C.P. No. 780

C.P. No. 780

MINISTRY OF AVIATION

AERONAUTICAL RESEARCH COUNCIL

CURRENT PAPERS

Measurement of Aerodynamic Heat Transfer in Intermittent Wind-Tunnels

by

A. Naysmith

LONDON: HER MAJESTY'S STATIONERY OFFICE

1965

PRICE 4s 6d NET

U.D.C. No. 533.6.011.6:533.6.071.011.5

C.P. No. 780

January 1964

MEASUREMENT OF AERODYNAMIC HEAT TRANSFER IN INTERMITTENT WIND-TUNNELS

by

A. Naysmith

SUMMARY

The transient technique of measurement of aerodynamic heat transfer is described, and the major sources of error investigated in detail. Methods of testing are recommended that allow errors either to be corrected or avoided, and graphs are given to facilitate estimation of errors.

Replaces R.A.E. Tech Note No. Aero 2942 - A.R.C. 25839.

CONTENTS

ILLUSTRATIONS

1 INTRODUCTION

There has been a considerable increase during the past decade in the quantity of experimental measurements of aerodynamic heat transfer. Unfortunately, even a casual inspection of the available results reveals that the quality of the measurements does not always reach a high standard, the consistency of the results being much less than is customary in measurements of forces or pressures. This is perhaps understandable because heat transfer rates are more difficult to measure than either forces or pressures, and the instrumentation tends to be more elaborate. Even so, there is no reason why the accuracy of measurements could not be improved if the major sources of error are thoroughly understood.

A large number of supersonic wind tunnels are of the intermittent type since this type of tunnel is easier and cheaper to build (and to run) than a continuous wind tunnel of the same size. The method of heat transfer measurement in such a tunnel is known as the "transient" or "heat-pulse" technique and is described in this report; the errors involved in the assumptions that have to be made if this technique is used are investigated in detail, and methods are suggested for correcting or avoiding these errors.

As this Note is intended to be an introduction to the technique for those meeting it for the first time, a list of definitions of the more important quantities is given in Section 2 together with comments upon their physical significance.

2 DEFINITIONS

When air flows over a body, heat is exchanged between the air and the body until thermal equilibrium is established. When this has occurred, each point on the body is at an equilibrium temperature which will vary from point to point. In general, a point on the surface of the body that is exposed to the airstream will receive heat from (or lose it to) the air - the process is known as forced convection or aerodynamic heat transfer. The point will also gain or lose heat by conduction from other parts of the body, and by radiation.

The aerodynamicist attempts to forecast the heat transferred by forced convection, and to express his results as a function of the surface temperature. The structural engineer then uses these results as a boundary condition in his analysis of heat conduction within the structure; he also takes into account heat transferred by radiation.

In order to discuss the subject of aerodynamic heat transfer in general terms it is necessary to anticipate one of the definitions given below and to consider the maximum temperature that each point on the surface of the body would reach in the absence of conduction and radiation. This is known as the recovery temperature. It follows that if the temperature of the surface is greater than the recovery temperature at any point, heat will be convected to the airstream and vice versa. The aerodynamicist, then, has to forecast the recovery temperature, and the way that the heat transferred by forced convection varies if the surface temperature is different from the recovery temperature. This he does by giving either the heat transfer factor, or the Stanton number - both of which are defined below.

In the theoretical study of forced convection, it is necessary to define a thermal boundary layer. Just as in the familiar boundary layer, the velocity gradient normal to the wall is large, so in the thermal boundary layer the temperature gradient normal to the wall is large. The temperature within the thermal boundary layer, however, is greater than that in the external stream, and the temperature profile varies considerably with the surface temperature of the body. The thickness of the thermal boundary layer can be defined, in an analogous way to the ordinary boundary layer thickness, as the distance from the wall at which the temperature reaches, say, 99% of the value in the external stream. The two boundary layers have thicknesses which are of the same order, but they are not identical.

The following definitions assume that a body is situated in an airstream and that heat is being transferred from the air to the body. Alternative names for some of the terms are shown in parentheses after the symbols, and where possible typical values of non-dimensional quantities have been given. It has been assumed that temperatures are low enough for the specific heat of air at constant pressure to be constant. If temperatures are greater than about 700°K, it is necessary to use enthalpy instead of temperature in the definitions given below.

2.1 Wall or surface temperature T_w (parietal temperature)

is the temperature of the surface of the body in contact with the airstream. It is also the temperature of the immediately adjacent air in the boundary layer.

In general, T_w will vary over the surface of the body; its value will depend upon the rate at which heat is transferred from the air, upon the amount of heat conduction within the body, upon the amount of radiation from its surface, and upon the history of these quantities from the commencement of flow over the body.

2.2 Equilibrium wall temperature T_e

at a point is the value of T_w when the body is in thermal equilibrium.

T_e depends upon the same quantities as does T_w.

2.3 Recovery temperature T_r (zero heat transfer wall temperature, adiabatic wall temperature)

is the value of T_w at which there is no aerodynamic heat transfer between the body and the airstream.

T_r will vary over the surface of the body. It depends only upon the aerodynamics and upon the shape of the body. If the wall temperature at any point is less than the recovery temperature, heat will flow in from the passing air. If a body has been in the airstream for a long time and is in thermal equilibrium, the heat lost by radiation will ensure that $\bar{T}_e < \bar{T}_r$, where the bars denote mean values. Locally, T_e could exceed T_r if heat was being supplied by conduction from a region with a higher recovery temperature.

2.4 Recovery factor r_T (temperature recovery factor, boundary layer recovery factor)

is defined by the equation $r_T = (T_r - T_1)/(T_o - T_1)$, where T_o is the stagnation temperature of the air just outside the local boundary layer.

$T_o - T_1$ is the temperature rise that would be experienced if the air just outside the boundary layer were brought to rest isentropically, whereas $T_r - T_1$ is the temperature rise that takes place through the boundary layer where the air is brought to rest at the surface of the body by a non-isentropic process. The recovery factor is therefore an efficiency factor that takes into account the dissipation in the boundary layer. It has a value of about 0.85 in laminar flow, and 0.9 in turbulent flow.

By expressing T_o in terms of the local Mach number (viz. $T_o/T_1 = 1 + \frac{1}{2}(\gamma - 1) M_1^2$) an alternative expression for r_T can be obtained: $T_r/T_1 = 1 + \frac{1}{2} r_T(\gamma-1) M_1^2$.

2.5 Driving temperature $T_r - T_w$

is the difference between the recovery temperature at any point on the surface of the body and the actual temperature there. It can be positive or negative.

2.6 Heat transfer rate Q

is the quantity of heat transferred in unit time between the airstream and unit area of the body surface by aerodynamic means. It excludes any heat transferred by radiation, and can be positive or negative.

2.7 Heat transfer factor h (heat transfer coefficient)

at a point on the surface is the heat transfer rate per unit driving temperature. It is a dimensional quantity.

Since Q has the same sign as $T_r - T_w$, the heat transfer factor is always positive and, paradoxically, can have a finite value even when no heat is being transferred since the ratio remains finite as $T_r - T_w \to 0$. It is often assumed to be independent of T_w, although it in fact varies with $(T_w/T_r)^n$, where n lies between 0 and $\frac{1}{2}$; it also varies slightly with Mach number. The equation $Q = h\ (T_r - T_w)$ is in effect a statement of Newton's Cooling Law.

2.8 Stanton number St (heat transfer coefficient, nombre de Margoulis)

is defined by the equation $St = h/\rho_1\ u_1\ c_p$, where ρ_1, u_1 and c_p are respectively the density, velocity, and specific heat of the air just outside of the boundary layer passing the point in question.

The Stanton number is the non-dimensional form of the heat transfer factor. It can be thought of as the ratio between Q, the amount of heat actually transferred to the body, and $\rho_1 u_1 c_p (T_r - T_w)$, which can be considered as the quantity of heat in the passing airstream that is available for transfer. In other words, it is a sort of efficiency factor. Values are typically of the order 10^{-3}.

2.9 Nusselt number Nu

is defined by $Nu = h\ell/k$, where k is the thermal conductivity of the air, and ℓ is a characteristic length. It is non-dimensional.

If ℓ is taken as the local boundary layer thickness δ, we can obtain

$$Q = Nu_\delta \, k(T_r - T_w)/\delta.$$

Thus the Nusselt number would be unity if the heat transfer across the boundary layer were entirely by conduction. One could also say that $k(T_r - T_w)/Q$ is a measure of the local thermal boundary layer thickness, and that the Nusselt number is the characteristic length divided by this thickness.

3 THE TRANSIENT TECHNIQUE

Models are made with a thin metallic skin - the insides are usually left empty (i.e. filled with air at low pressure during a test). In an experiment, a model is exposed suddenly to the tunnel airstream and the skin temperature recorded as a function of time at several points on the model.

If the skin is of constant thickness, if the temperature gradient through it is negligible, if no heat is transferred to the interior of the model, and if there is no heat conducted along the surface of the model, then it can be shown (Appendix 1) that

$$Q = \rho \, d \, c \frac{dT_w}{dt}, \qquad (1)$$

where ρ, d, and c are respectively the density, thickness and specific heat of the skin of the model. Of the four assumptions made in deriving (1), two are almost invariably justified: the model can be made with the skin thickness controlled to very close tolerances, and air is such a poor heat conductor (especially at low pressure) that heat transfer into the interior of the model can safely be ignored. The corrections involved if the other assumptions are not valid are discussed below.

3.1 Correction for thick skins

It is difficult to measure the temperature of the model skin except at the inner surface, and we will denote by $Q_{t.s.}$ the heat transfer rate obtained from measurements there using the thin skin approximation. If T is the temperature at distance x from the inner surface of a thick skin, we have

$$Q_{t.s.} = \rho c d \, (\partial T/\partial t)_{x=0} . \tag{2}$$

The heat transfer rate across the outer surface of the skin in

$$Q = k \, (\partial T/\partial x)_{x=d} , \tag{3}$$

where k is the thermal conductivity of the model,

and we can define a 'thick skin correction factor' F, where $Q = F.Q_{t.s.}$. From (2) and (3) we have

$$F = \frac{Kd}{\lambda} \cdot \frac{(\partial T/\partial x)_{x=d}}{(\partial T/\partial t)_{x=0}} . \tag{4}$$

The non-dimensional parameters $\lambda (= hd/k)$ and Kt have been introduced, where $K = h/\rho c d$, for convenience in subsequent expressions.

The temperature distribution through a homogeneous flat slab of thickness d that is heated by forced convection (with constant h and T_r) is given in Ref.1 p.100

$$\frac{T - T_a}{T_r - T_a} = 1 - \sum_{n=1}^{\infty} \frac{2\lambda \cos (\beta_n x/d)}{(\lambda^2 + \lambda + \beta_n^2) \cos \beta_n} \exp (- \beta_n^2 Kt/\lambda) , \tag{5}$$

where T_a is the initial temperature of the slab and the β_n are the roots of $\beta \tan \beta = \lambda$. For very small values of Kt this does not converge very rapidly, and it is better to use an approximate solution given in Ref.1 p.252

$$\frac{T - T_a}{T_r - T_a} = \text{erfc} \left(\frac{1 - x/d}{2 \sqrt{Kt/\lambda}}\right) + \text{erfc} \left(\frac{1 + x/d}{2 \sqrt{Kt/\lambda}}\right) - \text{erfc} \left(\sqrt{\lambda Kt} + \frac{1 - x/d}{2 \sqrt{Kt/\lambda}}\right) \exp [\lambda(1-x/d) + \lambda Kt]$$

$$- \text{erfc} \left(\sqrt{\lambda Kt} + \frac{1 + x/d}{2\sqrt{Kt/\lambda}}\right) \exp [\lambda(1 + x/d) + \lambda Kt] . \tag{6}$$

From (4) and (5) we can derive

$$F = \frac{\displaystyle\sum_{n=1}^{\infty} \frac{1}{\lambda^2 + \lambda + \beta_n^2} \exp\left(-\beta_n^2 \, Kt/\lambda\right)}{\displaystyle\sum_{n=1}^{\infty} \frac{\beta_n \operatorname{cosec} \beta_n}{\lambda^2 + \lambda + \beta_n^2} \exp\left(-\beta_n^2 \, Kt/\lambda\right)} \qquad (7)$$

and for very small values of Kt, using (4) and (6), we have

$$F = \frac{\left\{\operatorname{erfc} \sqrt{\lambda Kt} - \operatorname{erfc}\left(\sqrt{\lambda Kt} + \sqrt{\lambda/Kt}\right) \exp 2\lambda\right\} \exp \lambda Kt}{\frac{2}{\sqrt{\pi}} \exp\left(-\lambda/4Kt\right) - \lambda \exp\left(\lambda + \lambda Kt\right) \operatorname{erfc}\left(\sqrt{\lambda Kt} + \frac{1}{2}\sqrt{\lambda Kt}\right)} \, . \qquad (8)$$

Values of F have been computed for a range of values of λ and are shown in Fig.1. It will be seen that the correction factor can be very large for small values of Kt, but that it rapidly decays to a constant value less than unity for large Kt. In other words, for a short time after the air has started to flow over the model the temperature of the inner surface will not deviate from its initial value, and F will be infinitely large. However, once the temperature of the inner surface has started to rise, the temperature difference between the outer and inner surfaces will fall. It follows that the rate of change of temperature with time at any instant will have its maximum value at the inner face, so that heat transfer rates derived from temperature-time derivatives at the inner face will be over-estimates.

For each curve in Fig.1 there is a minimum value of Kt, denoted by $(Kt)^*$, such that F is constant ($= F^*$) for $Kt > (Kt)^*$. Providing that measurements are made for $Kt > (Kt)^*$ it is easy to correct values of heat transfer rates obtained by the thin skin approximation for the effects of skin thickness, because the correction factor is then a function of λ only. A graph of $(Kt)^*$ as a function of λ is shown in Fig.2, and it will be seen that for all λ, $\lambda > (Kt)^*$. The method of making measurements which immediately suggests itself is to take $Kt = \lambda$, as this ensures that $Kt > (Kt)^*$. If we put $t^* = \lambda/K$ it is then possible to eliminate h and obtain

$$t^* = \frac{d^2}{\kappa} \qquad (9)$$

where κ ($= k/\rho c$) is the thermal diffusivity of the model, and t^* is the earliest time after flow is established over the model at which measurements should be made. For a copper model with a skin thickness of 2 mm t^* would be 0.034 seconds; for a stainless steel model with a skin thickness of 1 mm t^* would be 0.24 seconds.

If $Kt \geq \lambda$, the second and subsequent terms in the two series in (7) are so small that they can be neglected (this is because $\beta_2^2 \approx 10$ for all λ, and $\beta_3 > \beta_2$ etc.) and so (7) reduces to

$$F^* = \frac{\sin \beta_1}{\beta_1} . \tag{10}$$

In order to evaluate the heat transfer factor h, it is necessary to allow for the fact that the temperature measured on the inside of the skin (T_i) is not the same as the surface temperature T_w. If $h_{t.s.}$ is the heat transfer factor obtained by measurements on the inside of the skin at time t^*,

$$h_{t.s.} = \frac{Q_{t.s.}}{T_r - T_i^*} . \tag{11}$$

We can now define a correction factor F_1, such that

$$h = \frac{Q}{T_r - T_w^*} = F_1 \, F^* \, h_{t.s.} \tag{12}$$

and therefore

$$F_1 = \frac{T_r - T_i^*}{T_r - T_w^*} . \tag{13}$$

From (5) and (13), and taking the first terms only of the series as before, we obtain

$$F_1 = \sec \beta_1 . \tag{14}$$

And from (10), (12) and (14) we can obtain

$$h = \frac{\tan \beta_1}{\beta_1} . h_{t.s.} . \qquad (15)$$

If λ is small, β_1 is small, and $\tan \beta_1$ can be expanded to give

$$h = \left\{1 + \frac{\beta_1^2}{3} + \frac{2}{15}\beta_1^4 + \ldots\ldots\right\} h_{t.s.} \qquad (16)$$

and we can also make the approximation $\beta_1^2 = \lambda$ and obtain

$$h = \left\{1 + \frac{\lambda}{3} + \frac{2}{15}\lambda^2 + \ldots\right\} h_{t.s.} . \qquad (17)$$

In Fig.3, a graph of $\frac{\tan \beta_1}{\beta_1}$ is given as a function of λ, with the straight line $1 + \frac{\lambda}{3}$ derived from the first two terms of (17) shown as well. It will be seen that the approximation is sufficiently accurate for all except large values of λ. If (15) or (17) is used to correct values of $h_{t.s.}$ it would be possible to use the corrected value of h to obtain a new value of λ (= hd/k) and hence an even more correct value for h;

$$h = \left\{1 + \frac{\lambda}{3} + \frac{2\lambda^2}{9} + \ldots\right\} h_{t.s.} \qquad (18)$$

so that providing λ^2 can be neglected, this refinement is not necessary.

3.2 Effect of heat conduction

If the requirement concerning no heat conduction along the surface of the model is removed, equation (1) no longer applies. Instead, we have for a body of revolution (Appendix 1)

$$Q = \rho c d \frac{\partial T}{\partial t} - kd\left(\frac{\partial^2 T}{\partial s^2} + \frac{1}{r}\frac{\partial r}{\partial s}\frac{\partial T}{\partial s}\right) \qquad (19)$$

where s is the distance along the surface from the nose of the model, and r is the radius. For models that are not bodies of revolution the equation for Q will in general be impossibly complicated.

It is obviously easier to use (1) than to use (19). In addition, to evaluate $\partial^2T/\partial s^2$ it is necessary to provide several equally spaced measuring points in a straight line. This could be inconvenient, and since (19) does not apply when the model is at incidence to the airstream it is best to arrange the experiment so that heat conduction along the surface is negligible. This is done by choosing a model material such that t^* is small. Measurements are made at $t = t^*$, and if t^* is small enough the model will not have heated up sufficiently for heat conduction to be present to any marked extent.

An approximate solution of (19) is given by Conti in Ref.2. He considers several different bodies, and discusses in particular the problem of high heating rates near the noses of bodies. George and Reineche, in Ref.3, estimate the effect on wall temperature of conduction along a thermally thin skin.

4 METHODS OF TESTING

There are three ways of exposing the model suddenly to the airflow: the tunnel can be started very quickly, the model can be covered with shields that are removed when the tunnel flow is established, or the airflow can be established in an otherwise empty tunnel and the model rapidly brought into the working section.

If the tunnel can be started quickly it is usual to make use of this property. There are, however, some disadvantages. The most serious is that just after starting the temperature response in the working section is often much more sluggish than the pressure response. In particular, if the air has to be heated in order to avoid liquefaction in the working section then the pipework between the heater and the working section will absorb heat from the airstream. This can mean that for a short time after flow is established the air in the working section will be below liquefaction temperature. The resulting heat transfer rate, which depends upon Reynolds number (which in turn varies with temperature) will then vary in a way which defies analysis. This difficulty has been successfully avoided in some instances by having a changeover valve just upstream of the tunnel throat: the flow runs through a pipe by-passing the working section until everything has warmed up, and then the changeover valve is operated. The practice of making measurements at a time after flow has started when the stagnation temperature has risen to a more or less steady value is not to be recommended because heat conduction will then have to be taken into account.

In many tunnels the above difficulties are avoided by enclosing the model by a pair of shields before a run - often incorporating heating or cooling in the shields in order to vary the driving temperature. The shields are retracted very quickly as soon as the stagnation temperature has ceased to rise. This is probably the most widely used method at the present time.

A variant of this technique has been used in the helium hypersonic tunnel at Princeton. A small hand-press is used to press powdered solid carbon dioxide into a conical shield that fits over the model. While this cools the model, a second shield is made. When the first shield has almost disappeared - having cooled the model - the second one is placed over the model, the working section closed, and the tunnel started. The shield very soon disintegrates, exposing the model to the airflow. Although the shield usually disintegrates so suddenly that all of the model is exposed to the flow at substantially the same time, wires can be embedded in the shield during its manufacture and be subsequently pulled to enforce complete disintegration.

The rapid projection of a model into a working section in which flow is already established presents some engineering problems; in particular, it is necessary to ensure that the rapid acceleration and deceleration of the model do not damage or dislodge any instruments mounted within it. The disadvantages are sufficient for the method to be used comparatively rarely for heat transfer measurements.

5 ACCURACY OF MEASUREMENT

In order to obtain a value for Q, it is necessary to measure ρ, c, d, and $\partial T/\partial t$. Since Q is the product of these quantities, the overall accuracy is determined by the least accurate of the measurements - an elementary fact that is sometimes overlooked. In addition, any variation in d will affect the accuracy of the results. Since it is necessary to make measurements as soon as possible after the start of the flow, $t^*(= d^2/\kappa)$ has to be small. However, if d is made too small, small absolute variations in d can mean large percentage variations, with corresponding loss of accuracy. It is therefore advisable to make the model from a material with a high value of κ - and of the available materials copper seems to be the best. If results of high accuracy are required it may even be advisable to cut the model open after the experiments are finished and to measure the local skin thickness at the points where the measuring instruments are situated.

The density and specific heat can be measured at leisure before the aerodynamic experiment takes place, using samples of the material from which the model has been made. Previous measurements of these properties are not necessarily reliable - the actual values can vary quite considerably from batch to batch of nominally identical materials.

It is not necessary to measure thermal conductivity to the same accuracy because it only affects the correction terms, which are (or should be) only a few percent of the total. Indeed, published values are probably sufficiently accurate for most experiments.

Since $\partial T/\partial t$ has to be obtained from numerical differentiation, which leads to a decrease in the accuracy of the results - the error in $\partial T/\partial t$ is about double that in either T or t - the temperature and the time have to be recorded more accurately than the other quantities.

6 INSTRUMENTATION OF MODELS

Skin temperature is easily measured with thermocouples attached to the inside of the model. Since effects of skin thickness can be allowed for, there is no need to attempt to place the thermocouple bead near the outer surface by drilling through from the back. In fact, the disturbance to the model skin could cause the local temperature to deviate from its undisturbed value.

In the author's experience, spot welding of the thermocouples on to the skin produces the most consistent results because the model skin is almost undisturbed. Providing the work is done with care, the mechanical strength of the weld will be sufficient to avoid accidental disturbance. A circuit of a spot welder is shown in Fig.4 and the method of operation described in Appendix 2.

It is always advisable to calibrate the thermocouples after they have been fixed to the model. This can conveniently be done by immersing the model in a heated oil bath and using either a calibrated mercury-in-glass thermometer or a platinum resistance thermometer as reference.

It is beyond the scope of this report to discuss methods of recording the millivolt output of the thermocouples.

6.1 Size of thermocouple wires

If the diameter of a thermocouple wire is large, or if it is a good heat conductor, it will measure a lower temperature than would have existed in its absence. On the other hand, small diameter wires are difficult to attach to the model and are easily broken. It is therefore desirable to use a wire with as large a diameter as possible without causing a surface temperature depression of more than (say) 1%. As far as the author is aware, no systematic investigation of this problem has been published. A rough and ready rule, based on an incomplete experimental investigation, is to use wires of diameter less than 20% of the model skin thickness, and to avoid copper as a thermocouple material because of its high thermal conductivity.

7 MEASUREMENT OF RECOVERY TEMPERATURE

Since the maximum running time of an intermittent wind tunnel is rarely long enough for a normal model to reach recovery temperature, this quantity has to be measured separately from the other temperatures. One method is to make a second model having the same shape as the first, but with an extremely thin metallic skin sometimes mounted on an insulator. The temperature of the skin is recorded as a function of time, and should reach equilibrium before the tunnel flow has to be stopped. If the recovery temperature is high it will be necessary to take radiation losses into account. If the emissivity of the model is ε, and σ is the Stefan-Boltzmann constant,

$$Q_{net} = h(T_r - T_w) - \varepsilon \sigma T_w^4 \qquad (20)$$

where Q_{net} is the total inward heat flow. If the model is in thermal equilibrium for $T_w = T_e$, $Q_{net} = 0$ and we can obtain from (20)

$$T_r = T_e + \frac{\varepsilon\sigma}{h} T_e^4 . \qquad (21)$$

The value of h used in (21) will have to be obtained by assuming that $T_r = T_e$ in its derivation, which should be sufficiently accurate in nearly all experiments.

No allowance has been made for radiant heating of the model from the hot walls of the tunnel, because the amount of heat received in this way is likely to be small - the working section walls in most wind tunnels heat up so slowly in comparison with the model that the heat radiated from them is negligible. Conduction errors and their corrections are discussed in detail in Ref.3.

8 CONCLUSIONS

A detailed analysis of the transient technique for measurement of aerodynamic heat transfer has shown that effects of skin thickness can be taken into account if measurements are made after a short time determined by the skin thickness and the properties of the material from which the model is made.

In order to obtain accurate results and to minimise errors due to conduction within the model, it is recommended that the model be made from a material with a high value of thermal diffusivity, and of the available materials copper seems to be the best.

Measurements of recovery temperature present the most difficulty, but the use of a separate model is usually sufficient to ensure accurate measurements.

LIST OF SYMBOLS

c	specific heat
d	thickness of model skin
F	correction factor defined in 3.1
F*	value of F when $Kt \geq (Kt)^*$
F_1	defined by (12) in 3.1
h	heat transfer factor (see 2.7)
$h_{t.s.}$	defined by (11) in 3.1
K	$h/\rho\, c\, d$
$(Kt)^*$	value of Kt defined in 3.1
k	thermal conductivity of model (except in 2.9)
M_1	Mach number just outside of the boundary layer
Nu	Nusselt number (see 2.9)

SYMBOLS (Contd)

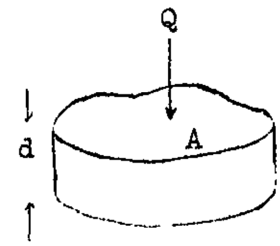
Q	heat transfer rate (see 2.6)
$Q_{t.s.}$	defined by (2) in 3.1
r	radius of the model
r_T	recovery factor (see 2.4)
St	Stanton number (see 2.8)
s	distance from nose of model, measured along the surface
T_a	model temperature before air flow starts
T_e	equilibrium temperature (see 2.2)
T_i	temperature of the inside of the model skin
T_i^*	value of T_i when $t = t^*$
T_r	recovery temperature (see 2.3)
T_w	wall temperature (see 2.1)
T_w^*	value of T_w when $t = t^*$
T_o	stagnation temperature just outside the boundary layer
T_1	temperature of air just outside the boundary layer
t	time, measured from the start of flow over the model
t^*	d^2/κ
x	distance through the model skin, measured from the inner surface
β_n	positive roots of $\beta \tan \beta = \lambda$
γ	ratio of the specific heats of air
ε	emissivity of model surface
κ	thermal diffusivity of model = $k/\rho c$
λ	hd/k
ρ	density of model skin
σ	Stefan-Boltzmann constant

REFERENCES

No.	Author	Title, etc.
1	Carslaw, H.S. Jaeger, J.C.	Conduction of heat in solids. Clarendon Press, Oxford. 1947.
2	Conti, Raul, J.	Approximate temperature distributions and stream-wise heat conduction effects in the transient aerodynamic heating of thin-skinned bodies. NASA Technical Note D895. September 1961.
3	George, A.R. Reinecke, W.G.	Conduction in thin-skinned heat transfer and recovery temperature models. AIAA Journal 1 No.8, pp.1956-58. August 1963.

APPENDIX 1

(i) Consider a small element of skin area A with heat being transferred to it. In time δt the heat input is $QA\,\delta t$. If the temperature rise is δT_w then



$$QA\,\delta t = A\,d\rho c\,\delta T_w$$

where d is the thickness, c the specific heat, and ρ the density of the skin.

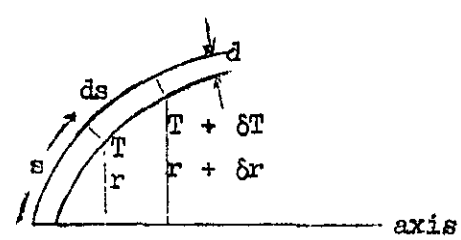
As $\delta t \to 0$, we get $\quad Q = \rho c d \dfrac{dT_w}{dt}$.

(ii) Consider a thin-walled body of revolution at zero incidence to an air-stream, so that there is no heat flow round a section normal to the axis.

Distance s is measured along the surface from the nose, and we consider a small element ds bounded by normals to the surface.

The heat flow into the element from the air per unit time is $Q\,2\pi r\,ds$.

The heat conducted into the element per unit time is $-k\,2\pi r\,d\,.\,\partial T/\partial s$, where k is the thermal conductivity of the wall and d its thickness.



The heat conducted away from the element is $-k\,2\pi(r+\delta r)\,d\,.\,\partial/\partial s\;(T_w + \delta T_w)$, where $(T_w + \delta T_w)$ is the temperature of the right hand end of the element, and r the radius of the cross-section normal to the axis.

If the element experiences a temperature rise of ΔT in time Δt,

$$2\pi r.\delta s.\rho d\,c\,.\,\Delta T = Q\,.\,2\pi r\,\delta s\,.\,\Delta t - 2\pi d k\left\{(r+\delta r)\,\frac{\partial}{\partial s}\,(T+\delta T) - r\,\frac{\partial T}{\partial s}\right\}\Delta t\ ,$$

where ρ and c are the density and specific heat of the wall material.

In the limit, as $\Delta t \to 0$ and δs, δr, and $\delta T \to 0$ we get

$$Q = \rho c d\,\frac{\partial T}{\partial t} - kd\left\{\frac{\partial^2 T}{\partial s^2} + \frac{1}{r}\cdot\frac{dr}{ds}\cdot\frac{\partial T}{\partial s}\right\}.$$

APPENDIX 2

OPERATION OF A THERMOCOUPLE WELDER

The circuit of a thermocouple welder, used in the R.A.E. for welding thermocouples to wind-tunnel models is shown in Fig.4. In use, the instrument is switched on, whereupon the capacitor C_2 becomes charged. The model is connected to one of the output terminals and the other terminal is connected to a thermocouple wire. The wire is held with its end touching the model, and the foot switch operated. The capacitor C_2 discharges, welding the wire to the model. A thermocouple is formed by welding two wires on to the model as close together as possible.

The correct operating voltage is chosen by trial and error by altering the variable resistor R_4, and if necessary the current can be controlled by means of R_6. If the wires to be welded are small, it has been found that the strongest welds were obtained when the connecting lead from the welder was attached to the thermocouple wire close to the end to be welded. An easy way to attach the lead to the wire is to fit a spring clip (crocodile clip) to the end of the lead and to hold the wire with it. It is advisable to cover the jaws of the clip with solder, which is soft and so the wire is not damaged when it is gripped by the clip.

Printed in England for Her Majesty's Stationery Office by the Royal Aircraft Establishment, Farnborough.

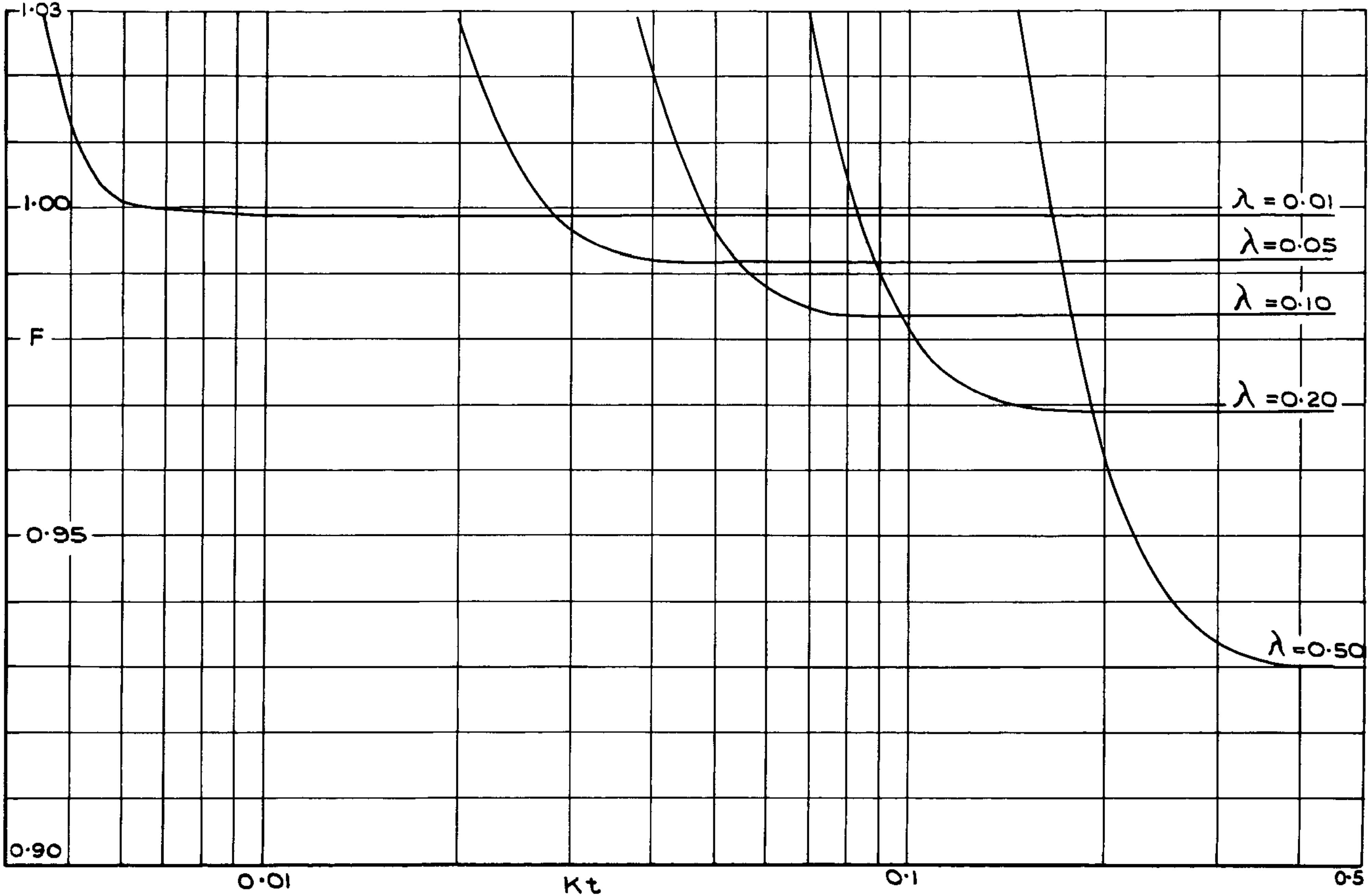


FIG. 1. VARIATION OF THICK SKIN CORRECTION FACTOR

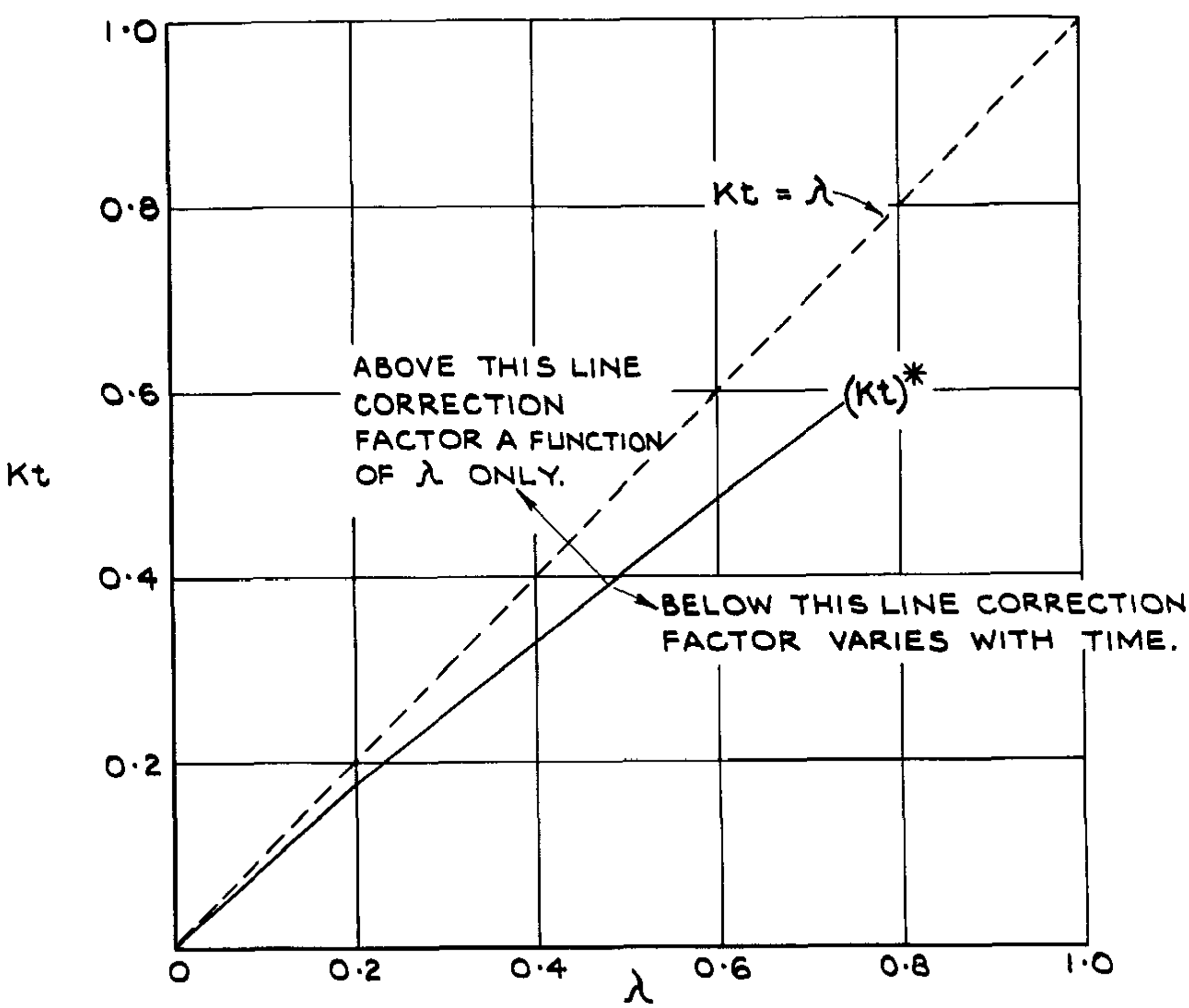


FIG. 2. VARIATION OF $(Kt)^*$ WITH λ

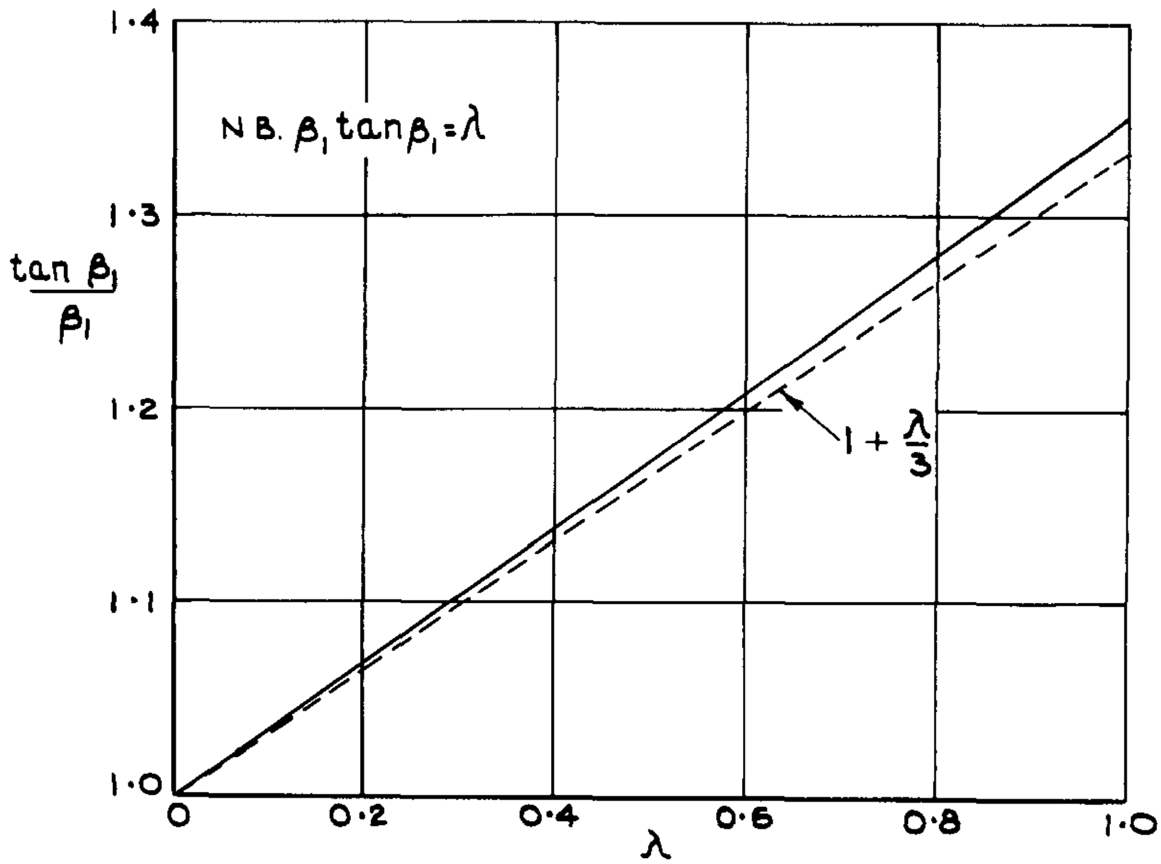
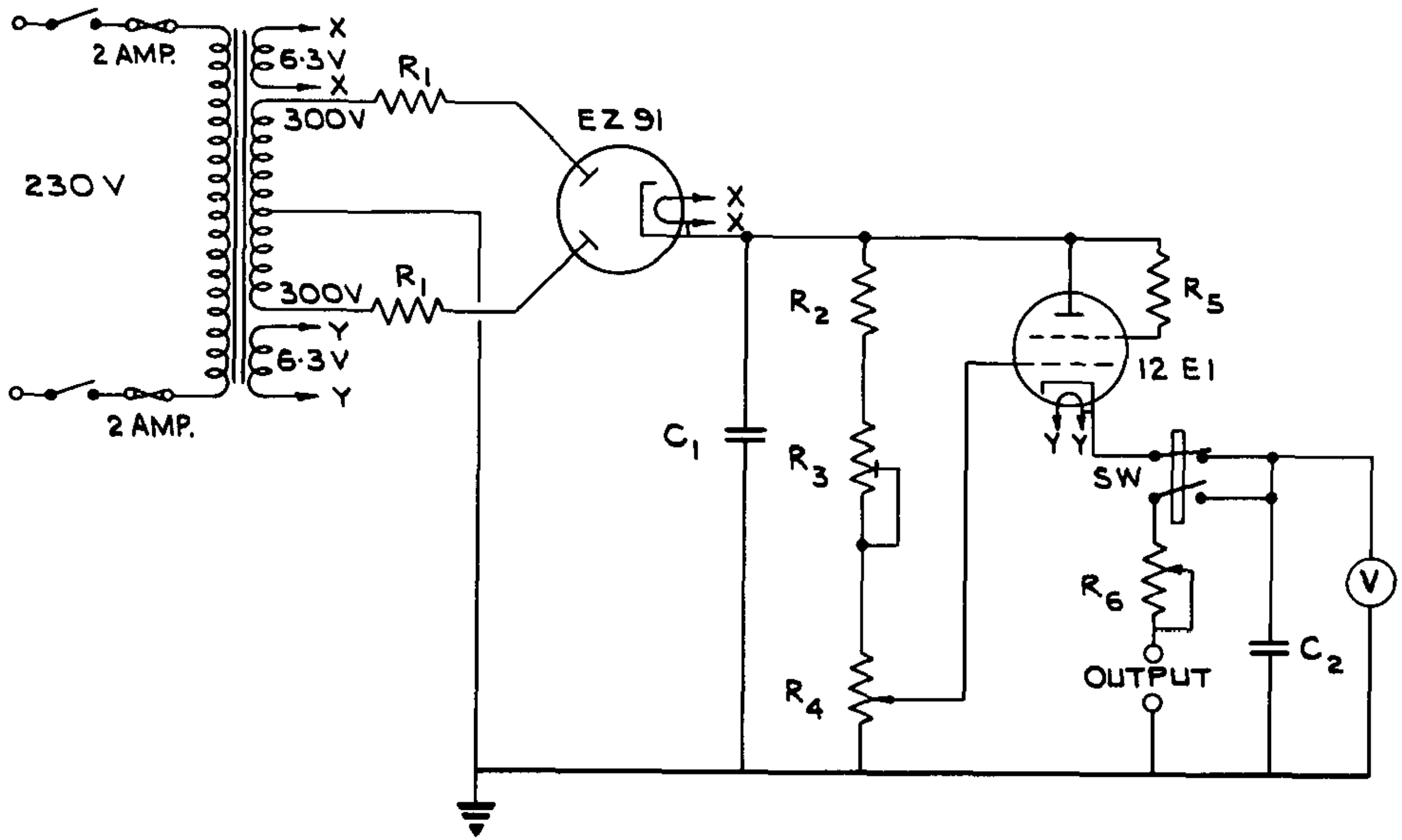


FIG. 3. VARIATION OF $\frac{\tan\beta_1}{\beta_1}$ WITH λ



R_1	220Ω	$1\frac{1}{2}$ W
R_2	100KΩ	$\frac{1}{2}$ W
R_3	25KΩ	$\frac{1}{2}$ W
R_4	100KΩ	1W
R_5	100Ω	$\frac{1}{4}$ W
R_6	5Ω	1W
C_1	8μF	600V
C_2	640μF	450V
SW	CHANGEOVER FOOT SWITCH	

N.B. OPERATION OF THIS EQUIPMENT IS DESCRIBED IN APPENDIX II

FIG 4. THERMOCOUPLE WELDER CIRCUIT.

A.R.C. C.P. No. 780

533.6.011.6 :
533.6.071.011.5

MEASUREMENT OF AERODYNAMIC HEAT TRANSFER IN INTERMITTENT WIND-TUNNELS.
Naysmith, A. January 1964.

The transient technique of measurement of aerodynamic heat transfer is described, and the major sources of error investigated in detail. Methods of testing are recommended that allow errors either to be corrected or avoided, and graphs are given to facilitate estimation of errors.

A.R.C. C.P. No. 780

533.6.011.6 :
533.6.071.011.5

MEASUREMENT OF AERODYNAMIC HEAT TRANSFER IN INTERMITTENT WIND-TUNNELS.
Naysmith, A. January 1964.

The transient technique of measurement of aerodynamic heat transfer is described, and the major sources of error investigated in detail. Methods of testing are recommended that allow errors either to be corrected or avoided, and graphs are given to facilitate estimation of errors.

C.P. No. 780

Published by
HER MAJESTY'S STATIONERY OFFICE

To be purchased from
York House, Kingsway, London w.c.2
423 Oxford Street, London w.1
13A Castle Street, Edinburgh 2
109 St Mary Street, Cardiff
39 King Street, Manchester 2
50 Fairfax Street, Bristol 1
35 Smallbrook, Ringway, Birmingham 5
80 Chichester Street, Belfast 1
or through any bookseller

C.P. No. 780

S.O. CODE No. 23-9015-80