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# Part III. Approximate Designs of Symmetrical Aerofoils for Specified Pressure Distributions

By

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# Approximate Two-Dimensional Aerofoil Theory
## Part III. Approximate Designs of Symmetrical Aerofoils for Specified Pressure Distributions.

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24th October, 194

We saw in Parts I[1] and II[2] that on a purely linear theory, in which all squares and products of the thickness, camber and lift coefficient are neglected, the velocity $q$ at the surface of an aerofoil in an unlimited stream $U$ is given by Approximation I, namely,

$$\frac{q}{U} = 1 + g = 1 + g_s + g_c + g_L. \qquad \ldots\ldots (1)$$

We use the same notation as in Parts I and II, so that

$$x = \tfrac{1}{2}(1 - \cos\theta), \qquad \ldots\ldots (2)$$

and $\theta$ is positive on the upper and negative on the lower surface, and is zero at the leading edge and $\pm\pi$ at the trailing edge. Then $g_s$ is an even function and $g_c + g_L$ an odd function of $\theta$, so that if we know Approximation I for $q/U$ on both surfaces we know $g_s$ and $g_c + g_L$ separately. From a knowledge of $g_s$ we can design the fairing and from a knowledge of $g_c + g_L$ the centre line; this is sufficient for many purposes.

The subject of this report is the design of the fairing when $g_s$ is known; but we may, as a foreword, make a brief reference to the more general problem of finding the aerofoil when the distribution of pressure on the surface is given. In the first place, if we know the pressure then from Bernoulli's equation we know the velocity. From the "exact" velocity distribution we have next to derive Approximation I. We may, if we wish, simply begin with a guess at the difference between the exact distribution and Approximation I, since in any case in exact work our first result for the aerofoil shape will have to be modified; but we may obtain some guidance in "guessing" by considering the exact distribution as Approximation II, namely,

$\frac{q}{U}$ /

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$$\frac{q}{U} = \frac{(1 + \frac{1}{2}C_0{}^2)|\sin\theta|}{(\psi^2 + \sin^2\theta)^{\frac{1}{2}}}(1 + g), \qquad \ldots\ldots (3)$$

where

$$C_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\psi(\theta)d\theta = \frac{1}{\pi}\int_0^{\pi}\psi_s(\theta)d\theta, \qquad \ldots\ldots (4)$$

and multiplying the given values of $q/U$ by

$$\frac{(\psi^2 + \sin^2\theta)^{\frac{1}{2}}}{(1 + \frac{1}{2}C_0{}^2)|\sin\theta|} \qquad \ldots\ldots (5)$$

to obtain the values on Approximation I. Since the aerofoil ordinates are unknown $\psi$ is, however, unknown. But $\psi^2$ and $C_0{}^2$ are small, and the main effect is restricted to the neighbourhoods of the leading and trailing edges, where $|\sin\theta|$ is small. At the leading edge

$$\psi = \psi_L = \lim_{\theta\to 0}\frac{2y}{\sin\theta} = \lim_{x\to 0}\frac{y}{x^{\frac{1}{2}}} = (2\rho_L)^{\frac{1}{2}}, \qquad \ldots\ldots (6)$$

and at the trailing edge

$$\psi = \psi_T = \lim_{\theta\to\pi}\frac{2y}{\sin\theta} = (2\rho_T)^{\frac{1}{2}}, \qquad \ldots\ldots (7)$$

where $\rho_L$, $\rho_T$ are the radii of curvature at the leading and trailing edges, respectively. If for $\psi$ we take any function which remains small and has the values $\psi_L$ and $\psi_T$ at the leading and trailing edges (e.g., a linear function), and use the factor (5), we shall get a reasonable first approximation to the values of $1 + g$. Now $(2\rho_L)^{\frac{1}{2}}$ and $(2\rho_T)^{\frac{1}{2}}$ are easily found in terms of $g_s$ from formulae given in §2, and in finding preliminary values we may take the given $q/U$, or any reasonable guess, as $1 + g$.

In this way, treating the given values of $q/U$ as Approximation II, we could, if we wished, proceed by successive approximation, using in the factor (5) the values of $\psi$ from our first attempt to provide a better value of $1 + g$, and so on. It is doubtful, however, if this will ever be worth while. When once we have determined an approximate shape for our aerofoil we find accurately (probably Approximation III will be accurate enough for most purposes) the velocity distribution at the surface of the aerofoil so found; when this is reasonably near the required distribution we correct the ordinates of the aerofoil by a theory which relates small changes in ordinates and small changes in velocity.

From now on we assume that $g_s$ is known, and proceed to show in detail how to calculate the ordinates of the fairing, its leading and trailing-edge radii of curvature, and those quantities (viz., $C_0$, $\varepsilon_s$, $\varepsilon_s'$) which are required to find the velocity distribution on Approximation III.

Certain restrictions on $g_s$ are necessary to ensure that the aerofoil boundary is a curve which does not cross itself between the leading and trailing edges; these restrictions will be briefly mentioned later.

2./

2. The Solution of the Problem in General Terms.

We saw in Part I, that, for a symmetrical aerofoil, if

$$\psi_s = \sum_{n=0}^{\infty} C_n \cos n\theta, \qquad \ldots\ldots (8)$$

and if the ordinate $y_s(x)$ expressed as a function of $\theta$ is denoted by $f(\theta)$, then

$$2y_s(x) = 2f(\theta) = (C_0 - \tfrac{1}{2}C_2)\sin\theta + \sum_{n=2}^{\infty} \tfrac{1}{2}(C_{n-1} - C_{n+1})\sin n\theta, \qquad \ldots\ldots (9)$$

$$2f'(\theta) = (C_0 - \tfrac{1}{2}C_2)\cos\theta + \sum_{n=2}^{\infty} \tfrac{1}{2}n(C_{n-1} - C_{n+1})\cos n\theta, \quad \ldots (10)$$

and

$$g_s(\theta)\sin\theta = (C_0 - \tfrac{1}{2}C_2)\sin\theta + \sum_{n=2}^{\infty} \tfrac{1}{2}n(C_{n-1} - C_{n+1})\sin n\theta. \qquad (11)$$

The Fourier series for $g_s \sin\theta$ and $2f'(\theta)$ are therefore conjugate*, and from Part I, Appendix, Lemma (6),

$$2f'(\theta) = \frac{1}{\pi} P\int_0^{\pi} \frac{g_s(t)\sin^2 t}{\cos\theta - \cos t}\,dt, \qquad \ldots\ldots (12)$$

where, as usual, P denotes that the principal value of the integral is to be taken.

For the radii of curvature at the leading and trailing edges we have, from (6) and (7), the following equations

$$(2\rho_L)^{\frac{1}{2}} = \lim_{\theta\to 0} \frac{2f(\theta)}{\sin\theta} = 2f'(0) = \frac{1}{\pi}\int_0^{\pi} g_s(t)(1 + \cos t)\,dt, \qquad (13)$$

$$(2\rho_T)^{\frac{1}{2}} = \lim_{\theta\to \pi} \frac{2f(\theta)}{\sin\theta} = -2f'(\pi) = \frac{1}{\pi}\int_0^{\pi} g_s(t)(1 - \cos t)\,dt. \qquad (14)$$

Equations (13) and (14) are the equations we require for $\rho_L$ and $\rho_T$. Equation (12) gives us the derivative with respect to $\theta$ of the aerofoil ordinate; if, with the chord divided into any number of segments, in each segment $g_s$ is given by a suitable algebraic formula, then the necessary integrals may be evaluated in terms of simple functions by methods used in Parts I and II. Moreover, the improper integral in (12) may be changed into a proper integral by the use of Lemma (7) of Part I (Appendix), namely

$$P\int_0^{\pi} \frac{dt}{\cos\theta - \cos t} = 0. \qquad \ldots\ldots (15)$$

Thus/

*The relation between the Fourier series for $g_s \sin\theta$ and $2f'(\theta)$ has also been noted by Squire[3] who proposes from a knowledge of $g_s$ to find the series in (11), and hence to deduce and sum the series in (9).

Thus

$$2f'(\theta) = \frac{1}{\pi}\int_0^\pi \frac{g_s(t)\sin^2 t - g_s(\theta)\sin^2\theta}{\cos\theta - \cos t}\,dt, \quad \ldots\ldots (16)$$

and the value of this integral may be found by numerical integration. Analytical methods should, however, be used wherever possible. In any case it is the value of $f(\theta)$, not of $f'(\theta)$, that is required, and it is generally easier to find $f(\theta)$ directly as follows.

Denote temporarily by $F(\theta)$ the sum of the Fourier series conjugate to that for $2f(\theta)$ in (9), i.e., let

$$F(\theta) = (C_0 - \tfrac{1}{2}C_2)\cos\theta + \sum_{n=2}^{\infty} \tfrac{1}{2}(C_{n-1} - C_{n+1})\cos n\theta \quad \ldots\ldots (17)$$

Also let*

$$G(\theta) = \int_0^\theta g_s(\theta)\sin\theta\, d\theta. \quad \ldots\ldots (18)$$

Then

$$F'(\theta) = -g_s(\theta)\sin\theta \quad \text{and} \quad \int_0^\pi F(\theta)d\theta = 0. \quad \ldots\ldots (19)$$

Hence

$$F(\theta) = \frac{1}{\pi}\int_0^\pi G(\theta)d\theta - G(\theta). \quad \ldots\ldots (20)$$

From Part I, Appendix, Lemma (6),

$$2f(\theta) = -\frac{\sin\theta}{\pi} P\int_0^\pi \frac{F(t)}{\cos\theta - \cos t}\,dt, \quad \ldots\ldots (21)$$

and from (15) it then follows that

$$y_s(x) = f(\theta) = \frac{\sin\theta}{2\pi} P\int_0^\pi \frac{G(t)}{\cos\theta - \cos t}\,dt. \quad \ldots\ldots (22)$$

Equation (22), with $G(\theta)$ defined in (18), is the equation we shall use to find the aerofoil ordinates. By the use of (15) the improper integral in (22) may be changed into a proper integral, namely

$$y_s(x) = f(\theta) = \frac{\sin\theta}{2\pi}\int_0^\pi \frac{G(t) - G(\theta)}{\cos\theta - \cos t}\,dt, \quad \ldots\ldots (23)$$

and numerical integration used, but wherever possible analytical methods should be employed to evaluate the integral in (22); a method of analytical evaluation will be considered in §3.

The aerofoil contour must not cross itself, and $y_s(x)$ must be positive between the leading and trailing edges. Since $y_s$ vanishes at the leading and trailing edges, $f'(0)$ must be positive and $f'(\pi)$ negative; in other words the values of $(2\rho_L)^{\frac{1}{2}}$ and $(2\rho_T)^{\frac{1}{2}}$, given by/

*Note that, in terms of x, $G = 2\int_0^x g_s\,dx$.

by (13) and (14), must be positive, with the proviso that for a sharp trailing edge $\rho_T = 0$. (If $g_s$ is finite at the trailing edge, then a sharp trailing edge will be a cusp, since, as we saw in Part I, if there is not a logarithmic infinity in $g_s$ at the trailing edge then the term in $1 - x$ in the expansion of $y_s$ in powers of $(1 - x)^{\frac{1}{2}}$ is zero*). For a normal aerofoil shape $f'(\theta)$ is positive over the front of the aerofoil, zero at the position of maximum thickness, and negative over the rear. The two most important restrictions in practice are, however,

$$\left.\begin{aligned} (2\rho_L)^{\frac{1}{2}} &= \frac{1}{\pi}\int_0^\pi g_s(t)(1 + \cos t)\,dt > 0, \\ (2\rho_T)^{\frac{1}{2}} &= \frac{1}{\pi}\int_0^\pi g_s(t)(1 - \cos t)\,dt \geqslant 0 \end{aligned}\right\} \qquad \ldots\ldots (24)$$

Having obtained formulae for the aerofoil ordinates and for $\rho_L$ and $\rho_T$, we proceed to find formulae for $C_0$, $\varepsilon_s$, $\varepsilon_s'$, which are required in calculating the velocity distribution, on Approximation III, of the aerofoil obtained. We also require $\psi_s(\theta)$, which is found from the aerofoil ordinates by the equation

$$\psi_s(\theta) = 2y_s \operatorname{cosec} \theta. \qquad \ldots\ldots (25)$$

$C_0$, $\varepsilon_s$, $\varepsilon_s'$, however, are more easily found from the values of $g_s$ than from those of $y_s$ or $\psi_s$. From Part I, equation (13), we see that

$$\varepsilon_s'(\theta) + \varepsilon_s(\theta)\cot\theta + C_0 = g_s(\theta). \qquad \ldots\ldots (26)$$

Hence

$$\frac{d}{d\theta}(\varepsilon_s \sin\theta) = g_s \sin\theta - C_0 \sin\theta \qquad \ldots\ldots (27)$$

$\varepsilon_s \sin\theta = 0$ at $\theta = 0$, so

$$\varepsilon_s \sin\theta = G(\theta) - C_0(1 - \cos\theta) \qquad \ldots\ldots (28)$$

and since $\varepsilon_s \sin\theta = 0$ at $\theta = \pi$,

$$C_0 = \tfrac{1}{2}G(\pi). \qquad \ldots\ldots (29)$$

Also/

*With a cusp $\psi_s = O(\pi - \theta)^2$ near the trailing edge, and

$$\lim_{\theta\to\pi} \frac{\sin\theta}{(\psi^2 + \sin^2\theta)^{\frac{1}{2}}} = 1.$$

Hence at a cusped trailing edge, on Approximation III

$$q/U = e^{C_0}(1 + \varepsilon_s')^2$$

for a symmetrical aerofoil when $C_L = 0$; when $C_L \neq 0$, $q/U \to -\infty$ at the trailing edge if $a_0 < 2\pi e^{C_0}$; for $a_0 = 2\pi e^{C_0}$,

$$q/U = (1 - C_L^2/a_0^2)^{\frac{1}{2}} e^{C_0}(1 + \varepsilon_s')^2.$$

Also

$$
\begin{aligned}
\varepsilon_s &= G(\theta)\,\mathrm{cosec}\,\theta - C_0 \tan\tfrac{1}{2}\theta \\
&= \tan\tfrac{1}{2}\theta \left\{ \frac{G(\theta)}{1-\cos\theta} - C_0 \right\} \\
&= \cot\tfrac{1}{2}\theta \left\{ C_0 - \frac{G(\pi) - G(\theta)}{1+\cos\theta} \right\} \qquad \ldots\ldots (30)
\end{aligned}
$$

From the definition of $G(\theta)$ in (18) we see that if $g_s$ is finite at the leading and trailing edges, $G(\theta)$ has a double zero at the leading edge and $G(\theta) - G(\pi)$ has a double zero at the trailing edge; the last two expressions in (30) therefore show that $\varepsilon_s$ is zero at the leading and trailing edges respectively.

Finally we find that

$$
\begin{aligned}
\varepsilon_s'(\theta) &= g_s(\theta) - \frac{G(\theta)\cos\theta}{\sin^2\theta} - \frac{C_0}{1+\cos\theta} = g_s(\theta) \\
&\qquad - \frac{1}{1+\cos\theta}\left[\frac{G(\theta)\cos\theta}{1-\cos\theta} + C_0\right] \\
&= g_s(\theta) + \frac{[G(\pi) - G(\theta)]\cos\theta}{\sin^2\theta} - \frac{C_0}{1-\cos\theta}. \\
&= g_s(\theta) - \frac{1}{1-\cos\theta}\left[C_0 - \frac{\{G(\pi) - G(\theta)\}\cos\theta}{1+\cos\theta}\right] \ldots\ldots (31)
\end{aligned}
$$

From (18)

$$
\lim_{\theta\to 0} \frac{G(\theta)}{\sin^2\theta} = \frac{g_s(0)}{2}, \quad \lim_{\theta\to\pi} \frac{G(\pi) - G(\theta)}{\sin^2\theta} = \frac{g_s(\pi)}{2}; \qquad \ldots\ldots (32)
$$

hence

$$
\varepsilon_s'(0) = \tfrac{1}{2}\{g_s(0) - C_0\}, \quad \varepsilon_s'(\pi) = \tfrac{1}{2}\{g_s(\pi) - C_0\}. \ \ldots\ldots (33)
$$

We have now found all the formulae required. $y_s$ is given by (22), $\rho_L$ and $\rho_T$ by (13) and (14), the most important restrictions on $g_s$ by (24), $C_0$ by (29), $\varepsilon_s$ by (30), $\varepsilon_s'$ by (31) and its values at $\theta = 0$ and $\theta = \pi$ by (33).

## 3. General Foundation for Special Cases.

In carrying out the analysis for special cases it is more convenient, in general, to work in terms of $\theta$, as in Part II, than in terms of $x$ as in Part I.

To consider evaluations for special cases it is probably sufficient to suppose that, with the chord divided into any number of segments, in each segment $g_s$ is a polynomial* in $x$. Then

as/

*The analysis may be carried out in other cases, for example, if the expressions for $g_s$ contain odd powers of $x^{\frac{1}{2}}$ or $(1-x)^{\frac{1}{2}}$, but it is doubtful if such results will be required.

as we saw in Part II, by expressing powers of $x$ in the manner shown in Lemma 3 of Part II (Appendix), the interval $(0,\pi)$ for $\theta$ may be divided into any number $s$ of intervals, $(0,\theta_1)$, $(\theta_1, \theta_2)$, $(\theta_2, \theta_3)$, ..., $(\theta_{r-1}, \theta_r)$, ... $(\theta_{s-1}, \pi)$ in any one of which, say $(\theta_{r-1}, \theta_r)$, $g_s$ may be represented by an expression of the form

$$g_s = \sum_{n=0}^{m_r} a_{nr} \cos n\theta \quad (\theta_{r-1} < \theta < \theta_r). \qquad \ldots\ldots(34)$$

Thus the expressions for $f(\theta)$ etc., will each be the sum of a number of contributions, of which a typical one is $a_{nr}$ multiplied by the contribution from a term $\cos n\theta$ in the expression for $g_s$ in the interval $(\theta_{r-1}, \theta_r)$. These contributions must be summed for $n$ from $n = 0$ to $n = m_r$, and then the results for all the intervals must be summed. We therefore proceed to find the contributions from a term $\cos n\theta$ in the expression for $g_s$ in the interval $(\theta_{r-1}, \theta_r)$.

The contribution to $G(\theta)$ is as follows

$$\left.\begin{aligned} G(\theta) &= 0 \quad \text{for} \quad 0 < \theta \leqslant \theta_{r-1} \\ &= \int_{\theta_{r-1}}^{\theta} \cos n\theta \sin\theta \, d\theta \quad \text{for} \quad \theta_{r-1} < \theta < \theta_r \\ &= \int_{\theta_{r-1}}^{\theta_r} \cos n\theta \sin\theta \, d\theta \quad \text{for} \quad \theta_r < \theta < \pi \end{aligned}\right\} \quad \ldots (35)$$

Hence, <u>for $n = 0$</u>, the contribution to

$$\left.\begin{aligned} G(\theta) &= 0 \quad \text{for} \quad 0 < \theta < \theta_{r-1} \\ &= \cos\theta_{r-1} - \cos\theta \quad \text{for} \quad \theta_{r-1} < \theta < \theta_r \\ &= \cos\theta_{r-1} - \cos\theta_r \quad \text{for} \quad \theta_r < \theta < \pi; \end{aligned}\right\} \quad \ldots. (36)$$

for <u>$n = 1$</u>, the contribution to

$$\left.\begin{aligned} G(\theta) &= 0 \quad \text{for} \quad 0 < \theta < \theta_{r-1} \\ &= \tfrac{1}{4}(\cos 2\theta_{r-1} - \cos 2\theta) \quad \text{for} \quad \theta_{r-1} < \theta < \theta_r \\ &= \tfrac{1}{4}(\cos 2\theta_{r-1} - \cos 2\theta_r) \quad \text{for} \quad \theta_{r-1} < \theta < \pi, \end{aligned}\right\} \quad .. (37)$$

and for <u>$n \geqslant 2$</u>, the contribution to

G(θ)/

$$G(\theta) = 0 \quad \text{for} \quad 0 < \theta < \theta_{r-1}$$

$$= \frac{1}{2(n+1)}\{\cos(n+1)\theta_{r-1} - \cos(n+1)\theta\} - \frac{1}{2(n-1)}\{\cos(n-1)\theta_{r-1} - \cos(n-1)\theta\}$$

$$\text{for} \quad \theta_{r-1} < \theta < \theta_r,$$

$$= \frac{1}{2(n+1)}\{\cos(n+1)\theta_{r-1} - \cos(n+1)\theta_r\} - \frac{1}{2(n-1)}\{\cos(n-1)\theta_{r-1} - \cos(n-1)\theta_r\}$$

$$\text{for} \quad \theta_r < \theta < \pi. \qquad \ldots\ldots (38)$$

Now write

$$I_n(\theta_r) = P\int_0^{\theta_r} \frac{\cos nt}{\cos\theta - \cos t}\,dt. \qquad \ldots\ldots (39)$$

Then $I_0(\pi) = 0$, and general expressions for $I_n(\theta_r)\sin\theta$ were given in Lemma 4 of Part II (Appendix).

It now follows from (22) that <u>for $n = 0$</u> the contribution to

$$2\pi y_s = -\cos\theta_{r-1} I_0(\theta_{r-1})\sin\theta + \cos\theta_r I_0(\theta_r)\sin\theta$$
$$- [I_1(\theta_r)\sin\theta - I_1(\theta_{r-1})\sin\theta]$$
$$= (\cos\theta - \cos\theta_{r-1})\log_e\frac{\sin\frac{1}{2}|\theta - \theta_{r-1}|}{\sin\frac{1}{2}(\theta + \theta_{r-1})}$$
$$- (\cos\theta - \cos\theta_r)\log_e\frac{\sin\frac{1}{2}|\theta - \theta_r|}{\sin\frac{1}{2}(\theta + \theta_r)}$$
$$+ (\theta_r - \theta_{r-1})\sin\theta; \qquad \ldots\ldots (40)$$

<u>for $n = 1$</u> the contribution to

$$2\pi y_s = -\tfrac{1}{4}\cos 2\theta_{r-1} I_0(\theta_{r-1})\sin\theta + \tfrac{1}{4}\cos 2\theta_r I_0(\theta_r)\sin\theta$$
$$- \tfrac{1}{4}[I_2(\theta_r)\sin\theta - I_2(\theta_{r-1})\sin\theta]$$
$$= \tfrac{1}{4}(\cos 2\theta - \cos 2\theta_{r-1})\log_e\frac{\sin\frac{1}{2}|\theta - \theta_{r-1}|}{\sin\frac{1}{2}(\theta + \theta_{r-1})}$$
$$- \tfrac{1}{4}(\cos 2\theta - \cos 2\theta_r)\log_e\frac{\sin\frac{1}{2}|\theta - \theta_r|}{\sin\frac{1}{2}(\theta + \theta_r)}$$
$$+ \tfrac{1}{2}(\sin\theta_r - \sin\theta_{r-1})\sin\theta + \tfrac{1}{4}(\theta_r - \theta_{r-1})\sin 2\theta;$$
$$\ldots\ldots (41)$$

and/

and for $n > 2$ the contribution to

$$2\pi y_s = -\tfrac{1}{2}\left\{\frac{\cos(n+1)\theta_{r-1}}{n+1} - \frac{\cos(n-1)\theta_{r-1}}{n-1}\right\} I_0(\theta_{r-1}) \sin\theta$$

$$+ \tfrac{1}{2}\left\{\frac{\cos(n+1)\theta_r}{n+1} - \frac{\cos(n-1)\theta_r}{n-1}\right\} I_0(\theta_r) \sin\theta$$

$$- \frac{1}{2(n+1)}\left[I_{n+1}(\theta_r)\sin\theta - I_{n+1}(\theta_{r-1})\sin\theta\right]$$

$$+ \frac{1}{2(n-1)}\left[I_{n-1}(\theta_r)\sin\theta - I_{n-1}(\theta_{r-1})\sin\theta\right]$$

$$= \left\{\frac{\cos(n+1)\theta - \cos(n+1)\theta_{r-1}}{2(n+1)} - \frac{\cos(n-1)\theta - \cos(n-1)\theta_{r-1}}{2(n-1)}\right\}\log_e \frac{\sin\frac{1}{2}|\theta-\theta_{r-1}|}{\sin\frac{1}{2}(\theta+\theta_{r-1})}$$

$$- \left\{\frac{\cos(n+1)\theta - \cos(n+1)\theta_r}{2(n+1)} - \frac{\cos(n-1)\theta - \cos(n-1)\theta_r}{2(n-1)}\right\}\log_e \frac{\sin\frac{1}{2}|\theta-\theta_r|}{\sin\frac{1}{2}(\theta+\theta_r)}$$

$$+ \frac{\theta_r - \theta_{r-1}}{2(n+1)}\sin(n+1)\theta - \frac{\theta_r - \theta_{r-1}}{2(n-1)}\sin(n-1)\theta$$

$$+ \frac{1}{n+1}\sum_{s=0}^{n-1}\frac{\sin(n-s)\theta_r - \sin(n-s)\theta_{r-1}}{n-s}\sin(s+1)\theta$$

$$- \frac{1}{n-1}\sum_{s=0}^{n-3}\frac{\sin(n-s-2)\theta_r - \sin(n-s-2)\theta_{r-1}}{n-s-2}\sin(s+1)\theta, \qquad \ldots\ldots (42)$$

where for $n = 2$ the last line in (42) is to be taken as zero.

For the purpose of computing $y_s$ from (40), (41) and (42) we may note here that

$$\frac{\sin\frac{1}{2}|\theta-\theta_r|}{\sin\frac{1}{2}(\theta+\theta_r)} = \left|\frac{1 - \cos\theta_r\cos\theta - \sin\theta_r\sin\theta}{\cos\theta_r - \cos\theta}\right|. \qquad \ldots\ldots (43)$$

From (13) and (14) it follows that the contribution to

$$(2\rho_L)^{\frac{1}{2}} = \frac{1}{\pi}\int_{\theta_{r-1}}^{\theta_r} \cos nt(1+\cos t)dt. \qquad \ldots\ldots (44)$$

and/

and the contribution to

$$2(\rho_T)^{\frac{1}{2}} = \frac{1}{\pi}\int_{\theta_{r-1}}^{\theta_r} \cos nt(1 - \cos t)dt. \qquad \ldots\ldots (45)$$

Hence the contribution to

$$\begin{aligned}(2\rho_L)^{\frac{1}{2}} &= \frac{1}{\pi}(\theta_r - \theta_{r-1} + \sin\theta_r - \sin\theta_{r-1}) \quad \text{for } n = 0 \\ &= \frac{1}{\pi}\left(\sin\theta_r - \sin\theta_{r-1} + \frac{\theta_r - \theta_{r-1}}{2} + \frac{\sin 2\theta_r - \sin 2\theta_{r-1}}{4}\right) \quad \text{for } n = 1 \\ &= \frac{1}{\pi}\left\{\frac{\sin n\theta_r - \sin n\theta_{r-1}}{n} + \frac{\sin(n-1)\theta_r - \sin(n-1)\theta_{r-1}}{2(n-1)}\right. \\ &\qquad \left. + \frac{\sin(n+1)\theta_r - \sin(n+1)\theta_{r-1}}{2(n+1)}\right\} \quad \text{for } n \geqslant 2,\end{aligned} \qquad \ldots\ldots (46)$$

and the contribution to

$$\begin{aligned}(2\rho_T)^{\frac{1}{2}} &= \frac{1}{\pi}(\theta_r - \theta_{r-1} - \sin\theta_r + \sin\theta_{r-1}) \quad \text{for } n = 0 \\ &= \frac{1}{\pi}\left(\sin\theta_r - \sin\theta_{r-1} - \frac{\theta_r - \theta_{r-1}}{2} - \frac{\sin 2\theta_r - \sin 2\theta_{r-1}}{4}\right) \quad \text{for } n = 1 \\ &= \frac{1}{\pi}\left\{\frac{\sin n\theta_r - \sin n\theta_{r-1}}{n} - \frac{\sin(n-1)\theta_r - \sin(n-1)\theta_{r-1}}{2(n-1)}\right. \\ &\qquad \left. - \frac{\sin(n+1)\theta_r - \sin(n+1)\theta_{r-1}}{2(n+1)}\right\} \quad \text{for } n \geqslant 2.\end{aligned} \qquad \ldots\ldots (47)$$

From (29) and (36), (37), (38) it follows that the contribution to

$$\begin{aligned}C_0 &= \tfrac{1}{2}(\cos\theta_{r-1} - \cos\theta_r) \quad \text{for } n = 0 \\ &= \tfrac{1}{8}(\cos 2\theta_{r-1} - \cos 2\theta_r) \quad \text{for } n = 1 \\ &= \frac{1}{4(n+1)}\{\cos(n+1)\theta_{r-1} - \cos(n+1)\theta_r\} \\ &\quad - \frac{1}{4(n-1)}\{\cos(n-1)\theta_{r-1} - \cos(n-1)\theta_r\} \quad \text{for } n \geqslant 2.\end{aligned} \qquad (48)$$

The contribution to $\varepsilon_s$ is then given by (30) with the contributions to $G(\theta)$ and $C_0$ written out above (equations (36), (37), (38) and (48)); similarly the contributions to $\varepsilon_s'(\theta)$, $\varepsilon_s'(0)$, $\varepsilon_s'(\pi)$ are given by (31) and (33), with the contribution to $G(\pi)$ equal to twice the contribution to $C_0$ and the contribution to

$g_s(\theta)$/

$$\left.\begin{aligned} g_s(\theta) &= 0 && \text{for} \quad 0 \leqslant \theta \leqslant \theta_{r-1} \\ &= \cos n\theta && \text{for} \quad \theta_{r-1} \leqslant \theta \leqslant \theta_r \\ &= 0 && \text{for} \quad \theta_r \leqslant \theta \leqslant \pi. \end{aligned}\right\} \quad \ldots\ldots (49)$$

4. Approximate Velocity Distribution Represented by a Quadratic in Each of Three Segments.

It appears to be sufficiently general for all immediate purposes to consider the calculations when, with the chord divided into three segments, in each segment $g_s$ is a quadratic in $x$. As special cases $g_s$ may be quadratic in each of two segments into which the chord is divided, or linear in each of two or three segments.

If $g_s$ is quadratic in $x$ in each of the three segments $0 \leqslant x \leqslant X_1$, $X_1 \leqslant x \leqslant X_2$, $X_2 \leqslant x \leqslant 1$, then if we write

$$\theta_1 = 2 \sin^{-1} X_1^{\frac{1}{2}}, \qquad \theta_2 = 2 \sin^{-1} X_2^{\frac{1}{2}}, \qquad \ldots\ldots (50)$$

and in the expressions for $g_s$ substitute

$$x = \tfrac{1}{2}(1 - \cos\theta), \qquad x^2 = \tfrac{1}{8}(3 - 4\cos\theta + \cos 2\theta), \quad \ldots\ldots (51)$$

we find $g_s$ in the following form

$$\left.\begin{aligned} g_s &= a_0 + a_1 \cos\theta + a_2 \cos 2\theta && \text{for} \quad 0 \leqslant \theta \leqslant \theta_1 \\ &= b_0 + b_1 \cos\theta + b_2 \cos 2\theta && \text{for} \quad \theta_1 \leqslant \theta \leqslant \theta_2 \\ &= c_0 + c_1 \cos\theta + c_2 \cos 2\theta && \text{for} \quad \theta_2 \leqslant \theta \leqslant \pi. \end{aligned}\right\} \quad \ldots\ldots (52)$$

In the formulae of the preceding section we first put $\theta_{r-1} = 0$, $\theta_r = \theta_1$, and taking $n = 0, 1, 2$ in order multiply the results by $a_0$, $a_1$, $a_2$ respectively; we then take $\theta_{r-1} = \theta_1$ and $\theta_r = \theta_2$ and with $n = 0, 1, 2$ in order multiply by $b_0$, $b_1$, $b_2$ respectively; and finally with $\theta_{r-1} = \theta_2$ and $\theta_r = \pi$ and $n = 0, 1, 2$ in order we multiply by $c_0$, $c_1$, $c_2$ respectively and add together all nine contributions. In this way we find the following results, in which we have written

$$a_r - b_r = k_r, \qquad b_r - c_r = \ell_r \quad (r = 0, 1, 2). \quad \ldots\ldots (53)$$

$2\pi y_s$/

$$2\pi y_s = -\frac{1}{12}\{2k_2(\cos 3\theta - \cos 3\theta_1) + 3k_1(\cos 2\theta - \cos 2\theta_1) + 6(2k_0 - k_2)(\cos\theta - \cos\theta_1)\}$$

$$\times \log_e \frac{\sin\frac{1}{2}|\theta - \theta_1|}{\sin\frac{1}{2}(\theta + \theta_1)}$$

$$-\frac{1}{12}\{2\ell_2(\cos 3\theta - \cos 3\theta_2) + 3\ell_1(\cos 2\theta - \cos 2\theta_2) + 6(2\ell_0 - \ell_2)(\cos\theta - \cos\theta_2)\}$$

$$\times \log_e \frac{\sin\frac{1}{2}|\theta - \theta_2|}{\sin\frac{1}{2}(\theta + \theta_2)}$$

$$+\frac{1}{6}\{3(2c_0 - c_2)\pi + 3(2k_0 - k_2)\theta_1 + 3(2\ell_0 - \ell_2)\theta_2 + 3k_1\sin\theta_1 + 3\ell_1\sin\theta_2$$

$$+ k_2\sin 2\theta_1 + \ell_2\sin 2\theta_2\}\sin\theta$$

$$+\frac{1}{12}\{3c_1\pi + 3k_1\theta_1 + 3\ell_1\theta_2 + 4k_2\sin\theta_1 + 4\ell_2\sin\theta_2\}\sin 2\theta$$

$$+\frac{1}{6}\{c_2\pi + k_2\theta_1 + \ell_2\theta_2\}\sin 3\theta; \qquad \ldots\ldots (54)$$

$$(2\rho_L)^{\frac{1}{2}} = \tfrac{1}{2}(2c_0 + c_1) + \frac{\theta_1}{2\pi}(2k_0 + k_1) + \frac{\theta_2}{2\pi}(2\ell_0 + \ell_1) + \frac{\sin\theta_1}{2\pi}(2k_0 + 2k_1 + k_2)$$

$$+ \frac{\sin\theta_2}{2\pi}(2\ell_0 + 2\ell_1 + \ell_2) + \frac{\sin 2\theta_1}{4\pi}(k_1 + 2k_2) + \frac{\sin 2\theta_2}{4\pi}(\ell_1 + 2\ell_2)$$

$$+ \frac{k_2\sin 3\theta_1}{6\pi} + \frac{\ell_2\sin 3\theta_2}{6\pi}; \qquad \ldots\ldots (55)$$

$$(2\rho_T)^{\frac{1}{2}} = \tfrac{1}{2}(2c_0 - c_1) + \frac{\theta_1}{2\pi}(2k_0 - k_1) + \frac{\theta_2}{2\pi}(2\ell_0 - \ell_1) - \frac{\sin\theta_1}{2\pi}(2k_0 - 2k_1 + k_2)$$

$$- \frac{\sin\theta_2}{2\pi}(2\ell_0 - 2\ell_1 + \ell_2) - \frac{\sin 2\theta_1}{4\pi}(k_1 - 2k_2) - \frac{\sin 2\theta_2}{4\pi}(\ell_1 - 2\ell_2)$$

$$- \frac{k_2\sin 3\theta_1}{6\pi} - \frac{\ell_2\sin 3\theta_2}{6\pi}; \qquad \ldots\ldots (56)$$

$$C_0 = \frac{1}{24}(12a_0 + 3a_1 - 4a_2 + 12c_0 - 3c_1 - 4c_2) - \frac{1}{4}(2k_0 - k_2)\cos\theta_1$$

$$- \frac{1}{4}(2\ell_0 - \ell_2)\cos\theta_2 - \frac{1}{8}k_1\cos 2\theta_1 - \frac{1}{8}\ell_1\cos 2\theta_2$$

$$- \frac{1}{12}k_2\cos 3\theta_1 - \frac{1}{12}\ell_2\cos 3\theta_2. \qquad \ldots\ldots (57)$$

Also/

Also

$$
\left.
\begin{aligned}
G(\theta) &= a_0 + \tfrac{1}{4}a_1 - \tfrac{1}{3}a_2 - (a_0 - \tfrac{1}{3}a_2)\cos\theta - \tfrac{1}{4}a_1 \cos 2\theta - \tfrac{1}{6}a_2 \cos 3\theta \\
&\qquad\qquad \text{for } 0 \leqslant \theta \leqslant \theta_1 \\
&= a_0 + \tfrac{1}{4}a_1 - \tfrac{1}{3}a_2 - (k_0 - \tfrac{1}{3}k_2)\cos\theta_1 - \tfrac{1}{4}k_1 \cos 2\theta_1 - \tfrac{1}{6}k_2 \cos 3\theta_1 \\
&\quad - (b_0 - \tfrac{1}{3}b_2)\cos\theta - \tfrac{1}{4}b_1 \cos 2\theta - \tfrac{1}{6}b_2 \cos 3\theta \\
&\qquad\qquad \text{for } \theta_1 < \theta < \theta_2 \\
&= a_0 + \tfrac{1}{4}a_1 - \tfrac{1}{3}a_2 - (k_0 - \tfrac{1}{3}k_2)\cos\theta_1 - \tfrac{1}{4}k_1 \cos 2\theta_1 - \tfrac{1}{6}k_2 \cos 3\theta_1 \\
&\quad - (\ell_0 - \tfrac{1}{3}\ell_2)\cos\theta_2 - \tfrac{1}{4}\ell_1 \cos 2\theta_2 - \tfrac{1}{6}\ell_2 \cos 3\theta_2 \\
&\quad - (c_0 - \tfrac{1}{3}c_2)\cos\theta - \tfrac{1}{4}c_1 \cos 2\theta - \tfrac{1}{6}c_2 \cos 3\theta \\
&\qquad\qquad \text{for } \theta_2 \leqslant \theta \leqslant \pi.
\end{aligned}
\right\}
\quad \ldots\ldots (58)
$$

Hence from (30) we find, with the help of Lemma 6 of Part II (Appendix) that

$$
\left.
\begin{aligned}
\varepsilon_s &= \tan\tfrac{1}{2}\theta\,\{a_0 + \tfrac{1}{2}a_1 - C_0 + \tfrac{1}{6}(3a_1 + 4a_2)\cos\theta + \tfrac{1}{3}a_2 \cos 2\theta\} \\
&\qquad\qquad \text{for } 0 \leqslant \theta \leqslant \theta_1 \\
&= \operatorname{cosec}\theta\,\{a_0 + \tfrac{1}{4}a_1 - \tfrac{1}{3}a_2 - (k_0 - \tfrac{1}{3}k_2)\cos\theta_1 - \tfrac{1}{4}k_1 \cos 2\theta_1 \\
&\qquad - \tfrac{1}{6}k_2 \cos 3\theta_1 - (b_0 - \tfrac{1}{3}b_2)\cos\theta - \tfrac{1}{4}b_1 \cos 2\theta \\
&\qquad - \tfrac{1}{6}b_2 \cos 3\theta\} - C_0 \tan\tfrac{1}{2}\theta \qquad \text{for } \theta_1 < \theta < \theta_2 \\
&= \cot\tfrac{1}{2}\theta\,\{C_0 - c_0 + \tfrac{1}{2}c_1 - \tfrac{1}{6}(3c_1 - 4c_2)\cos\theta - \tfrac{1}{3}c_2 \cos 2\theta\} \\
&\qquad\qquad \text{for } \theta_2 < \theta \leqslant \pi,
\end{aligned}
\right\}
\quad \ldots\ldots (59)
$$

and similarly from (31)

$$
\begin{aligned}
\varepsilon_s'(\theta) &= \frac{a_1}{2}\cos\theta + \frac{2}{3}a_2 \cos 2\theta + \frac{1}{1+\cos\theta}\left(a_0 - \frac{a_2}{3} - C_0\right) \\
&\qquad\qquad \text{for } 0 \leqslant \theta \leqslant \theta_1 \\
&= b_0 + b_1 \cos\theta + b_2 \cos 2\theta - \frac{C_0}{1+\cos\theta} \\
&\quad - \frac{\cos\theta}{\sin^2\theta}\{a_0 + \tfrac{1}{4}a_1 - \tfrac{1}{3}a_2 - (k_0 - \tfrac{1}{3}k_2)\cos\theta_1 - \tfrac{1}{4}k_1 \cos 2\theta_1 \\
&\quad - \tfrac{1}{6}k_2 \cos 3\theta_1 - (b_0 - \tfrac{1}{3}b_2)\cos\theta - \tfrac{1}{4}b_1 \cos 2\theta - \tfrac{1}{6}b_2 \cos 3\theta\} \\
&\qquad\qquad \text{for } \theta_1 < \theta < \theta_2 \\
&= \frac{c_1}{2}\cos\theta + \frac{2}{3}c_2 \cos 2\theta + \frac{1}{1-\cos\theta}\left(c_0 - \frac{c_2}{3} - C_0\right) \\
&\qquad\qquad \text{for } \theta_2 < \theta \leqslant \pi.
\end{aligned}
$$

Formulae appropriate to the case in which $g_s$ is quadratic in each of two segments into which the chord is divided are obtainable immediately from the above formulae by putting $\theta_1 = \theta_2$, $b_r = c_r$ and $\ell_r = 0$ for $r = 0, 1, 2$.

A different simplification, in which the number of parameters in the expression for $g_s$ is considerably reduced, is obtained by taking $g$ linear in each of three segments; this simplification will be considered in the next section.

## 5. Approximate Velocity Distribution Linear in Each of Three Segments.

If $g_s$ is linear in each of the three segments considered in §4 and is equal to $a$, $b$, $c$, $d$ at $\theta = 0, \theta_1, \theta_2, \pi$ respectively, then, in the notation of §4,

$$\left.\begin{aligned}
a_0 &= \frac{b - a\cos\theta_1}{1 - \cos\theta_1}, & a_1 &= -\frac{b - a}{1 - \cos\theta_1}, & a_2 &= 0, \\
b_0 &= \frac{c\cos\theta_1 - b\cos\theta_2}{\cos\theta_1 - \cos\theta_2}, & b_1 &= \frac{b - c}{\cos\theta_1 - \cos\theta_2}, & b_2 &= 0, \\
c_0 &= \frac{\cos\theta_2}{1 + \cos\theta_2}, & c_1 &= \frac{c - d}{1 + \cos\theta_2}, & c_2 &= 0.
\end{aligned}\right\} \quad \ldots\ldots (61)$$

Hence

$$\left.\begin{aligned}
k_2 = 0,\ k_1 &= -\frac{b-c+(c-a)\cos\theta_1-(b-a)\cos\theta_2}{(1-\cos\theta_1)(\cos\theta_1-\cos\theta_2)}, & k_0 &= -k_1\cos\theta_1 \\
\ell_2 = 0,\ \ell_1 &= \frac{b-c-(c-d)\cos\theta_1+(b-d)\cos\theta_2}{(1+\cos\theta_2)(\cos\theta_1-\cos\theta_2)}, & \ell_c &= -\ell_1\cos\theta_2
\end{aligned}\right\} \quad (62)$$

By substituting these values into the formulae of §4 we find that

$y_s$/

$$y_s = \frac{1}{4\pi}\,\frac{b - c + (c - a)\cos\theta_1 - (b - a)\cos\theta_2}{(1 - \cos\theta_1)(\cos\theta_1 - \cos\theta_2)}$$

$$\times \left\{(\cos\theta - \cos\theta_1)^2 \log_e \frac{\sin\frac{1}{2}|\theta - \theta_1|}{\sin\frac{1}{2}(\theta + \theta_1)}\right.$$

$$\left. - (\sin\theta_1 - 2\theta_1\cos\theta_1)\sin\theta - \tfrac{1}{2}\theta_1 \sin 2\theta\right\}$$

$$- \frac{1}{4\pi}\,\frac{b - c - (c - d)\cos\theta_1 + (b - d)\cos\theta_2}{(1 + \cos\theta_2)(\cos\theta_1 - \cos\theta_2)}$$

$$\times \left\{(\cos\theta - \cos\theta_2)^2 \log_e \frac{\sin\frac{1}{2}|\theta - \theta_2|}{\sin\frac{1}{2}(\theta + \theta_2)}\right.$$

$$\left. - (\sin\theta_2 - 2\theta_2\cos\theta_2)\sin\theta - \tfrac{1}{2}\theta_2 \sin 2\theta\right\}$$

$$+ \frac{c + d\cos\theta_2}{2(1 + \cos\theta_2)}\sin\theta + \frac{c - d}{8(1 + \cos\theta_2)}\sin 2\theta; \quad (63)$$

$$(2\rho_U)^{\frac{1}{2}} = \frac{3c - d(1 - 2\cos\theta_2)}{2(1 + \cos\theta_2)}$$

$$- \frac{1}{2\pi}\,\frac{b - c + (c - a)\cos\theta_1 - (b - a)\cos\theta_2}{(1 - \cos\theta_1)(\cos\theta_1 - \cos\theta_2)}$$

$$\times \{\theta_1(1 - 2\cos\theta_1) + 2\sin\theta_1 - \tfrac{1}{2}\sin 2\theta_1\}$$

$$+ \frac{1}{2\pi}\,\frac{b - c - (c - d)\cos\theta_1 + (b - d)\cos\theta_2}{(1 + \cos\theta_2)(\cos\theta_1 - \cos\theta_2)}$$

$$\times \{\theta_2(1 - 2\cos\theta_2) + 2\sin\theta_2 - \tfrac{1}{2}\sin 2\theta_2\}; \quad \ldots\ldots \quad (64)$$

and

$$(2\rho_T)^{\frac{1}{2}} = \frac{c + d(1 + 2\cos\theta_2)}{2(1 + \cos\theta_2)}$$

$$+ \frac{1}{2\pi}\,\frac{b - c + (c - a)\cos\theta_1 - (b - a)\cos\theta_2}{(1 - \cos\theta_1)(\cos\theta_1 - \cos\theta_2)}$$

$$\times \{\theta_1(1 + 2\cos\theta_1) - 2\sin\theta_1 - \tfrac{1}{2}\sin 2\theta_1\}$$

$$- \frac{1}{2\pi}\,\frac{b - c - (c - d)\cos\theta_1 + (b - d)\cos\theta_2}{(1 + \cos\theta_2)(\cos\theta_1 - \cos\theta_2)}$$

$$\times \{\theta_2(1 + 2\cos\theta_2) - 2\sin\theta_2 - \tfrac{1}{2}\sin 2\theta_2\}. \quad (65)$$

Also/

Also*

$$C_0 = \tfrac{1}{4}\{a + b + c + d + (c - a)\cos\theta_1 - (b - d)\cos\theta_2\}, \quad \ldots (66)$$

$$\left.\begin{aligned}
\varepsilon_s &= \tan\tfrac{1}{2}\theta\left\{\frac{a(1 - 2\cos\theta_1) + b}{2(1 - \cos\theta_1)} - C_0 - \frac{b - a}{2(1 - \cos\theta_1)}\cos\theta\right\} \\
&\qquad\qquad \text{for } 0 \leqslant \theta \leqslant \theta_1 \\
&= \operatorname{cosec}\theta\left\{\tfrac{1}{2}(a+b)(1-\cos\theta_1) + b(\cos\theta_1-\cos\theta)\right. \\
&\qquad \left. - \frac{b - c}{2(\cos\theta_1-\cos\theta_2)}(\cos\theta_1-\cos\theta)^2\right\} - C_0\tan\tfrac{1}{2}\theta \\
&\qquad\qquad \text{for } \theta_1 \leqslant \theta \leqslant \theta_2 \\
&= \cot\tfrac{1}{2}\theta\left\{C_0 - \frac{c+d(1+2\cos\theta_2)}{2(1+\cos\theta_2)} - \frac{c - d}{2(1+\cos\theta_2)}\cos\theta\right\} \\
&\qquad\qquad \text{for } \theta_2 \leqslant \theta \leqslant \pi,
\end{aligned}\right\} \quad (67)$$

and

$$\left.\begin{aligned}
\varepsilon_s'(\theta) &= -\frac{1}{1 + \cos\theta}\left\{\frac{b - a\cos\theta_1}{1 - \cos\theta_1} - C_0\right\} - \frac{b - a}{2(1 - \cos\theta_1)}\cos\theta \\
&\qquad\qquad \text{for } 0 \leqslant \theta \leqslant \theta_1 \\
&= \frac{b(\cos\theta - \cos\theta_2) - c(\cos\theta - \cos\theta_1)}{\cos\theta_1 - \cos\theta_2} - \frac{C_0}{1 + \cos\theta} \\
&\quad - \frac{\cos\theta}{\sin^2\theta}\left\{\frac{1}{2}(a+b)(1-\cos\theta_1)+b(\cos\theta_1-\cos\theta)\right. \\
&\quad \left. - \frac{b - c}{2(\cos\theta_1-\cos\theta_2)}(\cos\theta_1 - \cos\theta)^2\right\} \text{ for } \theta_1 \leqslant \theta \leqslant \theta_2 \\
&= \frac{1}{1 - \cos\theta}\left\{\frac{c + d\cos\theta_2}{1 + \cos\theta_2} - C_0\right\} + \frac{c - d}{2(1 + \cos\theta_2)}\cos\theta \\
&\qquad\qquad \text{for } \theta_2 \leqslant \theta \leqslant \pi.
\end{aligned}\right\} \quad (68)$$

6. Approximate Velocity Distribution Linear in Each of Two Segments.

A further considerable simplification is obtained by supposing that $g_s$ is linear in each of two segments into which the/

---

*$C_0$ is most easily found by using the formula

$$C_0 = \tfrac{1}{2}C(\pi) = \int_0^1 g_s\,dx.$$

(See footnote on p. 4.)

the chord is divided, the corresponding intervals for $x$ being, say $(0, X_1)$ and $(X_1, 1)$, where

$$X_1 = \tfrac{1}{2}(1 - \cos \theta_1) = \sin^2 \tfrac{1}{2}\theta_1. \qquad \ldots\ldots (69)$$

The graph of $g_s$ against $x$ is then a straight line from $x = 0$ to $x = X_1$, and a second straight line from $x = X_1$ to $x = 1$. We denote the values of $g_s$ at $x = 0, X_1, 1$ by $a$, $b$, $c$ respectively. The results may then be obtained from those of the preceding section by first putting $d = c$ and then letting $\theta_2 \to \pi$; they may also be obtained independently, or as a special case of two quadratics. The results now depend only on four parameters, namely, $a$, $b$, $c$ and $X_1$ or $\theta_1$, and are linear in $a$, $b$ and $c$; it will be found that they may be expressed in the following forms:

$$y_s = af_0 + bf_1 + cf_2, \qquad \ldots\ldots (70)$$

where

$$f_0 = -\frac{(\cos\theta - \cos\theta_1)^2}{4\pi(1 - \cos\theta_1)} \log_e \frac{\sin\frac{1}{2}|\theta - \theta_1|}{\sin\frac{1}{2}(\theta + \theta_1)} + \frac{\sin\theta_1 - 2\theta_1\cos\theta_1}{4\pi(1 - \cos\theta_1)} \sin\theta + \frac{\theta_1}{8\pi(1 - \cos\theta_1)} \sin 2\theta, \quad \ldots (71)$$

$$f_1 = \frac{(\cos\theta - \cos\theta_1)^2}{2\pi\sin^2\theta_1} \log_e \frac{\sin\frac{1}{2}|\theta - \theta_1|}{\sin\frac{1}{2}(\theta + \theta_1)} + \left[\frac{1}{2(1 + \cos\theta_1)} - \frac{\sin\theta_1 - 2\theta_1\cos\theta_1}{2\pi\sin^2\theta_1}\right] \sin\theta + \left[\frac{1}{8(1 + \cos\theta_1)} - \frac{\theta_1}{4\pi\sin^2\theta_1}\right] \sin 2\theta, \qquad \ldots\ldots (72)$$

$$f_2 = -\frac{(\cos\theta - \cos\theta_1)^2}{4\pi(1 + \cos\theta_1)} \log_e \frac{\sin\frac{1}{2}|\theta - \theta_1|}{\sin\frac{1}{2}(\theta + \theta_1)} + \frac{\sin\theta_1 + 2(\pi - \theta_1)\cos\theta_1}{4\pi(1 + \cos\theta_1)} \sin\theta - \frac{\pi - \theta_1}{8\pi(1 + \cos\theta_1)} \sin 2\theta; \qquad \ldots\ldots (73)$$

also/

also

$$(2\rho_L)^{\frac{1}{2}} = \frac{a}{2\pi(1-\cos\theta_1)}\{2\sin\theta_1 - 2\theta_1\cos\theta_1 + \theta_1 - \sin\theta_1\cos\theta_1\}$$

$$+ \frac{b}{1+\cos\theta_1}\left\{\frac{3}{2} - \frac{1}{\pi(1-\cos\theta_1)}(2\sin\theta_1 - 2\theta_1\cos\theta_1 + \theta_1 - \sin\theta_1\cos\theta_1)\right\}$$

$$+ \frac{c}{1+\cos\theta_1}\left\{\frac{1}{2\pi}(2\sin\theta_1 - 2\theta_1\cos\theta_1 + \theta_1 - \sin\theta_1\cos\theta_1) - \tfrac{1}{2} + \cos\theta_1\right\}; \quad \ldots\ldots (74)$$

$$(2\rho_T)^{\frac{1}{2}} = \frac{a}{2\pi(1-\cos\theta_1)}\{2\sin\theta_1 - 2\theta_1\cos\theta_1 - \theta_1 + \sin\theta_1\cos\theta_1\}$$

$$+ \frac{b}{1+\cos\theta_1}\left\{\frac{1}{2} - \frac{1}{\pi(1-\cos\theta_1)}(2\sin\theta_1 - 2\theta_1\cos\theta_1 - \theta_1 + \sin\theta_1\cos\theta_1)\right\}$$

$$+ \frac{c}{1+\cos\theta_1}\left\{\frac{1}{2} + \cos\theta_1 + \frac{1}{2\pi}(2\sin\theta_1 - 2\theta_1\cos\theta_1 - \theta_1 + \sin\theta_1\cos\theta_1)\right\}, \quad \ldots (75)$$

$$C_0 = a\frac{1-\cos\theta_1}{4} + \frac{b}{2} + c\frac{1+\cos\theta_1}{4}, \quad \ldots\ldots (76)$$

$$\left.\begin{aligned} \varepsilon_s &= \tan\tfrac{1}{2}\theta\left\{\frac{(b-a)(\cos\theta_1-\cos\theta)}{2(1-\cos\theta_1)} + \frac{a-c}{4}(1+\cos\theta_1)\right\} \\ &\qquad\qquad \text{for } 0 \leqslant \theta \leqslant \theta_1 \\ &= \cot\tfrac{1}{2}\theta\left\{\frac{(b-c)(\cos\theta_1-\cos\theta)}{2(1+\cos\theta_1)} + \frac{a-c}{4}(1-\cos\theta_1)\right\} \\ &\qquad\qquad \text{for } \theta_1 \leqslant \theta \leqslant \pi, \end{aligned}\right\} (77)$$

and/

and

$$\left.\begin{aligned}
\varepsilon_s'(\theta) &= \frac{1+\cos\theta_1}{1+\cos\theta}\left\{\frac{b}{2(1-\cos\theta_1)} - \frac{a(1+\cos\theta_1)}{4(1-\cos\theta_1)} - \frac{c}{4}\right\} \\
&\quad - \frac{b-a}{2(1-\cos\theta_1)}\cos\theta \qquad \text{for } 0 \leqslant \theta \leqslant \theta_1 \\
&= \frac{1-\cos\theta_1}{1-\cos\theta}\left\{\frac{b}{2(1+\cos\theta_1)} - \frac{a}{4} - \frac{c(1-\cos\theta_1)}{4(1+\cos\theta_1)}\right\} \\
&\quad + \frac{b-c}{2(1+\cos\theta_1)}\cos\theta \qquad \text{for } \theta_1 \leqslant \theta \leqslant \pi.
\end{aligned}\right\} \qquad (78)$$

7. <u>Numerical Examples with the Approximate Velocity Distribution Linear in Each of Two Segments.</u>

When the ordinates of an aerofoil are to be calculated it will usually be most convenient if they can be calculated at exact values of $x$; hence in Table 1 we have tabulated for a moderately small number of exact values of $x$ powers of $x$ and values of $\theta$ and certain functions of $\theta$ which will be found useful in such computations. This table may also be regarded as a supplement to Table 1 of Part I, which, as an aid to computing the velocity at the surface, gave values of certain functions at integral multiples of $20\theta/\pi$.

In Table 2(a) the values of the functions $f_0$ $f_1$, $f_2$ of the preceding section (see equations (71), (72) and (73)) are tabulated for $X_1 = 0{\cdot}5$, $0{\cdot}6$ and $0{\cdot}7$ for the same values of $x$ as in Table 1. From tables of $f_0$, $f_1$, $f_2$ the ordinate $y$ is very easily computed for any values of $a$, $b$ and $c$ from equation (70). For careful construction values at a greater number of stations along the chord will be required than can be obtained from Tables 1 and 2(a); for $X_1 = 0{\cdot}6$ tables of $f_0$, $f_1$, $f_2$ are given for a greater number of stations in Table 2(b).

In examples we always take $b > a$ and $b > c$, so on the simple linear theory $1 + b$ is the maximum value of $q/U$ on the aerofoil surface, and $x = X_1$ is the position of that maximum. For fixed values of $X_1$, $a/b$ and $c/b$ the thickness and all ordinates of the aerofoil are directly proportional to b. Over the forward part of the aerofoil the slope of the velocity graph against $x$ on the linear theory is $U(b-a)/X_1$ for a chord of unit length, i.e.,

$$s = \frac{U}{c}\,\frac{b-a}{X_1} \qquad \ldots\ldots (79)$$

for a chord of length $c$.

The resulting aerofoil has a sharp trailing edge if we put $\rho_T = 0$, which for specified values of $X_1$, $a$ and $b$ (i.e., a specified maximum value of $q/U$ on the surface, a

specified/

specified position of that maximum, and a specified slope over the forward portion of the aerofoil) gives a definite value of c. As we saw in §2, such a sharp trailing edge is always a cusp.

For illustrative purposes we have computed a short list of aerofoil ordinates for eight aerofoils, with $b = 0{\cdot}2$ for all of them, with $X_1 = 0{\cdot}5$ for four of them and $X_1 = 0{\cdot}6$ for the other four, and with various values of a and c. For each aerofoil we have also computed the velocity at the surface at zero incidence according to Approximation III. Particulars of the aerofoils are given in Tables 3(a) to 3(h). At the head of each table we give the values of $X_1$, a, b, c and $(b - a)/X_1$, together with the values of $\rho_L$ and $\rho_T$, $C$ and $e^{C_0}$. Aerofoil C (Table 3(c)) has a cusp at the trailing edge; aerofoil D (Table 3(d) has a very small radius of curvature at the trailing edge, which is therefore almost a cusp. In each table we give the ordinates in the second column and the values at zero incidence of $q/U$ at the surface on Approximation III in the last column; in the third, fourth and fifth columns we also give values of $\psi_s$, $\varepsilon_s$, $\varepsilon_s'$ which are used in finding $q/U$ on Approximation III and which will be required also in finding $q/U$ for cambered aerofoils which have these thickness distributions (see Part II).

The results for the eight symmetrical aerofoils are illustrated in Figs. 1(a) – 1(h). The aerofoil designations are the same as in the tables; in each figure the aerofoil shape is shown below, the assumed velocity distribution on the linear theory is shown dotted and the full curves give the velocity distribution on Approximation III, these velocity distributions being for zero incidence.

We conclude by setting out numerical formulae for $X_1 = 0{\cdot}5$, $0{\cdot}6$ and $0{\cdot}7$.

For $\underline{X_1 = 0{\cdot}5}$, $\cos\theta_1 = 0$, $\theta_1 = \tfrac{1}{2}\pi$, $\sin\theta_1 = 1$,

$$f_0 = -\frac{1}{4\pi}\cos^2\theta\,\log_e\frac{1-\sin\theta}{|\cos\theta|} + \frac{1}{4\pi}\sin\theta + \frac{1}{16}\sin 2\theta,$$

$$f_1 = \frac{1}{2\pi}\cos^2\theta\,\log_e\frac{1-\sin\theta}{|\cos\theta|} + \left(\frac{1}{2} - \frac{1}{2\pi}\right)\sin\theta,$$

$$f_2 = -\frac{1}{4\pi}\cos^2\theta\,\log_e\frac{1-\sin\theta}{|\cos\theta|} + \frac{1}{4\pi}\sin\theta - \frac{1}{16}\sin 2\theta,$$

$$(2\rho_L)^{\frac{1}{2}} = a\left(\frac{1}{\pi} + \frac{1}{4}\right) + b\left(1 - \frac{2}{\pi}\right) + c\left(\frac{1}{\pi} - \frac{1}{4}\right),$$

$$(2\rho_T)^{\frac{1}{2}} = a\left(\frac{1}{\pi} - \frac{1}{4}\right) + b\left(1 - \frac{2}{\pi}\right) + c\left(\frac{1}{\pi} + \frac{1}{4}\right),$$

$C_0$/

$$\left.\begin{aligned} C_0 &= \tfrac{1}{4}a + \tfrac{1}{2}b + \tfrac{1}{4}c, \\ \varepsilon &= \tan\tfrac{1}{2}\theta\{\tfrac{1}{4}(a-c) - \tfrac{1}{2}(b-a)\cos\theta\} \qquad \text{for } 0 < \theta < \tfrac{1}{2}\pi \\ &= \cot\tfrac{1}{2}\theta\{\tfrac{1}{4}(a-c) - \tfrac{1}{2}(b-c)\cos\theta\} \qquad \text{for } \tfrac{1}{2}\pi \leqslant \theta \leqslant \pi \end{aligned}\right\}.$$

and

$$\left.\begin{aligned} \varepsilon' &= \frac{1}{1+\cos\theta}(\tfrac{1}{2}b - \tfrac{1}{4}a - \tfrac{1}{4}c) - \tfrac{1}{2}(b-a)\cos\theta \\ &\qquad\qquad \text{for } 0 < \theta < \tfrac{1}{2}\pi \\ &= \frac{1}{1-\cos\theta}(\tfrac{1}{2}b - \tfrac{1}{4}a - \tfrac{1}{4}c) + \tfrac{1}{2}(b-c)\cos\theta \\ &\qquad\qquad \text{for } \tfrac{1}{2}\pi < \theta < \pi. \end{aligned}\right\}$$

For $\underline{X_1 = 0{\cdot}6}$, $\cos\theta_1 = -0{\cdot}2$, $\theta_1 = 1{\cdot}77215425$ radians, $\sin\theta_1 = 0{\cdot}97979590$,

$$f_0 = -\frac{0{\cdot}208\dot{3}}{\pi}(0{\cdot}2+\cos\theta)^2 \log_e\left|\frac{1 + 0{\cdot}2\cos\theta - 0{\cdot}97979590\sin\theta}{0{\cdot}2+\cos\theta}\right|$$

$$+ 0{\cdot}11198258_5 \sin\theta + 0{\cdot}05875981\sin 2\theta,$$

$$f_1 = \frac{0{\cdot}5208\dot{3}}{\pi}(0{\cdot}2+\cos\theta)^2 \log_e\left|\frac{1 + 0{\cdot}2\cos\theta - 0{\cdot}97979590\sin\theta}{0{\cdot}2+\cos\theta}\right|$$

$$+ 0{\cdot}34504354\sin\theta + 0{\cdot}00935046\sin 2\theta,$$

$$f_2 = -\frac{0{\cdot}3125}{\pi}(0{\cdot}2+\cos\theta)^2 \log_e\left|\frac{1 + 0{\cdot}2\cos\theta - 0{\cdot}97979590\sin\theta}{0{\cdot}2+\cos\theta}\right|$$

$$+ 0{\cdot}04297388\sin\theta - 0{\cdot}06811028\sin 2\theta,$$

$$(2\rho_L)^{\frac{1}{2}} = 0{\cdot}61494379a + 0{\cdot}33764053b + 0{\cdot}04741568c,$$

$$(2\rho_T)^{\frac{1}{2}} = 0{\cdot}09288549a + 0{\cdot}39278628b + 0{\cdot}51432823c,$$

$$C_0 = 0{\cdot}3a + 0{\cdot}5b + 0{\cdot}2c,$$

$$\left.\begin{aligned} \varepsilon &= \tan\tfrac{1}{2}\theta\{0{\cdot}28\dot{3}a - 0{\cdot}08\dot{3}b - 0{\cdot}2c - 0{\cdot}41\dot{6}(b-a)\cos\theta\} \\ &\qquad\qquad \text{for } 0 < x < 0{\cdot}6 \\ &= \cot\tfrac{1}{2}\theta\{0{\cdot}3a - 0{\cdot}125b - 0{\cdot}175c - 0{\cdot}625(b-c)\cos\theta\} \\ &\qquad\qquad \text{for } 0{\cdot}6 < x < 1, \end{aligned}\right\}$$

and/

and

$$\left.\begin{aligned} \varepsilon' &= \frac{1}{1 + \cos\theta}\{0{\cdot}\dot{3}b - 0{\cdot}1\dot{3}a - 0{\cdot}2c\} - 0{\cdot}41\dot{6}(b - a)\cos\theta \\ &\qquad\qquad\qquad\qquad \text{for} \quad 0 \leqslant x \leqslant 0{\cdot}6 \\ &= \frac{1}{1 - \cos\theta}\{0{\cdot}75b - 0{\cdot}3a - 0{\cdot}45c\} + 0{\cdot}625(b - c)\cos\theta \\ &\qquad\qquad\qquad\qquad \text{for} \quad 0{\cdot}6 \leqslant x \leqslant 1. \end{aligned}\right\}$$

For $\underline{X_1 = 0{\cdot}7}$, $\cos\theta_1 = -0{\cdot}4$, $\theta_1 = 1{\cdot}98231317$,
$\sin\theta_1 = 0{\cdot}91651514$,

$$f_0 = -\frac{0{\cdot}17\dot{8}5714\dot{2}}{\pi}(0{\cdot}4 + \cos\theta)^2 \log_e\left|\frac{1 + 0{\cdot}4\cos\theta - 0{\cdot}91651514\sin\theta}{0{\cdot}4 + \cos\theta}\right|$$

$$+ 0{\cdot}14223710\sin\theta + 0{\cdot}05633838\sin 2\theta,$$

$$f_1 = \frac{0{\cdot}5\dot{9}52380\dot{9}}{\pi}(0{\cdot}4 + \cos\theta)^2 \log_e\left|\frac{1 + 0{\cdot}4\cos\theta - 0{\cdot}91651514\sin\theta}{0{\cdot}4 + \cos\theta}\right|$$

$$+ 0{\cdot}35920968\sin\theta + 0{\cdot}02053873\sin 2\theta,$$

$$f_2 = -\frac{0{\cdot}41\dot{6}}{\pi}(0{\cdot}4 + \cos\theta)^2 \log_e\left|\frac{1 + 0{\cdot}4\cos\theta - 0{\cdot}91651514\sin\theta}{0{\cdot}4 + \cos\theta}\right|$$

$$- 0{\cdot}00144678\sin\theta - 0{\cdot}07687711\sin 2\theta,$$

$$(2\rho_L)^{\frac{1}{2}} = 0{\cdot}65569563_5 a + 0{\cdot}31434788b + 0{\cdot}02995648c,$$

$$(2\rho_T)^{\frac{1}{2}} = 0{\cdot}12163548a + 0{\cdot}42788172b + 0{\cdot}45048279c,$$

$$C_0 = 0{\cdot}35a + 0{\cdot}5b + 0{\cdot}15c,$$

$$\varepsilon = \tan\tfrac{1}{2}\theta\{0{\cdot}29\dot{2}8571\dot{4}a - 0{\cdot}\dot{1}4285\dot{7}b - 0{\cdot}15c - 0{\cdot}\dot{3}57142\dot{8}(b - a)\cos\theta$$

$$\text{for} \quad 0 \leqslant x \leqslant 0{\cdot}7$$

$$= \cot\tfrac{1}{2}\theta\{0{\cdot}35a - 0{\cdot}\dot{3}b - 0{\cdot}01\dot{6}c - 0{\cdot}8\dot{3}(b - c)\cos\theta\}$$

$$\text{for} \quad 0{\cdot}7 \leqslant x \leqslant 1$$

and

$$\left.\begin{aligned} \varepsilon' &= \frac{1}{1 + \cos\theta}\{0{\cdot}2\dot{1}4285\dot{7}b - 0{\cdot}06\dot{4}2857\dot{1}a - 0{\cdot}15c\} \\ &\quad - 0{\cdot}\dot{3}57142\dot{8}(b - a)\cos\theta \qquad \text{for} \quad 0 \leqslant x \leqslant 0{\cdot}7 \\ &= \frac{1}{1 - \cos\theta}\{1{\cdot}1\dot{6}b - 0{\cdot}35a - 0{\cdot}81\dot{6}c\} + 0{\cdot}8\dot{3}(b - c)\cos\theta \\ &\qquad\qquad\qquad\qquad \text{for} \quad 0{\cdot}7 \leqslant x \leqslant 1. \end{aligned}\right\}$$

Summary./

8. Summary.

If the velocity distribution at the surface of a symmetrical aerofoil according to the purely linear theory (Approximation I of Part I) is specified, then the aerofoil shape may be easily calculated. General formulae are given for the aerofoil ordinates, both in terms of Fourier series and as an integral. If the velocity distribution is expressible as a single polynomial in $x$ over the whole chord (where $x$ is the distance, in fractions of the chord, from the leading edge measured along the chord), then the Fourier series terminate and should be used. It is shown how the ordinates may be obtained analytically if, with the chord divided into any number of segments, the velocity distribution is expressible as a polynomial in $x$ in each segment. Simple general formulae are also obtained for $\rho_L$ and $\rho_T$, the radii of curvature at the leading and trailing edges, and for those quantities ($C_0$, $\varepsilon_s$, $\varepsilon_s'$) which are required for calculating a much closer approximation to the velocity distribution (Approximation III) for the actual aerofoil obtained.

Formulae for the ordinates and for $\rho_L$, $\rho_T$, $C_0$, $\varepsilon_s$, $\varepsilon_s'$ are set out if the approximate velocity distribution is expressible as a quadratic in $x$ in each of three segments into which the chord is divided, and the results for the case in which, with the chord divided into two segments only, the velocity distribution is quadratic in $x$ in each of them are immediately obtainable. The formulae are then simplified for the case in which the velocity is a linear function of $x$ in each of three segments; the results for this case are expressed in terms of the given values of the approximate velocity at the leading and trailing edges and at the joins of the segments. Finally the formulae are simplified still further for the case in which the velocity is a linear function of $x$ in each of two segments. For this case tables are provided from which the ordinates may easily be computed if the join between the segments, which will normally be the position of maximum velocity, is at $x = 0{\cdot}5$, $0{\cdot}6$ or $0{\cdot}7$. For eight aerofoils of this class, four with the join at $x = 0{\cdot}5$ and four with the join at $x = 0{\cdot}6$, numerical values of the ordinates and of $\rho_L$, $\rho_T$, $C_0$, $\psi_s$, $\varepsilon_s$, $\varepsilon_s'$ and the velocity at zero lift on Approximation III are given in the tables; the results are illustrated by graphs of the aerofoil shapes and of the velocity distributions.

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References

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2. Approximate Two-dimensional Aerofoil Theory. Part II. Velocity Distributions for Cambered Aerofoils.— S. Goldstein. Current Paper No. 69.

3. Review of Calculations on Low Drag Wing Sections.— H. B. Squire. A.R.C. 5865 — unpublished.

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Table 1./

Table 1.

| $x$ | $x^2$ | $x^3$ | $x^4$ | $x^5$ | $x^{1/2}$ | $x^{3/2}$ | $x^{5/2}$ | $x^{7/2}$ | $x^{9/2}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.005 | 0.000025 | 0.00000012 | 0.00000000 | 0.00000000 | 0.07071068 | 0.00035355 | 0.00000177 | 0.00000001 | 0.00000000 |
| 0.0075 | 0.00005625 | 0.00000042 | 0.00000000 | 0.00000000 | 0.08660254 | 0.00064952 | 0.00000487 | 0.00000004 | 0.00000000 |
| 0.0125 | 0.00015625 | 0.00000195 | 0.00000002 | 0.00000000 | 0.11180340 | 0.00139754 | 0.00001747 | 0.00000022 | 0.00000000 |
| 0.025 | 0.000625 | 0.00001562 | 0.00000039 | 0.00000001 | 0.15811388 | 0.00395285 | 0.00009882 | 0.00000247 | 0.00000006 |
| 0.05 | 0.0025 | 0.000125 | 0.00000625 | 0.00000031 | 0.22360680 | 0.01118034 | 0.00055902 | 0.00002795 | 0.00000140 |
| 0.075 | 0.005625 | 0.00042187 | 0.00003164 | 0.00000237 | 0.27386128 | 0.02053950 | 0.00154047 | 0.00011553 | 0.00000866 |
| 0.1 | 0.01 | 0.001 | 0.0001 | 0.00001 | 0.31622777 | 0.03162278 | 0.00316223 | 0.00031623 | 0.00003162 |
| 0.15 | 0.0225 | 0.003375 | 0.00050625 | 0.00007594 | 0.38729833 | 0.05809475 | 0.00871421 | 0.00130713 | 0.00019607 |
| 0.2 | 0.04 | 0.008 | 0.0016 | 0.00032 | 0.44721360 | 0.08944272 | 0.01788854 | 0.00357771 | 0.00071554 |
| 0.25 | 0.0625 | 0.015625 | 0.00390625 | 0.00097656 | 0.5 | 0.125 | 0.03125 | 0.0078125 | 0.001953125 |
| 0.3 | 0.09 | 0.027 | 0.0081 | 0.00243 | 0.54772256 | 0.16431677 | 0.04929503 | 0.01478851 | 0.00443655 |
| 0.35 | 0.1225 | 0.042875 | 0.01500625 | 0.00525219 | 0.59160798 | 0.20706279 | 0.07247198 | 0.02536519 | 0.00887782 |
| 0.4 | 0.16 | 0.064 | 0.0256 | 0.01024 | 0.63245553 | 0.25298221 | 0.10119288 | 0.04047715 | 0.01619086 |
| 0.45 | 0.2025 | 0.091125 | 0.04100625 | 0.01845281 | 0.67082039 | 0.30186918 | 0.13584113 | 0.06112851 | 0.02750783 |
| 0.5 | 0.25 | 0.125 | 0.0625 | 0.03125 | 0.70710678 | 0.35355339 | 0.17677669 | 0.08838835 | 0.04419417 |
| 0.55 | 0.3025 | 0.166375 | 0.09150625 | 0.05032844 | 0.74161985 | 0.40789092 | 0.22434000 | 0.12338700 | 0.06786285 |
| 0.6 | 0.36 | 0.216 | 0.1296 | 0.07776 | 0.77459667 | 0.46475800 | 0.27885480 | 0.16731288 | 0.10038773 |
| 0.65 | 0.4225 | 0.274625 | 0.17850625 | 0.11602906 | 0.80622577 | 0.52404675 | 0.34063039 | 0.22140975 | 0.14391634 |
| 0.7 | 0.49 | 0.343 | 0.2401 | 0.16807 | 0.83666003 | 0.58566202 | 0.40996341 | 0.28697439 | 0.20088207 |
| 0.75 | 0.5625 | 0.421875 | 0.31640625 | 0.23730469 | 0.86602540 | 0.64951905 | 0.48713929 | 0.36535447 | 0.27401585 |
| 0.8 | 0.64 | 0.512 | 0.4096 | 0.32768 | 0.89442719 | 0.71554175 | 0.57243340 | 0.45794672 | 0.36635738 |
| 0.85 | 0.7225 | 0.614125 | 0.52200625 | 0.44370531 | 0.92195445 | 0.78366128 | 0.66611209 | 0.56619527 | 0.48126598 |
| 0.9 | 0.81 | 0.729 | 0.6561 | 0.59049 | 0.94868330 | 0.85381497 | 0.76843347 | 0.69159012 | 0.62243111 |
| 0.925 | 0.855625 | 0.79145312 | 0.73209414 | 0.67718708 | 0.96176920 | 0.88963651 | 0.82291377 | 0.76119523 | 0.70410560 |
| 0.95 | 0.9025 | 0.857375 | 0.81450625 | 0.77378094 | 0.97467943 | 0.92594546 | 0.87964819 | 0.83566578 | 0.79388249 |
| 0.975 | 0.950625 | 0.92685937 | 0.90368789 | 0.88109569 | 0.98742088 | 0.96273536 | 0.93866697 | 0.91520029 | 0.89232029 |
| 0.9875 | 0.97515625 | 0.96296680 | 0.95092971 | 0.93904309 | 0.99373035 | 0.98130872 | 0.96904236 | 0.95692934 | 0.94496771 |

Table 1 Contd./

Table 1 Contd.

| x | θ | Cosθ | Cos 2θ | Cos 3θ | Sinθ | Sin 2θ | Sin 3θ | Cosecθ | Tan 1/2 θ | Cot 1/2 θ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | ∞ | 0 | ∞ |
| 0.005 | 0.141539 | 0.99 | 0.9602 | 0.911196 | 0.1410674 | 0.2793135 | 0.4119733 | 7.0888100 | 0.0708881 | 14.1067360 |
| 0.0075 | 0.173422 | 0.985 | 0.94045 | 0.8676865 | 0.1725543 | 0.3399320 | 0.4971117 | 5.7952772 | 0.0869291 | 11.5036225 |
| 0.0125 | 0.224075 | 0.975 | 0.90125 | 0.7824375 | 0.2222049 | 0.4332996 | 0.6227293 | 4.5003508 | 0.1125088 | 8.8881944 |
| 0.025 | 0.317560 | 0.95 | 0.805 | 0.5795 | 0.3122499 | 0.5932748 | 0.8149722 | 3.2025631 | 0.1601281 | 6.2449980 |
| 0.05 | 0.451027 | 0.9 | 0.62 | 0.216 | 0.4358899 | 0.7846018 | 0.9763934 | 2.2941573 | 0.2294157 | 4.3588989 |
| 0.075 | 0.554811 | 0.85 | 0.445 | -0.0935 | 0.5267827 | 0.8955306 | 0.9956193 | 1.8983159 | 0.2847474 | 3.5118846 |
| 0.1 | 0.643501 | 0.8 | 0.28 | -0.352 | 0.6 | 0.96 | 0.936 | 1.6666667 | 0.3333333 | 3 |
| 0.15 | 0.795399 | 0.7 | -0.02 | -0.728 | 0.7141428 | 0.9998000 | 0.6855771 | 1.4002802 | 0.4200840 | 2.3804761 |
| 0.2 | 0.927295 | 0.6 | -0.28 | -0.936 | 0.8 | 0.96 | 0.352 | 1.25 | 0.5 | 2 |
| 0.25 | 1.047198 | 0.5 | -0.5 | -1 | 0.8660254 | 0.8660254 | 0 | 1.1547005 | 0.5773503 | 1.7320508 |
| 0.3 | 1.159279 | 0.4 | -0.68 | -0.944 | 0.9165151 | 0.7332121 | -0.3299454 | 1.0910895 | 0.6546537 | 1.5275252 |
| 0.35 | 1.266104 | 0.3 | -0.82 | -0.792 | 0.9539392 | 0.5723635 | -0.6105211 | 1.0482848 | 0.7337994 | 1.3627703 |
| 0.4 | 1.369438 | 0.2 | -0.92 | -0.568 | 0.9797959 | 0.3919184 | -0.8230285 | 1.0206207 | 0.8164966 | 1.2247449 |
| 0.45 | 1.470629 | 0.1 | -0.98 | -0.296 | 0.9949874 | 0.1989975 | -0.9551879 | 1.0050379 | 0.9045340 | 1.1055416 |
| 0.5 | $\frac{1}{2}\pi$ | 0 | -1 | 0 | 1 | 0 | -1 | 1 | 1 | 1 |
| 0.55 | π-1.470629 | -0.1 | -0.98 | 0.296 | 0.9949874 | -0.1989975 | -0.9551879 | 1.0050379 | 1.1055416 | 0.9045340 |
| 0.6 | π-1.369438 | -0.2 | -0.92 | 0.568 | 0.9797959 | -0.3919184 | -0.8230285 | 1.0206207 | 1.2247449 | 0.8164966 |
| 0.65 | π-1.266104 | -0.3 | -0.82 | 0.792 | 0.9539392 | -0.5723635 | -0.6105211 | 1.0482848 | 1.3627703 | 0.7337994 |
| 0.7 | π-1.159279 | -0.4 | -0.68 | 0.944 | 0.9165151 | -0.7332121 | -0.3299454 | 1.0910895 | 1.5275252 | 0.6546537 |
| 0.75 | π-1.047198 | -0.5 | -0.5 | 1 | 0.8660254 | -0.8660254 | 0 | 1.1547005 | 1.7320508 | 0.5773503 |
| 0.8 | π-0.927295 | -0.6 | -0.28 | 0.936 | 0.8 | -0.96 | 0.352 | 1.25 | 2 | 0.5 |
| 0.85 | π-0.795399 | -0.7 | -0.02 | 0.728 | 0.7141428 | -0.9998000 | 0.6855771 | 1.4002802 | 2.3804761 | 0.4200840 |
| 0.9 | π-0.643501 | -0.8 | 0.28 | 0.352 | 0.6 | -0.96 | 0.936 | 1.6666667 | 3 | 0.3333333 |
| 0.925 | π-0.554811 | -0.85 | 0.445 | 0.0935 | 0.5267827 | -0.8955306 | 0.9956193 | 1.8983159 | 3.5118846 | 0.2847474 |
| 0.95 | π-0.451027 | -0.9 | 0.62 | -0.216 | 0.4358899 | -0.7846018 | 0.9763934 | 2.2941573 | 4.3588989 | 0.2294157 |
| 0.975 | π-0.317560 | -0.95 | 0.805 | -0.5795 | 0.3122499 | -0.5932748 | 0.8149722 | 3.2025631 | 6.2449980 | 0.1601281 |
| 0.9875 | π-0.224075 | -0.975 | 0.90125 | -0.7824375 | 0.2222049 | -0.4332996 | 0.6227293 | 4.5003508 | 8.8881944 | 0.1125088 |
| 1 | π | -1 | 1 | -1 | 0 | 0 | 0 | ∞ | ∞ | 0 |

Table 1 Contd./

Table 1 Contd.

| x | $\frac{1}{\pi}\log_e \frac{\sin\frac{1}{2}\lvert\theta-\theta_1\rvert}{\sin\frac{1}{2}(\theta+\theta_1)}$ where $\theta_1 = 2\sin^{-1}X_1^{\frac{1}{2}}$, for | | |
|---|---|---|---|
| | $X_1 = 0.5$ | $X_1 = 0.6$ | $X_1 = 0.7$ |
| 0 | 0 | 0 | 0 |
| 0.005 | 0.0452046 | 0.0368887 | 0.0295649 |
| 0.0075 | 0.0554808$_5$ | 0.0452616$_5$ | 0.0362682 |
| 0.0125 | 0.0719298 | 0.0586471$_5$ | 0.0469749 |
| 0.025 | 0.1028257 | 0.0837135 | 0.0669820 |
| 0.05 | 0.1486969$_5$ | 0.1206747 | 0.0963413 |
| 0.075 | 0.1864283 | 0.1507679 | 0.1200769 |
| 0.1 | 0.2206356 | 0.1777450 | 0.1411922 |
| 0.15 | 0.2850746 | 0.2275823 | 0.1797020 |
| 0.2 | 0.3496992 | 0.2759794 | 0.2163434 |
| 0.25 | 0.4192007 | 0.3258695 | 0.2531727 |
| 0.3 | 0.4987277 | 0.3797153 | 0.2916644 |
| 0.35 | 0.5964555 | 0.4404211 | 0.3332602 |
| 0.4 | 0.7297037 | 0.5123000 | 0.3797153 |
| 0.45 | 0.9527724$_5$ | 0.6030569 | 0.4335256 |
| 0.5 | $\infty$ | 0.7297037 | 0.4987276 |
| 0.55 | 0.9527724$_5$ | 0.9463087 | 0.5828055 |
| 0.6 | 0.7297037 | $\infty$ | 0.7025653 |
| 0.65 | 0.5964555 | 0.9329715 | 0.9118480 |
| 0.7 | 0.4987277 | 0.7025653 | $\infty$ |
| 0.75 | 0.4192007 | 0.5510943 | 0.8812915 |
| 0.8 | 0.3496992 | 0.4537243 | 0.6399199 |
| 0.85 | 0.2350746 | 0.3621252 | 0.4844927 |
| 0.9 | 0.2206356 | 0.2759794 | 0.3575355 |
| 0.925 | 0.1864283 | 0.2317372 | 0.2966626 |
| 0.95 | 0.1486969$_5$ | 0.1858183 | 0.2329667 |
| 0.975 | 0.1028257 | 0.1264900 | 0.1589390 |
| 0.9875 | 0.0719298 | 0.0862844 | 0.1105062 |
| 1 | 0 | 0 | 0 |

Table 2(a)/

## Table 2.(a)

Values of $f_0$, $f_1$, $f_2$.

| x | $X_1 = 0.5$ | | | $X_1 = 0.6$ | | |
|---|---|---|---|---|---|---|
| | $f_o$ | $f_1$ | $f_2$ | $f_o$ | $f_1$ | $f_2$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.005 | 0.0397591 | 0.0259296 | 0.0048450 | 0.0430924 | 0.0240787 | 0.0033625 |
| 0.0075 | 0.0484344 | 0.0318998$_5$ | 0.0059429 | 0.0525386 | 0.0296144 | 0.0041242 |
| 0.0125 | 0.0618583 | 0.0415483 | 0.0076958$_5$ | 0.0672124 | 0.0385502 | 0.0053399 |
| 0.025 | 0.0851278 | 0.0600287 | 0.0109684 | 0.0928921 | 0.0556252 | 0.0076077 |
| 0.05 | 0.1138358 | 0.0883486$_5$ | 0.0157605 | 0.1253352 | 0.0816872 | 0.0109225$_5$ |
| 0.075 | 0.1315643 | 0.1122041 | 0.0196230 | 0.1462412 | 0.1035628 | 0.0135873 |
| 0.1 | 0.1430482 | 0.1339036$_5$ | 0.0230482 | 0.1606292 | 0.1234271 | 0.0159438 |
| 0.15 | 0.1542388 | 0.1735688 | 0.0292638$_5$ | 0.1771241 | 0.1597477 | 0.0201996 |
| 0.2 | 0.1551349 | 0.2097302 | 0.0351349 | 0.1827927 | 0.1930181$_5$ | 0.0241891 |
| 0.25 | 0.1492427 | 0.2427804 | 0.0409896 | 0.1811331 | 0.2237496 | 0.0281300 |
| 0.3 | 0.1387088 | 0.2724914$_5$ | 0.0470573 | 0.1741958 | 0.2518969 | 0.0321649 |
| 0.35 | 0.1251050 | 0.2933050 | 0.0535596 | 0.1633951$_5$ | 0.2771559 | 0.0364185 |
| 0.4 | 0.1097616 | 0.3193645 | 0.0607718 | 0.1498258 | 0.2990452 | 0.0410270 |
| 0.45 | 0.0939979 | 0.3343727 | 0.0691232 | 0.1344216 | 0.3169064 | 0.0461657 |
| 0.5 | 0.0795775 | 0.3408451 | 0.0795775 | 0.1180635 | 0.3298414 | 0.0520952 |
| 0.55 | 0.0691232 | 0.3343727 | 0.0939979 | 0.1016997 | 0.3365246 | 0.0592694$_5$ |
| 0.6 | 0.0607718 | 0.3193645 | 0.1097616 | 0.0866910 | 0.3344076 | 0.0687993 |
| 0.65 | 0.0535596 | 0.2983050 | 0.1251050 | 0.0751363 | 0.3189395 | 0.0828938 |
| 0.7 | 0.0470573 | 0.2724914$_5$ | 0.1387088 | 0.0654050$_5$ | 0.2947450 | 0.0981076 |
| 0.75 | 0.0409896 | 0.2427804 | 0.1492427 | 0.0566128 | 0.2644174 | 0.1119825 |
| 0.8 | 0.0351349 | 0.2097302 | 0.1551349 | 0.0483008 | 0.2292480 | 0.1224512 |
| 0.85 | 0.0292638$_5$ | 0.1735688 | 0.1542388 | 0.0400842 | 0.1899101 | 0.1270772 |
| 0.9 | 0.0230482 | 0.1339036$_5$ | 0.1430482 | 0.0314786 | 0.1463035$_5$ | 0.1222179 |
| 0.925 | 0.0196230 | 0.1122041 | 0.1315643 | 0.0267670 | 0.1223951 | 0.1142293 |
| 0.95 | 0.0157605 | 0.0883486$_5$ | 0.1138358 | 0.0214738 | 0.0961526 | 0.1003185 |
| 0.975 | 0.0109684 | 0.0600287 | 0.0851278 | 0.0149289 | 0.0651348 | 0.0760613 |
| 0.9875 | 0.0076958$_5$ | 0.0415483 | 0.0618583 | 0.0104695 | 0.0450012 | 0.0556317 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |

| x | $X_1 = 0.7$ | | | x | $X_1 = 0.7$ | | |
|---|---|---|---|---|---|---|---|
| | $f_o$ | $f_1$ | $f_2$ | | $f_o$ | $f_1$ | $f_2$ |
| 0 | 0 | 0 | 0 | | | | |
| 0.005 | 0.0460015 | 0.0224081 | 0.0021241 | 0.5 | 0.1564865 | 0.3117118 | 0.0318017 |
| 0.0075 | 0.0561182 | 0.0275539 | 0.0026051 | 0.55 | 0.1396794 | 0.3221002 | 0.0357140 |
| 0.0125 | 0.0718764 | 0.0358533 | 0.0033727 | 0.6 | 0.1223016 | 0.3271749 | 0.0404214 |
| 0.025 | 0.0996367 | 0.0516848 | 0.0048034 | 0.65 | 0.1050678 | 0.3254809 | 0.0464209 |
| 0.05 | 0.1352773 | 0.0757758 | 0.0068918 | 0.7 | 0.0890545 | 0.3141619 | 0.0550412 |
| 0.075 | 0.1588844 | 0.0959398 | 0.0085671 | 0.75 | 0.0759642 | 0.2880519 | 0.0689966 |
| 0.1 | 0.1757337 | 0.1142211 | 0.0100452 | 0.8 | 0.0642757 | 0.2524144 | 0.0833099 |
| 0.15 | 0.1967332 | 0.1476334 | 0.0127048 | 0.85 | 0.0530370 | 0.2100374 | 0.0939970 |
| 0.2 | 0.2065073 | 0.1783091 | 0.0151836 | 0.9 | 0.0414727 | 0.1617576 | 0.0967697 |
| 0.25 | 0.2085910 | 0.2068064 | 0.0176153 | 0.925 | 0.0352028 | 0.1350739 | 0.0931146 |
| 0.3 | 0.2050035 | 0.2331701 | 0.0200840 | 0.95 | 0.0281968 | 0.1057935 | 0.0839546 |
| 0.35 | 0.1970918 | 0.2572189 | 0.0226588 | 0.975 | 0.0195749 | 0.0713596 | 0.0651904 |
| 0.4 | 0.1858536 | 0.2786341 | 0.0254102 | 0.9875 | 0.0137187 | 0.0491710 | 0.0482127 |
| 0.45 | 0.1720891 | 0.2969835 | 0.0284210 | 1 | 0 | 0 | 0 |

Table 2(b)/

## Table 2(b).

Values of $f_0$, $f_1$, $f_2$ for $X_1 = 0.6$.

| x | $f_0$ | $f_1$ | $f_2$ | x | $f_0$ | $f_1$ | $f_2$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | | | | |
| 0.001 | 0.0194112 | 0.0106954 | 0.0015003$_5$ | 0.32 | 0.1702728 | 0.2623704 | 0.0338329$_5$ |
| 0.002 | 0.0274023 | 0.0151515 | 0.0021229 | 0.34 | 0.1658087 | 0.2723559 | 0.0355441$_5$ |
| 0.003 | 0.0335002 | 0.0185883 | 0.0026015$_5$ | 0.36 | 0.1608713 | 0.2818217 | 0.0373070 |
| 0.004 | 0.0386128 | 0.0215003 | 0.0030058$_5$ | 0.38 | 0.1555238 | 0.2907316 | 0.0391310$_5$ |
| 0.005 | 0.0430924 | 0.0240787 | 0.0033625 | 0.4 | 0.1498258 | 0.2990452 | 0.0410270 |
| 0.006 | 0.0471200 | 0.0264214 | 0.0036855$_5$ | 0.42 | 0.1438349 | 0.3067168 | 0.0430068 |
| 0.007 | 0.0508032 | 0.0285863 | 0.0039831$_5$ | 0.44 | 0.1376070 | 0.3136951 | 0.0450848$_5$ |
| 0.008 | 0.0542124 | 0.0306112 | 0.0042606$_5$ | 0.46 | 0.1311979 | 0.3199219 | 0.0472776$_5$ |
| 0.009 | 0.0573965 | 0.0325222 | 0.0045217$_5$ | 0.48 | 0.1246639 | 0.3253300 | 0.0496059$_5$ |
| 0.01 | 0.0603912 | 0.0343383 | 0.0047692$_5$ | 0.5 | 0.1180635 | 0.3298414 | 0.0520952 |
| 0.012 | 0.0659145 | 0.0377402 | 0.0052305$_5$ | 0.52 | 0.1114590 | 0.3333624 | 0.0547784$_5$ |
| 0.014 | 0.0709362 | 0.0408981 | 0.0056561$_5$ | 0.54 | 0.1049199 | 0.3357773 | 0.0577002$_5$ |
| 0.016 | 0.0755567 | 0.0438646 | 0.0060538 | 0.56 | 0.0985280 | 0.3369346 | 0.0609243$_5$ |
| 0.018 | 0.0798463 | 0.0466761 | 0.0064287 | 0.58 | 0.0923906 | 0.3366158 | 0.0643521 |
| 0.02 | 0.0838563 | 0.0493593 | 0.0067844 | 0.6 | 0.0866910 | 0.33440768 | 0.0687993 |
| 0.025 | 0.0928921 | 0.0556252 | 0.0076077 | 0.62 | 0.0817391 | 0.3295774 | 0.0740699$_5$ |
| 0.03 | 0.1008170 | 0.0614115 | 0.0083587 | 0.64 | 0.0772549 | 0.3228627 | 0.0798824 |
| 0.035 | 0.1078816 | 0.0668426 | 0.0090555$_5$ | 0.66 | 0.0730849 | 0.3146819 | 0.0859419$_5$ |
| 0.04 | 0.1142509 | 0.0719982 | 0.0097101 | 0.68 | 0.0691521 | 0.3052577 | 0.0920663$_5$ |
| 0.05 | 0.1253352 | 0.0816872 | 0.0109225$_5$ | 0.7 | 0.0654050$_5$ | 0.2947450 | 0.0981076 |
| 0.06 | 0.1346857 | 0.0907620 | 0.0120391$_5$ | 0.72 | 0.0618049 | 0.2832617 | 0.1039323 |
| 0.07 | 0.1426757 | 0.0993861 | 0.0130852 | 0.74 | 0.0583204 | 0.2709016 | 0.1094122$_5$ |
| 0.08 | 0.1495532 | 0.1076625 | 0.0140775 | 0.76 | 0.0549244 | 0.2577409 | 0.1144178$_5$ |
| 0.09 | 0.1554941 | 0.1156601 | 0.0150275$_5$ | 0.78 | 0.0515924 | 0.2438406 | 0.1188133 |
| 0.1 | 0.1606292 | 0.1234271 | 0.0159438 | 0.8 | 0.0483008 | 0.2292480 | 0.1224512 |
| 0.12 | 0.1688639 | 0.1383989 | 0.0176987$_5$ | 0.82 | 0.0450253 | 0.2139960 | 0.1251661$_5$ |
| 0.14 | 0.1748468 | 0.1527610 | 0.0193792$_5$ | 0.84 | 0.0417399 | 0.1981003 | 0.1267658$_5$ |
| 0.16 | 0.1789793 | 0.1666167 | 0.0210100$_5$ | 0.86 | 0.0384142 | 0.1815543 | 0.1270185$_5$ |
| 0.18 | 0.1815537 | 0.1800251 | 0.0226086$_5$ | 0.88 | 0.0350105 | 0.1643189 | 0.1256321$_5$ |
| 0.2 | 0.1827927 | 0.1930181$_5$ | 0.0241891 | 0.9 | 0.0314786 | 0.1463035$_5$ | 0.1222179 |
| 0.22 | 0.1828735 | 0.2056103 | 0.0257625 | 0.92 | 0.0277446 | 0.1273262 | 0.1162224 |
| 0.24 | 0.1819408 | 0.2178039 | 0.0273384$_5$ | 0.94 | 0.0236877 | 0.1070215 | 0.1067776$_5$ |
| 0.26 | 0.1801162 | 0.2295926 | 0.0289254$_5$ | 0.96 | 0.0190755 | 0.0845838 | 0.0922999 |
| 0.28 | 0.1775041 | 0.2409632 | 0.0305316 | 0.98 | 0.0133085 | 0.0577287 | 0.0689628 |
| 0.3 | 0.1741958 | 0.2518969 | 0.0321649 | 1 | 0 | 0 | 0 |

## Table 3 (a)

### Aerofoil A.

$X_1 = 0.5$

$a = 0.11667$, $b = 0.2$, $c = -0.11$, $(b - a)/X_1 = 0.16666$

$\rho_L = 0.008642$  $\rho_T = 0.000164$

$C_o = 0.1016675$,  $e^{C_o} = 1.10701$

| x | $y_s$ | $\psi_s$ | $\varepsilon_s$ | $\varepsilon'_s$ | q/U for $C_L = 0$. |
|---|---|---|---|---|---|
| 0 | 0 | 0.1315 | 0 | 0.0075 | 0 |
| 0.005 | 0.0092917 | 0.1317 | 0.0011 | 0.0082 | 0.8220 |
| 0.0075 | 0.0113771 | 0.1319 | 0.0014 | 0.0085 | 0.8940 |
| 0.0125 | 0.0146801 | 0.1321 | 0.0018 | 0.0092 | 0.9679 |
| 0.025 | 0.0207311 | 0.1328 | 0.0027 | 0.0108 | 1.0383 |
| 0.05 | 0.0292173 | 0.1341 | 0.0044 | 0.01425 | 1.0830 |
| 0.075 | 0.0356319 | 0.1353 | 0.00605 | 0.0177 | 1.1018 |
| 0.1 | 0.0409349 | 0.13645 | 0.0078 | 0.0213 | 1.1138 |
| 0.15 | 0.0494898 | 0.1386 | 0.0116 | 0.0287 | 1.1305 |
| 0.2 | 0.0561808 | 0.14045 | 0.0158 | 0.0365 | 1.1434 |
| 0.25 | 0.0614594 | 0.1419 | 0.0207 | 0.0447 | 1.1546 |
| 0.3 | 0.0655052 | 0.1429 | 0.0262 | 0.0536 | 1.1652 |
| 0.35 | 0.0683654 | 0.1433 | 0.0324 | 0.0631 | 1.1751 |
| 0.4 | 0.0699939 | 0.1429 | 0.0395 | 0.0736 | 1.1846 |
| 0.45 | 0.0702377 | 0.1412 | 0.0475 | 0.0852 | 1.1938 |
| 0.5 | 0.0686998 | 0.1374 | 0.0567 | 0.0983 | 1.2027 |
| 0.55 | 0.0645994 | 0.12985 | 0.0653 | 0.0739 | 1.1686 |
| 0.6 | 0.0588894 | 0.1202 | 0.0716 | 0.0509 | 1.1350 |
| 0.65 | 0.0521482 | 0.1093 | 0.0757 | 0.0291 | 1.1017 |
| 0.7 | 0.0447305 | 0.0976 | 0.0777 | +0.0082 | 1.0689 |
| 0.75 | 0.0369216 | 0.0853 | 0.0775 | -0.0119 | 1.0366 |
| 0.8 | 0.0289804 | 0.07245 | 0.0748 | -0.0315 | 1.0049 |
| 0.85 | 0.0211617 | 0.0593 | 0.0694 | -0.0507 | 0.9736 |
| 0.9 | 0.0137345 | 0.0458 | 0.0602 | -0.0694 | 0.9430 |
| 0.925 | 0.0102582 | 0.03895 | 0.05365 | -0.0786 | 0.9277 |
| 0.95 | 0.0069866 | 0.0321 | 0.0450 | -0.0877 | 0.9126 |
| 0.975 | 0.0039214 | 0.0251 | 0.0327 | -0.0968 | 0.8972 |
| 0.9875 | 0.0024031 | 0.0216 | 0.0234 | -0.1013 | 0.8881 |
| 1 | 0 | 0.0181 | 0 | -0.1058 | 0 |

Table 3 (b)/

Table 3 (b)

Aerofoil B.

$X_1 = 0.5$

a = 0.15833, b = 0.2, c = -0.11, $(b-a)/X_1$ = 0.08334

$\rho_L$ = 0.012035, $\rho_T$ = 0.000220

$C_o$ = 0.1120825, $e^{C_o}$ = 1.11861

| x | $y_s$ | $\psi_s$ | $\varepsilon_s$ | $\varepsilon'_s$ | q/U for $C_L$ = 0. |
|---|---|---|---|---|---|
| 0 | 0 | 0.1551 | 0 | 0.0231 | 0 |
| 0.005 | 0.0109480 | 0.1552 | 0.0033 | 0.0235$_5$ | 0.7879 |
| 0.0075 | 0.0133949 | 0.1552$_5$ | 0.0040 | 0.0238 | 0.8710 |
| 0.0125 | 0.0172571 | 0.1553 | 0.0053 | 0.0242 | 0.9607 |
| 0.025 | 0.0242775 | 0.1555 | 0.0076 | 0.0253 | 1.0502 |
| 0.05 | 0.0339597 | 0.1558 | 0.0111 | 0.0275 | 1.1070 |
| 0.075 | 0.0411129 | 0.1561 | 0.0141 | 0.0298 | 1.1295 |
| 0.1 | 0.0468942 | 0.1563 | 0.0168 | 0.0322 | 1.1422 |
| 0.15 | 0.0559154 | 0.1566 | 0.0221 | 0.0371 | 1.1575 |
| 0.2 | 0.0626437 | 0.1566 | 0.0273 | 0.0424 | 1.1674 |
| 0.25 | 0.0676768 | 0.1563 | 0.0327 | 0.0482 | 1.1751 |
| 0.3 | 0.0712838 | 0.1555$_5$ | 0.0335 | 0.0545 | 1.1816 |
| 0.35 | 0.0735773 | 0.1543 | 0.0446 | 0.0614 | 1.1873 |
| 0.4 | 0.0745666 | 0.1522 | 0.0514 | 0.0691 | 1.1925 |
| 0.45 | 0.0741537 | 0.1490$_5$ | 0.0588 | 0.0778 | 1.1974 |
| 0.5 | 0.0720150 | 0.1440 | 0.0671 | 0.0879 | 1.2018 |
| 0.55 | 0.0674790 | 0.1356 | 0.0747 | 0.0644 | 1.1676 |
| 0.6 | 0.0614211 | 0.1254 | 0.0801 | 0.0423 | 1.1338 |
| 0.65 | 0.0543795 | 0.1140 | 0.0833 | 0.0211 | 1.1005 |
| 0.7 | 0.0466909 | 0.1019 | 0.0845 | +0.0008 | 1.0677 |
| 0.75 | 0.0386293 | 0.0892 | 0.0835 | -0.0189 | 1.0354 |
| 0.8 | 0.0304441 | 0.0761 | 0.0800 | -0.0380$_5$ | 1.0036 |
| 0.85 | 0.0223808 | 0.0627 | 0.0738 | -0.0568 | 0.9723 |
| 0.9 | 0.0146946 | 0.0490 | 0.0637 | -0.0752 | 0.9416 |
| 0.925 | 0.0110757 | 0.0420$_5$ | 0.0566 | -0.0842 | 0.9262 |
| 0.95 | 0.0076432 | 0.0351 | 0.0474 | -0.0932 | 0.9110 |
| 0.975 | 0.0043783 | 0.0280 | 0.0343 | -0.1022 | 0.8954 |
| 0.9875 | 0.0027237 | 0.0245 | 0.0245$_5$ | -0.1066 | 0.8865 |
| 1 | 0 | 0.0210 | 0 | -0.1110 | 0 |

Table 3 (c)/

## Table 3 (c)

## Aerofoil C.

## $X_1 = 0.5$

$a = 0.11667, \quad b = 0.2, \quad c' = -0.14190455, \quad (b - a)/X_1 = 0.16666$

$\rho_L = 0.008358, \quad \rho_T = 0$

$C_0 = 0.0936914, \quad e^{C_0} = 1.09822$

| x | $y_s$ | $\psi_s$ | $\varepsilon_s$ | $\varepsilon'_s$ | q/U for $C_L = 0$. |
|---|---|---|---|---|---|
| 0 | 0 | 0.1293 | 0 | 0.0115 | 0 |
| 0.005 | 0.0091371 | 0.1295 | 0.0017 | 0.0122 | 0.8282 |
| 0.0075 | 0.0111875 | 0.1297 | $0.0020_5$ | 0.0125 | 0.8993 |
| 0.0125 | 0.0144346 | 0.1299 | 0.0027 | 0.0132 | 0.9720 |
| 0.025 | 0.0203811 | 0.1305 | 0.0040 | 0.0149 | $1.0409_5$ |
| 0.05 | 0.0287145 | $0.1317_5$ | 0.0062 | $0.0184_5$ | 1.0844 |
| 0.075 | 0.0350058 | 0.1329 | 0.0083 | $0.0220_5$ | 1.1029 |
| 0.1 | 0.0401995 | 0.1340 | 0.0104 | 0.0257 | 1.1146 |
| 0.15 | 0.0485561 | 0.1360 | 0.0149 | 0.0334 | 1.1310 |
| 0.2 | 0.0550598 | $0.1376_5$ | 0.0198 | 0.0414 | 1.1437 |
| 0.25 | 0.0601516 | 0.1389 | 0.0253 | 0.0500 | $1.1548_5$ |
| 0.3 | 0.0640038 | 0.1397 | 0.0314 | 0.0593 | 1.1652 |
| 0.35 | 0.0666566 | $0.1397_5$ | 0.0383 | 0.0693 | 1.1751 |
| 0.4 | 0.0680550 | 0.1389 | 0.0460 | 0.0803 | 1.1844 |
| 0.45 | 0.0680324 | $0.1367_5$ | 0.0547 | 0.0925 | 1.1934 |
| 0.5 | 0.0661609 | 0.1323 | 0.0646 | 0.1063 | 1.2020 |
| 0.55 | 0.0616004 | 0.1238 | 0.0739 | $0.0795_5$ | 1.1646 |
| 0.6 | 0.0553875 | 0.1131 | 0.0807 | 0.0544 | $1.1276_5$ |
| 0.65 | 0.0481568 | 0.1010 | 0.0851 | 0.0305 | 1.0913 |
| 0.7 | 0.0403051 | $0.0879_5$ | 0.0871 | $+0.0075_5$ | 1.0555 |
| 0.75 | 0.0321601 | 0.0743 | 0.0867 | -0.0146 | 1.0203 |
| 0.8 | 0.0240309 | 0.0601 | 0.0836 | -0.0361 | 0.9858 |
| 0.85 | 0.0162408 | 0.0455 | 0.0774 | -0.0571 | 0.9519 |
| 0.9 | 0.0091706 | 0.0306 | 0.0671 | -0.0777 | 0.9188 |
| 0.925 | 0.0060607 | 0.0230 | 0.0598 | -0.0878 | 0.9025 |
| 0.95 | 0.0033547 | 0.0154 | 0.0501 | -0.0979 | 0.8864 |
| 0.975 | 0.0012054 | 0.0077 | 0.0364 | -0.1079 | 0.8704 |
| 0.9875 | 0.0004296 | 0.0039 | 0.0260 | $-0.1128_5$ | 0.8628 |
| 1 | 0 | 0 | 0 | -0.1178 | 0.8547 |

Table 3 (d)/

Table 3 (d)

Aerofoil D.

$X_1 = 0.5$

a = 0.15833, b = 0.2, c = -0.14691, (b - a)/$X_1$ = 0.08334

$\rho_L$ = 0.011647 $\rho_T$ = 0.000000

$C_0$ = 0.102855, $e^{C_0}$ = 1.10833

| x | $y_s$ | $\psi_s$ | $\varepsilon_s$ | $\varepsilon'_s$ | q/U for $C_L = 0$ |
|---|---|---|---|---|---|
| 0 | 0 | 0.1526 | 0 | 0.0277 | 0 |
| 0.005 | 0.0107692 | 0.1527 | 0.00395 | 0.0282 | 0.7948 |
| 0.0075 | 0.0131755 | 0.1527 | 0.00485 | 0.0284 | 0.8771 |
| 0.0125 | 0.0169731 | 0.1528 | 0.0063 | 0.0289 | 0.9656 |
| 0.025 | 0.0238727 | 0.1529 | 0.00905 | 0.0300 | 1.0534 |
| 0.05 | 0.0333780 | 0.15315 | 0.0132 | 0.0324 | 1.1089 |
| 0.075 | 0.0403886 | 0.1533 | 0.0167 | 0.0348 | 1.13065 |
| 0.1 | 0.0460435 | 0.1535 | 0.0199 | 0.0373 | 1.1431 |
| 0.15 | 0.0548352 | 0.1536 | 0.0259 | 0.0426 | 1.1580 |
| 0.2 | 0.0613469 | 0.1534 | 0.0319 | 0.0482 | 1.1677 |
| 0.25 | 0.0661639 | 0.1528 | 0.0380 | 0.0543 | 1.1752 |
| 0.3 | 0.0695469 | 0.1518 | 0.0445 | 0.0611 | 1.1816 |
| 0.35 | 0.0716004 | 0.1501 | 0.0514 | 0.0685 | 1.1872 |
| 0.4 | 0.0723235 | 0.1476 | 0.0589 | 0.0768 | 1.1923 |
| 0.45 | 0.0716023 | 0.1439 | 0.0671 | 0.0862 | 1.1969 |
| 0.5 | 0.0690778 | 0.1382 | 0.0763 | 0.0971 | 1.2010 |
| 0.55 | 0.0640096 | 0.1287 | 0.0847 | 0.0710 | 1.16295 |
| 0.6 | 0.0573698 | 0.1171 | 0.0906 | 0.0463 | 1.1254 |
| 0.65 | 0.0497619 | 0.1043 | 0.0942 | 0.0227 | 1.0884 |
| 0.7 | 0.0415712 | 0.0907 | 0.0954 | +0.0000 | 1.0521 |
| 0.75 | 0.0331207 | 0.0765 | 0.0941 | -0.0220 | 1.0164 |
| 0.8 | 0.0247181 | 0.0618 | 0.0902 | -0.0434 | 0.9814 |
| 0.85 | 0.0166379 | 0.0467 | 0.0831 | -0.0643 | 0.94715 |
| 0.9 | 0.0094147 | 0.0314 | 0.0717 | -0.0848 | 0.9135 |
| 0.925 | 0.0062196 | 0.0236 | 0.0637 | -0.0949 | 0.8972 |
| 0.95 | 0.0034415 | 0.0158 | 0.0533 | -0.1050 | 0.8809 |
| 0.975 | 0.0012362 | 0.0079 | 0.0386 | -0.1150 | 0.8648 |
| 0.9875 | 0.0004405 | 0.0040 | 0.0276 | -0.1199 | 0.8568 |
| 1 | 0 | 0.0000 | 0 | -0.1249 | 0 |

Table 3 (e)/

Table 3 (e)

Aerofoil E.

$X_1 = 0.6$

a = 0.1, b = 0.2, c = -0.11, $(b - a)/X_1$ = 1/6

$\rho_L$ = 0.007664 $\rho_T$ = 0.000489

$C_o$ = 0.108 $e^{C_o}$ = 1.11405 .

| x | $y_s$ | $\psi_s$ | $\varepsilon_s$ | $\varepsilon'_s$ | q/U for $C_L$ = 0. |
|---|---|---|---|---|---|
| 0 | 0 | 0.1238 | 0 | -0.0040 | 0 |
| 0.005 | 0.0087551 | 0.1241 | -0.0005 | -0.0034 | 0.8304 |
| 0.0075 | 0.0107251 | 0.1243 | -0.0006 | -0.0031 | 0.8976 |
| 0.0125 | 0.0138439 | 0.1246 | -0.0008 | -0.0025 | 0.9659 |
| 0.025 | 0.0195774 | 0.1254 | -0.0009 | $-0.0009_5$ | 1.0297 |
| 0.05 | 0.0276695 | 0.1270 | -0.0009 | $+0.0021_5$ | 1.0700 |
| 0.075 | 0.0338421 | 0.1285 | -0.0005 | 0.0053 | 1.0872 |
| 0.1 | 0.0389945 | 0.1300 | +0.0001 | 0.0085 | 1.0982 |
| 0.15 | 0.0474400 | 0.1329 | 0.0019 | 0.0151 | 1.1139 |
| 0.2 | 0.0542221 | 0.1356 | 0.0043 | 0.0221 | 1.1263 |
| 0.25 | 0.0597689 | 0.1380 | 0.0074 | 0.0294 | 1.1373 |
| 0.3 | 0.0642608 | 0.1402 | 0.0111 | 0.0371 | 1.1476 |
| 0.35 | 0.0677647 | 0.1421 | 0.0155 | 0.0454 | 1.1575 |
| 0.4 | $0.0702786_5$ | 0.1435 | 0.0207 | 0.0544 | 1.1670 |
| 0.45 | 0.0717452 | 0.1442 | 0.0267 | 0.0643 | 1.1762 |
| 0.5 | $0.0720441_5$ | 0.1441 | 0.0337 | 0.0753 | 1.1850 |
| 0.55 | $0.0709552_5$ | 0.1426 | 0.0418 | 0.0879 | 1.1936 |
| 0.6 | 0.0679827 | 0.1388 | 0.0514 | 0.1025 | 1.2018 |
| 0.65 | 0.0621832 | 0.1304 | 0.0604 | 0.0723 | 1.1589 |
| 0.7 | 0.0546977 | 0.1194 | 0.0666 | 0.0436 | 1.1168 |
| 0.75 | 0.0462267 | 0.1068 | 0.0699 | +0.0161 | 1.0754 |
| 0.8 | $0.0372100_5$ | 0.0930 | $0.0702_5$ | -0.0103 | 1.0348 |
| 0.85 | 0.0280119 | 0.0784 | 0.0672 | -0.0359 | 0.9949 |
| 0.9 | 0.0189646 | 0.0632 | $0.0597_5$ | -0.0608 | 0.9558 |
| 0.925 | 0.0145905 | 0.0554 | 0.0538 | -0.0731 | 0.9363 |
| 0.95 | 0.0103429 | 0.0475 | 0.0456 | -0.0852 | 0.9168 |
| 0.975 | 0.0061531 | 0.0394 | 0.0334 | -0.0971 | 0.8962 |
| 0.9875 | 0.0039277 | 0.0354 | 0.0240 | -0.1031 | 0.8828 |
| 1 | 0 | 0.0313 | 0 | -0.1090 | 0 |

Table 3 (f)/

## Table 3 (f)

### Aerofoil F.

### $X_1 = 0.6$

$a = 0.15, \quad b = 0.2, \quad c = -0.11, \quad (b - a)/X_1 = 1/12.$

$\rho_L = 0.011943, \qquad \rho_T = 0.000645$

$C_0 = 0.123, \qquad e^{C_0} = 1.13089$

| x | $y_s$ | $\psi_s$ | $\varepsilon_s$ | $\varepsilon'_s$ | q/U for $C_L$ = 0. |
|---|---|---|---|---|---|
| 0 | 0 | 0.1546 | 0 | 0.0135 | 0 |
| 0.005 | 0.0109097 | 0.1547 | 0.0019 | 0.0139 | 0.7851 |
| 0.0075 | 0.0133500 | 0.1547 | 0.0024 | 0.0141 | 0.8654 |
| 0.0125 | 0.0172045 | 0.1549 | 0.0031 | 0.0145 | 0.9540 |
| 0.025 | 0.0242220 | 0.1551 | 0.0045 | 0.0154 | 1.0424 |
| 0.05 | 0.0339362 | 0.1557 | 0.0067 | 0.0174 | 1.0984 |
| 0.075 | 0.0411541 | 0.1562 | 0.0086 | 0.0194 | 1.1206 |
| 0.1 | 0.0470260 | 0.1568 | 0.0104 | 0.0215 | 1.1331 |
| 0.15 | 0.0562962 | 0.1577 | 0.0140 | 0.0258 | 1.1482 |
| 0.2 | 0.0633617 | 0.1584 | 0.0177 | 0.0304 | 1.1581 |
| 0.25 | 0.0688256 | 0.1589 | 0.0216 | 0.0354 | $1.1657_5$ |
| 0.3 | 0.0729706 | 0.1592 | 0.0259 | 0.0407 | 1.1723 |
| 0.35 | 0.0759344 | 0.1592 | 0.0305 | 0.0466 | 1.1781 |
| 0.4 | 0.0777699 | 0.1587 | 0.0357 | 0.0531 | 1.1834 |
| 0.45 | 0.0784663 | 0.1577 | 0.0414 | 0.0603 | $1.1882_5$ |
| 0.5 | 0.0779473 | 0.1559 | 0.0478 | 0.0687 | 1.1927 |
| 0.55 | 0.0760402 | 0.1528 | 0.0552 | 0.0784 | 1.1969 |
| 0.6 | 0.0723173 | 0.1476 | 0.0637 | 0.0900 | 1.2006 |
| 0.65 | 0.0659400 | 0.1382 | 0.0715 | 0.0607 | 1.1575 |
| 0.7 | 0.0579679 | 0.1265 | 0.0764 | 0.0329 | 1.1152 |
| 0.75 | 0.0490573 | 0.1133 | 0.0786 | +0.0061 | 1.0736 |
| 0.8 | 0.0396251 | 0.0991 | $0.0777_5$ | -0.0197 | 1.0328 |
| 0.85 | $0.0300161_5$ | 0.0841 | 0.0735 | -0.0447 | 0.9928 |
| 0.9 | 0.0205385 | 0.0685 | $0.0647_5$ | -0.0692 | 0.9535 |
| 0.925 | $0.0159288_5$ | 0.0605 | 0.0581 | -0.0812 | 0.9337 |
| 0.95 | 0.0114166 | 0.0524 | 0.0490 | -0.0931 | 0.9142 |
| 0.975 | $0.0068995_5$ | 0.0442 | 0.0358 | -0.1048 | 0.8928 |
| 0.9875 | 0.0044512 | 0.0401 | 0.0257 | -0.1107 | 0.8779 |
| 1 | 0 | 0.0359 | 0 | -0.1165 | 0 |

Table 3 (g)/

## Table 3 (g)

## Aerofoil G.

$X_1 = 0.6$

$a = 0.1, \quad b = 0.2, \quad c = -0.14, \quad (b \quad a)/X_1 = 1/6.$

$\rho_L = 0.007489, \qquad \rho_T = 0.000125$

$C_o = 0.102, \qquad e^{C_o} = 1.10738$

| x | $y_s$ | $\psi_s$ | $\varepsilon_s$ | $\varepsilon'_s$ | q/U for $C_L$ = 0. |
|---|---|---|---|---|---|
| 0 | 0 | 0.1224 | 0 | -0.0010 | 0 |
| 0.005 | 0.0086542 | 0.1227 | -0.0001 | -0.0004 | 0.8346 |
| 0.0075 | $0.0105993_5$ | 0.1229 | -0.0001 | -0.0001 | 0.9014 |
| 0.0125 | 0.0136837 | 0.1232 | -0.0001 | +0.0006 | $0.9683_5$ |
| 0.025 | 0.0193492 | 0.1239 | 0.0000 | 0.0021 | 1.0316 |
| 0.05 | 0.0273418 | 0.1255 | +0.0005 | 0.0053 | $1.0709_5$ |
| 0.075 | 0.0334345 | 0.1269 | 0.0012 | $0.0085_5$ | 1.0879 |
| 0.1 | 0.0385162 | 0.1284 | 0.0021 | $0.0118_5$ | 1.0938 |
| 0.15 | 0.0468340 | 0.1312 | 0.0044 | 0.0187 | 1.1143 |
| 0.2 | 0.0534964 | 0.1337 | 0.0075 | 0.0258 | 1.1266 |
| 0.25 | 0.0589250 | 0.1361 | 0.0109 | 0.0334 | 1.1375 |
| 0.3 | 0.0632959 | 0.1381 | 0.0151 | 0.0414 | 1.1478 |
| 0.35 | 0.0666721 | 0.1398 | 0.0199 | 0.0501 | 1.1575 |
| 0.4 | 0.0690478 | 0.1409 | 0.0256 | 0.0594 | $1.1669_5$ |
| 0.45 | $0.0705602_5$ | 0.1414 | 0.0321 | 0.0698 | $1.1760_5$ |
| 0.5 | 0.0704813 | 0.1410 | 0.0397 | 0.0813 | 1.1848 |
| 0.55 | 0.0691772 | 0.1391 | 0.0485 | 0.0945 | 1.1931 |
| 0.6 | 0.0659187 | 0.1346 | 0.0588 | 0.1100 | 1.2011 |
| 0.65 | 0.0596964 | 0.1252 | 0.0634 | 0.0770 | 1.1543 |
| 0.7 | 0.0517544 | 0.1129 | 0.0750 | 0.0457 | 1.1085 |
| 0.75 | 0.0428672 | 0.0990 | 0.0784 | $+0.0157_5$ | 1.0635 |
| 0.8 | 0.0335365 | 0.0838 | 0.0785 | -0.0131 | 1.0196 |
| 0.85 | 0.0241996 | 0.0678 | 0.0749 | -0.0411 | 0.9766 |
| 0.9 | 0.0152981 | 0.0510 | 0.0665 | -0.0683 | 0.9345 |
| 0.925 | 0.0111636 | 0.0424 | 0.0598 | -0.0817 | 0.9140 |
| 0.95 | 0.0073333 | 0.0336 | 0.0506 | -0.0949 | 0.8935 |
| 0.975 | 0.0038713 | 0.0248 | $0.0370_5$ | -0.1080 | 0.8732 |
| 0.9875 | $0.0022587_5$ | 0.0203 | 0.0266 | -0.1145 | 0.8622 |
| 1 | 0 | 0.0158 | 0 | -0.1210 | 0 |

Table 3 (h)

Aerofoil H.

$X_1 = 0.6$

a = 0.15, b = 0.2, c = -0.146, $(b - a)/X_1$ = 1/12,

$\rho_L$ = 0.011681, $\rho_T$ = 0.000151

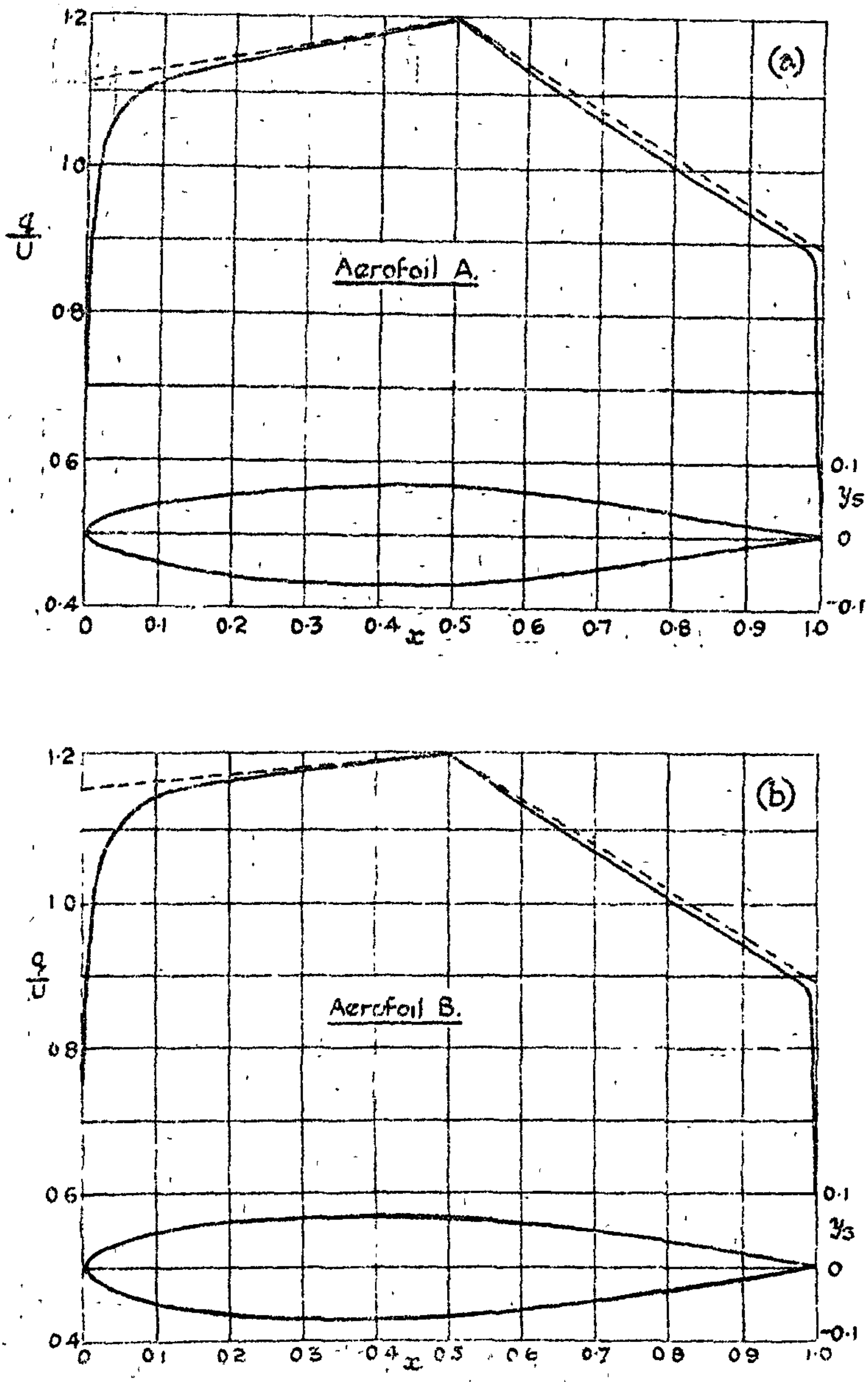
$c_Q$ = 0.1158, $c^{Co}$ = 1.12277

| x | $y_s$ | $\psi_s$ | $\varepsilon_s$ | $\varepsilon'_s$ | q/U for $C_L$ = 0. |
|---|---|---|---|---|---|
| 0 | 0 | 0.1528 | 0 | 0.0171 | 0 |
| 0.005 | 0.0107887 | 0.1530 | 0.0024 | 0.0175 | $0.7877_5$ |
| 0.0075 | 0.0132015 | 0.1530 | 0.0030 | 0.0177 | 0.8696 |
| 0.0125 | 0.0170123 | 0.1531 | 0.0039 | 0.0181 | 0.9571 |
| 0.025 | 0.0239481 | 0.1534 | 0.0056 | 0.0191 | 1.0446 |
| 0.05 | 0.0335430 | 0.1539 | 0.0083 | 0.0212 | 1.0996 |
| 0.075 | 0.0406650 | 0.1544 | 0.0106 | 0.0233 | 1.1214 |
| 0.1 | 0.0464520 | 0.1548 | 0.0128 | 0.0255 | $1.1337_5$ |
| 0.15 | 0.0555690 | 0.1556 | 0.0170 | 0.0300 | $1.1486_5$ |
| 0.2 | 0.0624909 | 0.1562 | 0.0213 | 0.0349 | 1.1583 |
| 0.25 | 0.0678129 | 0.1566 | 0.0258 | 0.0402 | $1.1658_5$ |
| 0.3 | 0.0718127 | 0.1567 | 0.0306 | 0.0459 | 1.1724 |
| 0.35 | $0.0746233_5$ | $0.1564_5$ | 0.0358 | 0.0521 | 1.1781 |
| 0.4 | 0.0762930 | 0.1557 | 0.0415 | 0.0591 | 1.1833 |
| 0.45 | 0.0768043 | 0.1544 | 0.0479 | 0.0669 | 1.1881 |
| 0.5 | 0.0760719 | 0.1521 | 0.0550 | 0.0759 | 1.1924 |
| 0.55 | 0.0739065 | 0.1486 | 0.0631 | 0.0864 | 1.1963 |
| 0.6 | 0.0698405 | 0.1426 | 0.0725 | 0.0990 | 1.1998 |
| 0.65 | 0.0629558 | 0.1320 | 0.0810 | 0.0664 | 1.1520 |
| 0.7 | $0.0544360_5$ | 0.1188 | $0.0864_5$ | 0.0354 | $1.1051_5$ |
| 0.75 | 0.0450260 | 0.1040 | 0.0887 | +0.0057 | 1.0593 |
| 0.8 | $0.0352168_5$ | 0.0880 | $0.0876_5$ | -0.0231 | 1.0145 |
| 0.85 | 0.0254414 | $0.0712_5$ | 0.0827 | -0.0510 | 0.9700 |
| 0.9 | 0.0161387 | 0.0538 | $0.0728_5$ | -0.0782 | 0.9281 |
| 0.925 | 0.0118166 | 0.0449 | 0.0653 | -0.0915 | 0.9072 |
| 0.95 | 0.0078051 | 0.0358 | 0.0551 | -0.1048 | 0.8862 |
| 0.975 | $0.0041613_5$ | 0.0267 | 0.0402 | -0.1179 | 0.8653 |
| 0.9875 | 0.0024484 | 0.0220 | $0.0288_5$ | -0.1244 | 0.8541 |
| 1 | 0 | 0.0174 | 0 | -0.1309 | 0 |

---

O'L.
W
AH.

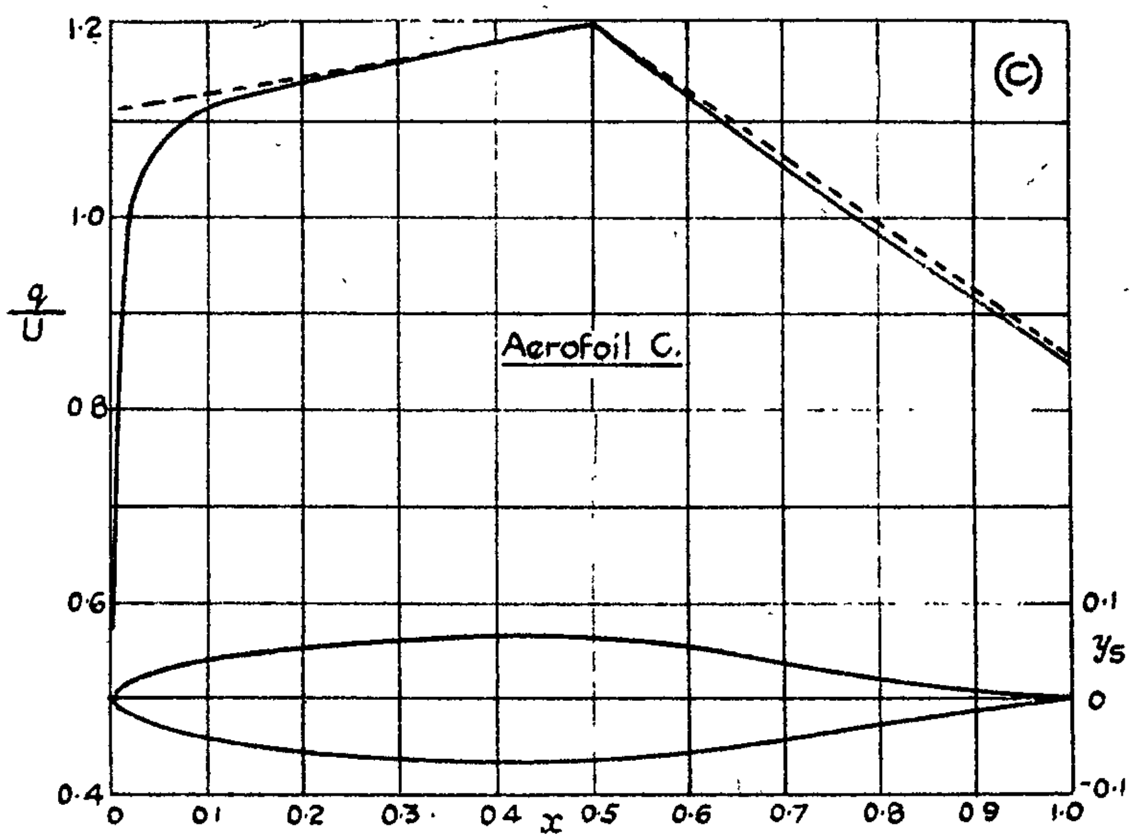
FIG. 1 (a & b)



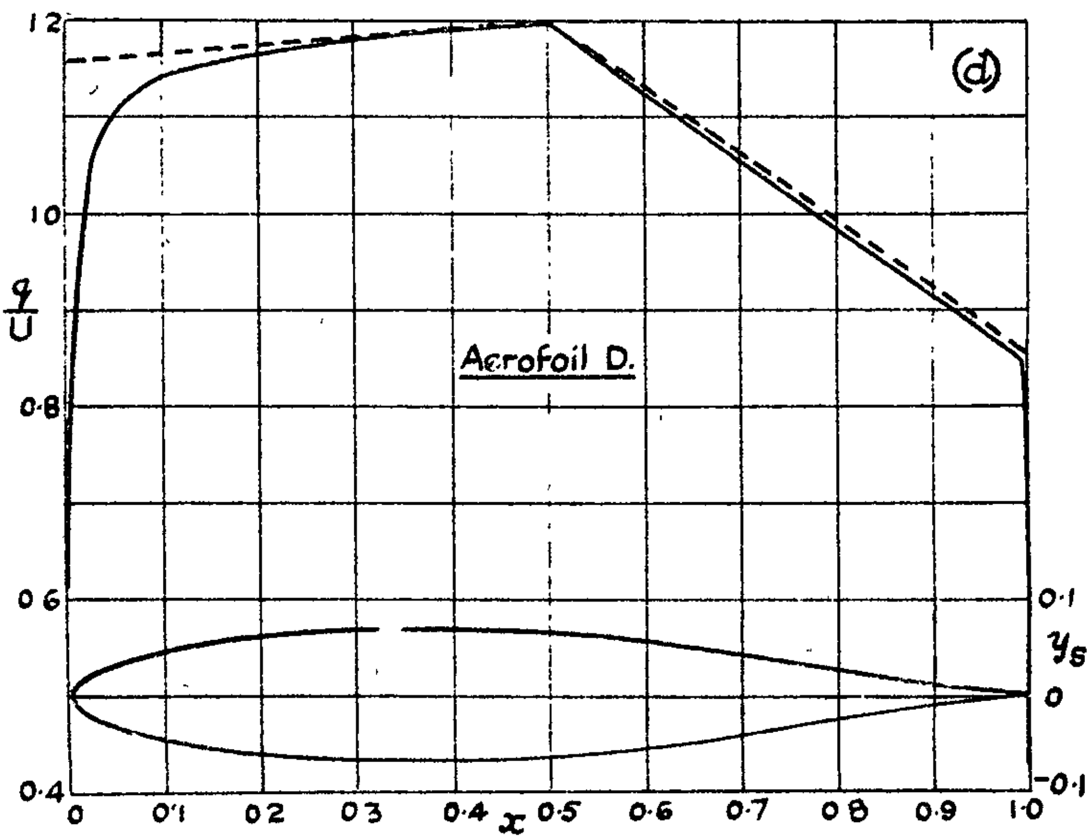
1·2
1·0
0·8
0·6
0·4
q/U
(a)
Aerofoil A.
0·1
y_s
0
-0·1
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
x
1·2
1·0
0·8
0·6
0·4
q/U
(b)
Aerofoil B.
0·1
y_s
0
-0·1
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
x


V.S

FIG. 1 (c & d)

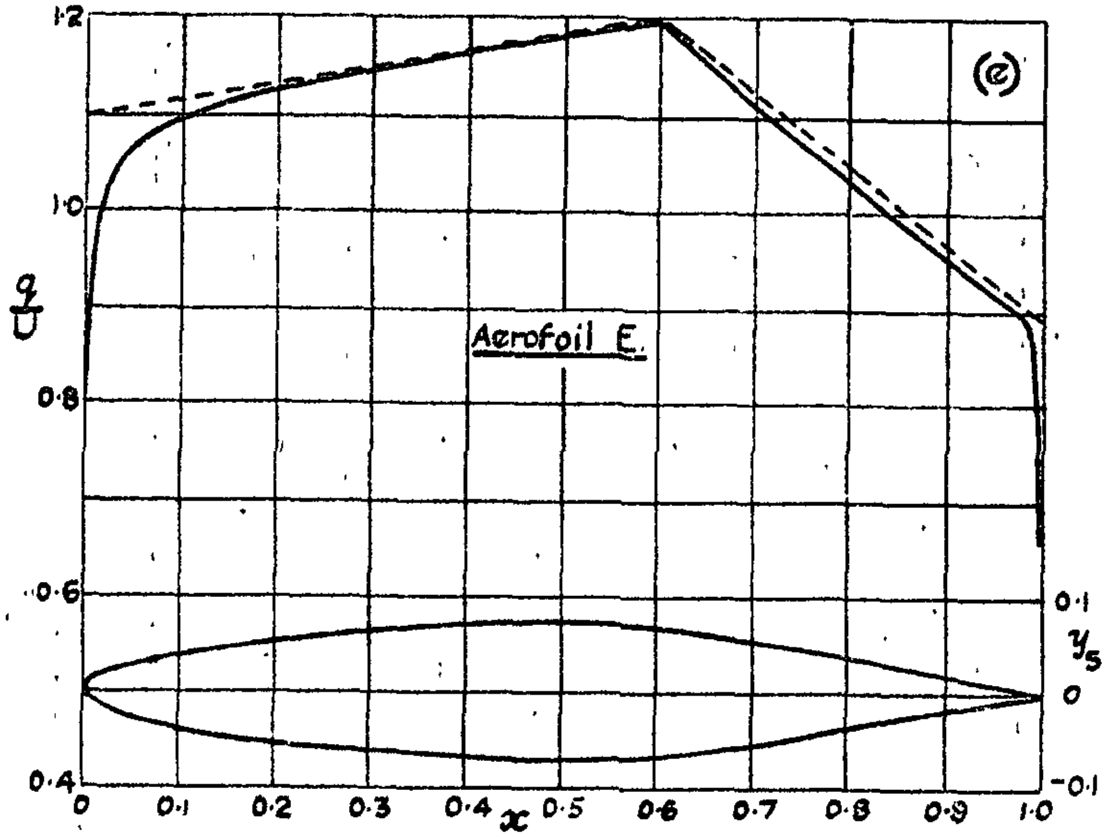


(c)
Aerofoil C.
q/U
1·2
1·0
0·8
0·6
0·4
0·1
$y_s$
0
-0·1
0
0·1
0·2
0·3
0·4
$x$
0·5
0·6
0·7
0·8
0·9
1·0

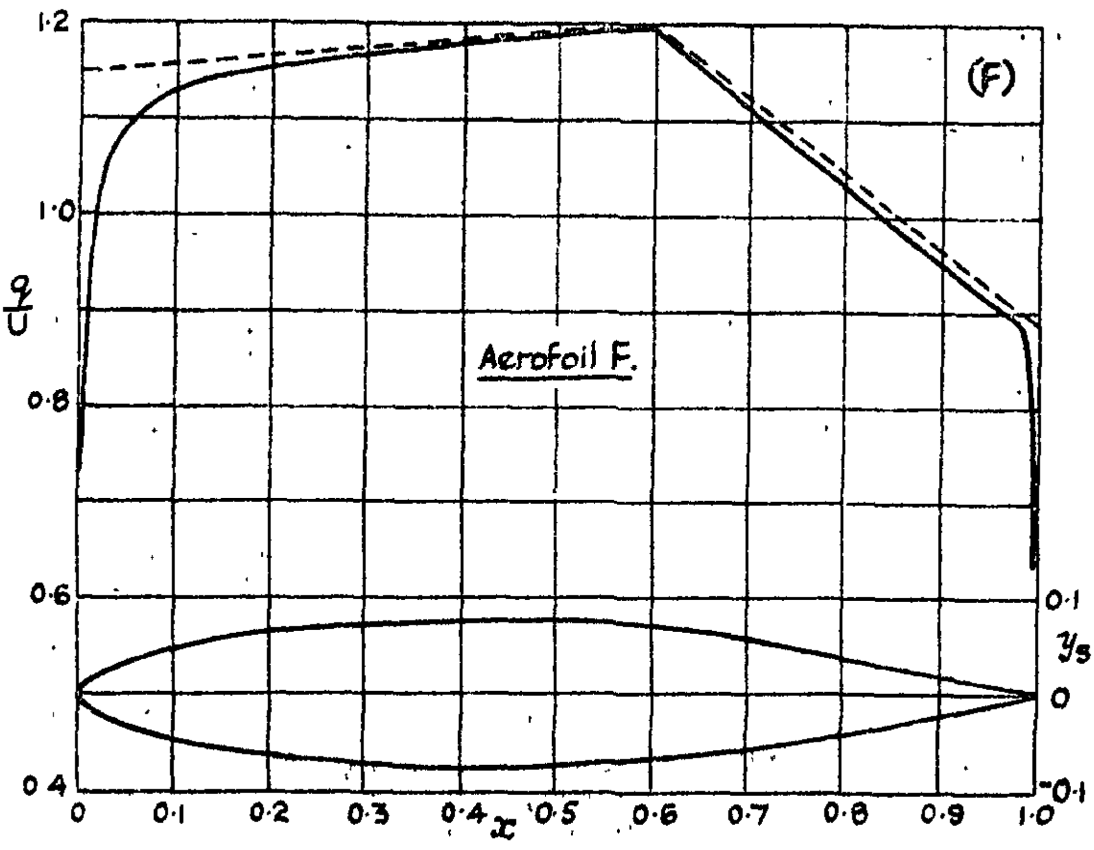



(d)
Aerofoil D.
q/U
1·2
1·0
0·8
0·6
0·4
0·1
$y_s$
0
-0·1
0
0·1
0·2
0·3
0·4
$x$
0·5
0·6
0·7
0·8
0·9
1·0

Fig. 1 (e & f).



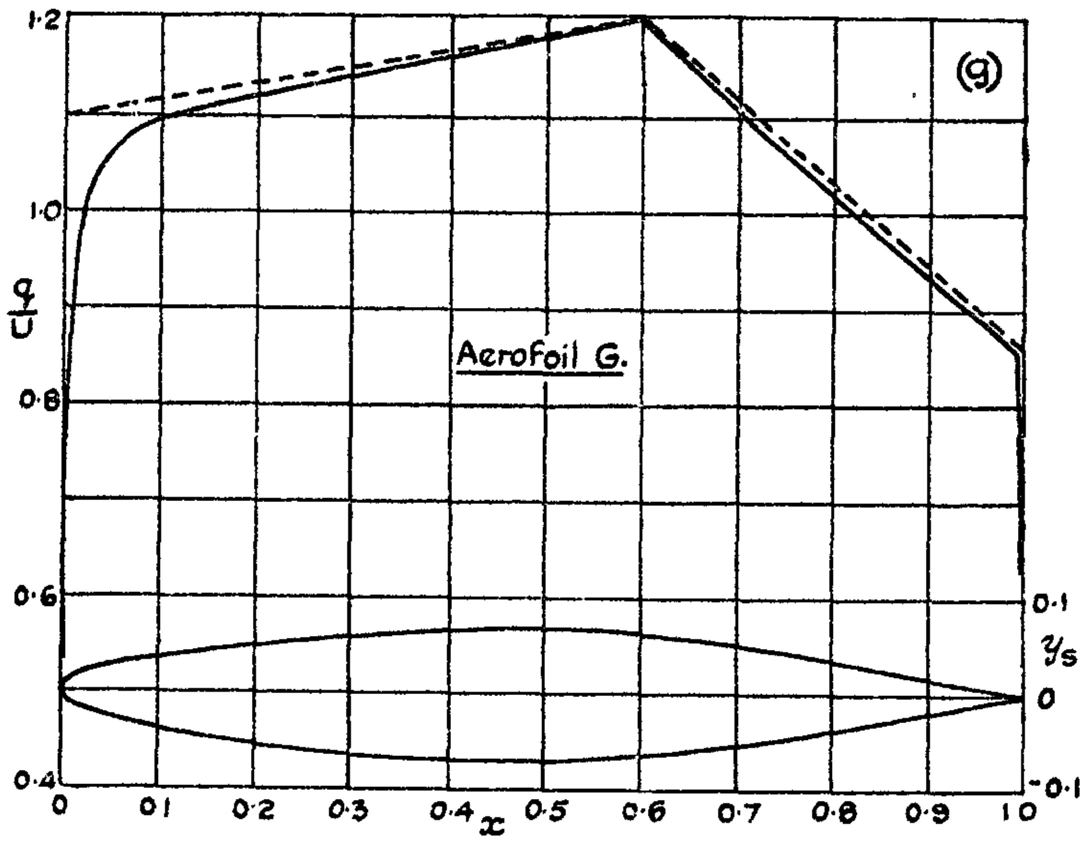
(e)
Aerofoil E.
q/U
1·2
1·0
0·8
0·6
0·4
0·1
$y_s$
0
−0·1
0
0·1
0·2
0·3
0·4
x
0·5
0·6
0·7
0·8
0·9
1·0

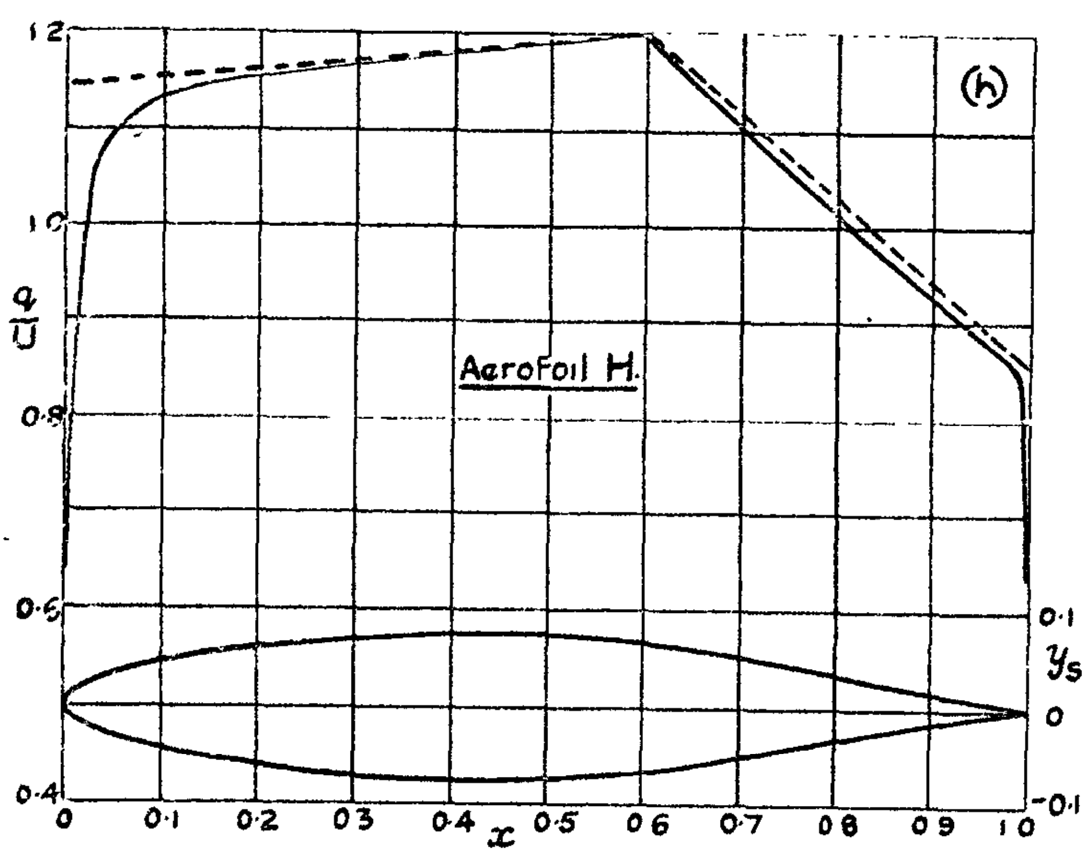
(F)
Aerofoil F.
q/U
1·2
1·0
0·8
0·6
0·4
0·1
$y_s$
0
−0·1
0
0·1
0·2
0·3
0·4
x
0·5
0·6
0·7
0·8
0·9
1·0


V.S.

FIG. 1 (g & h).



1·2
1·0
q/U
0·8
0·6
0·4
(g)
Aerofoil G.
0·1
$y_s$
0
-0·1
0
0·1
0·2
0·3
0·4
x
0·5
0·6
0·7
0·8
0·9
1·0

1·2
1·0
q/U
0·8
0·6
0·4
(h)
Aerofoil H.
0·1
$y_s$
0
-0·1
0
0·1
0·2
0·3
0·4
x
0·5
0·6
0·7
0·8
0·9
1·0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