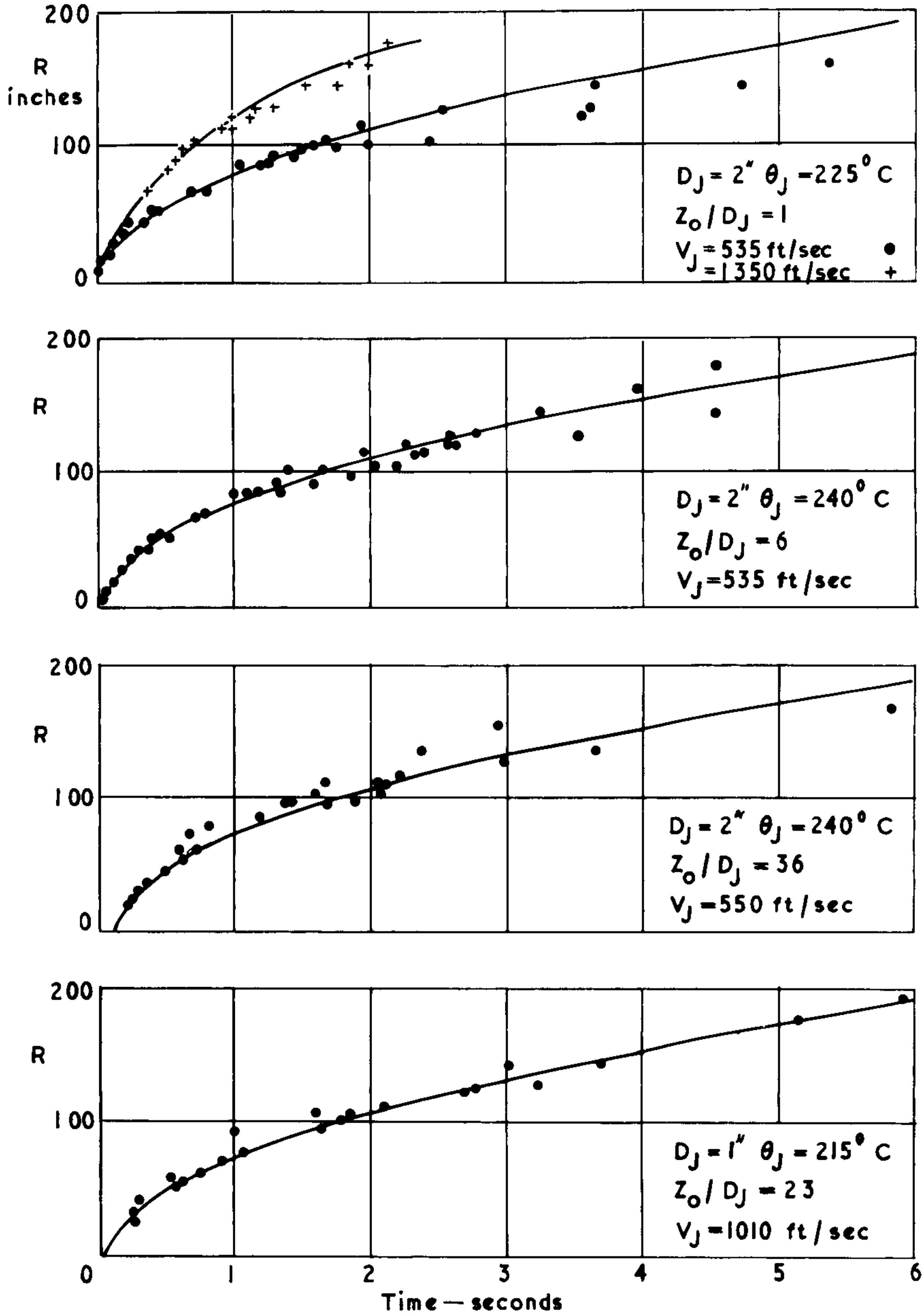


FIG. 3 Initial progress of jet across ground

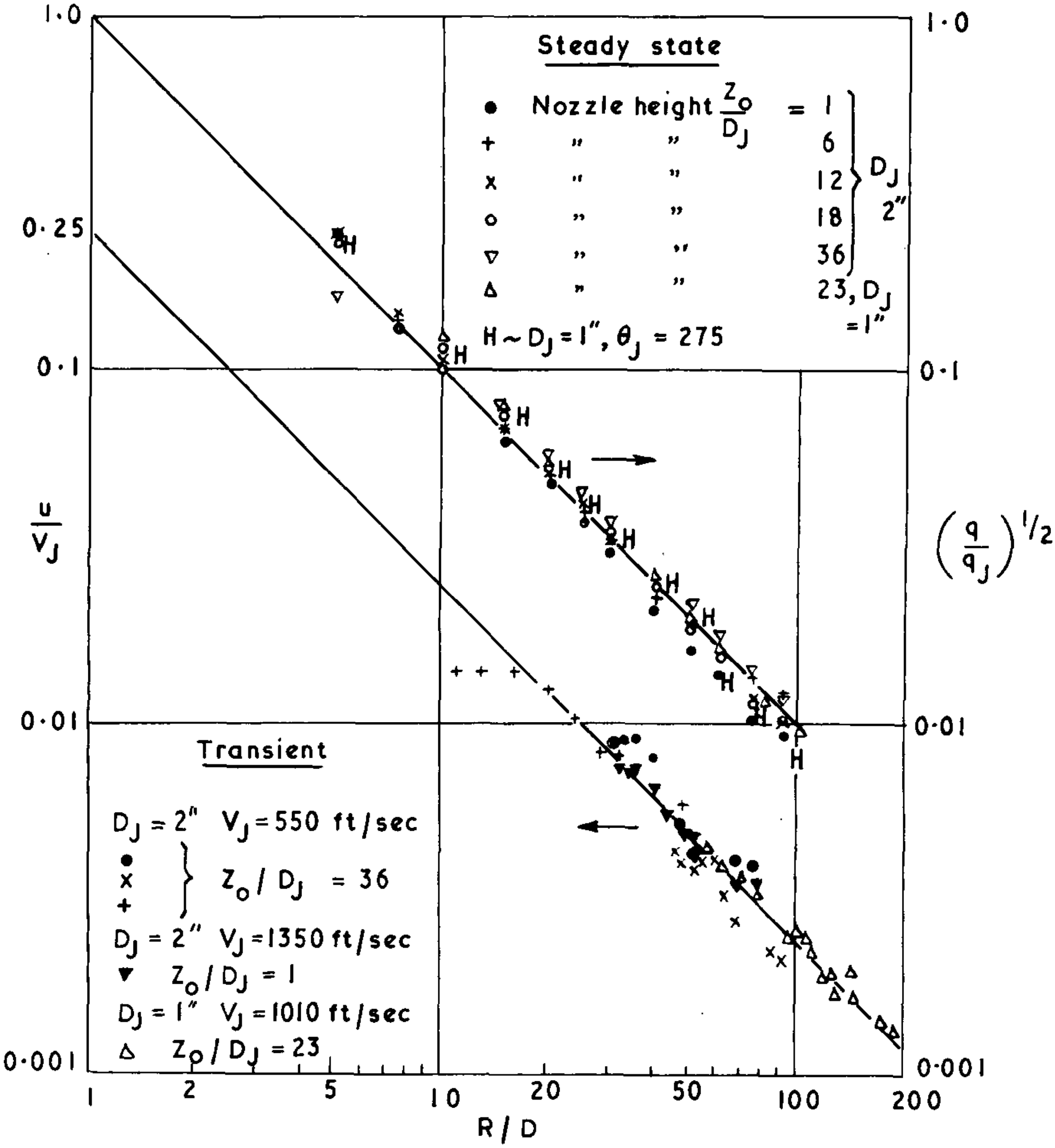


FIG. 4 Relation between velocity at ground level and distance from axis of vertical jet in transient and steady state conditions

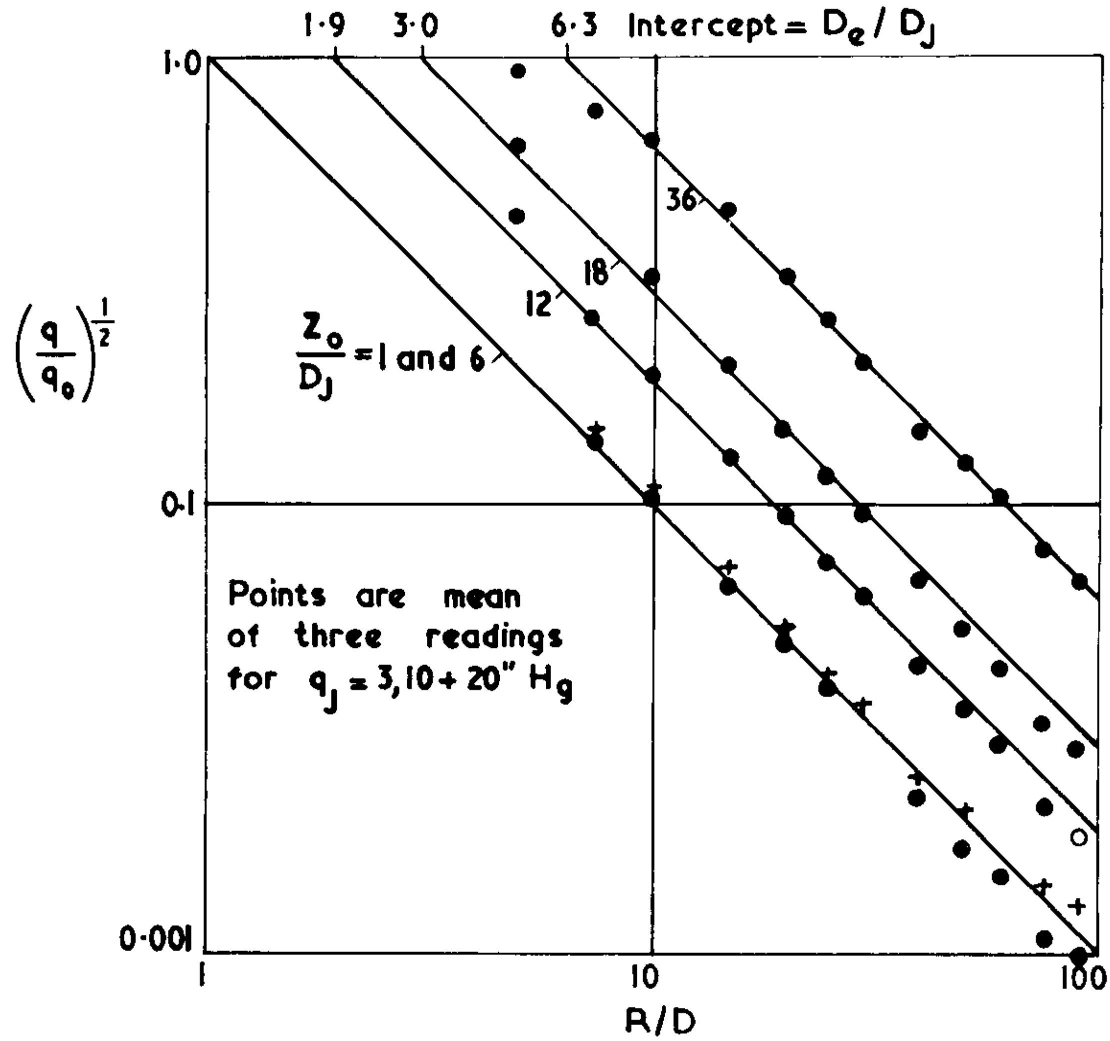


FIG.5. Variation of velocity at ground level related to total pressure at point of impingement

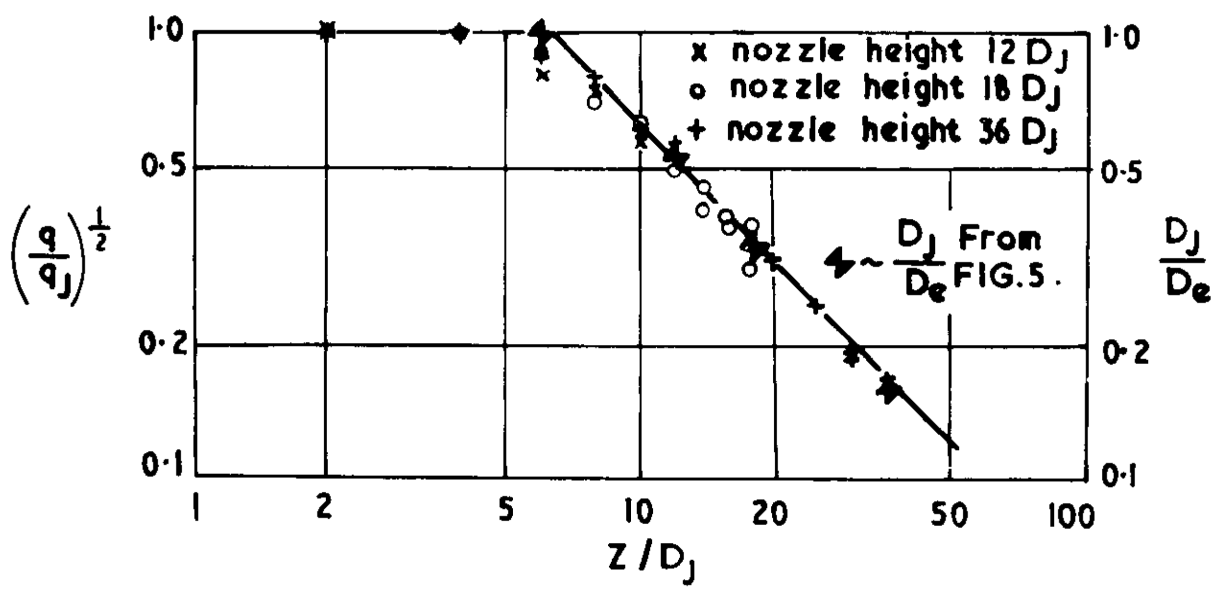


FIG.6. Variation of velocity along axis of jet.

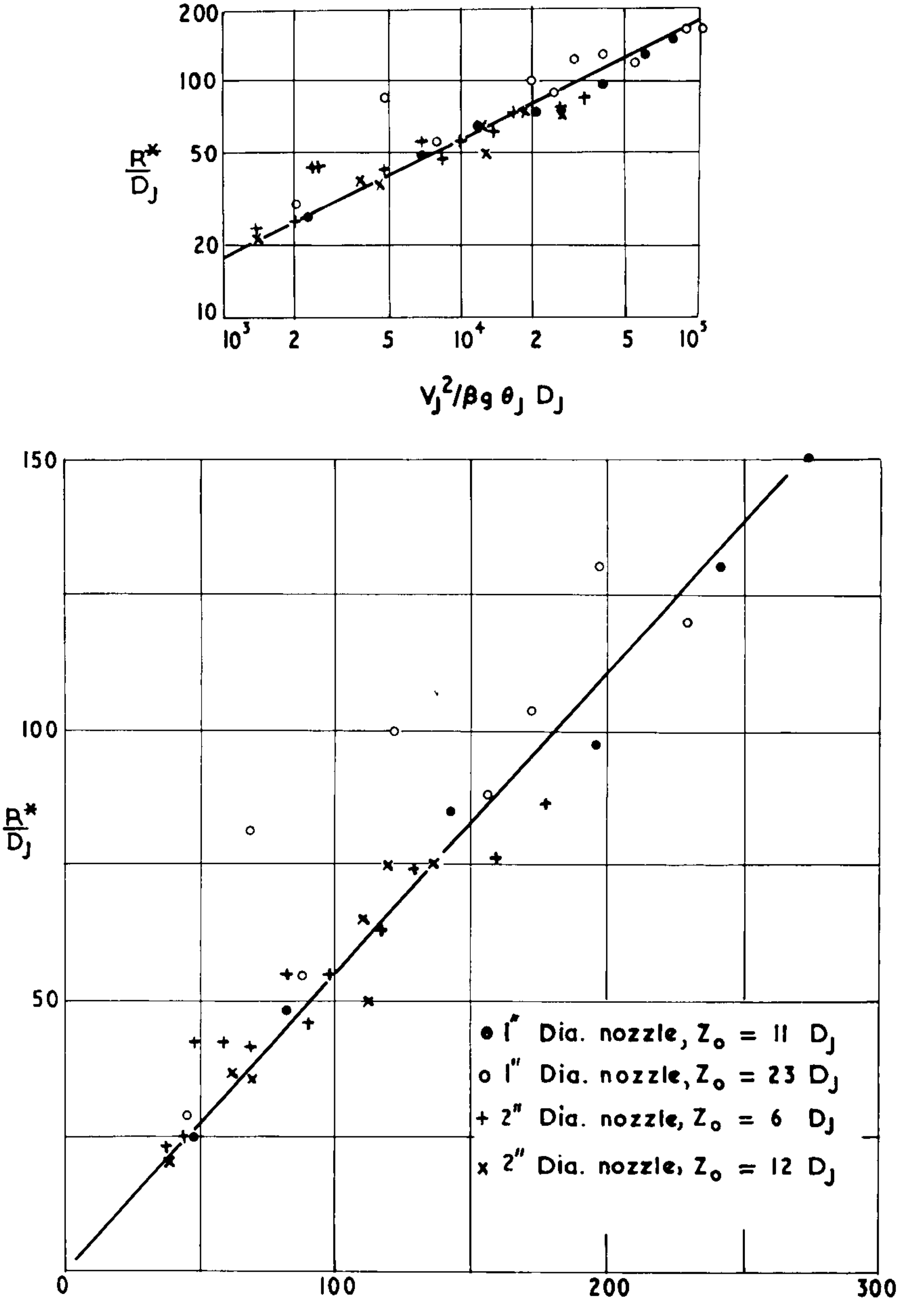
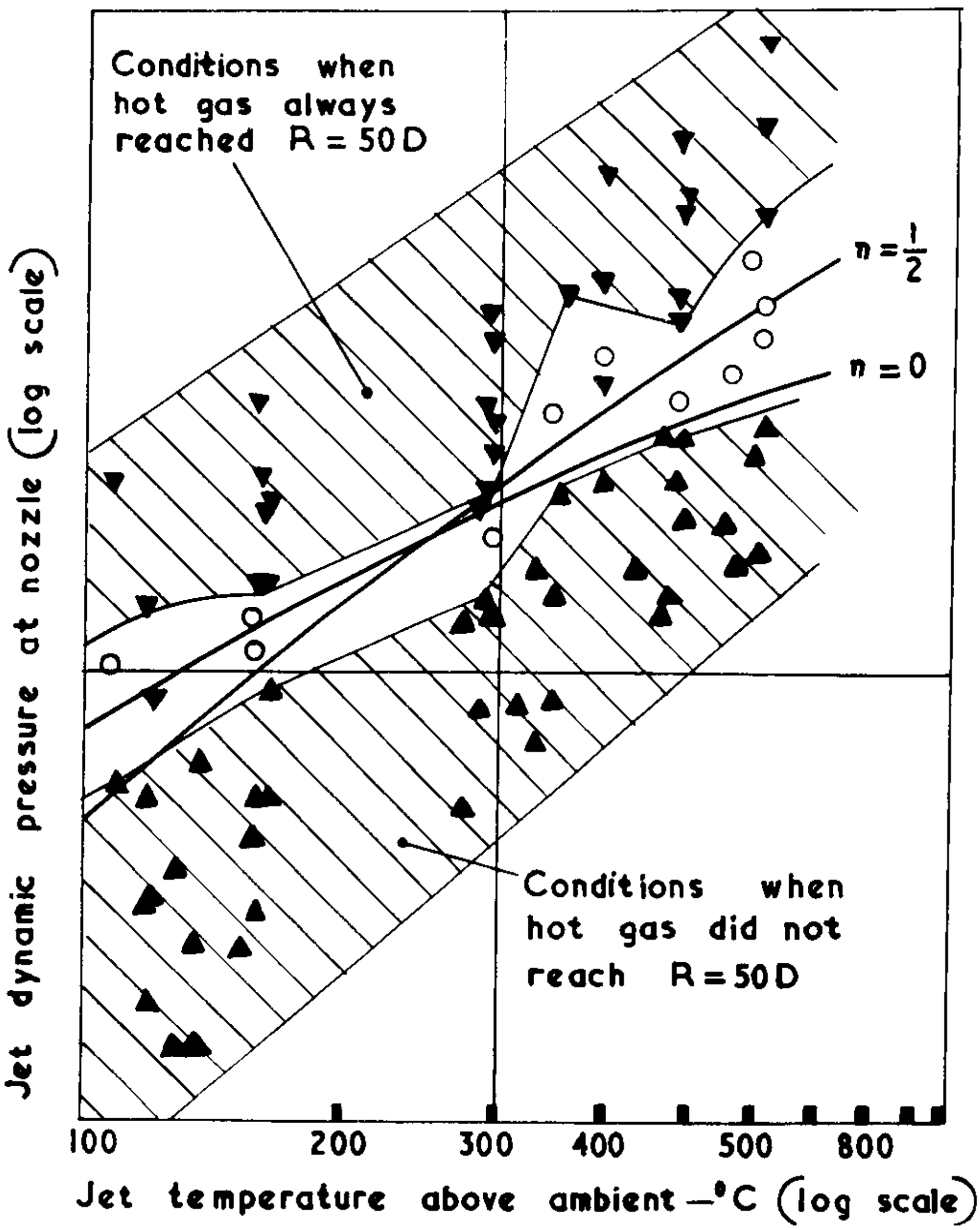


FIG.7. Horizontal extent of hot jet at ground level



▲ No hot gas detected over period of 30 seconds

▼ Hot gas always present over period of 30 seconds

o Hot gas detected intermittently

FIG. 8. Relation between jet dynamic pressure and temperature for constant horizontal extent of jet.

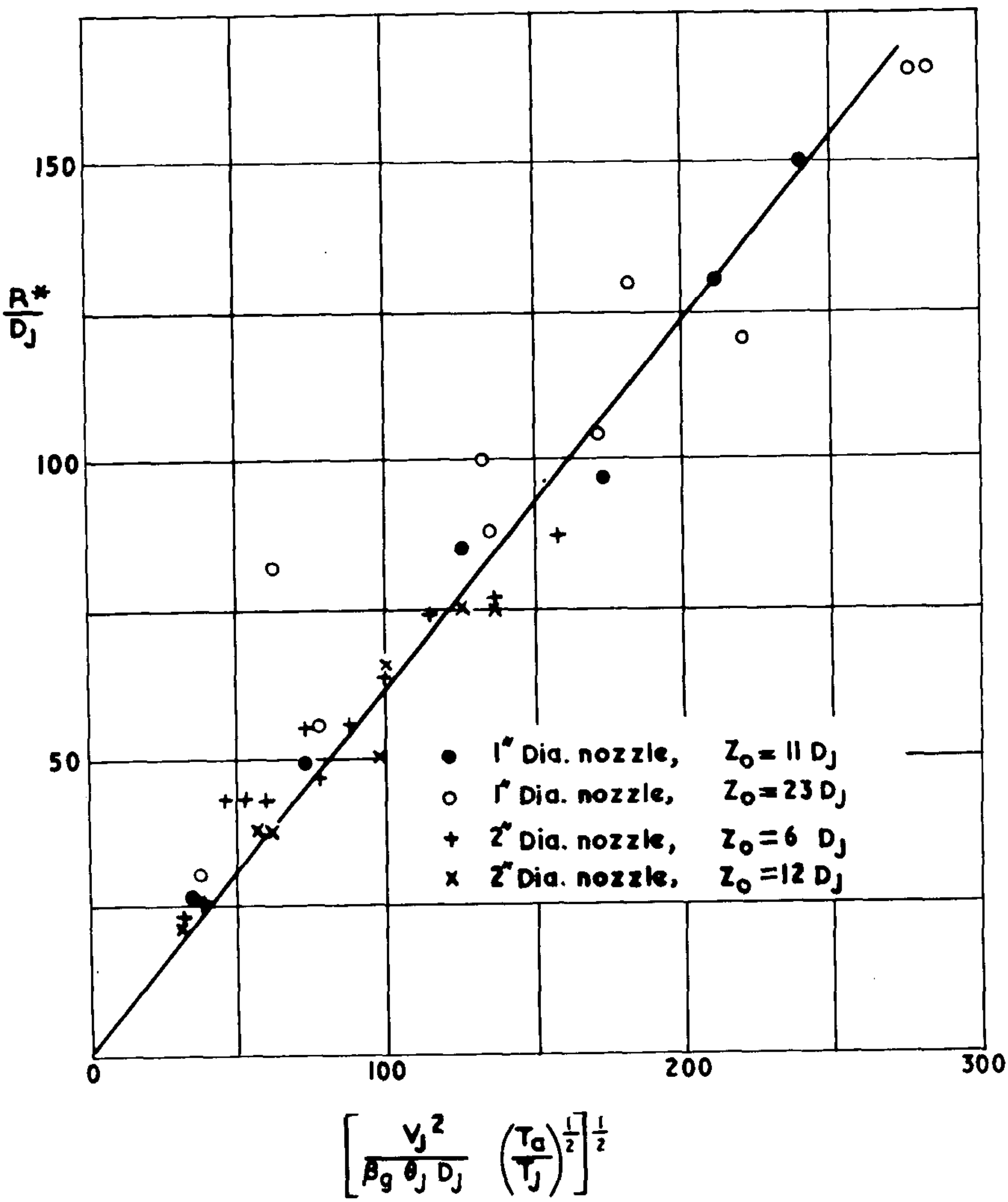


FIG.11. Horizontal extent of hot jet correlating parameter includes temperature ratio term

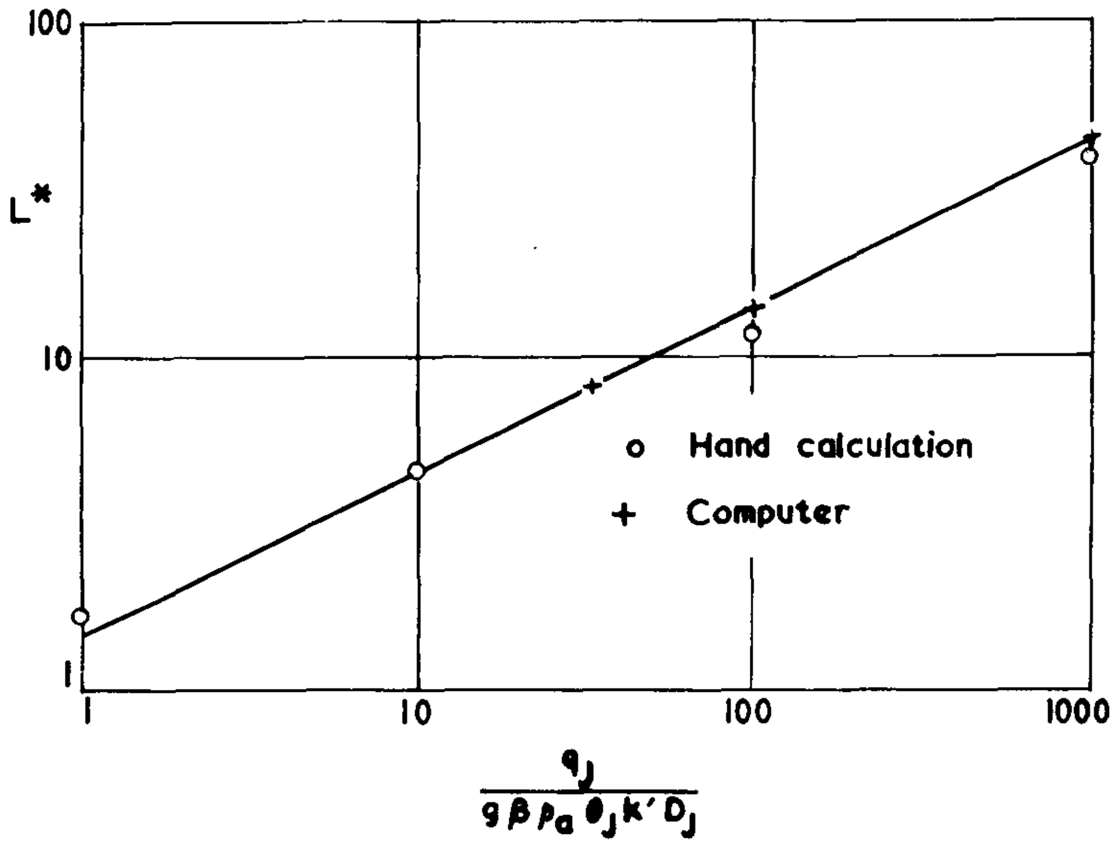
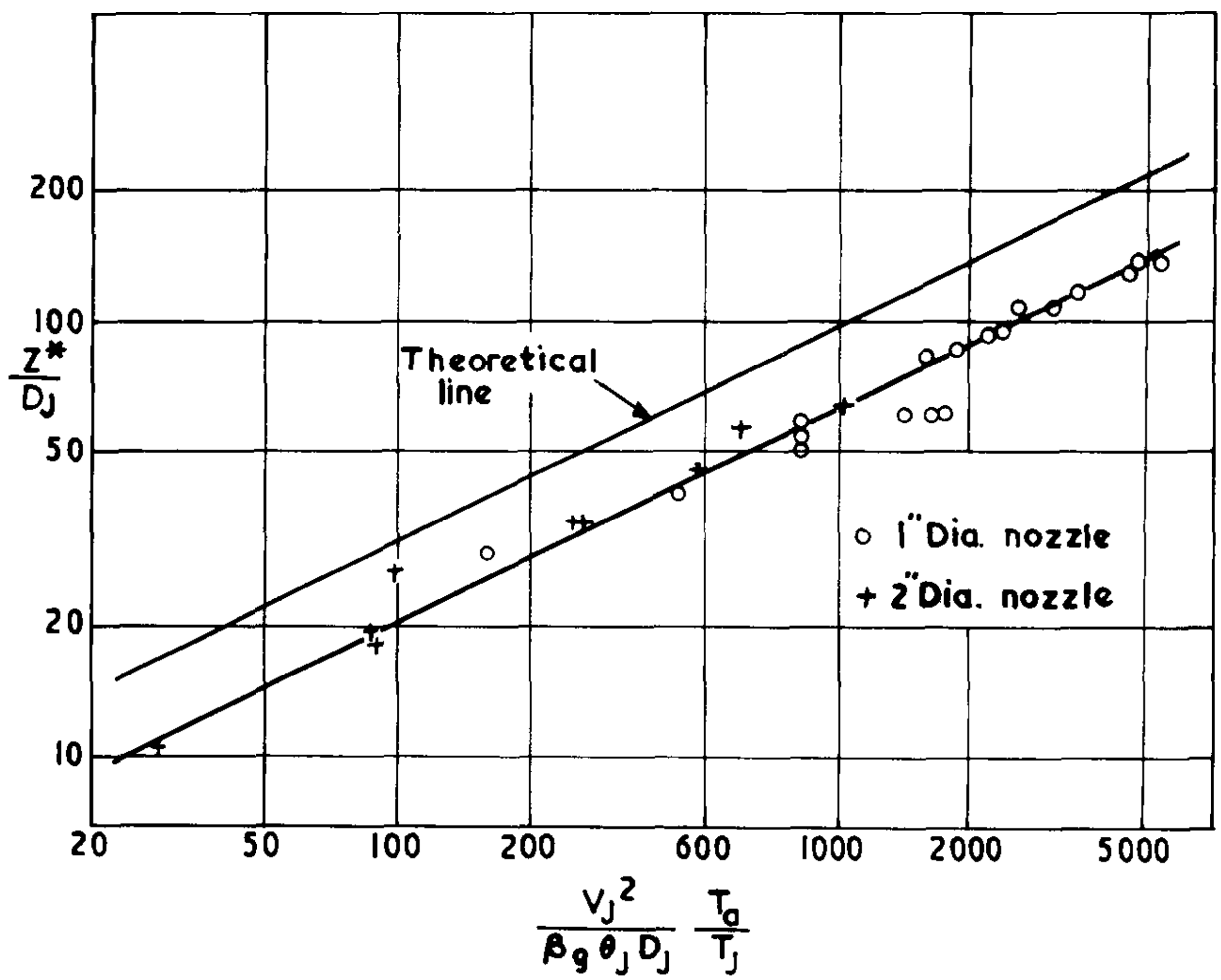


FIG.12. Calculated penetration of vertical jet



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October 1964
Cox, M. and Abbott, W. A.

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DOWNWARDS UPON A HORIZONTAL SURFACE

The velocity and temperature conditions in the space surrounding a single nozzle discharging hot gas vertically downwards on to a horizontal surface have been studied. Except a very low exit velocities, the amount of recirculation to an intake situated on the axis of the jet is small. Within limits, the dynamic head at ground level at a point away from the axis is independent of the height of the nozzle above the ground. These limits are decided by the spread of the jet before it reaches the ground, and by the lateral extent of the jet before it separates and rises from the ground.

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P.T.O.

DETACHABLE ABSTRACT CARDS

A parameter including the initial velocity of the jet and its temperature is used to define both the vertical penetration of the jet and its lateral extent if it strikes the ground. The rate of decay of dynamic head in the jet along its axis and across the ground has been studied. In transient experiments, the rate of progress of the initial spread of the jet across the ground has been determined.

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