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The Problem of Panel Flutter with Reference to the Blue Streak and Black Knight Vehicles

by

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1966

FIVE SHILLINGS NET

U.D.C. No. 623.651-519 : 531.258 : 533.6.013.422

C.P. No. 848

May, 1961

THE PROBLEM OF PANEL FLUTTER WITH REFERENCE TO THE BLUE STREAK AND BLACK KNIGHT VEHICLES

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G. Z. Harris, B.Sc., Ph.D.

SUMMARY

The problem of panel flutter is discussed, and the theory of Miles for cylindrical panels is applied in assessing the panel flutter problem on the Blue Streak and Black Knight vehicles.

Replaces R.A.E. Tech. Note Structures 296 - A.R.C. Report No. 23090

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1 INTRODUCTION

The term "panel flutter" applies to those panel oscillations in a flow in which the aerodynamic force due to the panel motion plays a decisive part. Panel vibration excited by any type of noise or by buffeting is not considered under this heading. The flutter of a flag in the wind is a well known example of panel flutter. This is common experience at subsonic speeds but flutter of aircraft and missile panels is thought to be a problem only at supersonic speeds. It is thought to have occurred, for example, on German V-2 missiles where failures were eventually attributed to panel flutter. Panel failure occurs ultimately due to fatigue, but other than this the flutter has an influence on drag.

In this Note a brief assessment is first made of the general problem of panel flutter. The theory of Miles is then summarised and applied in the detailed consideration of the panel flutter problem on the Blue Streak and Black Knight vehicles.

2 THE PROBLEM OF PANEL FLUTTER

Panel flutter is a practical problem supersonically, and then only above a certain flutter speed. For an unbuckled panel, as the airspeed increases the panel is first stable and then becomes subject to an oscillation when the flutter speed is passed. This oscillation becomes higher in frequency with further increases in airspeed, although experiment shows that the flutter can exhibit marked discontinuities in nodal pattern and frequency with continuously increasing airspeed. The experimental evidence is that failure occurring as a result of panel flutter is due to fatigue and is not a quasi-static failure as in commoner forms of flutter. For example, Sylvester[1] reports wind tunnel tests on buckled flat panels where stresses of the order of ±10,000 p.s.i. to ±15,000 p.s.i. were measured near the trailing edge of an aluminium alloy panel (endurance limit, based on 500,000,000 cycles of reversed bending, about 12,000 p.s.i.) when failure occurred due to fatigue. Apart from fatigue, panel flutter is undesirable because of its effect on drag, although no work has been reported on this aspect.

For flat and buckled panels there is a large body of literature concerned with methods of finding the flutter speed and with methods of suppressing or delaying the onset of flutter. For example, longitudinal tensile stress of the panel is well known to be favourable. A feature of some of the methods of predicting the flutter speed is the low answer given for existing practical configurations. The three-dimensional theory of Hedgepeth[2] implies that a 28 inch long aluminium panel of aspect ratio $\frac{1}{2}$ at sea level will flutter at $M = 2$ for thicknesses less than 0.128 inches, which is contrary to experience on existing aircraft. Indeed on the basis of this theory it seems that to design flutter-free panels on high speed vehicles without some artifice being employed a thickness impracticable from other considerations must be used. The other approach to the problem is to assess the fatigue problem involved.

Recent work by Miles[7] indicates that the boundary layer may alleviate the severity of the flutter if the amplitude of the panel motion is small compared with the boundary layer thickness. This work is, however, somewhat tentative.

The consideration of panel flutter from a fatigue point of view is a very difficult problem. Any calculation should estimate the oscillatory stresses involved and some knowledge of the amplitudes concerned is thus needed. Most of the existing work uses linearised theories which merely assume the amplitude small and give a flutter speed above which the small amplitude oscillation becomes undamped or negatively damped. At any given speed above this speed, since experiment has shown oscillation to continue without growing in amplitude, something must limit the amplitude and this must come from the non-linear terms which have been neglected. Thus only a non-linear theory can predict the amplitudes of oscillation and hence the stresses involved. Within the required accuracy, linear theory can predict the number of cycles occurring during the flight of the panel.

3 FLUTTER OF CYLINDRICAL PANELS

Panel flutter of the circular cylinder has been considered theoretically by Leonard and Hedgepeth[3] and by Miles[4,5]. Experiments on pressurised circular cylinders were reported by Fung, Miles and Kaplan and some quantitative agreement was found with the theory of Miles. Miles's theory was applied to assess the panel flutter problem on a U.S. missile, and flutter was predicted to occur first at a Mach number of about 1.8, 55 seconds after launching.

The theory of Miles considers the stability of aerodynamically induced travelling waves of small wavelength on an infinite cylindrical shell containing either fluid or a gas under pressure. For an airspeed U four possible waves of speeds approximating $\pm V_o$ and $U \pm a$ are predicted (considering, for the moment, only axially symmetric waves), V_o being the axial speed at which transverse waves travel along the shell in the absence of aerodynamic forces. In still air, waves are possible at any given wavelength (or frequency) at some speed above a minimum wave speed $(V_o)_{min}$, so continuous spectra of possible waves in fact correspond to the speeds $\pm V_o$ above. At subsonic airspeeds all the possible waves are damped out with the assumption of structural damping. At supersonic speeds unstable waves can occur. The only serious instability, however, is predicted when

$$V_o = U - a .$$

So the lowest airspeed at which instability occurs is

$$U_f = a + (V_o)_{min} ,$$

and this is the flutter speed. In fact, for an isotropic prestressed shell,

$$U_f = a + \sqrt{\frac{1}{\rho_1}\left(\sigma_x + \frac{E\,h}{R\sqrt{3(1-\nu^2)}}\right)} , \qquad (1)$$

σ_x being the tensile axial stress. It is known (see, e.g., Ref.6, p.407) that if σ_x arises from a pressure differential p,

$$\sigma_x = \frac{p\,R}{2h} . \qquad (2)$$

For airspeeds above the flutter speed U_f the instability continues, although the waves are no longer necessarily axially symmetric as they are at the onset of flutter. The prestressed and the non-prestressed shell must now be considered separately, since the expressions for the flutter frequency are different.

(a) For the non-prestressed shell, flutter occurs initially at a frequency

$$\frac{1}{\pi R}\sqrt{\frac{E}{2\rho_1}} \tag{3}$$

while at speeds in excess of the flutter speed the frequency is

$$\frac{1}{\pi R}\sqrt{\frac{E}{2\rho_1}}\left(M - \frac{1}{a\sqrt[4]{3(1-\nu^2)}}\sqrt{\frac{Eh}{R\rho_1}}\right)^{-2} . \tag{4}$$

(b) For the prestressed shell the flutter continues to be axially symmetric from the time of onset until a Mach number

$$M_1 \doteq 1 + \sqrt{\frac{5\,\sigma_x}{3\,\rho_1\,a^2}} \tag{5}$$

at a frequency

$$\frac{(M-1)\sqrt{E}}{2\pi R}\left\{\rho_1 (M-1)^2 - \frac{\sigma_x}{a^2}\right\}^{-\frac{1}{2}} . \tag{6}$$

Above M_1 the flutter is no longer axially symmetric and the frequency is

$$\frac{C^2(C-1)\sqrt{E}}{2\pi\,M^2\,R}\left\{\rho_1 (C-1)^2 - \frac{2\sigma_x}{a^2}\left(1 - \frac{C^2}{2M^2}\right)\right\}^{-\frac{1}{2}} \tag{7}$$

where C is a root of the equation

$$\frac{\sigma_x}{\rho_1\,a^2}\left\{18C - 8 - \frac{C^2}{M^2}(7C-2)\right\} - (7C-4)(C-1)^2 = 0 . \tag{8}$$

Roots of this equation are plotted in Ref. 5 for different values of the parameter $\sigma_x/\rho_1\,a^2$. It should be noted that the expressions (6), (7) and (8) are based on the assumption that $E\,h/R\,\sigma_x$ is small, so that one cannot set $\sigma_x = 0$ in (6) and expect agreement with (3).

The wavelength at flutter is shown to be small compared with the radius of the cylinder, so extrapolation of the theory to finite shells is justified since in practical configurations this will also imply the wavelength small compared with the length of the cylinder.

For a more complete exposition of Miles's work, of which the foregoing is a brief summary embodying the results needed here, the reader is referred to Refs.4 and 5.

4 PANEL FLUTTER OF THE BLUE STREAK VEHICLE

4.1 Description of vehicle and trajectory

A schematic general arrangement of Blue Streak is given in Fig.1. The oxidant tank is stabilised both on the ground and in flight by internal pressure, and for the purposes of this work the gauge pressure in the oxidant tank will be taken as 30 p.s.i. The main tank structure is of thin unstiffened stainless steel sheet which is 0.019 inch thick in the oxidant tank. External stringers are used to stabilise the unpressurised guidance bay.

The vehicle is launched vertically and climbs for 20 seconds. In the two standard trajectories the vehicle turns gradually at 0.8 degrees per second to achieve ultimate angles with the launcher horizontal of 36° and 74° respectively. The speed of sound is achieved in these trajectories after about 57 seconds and 58 seconds respectively.

4.2 Assessment of the panel flutter problem

The parts of the vehicle which appear susceptible to panel flutter are that part of the oxidant tank which contains fuel, that part above the fuel containing gas under pressure and the guidance bay. In similar U.S. calculations flutter of the fuel filled part of the tank was found to give very few cycles of flutter, and for the vertically stiffened section only a very mild instability was found, and then over a limited part of the trajectory. Since panel flutter of the gas pressurised portion will here be shown to occur for a large number of cycles over much of the trajectory, detailed consideration has not been given to these other possibilities.

Flutter of this part of the cylinder will now be considered.

The thrust causes compressive stresses and bending gives rise to tensile and compressive stresses in the shell. Hence the lowest value of σ_x will occur where the tensile stress due to internal pressure is offset to the greatest extent by compression due to thrust and bending. From equation (1) the lowest flutter speed is seen to correspond to the lowest value of σ_x, so if the effects of thrust and bending are ignored and the axial tension assumed to be due solely to internal pressure the flutter speed will be overestimated. The vehicle acceleration is, however, so large that the estimate of the time of onset of flutter is insensitive to this error. No account will be taken of temperature effects.

To evaluate the flutter speed and frequency values $E = 26 \times 10^6$ p.s.i., $\rho_1 = 0.28$ lb per cu in., $\nu = 0.3$, $p = 30$ p.s.i. are assumed, while $R = 60$ inches and $h = 0.019$ inch. Hence equations (1) and (2) in conjunction with the trajectory data predict flutter at $M = 1.73$, when

height = 42,500 feet, $t = 76$ seconds for the 36° trajectory

and

height = 49,000 feet, $t = 78$ seconds for the 74° trajectory.

At these times the vehicle tanks are 57 per cent full in each case. The flutter frequency as given by equation (6) is 1650 c.p.s. in each case.

To estimate the number of cycles of flutter only the 36° trajectory is considered; the number of cycles for the 74° trajectory will not be much different. The frequency falls sharply at first to 890 c.p.s. at 78 seconds and then falls steadily to 45 c.p.s. at 150 seconds. This variation of frequency with time from launching is shown in Fig. 2. The total number of cycles of flutter is predicted to be 23,500: this is actually the number predicted from the onset of flutter to time 150 seconds, but is little different from the number over the whole trajectory since the frequency is small at 150 seconds and still decreasing.

Since flutter is shown to occur, the more obvious methods of suppressing or delaying it, namely increasing the internal pressure and varying the skin thickness of the oxidant tank, were assessed. These calculations were performed only for the 36° trajectory.

The effect on the critical Mach number for flutter of altering the internal pressure from 15 to 50 p.s.i., i.e. from a half to over one and a half times the actual pressure, is shown in Fig. 3. It will be seen that the change has little significance in practice, since although the Mach number varies from 1.53 to 1.92, the onset of flutter at 76 seconds is advanced or delayed by only 4 seconds on account of the large vehicle acceleration.

The effect of varying the skin thickness is shown in Fig. 4 where the Mach number and time of onset of flutter are plotted against a parameter h_1, where the assumed skin thickness is $0.019\ h_1$ inches and $h_1 = 1$ corresponds to the actual skin thickness. It is seen that increasing the skin thickness from $h_1 = 1$ (0.019 in.) to $h_1 = 3$ (0.057 in.) actually causes a decrease in the flutter Mach number. Further increases above $h_1 = 3$ give an increase in flutter Mach number.

The reason for this behaviour may be seen from (1) and (2). The terms within the square root of (1) take values

$$\frac{\sigma_x}{\rho_1} = \frac{pR}{2h\,\rho_1} = 0.482\,\frac{a^2}{h_1} \qquad \text{and} \qquad \frac{E\,h}{R\,\rho_1\sqrt{3(1-\nu^2)}} = 0.051\,a^2\,h_1\,.$$

These are respectively the contributions of the axial tension and the stiffness of the unpressurised cylinder. When flutter occurs initially at $h_1 = 1$ it is the pressurisation term which plays the major part and this continues to be so until the minimum at $h_1 = 3$. To the right of the minimum it is the bending term which predominates. Again, because of the high vehicle acceleration, the time of onset of flutter is relatively insensitive to thickness changes.

5 PANEL FLUTTER OF THE BLACK KNIGHT VEHICLE

5.1 Description of vehicle

A schematic general arrangement of Black Knight is shown in Fig. 5. The main tank structure is essentially a thin-walled circular cylinder provided with ring frames, and is of 0.036 inch aluminium alloy. The frames are at a pitch varying between 16.6 and 22.5 inches in the H.T.P. tank and between 10.5 and 13.5 inches in the kerosene tank. Further stabilisation is provided by excess internal pressure, of 7.5 p.s.i. in the H.T.P. tank, rising from zero at launch to 8 p.s.i. at 50 seconds in the electronics bay and becoming 2 p.s.i. at 60 seconds in the kerosene tank. The equipment bay and the head separation compartment are not pressurised, but the head separation compartment has vertical stiffening and the equipment bay contains an access panel, the seating for which provides axial stiffening.

5.2 Assessment of the panel flutter problem

The parts of the vehicle susceptible to panel flutter are the head separation compartment, equipment bay and electronics bay together with the fuel and gas filled parts of the H.T.P. and kerosene tanks. We expect flutter of the fuel filled parts of the tank to occur for only a few cycles and flutter of the vertically stiffened portions, such as the head separation compartment and equipment bay, is anticipated to be very mild. The gas filled portion of the kerosene tank is pressurised to 2 p.s.i. and we may expect it to encounter panel flutter at a lower speed than either the H.T.P. tank or the electronics bay, these being pressurised to 7.5 p.s.i. and 8 p.s.i. respectively.

Hence flutter of the kerosene tank will now be considered. Following 4.2 the effects of thrust and bending will be ignored, and this will incur little error in the estimate of time of onset of flutter.

Assuming values $E = 10 \times 10^6$ p.s.i., $\rho_1 = 0.1$ lb per cu in. and $\nu = 0.3$, with $R = 18$ inches, $h = 0.036$ inch and $p = 2$ p.s.i., equation (1) predicts flutter at a Mach number of 1.6. From the trajectory data, this occurs 74 seconds after launch when the vehicle is at a height of 45,000 feet.

The flutter frequency will now be considered. In the expression (1) for the flutter speed the values of the terms within the square root are

$$\frac{E\,h}{\rho_1\ R\ \sqrt{3(1-\nu^2)}} = 0.347\ a^2 \qquad \text{and} \qquad \frac{\sigma_x}{\rho_1} = 0.014\ a^2\ ;$$

these are the contributions of shell stiffness and pressurisation respectively. The pressurisation term thus contributes only 4% to the expression within the square root and 2% to the square root itself. We thus ignore the effect of pressurisation in determining the flutter frequency and use the expressions (3) and (4), true for an unpressurised shell.

The frequency at flutter is thus 2,458 c.p.s. and the frequency above the flutter speed is shown in Fig. 6. It falls rapidly at first to 384 c.p.s. at 100 seconds and continues more slowly to fall to 34 c.p.s. at 140 seconds. The number of cycles from the onset of flutter to time 140 seconds is 35,000.

The effect of skin thickness variations is shown in Fig. 7 where the Mach number and time of onset of flutter are plotted against a thickness parameter h_2. The skin thickness considered is 0.036 h_2 inches so that $h_2 = 1$ corresponds to the actual skin thickness of the vehicle Increasing the skin thickness delays the onset of flutter but, again on account of the large vehicle accelerations, the time of onset is delayed only little; for example, doubling the skin thickness delays it only from 74 to 79 seconds.

6 CONCLUSIONS

The danger of panel flutter is fatigue failure rather than a quasi-static failure as with commoner forms of flutter.

On the basis of the theory of Miles it seems possible that a fatigue problem may be involved on both the Blue Streak and Black Knight vehicles. For each vehicle flutter of part of the main tank structure is predicted for a large number of cycles over a considerable part of the trajectory, although the types of flutter are different. On Blue Streak it is the internal pressurisation and the consequent axial tension which delays the onset of flutter: 23,500 cycles of flutter are predicted, and the stresses in the tank walls on which the oscillatory stresses are superimposed are already high. For Black Knight it is the tank stiffness which delays the onset of flutter, and 35,000 cycles of flutter are predicted. In neither case, however, is it possible to find the amplitudes and hence the flutter stresses involved and give a more exact assessment of the problem. Flutter is merely delayed, and not suppressed, by increasing the pressurisation and skin thickness, and considerable changes do not affect the time of onset of flutter greatly due to the large vehicle acceleration.

These conclusions are tentative, and their value is limited by the assumptions which have been made. In particular the theory, true for an airstream passing a cylinder at constant speed, has here been applied to an accelerating vehicle. Thus whether the oscillation predicted for unaccelerated flight conditions will establish itself has yet to be confirmed, and there is need for some empirical evidence on this. It is worth noting, however, that calculations on the U.S. missiles predicted panel flutter with a general picture comparable with that here while the latest information available gives no reason to suppose that severe panel flutter has occurred in practice. Moreover several firings of Black Knight have taken place since the present calculations were completed and there is no evidence of panel flutter in this case either.

7 NOTATION

C	defined by equation (8)
E	Young's modulus
M	Mach number
M_1	defined by equation (5)
M_f	flutter Mach number
R	radius of shell
U	air speed
U_f	flutter speed
V	wave speed
V_o	wave speed in absence of aerodynamic forces
$(V_o)_{min}$	minimum value of V_o

a	speed of sound in air
h	shell wall thickness
h_1	thickness parameter for Blue Streak = (actual thickness) ÷ (0.19 in.)
h_2	thickness parameter for Black Knight = (actual thickness) ÷ (0.036 in.)
p	internal pressure differential
t	time from launching in seconds
ρ_1	density of shell material
σ_x	axial stress
ν	Poisson's ratio

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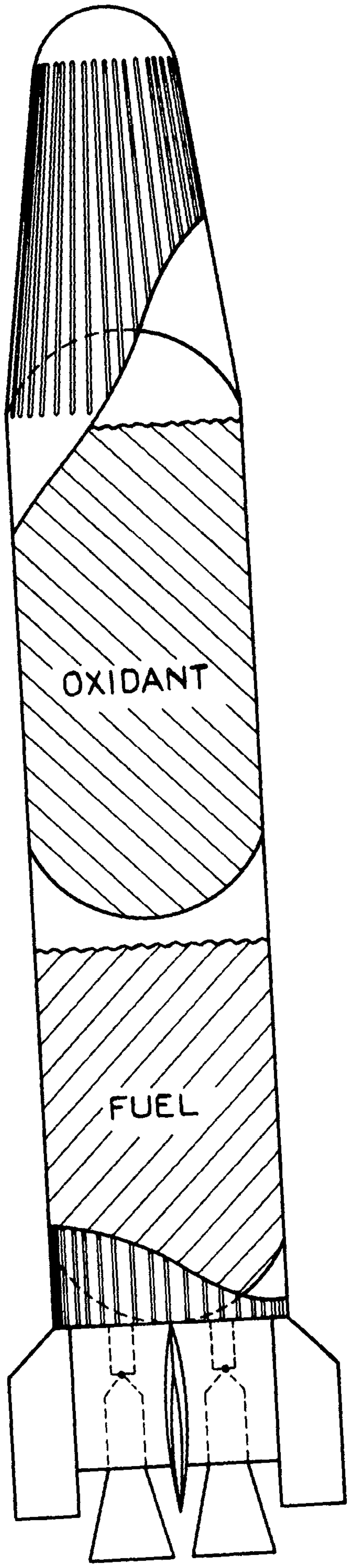

FIG. 1. SCHEMATIC GENERAL ARRANGEMENT OF BLUE STREAK.

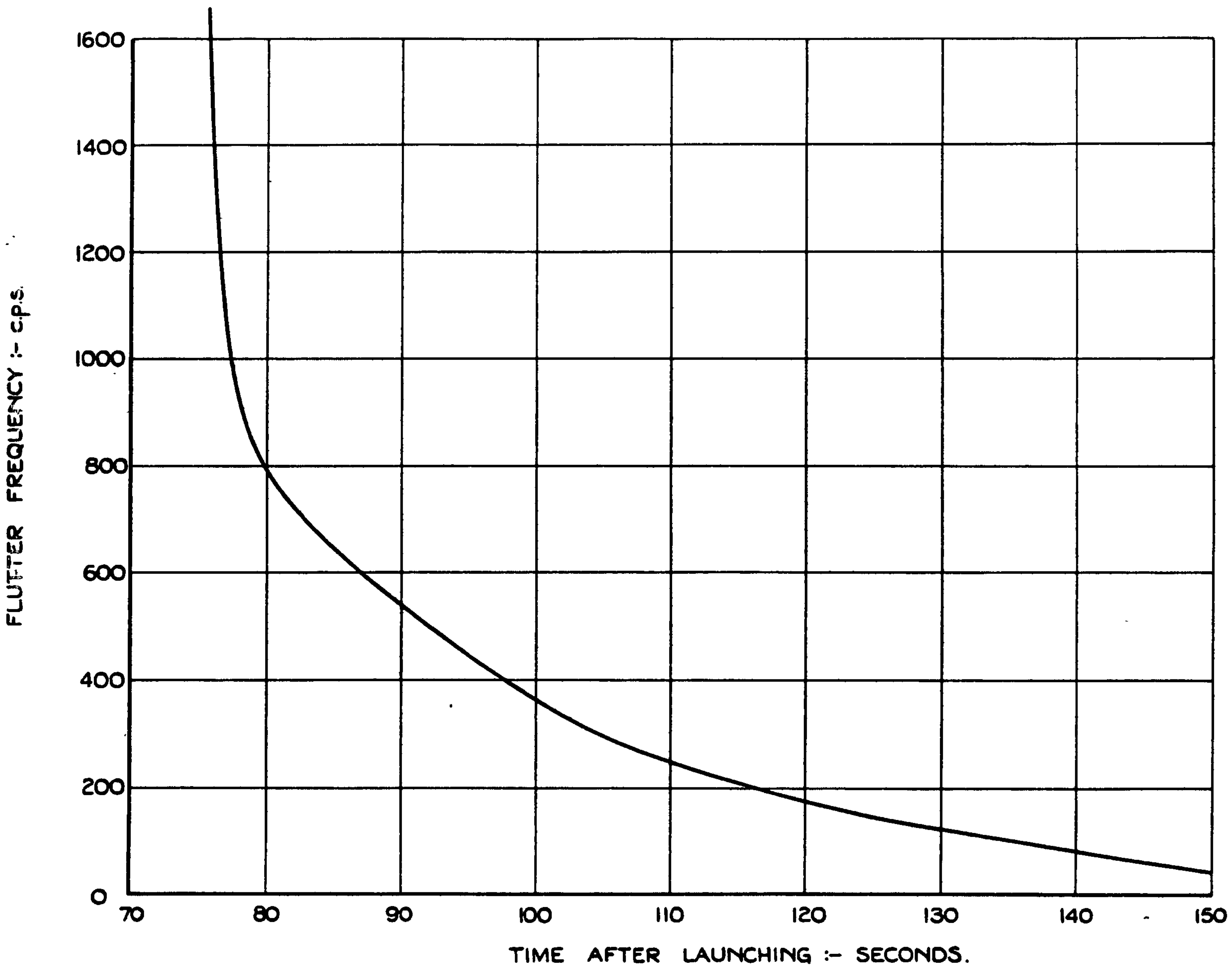

FIG. 2. VARIATION OF FLUTTER FREQUENCY WITH TIME AFTER LAUNCHING FOR BLUE STREAK.

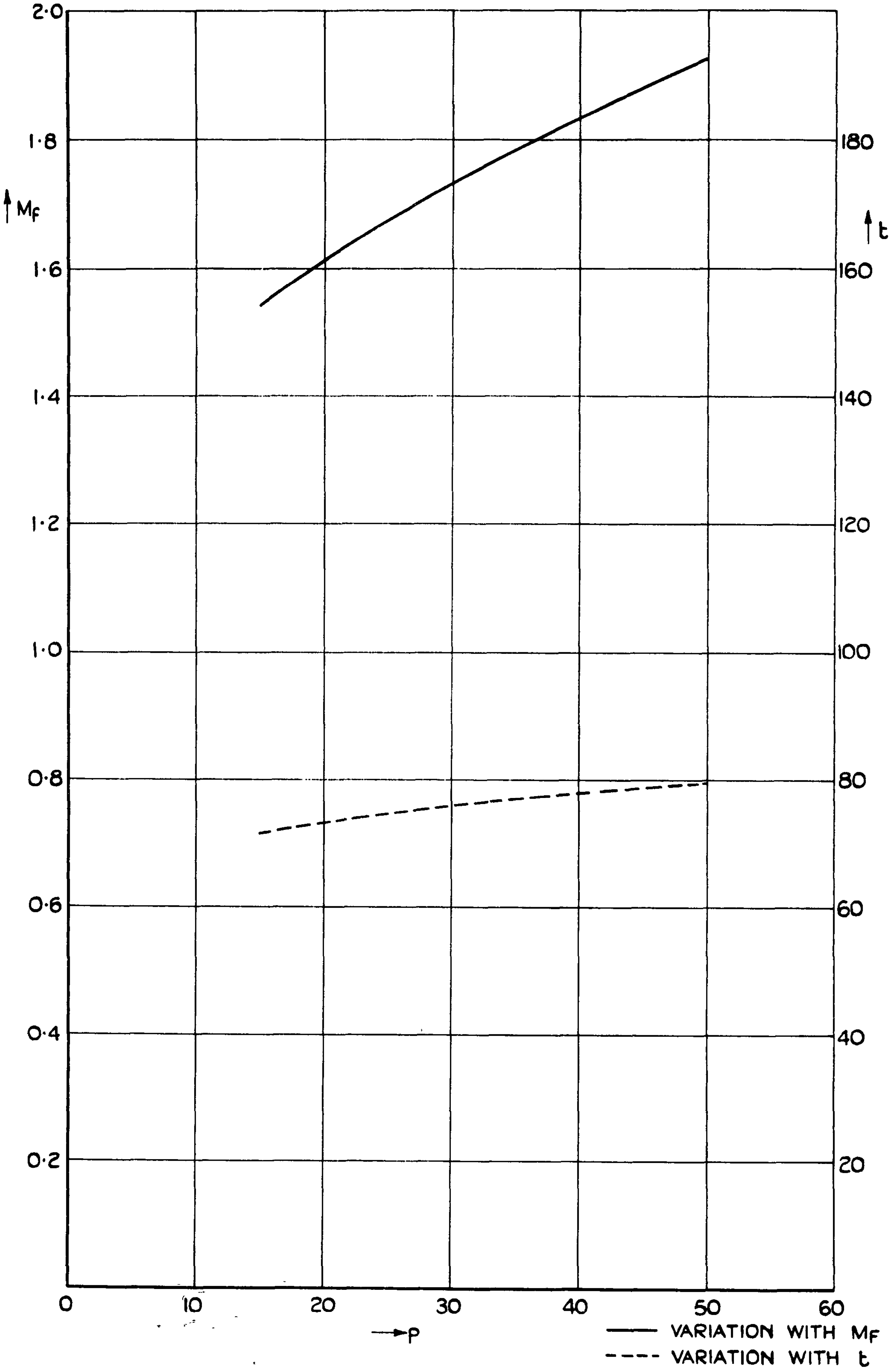

FIG. 3. VARIATION OF FLUTTER MACH NUMBER AND TIME OF ONSET OF FLUTTER WITH INTERNAL PRESSURE FOR BLUE STREAK.

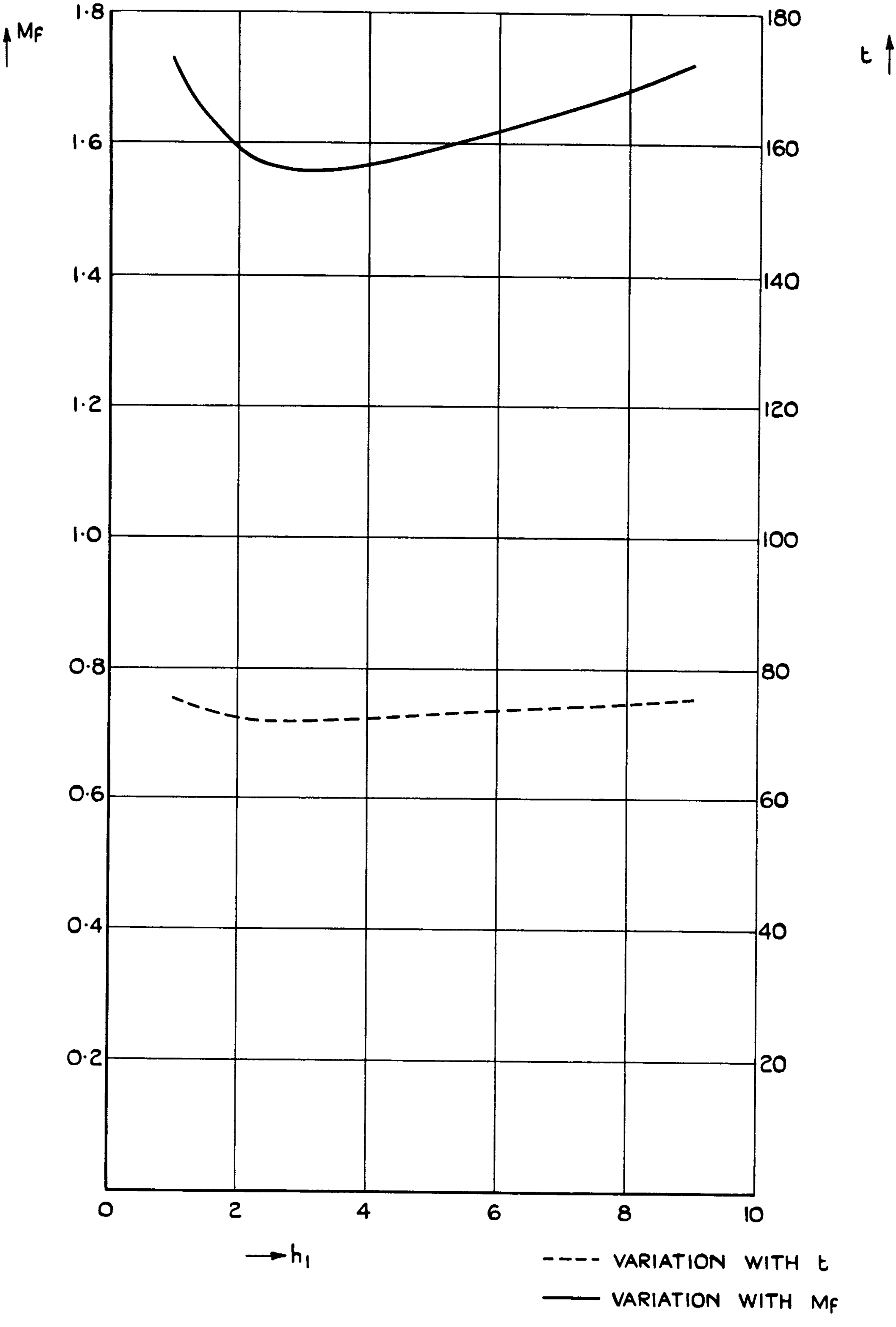

FIG. 4. VARIATION OF FLUTTER MACH NUMBER AND TIME OF ONSET OF FLUTTER WITH SKIN THICKNESS FOR BLUE STREAK.

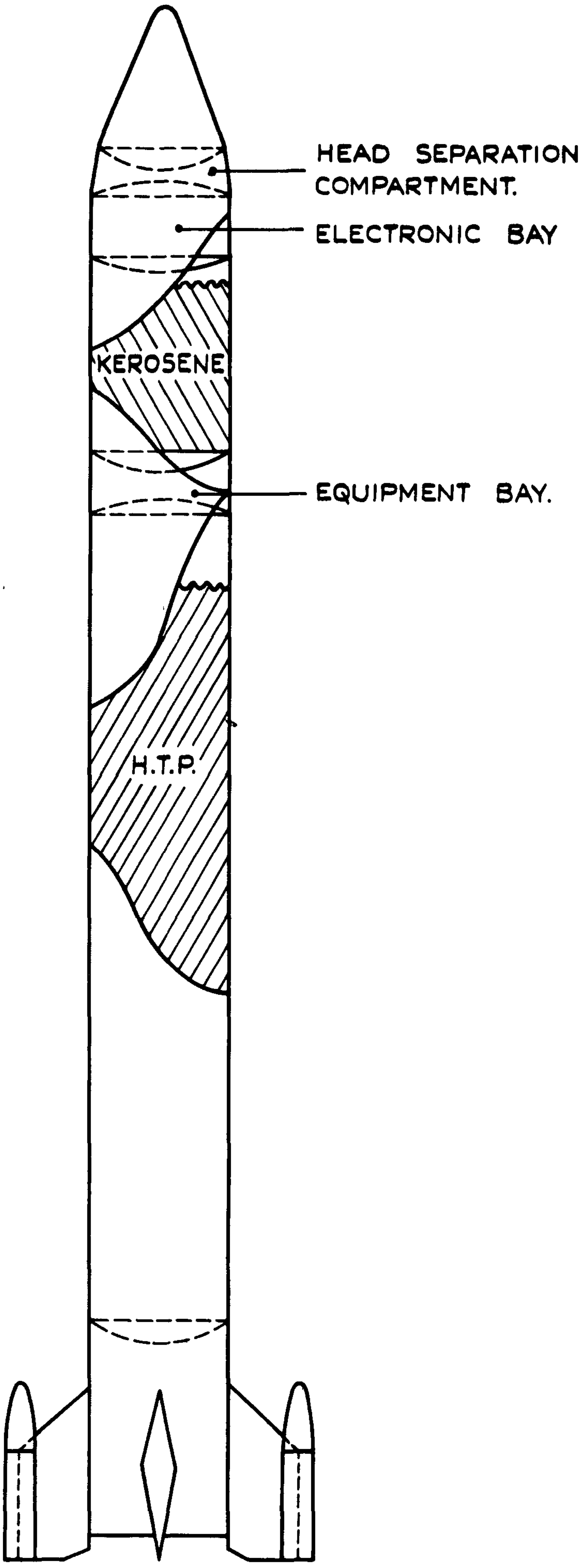

FIG. 5. SCHEMATIC GENERAL ARRANGEMENT OF BLACK KNIGHT.

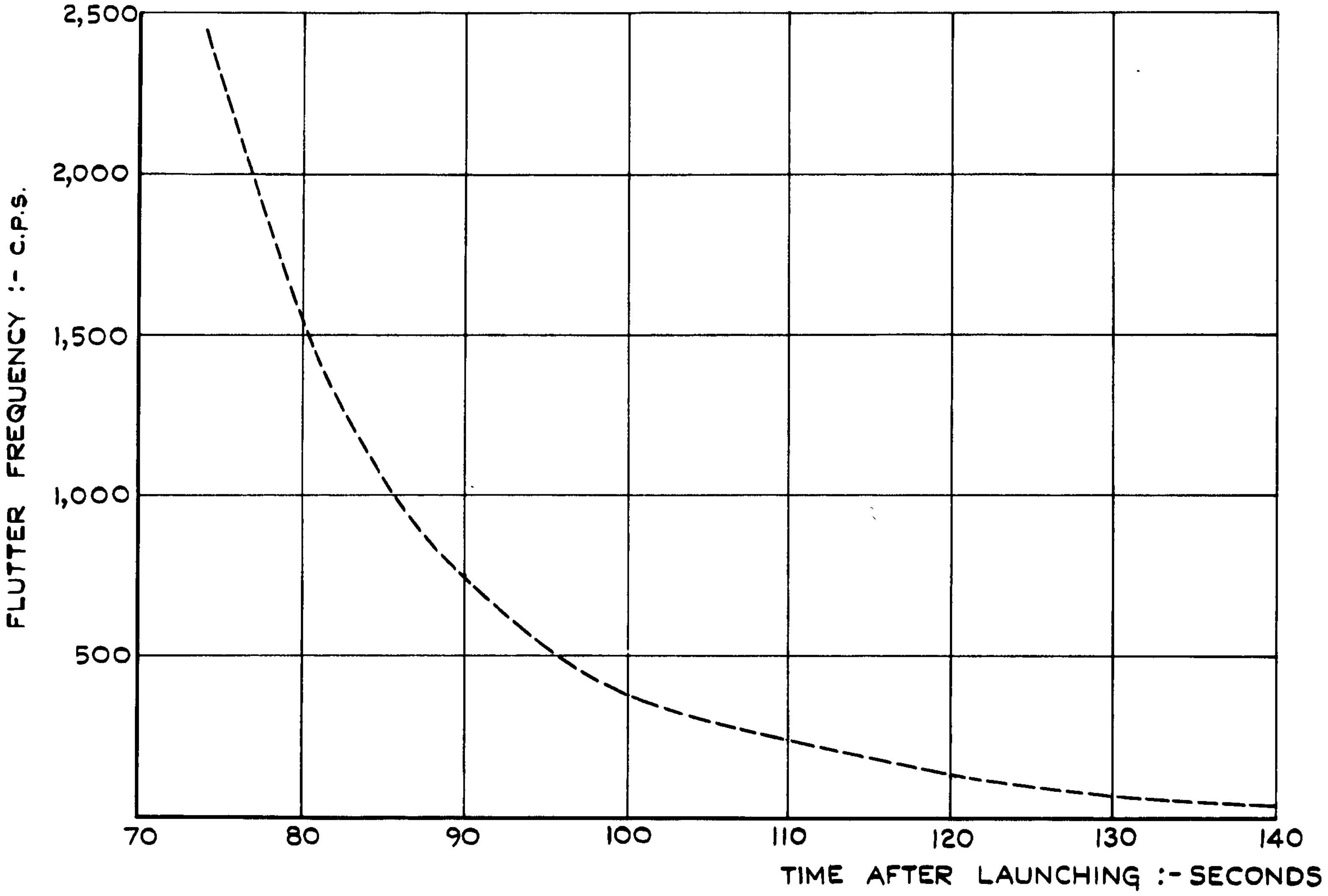

FIG. 6. VARIATION OF FLUTTER FREQUENCY WITH TIME AFTER LAUNCHING FOR BLACK KNIGHT.

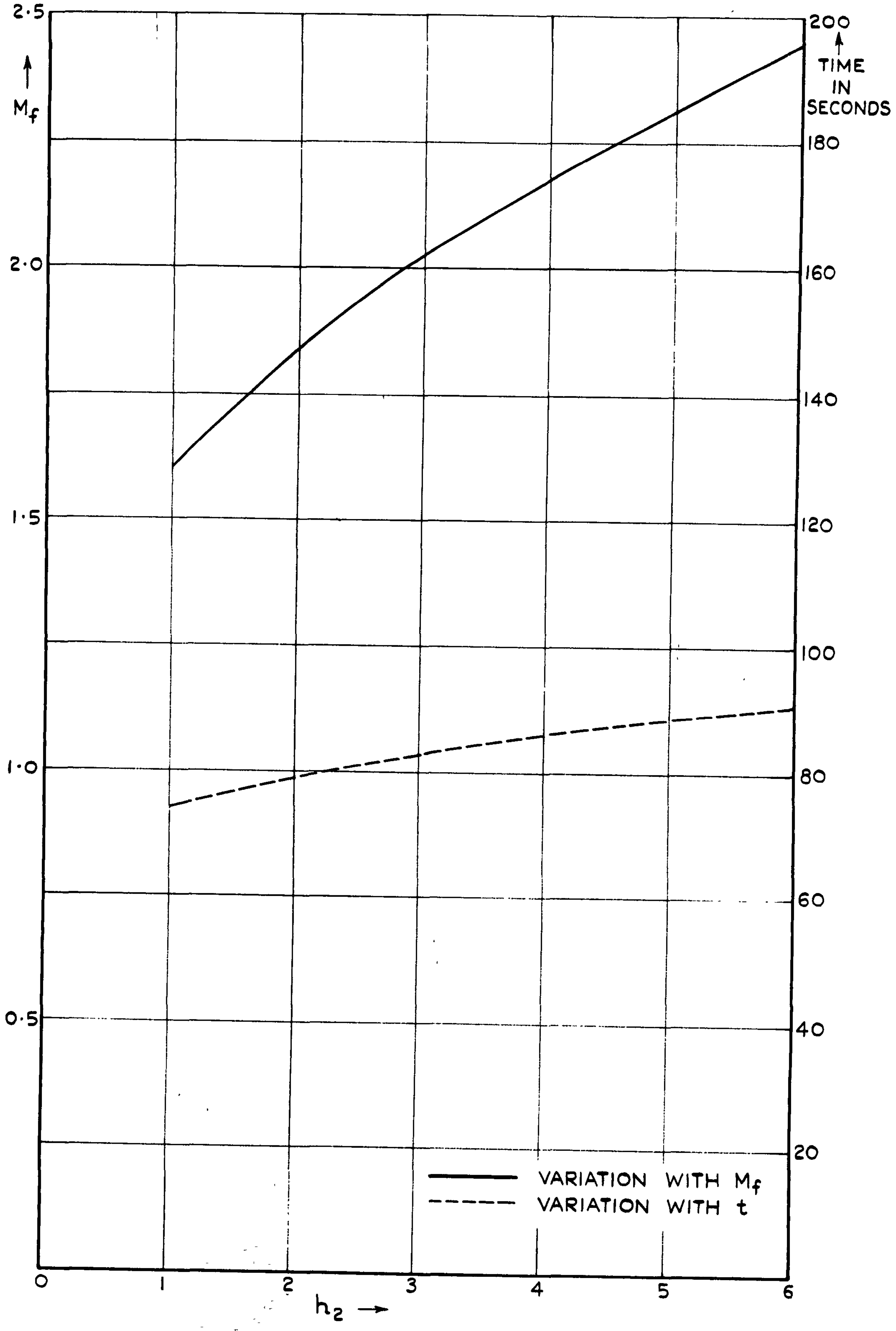

FIG. 7. VARIATION OF FLUTTER MACH NUMBER AND TIME OF ONSET OF FLUTTER WITH SKIN THICKNESS FOR BLACK KNIGHT.

Printed in England for Her Majesty's Stationery Office by the
Royal Aircraft Establishment, Farnborough. Dd.125875. K.4.

A.R.C. CP No.848

623.651-519 :
531.258 :
533.6.013.422

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