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Effect of Flow Curvature Due
to the Fuselage on the
Flapping Motion of a
Helicopter Rotor

by

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EFFECT OF FLOW CURVATURE DUE TO THE FUSELAGE ON THE
FLAPPING MOTION OF A HELICOPTER ROTOR

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M. A. P. Willmer

SUMMARY

The effect of flow curvature due to the fuselage on the flapping motion of a helicopter rotor has been investigated theoretically. This has been achieved by calculating the changes in the flow through the disc and hence the additional aerodynamic moment about the flapping hinge, using a Rankine solid to represent the actual fuselage. It has been found that in the tip speed range 0 to 0.6, only the higher harmonics are affected significantly and that the changes become more pronounced as the length of the fuselage is increased.

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1 INTRODUCTION

Although in the past a considerable amount of wind tunnel testing of helicopter rotors has been performed, very little is known about how the rotor flapping motion will be affected in flight by the presence of the fuselage. The airflow will now be forced upwards in the front, downwards at the rear and be unaffected at the sides. This disturbed airstream can cause the flow through the rotor to be changed and hence alter the rotor flapping motion and resultant vibration. The degree to which the flapping will be affected will depend upon, amongst other things, the shape and size of the fuselage as well as the relative position of the fuselage to the rotor or rotors. Attention has been confined in the present investigation to the single rotor helicopter. A representation of the Wessex fuselage has been used, followed by calculations on a lengthened fuselage.

In addition to distorting the main flow field, the presence of the solid body will modify the local blade loading due to "ground effect" phenomena, even though the interference effect is present on only a small part of the disk. This effect might be important in hovering but will become relatively less important with forward speed. It has not been included in the work described in this note.

In order to calculate the disturbed flow, the fuselage has been replaced by the equivalent Rankine solid. These solids, which are described in Ref.1, have the great advantage that the velocities perpendicular to their longitudinal axis can be easily determined. Their shape is also such that the main features of the fuselage are represented. However, like most theoretical inviscid flow solutions to practical problems of this type, no account is taken of the wake produced by the body. This wake, as in the case of flow past a cylinder, will have the effect of reducing the vertical velocities downstream of the fuselage. For present purposes, however, this effect will be neglected, for it is not expected to change the results significantly.

The theory used to determine the blade motion, is based upon that given in Ref.2 and, as in this reference, only the first six harmonics of flapping have been considered. It is valid in the region where simple aerodynamic rotor theory can be applied.

2 THE ANALYSIS

2.1 The fuselage as a Rankine solid

The determination of the flow around a three dimensional body is, in general, very difficult. However, the problem is greatly simplified when the body is a solid of revolution moving in the direction of the axis of revolution. In this case, there can be defined a Stokes' stream function, which is analogous to the stream function of two dimensional hydrodynamic theory.

In Ref.1, it has been shown that when a source and an equal sink are combined with a uniform stream in the negative direction of the x-axis, see Fig.1, the stream function corresponding to the flow past a closed solid of revolution of oval section is easily determined. Such a body is called a Rankine solid. It is solids of this type that this note uses to represent the

fuselage of a helicopter. In Fig.2 is shown a typical Rankine solid superimposed on the fuselage of the Wessex. It can be seen that a very fair approximation to the fuselage shape is obtained. Also, for comparison, Fig.2 gives a Rankine solid corresponding to a lengthened Wessex fuselage where the length of the body is taken to be equal to the diameter of the blades.

From Ref.1, it has been shown that the stream function for flow past a Rankine solid is given by

$$\bar{\psi} = \frac{1}{2} V r^2 \sin^2 \theta + m(\cos \theta_2 - \cos \theta_1) \quad (1)$$

The dividing streamline given by $\bar{\psi} = 0$, generates, by rotation about the x axis, the dividing stream surface. Thus, if the semi-length of the oval is ℓ and the semi-height h , see Fig.1, it can be shown that these are related to the strength and position of the source and sink by the equations

$$(\ell^2 - a^2)^2 = \ell h^2 \sqrt{a^2 + h^2} \quad (2)$$

$$b = \sqrt{\frac{2m}{V}} = \frac{(\ell^2 - a^2)}{\sqrt{2 a \ell}} \quad (3)$$

From equation (2) can be found the distance from the origin of the source and sink for a solid of given dimensions. These would be obtained from the length and height of the fuselage under consideration. Having found 'a' the source and sink strength can be calculated from equation (3). The details of how these calculations can be performed will be left until later in the note.

2.2 The aerodynamic moment on the blade induced by the fuselage

From equation (1) it is possible to obtain the velocities due to the presence of the fuselage in both x and the z' directions. However, these velocities will be the same in any plane which contains the x axis. It is only necessary, therefore, to consider a typical plane at an angle with the vertical in order to calculate the fuselage induced aerodynamic moment on the blades. In Fig.3, the rotor, which is assumed to be horizontal, lies in the plane APBO'. It is at a height k from the centre of the oval representing the fuselage. The point P is a typical point on the blade at spanwise station t at azimuth position ψ . Thus, it can be seen that it is necessary to determine principally the induced velocity at P in the z direction.

It can be shown that the velocities at a point (x, z') in the plane x O z' are given by

$$v_x = -\frac{1}{z'} \frac{\partial \bar{\psi}}{\partial z'} ; \quad v_z' = \frac{1}{z'} \frac{\partial \bar{\psi}}{\partial z'} \quad (4)$$

Thus at the point P the induced velocity in the z direction can be shown to be

$$\frac{v_z}{V} = \frac{b^2 z' \cos \varphi}{2} \left\{ \frac{1}{[(x-a)^2 + z'^2]^{3/2}} - \frac{1}{[(x+a)^2 + z'^2]^{3/2}} \right\}. \quad (5)$$

In order to calculate the aerodynamic moment, it is necessary to express v_z in terms of the rotor coordinates t and ψ . Hence,

$$\frac{v_z}{V}(t, \psi, k) = \frac{b^2 k}{2} \left\{ \frac{1}{[k^2 + t^2 + a^2 + 2at \cos \psi]^{3/2}} - \frac{1}{[k^2 + t^2 + a^2 - 2at \cos \psi]^{3/2}} \right\}. \quad \dots\dots (6)$$

From reference 2, it can be shown that this induced velocity increases the aerodynamic moment on the blade by

$$\Delta M = \frac{1}{2} \rho \bar{a} c \Omega^2 R^4 \int_0^B \frac{v_z}{\Omega R} s(s + \mu \sin \psi) ds \quad (7)$$

where

$$s = tR. \quad (8)$$

For convenience this increment of aerodynamic moment is written in the form

$$\frac{\Delta M}{\frac{1}{2} \rho \bar{a} c \Omega^2 R^4} = I_1 + I_2 \quad (9)$$

where

$$I_1 = \int_0^B \frac{v_z s^2 ds}{\Omega R} \quad (10)$$

$$I_2 = \int_0^B \frac{v_z}{\Omega R} s ds \mu \sin \psi. \quad (11)$$

From equations (6), (10) and (11) it can be shown that

$$I_1 = F_1(\psi) - F_1(\psi + \pi) \quad (12)$$

$$I_2 = F_2(\psi) + F_2(\psi + \pi) \quad (13)$$

where

$$F_1(\psi) = \frac{\mu b'^2 k'}{2} \left\{ \log \left(\frac{1 + a' \cos \psi + \sqrt{1 + 2a' \cos \psi + a'^2 + k'^2}}{a' \cos \psi + \sqrt{a'^2 + k'^2}} \right) + \frac{1}{k'^2 + a'^2 \sin^2 \psi} \left[\frac{a'^2 \cos 2\psi - k'^2 + a' \cos \psi (a'^2 + k'^2)}{\sqrt{a'^2 + k'^2 + 1 + 2a' \cos \psi}} - a' \cos \psi \sqrt{k'^2 + a'^2} \right] \right\} \dots\dots (14)$$

$$F_2(\psi) = \frac{\mu^2 b'^2 k'}{2} \sin \psi \times \frac{1}{k'^2 + a'^2 \sin^2 \psi} \left\{ \sqrt{k'^2 + a'^2} - \frac{a' \cos \psi + a'^2 + k'^2}{\sqrt{1 + 2a' \cos \psi + a'^2 + k'^2}} \right\} \dots\dots (15)$$

and

$$k' = k/R$$

$$a' = a/R$$

$$b' = b/R .$$

Using a typical value of k' based on the ratios of height of Wessex rotor above the fuselage to rotor radius, and taking $a' = 1$, i.e. the semi-distance between the sources equal to the rotor radius, the two components of I_1 and I_2 are plotted, against ψ in Fig.4. It can be seen that the function I_1 is the main contribution to ΔM and has maximum and minimum values when the blade lies in the direction of motion of the helicopter. This is as one would expect from physical considerations, the maximum occurring in the fore position when the flow is forced upwards by the presence of the fuselage and the minimum occurring in the aft position as the flow theoretically bends downwards following the curve of the body.

Fig.5 investigates these maximum and minimum values more closely and it can be seen that the ratio of the semi-fuselage length to the rotor radius is important. From the graph, I_1 is a maximum when $l' = l/R$ is just less than unity and falls away rapidly on either side. This feature is again not unexpected.

2.3 The effect on rotor flapping

In Ref.2, the equations connecting the coefficients of rotor flapping are given when no fuselage effect is present. The flapping of the rotor is assumed to be expressed in the form

$$\beta = a_0 - \sum_{n=1}^N (a_n \cos n \psi + b_n \sin n \psi) \quad (17)$$

when N harmonics are considered.

The equations which are obtained from equating the aerodynamic moment and the inertia moment about the flapping hinge, reduce to a set of simultaneous linear equations for the flapping coefficients a_0 , a_n , and b_n . The introduction of the fuselage effect increases the aerodynamic moment so that the moment equation can be written in the form

$$M_{nF} - M_{IN} = -\Delta M \quad (18)$$

where M_{nF} = aerodynamic moment with no fuselage present

M_{IN} = inertia moment

ΔM = additional aerodynamic moment due to the fuselage effect

Now the additional aerodynamic moment ΔM given by equations (9) - (15), can be written in the form

$$\frac{\Delta M}{\frac{1}{2} \rho \bar{a} c \Omega^2 R^4} = \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos n \psi + B_n \sin n \psi) \quad (19)$$

where

$$A_j = \frac{1}{\pi} \int_0^{2\pi} (I_1 + I_2) \cos j \psi \, d\psi \quad (j = 0, 1, 2, \dots) \quad (20)$$

$$B_j = \frac{1}{\pi} \int_0^{2\pi} (I_1 + I_2) \sin j \psi \, d\psi \quad (j = 1, 2, 3, \dots) \quad (21)$$

and where the functions I_1 and I_2 are given by equations (12) to (15). In this way, a set of linear simultaneous equations can be established which are similar to those of Ref. 2. It will be noticed from equation (18) that the only change occurs on the right hand sides of the new equations, which will now have the appropriate term of equations (19) to (21), instead of zero value which applies when the fuselage effect is not present.

The integrals of equations (20) and (21) are very difficult to determine analytically so in the cases considered the integrations were performed numerically.

2.4 The calculation of the Rankine solid parameters

Before the computation of the effect of a fuselage on the flapping motion of its rotors can be performed, it is necessary to calculate the parameters a' and b' from the fuselage dimensions. Three parameters correspond to the position and strength of the sink and source which determine the shape of the Rankine solid which will be used to represent the fuselage. Thus if it is assumed that the ℓ and h in equations (2) and (3) can be considered known, the next step is to determine a and b . Thus from equation (16) the non-dimensional forms a' and b' can be calculated. In general, it is a difficult problem to solve equation (2) for a in terms of ℓ and h analytically. However, by considering the dependent variable to be $(h/\ell)^2$ and the independent variable to be $(a/\ell)^2$ equation (2) reduces to a cubic. Thus the solutions are given by

$$(h/\ell)^2 = -\frac{1}{3} \left(\frac{a}{\ell}\right)^2 (1 + 2 \cos \bar{\theta}) \quad (22)$$

$$\bar{\theta} = \frac{1}{3} \cos^{-1} \left\{ 1 - \frac{27}{2} \frac{[1 - (a/\ell)^2]^4}{(a/\ell)^6} \right\} \quad (23)$$

or

$$(h/\ell)^2 = \frac{1}{3} (a/\ell)^2 (2 \cosh \bar{\theta} - 1) \quad (24)$$

$$\bar{\theta} = \frac{1}{3} \cosh^{-1} \left\{ \frac{27}{2} \frac{[1 - (a^2/\ell^2)^2]^4}{(a/\ell)^6} - 1 \right\}. \quad (25)$$

These solutions are plotted in Figs.6A and B where for a given value of $(h/l)^2$ the corresponding value of $(a/l)^2$ can be swiftly found. The value of b and hence b' can then be obtained from equations (3) and (16).

3 DISCUSSION OF RESULTS

Results of the theory are given in Figs.7 to 18 where the variation of the flapping coefficients with tip speed ratio is shown. Three cases have been considered viz. no fuselage present, the Wessex fuselage represented by the equivalent Rankine solid and a solid with the same height as the Wessex but with the same length as the rotor diameter.

Generally, the results show no major departures from the behaviour of an isolated rotor. When the solids representing the fuselages are introduced, the lower harmonic flapping coefficients are not affected. The influence of the shorter fuselage is first felt in the b_3/a_0 term, see Fig.12, but the changes are only very slight. Larger effects appear in the 5th and 6th harmonics, see Figs.15 to 18. From these figures it can be seen that the flapping is decreased by the introduction of the fuselage and in some cases the sign is reversed. In the case of the larger fuselage, Figs.15, 17 and 18 show that a maximum negative value is obtained for a_5/a_0 , a_6/a_0 , b_6/a_0 when the tip speed ratio lies between 0.35 and 0.5. Indeed, it can be seen that at $\mu = 0.6$ the numerical value of the coefficient (a_5/a_0) is less than its value at $\mu = 0.1$.

For the above cases only one value of k has been used. The selected value corresponds to a rotor 2 ft above the Wessex fuselage.

4 CONCLUSIONS

4.1 A theory has been developed to determine the effect of the curved flow field due to the presence of a helicopter fuselage on the harmonics of the flapping of the rotor.

4.2 It has been shown that the lower harmonics of the rotor flapping are not affected either by the presence of a fuselage of the Wessex type or by a long fuselage of the same length as the rotor diameter.

4.3. The fuselage affects the higher harmonics of flapping but not to any major extent. Both fuselages considered decreased the values for the fifth and sixth harmonic terms and sometimes the sign was reversed. In the case evaluated, the terms a_5/a_0 , a_6/a_0 and b_5/a_0 had maximum negative values when the long fuselage was considered. For these maximum values, the tip speed ratio was in the range 0.35 to 0.5.

LIST OF SYMBOLS

a	= position of source and sink
\bar{a}	= lift curve slope
a_o, a_n, b_n	= flapping coefficients
A_o, A_n, B_n	= harmonic components of aerodynamic moment due to the presence of the fuselage
b	= sink and source parameter $\sqrt{\frac{2m}{V}}$
B	= tip loss factor
c	= chord
h	= semi-height of Rankine solid
k	= height of rotor above centre of fuselage
ℓ	= semi length of Rankine solid
m	= strength of sink and source
M_{nF}	= lift moment about flapping hinge when no fuselage is present
M_{IN}	= inertia moment about flapping hinge when no fuselage is present
ΔM	= increment of lift moment about flapping hinge due to presence of fuselage
N	= number of harmonics considered
r, θ	= coordinates of point P in OXZ' plane
R	= blade radius
t	= spanwise rotor coordinate
V	= forward velocity
$v_x, v_{z'}$	= velocities in x and z' directions respectively
v_z	= velocity of flow in z direction
x, z'	= coordinates of a typical point P
z	= coordinate perpendicular to direction of flight
β	= flapping angle

LIST OF SYMBOLS (Contd.)

Ω = speed of rotation

ρ = density of air

ϕ = angle between OXZ' and OXZ planes

ψ = blade azimuth position measured from downward position in direction of rotation

\bar{O} = defined by equations (23) and (25)

μ = tip speed ratio

$\bar{\Psi}$ = stream function

LIST OF REFERENCES

<u>No.</u>	<u>Author</u>	<u>Title, etc.</u>
1	Milne-Thomson, L.M.	Theoretical hydrodynamics. Macmillan and Co. Ltd. 1949.
2	Stewart, W.	Higher harmonics of flapping on the helicopter rotor. A.R.C. C.P. 121. March, 1952.

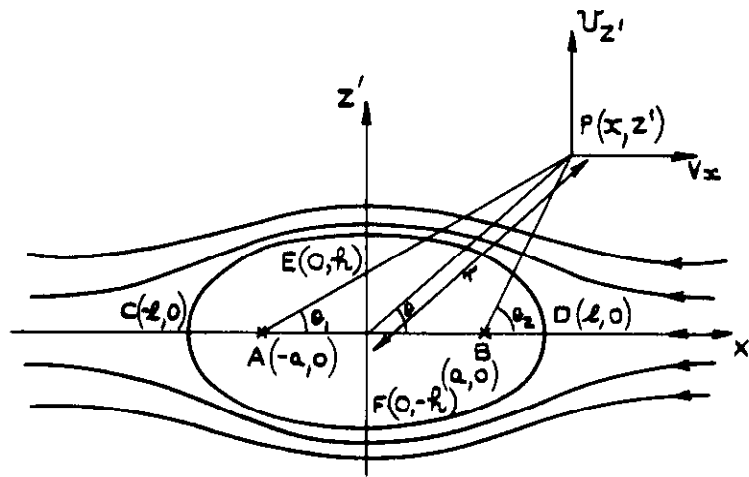


FIG. 1. THE FLOW PAST A RANKINE SOLID.

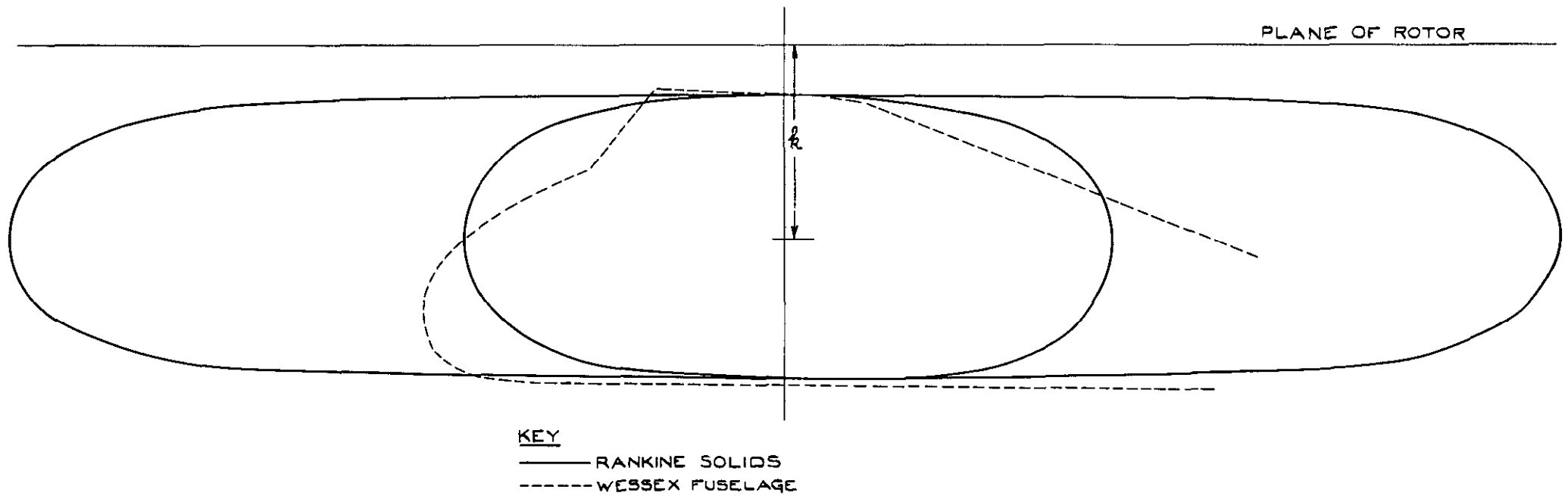


FIG 2. THE COMPARISON BETWEEN RANKINE SOLIDS AND THE WESSEX FUSELAGE.

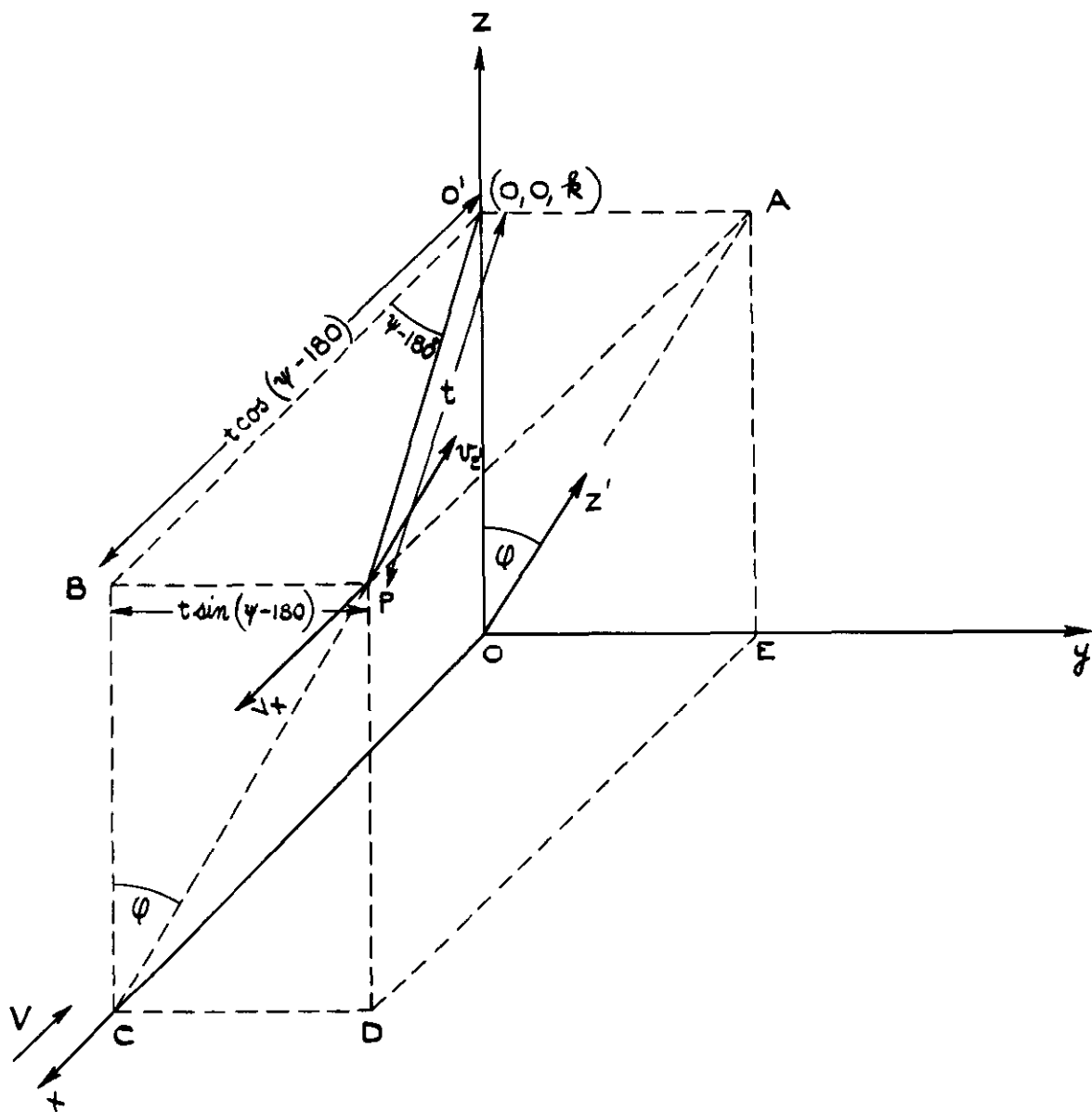


FIG. 3. DIAGRAM SHOWING THE COORDINATE SYSTEMS.

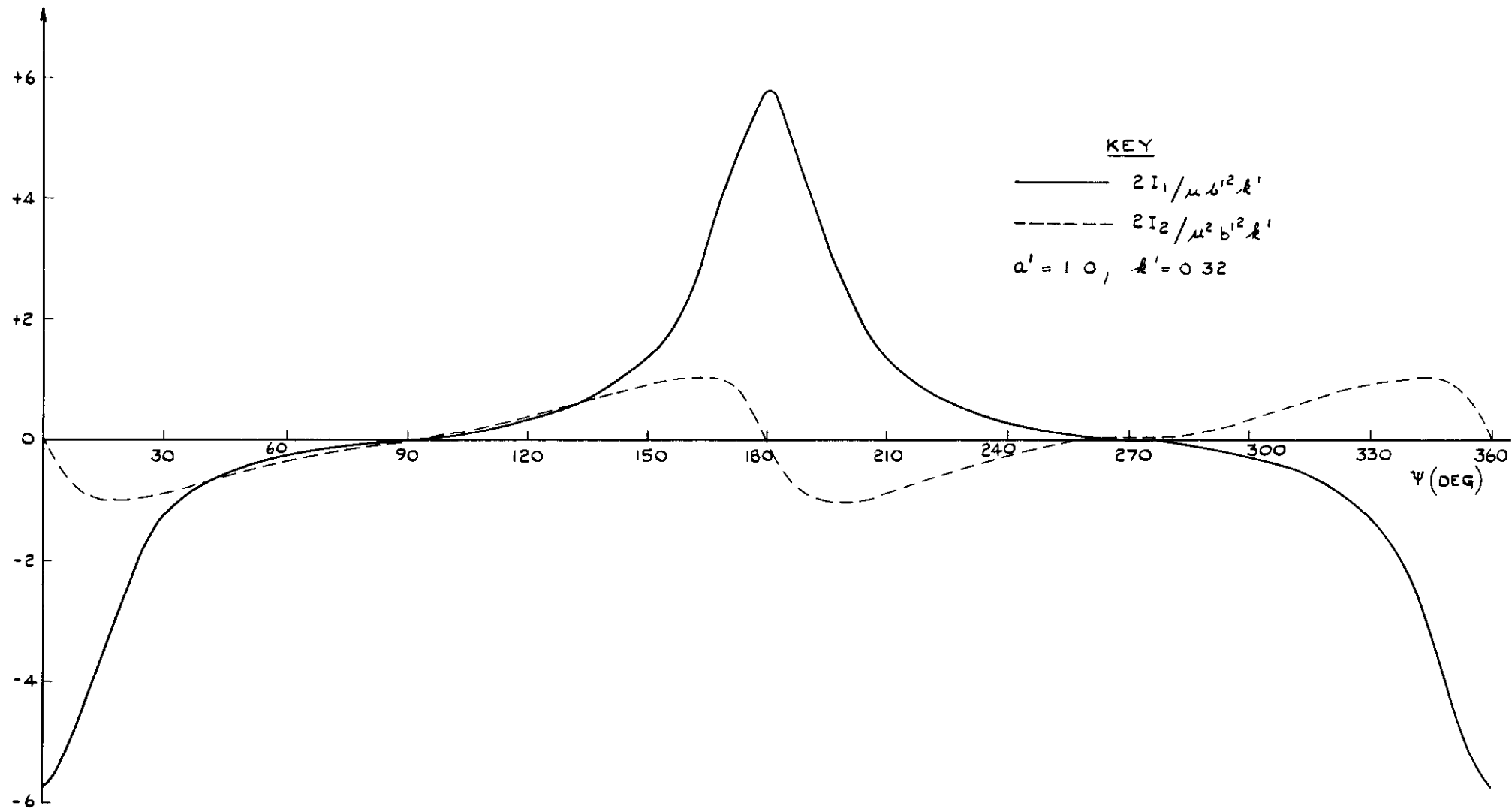


FIG. 4. VARIATION OF I_1 & I_2 WITH ψ .

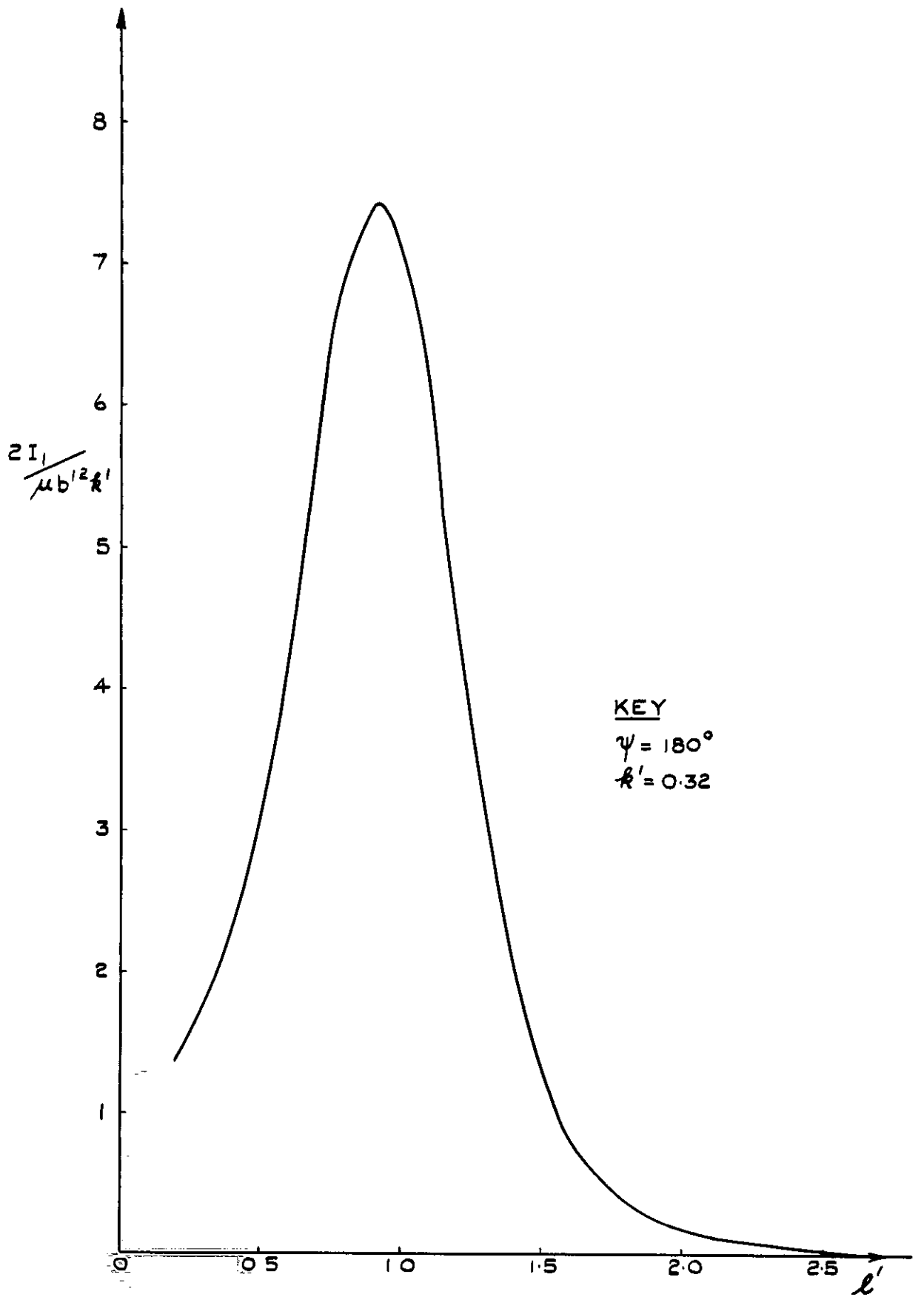


FIG. 5. VARIATION OF I_1 WITH ℓ' AT $\psi = 180^\circ$.

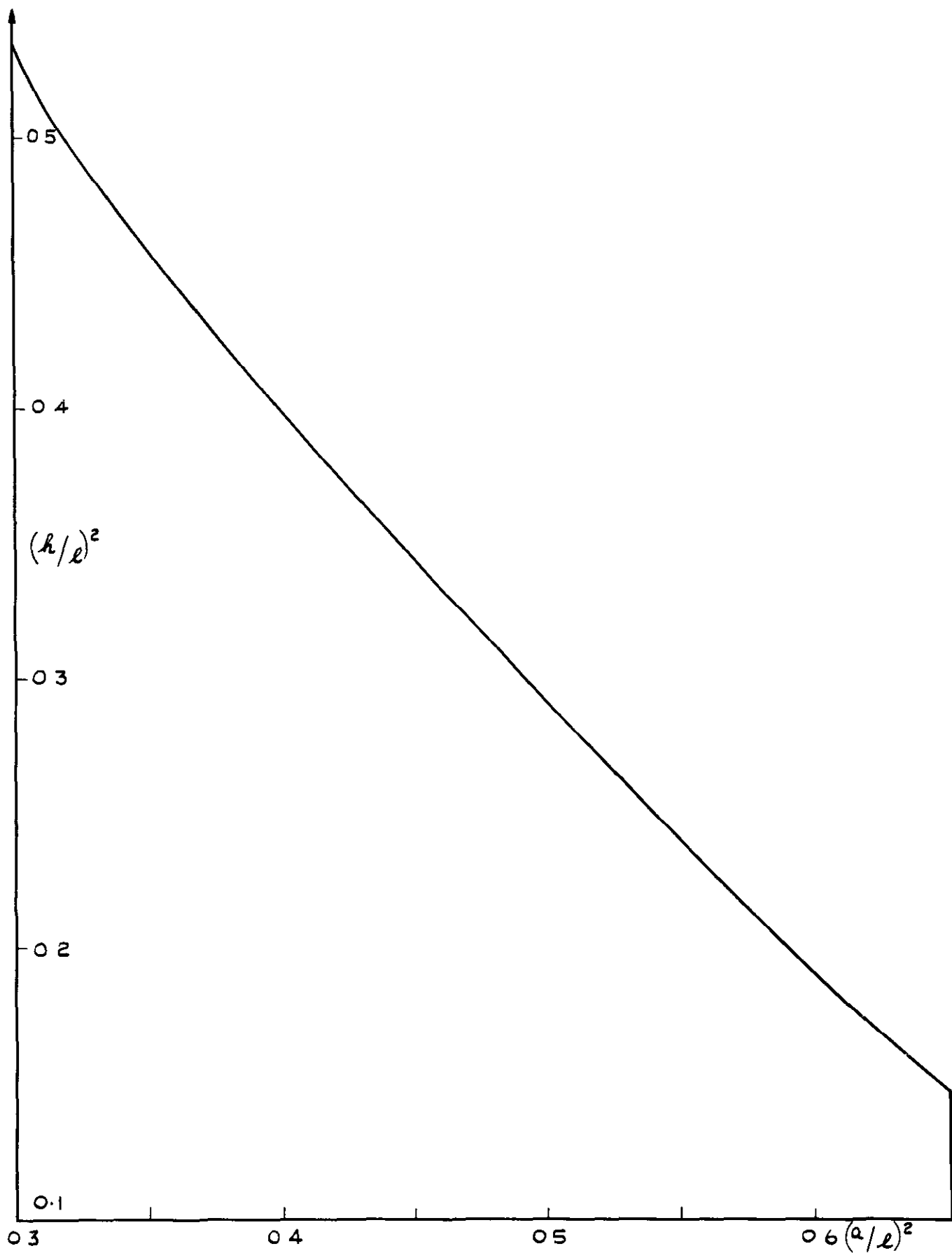


FIG. 6.(a) VARIATION OF $(k/l)^2$ WITH $(a/l)^2$; $0.3 \leq (a/l)^2 \leq 0.65$.

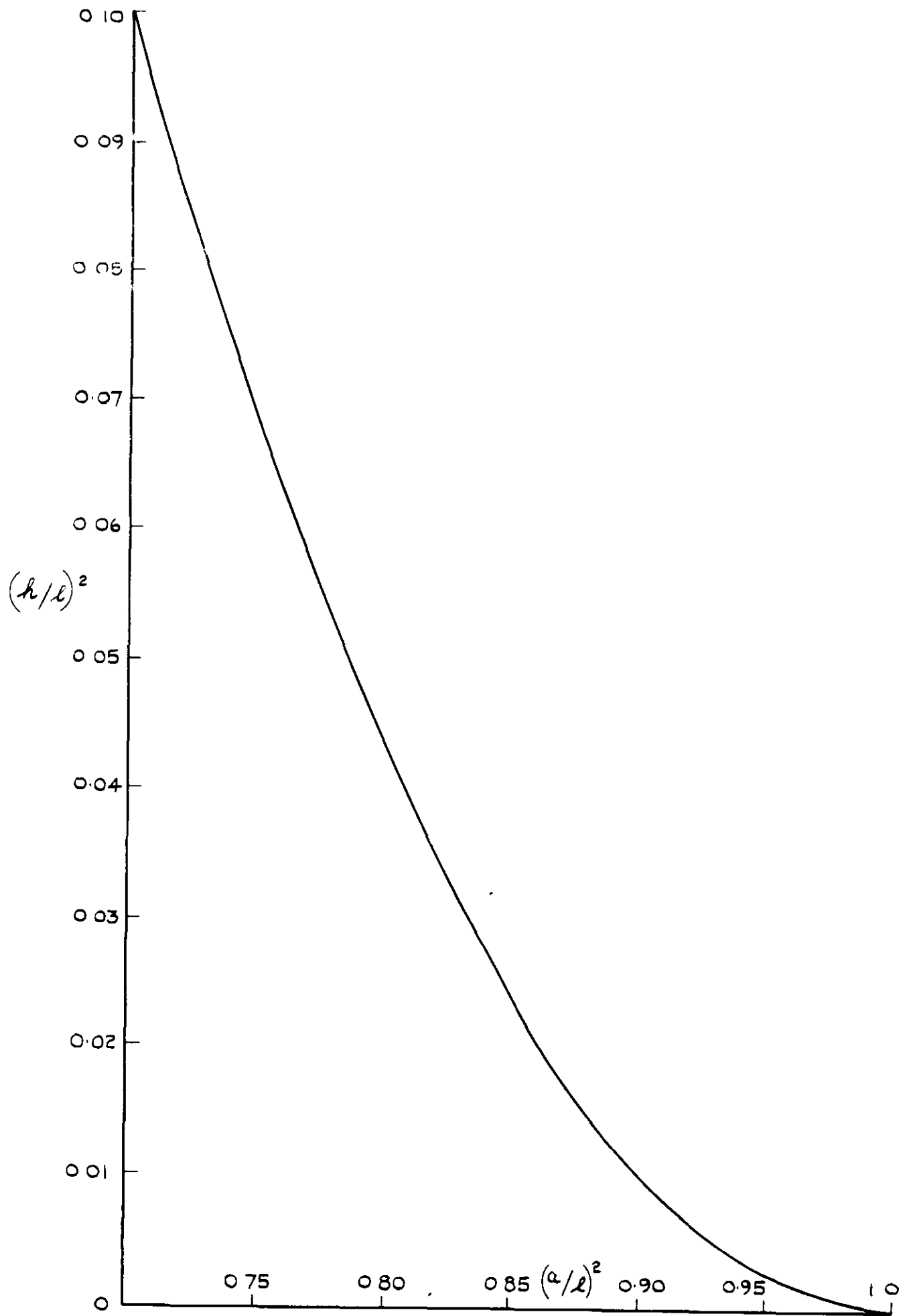


FIG. 6.(b). VARIATION OF $(h/l)^2$ WITH $(a/l)^2$; $0.75 \leq (a/l)^2 \leq 1.0$.

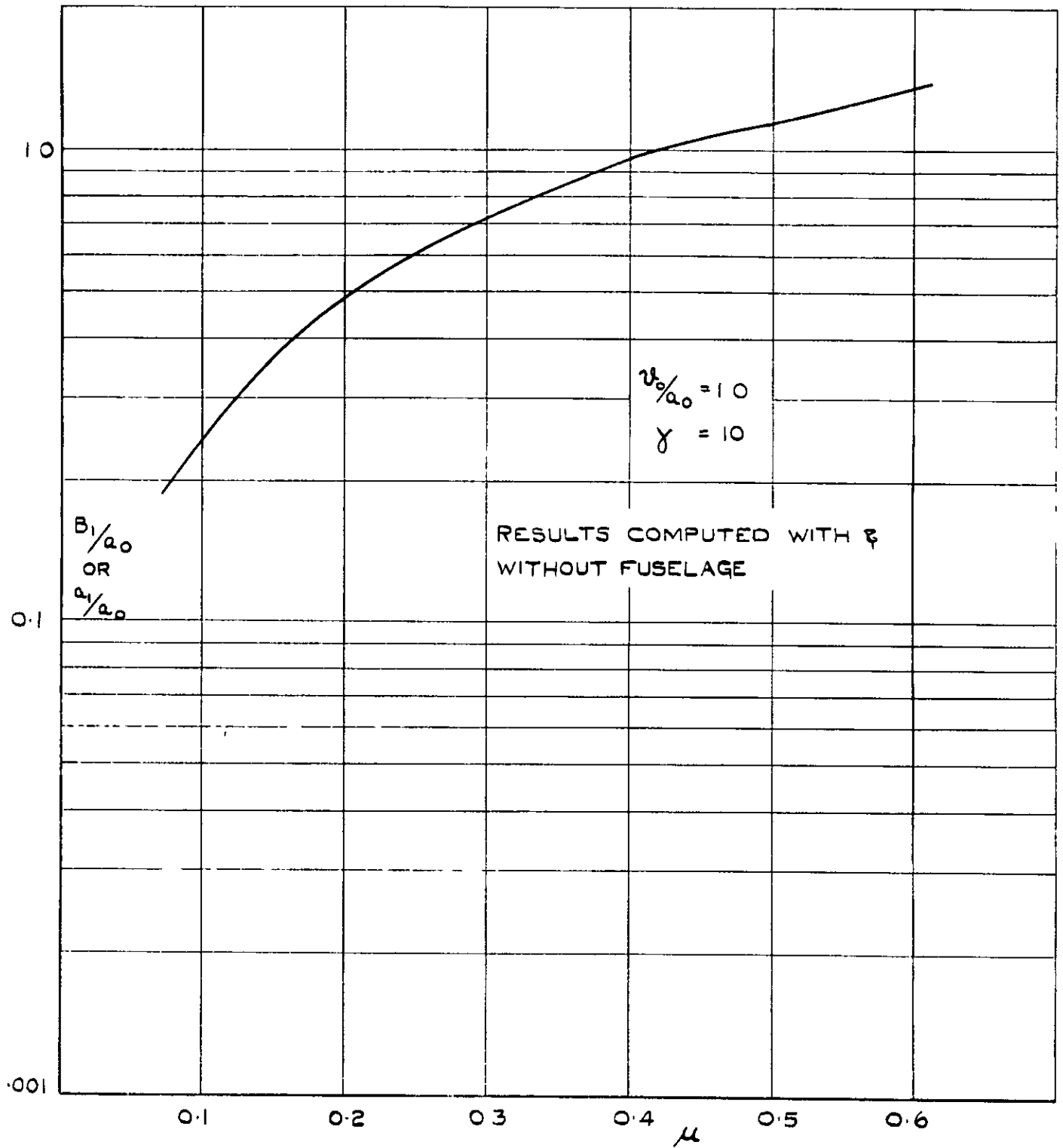


FIG. 7. B_1/a_0 OR a_1/a_0 v TIP SPEED RATIO.

$$\frac{v_0}{a_0} = 1.0$$

$$\gamma = 10$$

RESULTS COMPUTED WITH AND
WITHOUT FUSELAGE

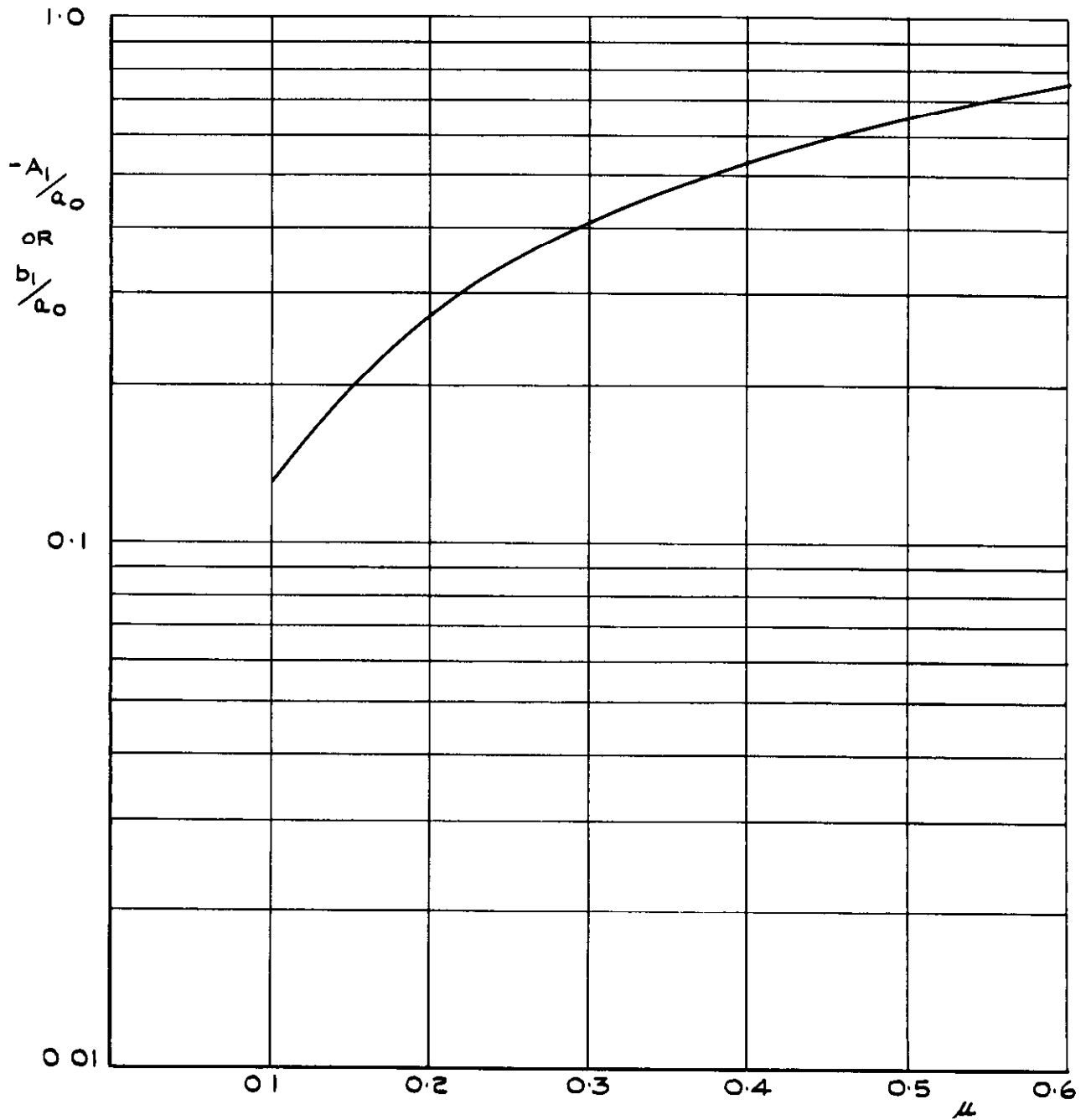


FIG. 8. $-A_1/a_0$ OR b_1/a_0 vs. TIP SPEED RATIO.

$$U_0/a_0 = 1.0$$

$$\gamma = 10$$

RESULTS COMPUTED WITH AND
WITHOUT FUSELAGE

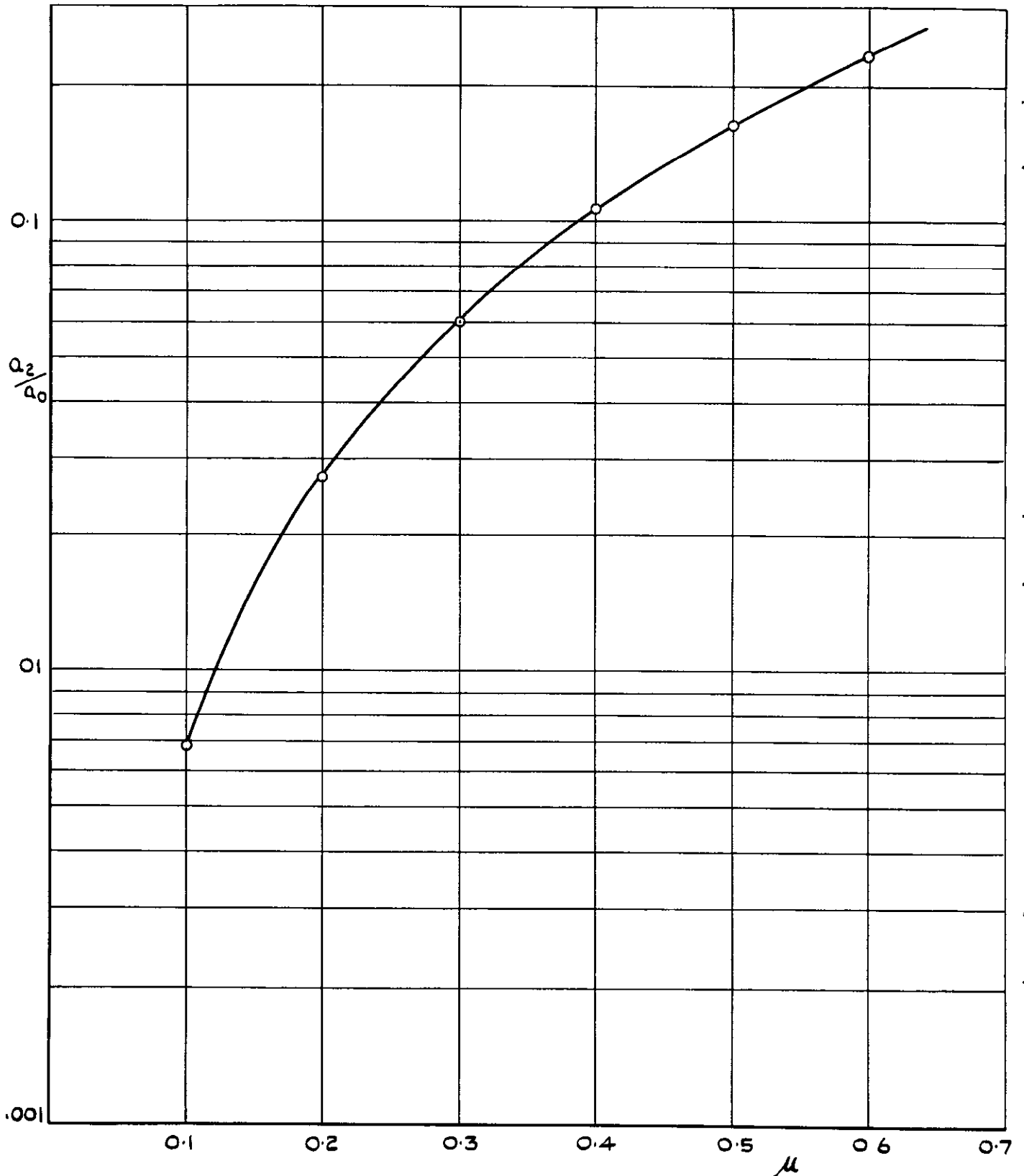


FIG. 9. a_2/a_0 vs TIP SPEED RATIO.

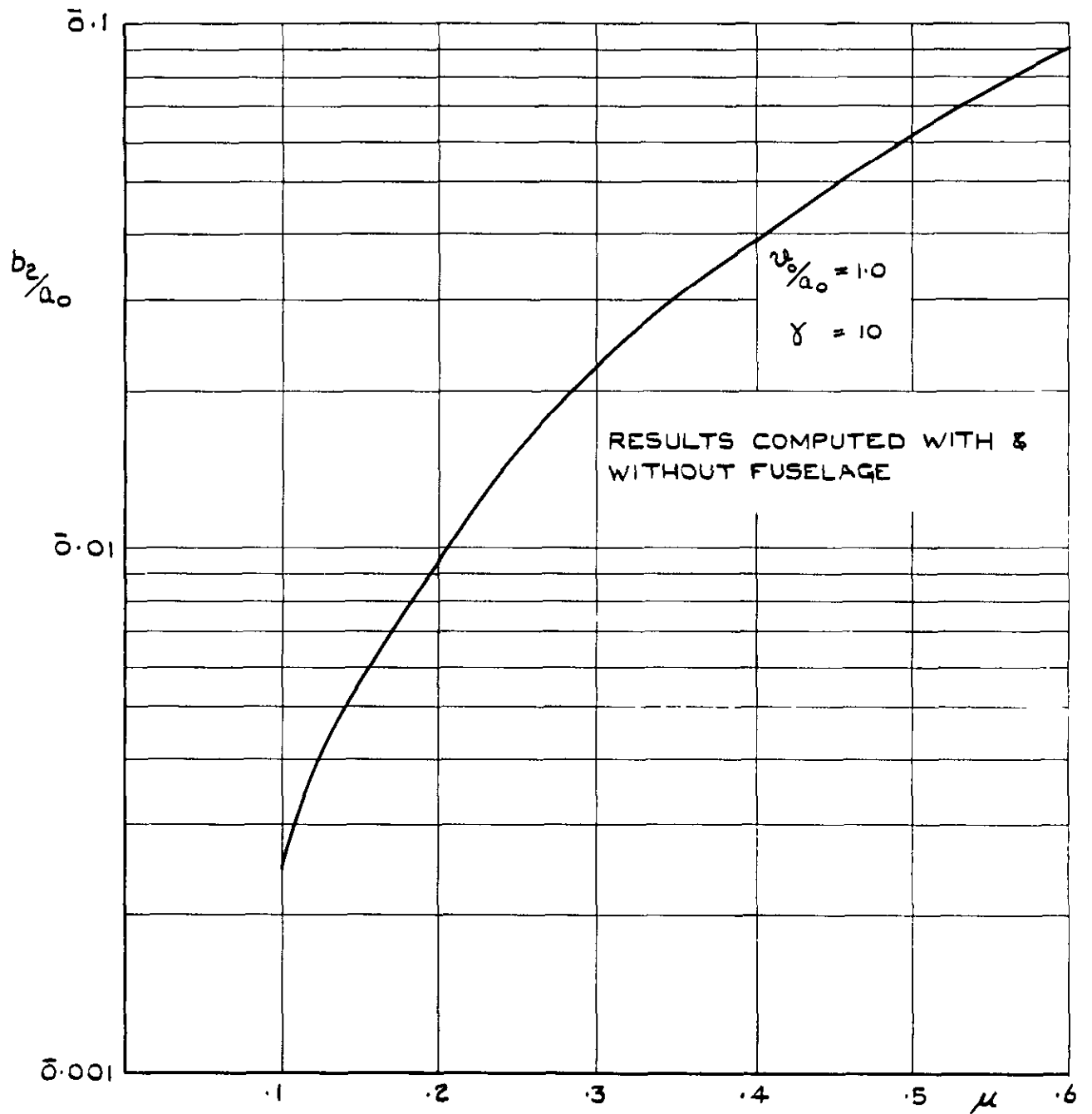


FIG 10. b_2/a_0 v TIP SPEED RATIO

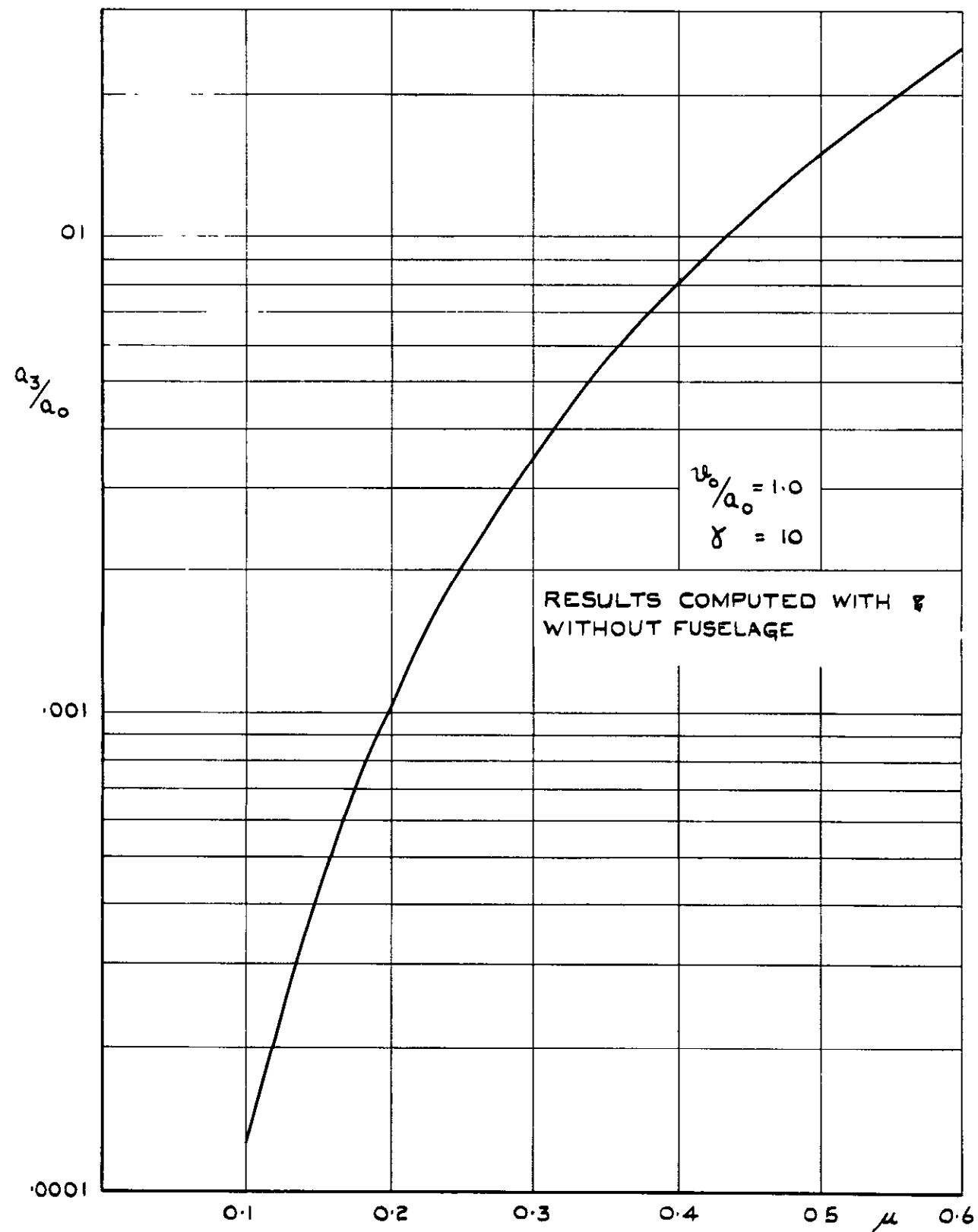


FIG. 11. a_3/a_0 v TIP SPEED RATIO

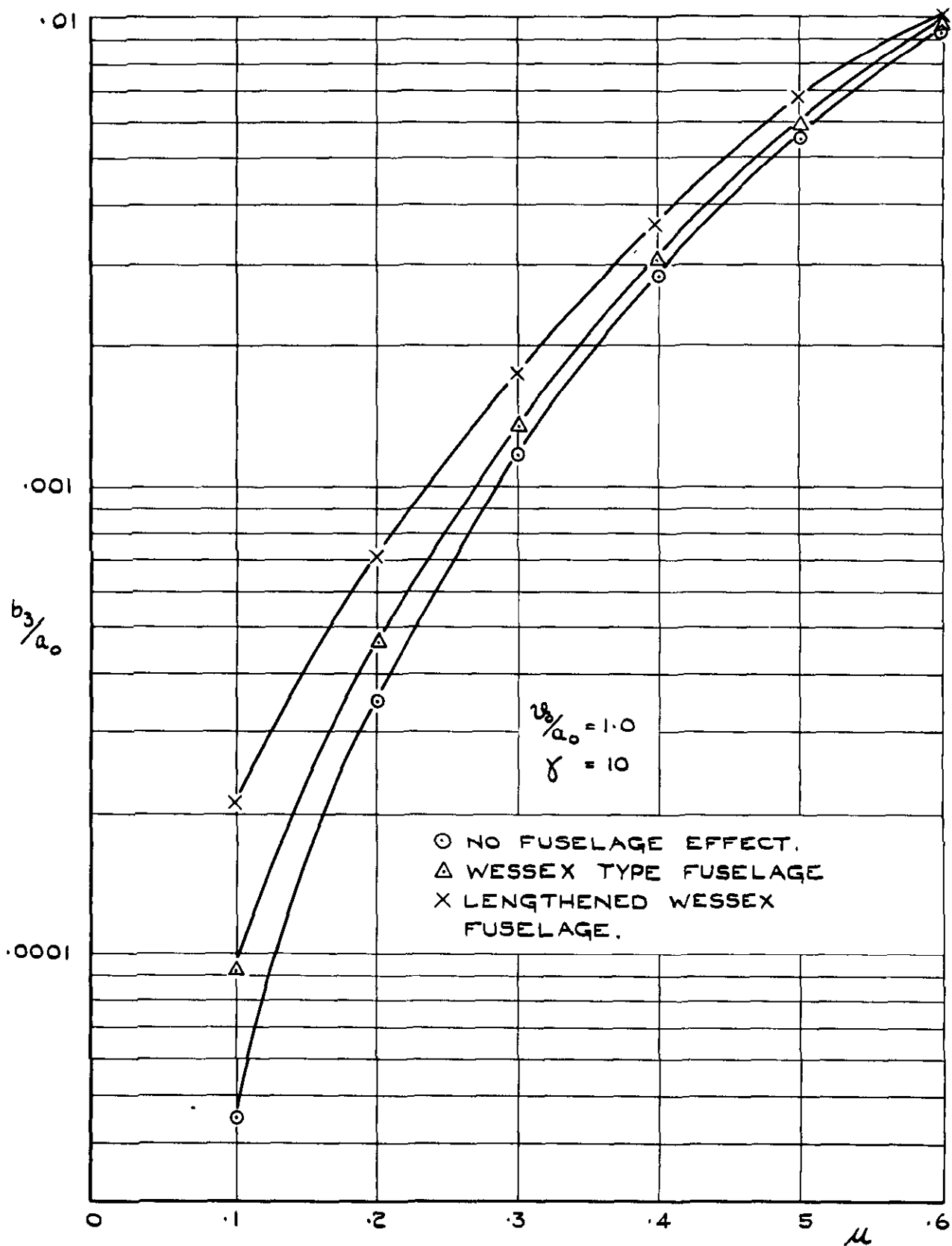


FIG. 12. b_3/a_0 v TIP SPEED RATIO.

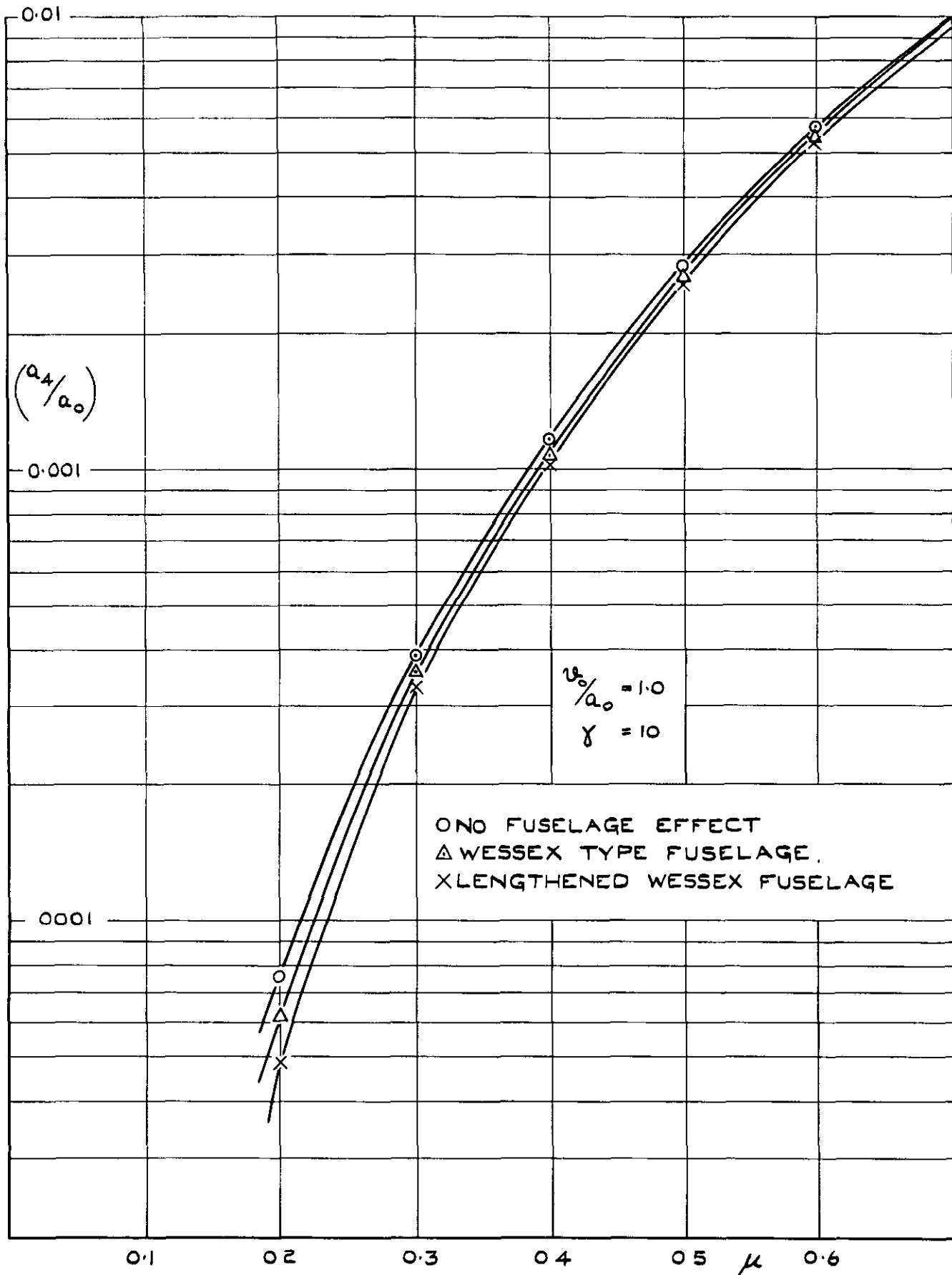


FIG. 13. a_4/a_0 v TIP SPEED RATIO.

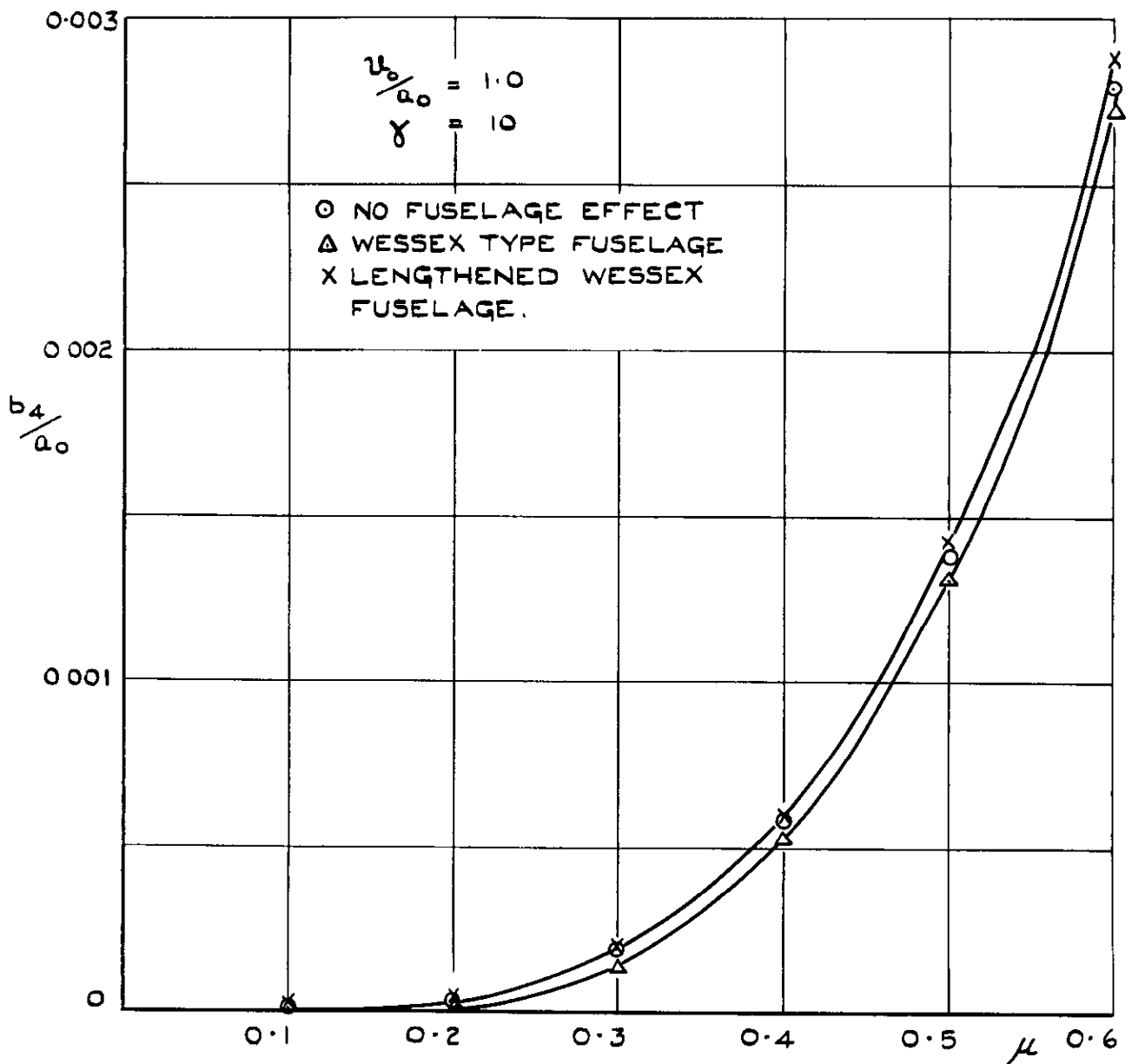


FIG. 14. $\frac{b_4}{a_0}$ v TIP SPEED RATIO.

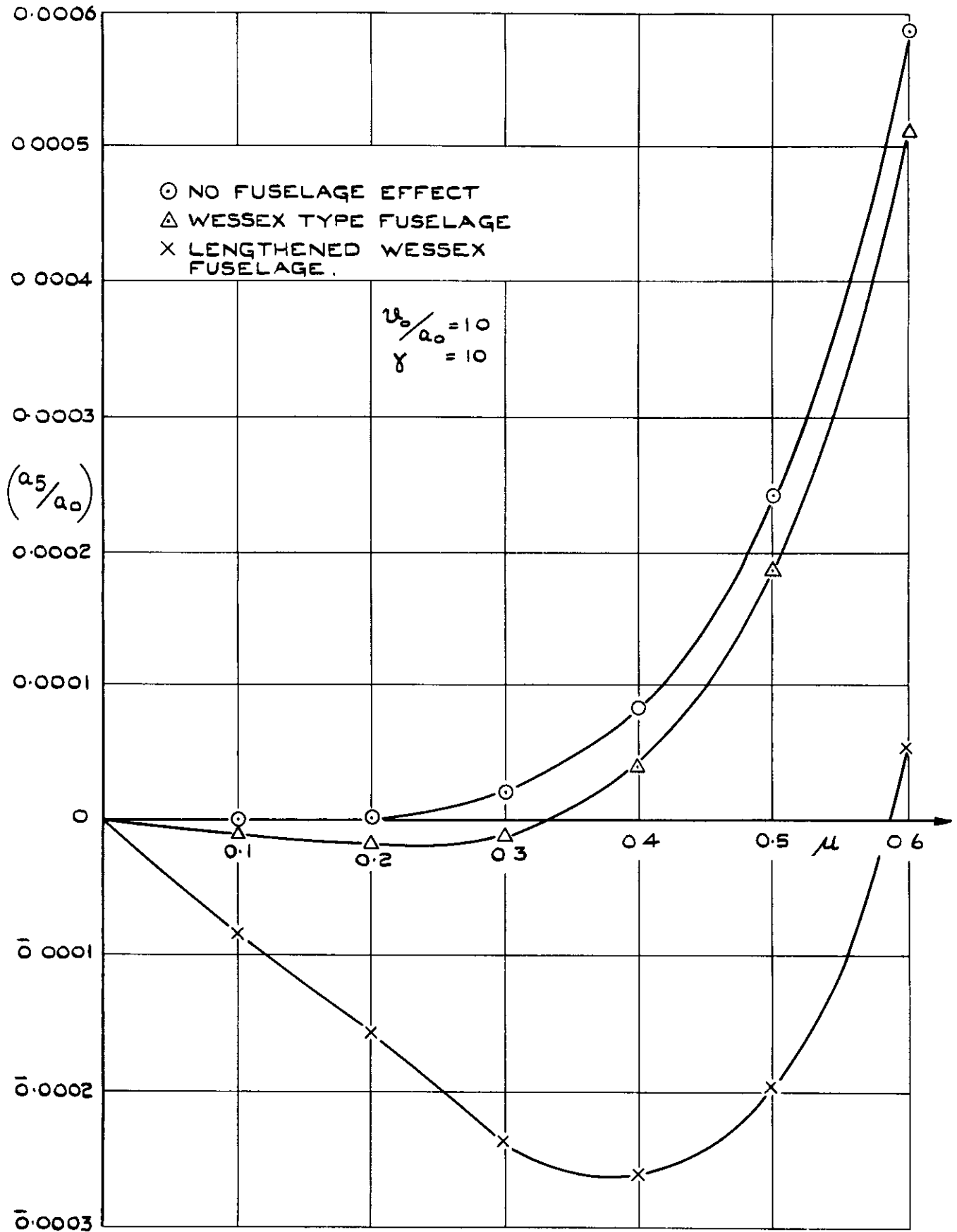


FIG. 15. a_5/a_0 v TIP SPEED RATIO.

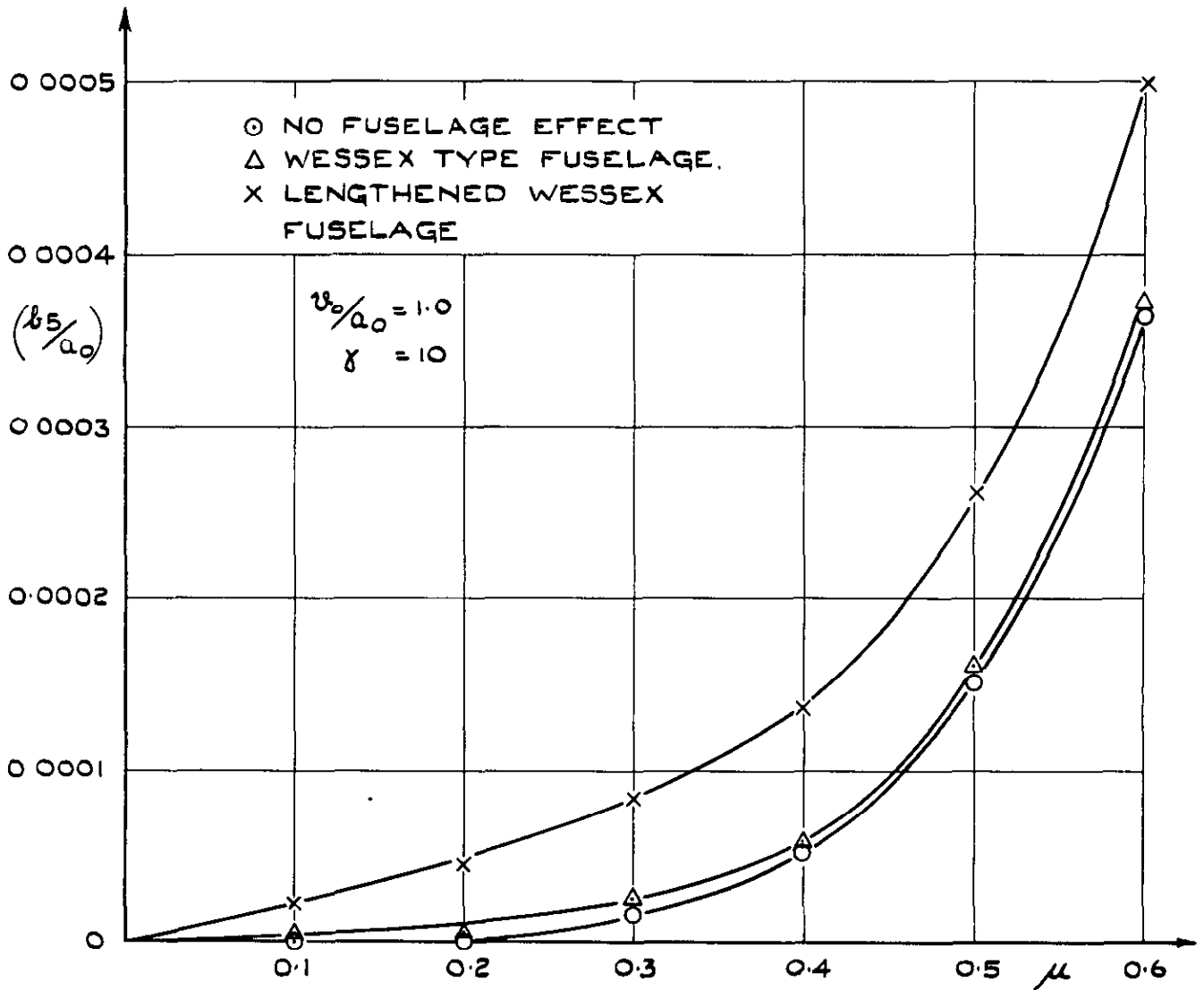


FIG. 16. b_5/a_0 v TIP SPEED RATIO.

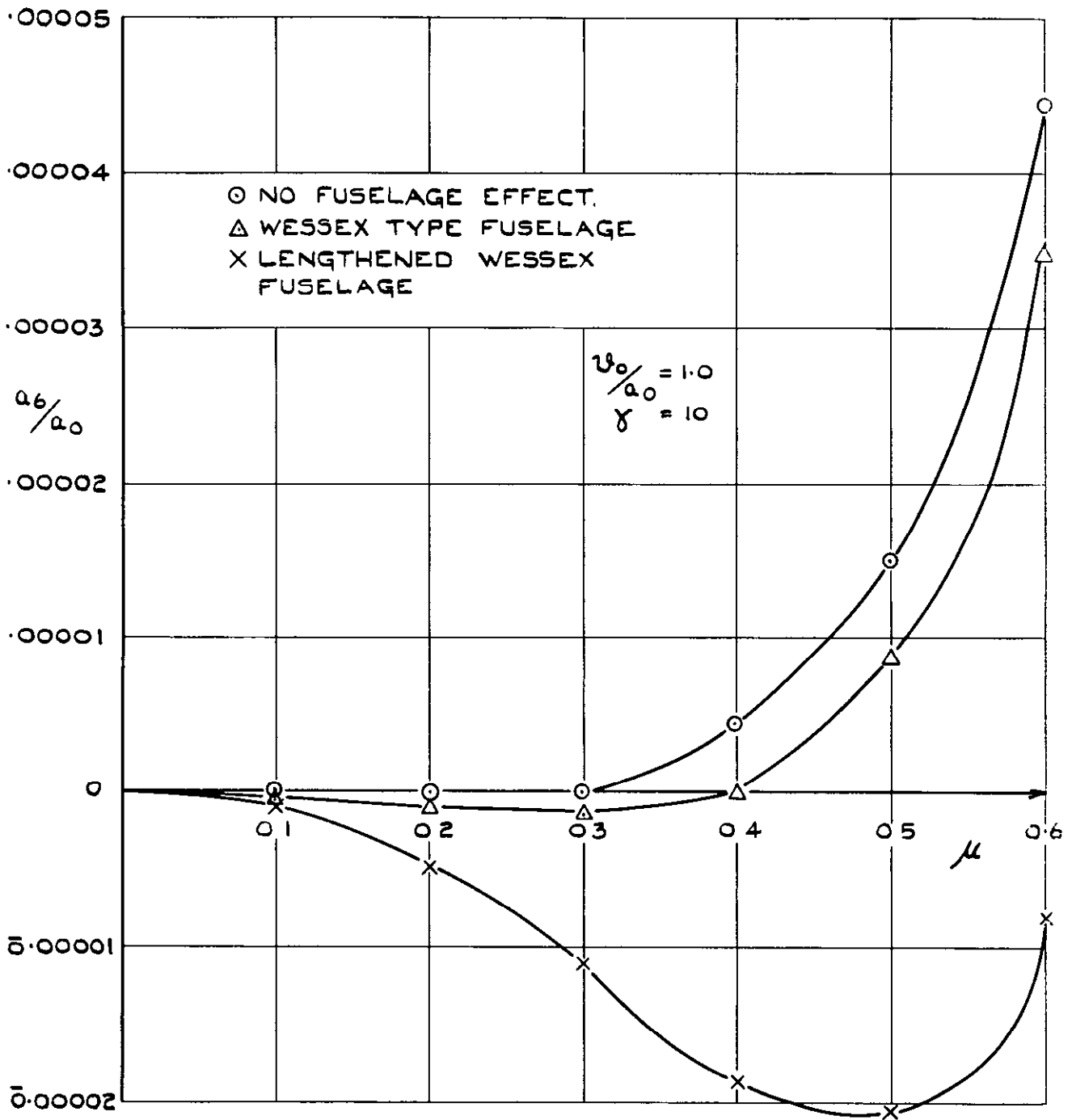


FIG. 17. a_6/a_0 v TIP SPEED RATIO.

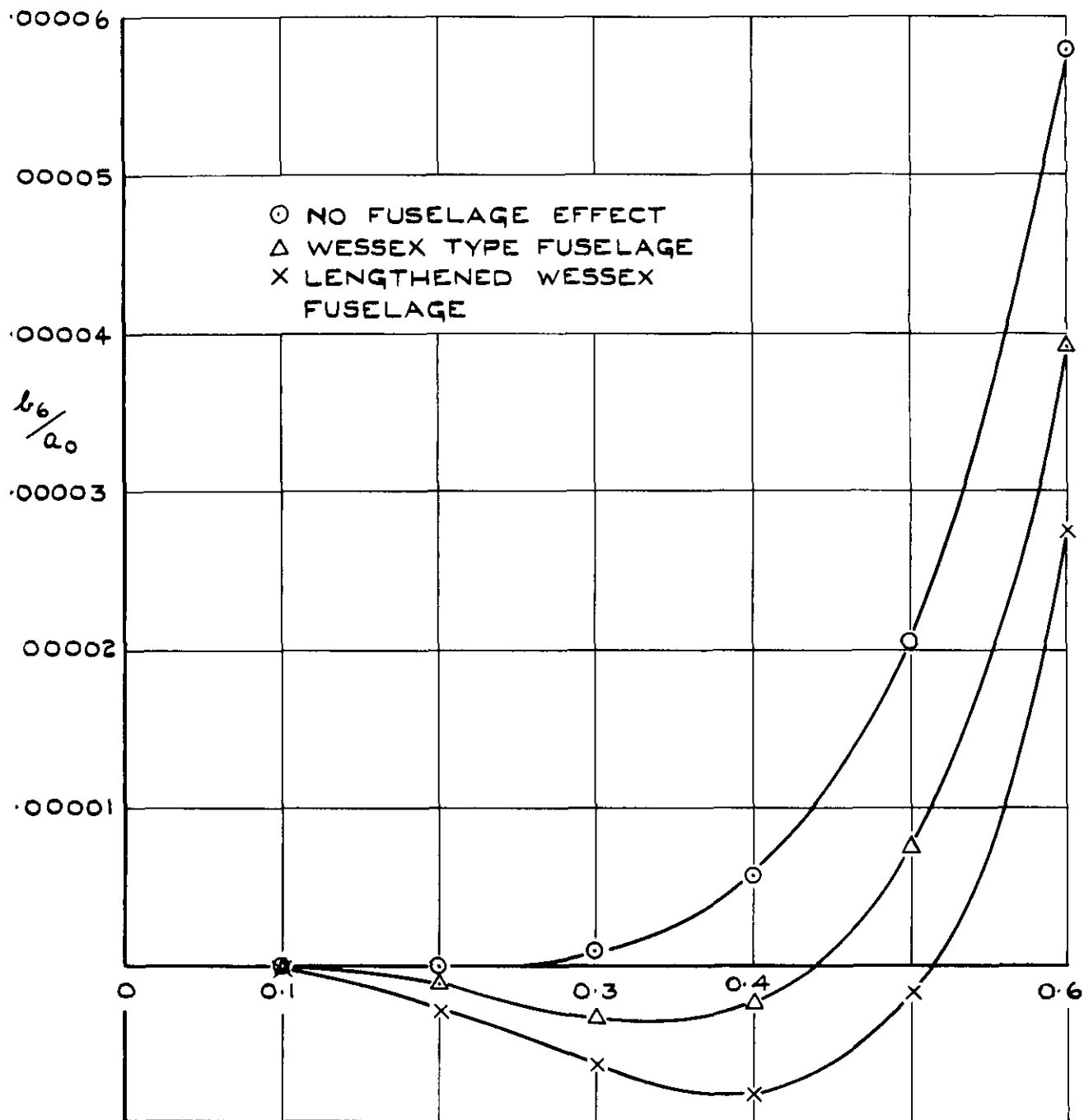


FIG. 18. b_6/a_0 v TIP SPEED RATIO.

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EFFECT OF FLOW CURVATURE DUE TO THE FUSELAGE ON THE FLAPPING MOTION OF A HELICOPTER ROTOR. Willmer, M.A.P. July 1963.

The effect of flow curvature due to the fuselage on the flapping motion of a helicopter rotor has been investigated theoretically. This has been achieved by calculating the changes in the flow through the disc and hence the additional aerodynamic moment about the flapping hinge, using a Rankine solid to represent the actual fuselage. It has been found that in the tip speed range 0 to 0.6, only the higher harmonics are affected significantly and that the changes become more pronounced as the length of the fuselage is increased.

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