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The Unsteady Motion of Slender Wings with Leading-Edge Vortices

BY

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The Unsteady Motion of Slender Wings with Leading-Edge Vortices

- By -
B. D. Dore

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SUMMARY

The unsteady motion of thin, slender wings with leading-edge vortioes is considered. Incompressible flow and slender-wing theory are assumed throughout, and use is made of an analogy between the unsteady flow and related steady flows. The particular motions treated are:

(i) a sudden plunging motion, or change of incidence,

(ii) entry into a gust,

For the first case, it is shown that the flow in any transverse plane moving with the wing reaches its steady state as soon as that plane has travelled past the initial position of the leading apex of the wing. An extension of the theory of Brown and Michael is given to determine the strength of the vortex and its path from the leading-edge to the steady-state position, Preliminary results obtained are compared with experiment,

For the second case, use of the analogy shows that quasi-steady theory holds, The flow in any transverse plane is instantaneously in the steady state corresponding to the effective incidence produced there by the gust.

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CONTENTS

1. auction

The flight of slender wings with sharp leading edges is likely to involve flow separation. This phenomenon gives rise to regions of concentrated vorticity above and slightly inboard of the edges, and joined to them by vortex sheets,

The theory of leading-edge separation in steady flow has been considered by several authors [1,2,3] One of the simplest approaches, which forms the basis of the present work, is that given by Brown and Michael[1] for flat delta wings. They replace the concentrated vorticity by an isolated vortex and the vortex sheet by a "cut" joining the vortex to the leading edge. Thus, the velocity potential is one-valued but there is a discontinuity in pressure across tho cut. Brown and Michael use the approximations of slender-wing theory. In any oross-flow plane there are three unknown quantities — the co-ordinates and strength of the vortex. These are determined from the following conditions:

(i) the two components of the force acting on the vortex and cut must be zero,

(ii) the fluid velocity must be finite at the leading edge.

The extension of the Brown and Michael theory to flat wings with curved leading edges has been given by Smith[3].

A/

A theory of leading-edge separation for slender wings in oscillatory motion has been presented by Randall[4,5]. The method is a logical extension of the Brown and Michael theory and is valid when the amplitude of oscillation is small compared with the mean incidence. Hancock[6] has discussed the transient motion of slender delta wings with leading-edge separation, in particular the cases of entry into a sharp-edged gust and of a sudden change of incidence. On the basis of slender-wing theory, he concludes that the solution to the gust problem is trivial; the flow in any transverse plane changes instantaneously from its initial steady state to its final steady state as that plane enters the gust. The argument put forward by Hancock for the case of a sudden change of incidence is based on considerable supposition and will not be discussed here,

In the present note, use is made of an analogy* which relates the unsteady motion of a slender wing with leading-edge vortices to a sequence of steady flows past slender wings of identical planform but with different camber, The usual slender-wing approximations are made and an incompressible fluid is assumed.

2. Notation

C_p	pressure coefficient
k	$= \tan \varepsilon$
p	pressure
p_∞	free-stream pressure
t	time
U	free-stream velocity
x, y, z	Cartesian co-ordinates fixed in wing
x', y', z'	oartesian co-ordinates fixed in space
X, Y, Z	Cartesian co-ordinates in the analogy
x_o	value of x' in reference plane (Fig.1)
Y_v, Z_v	position of vortex in cross-flow plane
α	incidence
γ	defined by equation (23)
Γ	strength of vortex
ε	semi-apex angle of delta wing

$\eta, \zeta/$

---

*In a recent paper (A.R.C.25 118) Lowson has independently adopted the same analogy on the basis of physical considerations and has applied it to oscillatory motion.

2. Notation (continued)

η, ζ	defined by equation (24)
λ	defined by equation (19)
ξ	defined by equation (22)
ρ	free-stream density
σ, τ	defined by equations (27) and (28)
ϕ, ϕ', Φ	perturbation velocity potentials

3. Analogy for Sudden Plunging Motion

Suppose the motion is referred to axes oxyz fixed in a flat, slender wing; origin O at the apex, ox along the wing centre-line, oz normal to the wing, oy completing a right-handed set (Fig.1). For time $t < 0$ the wing moves in the direction of tho negative x-axis with constant velocity U and at zero incidence. At $t = 0$ the wing starts to plunge with uniform velocity $U\alpha$ in the direction of the negative z-axis, so that the incidence changes suddenly from 0 to α. If the total velocity potential is $Ux + U\alpha z + \phi$, the perturbation velocity potential $\phi(x,y,z,t)$ satisfies*

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \qquad \ldots (1)$$

and, for $t < 0$,

$$\phi \equiv 0. \qquad \ldots (2)$$

If the motion is referred to axes fixed in space such that

$$x - Ut = x', \quad y = y', \quad z - U\alpha t = z', \qquad \ldots (3)$$

then

$$\phi(x,y,z,t) = \phi(x' + Ut, y', z' + U\alpha t, t) = \phi'(x',y',z',t) \text{ say}, \qquad \ldots (4)$$

where

$$\frac{\partial^2 \phi'}{\partial y'^2} + \frac{\partial^2 \phi'}{\partial z'^2} = 0 \qquad \ldots (5)$$

and

$$\phi' \equiv 0 \ (t < 0). \qquad \ldots (6)$$

Since (5) is independent of x', each plane $x' =$ constant may be considered separately. Thus, the remaining conditions on ϕ' for each plane $x' =$ constant are as follows:

(i)/

---

*As in both linear unsteady flow and non-linear steady flow it is assumed that $\partial^2\phi/\partial x^2$ may be neglected in the partial differential equation. No rigorous justification is yet available.

(i) at infinity, $$\frac{\partial\phi'}{\partial y'} = 0, \quad \frac{\partial\phi'}{\partial z'} = 0, \qquad \ldots (7)$$

(ii) on the wing, $$\left.\begin{aligned} \frac{\partial\phi'}{\partial z'} &= 0 \quad (t < 0) \\ &= -U\alpha \quad (t \geq 0) \end{aligned}\right\}, \qquad \ldots (8)$$

(iii) there is no flow through the leading-edge vortex sheet, so that

$$\frac{\partial f'}{\partial t} + \frac{\partial\phi'}{\partial x'}\frac{\partial f'}{\partial x'} + \frac{\partial\phi'}{\partial y'}\frac{\partial f'}{\partial y'} + \frac{\partial\phi'}{\partial z'}\frac{\partial f'}{\partial z'} = 0,$$

where $f'(x',y',z',t) = 0$ is the equation of the sheet. Hence, within the concepts of slender-wing theory,

$$\frac{\partial f'}{\partial t} + \frac{\partial\phi'}{\partial y'}\frac{\partial f'}{\partial y'} + \frac{\partial\phi'}{\partial z'}\frac{\partial f'}{\partial z'} = 0, \qquad \ldots (9)$$

(iv) there is no diffarcnce in pressure across the vortex sheet, so that

$$\text{on } f' = 0, \qquad \Delta\left(\frac{\partial\phi'}{\partial t} + \tfrac{1}{2}\left[\left(\frac{\partial\phi'}{\partial y'}\right)^2 + \left(\frac{\partial\phi'}{\partial z'}\right)^2\right]\right) = 0, \qquad \ldots (10)$$

(v) it is required that the flow separates tangentially at the leading edge at each Instant of time.

Consider the situation in some cross-flow plane $x' = x_0$ = constant which contains a section of the wing at $t = 0$ (Fig.1). If

$$X = Ut, \quad Y = y', \quad Z = z', \qquad \ldots (11)$$

tho flow in this plane is analogous to the steady flow U past a wing of the same planform, with local incidence *zero* for $X < 0$ and α for $X \geq 0$ (Fig.2). The analogy exists because the perturbation velocity potential $\Phi(X,Y,Z)$ of the steady problem satisfies

$$\frac{\partial^2\Phi}{\partial Y^2} + \frac{\partial^2\Phi}{\partial Z^2} = 0 \qquad \ldots (12)$$

and $$\Phi \equiv 0 \quad (X < 0). \qquad \ldots (13)$$

The remaining conditions in any cross-flow plane X = constant are

(i) at infinity, $$\frac{\partial\Phi}{\partial Y} = 0, \quad \frac{\partial\Phi}{\partial Z} = 0, \qquad \ldots (14)$$

(ii)/

(ii) on the wing,

$$\frac{\partial\Phi}{\partial Z} = 0 \quad (X < 0),$$
$$= -U\alpha \quad (X \geqslant 0), \qquad \ldots (15)$$

(iii) if $F(X,Y,Z) = 0$ is the equation of the leading-edge vortex sheet,

$$U\frac{\partial F}{\partial X} + \frac{\partial\Phi}{\partial Y}\frac{\partial F}{\partial Y} + \frac{\partial\Phi}{\partial Z}\frac{\partial F}{\partial Z} = 0, \qquad \ldots (16)$$

(iv) there is no difference in pressure across the sheet, so that

$$\text{on } F = 0, \qquad \Delta\left(U\frac{\partial\Phi}{\partial X} + \tfrac{1}{2}\left[\left(\frac{\partial\Phi}{\partial Y}\right)^2 + \left(\frac{\partial\Phi}{\partial Z}\right)^2\right]\right) = 0, \qquad \ldots (17)$$

(v) there is smooth outflow at the leading edge of the wing.

The equivalence of the two corresponding sets of equations (5) to (10) and (12) to (17) is apparent if, in the latter, X is replaced by Ut. The solution of the steady problem may therefore be applied to the unsteady plunging motion. The steady flow characteristics for X = constant give those in the plane $x = x_0 + X$ at time

$$t = \frac{X}{U} = \frac{x - x_0}{U},$$

where $x' = x_0$ is the reference plane in Figs.1 and 2.

An important consequence of the analogy is found when $x_0 = 0$. Then the analogy is with the steady flow past the actual wing at uniform incidence α. The characteristics in any plane X = constant give those in the plane $x = X$ for time $t = x/U$. In other words, for each section x = constant, the steady-state is reached in time x/U. That this state is maintained for $Ut > x$ is easily seen by considering negative x. It is pointed out that the same property holds for slender wings of arbitrary planform and camber, whatever the initial incidence may be.

4. Application to Delta Wing

Application of the analogy is particularly well-suited to delta wings in plunging motion. Consider a delta wing of semi-apex angle ε and leading edge $y = kx$, where $k = \tan\varepsilon$ (Fig.1). Since the incidence is zero for $t < 0$, the analogy leads us to study the steady flow past a trapezium wing at an incidence α (Fig.2). The similarity of the trapezium wings resulting from arbitrary x_0 shows that the flow in each plane X = constant depends only on X/s_0 where $s_0 = kx_0$. The solution for the unsteady plunging motion is then known if the solution is known for one trapezium wing for all positive X. In fact the flow characteristics in the plane

X/

$$X = \frac{\lambda x_o}{1-h}, \qquad \ldots (18)$$

where
$$\lambda = \frac{Ut}{x} \leqslant 1, \qquad \ldots (19)$$

give those in *any* plane x = constant at time t = Xx/U. Thus λ is a unifying parameter in the unsteady problem, This is also apparent from non-dimensional arguments since, for the trapezium wing,

$$\Phi(X,Y,Z) = Us_o\, G\left(\frac{X}{s_o}, \frac{Y}{s_o}, \frac{Z}{s_o}\right). \qquad \ldots (20)$$

The transformation from (X,Y,Z)-space back to (x,y,z)-space shows that, in the unsteady problem, the perturbation velocities and pressure in any cross-flow plane are functions of λ, y/x, z/x. According to (18) the flow characteristics for λ = 1 are found from those in the plane X = ∞. Thus, the steady state is reached when λ = 1 as deduced in the last paragraph of Section 3.

5. Trapezium Wing in Steady Flow

A solution is based on the method of Brown and Michael. The approach will only be briefly outlined here — for a full account the reader is referred to Section 2 of Ref.5.

The equation of the leading edge is

$$\left.\begin{aligned} |Y| &\leq s_o, & X &= 0 \\ |Y| &= s_o(1+k\xi), & X &> 0 \end{aligned}\right\}, \qquad \ldots (21)$$

where
$$\xi = \frac{X}{s_o} = \frac{X}{kx_o}. \qquad \ldots (22)$$

For any plane normal to the wing centre-line let the strength and co-ordinates of the leading-edge vortex be given by

$$\Gamma(\xi) = 2\pi U s_o(1+k\xi)\gamma(\xi), \qquad \ldots (23)$$

$$Y_v + iZ_v = s_o(1+k\xi)(\eta+i\zeta). \qquad \ldots (24)$$

Conditions (i) and (ii) of Section 1 lead to the following two differential equations for $\eta(\xi), \zeta(\xi)$, (cf. Ref.5, equations (20)):

4(1+kξ)/

$$4(1+k\xi)\{\sigma(3\tau^2-\sigma^2)\eta + \tau(\tau^2-3\sigma^2)\zeta + \sigma(\sigma^2+\tau^2)^2\}\zeta\eta'$$
$$+ 4(1+k\xi)\{\tau(\tau^2-3\sigma^2)\eta\zeta - \sigma(3\tau^2-\sigma^2)\zeta^2 - \sigma(\sigma^2+\tau^2)^2\eta\}\zeta'$$
$$= -\alpha(\sigma^2+\tau^2)[2\sigma^2-(\sigma^2+\tau^2)(\tau^2-3\sigma^2)]$$
$$+4\sigma(\sigma^2+\tau^2)^2\left[k + \frac{\alpha'(1+k\xi)}{a}\right]\zeta\ , \qquad \ldots (25)$$

and
$$4\sigma(1+k\xi)\{[\sigma(3\tau^2-\sigma^2)\eta + \tau(\tau^2-3\sigma^2)\zeta][\eta(\eta-1)+\zeta^2] - \sigma(\sigma^2+\tau^2)^2\eta\}\eta'$$
$$+ 4\sigma(1+k\xi)\{[\tau(\tau^2-3\sigma^2)\eta - \sigma(3\tau^2-\sigma^2)\zeta][\eta(\eta-1)+\zeta^2] - \sigma(\sigma^2+\tau^2)^2\zeta\}\zeta'$$
$$= -\alpha\tau(\sigma^2+\tau^2)[(\tau^2-\sigma^2) - (\sigma^2+\tau^2)(\tau^2-3\sigma^2)]$$
$$+4\sigma^2(\sigma^2+\tau^2)^2\left(k[(2\eta-1)\eta+2\zeta^2] + \frac{\alpha'(1+k\xi)}{a}[\eta(\eta-1)+\zeta^2]\right),$$
$$\ldots (26)$$

the dashes representing differentiation with respect to ξ. Here, σ and τ are related to η and ζ by a oonformal transformation such that

$$\sigma^2 - \tau^2 = \eta^2 - \zeta^2 - 1\ , \qquad \ldots (27)$$

$$\sigma\tau = \text{l-g}. \qquad \ldots (28)$$

The vortex strength is found from equation (23) with

$$\gamma(\xi) = \frac{\alpha(\sigma^2 + \tau^2)}{2\sigma}\ . \qquad \ldots (29)$$

Ths solution of equations (25) and (26) can be determined numerioally. However, a certain amount of information can be obtained analytically. For the case of a sudden plunging motion a is constant so that the differential equations simplify slightly with $\alpha' = 0$. The following series expansions then hold for sufficiently small values of ξ:

$$\eta = 1 + \sum_{n=0}^{N} a_{n+1}\ \xi^{n/3}\ , \qquad \ldots (30)$$

$$\zeta = \xi^{2/3}\sum_{n=0}^{N} b_n\ \xi^{n/3}\ , \qquad \ldots (31)$$

where/

where

$$\left.\begin{aligned} a_1 &= -\frac{3}{7}k \\ a_2 &= \left(\frac{\alpha}{4}\right)^{4/3}\left[\frac{9}{16}+\frac{114}{49}\frac{k^2}{\alpha^2}\right] \end{aligned}\right\},$$

$$\left.\begin{aligned} b_0 &= \left(\frac{\alpha}{4}\right)^{2/3} \\ b_1 &= -\frac{1}{7}k \\ b_2 &= \left(\frac{\alpha}{4}\right)^{4/3}\left[\frac{197}{240}-\frac{18}{49}\frac{k^2}{\alpha^2}\right] \end{aligned}\right\}.$$

If $k = 0$ equations (30) and (3-l) give the expansions for a rectangular wing In which case $a_3 = b_3 = 0$.

When the solution in η and ζ has been obtained for the trapezium wing up to some small value of ξ, it can be continued, say, by a Bunge-Kutta process. However, the values of η and ζ at $\xi = \infty$ are those in the steady motion of tho delta wing and are given for example in Table 5 of Ref.5. The values of η' and ζ' at $\xi = \infty$ are zero, but the limiting ratio of ζ' to η' may be found as follows. The ratio of the left-hand-sides of (25) and (26) is equated to the ratio of their right-hand-sides, the limiting value of which is found by differentiating numerator and denominator with respect to ξ (extension of L'Hôpital's rule). This leads to a quadratic in ζ'/η', the coefficients of which are known in terms of $\eta(\infty)$ and $\zeta(\infty)$.

When η and ζ are known the strength of the leading-edge vortex is given by equations (23), (27) to (29). The perturbation velocity potential $\Phi(X,Y,Z)$ can then be found for each cross-flow plane and η',ζ' can be calculated from the differential equations. Hence the pressure coefficient* on the wing

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2} = -\frac{2}{U}\frac{\partial\Phi}{\partial X} - \frac{1}{U^2}\left(\frac{\partial\Phi}{\partial Y}\right)^2 - \alpha^2 , \qquad \ldots(32)$$

can be evaluated.

6./

*This differs from equation (21) of Ref.1 in the sign of the last term, since the X-axis has been-taken parallel to the stream and not along the wing centre-line (Fig.2).

6. Comparison with Experiment

Some experiments/ have been carried out in the N.P.L. Water Tunnel to observe the behaviour of the leading-edge vortices following the sudden plunge of a flat, sharp-edged delta wing with semi-apex angle $\varepsilon = 20°$. Tho particular case $a = 11.3°$ was analysed in detail. Measurements of the vortex position appear to confirm the theoretical conclusion that the flow in each plane x = constant reaches its steady state in time $t = x/U$. Moreover, records of the vortex movements in three such planes tend to indicate that Ut/x is a unifying parameter for the delta wing.

In Figs. 3 and 4 theoretical curves of ζ against η are given to show the effects of incidence and apex angle on the vortex positions during a sudden plunge. The initial portions result from combined use of equations (18), (22), (30) and (31) and the final portions from the limiting values of ζ'/η' (Section 5). The dotted central portions of the curves are roughly faired to these end portions. Figs, 3 and 4 give no indication of the rate of vortex movement.

For the purpose of comparison with experiment η and ζ may be written as

$$\eta = 1 - \frac{3}{7}\lambda + \left[\frac{9}{16} + \frac{114}{49}\frac{k^2}{\alpha^2}\right]\left(\frac{\alpha\lambda}{4k}\right)^{4/3} + 0(\lambda^{5/3}), \qquad \ldots (32)$$

$$\zeta = \left(\frac{\alpha\lambda}{4k}\right)^{2/3} - \frac{1}{7}\lambda + \left[\frac{197}{240} - \frac{18}{49}\frac{k?}{\alpha^2}\right]\left(\frac{ah}{4k}\right)^{4/3} + 0(\lambda^{5/3}), \quad . (33)$$

where $\lambda = Ut/x$ and equations (18), (22), (30) and (31) have been used. Thus, for sufficiently small time, η and ζ depend only on the unifying parameter λ and a single geometric parameter a/k. Consideration of equations (25) and (26) written in terms of λ shows, in fact, that the complete solutions for η and ζ depend only on λ and a/k. From equations (32) and (33) and the final vortex positions in Table 5 of Ref. 5, curves of η and ζ are plotted separately against Ut/x in Fig. 5 for the particular case $a/k = 0.542$ analysed in Ref. 7. These curves are compared with the experimental results from Figs. 6 and 9 of Ref. 7 for the plane $x = \frac{2}{3}$ (root chord). The steady-state vortex positions found experimentally by Alexander[8] are also shown and are seen to give better agreement with the theoretical positions. Further 'experiments in the N.P.L. Water Tunnel are planned; these should provide similar comparisons for more slender ($\varepsilon = 10°$) and less slender ($\varepsilon = 30°$) delta wings in plunging motion.

At this stage it is worth making the following comments:

(I) Considerable differences occur between the Brown and Michael theoretical and the final spanwise vortex positions found in experiment. This can be seen from the following table for $\alpha/k = 0.542$.

Table/

	η	ζ
Theory (Ref. 1)	0.897	0.131
Experiment (Ref. 7)	0.63	0.19
Experiment (Ref. 8)	0.77	0.14

A possible reason for discrepancy is secondary separation which normally occurs in practice. It has been shown by Alexander[8] that, when the secondary separation is eliminated by leading-edge blowing, the spanwise vortex position is closer to that predicted by theory, For example, with blowing, Alexander finds $\eta = 0.81$ when $\alpha/k = 0.542$.

(2) For a sudden plunge, the initial path of the vortex in the cross-flow plane is normal to the wing surface from theory, and apparently from experiment also. If, for example, a' is a non-zero constant in equations (25) and (26), the initial path would be inward and parallel to the wing surface,

(3) In the quadratic for $\zeta'(\infty)/\eta'(\infty)$ referred to in Section 5, the numerical solutions corresponding to Figs. 3 and 4 show one negative and one positive root, The former root has been chosen as more physically realistic in the present examples,

(4) Consideration of the expression for the pressure on the wing shows that, away from the edges, the initial pressure at $t = 0+$ is infinite as $t^{-\frac{1}{3}}$. By contrast, linearized theory gives infinite pressure at $t = 0$, but finite pressure at $t = \infty$.

7. Analogies for some Unsteady Motions

In dealing with the oase of sudden plunging motion the flow in planes x' = constant (fixed in space) was considered, and was referred to (t,y',z')-space. This procedure can be applied to arbitrary unsteady motion. The analogies to which it leads for some particular motions are discussed in this section.

7.1 Entry into a gust

Suppose a slender wing, moving with uniform velocity, enters a gust. Consider a reference plane x' = constant situated within the gust. By applying the procedure mentioned above, it is seen that the flow in this plane is analogous to the steady flow past the given wing at the effective incidence produced by the gust in the plane, Since this property holds for any plane x' = constant it is clear that quasi-steady theory holds for each plane

x/

$x =$ constant moving with the wing. In other words, the flow in each plane $x =$ constant is instantaneously in the steady state corresponding to the effective incidence produced by the gust. In particular, if the gust is sharp-edged, the flow in each plane $x =$ constant changes instantaneously from its initial steady state to its final steady state as the plane enters the gust, Hancock[6] has previously derived this result by a different argument.

Consider the case when the gust is not sharp-edged. Suppose, for example, that the gust velocity increases monotonically to its full value, and consider the variation of pressure at some point of the upper surface as the wing enters the gust. A curve of $-C_p$ against distance travelled into the gust can be plotted for this point using known steady-state pressure distributions, in the case of delta wings[1,2]. This curve will not rise monotonically for all points on the wing upper surface. In fact, if the chosen point lies between the leading edge and the projection on the wing of the final vortex position, $-C_p$ first rises, then falls, and then rises again to its final steady value. This can be seen from Fig.7 of Ref.1 or from theoretical calculations of Mangler and Smith in Fig.7 of Ref.2. The initial peak in $-C_p$ coincides approximately with the instant when the leading-edge vortex is above the chosen point, It is relevant to point out that the phenomenon of an initial peak in the pressure measured at the upper surface of a delta wing has been found experimentally (Fig.11(b) of Ref.9), although it is not clear whether the above theory is the correct explanation.

The case of entry into gusts by wings with curved leading edges is covered by the quasi-steady property referred to above, so that in principle the flow characteristics could be determined by means of Ref.3.

7.2 Other unsteady motions

The analogy can clearly be used to deal with sudden plunging motion of wings with curved leading edges. However, it is not nearly so convenient since Ut/x is no longer a unifying parameter. Consideration would need to be given to the analogy through a sequence of reference planes $x' = x_0$ in order to obtain the flow pattern. A glance at equations (30) and (31) shows that the initial path of the vortex is still normal to the wing surface. In addition it is known that the final position is reached at $Ut/x = 1$.

Other applications of analogies are to the cases of oscillating and deforming wings. However, Ut/x would not be a unifying parameter, even for delta wings. Compensating a little for this difficulty is the fact that the restrictions on the motion are somewhat less severe than in Randall's theory[4,5]. It is sufficient that

(1) the mean incidence is large enough to ensure that the effective local incidence is everywhere positive,

(2) Laplace's two-dimensional equation holds.

Although the first restriction imposes some limitations on amplitude of oscillation, there are several practical cases in which amplitude effects could be investigated. For compressible flow, the second restriction limits applications to oscillations at low frequency.

8./

8. Acknowledgement

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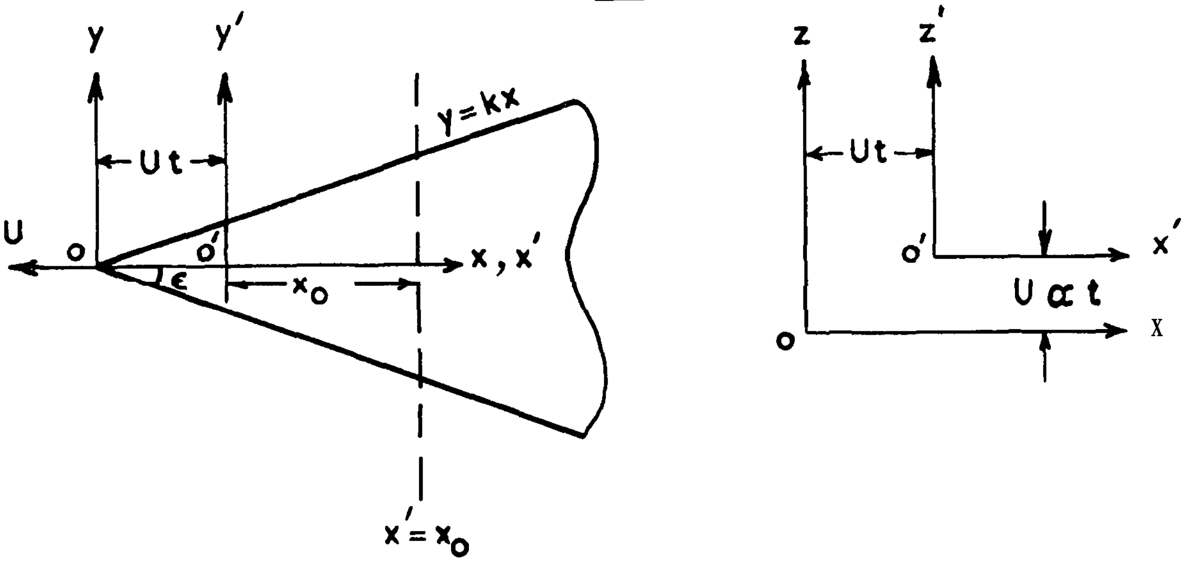
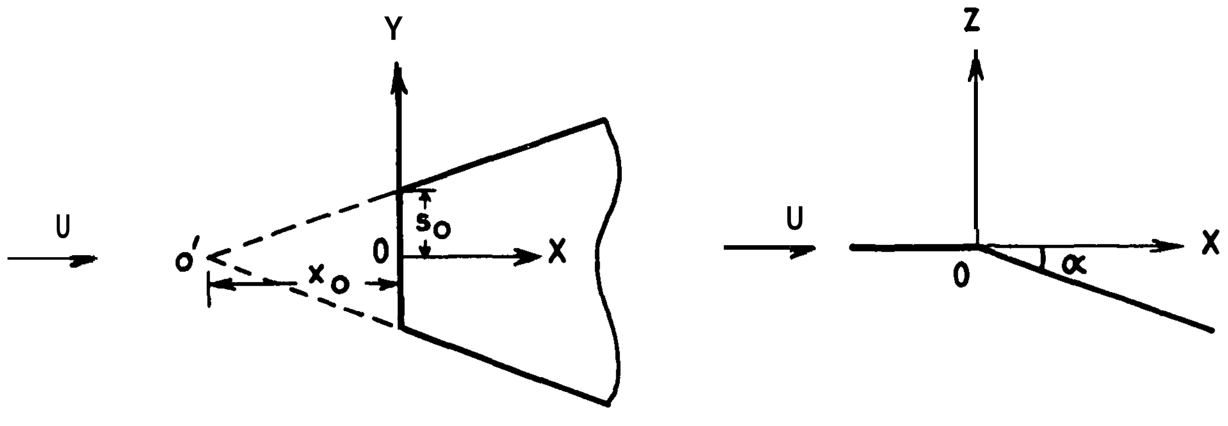
FIGS. 1 & 2

FIG. 1



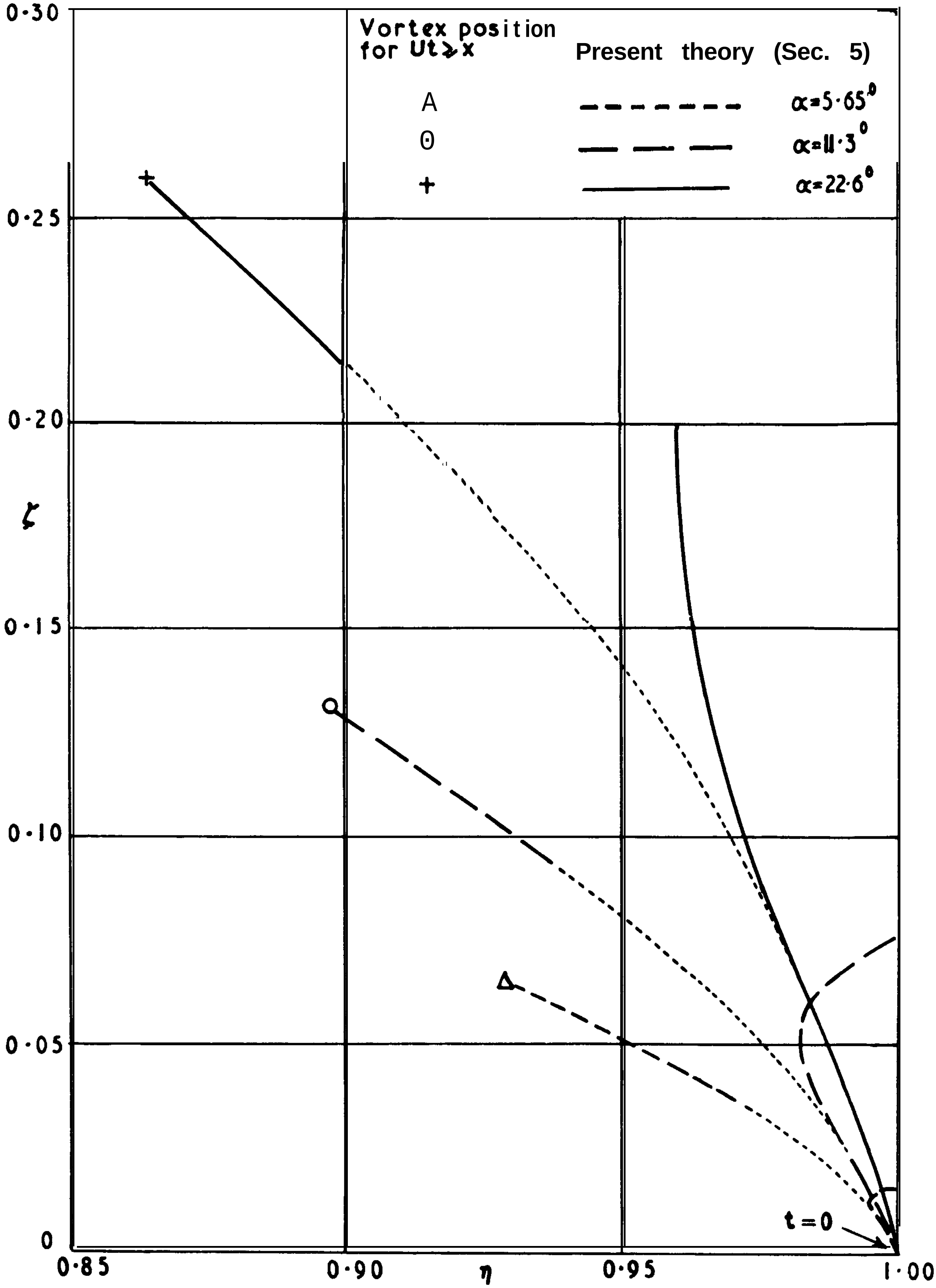
Co-ordinate systems

FIG. 2



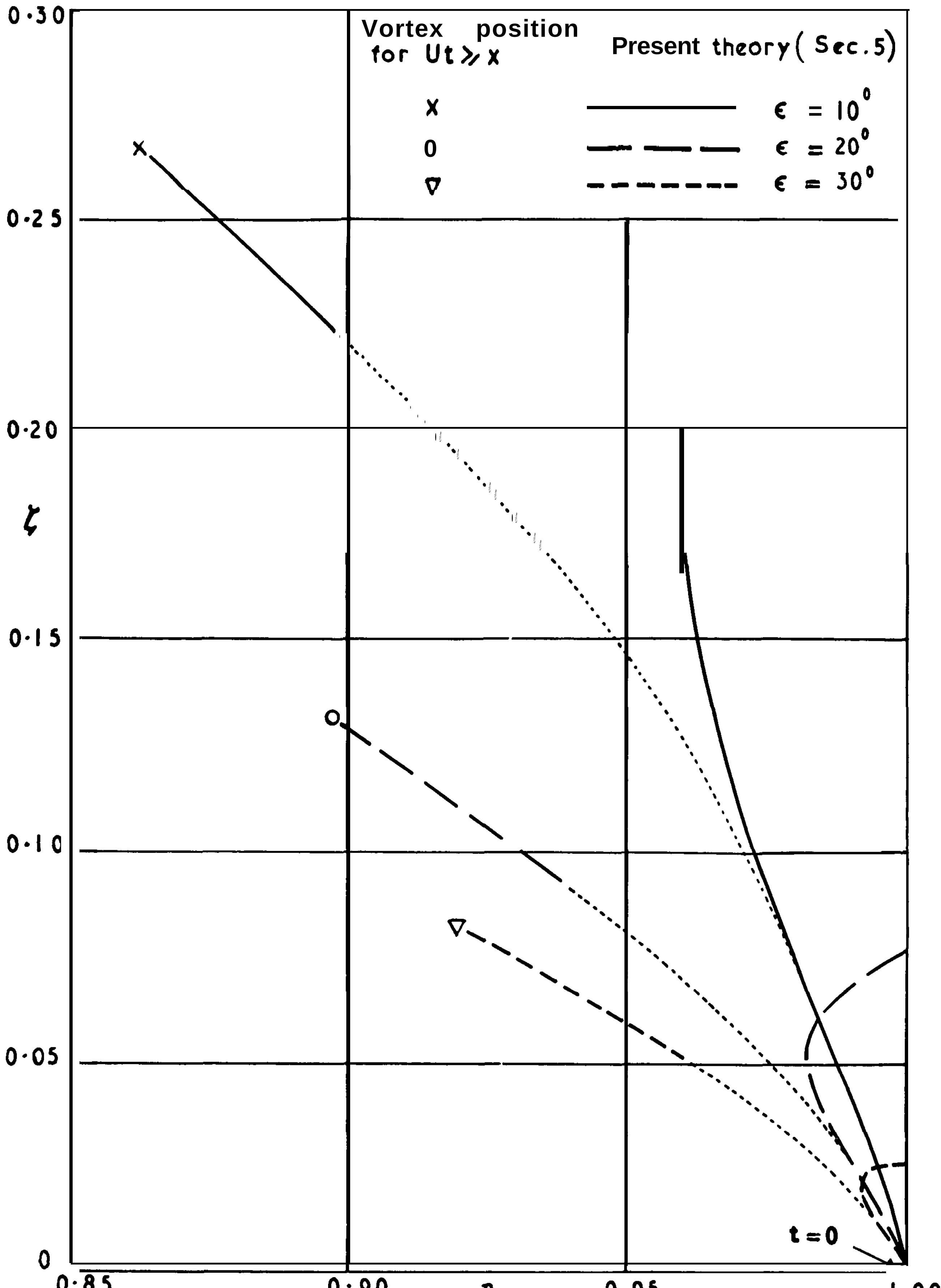
Co-ordinate system in the analogy

FIG.3



Effect of incidence on vortex positions ($\epsilon = 20°$)

FIG.4



Effect of apex angle on vortex positions ($\alpha = 11{\cdot}3°$)

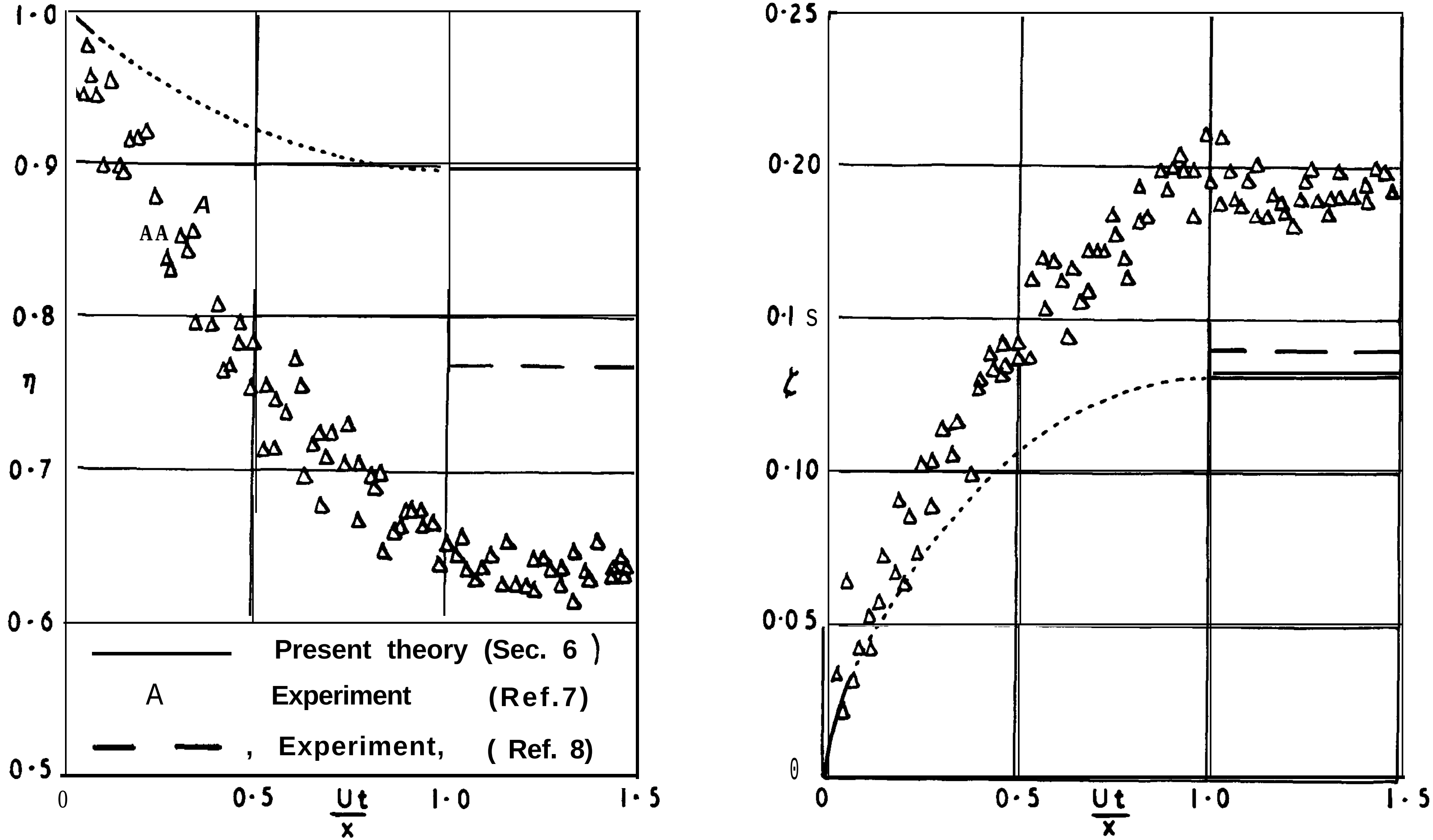


FIG. 5

Theoretical and experimental vortex movement ($\epsilon = 20^\circ$, $\alpha = 11 \cdot 3^\circ$)

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