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Handling Qualities of Aircraft with Marginal Longitudinal Stability

By

T. B. Saunders

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SUMMARY

The problem of assessing the handling qualities of aircraft with marginal or negative longitudinal stability is investigated. A mathematical model of the human pilot is used to study closed loop tracking and the results are checked by an experiment in a fixed base simulator. Some of the older handling qualities criteria such as manoeuvre margin and stick force per g are of limited use at marginal levels of stability but plots of pilot opinion contours against the stiffness and damping term of a second order system are useful provided the correct pair of dominant roots is chosen. However, where either root of the other pair has a time to double amplitude less than 3.5 sec the rating from the dominant pair may be modified by consideration of multi-loop control of pitch angle, speed and height. At present this can be done reliably only by testing the particular case in a flight simulator These facts are used as the basis of a procedure which can be used for assessing the longitudinal handling qualities of aircraft even when these have marginal stability Disagreement between theoretical predictions and simulator results leads to the hypothesis that the necessity to apply phase lag has little effect on pilot opinion. This conflicts with previously accepted pilot adaptation rules.

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Detachable Abstract Cards

1.0 INTRODUCTION

The design of automatic control systems for aircraft has advanced to such a stage in recent years that a change of emphasis has occurred in the study of aircraft handling qualities. Once the need for automatic controls is accepted, the achievement of optimum handling qualities is frequently only a matter of adjusting system gains. Design compromises are required only in the consideration of system failures where a lower level of autostabiliser reliability can be accepted if the basic aircraft is designed to have marginally acceptable handling qualities after a failure at any flight condition. Thus interest is focussed firstly on vehicle characteristics which would give optimum handling qualities and secondly on the minimum handling qualities that would ensure that the pilot could control the aircraft safely while changing to a safe flight condition after a failure. This study is concerned with one particular aspect of the problem of defining marginal handling qualities.

One of the basic design problems of supersonic aircraft is the difficulty of providing adequate longitudinal stability at subsonic speeds without having excessive stability and trim drag when the aerodynamic centre and centre of pressure move aft at supersonic speeds. Thus it is useful to consider the assessment of longitudinal handling qualities at low levels of stability. This general problem may also have some application to V.S.T.O.L. and hypersonic aircraft which may have low stability because of the low dynamic pressures at which they operate.

It is known (Ref. 1) that the form of longitudinal motion which is found at normal levels of stability, and on which most existing handling qualities criteria depend, does not occur at low stability. As stability is reduced by moving the C.G. aft, the familiar short period and phugoid modes start to interact and eventually combine. In spite of this, several simulator studies of low longitudinal stability (e.g. Refs. 2 - 4) have simulated only a simple second order system. The results can be applied in many cases, including some at low stability in which the handling qualities of an aircraft can be defined adequately by reference to a single dominant pair of roots (Ref. 5). However, a completely general assessment of handling qualities at low stability should contain some reference to the possibility of interaction between modes.

More realistic experiments on low stability handling qualities have been carried out in variable stability aircraft (Refs. 5 - 7) and have not encountered cases where more than two roots are important at any time. However, Refs. 8 and 9 found that an unstable phugoid mode can affect pilot opinion for given short period characteristics when the pilot's task is difficult or of long duration (i.e. instrument flying on airways in turbulence or approaching to land).

There is thus a requirement for more to be done on the problem of low stability handling qualities and this note describes, in Section 6, a series of fixed base simulator tests intended to add to current knowledge of the problem. The simulator experiments were designed on the basis of theoretical consideration of a quasi linear mathematical model of the pilot/aircraft system in Section 5. The method of servo analysis used there, and some relevant applications are described in Refs. 10 - 12. Section 3 and 4 describe preliminary work on aircraft open loop characteristics at low stability and existing, good stability, handling qualities criteria. A procedure for assessing low stability handling qualities is suggested in Section 8.

2.0 LIST OF SYMBOLS

Stability and control derivatives etc. as in Ref. 13.

ω_{sp}	undamped natural frequency of short period mode
ω_p	undamped natural frequency of phugoid mode
ζ_{sp}	damping ratio of short period mode
ζ_p	damping ratio of phugoid mode
T_{θ_1}, T_{θ_2}	Pitch control numerator time constants
$K_\theta / K_{\theta sp}$	Static to short period gain ratio ($= \frac{1}{T_{\theta_1} T_{\theta_2} \omega_p^2}$)
s	the Laplace transform variable $= \sigma + j\omega$
θ_D	demanded pitch angle
ε	pitch error
γ	flight path angle
u_G, w_G	horizontal longitudinal and vertical gust velocities
Ni(s)	numerator of i/η (s) transfer function (i = θ, h or u)
$N_{i_j}(s)$	numerator of $i/j_G(s)$ transfer function (j = u or w) (i = θ, h or u)
A_i, B_i, C_i, D_i	Coefficients of $N_i(s)$ (i = θ, h or u)
A_{ij}, B_{ij}, C_{ij}, D_{ij}	Coefficients of $N_{ij}(s)$ (i = θ, h or u) (j = u or w)
$\Delta(s)$	Longitudinal denominator (i.e. l.h.s. of stability quartic)
$\Delta_1(s)$	Part of longitudinal denominator independent of C.G. position
$\Delta_2(s)$	Coefficient of C.G. position in longitudinal denominator
B_1, C_1, D_1, E_1	Coefficient of the stability quartic (Δ) (Ref. 13.)
R_1	Rouths discriminant ($= B_1C_1D_1 - D_1^2 - B_1^2 E_1$)
σ, ω	Real and imaginary parts of s

3.0 AIRCRAFT DYNAMICS AT LOW STABILITY

a) Description of Aircraft Dynamics

The linearised longitudinal equations of motion on which subsequent calculations are based, are : -

$$\left.\begin{aligned}
&\hat{t}\dot{u} - x_u u - x_w w + g\hat{t}\theta = -x_u u_G - x_w w_G \\
&- z_u u + \hat{t}\dot{w} - z_w w - \mu_1 \ell q = V z_\eta \eta - z_u u_G - z_w w_G \\
&- \frac{m_u}{i_B} u - \hat{t}\frac{m_{\dot{w}}}{i_B}\dot{w} - \frac{m_w}{i_B} w + \hat{t}\ell\dot{q} - \frac{m_q}{i_B}\ell q = V\frac{m_\eta}{i_B}\eta - \frac{m_u}{i_B}u_G - \frac{m_w}{i_B}w_G - \hat{t}\frac{m_{\dot{w}}}{i_B}\dot{w}_G
\end{aligned}\right\} \quad (1)$$

where u_G, w_G represent the horizontal and vertical gust excitation velocities.

They are written above with reference to stability axes and real time. They can be transformed by the Laplace transformation into a set of linear algebraic simultaneous equations in the motion variables with the transform variable s as a parameter. The response of any motion variable to a given forcing input can then be described by a rational transfer function in the parameter s. Thus a set of such transfer functions could be used as an alternative to the equations (1) to describe the dynamics of the aircraft.

All the transfer functions have the same denominator and this defines the free motion of the system after the forcing input has stopped. A necessary and sufficient condition for stability of a linear system is that the real parts of all the roots of this denominator should be negative. The longitudinal denominator is given by the "stability quartic" :-

$$\Delta = s^4 + B_1 s^3 + C_1 s^2 + D_1 s + E_1 \quad (2)$$

where

$$\left.\begin{aligned}
B_1 &= -\frac{1}{\hat{t}}\left(z_w + \frac{\mu_1 m_{\dot{w}}}{i_B} + x_u + \frac{m_q}{i_B}\right) \\
C_1 &= \frac{1}{\hat{t}^2}\left\{x_u z_w - x_w z_u + \frac{z_w m_q}{i_B} - \frac{\mu_1 m_w}{i_B} + x_u\left(\frac{m_q}{i_B} + \mu_1\frac{m_{\dot{w}}}{i_B}\right)\right\} \\
D_1 &= \frac{1}{\hat{t}^3}\left\{-\frac{m_q}{i_B}\left(z_w x_u - x_w z_u\right) + \frac{\mu_1}{i_B}\left(x_u m_w - x_w m_u\right) + \frac{C_L \mu_1}{2 i_B}\left(z_u m_{\dot{w}} + m_u\right)\right\} \\
E_1 &= \frac{1}{\hat{t}^4}\frac{C_L \mu_1}{2 i_B}\left(z_u m_w - m_u z_w\right)
\end{aligned}\right\} \quad (3)$$

At an early stage in the study the methods of Ref. 14 were used to obtain approximate factors for the quartic (2) and the numerators of the following transfer functions :

$$\text{(i)} \quad \frac{\theta}{\eta}(s) = \frac{N_\theta(s)}{\Delta(s)} = \frac{1}{\Delta}\left(A_\theta s^2 + B_\theta s + C_\theta\right)$$

$$\text{(ii)} \quad \frac{h}{\eta}(s) = \frac{N_h(s)}{s\Delta(s)} = \frac{1}{s\Delta}\left(A_h s^3 + B_h s^2 + C_h s + D_h\right)$$

$$\text{(N.B.} \quad \dot{h} = V\theta - w)$$

$$\text{(iii)} \quad \frac{u}{\eta}(s) = \frac{N_u(s)}{\Delta} = \frac{1}{\Delta}\left(A_u s^2 + B_u s + C_u\right)$$

$$\text{(iv)} \quad \frac{u}{u_G}(s) = \frac{N_{uu}(s)}{\Delta(s)} = \frac{1}{\Delta}\left(A_{uu} s + B_{uu} s + C_{uu} s + D_{uu}\right)$$

$$\text{(v)} \quad \frac{u}{w_G}(s) = \frac{sN_{u_w}(s)}{\Delta(s)} = \frac{s}{\Delta}\left(A_{uw} s^2 + B_{us} s + C_{uw}\right)$$

$$\text{(vi)} \quad \frac{\theta}{w_G}(s) = \frac{sN_{\theta w}(s)}{\Delta(s)} = \frac{s}{\Delta}\left(A_{\ w} s^2 + B_{\ w} s + C_{\ w}\right)$$

$$\text{(vii)} \quad \frac{h}{w_G}(s) = \frac{N_{h_w}(s)}{\Delta(s)} = \frac{1}{\Delta}\left(A_{hw} s^3 + B_{hw} s^2 + C_{hw} s + D_{hw}\right)$$

Although some of these transfer functions are not referrred to again in the present report, their coefficients are recorded in full in Table I and the approximate factors in Table II.

It should be made clear at this stage that, in the present note, the terms "short period" and "phugoid" are applied to the corresponding pairs of transfer function poles. The use of these words does not imply that the poles described represent normal modes of the motion or that the aircraft is allowed to respond in such a way that a phugoid or short period oscillation could be distinguished in its motion. The combination of phugoid and short period poles which occurs at far aft C.G. positions is known as "third oscillation" after Ref 1 and the other pair of roots is then called the "dominant short period pair".

b) Low Stability Cases

Two types of low stability cases for fixed wing aircraft can be distinguished. The first results from poor damping of the short period oscillation and occurs at high altitude and high speed. Several supersonic aircraft including the X-15 (Ref. 15) exhibit this type of low stability. The way in which the poor damping arises can be explained as follows :-

The approximate expressions for short period frequency and damping from Table II are :

$$\left.\begin{aligned} \omega^2_{sp} &\simeq \frac{1}{\hat{t}^2 i_B}\left(z_w m_q - \mu_1 m_w\right) \\ 2\zeta_{sp}\,\omega_{sp} &\simeq -\frac{1}{\hat{t}}\left(z_w + \frac{m_q}{i_B} + \frac{\mu\, m_{\dot{w}}}{i_B}\right) \end{aligned}\right\} \qquad (4)$$

These can be further approximated;

$$\omega^2_{sp} \simeq -\frac{\mu_1}{\hat{t}\, i_B} m_w$$

$$2\zeta_{sp}\omega_{sp} \simeq -\frac{1}{\hat{t}}\left(z_w + \frac{m_q}{i_B}\right)$$

Thus, substituting for $\hat{t}$ and μ_1 :

$$\omega^2_{sp} \propto \rho V^2 \qquad \text{approximately}$$

and $\quad 2\zeta_{sp}\omega_{sp} \propto \rho V$ approximately

so that $\zeta_{sp} \propto \rho^{\frac{1}{2}}$ if we assume the derivatives to be roughly constant. At the same time the frequency is proportional to equivalent airspeed and can be high at high Mach numbers. All the derivatives, assumed constant, do in fact increase slightly with altitude but the effect is not as strong as the direct effects of speed and altitude. Since C_1 remains large, the small value of B_1 does not necessarily violate the conditions of validity of the approximate factors of the denominator in Table II.

The other form of instability is of more interest in this section because the approximate factors of Table II become invalid. It occurs when the C.G. is placed too far aft and is thus characteristic of supersonic aircraft when the aerodynamic centre moves forward at subsonic speeds. Many supersonic aircraft experience low stability of this type in certain flight conditions and configurations.

At forward C.G. positions the aircraft is stable and all the coefficients of the stability quartic are positive. As the C.G. moves aft the coefficients C_1 and E_1 decrease and eventually change sign. The absence of negative coefficients is a necessary but not sufficient condition for stability. Routh's criterion gives the following sufficient set of conditions on the coefficients of a quartic

$$\left.\begin{aligned} &B_1 > 0,\ C_1 > 0,\ D_1 > 0,\ E_1 > 0 \\ &R_1 = B_1 C_1 D_1 - D_1^2 - B_1^2 E_1 > 0 \end{aligned}\right\} \qquad (5)$$

Thus it can be seen that, if E_1 is still positive, R_1 will change sign before C_1. Since E_1 is still positive this instability will be in the form of an unstable oscillation whereas a negative value of E_1 will give a divergence. The conditions of validity in Table II show that the approximate factors can still be used with a small or even negative value of E_1. However, since D_1 is usually small enough for D_1^2 to be neglected R_1 can be written $B_1 (C_1D_1 - B_1E_1)$. When this becomes small, the last condition of validity for the denominator factors is violated. Since C_1 and E_1 decrease together this means that new approximate factors must be sought for the denominator in all cases of this type of low stability. The numerator approximate factors remain valid at low stability and do not vary much because quantities like

z_w and $\frac{m_\eta}{z_\eta}$ are not strongly affected by C.G. position.

c) Approximate Factors of the Stability Quartic at Aft C.G.'s

The physical justification for the denominator approximate factors of Table II lies in the difference in time scales of the motions involving change of incidence (short period) and change of height and speed (long period phugoid). This enables the two modes to be considered independently. As the C.G. moves aft the period of the short period oscillation increases and the two modes can no longer be assumed independent. Because the simple physical pattern has broken down there seems to be little hope of finding alternative approximate factors in literal terms as in Table II.

However, if the effect of moving the C.G. is considered, the stability quartic can be written approximately as a linear function of C.G. position m :

$$\Delta(s) = \Delta_1(s) + m\,\Delta_2(s) \qquad (6)$$

where $\Delta_1(s)$ is the stability quartic at some datum C.G. position and $\Delta_2(s)$ is the quadratic :

$$\Delta_2(s) = \frac{\bar{c}\,\mu_1}{100\,\hat{t}^2\,i_B}\left[- z_w s^2 + \frac{1}{\hat{t}}\left\{x_u z_w - \left(x_w - \frac{C_L}{2}\right)(z_u + C_L)\right\} s - \frac{C_L^2}{2}\,\frac{z_w}{\hat{t}^2}\right] \qquad (7)$$

If the C.G. datum is chosen such that $\Delta_1(s)$ can be factorised by the formulae of Table II, the factors of $\Delta(s)$ at any other C.G. position can be found by factorising the sum of two factorised polynomials. This is in effect the basic problem of closed loop servomechanism theory and can be conveniently solved by either root locus or generalised frequency response methods (Ref. 16). Although not as satisfactory as the literal expressions of Table II this method is one stage more useful than a purely numerical solution and is used in Appendix 1 to distinguish two types of instability.

Some idea of the accuracy of the method can be gained from Table 3 where numerical solutions of $\Delta = 0$ are compared with solutions of $\Delta_1 + \Delta_2 m = 0$ by the root locus method for a supersonic aircraft at $M = 0.9$, sea level. In this particular example the approximate factors are reasonably accurate over a wide range of C.G. positions. The next section will consider some existing handling qualities criteria in the light of the above description of the behaviour of aircraft transfer functions at low stability.

4.0 EXISTING HANDLING QUALITIES CRITERIA

Before going to a theoretical consideration of low stability handling qualities it is appropriate to look at the criteria which are currently available to assess longitudinal handling qualities of aircraft.

Provided the phugoid and short period modes are well separated, a constant stick force or elevator deflection gives a constant normal acceleration for an appreciable time in a pull-up as well as in a constant speed level coordinated turn. This case is shown in the plot of n vs. time of Fig. 1A. Thus stick force per g was used as a primary longitudinal handling qualities criterion. For highly manoeuvrable (e.g. fighter) aircraft a value of about 6 lb / g was considered optimum and anything below about 2 lb / g unacceptable. For larger bomber and transport aircraft with lower requirements for manoeuvrability and also lower limiting load factors, a natural safeguard against overstressing was built in by multiplying these figures by about 8.

However, where the phugoid frequency is nearer to that of the short period the pilot's action in pulling back the stick disturbs the phugoid oscillation and the steady normal acceleration is not achieved (Fig. 1B).

Thus stick force per g is in general a dynamic quantity, the value of which depends on the technique used by the pilot in the pull-up. In order to define the quantity more precisely and independently of pilot technique, the manoeuvre margin is used (Ref. 17). This is the distance of the C.G. ahead of the manoeuvre point and is proportional to stick deflection per g in an idealised pull up manoeuvre at constant speed. The manoeuvre margin is also approximately proportional to the coefficient C_1 of the stability quartic and hence to the stiffness ω_{sp}^2 of the approximate short period mode. Thus, although the concept of manoeuvre margin obviously breaks down at low stability it can be seen to represent a margin from instability of the negative R_1 or oscillatory type. Similarly the static margin is proportional to E_1 and hence refers to instability of the divergent type. This explanation is related to the approximate factorisation of Section 3c.

In the past it has been assumed that a reasonably large positive manoeuvre margin is necessary (i.e. consistent with good stick force per g) but that, given a good manoeuvre margin, the static margin could be slightly negative (i.e. a slight phugoid divergence is acceptable).

More recently the importance of short period damping has been generally recognised and longitudinal handling qualities have been assessed on the basis of empirical pilot opinion contours on an $\omega^2_{sp} \sim 2\zeta_{sp}\omega_{sp}$ plane, e.g. Refs. 8, 11 18. In these the ordinate is proportional to manoeuvre margin and the abcissa proportional to the inverse of time to half amplitude. An example from Ref. 18 is plotted in Figure 3.

None of these criteria is entirely satisfactory for assessing minimum acceptable handling qualities of aircraft with marginal stability. At low stability a constant stick force no longer gives constant normal acceleration in a pull-out since the speed changes appreciably before the steady state. (constant speed) value of normal acceleration is reached. Some physical meaning can still be credited to stick force per g by considering the stick force needed to sustain a steady level turn or pull-out at constant speed. However, this may not make stick force per g a useful parameter since speed control can obviously be an important factor in such a case. The same argument casts doubt on the usefulness of manoeuvre margin as a low stability handling qualities parameter expecially since the estimate of short period stiffness obtained from manoeuvre margin is no longer accurate.

The most promising of the criteria described above is that based on the characteristics of the short period oscillation. This can be generalised to consider the dominant pair of short period poles and is then undoubtedly an important parameter at low as well as high levels of stability. A reproduction of Figure 13 from Ref. 18 giving pilot opinion boundaries compiled from a number of different reports is given in Fig. 3. Although useful, this criterion may not be sufficient to define the handling qualities of an aircraft with marginal stability. At low stability the dominant pair of poles may not be easily identifiable and the other roots defining the motion may become more important. The effect of the other roots on low stability handling qualities is studied in the next section using the methods of servo analysis on a mathematical model of the pilot/aircraft system.

5.0 SYSTEMS ANALYSIS STUDY

The purpose of this section is to study theoretically the relative effects that certain characteristics of the aircraft dynamics may have on handling qualities at low levels of stability. The complicated nature of human behaviour would seem to make this a formidable task but there is reason to believe (Refs. 10, 11, 12, 19) that useful knowledge of the pilot's behaviour in closed loop tracking with random-appearing inputs can be gained from analysis of a simplified mathematical model in which the pilot is represented by a simple quasi-linear describing function.

a) Mathematical Model

It is important to bear in mind the limitations of the model used here. Some of these limitations will need to be taken into account in deriving realistic conclusions. Although some of the assumptions are necessary to obtain a useful theory, others might be relaxed as suggested in Section 9.

i) The aircraft is represented by the three longitudinal equations (1) describing small perturbations about a level unaccelerated flight condition. In considering poor phugoid dynamics, large speed perturbations might be encountered and then the linear equations assumed would not be adequate to describe the motion unless the variation of stability derivatives with speed were allowed for.

ii) The usual quasi-linear Gaussan input describing function is used to describe the behaviour of the pilot (Ref. 11).

$$Y_P = K_P \frac{1 + T_L s}{1 + T_I s} e^{-0.3s}$$

The constants K_P, T_I, T_L are determined by the task being carried out by the pilot. This is a reasonable approximation for easy closed loop control tasks with random-appearing inputs but provides only a rough indication of the low stability control boundary where the pilot behaves non-linearly.

iii) Only single loop control of pitch angle is considered in detail (Fig. 4). In circumstances in which the phugoid is important extensive use might be made of the throttle and trimmer in multi-loop control of attitude, speed and height and this simple analysis would then give misleading results.

iv) An essential condition of the theory used here is that it applies only to performance of a closed loop tracking task with a random-appearing input or forcing function. The task could be to reproduce the motions of a target which appears to the pilot to move at random, or else to eliminate motions of the aircraft due to some random-appearing disturbance such as turbulence. In either case the important point is that the forcing function (i.e. target motion or aircraft disturbance) should appear random to the pilot (i.e. should not be predictable by him). It does not need to be random in the precise mathematical sense. In the analysis in Section 5c and the simulator experiments we shall choose to consider the motion of a target since the forcing function is then presented to the pilot directly without being filtered by the aircraft dynamics as is the case with aircraft disturbances.

b) Method of Analysis

In considering each case, the fundamental problem is to determine the form of the describing function which would best describe the pilot's behaviour (i.e. to determine K_P, T_I and T_L) and hence to find the theoretical performance of the whole closed loop pilot-aircraft system in carrying out a closed loop tracking task.

Each case is analysed by iteration of these two steps. First, possible values of T_I, and T_L are chosen in accordance with a set of pilot adaptation rules, then the closed loop characteristics are determined as a function of the gain, K_P by methods used for analysing servomechanisms and the adaptation rules are again used, this time to choose the best value of gain.

i) Adaptation Rules

i) The adaptation rules used in this analysis are those of Ref. 12 and can be restated briefly as follows:

The pilot's describing function will contain equalisation terms (i.e. non-zero values of T_I or T_L) if and only if this is necessary to obtain either sufficient closed loop system stability or sufficiently good tracking performance.

In the target-following case which we are considering, good tracking performance implies a transfer funtion near to unity. Now it is not possible to obtain practical equipment, human or otherwise, with unit transfer function at all frequencies. Fortunately, practical forcing functions are similarly restricted to frequencies below a certain cut-off frequency so we need only consider the closed loop transfer function below this cut-off frequency. It is therefore possible to determine tracking performance from the low frequency characteristics or response of the closed loop system.

The pilot's gain K_p has to be chosen on the basis of a compromise between the conflicting aims of low frequency characteristics and system stability. For a given form of equalisation, high gain is needed to give good low frequency characteristics but this tends to reduce the system stability. Because the stability is usually governed by the characteristics of the transfer function at higher frequencies than those important for tracking performance, it is usually possible to alleviate the conflict and thus obtain a better quality of control by judicious choice of the equalisation terms.

Low frequency performance is specified quantitatively in Ref. 12 in terms of the amount of "low frequency droop". This is defined as "the amount of closed loop amplitude ratio below zero decibels" (i.e. less than unity) and should not exceed 2 db. below the forcing function cut-off frequency. Stability is specified quantitatively in terms of "phase margin" and the damping ratio of any oscillatory closed loop poles which may exist. The phase margin is 180° plus the phase angle of the transfer function at the crossover frequency where the amplitude ratio becomes less than unity (zero decibels) and should be between 60° and 110° if possible. The damping ratio ζ is the proportion of critical damping possessed by the oscillation and should be greater than 0.35.

The above rules are, to some extent, hypothetical and in any case only apply to combinations of forcing function and aircraft characteristics which appear fairly easy to the pilot. When the pilot is faced with a task beyond a certain level of difficulty he accepts a poorer quality of control, in the sense of tracking performance or amount of stability in order to reduce the amount of equalisation he needs to apply. However, the present method of analysis is generally unreliable in such cases and we shall consider handling qualities to be determined by the form of pilot's transfer function required to satisfy the above adaptation rules. Any amount of lead is supposed to degrade pilot opinion, the amount of degradation increasing with increases in the lead time constant T_L. A similar law applies to the effect of lag, the amount of degradation being probably rather less for a given value of T_I than for the same value of T_L. The gradient of pilot opinion versus pilot gain K_p is not steep within the limits 40 to 100 lb /rad./hand (Ref.11) so this factor will be assumed to be catered for by a variable feel system if necessary.

ii) Servo Analysis Methods

The characteristics of a servomechanism can be represented graphically by a number of methods which however fall into two groups. Analysis methods based on representation from both groups are used here to determine the characteristics of the closed loop system as a function of pilot's gain Kp

The first group concerns representation of the poles and zeros of the transfer function as points on the complex plane and is represented here by the root locus method. In this the loci of the closed loop poles as the loop gain varies are drawn on the complex plane by easy and rapid graphical methods. The loci all start from open loop poles at zero gain and finish at the open loop zeros or infinity at infinite gain. A gain value for which any part of the loci is in the right half plane is one at which the system would be unstable.

The second group of representations consists of those in which the magnitude and phase angle of the transfer function are plotted against some value of the complex Laplace transform variable S. Two such representations are used here. In the first, the magnitude in decibels and the phase angle of the open loop transfer function are plotted against the log of σ the real past of S $(= \sigma + j\omega)$. This plot is useful for finding real roots of the closed loop transfer function since these occur whenever the plot shows a value of -1, i.e. 0 db magnitude with a phase angle of -180°. With this and the root locus it is possible to draw a Bode plot of the closed loop transfer function. The Bode plot is a plot of the transfer function decibel magnitude and phase against $\log j\omega$, the imaginary part of S. The values given at any value of $j\omega$ are those which would be measured in response to a sinusoidal input at a frequency of ω radians/sec.

Both Bode and σ plots are easily constructed by adding corner corrections to rapidly drawn straight line asymptotes. The corrections for Bode plots are given in most text books on servomechanisms but the σ-plot corrections are less widely published and are reproduced in Fig. 5 in sufficient detail to enable a σ-plot to be drawn.

Further details of the above methods and a plea for their joint use in servo analysis are published in Ref. 16.

c) Theoretical Analysis

The effect of a transfer function with quadratic denominator (e.g. short period only) has been explained theoretically in Ref. 11 and empirical pilot opinion contours consistent with such an explanation are shown in Fig. 3. We shall therefore concentrate on the possible effect of other transfer function roots. Of these the numerator roots can be eliminated since they do not vary widely at low stability and Ref. 12 has shown their effect to be small.

The Phugoid Poles

In most cases, including those with good stability, the dominant pair of poles, considered in Fig. 3, will constitute the short period mode and we can call the remaining pair "phugoid poles". It has been shown (e.g. Ref. 8, etc.) that the phugoid does not affect pilot opinion in a simple tracking task in a conventional stable aeroplane. It is easy to explain this fact theoretically (Fig. 5)

The aircraft transfer function to be considered first is :

$$\frac{\theta}{\eta}(S) = \frac{5(S + 2)(S + .05)}{(S^2 + 6S + 20)(S^2 + .16S + .01)}$$

This has good short period characteristics (see Fig. 3) and no other unstable roots so we shall first take a pilot model with no equalisation :

$$\frac{\eta}{\theta}(S) = K_P \; e^{-0.3S}$$

From a study of the root locus at the bottom of Fig. 6 we choose a gain of

$$K_P = 1.6$$

which gives a damping ratio of 0.35 for the closed loop roots originating from the short period. At the top of the figures, the open loop asymptotes are drawn and the σ-plot constructed using the corner corrections of Figure 5. The advantage of using a decibel scale is that the gain can be changed by vertical translation of the plot, or, conversely, the scale. The scale shown at the left hand side of the figure corresponds to the chosen pilot gain of 1.6. The phase angle associated with the σ-plot is shown in figures immediately under the plot. We can now see that the σ-plot indicates two real closed loop poles at $1/T_{CL_1}$ and $1/T_{CL_2}$. The frequency of the remaining pair of complex conjugate poles can be obtained from the root locus. With the knowledge that the closed loop zeros are the same as the open loop ones, the asymptotes for the Bode plot of the closed loop system can then be drawn on the same axes as the σ-plot. The closed loop phase angle plot is drawn below the amplitude ratio plots. Assuming a forcing function cut-off frequency of 1 radian/second, the chosen pilot describing function can be seen to satisfy the adaptation rules given in Section 5.b(i) because :

a) The low frequency droop does not exceed 2 decibels below the cut-off frequency

b) the damping ratio of the oscillatory roots is not less than 0.35

c) the phase margin is certainly greater than 60° (to measure the exact value, corner corrections would have to be applied to the closed loop Bode amplitude ratio plot. In the present case this is not necessary to merely prove compliance with the adaptation rules). The above description of the process of analysis serves as an example of the methods outlined in Parts (a) and (b) and will not be repeated in subsequent examples.

The main significance of Fig. 6 is the insight it can give into the reason why the low frequency phugoid roots do not have a strong effect on the handling qualities. The amount of low frequency droop is determined by the level of the closed loop Bode asymptote at frequencies below the forcing function cut-off frequency. This in turn is governed by three factors (see Fig. 6)

i) the level at zero frequency. This is a function of the open loop static gain value which, as we have seen was fixed by consideration of the stability of the short period roots.

ii) any change in level caused by the group of phugoid roots at the left hand side of Fig. 6.

iii) Any change in level below the cut-off frequency due to short period roots.

Of these, (i) and (iii) are properties of the short period roots. The effect of the phugoid can be representated by (ii) and in this case is very small.

The reason for its smallness can be deduced from Fig. 6. Considering the open loop asymptotes, the two groups of roots (phugoid and short period) are connected by a line which slopes downwards with a slope of -20db per decade. Thus, when the two groups are widely separated in frequency as they are in this example and in most stable aircraft, the phugoid group will be considerably higher than the short period group. Now the gain and hence the level of the zero decibel line are governed by the short period roots so in the case we are considering the 0 db. line will lie a long way below the level of the open loop asymptote in the phugoid frequency range. Thus the zero decibel line will cut the σ-plot in the narrow part of the negative infinite peak at the zero $1/T_{\theta_1}$. The closed loop pole $1/T_{C_{L_1}}$ will therefore be very near to the closed loop zero $1/T_{\theta_1}$ and the change in level of the closed loop asymptote will be small. In the case of a stable phugoid shown in Figure 6 the change is positive and reduces the amount of droop. An unstable phugoid would produce an increase in droop which, however, would still be small in the normal case.

Thus the significance of the phugoid roots depends on the difference in open loop gain between points (A) and (B) in the top part of Figure 6. We should strictly ignore any contribution to this ratio from pilot equalisation, since this in itself would add to the degradation of pilot opinion. The important ratio is then called the "static to short period gain ratio" of the aircraft and is given by the expression

$$\frac{K_\theta}{K_{\theta_{SP}}} = \frac{1}{T_{\theta_1} \quad T_{\theta_2}\omega_p^2}$$

Normally this has a large value (10 in Figure 6) and indicates that the phugoid is not important. In cases where $K_\theta/K_{\theta_{SP}}$ is small we should expect handling qualities to be more strongly affected by the characteristics of the phugoid roots.

This other case is illustrated in the system analysis plots of Figures 7 and 8 in which the gain ratio is only 0.5. The various types of plot are in the same relative positions as they are in Figure 6. In the case of a stable phugoid shown in Figure 7, a few preliminary plots have shown that the adaptation rules can be satisfied by using a small amount of lag lead equalisation except for some excessive droop at frequencies below 0.02 radians per second.

It is interesting to note that this very low fequency droop was accepted as evidence of a degradation in handling qualities in Ref. 12. However, two arguments can be put forward for neglecting droop at such low frequencies.

1. Appreciable power in the θ excitation at very low frequencies will imply large excursions in height since :

Amplitude of height variation $\approx \theta$-amplitude $\times \frac{V}{\omega}$ assuming constant incidence. Thus, it is unlikely that much excitation will exist below 0.2 rad/sec in realistic tracking tasks.

2. The period of the motion causing this error is of the order of 5 minutes or more. It is doubtful whether a pilot's behaviour can ever be assumed to be stationary over such long periods. In this case some non-stationary datum adjustment could be used to avoid the very low frequency errors.

These could explain the absence of any worsening of pilot opinion found experimentally in Ref. 12.

In Figure 8 the same value of $K_\theta/K_{\theta_{SP}}$ is used but the phugoid is now divergent. Even with the large amount of lag-lead shown the low frequency error is very large. A significantly worse pilot opinion would be expected in this case because of the large amount of lag required and the poor tracking performance. We should therefore expect the phugoid to affect aircraft handling qualities only if $K_\theta/K_{\theta_{SP}}$ is small. The negative flight simulator results of Ref. 12 are not inconsistent with this idea since good phugoid dynamics ($\zeta_p = 0.25$) were used in that experiment.

Up to now we have considered only a simple compensatory pitch angle control task. In practice other factors tend to complicate the situation. Whenever the phugoid mode is excited speed variations occur and the pilot will try to prevent these by use of the throttle, i.e. by varying the engine thrust. This will have the effect of suppressing the phugoid but will also introduce a new control problem.

One situation where the phugoid is known to be important is the approach where safety margins in speed and height are small and these two quantities have to be carefully controlled. Ref. 9 suggests on the basis of tests with a variable stability aircraft, that handling qualities can be assessed by reference to the stability ("speed stability") of a simplified system in which the aircraft is assumed to be constrained to describe a straight glide path. This leaves only speed able to vary in accordance with the aircraft dynamics. The motion has a first order denominator with time constant

$$\tau = \frac{V}{2g}\left(\frac{C_D}{C_L} - \frac{dC_D}{dC_L}\right)^{-1}$$

The flight tests suggested that instrument approaches become unacceptably difficult if this time constant is less than 15 seconds in the unstable sense.

In fact, control of airspeed on the approach involves a complicated multi-loop control problem (Fig. 9A). Very little work has been done on this general problem but it is reasonable to suspect that an inner loop is closed round θ and η (Fig. 9B). Any further assumptions must be subject to considerable doubt but the classical "piston engine technique" of controlling height with the throttle and speed with the stick is represented in Figure 10A. The technique used must depend on the characteristics of the aircraft and it is doubtful whether the technique of Fig. 10A is widely used in jet aircraft. Ref. 10 uses a further simplification (Fig. 10B) in which speed is not directly controlled. It would appear that this technique would be insufficient to stabilise any configuration for which the constrained motion of Ref. 9 is unstable so the treatment is not sufficiently general.

Further work needs to be done on this interesting low stability problem. Any such work would have to rely heavily on controlled simulator experiments since virtually nothing is known about the behaviour of a human operator in a multi-loop tracking task.

The theoretical analysis described in this section is not based on sufficient experimental evidence to be accepted on its own merits. It provides some insight into the possible processes of control by a human pilot but the more definite conclusions must be checked by simulator experiments before they can be accepted as facts. A series of such tests was carried out and is described in the next section.

6.0 SIMULATOR TESTS

This section describes a short series of tests in a fixed base simulator designed to determine the effects of the phugoid roots on handling qualities. In particular it was intended to check the conclusion reached in Section 5 that the significance of this effect depends on the magnitude of the static to short period gain ratio $K_\theta/K_{\theta SP}$.

a) Description of Simulator

A block diagram of the simulator is presented in Fig. 11. The elevator deflection/pitch angle transfer function was set up on a LACE analogue computer. A voltage proportional to stick deflection was supplied from the cockpit which also contained the 11 ins. x 8 ins. cathode ray tube display shown in Fig. 12. This was driven by signals from the computer. The display tube was about 36 ins. from the pilot's eye and the display was assumed to subtend the same angles at the pilot's eye as the outside world. On this assumption the gain was adjusted so that, neglecting the phugoid roots, a constant stick force of 1 lb. would give a constant pitch rate of 0.74 deg /sec for all configurations. At M = 0.9 at Sea Level this would be equivalent to roughly 3 lb /g. This value was chosen on the basis of preliminary pilot opinion checks and is lower than the values usually found in fighter aircraft.

Roll control of the horizon line with ailerons was provided through a simple second order transfer function. This was intended merely to form a distraction for the pilot and possibly to add to the realism of the simulation.

Three random noise generators were tried in turn in an effort to obtain a realistic and consistent tracking task. A photoelectric wheel producing 10 sine waves was discarded because of its inconsistency at sufficiently low speeds and a filtered white noise generator was also abandoned because of the difficulty of filtering out the very low frequencies. In the end it was decided to use a photoelectric film reading device which had been developed to simulate terrain following radar equipment. In the present context the equipment merely produces a voltage proportional to the height of an opaque band on a strip of transparent $2\frac{1}{2}$ in. film. The ordinates of the noise were obtained by summing twenty sine waves with equal amplitudes and the frequencies given in Table 4. The summation was carried out on the DEUCE digital computer and the noise band was reproduced photographically after being painted on blank film by hand. A sample of the noise film is shown in Fig. 13 and a probability distribution histogram for a $16\frac{1}{2}$ minute sample in Figure 14. The distribution is not Gaussian but this does not matter as long as the noise appears random to the pilot. The standard deviation of the target motion was 1.64° and this resulted in a displacement of the dot which did not exceed about $\pm 5°$ i.e. about $\pm$ 3 in. on the display tube.

As shown in Fig. 11 the analogue computer also calculated the integrals with respect to time of :

Pitch angle demand θ_D

θ_D^2

Pitch angle error ε

ε^2

Modulus of elevator deflection

b) <u>Description of Experiment</u>

31 sets of aircraft dynamics were chosen initially and these were reduced to 11 as the experiment proceeded. The characteristics of the 11 configurations finally tested are listed in Table 5.

As stated above the pilot was given a pursuit type display, i.e. he was supplied with the actual value of pitch angle as well as the error. This departure from the compensatory case considered theoretically was justified by the necessity to retain some realism in a rather artificial controlled experiment. A brief check with a compensatory display showed no significant difference in tracking errors but the pilot opinion rating was more difficult to define in the practical terms of the Cooper scale, Table VI.

Each pilot was asked to give an opinion rating of each of six configurations, on the basis of a practice general handling and tracking run lasting as long as required, followed immediately by a five minutes tracking run during which the errors were measured. The Cooper scale Table VI, was used for assessment. One configuration in each group of six was repeated as a check on consistency. In order to limit the time used for the experiment a total of only 81 runs were flown by two test pilots and two engineers with past flying experience. Most of the flying was done by the two engineers and the error measurements were valid only in 35 of the runs.

c) Results

The results are included in the last two columns of Table V in the form of mean pilot opinion ratings and mean values of the tracking error ratio :

$$\frac{\int \varepsilon^2 \, dt}{\int \theta_D^2 \, dt}$$

taken over a 5 minute run. Because of the small number of results a thorough statistical analysis would be difficult and was not justified. It is sufficient to note that an analysis of some of the larger samples suggested that 5% confidence limits of

$\pm$ .015 on tracking error ratios

and $\pm$ .5 on pilot opinion rating

would be reasonable overall values.

The idea of statistical analysis having been abandoned, the results for different pilots were lumped together after crude corrections had been made to the results of the two test pilots to bring the mean levels of results for different pilots near together. This ensured that the comparisons between different configurations would not be affected by differences between pilots. 14 pilot opinion ratings obtained with a compensatory display were also included since these showed no significant difference from the pursuit display results. The tracking scores and opinion ratings given in Table 5 are consistent with one another and the results are discussed below in terms of opinion ratings only. The results demonstrate the following points :

i) Disagreement with the Theoretical Cases of Section 5

Configuration B2 is the basic reference configuration with good short period and phugoid dynamics and a high value of $K_\theta/K_{\theta_{SP}}$. It is the configuration studied theoretically in Fig. 6 and was given an average rating of 3 which is consistent with the theoretical analysis.

In configuration B7 the phugoid has been made unstable

$$(S + 0.1) \quad (S - 0.1)$$

and the static to short period gain ratio has been reduced to 0.5. The theoretical prediction given for this case in Fig. 8 is completely inconsistent with the experimental result. The average pilots opinion ratings and tracking scores for B7 were not significantly worse than those for B2 whereas the analysis had predicted that B7 should be quite difficult to control and certainly much worse than B2.

ii) Effect of more Unstable Phugoids

In configurations B10 the phugoid is made more unstable by halving the time constant of the divergence :

$(S + 0.2)\ (S - 0.2)$ (Time constant = 5 sec. instead of 10 sec.)

There is still no significant degradation in pilot opinion.

However, when the divergent time constant is further reduced to 2 sec. in B13

$(S + 0.5)\ (S - 0.5)$

the pilot's rating becomes significantly worse (6). Experience in flying these configurations suggested that this degradation in pilot opinion might be mainly due to the increased physical effort needed to resist the changing trim (stick) forces but this suggestion is contradicted by the fact that B11 (the stable phugoid with the same time constant as B13) was rated nearly as good as the basic standard B2. The negatively damped phugoid with the same stiffness (ω_p^2) as B11 and B13 is given by B12 and was rated between the other two.

(iii) Effect of $K_\theta/K_{\theta_{SP}}$ with Very Unstable Phugoid

In (ii) above the effect of a stiffer phugoid was investigated with a low value of $K_\theta/K_{\theta_{SP}}$ (0.5). Configurations B17, B18 and B19 were the same as B11, B12 and B13 except for a higher value of $K_\theta/K_{\theta_{SP}}$ (6.4). The only significant effect of this increase in static to short period gain ratio was to increase the degradation in pilot opinion due to the negatively damped phugoid so that it was given roughly the same average rating as the divergent one.

iv) Effect of Reducing the Short Period Frequency

Finally in B20 and B22 the same phugoid characteristics as B11 and B13 were tested with the short period frequency reduced from 4.47 to 1.5 and the same value of $K_\theta/K_{\theta_{SP}}$ (0.5). This was to find out to what extent the results depended on the relative frequencies of the short period and phugoid roots. With the frequencies brought closer together by this amount the effect of the divergent phugoid is slightly greater but the effect is barely significant.

This result is important for the guidance it gives concerning the correct choice of the dominant pair of roots. The modified short period roots are a pair of negative real roots (-5.6 and -.4) which happen to correspond to a C.G. position just forward of that at which the third oscillation is formed. The case with the stable phugoid is given one rating worse than the value of 4.5 given by the short period roots on Figure 2. This is the same discrepancy as is given in the case of configuration B2. Further, as stated above, the divergent phugoid produces the same difference in rating as it did with the higher frequency short period. Thus, although the phugoid has an effect, the short period roots can still be considered to be the dominant roots in this case. Since this is not the case once the third oscillation has formed, there are grounds for concluding that the formation of the third oscillation marks the point at which a change should be made in the choice of the dominant pair of roots,

7.0 DISCUSSION

a) Assumed Pilot Characteristic

The detection of a significant lack of agreement between the theoretically predicted and measured effects of $K_\theta/K_{\theta_{SP}}$ appears to throw doubt on the particular form of handling qualities theory used here. A similar discrepancy has been demonstrated before (Refs. 10 and 12) and it was thought before the tests described above that this could be explained by doubting the significance of tracking errors at very low frequencies. The simulator experiments were designed to check this and, although the neglect of very low frequency errors may well be valid, it is not sufficient to explain the discrepancy There remains a more fundamental problem.

In looking for an explanation, we note that low values of $K_\theta/K_{\theta_{SP}}$ affected neither pilot opinion rating nor tracking performance. Thus it seems that to obtain good tracking performance the pilot must have been applying a considerable amount of lag (Fig. 15) without degrading his opinion rating and we are forced back to the hypothesis of Ref. 10 that the effect on pilot opinion rating of a lag-lead type of pilot behaviour is much less than that given by the interpretation of Ref. 19 in Ref. 10.

This interpretation relies on the coincidence of the form of open loop pilot/aircraft transfer function for two controlled elements differing by a simple integrating factor 1/S, i.e. a lag term with infinite time constant. Each of these controlled elements was flown by two pilots and in Ref. 10 the ratings for the two pilots were averaged. Now the four results used are inconsistent:

Configuration Number	Controlled Element	Pilot Opinion I	Pilot Opinion II
6	5	Acceptable	Poor +
11	5/S	Acceptable +	Good

and little significance can be attached to the conclusion drawn from averaging over the two pilots. Thus, in view of the results of the present experiment and those mentioned in Ref. 10, it seems probable that large amounts of pilot lag can be applied without any significant degradation in pilot opinion.

This important hypothesis can be further justified by physical reasoning as follows. When flying a controlled element with a relatively stiff (i.e. highfrequency) phugoid the level of stick force required varies at the lowest input frequencies with higher frequency corrections being superimposed. Thus, at a certain instant in time the pilot may have to apply a large, relatively constant stick force corresponding to the peak of a low frequency variation while continuously tracking higher frequency inputs. In practical terms this slowly varying out of trim stick force is that needed to control trim changes due to changes in speed as the aircraft climbs or dives. The pilot's mode of operation is then to hold constant stick force and modify this by small increments proportional to the error. In other words, rate of increase of stick force is made proportional to error and zero error gives constant stick force rather than zero stick force. Apart from the physical effort of maintaining high levels of stick force, this form of control is not difficult and requires comparatively little mental effort.

The truth of this statement can be seen by thinking of the pilot as a sampled data system applying discrete steps of stick force level proportional to error and holding constant stick force between samples. The difficulty of such a process is much less than that of phase lead control which involves differentiating the error by remembering previous values in order to predict future values of the error. It should be noted that the above discussion only results in a reasonable hypothesis. The facts could be proved by a further series of experiments similar to those of Ref. 19.

b) Practical Consideration

Certain remarks in the first part of this section referred to speed variation and the physical effort required to maintain high levels of stick force. The importance of these two aspects limits the practical significance of some of the results.

Both the theoretical work and the simulator experiments concerned only single loop control of pitch attitude. In setting up the experiment it was found that, to detect the difference in phugoid characteristics, it was necessary to have some tracking input at very low frequencies (about 0.1 rad/sec). Now the value of this low frequency cut off is related to the range of height variation and hence speed variation, since constant thrust is assumed. In fact, at low frequencies it is reasonable to assume that variation of incidence α is small compared with variation of pitch angle . In this case, if the pitch angle varies in a sine wave, with frequency ω, the amplitude of height variation is approximately V/ω times the amplitude of pitch angle variation.

Thus the differences in tracking performance and pilot opinion depend on the presence of appreciable height and speed variations. In these circumstances the pilot would almost certainly use the throttle and trimmer controls with which he would be provided in a real aeroplane and which were missing in the simulator. All the pilots who flew in the simulator commented on this fact.

Thus the important conclusions are the ineffectiveness of $K_\theta/K_{\theta_{SP}}$ as a handling qualities parameter and the lack of interaction between phugoid and short period roots. The degradation in pilot opinion due to the stiffer phugoid roots is not a directly applicable result. The effect could be due to a combination of two causes :

a) discomfort caused by the high level of stick force to be maintained without a trimmer. This should occur equally with stable and unstable phugoids and the good pilot opinions obtained with a very stiff stable phugoid suggests that this first effect may be fairly unimportant.

b) Apprehension of loss of control in the case of a powerful unstable phugoid. This would be aggravated by the lack of any thrust control.

Handling problems of this type would appear in practice as speed control problems and require a study of the full multi-loop system which is beyond the scope of this note. Until such a study can be completed the empirical speed control criterion of Ref. 19 might be used to give a rough estimate of handling qualities for these cases (see Page 16)

8.0 PROCEDURE FOR ASSESSING LONGITUDINAL HANDLING QUALITIES

i) Choose the dominant pair of roots of the stability quartic. The method of doing this is illustrated in Fig. 16. In the conventional high stability short period phugoid dynamics (a) and the case where the short period roots are real (B), these are the short period roots. In the Case (C), where a third oscillation has formed, the two extreme roots on the real axis should be chosen.

ii) Assess the basic handling qualities from this pair of roots on the basis of Figure 3. If there are no other unstable roots with inverse time constant less than - 0.2 per sec this is the final assessment.

iii) If there are other roots with inverse timeconstant less than -.2 per sec. (time to double amplitude < 3.5 sec) calculate :

$$\frac{1}{\tau} = \frac{2g}{V}\left(\frac{C_D}{C_L} - \frac{dC_D}{dC_L}\right)$$

If

$$\frac{1}{\tau} \leq -\frac{1}{15} \text{ per sec}$$

the basic pilot opinion will probably be made much worse by this extra root.

If $\frac{1}{\tau} > -\frac{1}{15}$ per sec the basic pilot opinion may be

reasonably accurate but in any of these cases a flight simulator check of the particular case is desirable.

iv) Check airframe gain as constant speed stick force per g. This is still important for the special case of a constant speed turn and can be used to give some idea of overall airframe gain. It should be between 2 lb/g and 8 lb/g for high performance aircraft.

9.0 FURTHER WORK

The present investigation has concentrated mainly on a study of single loop attitude control. This has reached a stage where the handling qualities of low stability configurations can be assessed with more confidence than before. However, certain types of configuration would still require individual checks in a simulator and the necessity for further study is indicated in a number of areas.

a) Effect of Pilot Lag The hypothesis that the necessity to adopt a transfer function including large amounts of phase lag has only a small effect on pilot opinion should be checked by a specific simulator experiment.

b) Multi-loop Control of Attitude, Speed and Height

Handling qualities are still difficult to assess in certain cases where the phugoid is more effective and the assessment would depend on the characteristics of a multi-loop attitude/speed/height control system. Some work has been done on this problem in the approach case but there is ample justification for a further major effort starting with a general study of the behaviour of the human operator in multi-loop control systems.

c) The Lateral Low Stability Problem

The logical sequel to all this work is a study of lateral handling qualities at low stability. Here also the importance of multi-loop considerations can be anticipated and therefore item (b) would form a useful preliminary study.

10.0 CONCLUSIONS

1. Approximate factors of the longitudinal airframe transfer functions have been derived in British notation and are listed in Table II. At low levels of stability due to moving the C.G. aft the factors of the denominator are not valid and an alternative through less convenient method of factorisation has been derived (section 3c).

2. The manoeuvre margin is closely related to the "short period" approximate factor and is therefore useful only for assessing the handling qualities of conventionally stable aircraft. Also "stick force per g" relies for its usefulness on the wide separation of short period and phugoid roots and is thus not a sufficient criterion at low stability.

3. An iso-opinion plot of the stiffness and damping terms of two of the four denominator roots chosen as dominant does not depend on conventional levels of stability and is taken as a basis for the assessment of low stability handling qualities.

4. It was predicted theoretically that with an unstable phugoid, the static to short period gain ratio $K_\theta/K_{\theta_{SP}}$ would have a strong effect on both tracking performance and pilot opinion. However, the effect was not found in the results of a fixed base simulator experiment. In fact, no case could be found in which a mildly unstable phugoid affects pilot opinion or tracking performance.

5. The absence of any effect of $K_\theta/K_{\theta_{SP}}$ can be taken to imply that a pilot can apply a considerable amount of phase lag without any appreciable detriment to his opinion. This hypothesis conflicts with the interpretation of the results of Ref. 19 given in Ref. 10, but is consistent with the results of more recent work in the United States.

6. A phugoid mode with inverse time constant of 0.5 per sec has an adverse effect on pilot opinion if it is unstable but not if it is stable. In the unstable case it is not permissible to assess the handling qualities on the basis of single loop control and further work on multi-loop pitch angle/speed/height control is needed.

5. Thus, low stability handling qualities canbe assessed on the basis of iso opinion plots of the stiffness and damping term of the dominant mode unless the other mode has a time to double amplitude less than 3.5 sec. In that case a simulator experiment should be carried out.

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APPENDIX I

Behaviour of Longitudinal Poles as C.G. Moves Aft

If the root locus method of Section 3 is used to plot the locus of airframe transfer function poles as the C.G. moves aft, two distinct types of plot are obtained (Figures 2 A and B). In Figure 2A the initial instability is a divergence and in Figure 2B it is oscillatory with low frequency. The roots of $\Delta_2(s) = 0$ are usually near to the phugoid poles and the relative position of these two pairs of roots can be seen to govern the type of plot (A or B). This can be further related to the ratio of static margin to manoeuvre margin :

The distance of the phugoid roots from the origin is approximately $\omega_p \doteq \sqrt{\frac{E_1}{C_1}}$ (Table 2). The distance of the Δ_2 zeros from the origin is $\omega_2 = \dfrac{C_L}{\sqrt{2}\,\hat{t}}$ (from Equation 7).

The following approximate expressions for C and E in terms of manoeuvre and static margins can be deduced from Ref. 17 :-

$$C_1 \doteq - \frac{1}{\hat{t}^2} \frac{\mu_1}{i_B} \frac{\bar{c}}{2\ell} z_w H_m$$

$$E_1 \doteq - \frac{1}{\hat{t}^4} \frac{\mu_1}{i_B} \frac{\bar{c}}{2\ell} \frac{C_L^2}{2} z_w K_n$$

Therefore $$\frac{\omega_p^2}{\omega_2^2} \doteq \frac{2\hat{t}^2}{C_L^2} \frac{E_1}{C_1} \doteq \frac{K_n}{H_m} \begin{cases} > 1 \text{ for type A} \\ < 1 \text{ for type B} \end{cases}$$

Thus, the instability indicated by negative manoeuvre margin is not derived from the short period roots but could be an innocuous oscillatory instability of the phugoid. How unstable this mode has become when the manoeuvre margin changes sign is indicated by the (negative) value of R_1 at that time : $-(D_1^2 + B_1^2 E)$

Since D_1 is usually small the negative magnitude of R_1 can be very small at zero manoeuvre margin.

TABLE 1

Summary of Longitudinal Transfer Function Numerators

	A	B	C	D
Exact θ	$\frac{\mu_1}{t^2}\left(\frac{m_{\dot{w}}}{i_B} Z_\eta + \frac{m_\eta}{i_B}\right)$	$\frac{\mu_1}{t^3 i_B}\left\{m_w Z_\eta - Z_w m_\eta - x_u(m_\eta + z_\eta m_{\dot{w}})\right\}$	$\frac{\mu_1}{t^4 i_B}\left\{x_w(m_u z_\eta - z_u m_\eta) + x_u(z_w m_\eta - m_w z_\eta)\right\}$	
Approx.	$\frac{\mu_1 m_\eta}{t^2 i_B}$	$\frac{\mu_1}{t^3 i_B}(m_w z_\eta - z_w m_\eta)$		
Exact u	$\frac{V}{t^2} Z_\eta x_w$	$\frac{V}{t^3 i_B}\left\{-Z_\eta\left(x_w m_q + \frac{g\hat{t}}{V}\mu_1 m_{\dot{w}}\right) + \mu_1 m_\eta\left(x_w - \frac{g\hat{t}}{V}\right)\right\}$	$\frac{V}{t^4 i_B}\mu_1(z_w m_\eta - m_w z_\eta)\frac{g\hat{t}}{V}$ $= \frac{g\mu_1}{t^3 i_B}(z_w m_\eta - m_w z_\eta)$	
Approx.		$\frac{V\mu_1 m_\eta}{t^3 i_B}\left(x_w - \frac{g\hat{t}}{V}\right)$		
Exact h	$-\frac{VZ_\eta}{t}$	$\frac{V}{t^2}\left\{Z_\eta\left(\frac{\mu_1 m_{\dot{w}}}{i_B} + x_u + \frac{m_q}{i_B}\right)\right\}$	$\frac{V}{t^3 i_B}\left\{\mu_1(m_w z_\eta - z_w m_\eta) - x_u z_\eta(m_q + \mu_1 m_{\dot{w}})\right\}$	$\frac{V\mu_1}{t^4 i_B}\left\{\left(x_w - \frac{g\hat{t}}{V}\right)(m_u z_\eta - z_u m_\eta) + x_u(z_w m_\eta - m_w z_\eta)\right\}$
Approx.			$\frac{V\mu_1}{t^3 i_B}(m_w z_\eta - z_w m_\eta)$	$\frac{V\mu_1}{t^4 i_B} m_\eta\left\{x_u z_w - z_u\left(x_w - \frac{g\hat{t}}{V}\right)\right\}$

TABLE 1 (continued)

Summary of Longitudinal Transfer Function Numerators

	$A_{,u}$	$B_{,u}$	$C_{,u}$	$D_{,u}$
Exact $u\,u_G$	$-\frac{x_u}{\hat{t}}$	$\frac{1}{\hat{t}^2}\left\{x_u\left(z_w + \frac{m_q}{i_B} + \frac{\mu_1 m_{\dot{w}}}{i_B}\right) - z_u x_w\right\}$	$-\frac{1}{\hat{t}^3 i_B}\left\{x_u(z_w m_q - \mu_1 m_w) - z_u\left(\frac{g\hat{t}}{V}\mu_1 m_{\dot{w}} + x_w m_q\right) + \mu_1 m_u\left(x_w - \frac{g\hat{t}}{V}\right)\right\}$	$\frac{1}{\hat{t}^4 i_B}\frac{g\hat{t}}{V}\mu_1\left(z_u m_w - m_u z_w\right)$
Approx.		$\frac{x_u}{\hat{t}^2}\left(z_w + \frac{m_q}{i_B} + \frac{\mu_1 m_{\dot{w}}}{i_B}\right)$	$-\frac{1}{\hat{t}^3 i_B}\left\{x_u(z_w m_q - \mu_1 m_w) + \mu_1 m_u\left(x_w - \frac{g\hat{t}}{V}\right)\right\}$	

TABLE 1 (continued)

Summary of Longitudinal Transfer Function Numerators

	$A_{,w}$	$B_{,w}$	$C_{,w}$	$D_{,w}$
uw_G	$-\frac{x_w}{\hat{\tau}}$	$\frac{1}{\hat{\tau}^2 i_B} x_w m_q$	$\frac{1}{\hat{\tau}^3 i_B} \frac{g\hat{\tau}}{V} \mu_1 m_w$	
θw_G	$-\frac{\mu_1 m_{\dot{w}}}{\hat{\tau} V i_B}$	$-\frac{\mu_1}{\hat{\tau}^2 V i_B} (m_w - x_u m_{\dot{w}})$	$-\frac{\mu_1}{\hat{\tau}^3 V i_B} (x_u m_w - x_w m_u)$	
hw_G	$\frac{z_w}{\hat{\tau}}$	$-\frac{1}{\hat{\tau}^2} \left(z_w \frac{m_q}{i_B} + \frac{\mu_1 m_w}{i_B} + z_w x_u - z_u x_w \right)$	$-\frac{1}{\hat{\tau}^3} \left\{ \frac{m_q}{i_B} (z_u x_w - x_u z_w) + \frac{g\hat{\tau}}{V} \frac{\mu_1 m_{\dot{w}}}{i_B} z_u \right\}$	$-\frac{1}{\hat{\tau}^4} \frac{g\hat{\tau}}{V} \frac{\mu_1}{i_B} (z_u m_w - m_u z_w)$

TABLE II

SUMMARY OF LONGITUDINAL APPROXIMATE FACTORS

Factored Forms	Approximate Factors	Conditions of Validity
$\Delta(s)=\underbrace{(s^2+2\zeta_p\omega_p s+\omega_p^2)}(s^2+2\zeta_{sp}\omega_{sp}s+\omega_{sp}^2)$	$\omega_{sp}^2 \doteq \frac{1}{\hat{t}^2 i_B}(z_w m_q-\mu_1 m_w)$	$\left\lvert x_u z_w - x_w z_u + x_u\left(\frac{m_q}{i_B}+\mu_1\frac{m_{\dot{w}}}{i_B}\right)\right\rvert << \left\lvert z_w\frac{m_q}{i_B}-\mu_1\frac{m_w}{i_B}\right\rvert$
or	$2\zeta_{sp}\omega_{sp} \doteq -\frac{1}{\hat{t}}\left(z_w+\frac{m_q}{i_B}+\frac{\mu_1 m_{\dot{w}}}{i_B}\right)$	$\left\lvert +m_q x_w z_u+\frac{g\hat{t}}{V}z_u\mu_1 m_{\dot{w}}\right\rvert << \left\lvert x_u(z_w m_q-\mu_1 m_w) + \mu_1 m_u\left(x_w-\frac{g\hat{t}}{V}\right)\right\rvert$
$(s+\frac{1}{T_{p_1}})(s+\frac{1}{T_{p_2}})$	ω_p^2 or $\frac{1}{T_{p_1}T_{p_2}} \doteq \frac{g}{V}\frac{\mu_1}{\hat{t}}\frac{z_u m_w-m_u z_w}{z_w m_q-\mu_1 m_w}$	
	$2\zeta_p\omega_p$ or $\frac{1}{T_{p_1}}+\frac{1}{T_{p_2}} \doteq \frac{1}{\tau}\left\{-x_u-\frac{\mu_1 m_u(x_w-g\hat{t}/V)}{z_w m_q-\mu_1 m_w}\right\}$	$\left\lvert\frac{E_1}{C_1}+\left(\frac{D_1}{C_1}-\frac{B_1}{C_1}\frac{E_1}{C_1}\right)B_1\right\rvert << C_1$ i.e.
		$\lvert 2\zeta_{sp}\omega_{sp}\,2\zeta_p\omega_p+\omega_p^2\rvert << \lvert\omega_{sp}^2\rvert$
		$\left\lvert\frac{D_1}{C_1}-\frac{B_1}{C_1}\frac{E_1}{C_1}\right\rvert << \lvert B_1\rvert$, i.e. $\lvert 2\zeta_p\omega_p\rvert <<$
		$\lvert 2\zeta_{sp}\omega_{sp}\rvert$
		$\lvert B_1E_1\rvert << \lvert C_1D_1\rvert$ i.e. $\lvert\omega_p^2 2\zeta_{sp}\omega_{sp}\rvert << \lvert\omega_{sp}^2 2\zeta_p\omega$

TABLE II (continued)

SUMMARY OF LONGITUDINAL APPROXIMATE FACTORS

Factored Forms	Approximate Factors	Conditions of Validity
$N_\theta(s) = A_\theta\left(S + \frac{1}{T_{\theta_1}}\right)\left(S + \frac{1}{T_{\theta_2}}\right)$	$A_\theta \doteq \frac{\mu_1 m_\eta}{\hat{t}^2 i_B}$	$\lvert m_{\dot w}\rvert \ll \left\lvert \frac{m_\eta}{z_\eta} \right\rvert$
	$\frac{1}{T_{\theta_1}} \doteq \frac{1}{\hat{t}}\left\{-x_u + x_w \frac{m_u z_\eta - m_\eta z_u}{m_w z_\eta - m_\eta z_w}\right\}$	$\lvert x_u\rvert \ll \left\lvert z_w - m_w \frac{z_\eta}{m_\eta} \right\rvert$
	$\frac{1}{T_{\theta_2}} \doteq -\frac{1}{\hat{t}}\left(z_w - m_w \frac{z_\eta}{m_\eta}\right)$	$\left\lvert x_u - x_w \frac{z_u m_\eta - m_u z_\eta}{z_w m_\eta - m_w z_\eta} \right\rvert \ll \left\lvert z_w - m_w \frac{z_\eta}{m_\eta}\right\rvert$ i.e.
		$\left\lvert\frac{1}{T_{\theta_1}}\right\rvert \ll \left\lvert \frac{1}{T_{\theta_2}} \right\rvert$
$N_h(s) = A_h\left(S + \frac{1}{T_{h_1}}\right)\left(S + \frac{1}{T_{h_2}}\right)$	$\frac{1}{T_{h_1}} \doteq \frac{1}{\hat{t}}\left\{-x_u + \left(x_w - \frac{g\hat{t}}{V}\right)\frac{z_u}{z_w}\right\}$	$z_q \doteq 0 \quad z_u < 0$
$\left(S + \frac{1}{T_{h_3}}\right)$	$\frac{1}{T_{h_2}}, \frac{1}{T_{h_3}} \doteq \pm\frac{1}{\hat{t}}\sqrt{\frac{\mu_1 m_\eta z_w}{i_B z_\eta}}$	$\mu_1\left(m_w - z_w \frac{m_\eta}{z_\eta}\right) \gg x_u \left(m_q + \mu_1 m_{\dot w}\right)$
(Aft control)		$\frac{m_\eta}{z_\eta} \gg \frac{m_w}{z_w}, \frac{m_\eta}{z_\eta} \gg \frac{m_u}{z_u}$
		$\left\lvert \frac{\mu_1}{\hat{t}^2 i_B} \frac{m_\eta z_w}{z_\eta} \left\{\frac{x_u - \left(x_w - \frac{g\hat{t}}{V}\right) z_u/z_w}{x_u + \frac{\mu_1 m_{\dot w}}{i_B} + \frac{m_q}{i_B}}\right\}\right\rvert \gg 1$

TABLE II (continued)

SUMMARY OF LONGITUDINAL APPROXIMATE FACTORS

Factored Forms	Approximate Factors	Conditions of Validity
$N_u(s) = A_u\left(S + \frac{1}{T_{u_1}}\right)\left(S + \frac{1}{T_{u_2}}\right)$	$A_u = \frac{V}{\tau^2} z_\eta x_{\dot w}$	$z_q \doteq 0$
	$\frac{1}{T_{u_1}} \doteq -\frac{g}{Vm_\eta} \frac{(m_w z_\eta - z_w m_\eta)}{(x_w - \frac{g\hat{t}}{V})}$	$\frac{1}{T_{u_1}} << \frac{1}{T_{u_2}}$
	$\frac{1}{T_{u_2}} \doteq \frac{\mu_1 m_\eta (x_w - \frac{g\hat{t}}{V})}{\tau\, i_B z_\eta x_w}$	$\left\|\mu_1 m_\eta \left(x_w - \frac{g\hat{t}}{V}\right)\right\| >> \left\| z_\eta\left(\mu_1 m_{\dot w} \frac{g\hat{t}}{V} + x_w m_q\right)\right\|$
$N_{uu}(s) = A_{uu}\left(S + \frac{1}{T_{uu_1}}\right.$	$A_{uu} = -\frac{x_u}{\hat{t}}$	$z_w + \frac{m_q}{i_B} + \frac{\mu_1 m_{\dot w}}{i_B} - z_u \frac{x_{\dot w}}{x_u} < 0$
$\left. - \frac{1}{T_{uu_2}}\right)\left(S + \frac{1}{T_{uu_3}}\right)$	$\frac{1}{T_{uu_1}} \doteq -\frac{1}{\tau}\left\{\frac{\frac{g\hat{t}}{V}\frac{\mu_1}{x_u}(z_u m_w - m_u z_w)}{z_w m_q - \mu_1 m_w + \frac{\mu_1 m_u}{x_u}(x_w - \frac{g\hat{t}}{V})}\right\}$	$z_w m_q - \mu_1 m_w + \frac{\mu_1 m_u}{x_u}\left(x_w - \frac{g\hat{t}}{V}\right) > 0$
		$\frac{1}{x_u}(z_u m_w - m_u z_w) > 0$
	$\frac{1}{T_{uu_1}}, \frac{1}{T_{uu_2}} \doteq \frac{1}{\hat{t}}\left\{-\left(z_w + \frac{m_q}{i_B} + \frac{\mu_1 m_{\dot w}}{i_B}\right)\right.$	$\left\|\frac{z_u}{x_u}\left(x_w m_q + \frac{g\hat{t}}{V}\mu_1 m_{\dot w}\right)\right\| << \left\| z_w m_q - \mu_1 m_w + \frac{\mu_1 m_u}{x_u}\left(x_w - \frac{g\hat{t}}{V}\right)\right\|$
	$\pm \sqrt{\left(z_w + \frac{m_q}{i_B} + \frac{\mu_1 m_{\dot w}}{i_B}\right)^2 + \frac{4}{i_B}\left[z_w m_q - \mu_1 m_w + \right.}$	$\left\|\frac{1}{T_{uu_1}}\right\| << \left\|\frac{1}{T_{uu_2}} - \frac{1}{T_{uu_3}}\right\|$
	$\left.\left. + \frac{\mu_1 m_u}{x_u}\left(x_w - \frac{g\hat{t}}{V}\right)\right]\right\}$	$\left\|\frac{z_u x_w}{x_u}\right\| << \left\| z_w + \frac{m_q}{i_B} + \frac{\mu_1 m_{\dot w}}{i_B}\right\|$

TABLE II (continued)

SUMMARY OF LONGITUDINAL APPROXIMATE FACTORS

Factored Forms	Approximate Factors	Conditions of Validity
$N_{hw}(s) = A_{hw}\left(s + \frac{1}{T_{hw_1}}\right)$	$A_{hw} = \frac{z_w}{\hat{t}}$ $\frac{1}{T_{hw}} \doteq -\frac{1}{\hat{t}}\left(\frac{m_q}{i_B} + \frac{\mu_1 m_w}{i_B z_w} + x_u - x_w\frac{z_u}{z_w}\right)$	$\left\lvert\frac{m_q}{i_B} + \frac{\mu_1 m_w}{i_B z_w} + x_u - x_w\frac{z_u}{z_w}\right\rvert \gg \left\lvert\frac{1}{\hat{t} i_B}\left\{m_q\left(\frac{z_u x_w}{z_w} - x_u\right) + \frac{g\hat{t}}{V}\mu_1 m_{\dot{w}}\frac{z_u}{z_w}\right\}\right\rvert$
$(s^2 + 2\zeta_{hw}\omega_{hw}s + \omega_{hw_1}{}^2)$	$\omega_{hw}{}^2 \doteq \frac{1}{\hat{t}^2 i_B}\,\frac{\frac{g\hat{t}}{V}\mu_1(z_u m_w - m_u z_w)}{\frac{m_q}{i_B}z_w + \frac{\mu_1 m_w}{i_B} + x_u z_w - x_w z_u}$ $2\zeta_{hw}\omega_{hw} \doteq \frac{1}{\hat{t} i_B}\left\{\frac{m_q(z_u x_w - x_u) + \frac{g\hat{t}}{V}\mu_1 m_{\dot{w}} z_u}{\frac{m_q z_w}{i_B} + \frac{\mu_1 m_w}{i_B} + x_u z_w - x_w z_u}\right.$ $\left. - \frac{\frac{g\hat{t}}{V}\mu_1 z_w(z_u m_w - z_w m_u)}{\left(\frac{m_q z_w}{i_B} + \frac{\mu_1 m_w}{i_B} + x_u z_w - x_w z_u\right)^2}\right\}$	$\left\lvert\frac{m_q}{i_B} + \frac{\mu_1 m_w}{i_B z_w} + x_u - x_w\frac{z_u}{z_w}\right\rvert \gg \left\lvert\frac{1}{\hat{t}^2 i_B}\left\{\frac{g\hat{t}}{V}\mu_1\left(\frac{z_u m_w}{z_w} - m_u\right)\right\}\right\rvert$

Table II (continued)

SUMMARY OF LONGITUDINAL APPROXIMATE FACTORS

Factored Forms	Approximate Factors	Conditions of Validity
$N_{uw}(s) = A_{uw}s(S^2+2\zeta_{uw}\omega_{uw} + \omega_{uw}^2$ or $(s + \frac{1}{T_{uw_1}})(s + \frac{1}{T_{uw_2}})$	$A_{uw} = -\frac{x_w}{\hat{t}}$ $2\zeta_{uw}\omega_{uw}$ or $(\frac{1}{T_{uw_1}} + \frac{1}{T_{uw_2}}) = \frac{m_q}{\hat{t} i_B}$ ω_{uw}^2 or $\frac{1}{T_{uw_1}} \frac{1}{T_{uw_2}} = \frac{g\hat{t}}{V} \frac{\mu_1 m_w}{\hat{t}^2 i_B}$	Roots real if $(\frac{m_q}{i_B})^2 - 4\frac{g\hat{t}}{V}\mu_1 \frac{m_w}{i_A} \geq 0$
$N_{\theta w}(s) = A_{\theta w}s(s + \frac{1}{T_{\theta w_1}})(s + \frac{1}{T_{\theta w_2}})$	$A_{\theta w} = -\frac{\mu_1 m_{\dot{w}}}{\hat{t} V i_B}$ $\frac{1}{T_{\theta w_1}} \doteq +\frac{1}{\hat{t}}(\frac{m_w}{m_{\dot{w}}} - x_u)$ $\frac{1}{T_{\theta w_2}} \doteq +\frac{1}{\hat{t}}\left(\frac{x_u m_w - x_w m_u}{m_w - x_u m_{\dot{w}}}\right)$	$\left(\frac{m_w}{m_{\dot{w}}} - x_u\right)^2 >> 4\left(\frac{x_u m_w - x_w m_u}{m_{\dot{w}}}\right)$

TABLE III

Approximate Factors of the Stability Quartic at Aft C.G. Positions

Supersonic Aircraft at M = 0.9/Sea Lecel

Datum C.G. 37.2% $\bar{c}$

(Manoeuvre point 48% $\bar{c}$)

C.G. Position % $\bar{c}$	Roots of $\Delta(s)$ (Deuce)	Roots of $\Delta_1 + \Delta_2 m$ (Root Locus)
41	.0857 - .1006 (-1.1648 (±2.0047i	.082 -.095 (-1.19 (±1.98i
43.7	.1201 - .1383 (-1.1487 (±1.3829i	.110 - .123 (-1.185 (±1.400i
47	.2029 - .3734 - .5157 -1.5916	.2 - .27 - .58 -1.8
50	.5231 (- .0618 (± .2171i -2.6441	.4 (- .1 (± .18i -2.6

TABLE IV

Frequencies of Sine Waves Summed to Form Random Appearing Input Signal

Channel No.	Frequency ω(rad./sec.)
1	.105
2	.142
3	.161
4	.198
5	.251
6	.310
7	.363
8	.408
9	.453
10	.491
11	.556
12	.576
13	.658
14	.685
15	.754
16	.822
17	.860
18	.905
19	.963
20	1.000

All channels had the same amplitude

TABLE V

AIRCRAFT DYNAMICS TESTED IN SIMULATOR EXPERIMENT

$$\frac{\theta}{\eta}=\frac{A_\theta \left(S+\frac{1}{T_{\theta_1}}\right)\left(S+\frac{1}{T_{\theta_2}}\right)}{(S^2+2\zeta_p\omega_p S+\omega_p^2)(S^2+2\zeta_{SP}\omega_{SP}S+\omega_{SP}^2)}$$

Config. No.	S.P. Stiffness Term (ω_{SP}^2)	S.P. Damping Term ($2\zeta_{SP}\omega_{SP}$)	S.P. Numerator Inverse Time Constant ($1/T_{\theta_2}$)	Phugoid Stiffness Term (ω_P^2)	Phugoid Damping Term ($2\zeta_P\omega_P$)	Phugoid Inverse Time Constant ($1/T_{\theta_1}$)	Gain A_θ	Static to S.P. Gain Ratio $K_\theta/K_{\theta_{SP}}$	Tracking Error Ratio	Pilot Opinion
B2	20	6	2.0	.01	.16	.05	5.0	10.0	.144	3.0
B7	20	6	0.5	-.01	0	.01	20.0	0.5	.151	3.9
B10	20	6	2.0	-.04	0	.01	5.0	0.5	.150	3.6
B11	20	6	2.0	.25	.40	.0625	5.0	0.5	.146	3.5
B12	20	6	2.0	.25	-.30	.0625	5.0	0.5	.178	4.5
B13	20	6	2.0	-.25	0	.0625	5.0	0.5	.213	6.1
B17	20	6	4.0	.25	.40	.40	2.5	6.4	.145	3.0
B18	20	6	4.0	.25	-.30	.40	2.5	6.4	.189	5.2
B19	20	6	4.0	-.25	0	.40	2.5	6.4	.219	5.4
B20	2.25	6	2.0	.25	.40	.0625	5.0	0.5	-	5.5
B22	2.25	6	2.0	-.25	0	.0625	5.0	0.5	-	7.5

D 70355/1/Dd.125875 K4 5/66 XL/CL

TABLE VI

COOPER PILOT RATING SYSTEM

	Adjective Rating	Numerical Rating	Description	Primary Mission Accomplished	Can be Landed
Normal Operation	Satisfactory	1	Excellent, includes optimum	Yes	Yes
		2	Good, pleasant to fly	Yes	Yes
		3	Satisfactory, but with some mildly unpleasant characteristics	Yes	Yes
Emergency Operation	Unsatisfactory	4	Acceptable, but with unpleasant characteristics	Yes	Yes
		5	Unacceptable for normal operation	Doubtful	Yes
		6	Acceptable for emergency condition only*	Doubtful	Yes
No operation	Unacceptable	7	Unacceptable even for emergency condition*	No	Doubtful
		8	Unacceptable - dangerous	No	No
		9	Unacceptable - uncontrollable	No	No
	Catastrophic	10	Motions possibly violent enough to prevent pilot escape	No	No

* Failure of a stability augmenter

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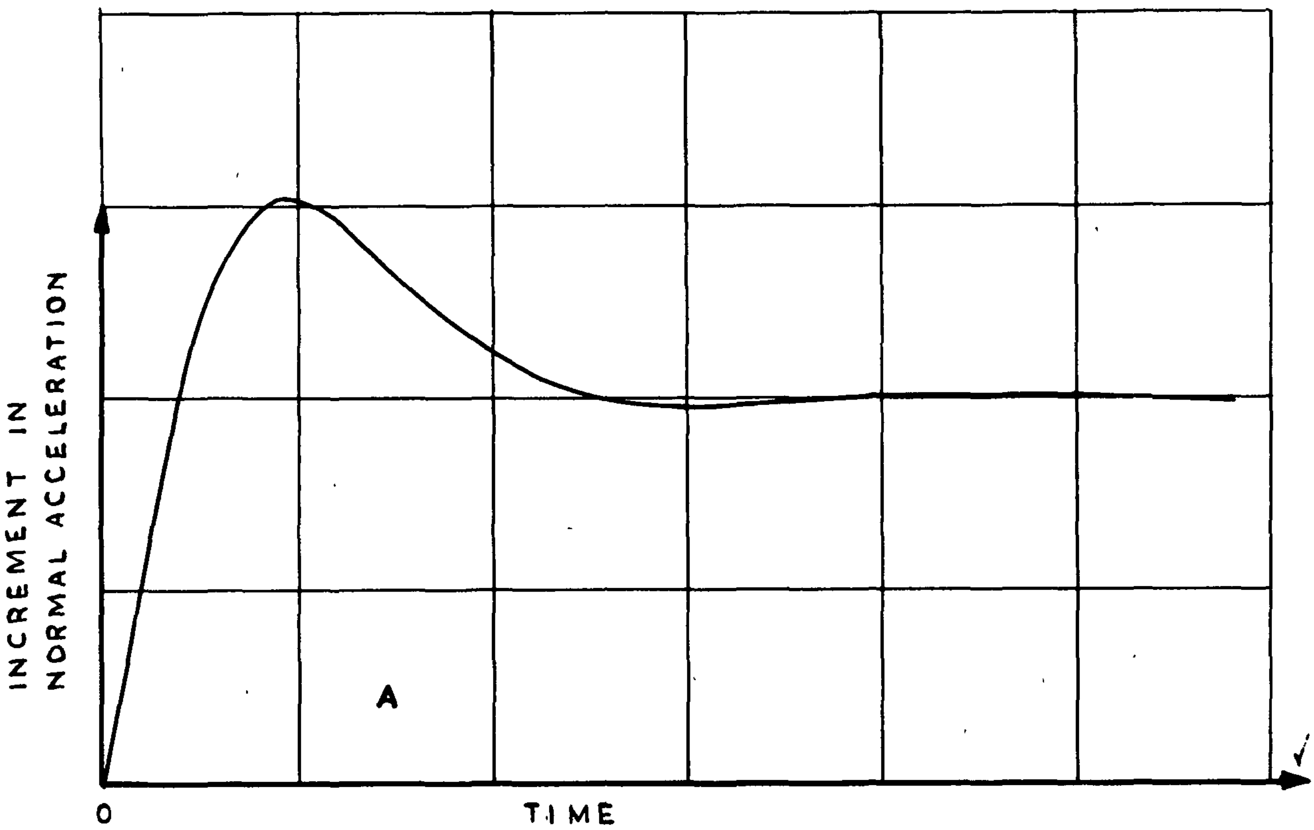
FIG. 1

EFFECT OF PHUGOID ON STEP RESPONSE.



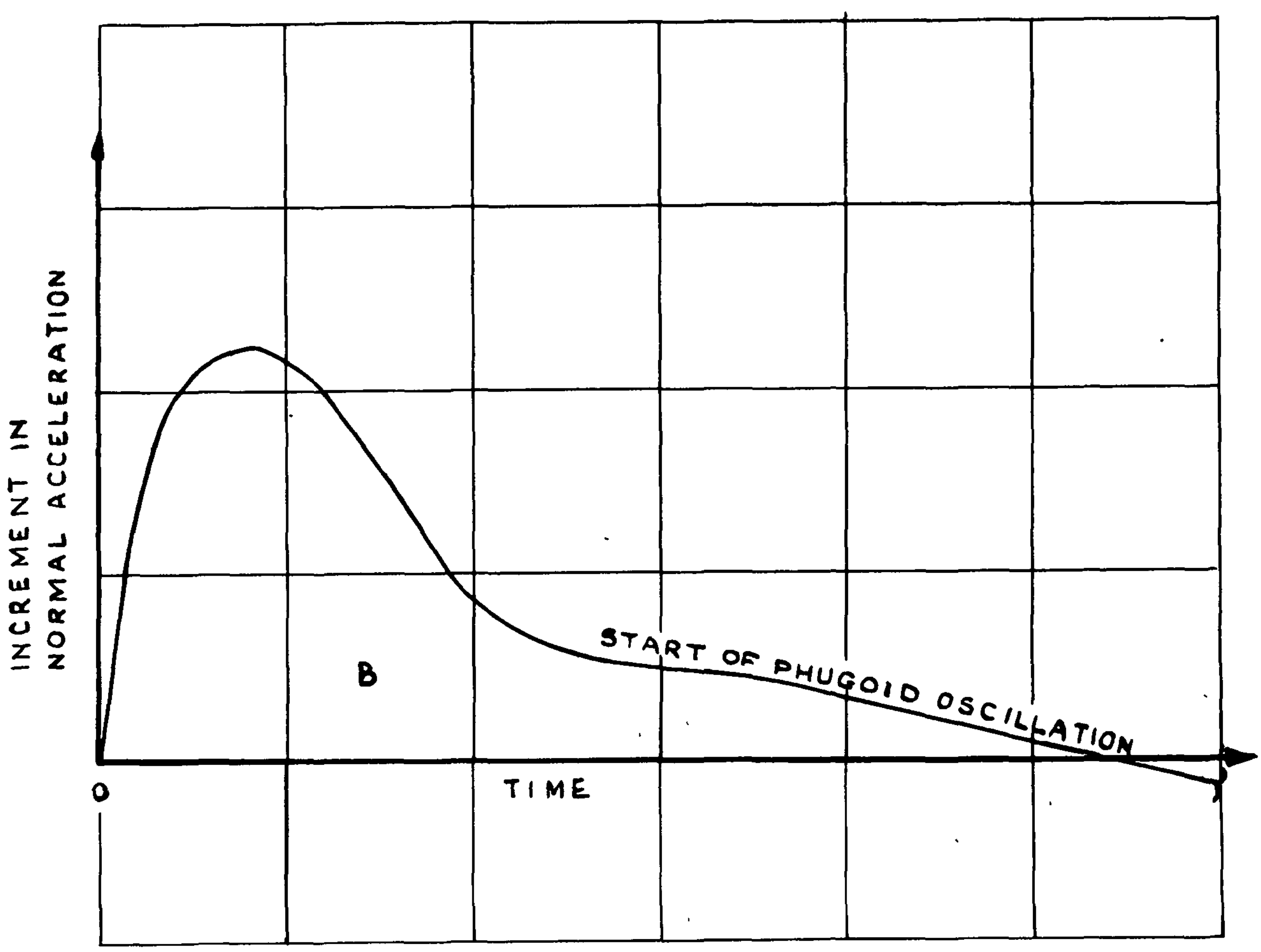


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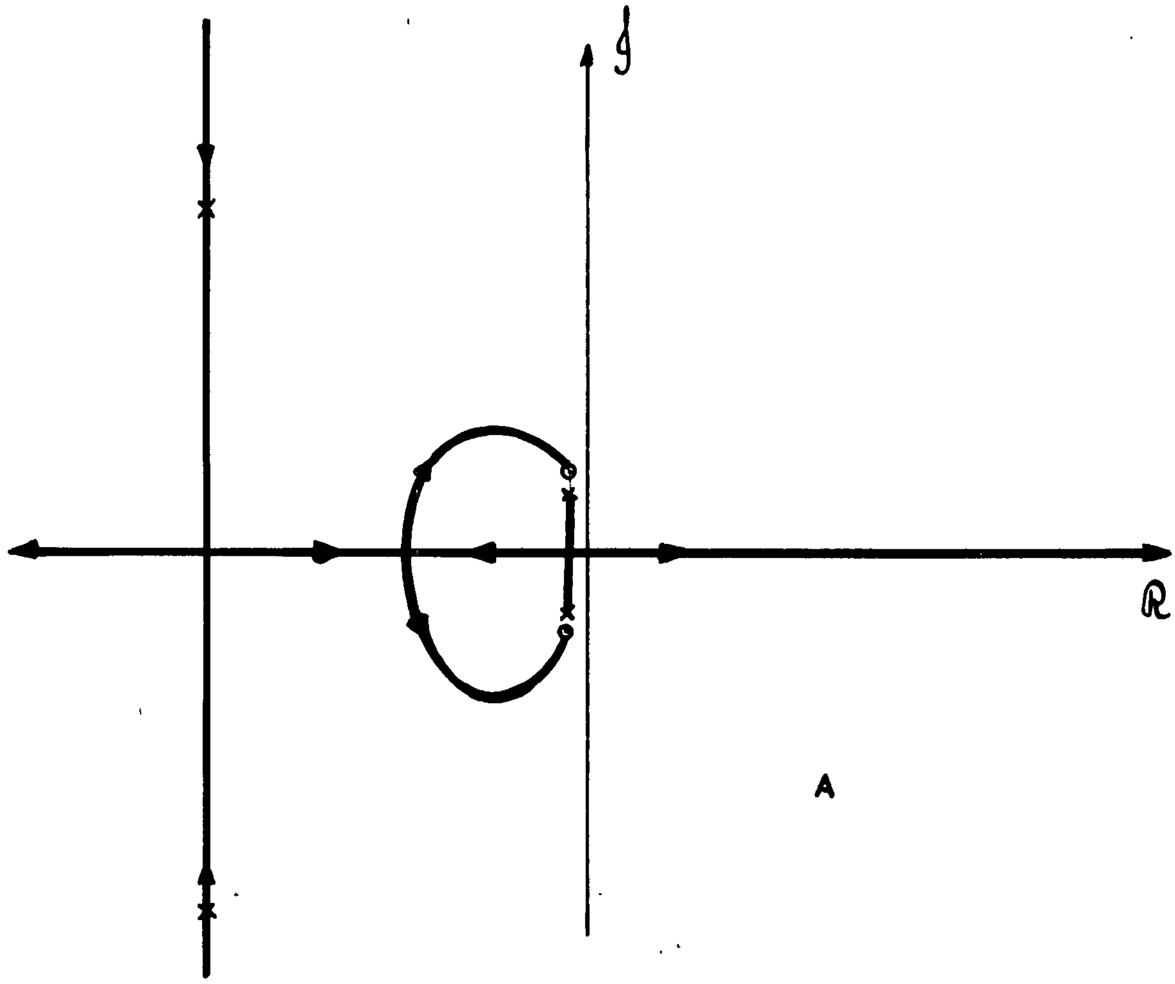
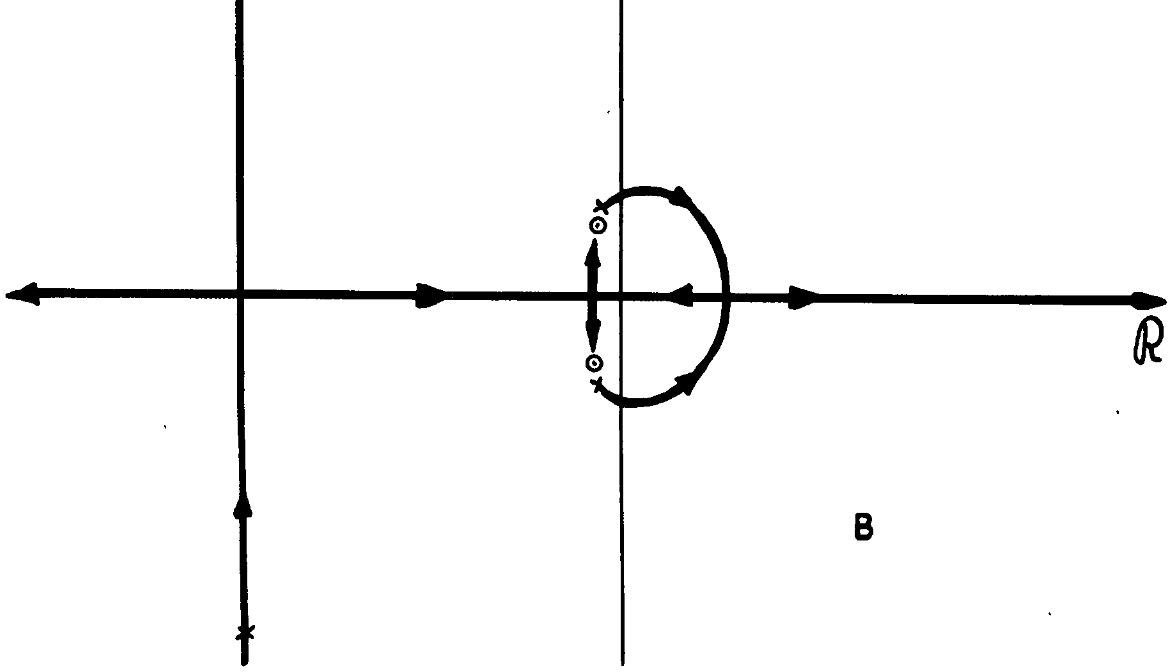
FIG. 2

TWO TYPES OF LOCUS OF DENOMINATOR ROOTS AS C.G. MOVES AFT.



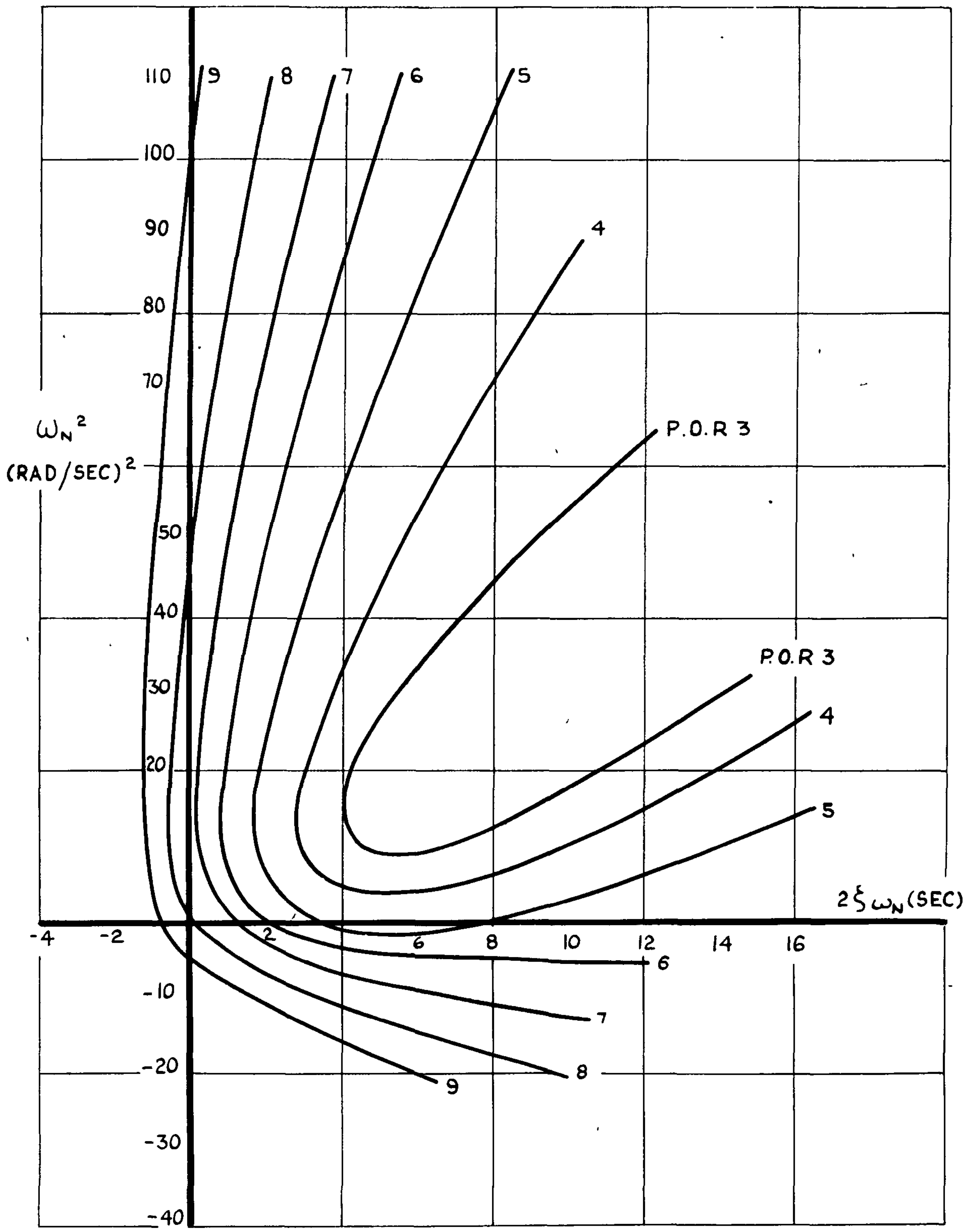
A



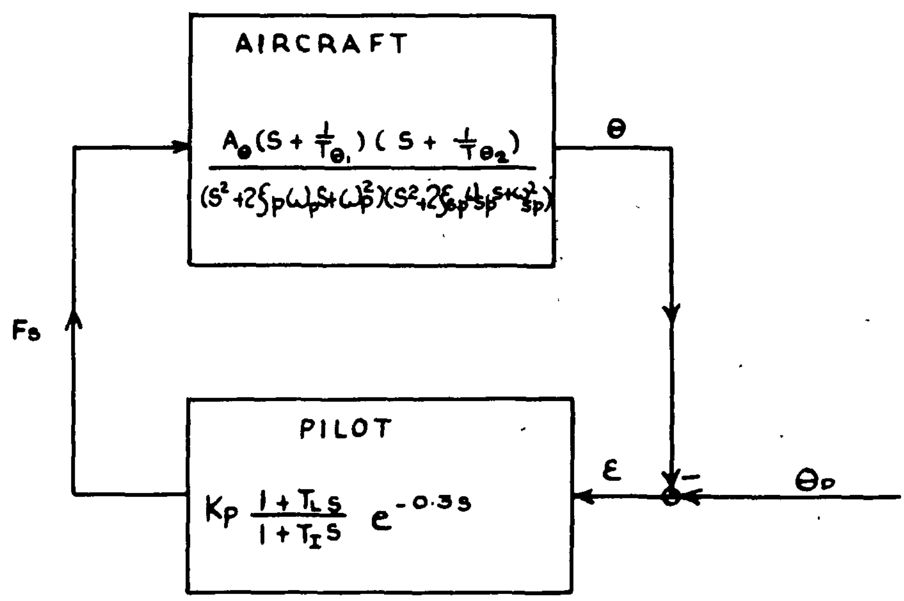
B

FIGHTER LONGITUDINAL DYNAMICS
- OPINION BOUNDARIES

STICK FORCE PER 'g' 6-8 LB.
STICK MOVEMENT PER 'g' 0 2 LB.



BLOCK DIAGRAM OF SINGLE LOOP PITCH ANGLE CONTROL.



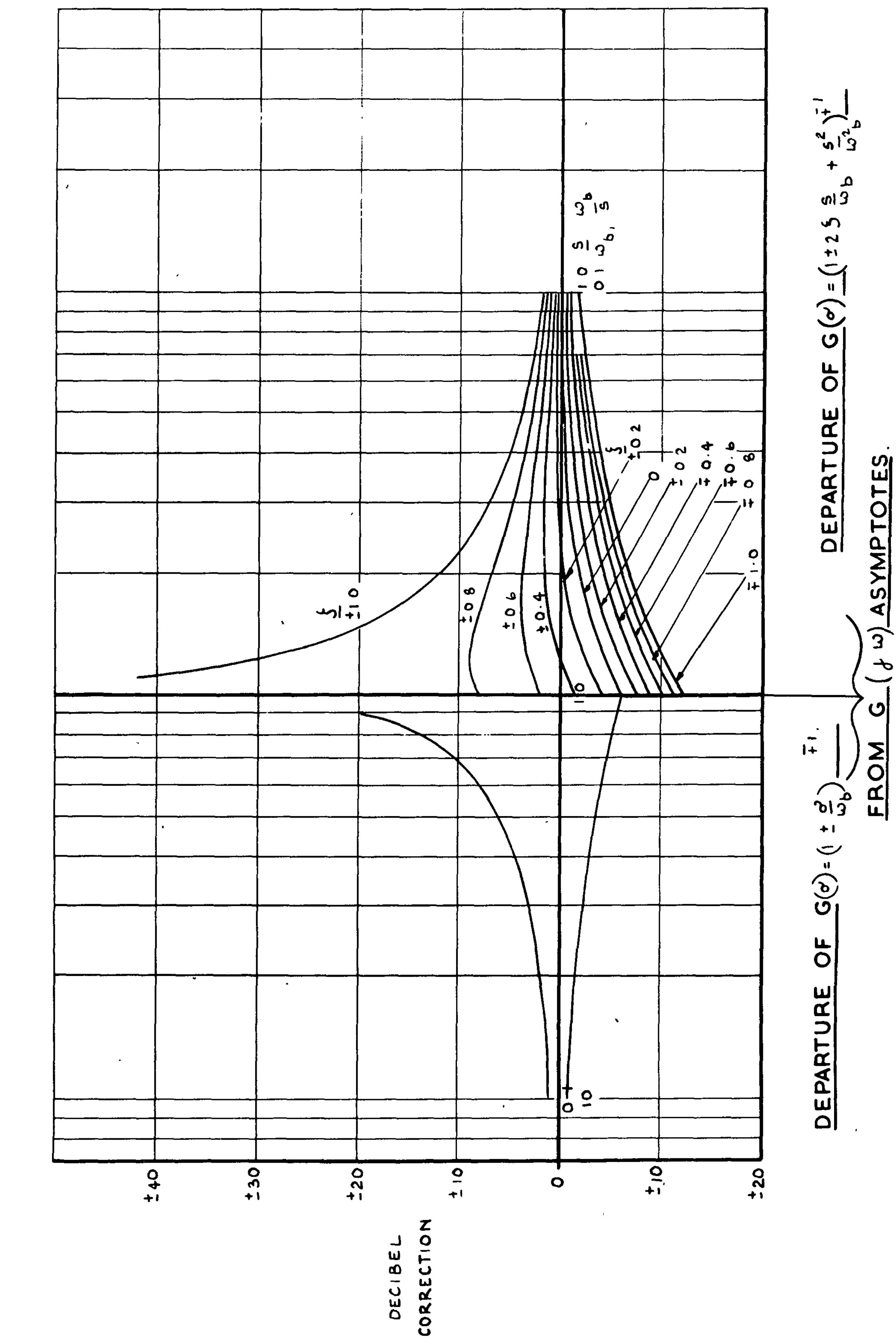
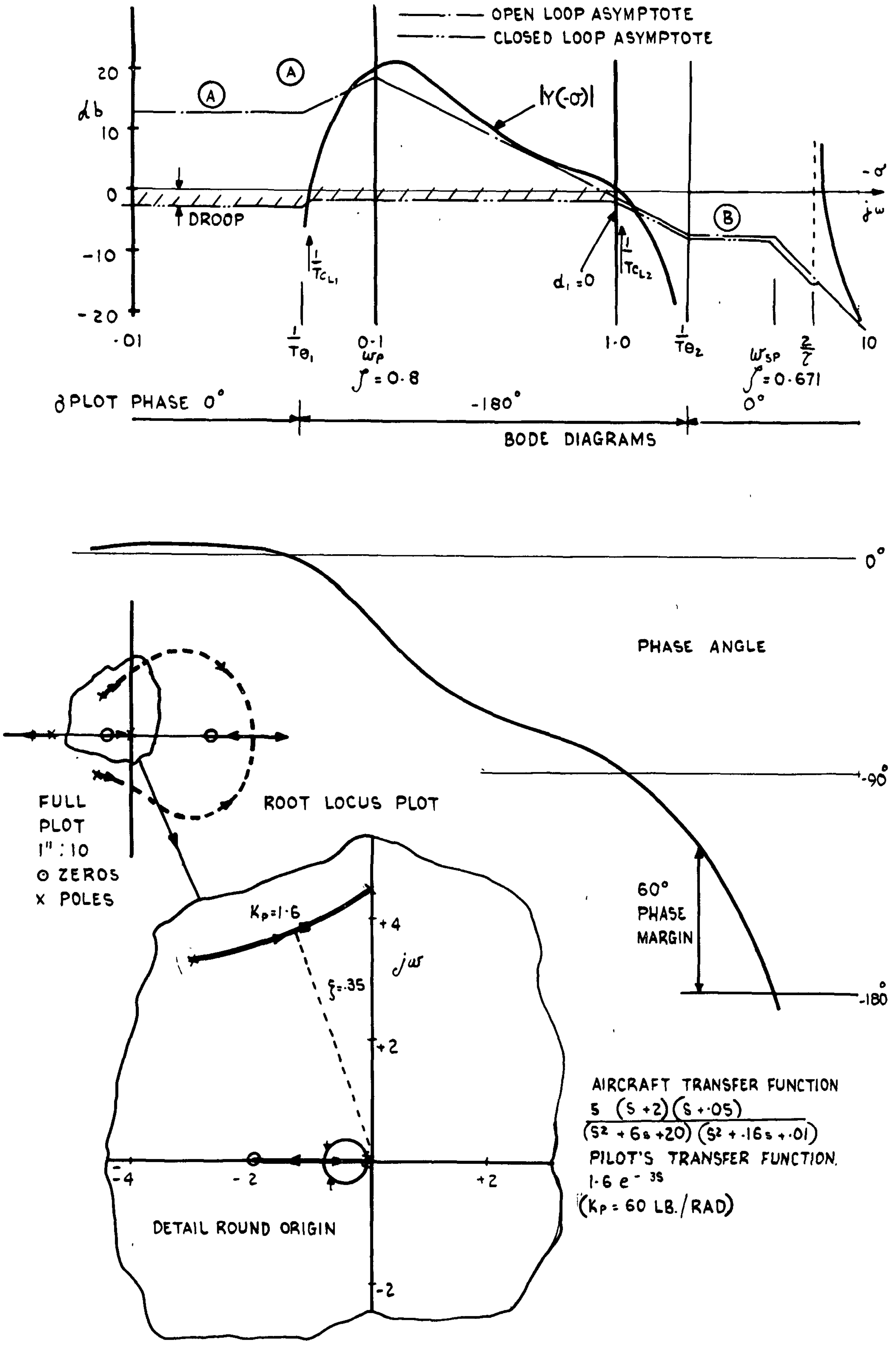

DEPARTURE OF G(σ) = (1 ± σ/ω_b)^∓1
DEPARTURE OF G(σ) = (1 ± 2ζ s/ω_b + s²/ω²_b)^∓1
FROM G(jω) ASYMPTOTES.
DECIBEL CORRECTION
±40
±30
±20
±10
0
∓10
∓20
ζ ±1.0
±0.8
±0.6
±0.4
±0.2
0
∓0.2
∓0.4
∓0.6
∓0.8
∓1.0
1.0
0.1
10
s/ω_b
ω_b/s

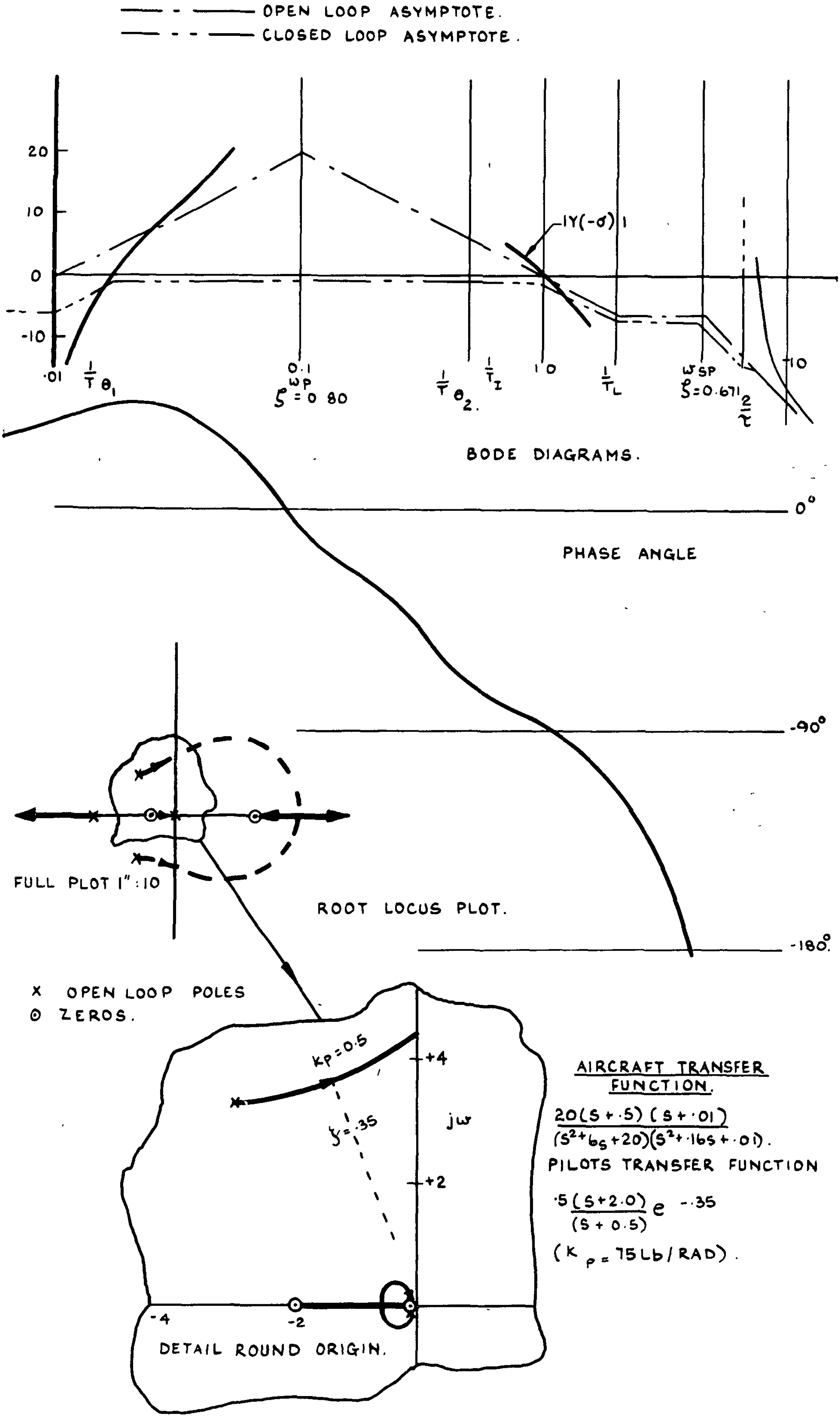
FIG 6

SYSTEM ANALYSIS-CONVENTIONAL CASE.



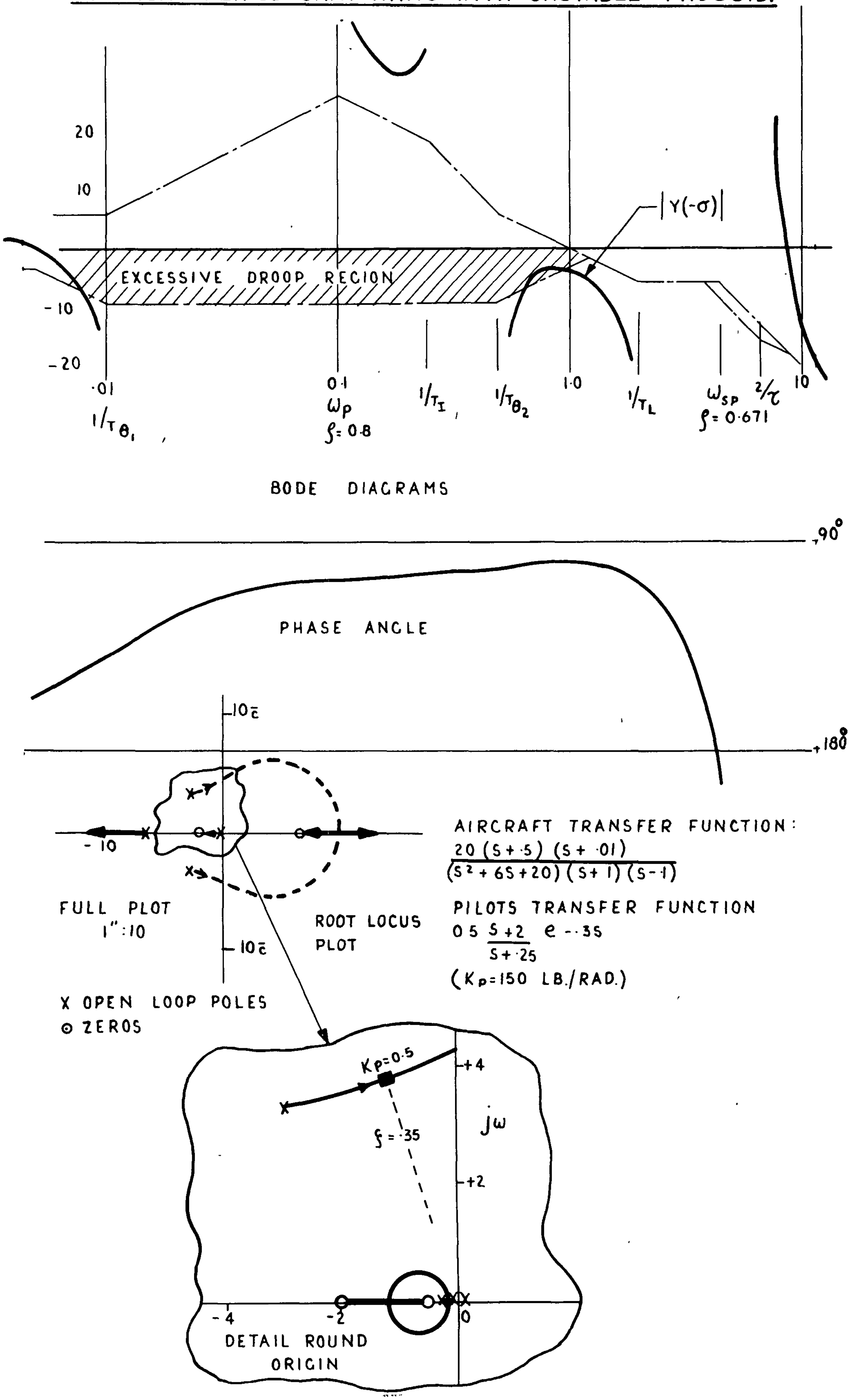
Ae. 197.
FIG. 7.

SYSTEM ANALYSIS - LOW STATIC TO SHORT PERIOD GAIN RATIO.



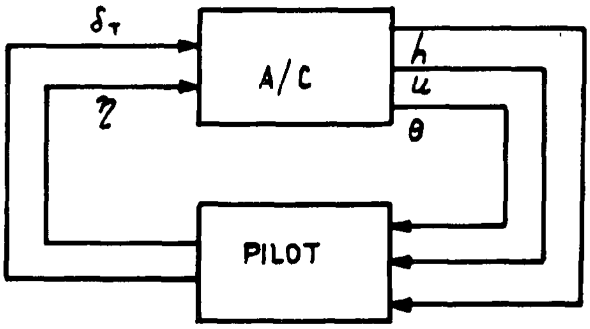
Ae. 197
FIG. 8

SYSTEM ANALYSIS - LOW STATIC. TO SHORT PERIOD GAIN RATIO WITH UNSTABLE PHUGOID.



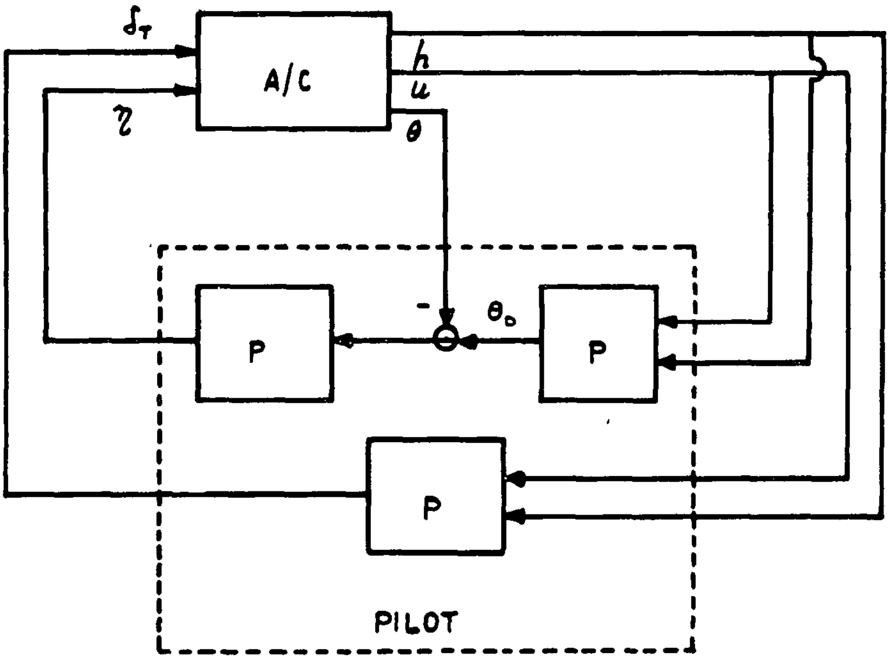
APPROACH CONTROL PROBLEM.

A



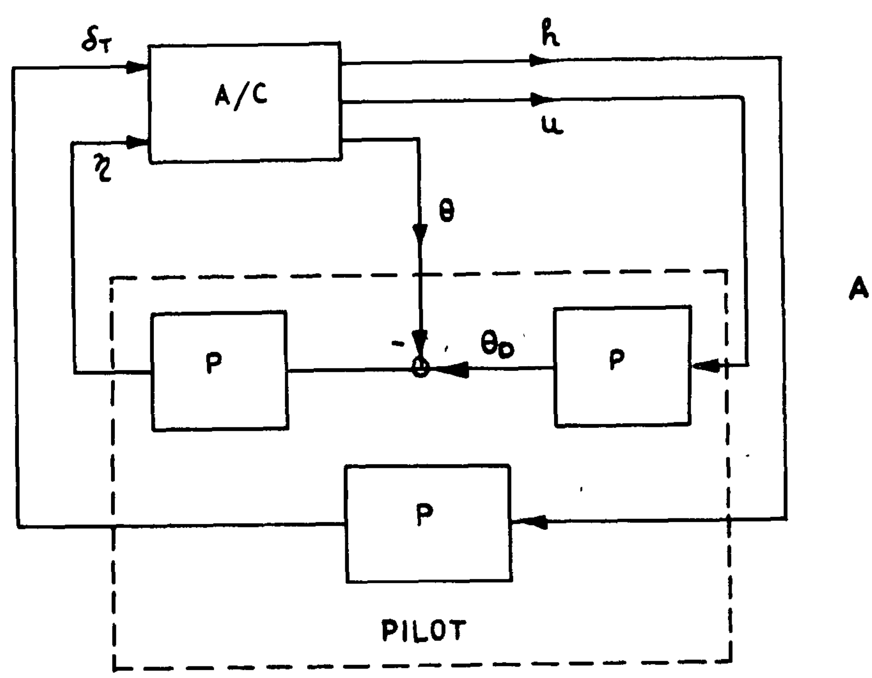
GENERAL PROBLEM

B

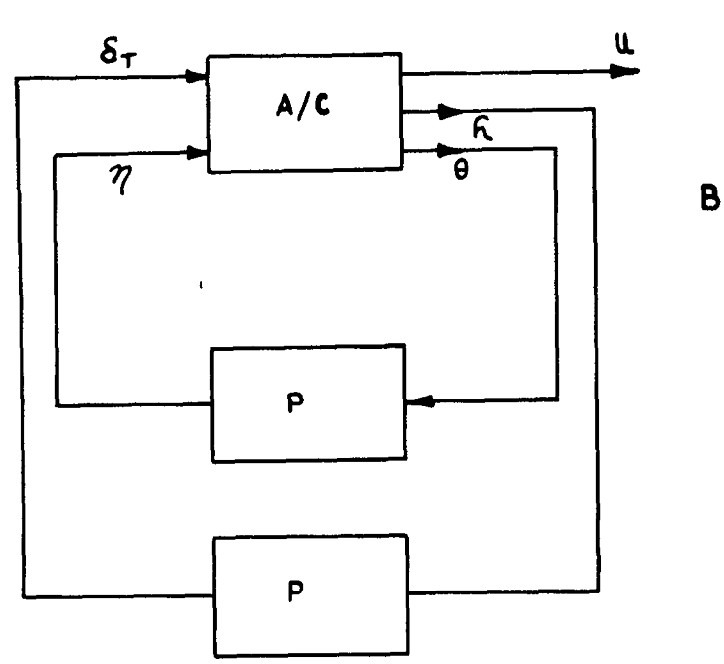


INNER LOOP ROUND Θ & η.

APPROACH CONTROL PROBLEM.



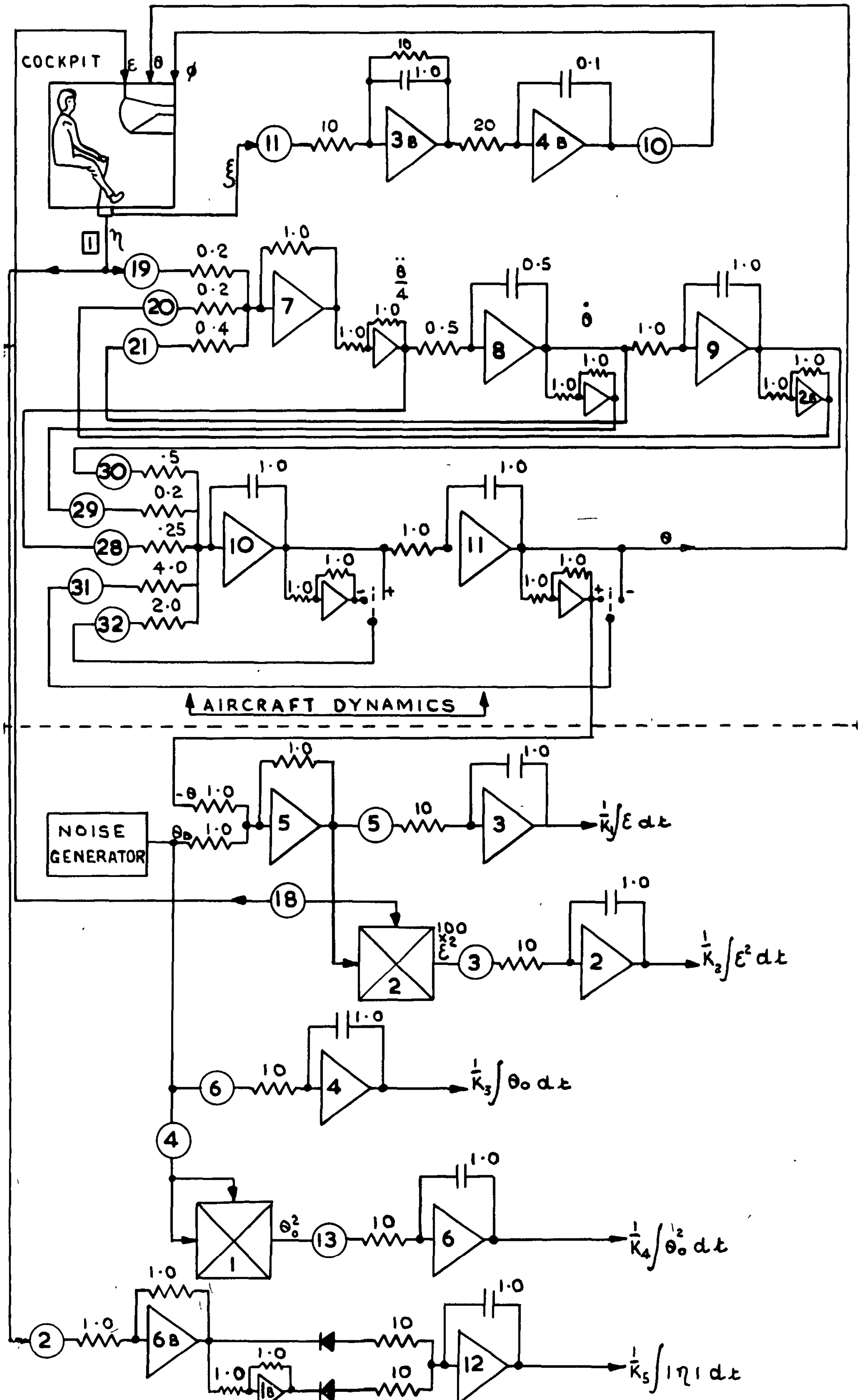
"HEIGHT WITH THROTTLE, SPEED WITH STICK" TECHNIQUE

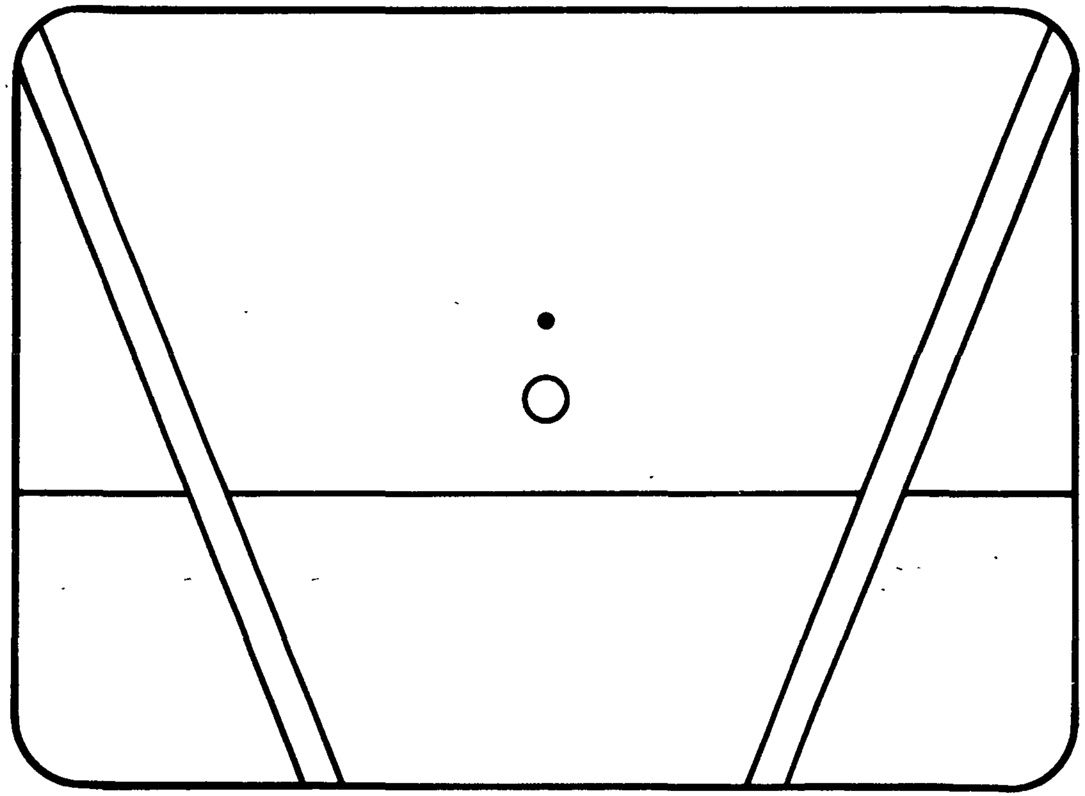


REF. 10. SPEED OPEN LOOP

FIG. 11

BLOCK DIAGRAM OF SIMULATOR



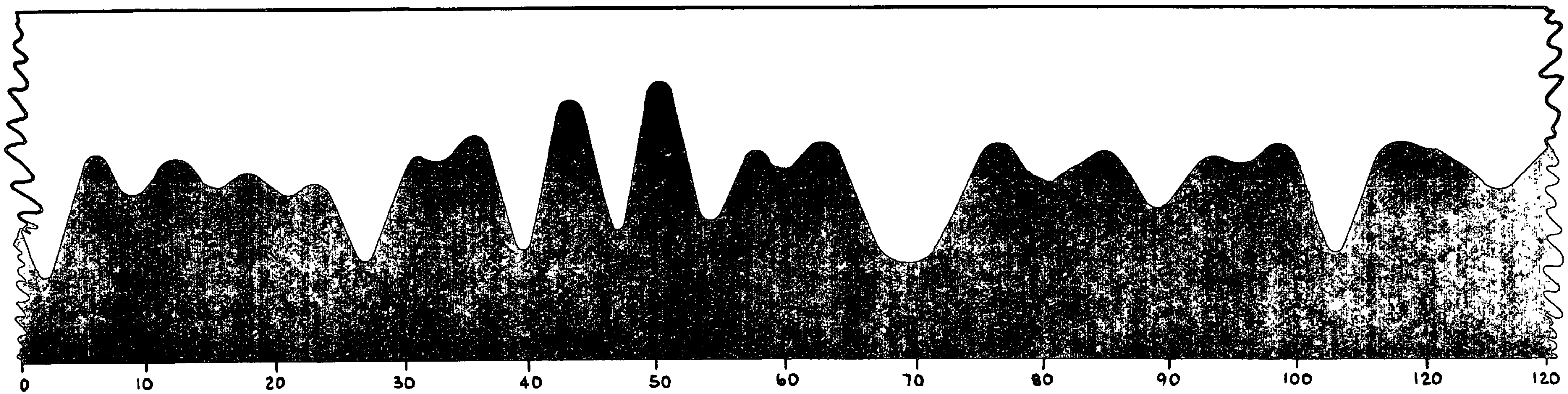


	VERTICAL DEFLECTIONS	ROTATIONS
TARGET SPOT	ε	——
AIRCRAFT CIRCLE	0	——
HORIZON LINE	$-\theta$	ϕ

SAMPLE INDICATION: NOSE DOWN ERROR. CORRECTIVE ACTION STICK BACK AIRCRAFT CLIMBING. ZERO BANK ANGLE.

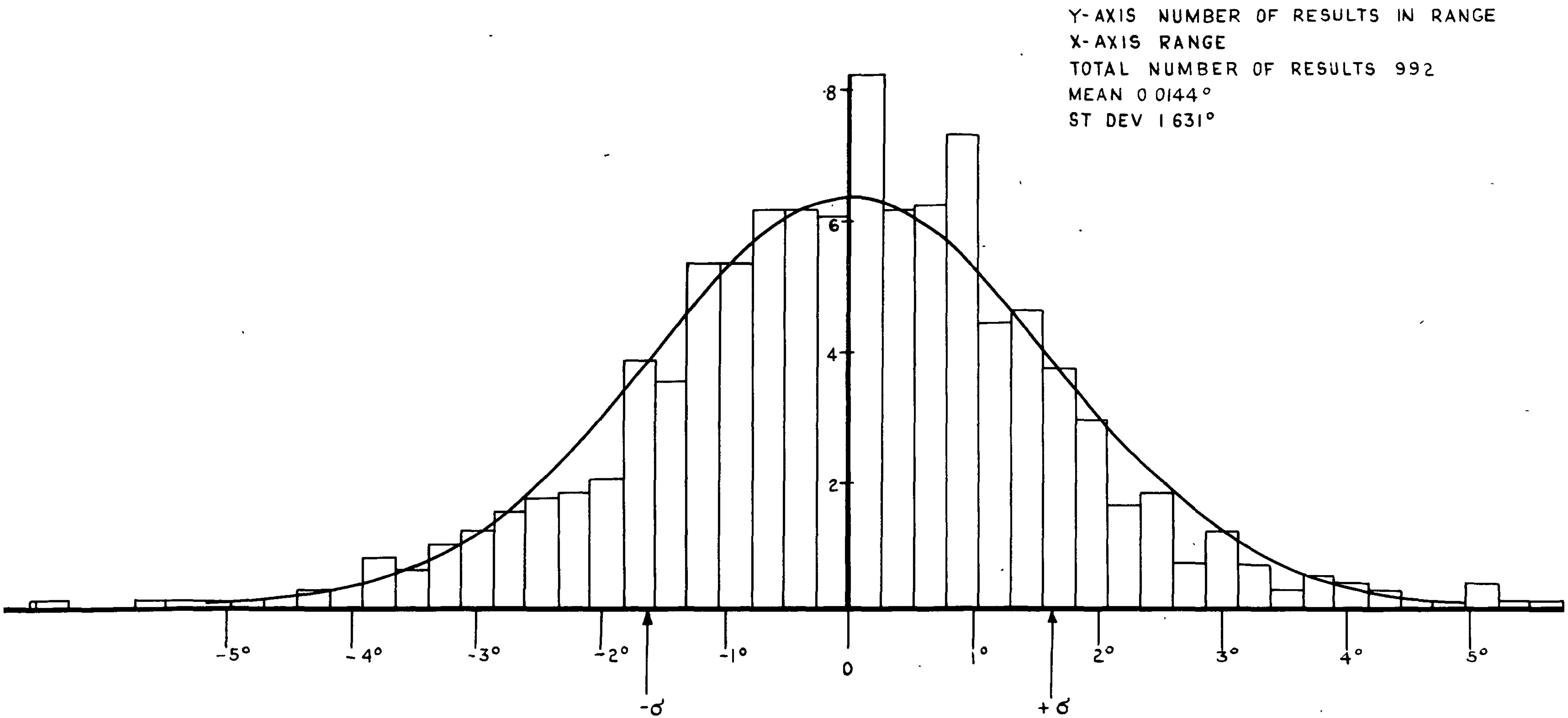
COCKPIT DISPLAY.

Ae.197
FIG.12



SAMPLE OF RANDOM NOISE FILM.

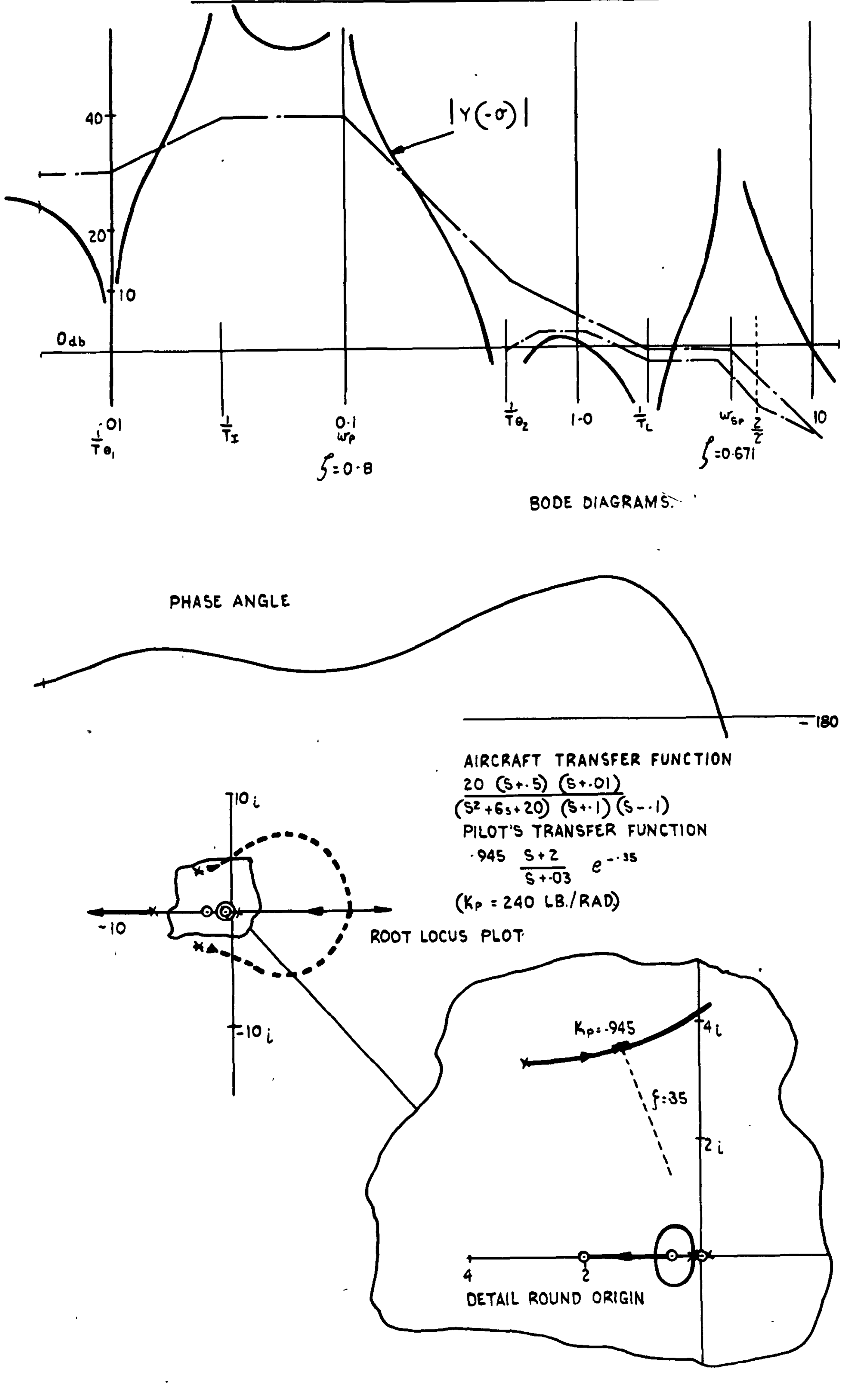
Ae. 197.
FIG. 13.



PROBABILITY DISTRIBUTION OF TRACKING INPUT.

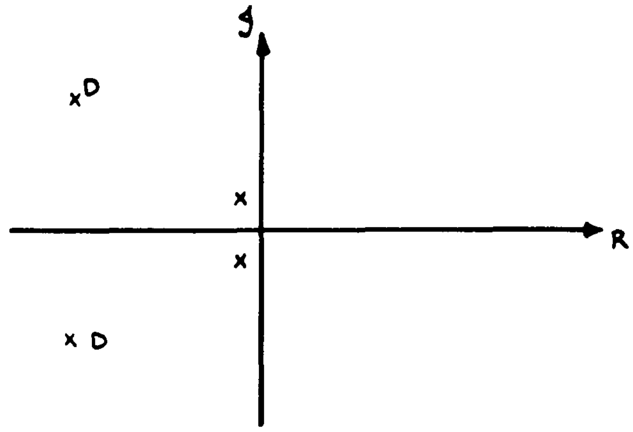
Ae 197
FIG 14

REVISED SYSTEM ANALYSIS
LOW GAIN RATIO WITH UNSTABLE PHUGOID
CONTROLLED BY LARGE AMOUNT OF LAG LEAD

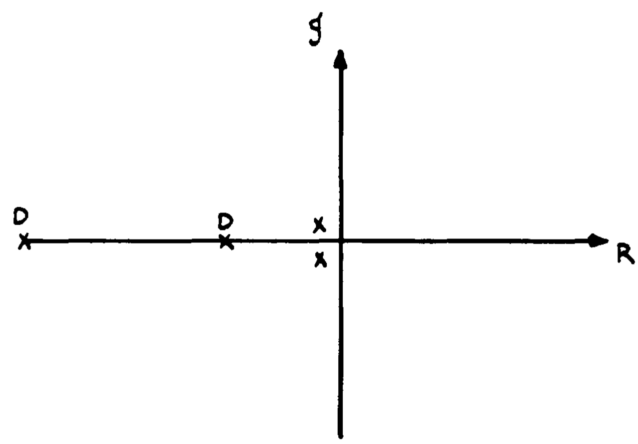


CHOICE OF DOMINANT PAIR OF ROOTS.

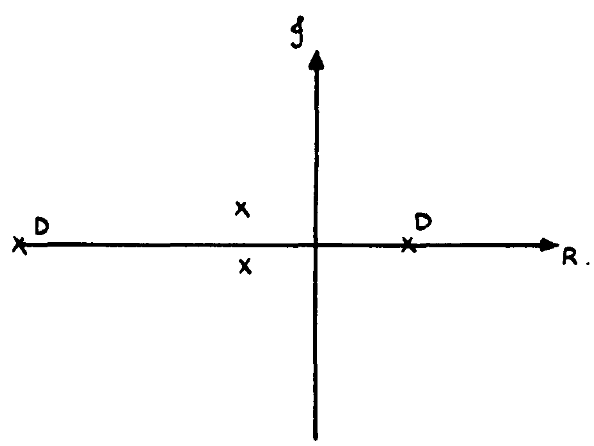
(DOMINANT ROOTS ARE INDICATED BY THE LETTER D)



A. CONVENTIONAL HIGH STABILITY CASE.



B. SHORT PERIOD ROOTS REAL



C. THIRD OSCILLATION FORMED

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April, 1964
Saunders, T. B.

HANDLING QUALITIES OF AIRCRAFT WITH
MARGINAL LONGITUDINAL STABILITY

The problem of assessing the handling qualities of aircraft with marginal or negative longitudinal stability is investigated. A mathematical model of the human pilot is used to study closed loop tracking and the results are checked by an experiment in a fixed base simulator. Some of the older handling qualities criteria such as manoeuvre margin and stick force per g are of limited use at marginal levels of stability but plots of pilot opinion contours against the stiffness and damping term of a second order system are useful provided the correct pair of dominant roots is chosen. However, where either root of the other pair has a time to double amplitude less than 3·5 sec the rating from the dominant pair may be modified by consideration of multi-loop control of pitch angle, speed and height. At present this
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can/

DETACHABLE ABSTRACT CARDS

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