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A Simple Method of Calculating the Flow Produced in an Annular Electric Arc Heater

by

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A SIMPLE METHOD OF CALCULATING THE FLOW PRODUCED IN AN ANNULAR ELECTRIC ARC HEATER

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SUMMARY

This note describes a simple model from which the flow in an annular electric arc heater can be calculated. It is intended to clarify ideas about such heaters, to direct thought to alternative possibly better types and to guide experimental work. When more experimental and theoretical results are available it can form the basis from which heaters might be designed.

It is shown that the simple annular type of heater will probably produce excessive swirl in the outlet air stream. Alternatives are suggested.

Account is taken in the theory of both the heat addition and the forces produced by the interaction of magnetic fields with the arc current.

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CONTENTS

ILLUSTRATIONS

SYMBOLS

A	heater annulus area
a	velocity of sound
B_L	longitudinal magnetic field strength
B_θ	circumferential magnetic field strength
C_p	specific heat of gas at constant pressure
E	electric field strength in column
F_L	longitudinal force on arc
F_R	radial force on arc
F_θ	circumferential force on arc
I	current in arc
ℓ	distance along arc
L	total arc length
P	pressure in gas
P'	power developed by F_R
Q	rate of heat addition in heater
r_1	radius of inner electrode
r_2	radius of outer electrode
T	temperature of gas
u	longitudinal velocity of gas
u_s	swirl velocity of gas at outlet
u_{arc}	circumferential component of arc velocity
V	voltage difference between electrodes
V^*	sum of voltage drops in electrode fall regions
v	velocity of gas relative to arc
w	the width of the arc wake

SYMBOLS (CONTD)

α angle between radius and normal to arc, Fig.3

γ ratio of specific heats

ρ gas density

Suffices

1 station upstream of arc

2 station downstream of arc

3 station immediately behind arc, introduced in Appendix 1

s a swirl component of velocity, introduced in Appendix 1

w indicates a mean wake velocity, introduced in Appendix 1

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1 INTRODUCTION

When we first became seriously interested in electric arc heaters, we used the empirical data then available to produce outline designs of heaters suitable for a range of proposed wind tunnels. However it was necessary to make a number of arbitrary assumptions and numerous difficulties appeared.

Since then the problems have been tackled experimentally and, within the very severe restrictions of our present experimental facilities, we have obtained more data[1] and an understanding of the shape of an electric arc in an annular gap[2]. To supplement and extend the experimental work the processes occurring in and around electric arcs both stationary and in a moving air stream have been investigated theoretically[3] and expressions obtained for the electric field strength, arc velocity etc. in terms of the pressure, current and applied magnetic field.

The present note is concerned with constructing a simple model of the flow and heat addition processes within an arc heater with a view to calculating conditions at the outlet and facilitating correct design. The heater is assumed to consist of an annular passage with the arc rotating in a plane normal to the duct axis. Attention is focussed firstly upon two planes remote from the arc where conditions are assumed to be uniform. Finally in Appendix 1 consideration is given to conditions close to the arc and an approximate model is proposed for one particular case. This model is shown to be consistent with the results of the main text.

A tentative analysis leads to the conclusion that an annular heater may produce a high degree of swirl at the exit.

2 FLOW MODEL

The basic heater geometry considered is shown in Fig.1. It is assumed that the gas is flowing in an annular passage from station (1) to station (2) with an arc rotating rapidly in a fixed plane normal to the axis and located between the two stations. The arc is driven round the central electrode (core of the annulus) by the uniform longitudinal magnetic field B_L. The arc is held against the mass flow $\rho_1 u_1 A$ by a uniform circumferential magnetic field B_θ. The air flow is uniform across the inlet annulus area A. The arc is of a shape like an involute i.e. I has components in the radial and circumferential directions but not in the longitudinal direction.

It will be assumed that there is no friction between the air and the electrodes or between the arc and the electrodes. There will therefore be no convective heat loss to the walls and the forces usually regarded as acting upon the arc may be regarded as acting on the gas as it passes the region of heat addition. Radiation from the arc column will be neglected except for that part which is absorbed by the gas in the arc region.

The force acting on the air will be treated as three separate components:

F_θ, the circumferential component produced by the radial component of I and B_L.

F_L, the longitudinal force produced by the radial component of I and B_θ.

F_R, the outward radial force produced by the circumferential component of I and B_L.

N.B. If there was a radial magnetic field this would also produce a force in the longitudinal direction when the current has a circumferential component. Such a field could be introduced to overcome any practical objection to assuming B_θ constant and to vary with radius the force opposing the longitudinal drag should this prove necessary.

The problem will now be simplified by adopting the approach used in simple outline turbine design. Conditions at the mean radius will be considered, the force F_R will be neglected as will the radial non-uniformities produced by F_R and by any swirl velocity. The heat added will be assumed uniform across the annulus. The resulting flow model is then as shown in Fig.2 where we have the uniform entry and exit conditions designated by suffices (1) and (2) respectively, a heat addition rate per unit area Q/A and forces per unit area acting on the gas F_θ/A and F_L/A. A is the total annulus area.

We then have the following equations*:-

(Continuity)

$$\rho_1 u_1 = \rho_2 u_2 \tag{1}$$

(Momentum)

$$\rho_1 u_1 (u_2 - u_1) = P_1 - P_2 - F_L/A \tag{2a}$$

$$\rho_1 u_1 (u_s - 0) = F_\theta/A \tag{2b}$$

(Energy)

$$\frac{u_1^2}{2} + C_p T_1 + \frac{Q}{A\rho_1 u_1} + \frac{F_\theta\, u_{arc}}{A\rho_1 u_1} = \frac{u_2^2}{2} + \frac{u_s^2}{2} + C_p T_2 \tag{3}$$

where u_{arc} is the circumferential component of the arc velocity.

The equations can be solved using the ideal equation of state and regarding P_2, T_2, ρ_2, u_2 and u_s as unknowns.

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*These equations apply exactly to the limiting case where $r_1 \to \infty$ and $(r_2 - r_1)$ remains finite. Under these conditions the arc will be radial and then $F_R = 0$.

$$u_2 = \frac{1}{u_1(\gamma+1)}\left\{a_1^2 + \gamma u_1^2 - \frac{\gamma F_L}{A\rho_1}\right\} \pm$$

$$\sqrt{\left[\frac{1}{u_1(\gamma+1)}\left\{a_1^2+\gamma u_1^2-\frac{\gamma F_L}{A\rho_1}\right\}\right]^2 + \frac{2(\gamma-1)}{(\gamma+1)}\left[\frac{1}{2}\left(\frac{F_\theta}{A\rho_1 u_1}\right)^2 - \frac{u_1^2}{2} - \frac{1}{A\rho_1 u_1}\left\{Q + F_\theta u_{arc}\right\} - \frac{a_1^2}{(\gamma-1)}\right]} \quad \ldots (4)$$

$$a_2^2 = a_1^2\frac{u_2}{u_1} - \gamma u_2(u_2 - u_1) - \frac{\gamma u_2 F_L}{A\rho_1 u_1} \quad (5)$$

$$u_s = \frac{F_\theta}{A\rho_1 u_1}\ . \quad (6)$$

The remaining unknowns P_2, T_2 and ρ_2 can be obtained from equations (1) and (5). However it will be found that when u_1 is small, u_2 is also small and equation (5) cannot be used to determine a_2, in that case a_2 is best determined from equation (3) through first calculating T_2.

To proceed further we need to know F_θ, F_L, u_{arc} and Q.

2.1 Determination of F_θ

The total force resulting from I and B_L will be normal to both and is shown as F in Fig.3. F_θ is the circumferential component of F. δF is the force acting on incremental length $\delta\ell$ of the arc.

$$\delta F = IB_L\,\delta\ell$$

and
$$\delta F_\theta = IB_L \sin\alpha\,\delta\ell = IB_L\,\delta r\ .$$

Therefore
$$F_\theta = IB_L(r_2 - r_1) \quad (7)$$

where α is the angle between the radius and the normal to the arc (Fig.3) and r_1 and r_2 are the radii of the inner and outer electrodes respectively. Note that F_θ is independent of the arc shape. This is not true of F_R which gets bigger as the arc becomes more swept or as α gets smaller.

Because $\delta F_\theta = I\,B_L\,\delta r$ we may write

$$\text{applied torque} = \int_{r_1}^{r_2} I\,B_L\,r dr = \frac{I\,B_L}{2}(r_2^2 - r_1^2) = A\rho_1 u_1\,\bar{u}_s\,\bar{r}\,,$$

the angular momentum gained by the fluid/unit time, where $\bar{u}_s$ is defined as the mean velocity of swirl and $\bar{r}$ is the mean radius.

Then $$\bar{u}_s = \frac{I\,B_L(r_2 - r_1)}{A\rho_1 u_1} = \frac{F_\theta}{A\rho_1 u_1} \quad \text{as before .}$$

2.2 Determination of F_L

As illustrated in Fig.3

$$\delta F_L = I\,B_\theta \sin\alpha\delta\ell = I\,B_\theta\,\delta r.$$

Therefore $$F_L = I\,B_\theta(r_2 - r_1)\,. \tag{8}$$

As with F_θ, F_L is also independent of arc shape. These are two most important and convenient results.

2.3 Determination of u_{arc}

Experimentally u_{arc} is one of the easiest quantities to measure and as a result numerous investigators have published results. This work has been reviewed by Adams[1] who deduced from his own results at one atmosphere that

$$u_c = 1{\cdot}87\,B_L^{0\cdot6}\,I^{0\cdot33} \tag{9}$$

where u_c is the cathode root velocity* and

*We assume here that the arc is of involute shape as described in Ref.2. This means that all parts of the arc travel at velocity u_c in a direction normal to the local current.

$$u_{arc} = u_c \sin \alpha \,. \qquad (10)$$

Here u_c is in ft/sec, B_L in gauss and I in amps.

There is an almost complete absence of results at pressures above one atmosphere. However Lord[3] has deduced theoretically that

$$u_c = \text{const}\, \frac{B_L^{0\cdot 58}\, I^{0\cdot 15}}{P^{0\cdot 42}} \,. \qquad (11)$$

We will therefore assume that

$$u_{arc} = 1{\cdot}87 \sin \alpha \, \frac{B_L^{0\cdot 6}\, I^{0\cdot 33}}{P^{0\cdot 42}} \qquad (12)$$

i.e. accept the practical index for I and the theoretical index for P. This will be good enough for present purposes.

2.4 Determination of Q

The required energy input to the gas for the production of a satisfactory heater for a particular wind tunnel is easily calculated[4]. However, even when ignoring heat losses to the walls, the energy input to the gas for a given geometry, current, magnetic field etc. is most difficult to determine.

Experimentally the voltage V between the electrodes, the current I and the applied magnetic field can all be measured. The total power expended within the chamber is VI. However this includes the power expended immediately adjacent to the electrodes in the electrode fall regions, also the mechanical power $F_\theta\, u_{arc}$ and that lost by radiation.

The model considered above assumes that the arc is all uniform column in as much as all electrode effects are neglected. To be consistent we must confine ourselves to the case where the electrode effects are very localised and it may then be assumed that the power expended within the electrode fall regions will be absorbed by the cooled electrodes and lost to the gas. Then (ignoring radiation from the column)

$$Q = \int_{\ell=0}^{\ell=L} I(E - u_{arc}\, B_L)\, d\ell \qquad (13)$$

where E is the electric field within the column (measured relative to the electrodes) and L is the column length.

Also

$$VI = V^*I + Q + F_\theta u_{arc} \qquad (14)$$

where V^* is the sum of the voltage drops at the two electrodes.

Clearly we need to study the electrode volt drop V^* and the column gradient E. We also need to know the conditions under which the electrode effects are localised and the radiation loss small.

3 MAGNITUDES OF QUANTITIES

To determine the magnitudes of the quantities involved, assumptions must be made about the geometry of the heater, the values of current, magnetic field and flow through the heater. It must be remembered that the values assumed for current and magnetic field may not be consistent with the assumed heat input, geometry and mass flow.

It will be assumed here that the heater is intended for a high density hypersonic tunnel with a gas flow of 3·6 lb/sec at a stagnation pressure of 1000 atmospheres and that u_1 is approximately 0·5 ft/sec.

3.1 Magnitude of F_θ

$$F_\theta = I\, B_L(r_2 - r_1) \, . \qquad (7)$$

Assume $1000 < I < 10{,}000$ amps

$1000 < B_L < 10{,}000$ gauss

and $(r_2 - r_1) \approx 2$ ins i.e. 5 cms

then $36 < F_\theta < 36 \times 10^2$ poundals .

3.2 Magnitude of F_L

$$F_L = I\, B_\theta(r_2 - r_1) \, . \qquad (8)$$

At the present time we have little direct knowledge of the value of B_θ required to maintain the arc in the correct plane. However we may assume that

$$\frac{B_\theta}{B_L} \approx \frac{u_1}{u_{arc}} \, .$$

If we assume B_L = 10,000 gauss, u_1 = 0·5 ft/sec and u_{arc} = 500 ft/sec, then B_θ = 10 gauss and for the above variation of current

$$36 \times 10^{-2} < F_L < 36 \times 10^{-1} \text{ poundals.}$$

3.3 Magnitude of u_{arc}

$$u_{arc} = 1 \cdot 87 \sin \alpha \frac{B_L^{0\cdot6}\, I^{0\cdot33}}{P^{0\cdot42}} \quad . \tag{12}$$

If we assume sin α = 1 then we obtain the following values for u_{arc}.

When B_L = 1,000 gauss, I = 1,000 amps and P = 1,000 atmos.
u_{arc} = 63 ft/sec

When B_L = 10,000 gauss, I = 10,000 amps and P = 1,000 atmos.
u_{arc} = 537 ft/sec

When B_L = 10,000 gauss, I = 10,000 amps and P = 1 atmos.
u_{arc} = 9,800 ft/sec

3.4 Magnitude of Q

From the wind tunnel performance we know that the required increase in stagnation enthalpy is about 6 × 10^7 ft poundals/sec. This we will assume is equal to Q.

3.5 Magnitude of u_2

$$u_2 = \frac{1}{u_1(\gamma+1)}\left\{a_1^2 + \gamma u_1^2 - \frac{\gamma F_L}{A\rho_1}\right\} \pm$$

$$\sqrt{\left[\frac{1}{u_1(\gamma+1)}\left\{a_1^2+\gamma u_1^2-\frac{\gamma F_L}{A\rho_1}\right\}\right]^2 + \frac{2(\gamma-1)}{(\gamma+1)}\left[\frac{1}{2}\left(\frac{F_\theta}{A\rho_1 u_1}\right)^2 - \frac{u_1^2}{2} - \frac{1}{A\rho_1 u_1}\left\{Q + F_\theta u_{arc}\right\} - \frac{a_1^2}{(\gamma-1)}\right]} \quad \ldots (4)$$

When u_1 is subsonic only the negative sign before the square-root is applicable. Assume F_θ = 36 × 10^2 poundals, F_L = 36 × 10^{-1} poundals (the maximum values) and u_{arc} = 500 ft/sec.

$F_\theta u_{arc} = 1{\cdot}8 \times 10^6$ compared with $Q = 60 \times 10^6$ ft poundals/sec, i.e. the mechanical work done by force F_θ is small compared with the heat added.

$$\frac{1}{A\rho_1 u_1}\{Q + F_\theta\, u_{arc}\} = 17 \times 10^6 \text{ ft}^2/\text{sec}^2 \quad \text{compared with}$$

$$\tfrac{1}{2}\left(\frac{F_\theta}{A\rho_1 u_1}\right)^2 = 0{\cdot}5 \times 10^6 \text{ ft}^2/\text{sec}^2 ,$$

$$\frac{u_1^2}{2} = 0{\cdot}125 \text{ and } \frac{a_1^2}{(\gamma-1)} = 3{\cdot}25 \times 10^6 \text{ ft}^2/\text{sec}^2$$

leading to

$$\frac{2(\gamma-1)}{(\gamma+1)}\left[\tfrac{1}{2}\left(\frac{F_\theta}{A\rho_1 u_1}\right)^2 - \frac{u_1^2}{2} - \frac{1}{A\rho_1 u_1}\{Q + F_\theta\, u_{arc}\} - \frac{a_1^2}{(\gamma-1)}\right] = -6{\cdot}6 \times 10^6 \text{ ft/sec}^2 .$$

Similarly

$$a_1^2 = 1{\cdot}3 \times 10^6 \text{ ft}^2/\text{sec}^2$$

$$\gamma u_1^2 = 0{\cdot}25 \text{ ft}^2/\text{sec}^2$$

and

$$\frac{\gamma F_L}{A\rho_1} = 0{\cdot}7 \text{ ft}^2/\text{sec}^2$$

leading to

$$\frac{1}{u_1(\gamma+1)}\left\{a_1^2 + \gamma u_1^2 - \frac{\gamma F_L}{A\rho_1}\right\} = 1{\cdot}1 \times 10^6$$

and

$$\left[\frac{1}{u_1(\gamma+1)}\left\{a_1^2 + \gamma u_1^2 - \frac{\gamma F_L}{A\rho_1}\right\}\right]^2 = 1{\cdot}2 \times 10^{12} .$$

Clearly we have a case where u_2 is small (in fact $u_2 = 3{\cdot}2$ ft/sec). This is because of the very low value chosen for u_1. If u_1 increases until it approaches a, a very different result will appear. The optimum value of u_1 to be used in a heater is not yet known and therefore the above formulae have been written in full. However obvious simplifications can be made when it is known that u_1 and F_L are small.

3.6 Magnitude of u_s

$$u_s = \frac{F_\theta}{A\rho_1 u_1} . \qquad (6)$$

From above $36 < F_\theta < 36 \times 10^2$ poundals and $A\rho_1 u_1 = 3{\cdot}6$ lb/sec so

$$10 < u_s < 10^3 \text{ ft/sec} .$$

These values of u_s must be compared with $u_1 = 0{\cdot}5$ and $u_2 = 3{\cdot}2$ ft/sec. It is clear that the gas at the heater exit may have a very large swirl component of velocity.

4 DISCUSSION OF RESULTS

4.1 Shortcomings of flow model

The major fault with the flow model presented is obviously the neglect of the radial component of force produced when the current has a circumferential component and also the neglect of the power developed by this force as the arc moves outwards. Unfortunately both these quantities depend upon the arc shape. However if we assume that the arc is of involute shape[2] then we can proceed. Fig.3 illustrates that

$$\delta F_R = I\, B_L \cos\alpha\, \delta \ell$$

also $$\delta r/\delta \ell = \sin\alpha$$

and for an involute $\sin\alpha \doteq r_1/r$ where r_1 is the radius of the inner electrode. Substitution gives

$$\delta F_R = I\, B_L \sqrt{\frac{r^2}{r_1^2} - 1}\ \delta r . \qquad (15)$$

Integrating from r_1 to r_2, where r_2 is the diameter of the outer electrode gives

$$F_R = I\, B_L \left[\frac{r_2}{2}\sqrt{\left(\frac{r_2}{r_1}\right)^2 - 1} - \frac{r_1}{2}\log_e\left\{\frac{r_2}{r_1} + \sqrt{\left(\frac{r_2}{r_1}\right)^2 - 1}\right\}\right] . \qquad (16)$$

This may be rewritten in terms of r_1 and A as

$$F_R = I\,B_L\left[\frac{1}{2}\sqrt{\frac{A^2}{\pi^2 r_1^2}+\frac{A}{\pi}} - \frac{r_1}{2}\log_e\left\{\sqrt{\frac{A}{\pi r_1^2}+1}+\sqrt{\frac{A}{\pi r_1^2}}\right\}\right]. \qquad (16a)$$

From which it can be seen that for a constant annulus area F_R decreases as r_1 increases.

The magnitude of F_R can be quite large. For the case considered above when $r_1 = 3$ inches and $r_2 = 5$ inches, $F_R = 31 \times 10^2$ poundals compared with $F_\theta = 36 \times 10^2$ poundals. This however is probably not so important as neglecting the power P' expended by F_R.

$$\delta P' = u_c\, I\, B_L \cos^2\alpha\, \delta\ell$$

which for an involute shaped arc becomes

$$\delta P' = u_c\, I\, B_L\left[\frac{r}{r_1} - \frac{r_1}{r}\right]\delta r \qquad (17)$$

which on integrating from r_1 to r_2 gives

$$P' = u_c\, I\, B_L\left[\frac{r_2^2 - r_1^2}{2r_1} - r_1\log_e\frac{r_2}{r_1}\right] \qquad (18)$$

and may be rewritten as

$$P' = u_c\, I\, B_L\left[\frac{A}{2\pi r_1} - r_1\log_e\sqrt{\frac{A}{\pi r_1^2}+1}\right]. \qquad (18a)$$

From which we again see that for a constant annulus area P' decreases as r_1 increases.

4.2 Practical interpretation

The major result appearing from the above is that a heater with a simple annular arrangement of fixed electrodes and using a large longitudinal magnetic field to produce rapid arc motion may produce an excessive swirl in the heated gas stream.

The swirl will produce a radial pressure gradient which will persist as long as the swirl exists. In addition there is an outward body force acting in the arc region which locally will increase the pressure gradient.

The results also show that with a low inlet velocity there will be very little change of longitudinal velocity through the arc region in spite of the inclusion of the body forces in the equations.

For use with a wind tunnel it will be necessary to remove the swirl before the gas enters the sonic throat presumably by inserting vanes into the flow. These will need to be cooled and must introduce large heat losses. Consideration should therefore be given to alternatives.

The original reason for introducing the rapid arc motion was to minimise electrode erosion and it has been suggested that

$$I_{max} = 250 \sqrt{u_c} \tag{19}$$

where I_{max} is the maximum arc current (amps) and u_c the arc velocity over the electrodes (ft/sec). No further information on this practical limit has reached the author. Accepting that there is such a limit and that a high value of u_c is essential then three alternatives to the simple annular arrangement appear possible. Firstly the electrodes could rotate relative to the gas stream, the arc remaining stationary or slow moving. Secondly a geometry could be evolved where only the ends of the arc move, the column being restrained by passing the arc through orifices and shielding the major part from the magnetic field. Thirdly swirl could be given to the air entering the chamber and removed by the arc.

The third suggestion appears the most practical but it has the disadvantage that for a given current and magnetic field u_c will be reduced.

Finally the outward radial force on the arc, always present with the annular geometry, is a practical nuisance. The obvious arrangement that avoids this problem is to use an inward flow type heater where the ring electrodes are of equal size and the arc, driven by a radial magnetic field, generates the surface of a cylinder as it rotates.

5 CONCLUSIONS

By treating the flow through an annular electric arc heater as that through a linear cascade, where the usual aerofoils are replaced by the electric arc, a method of calculating the resultant flow at a station downstream of the arc has been presented. The resultant electromagnetic force on the arc is equal and opposite to the drag on the arc which in turn is in the direction of the relative incident air stream. In spite of the arc having no lift (any force normal to the relative velocity), it is shown that when the drag and circumferential arc velocity are large a considerable amount of swirl is produced in the outgoing air stream.

The model neglects the radial component of the force acting on the arc and the work done by this component (para.4.1). However, in any practical design, efforts will be made to minimise this force for a number of reasons.

To determine the outlet flow the following must be known:-

the heater geometry,

the inlet temperature, pressure and mass flow,

the arc current,

the applied magnetic field,

and the voltage across the electrodes.

In addition experimental or theoretical relationships are required to determine the arc velocity, the voltage gradient in the column and the voltage drops at the electrodes.

To use the method for successful arc heater design an additional equation is required involving the electrode erosion such as equation (19).

The model will assist the rational consideration of alternative heater geometries and clarifies our need for more information on:-

u_c, the arc velocity,

E, the voltage gradient in the column,

I, the current,

V*, the electrode volt drops,

and I_{max}, the erosion limit.

All the above must be investigated over the full range of working conditions.

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APPENDIX 1

CONSIDERATION OF THE WAKE BEHIND THE ARC

1 INTRODUCTION

In the above note discussion is limited to conditions in the two planes (1) and (2) remote from the arc. When the throughput in the arc chamber is low and the arc advances partly into its own wake little else can be done. However, the author has been asked to consider the case where the throughput is sufficient to ensure that the rotating arc leaves a discrete wake. In this case some clarification of events between stations (1) and (2) is possible.

It has also become apparent that some readers find it easier to consider the case where the arc is regarded as stationary but then have difficulty in seeing how an arc, regarded as a solid body experiencing drag but no lift, can produce a swirl in the exit gas stream. To clarify these points the following discussion is based upon a stationary arc model and to avoid confusion with the main text it is presented in an Appendix.

2 FLOW MODEL

The approximate flow model adopted is illustrated in Fig.4. For simplicity the case where $r_1 \to \infty$ whilst $(r_2 - r_1)$ remains finite will be considered, thus avoiding all reference to curvature of the arc and effects thereby introduced. The arc is considered as a rigid body giving up heat to the air which flows past it.

In the centre of Fig.4 is shown the heater annulus straightened into a "cascade". The distance between the arcs is the circumference of the arc heater $2\pi r$ and $A = 2\pi r(r_2 - r_1)$ is the heater annulus area.

Consideration is again given to conditions at stations upstream (1) and downstream (2) of the heater plane. In addition a station (3) is considered immediately behind the arcs where discrete wakes exist but sufficiently far from the arcs for the static pressure to be constant across the station. At station (3) all the effects of the arc are assumed to be confined to the wake regions and the flow outside the wake is assumed to be unaffected. (This model is in fact never completely realised in a closed duct. It should be noted that the above assumptions imply that $A_3 > A_1$ so that the duct must be assumed to expand after station (1) returning to the original area at station (2).)

The air velocities relative to the arc are designated v where typically

$$\underline{v_1} = \underline{u_1} - \underline{u_{arc}} .$$

The air velocities relative to the chamber are designated u as before and suffices (1), (2) and (3) refer to the above stations. In addition suffix s refers to a swirl or circumferential component of velocity and suffix w to a mean wake velocity.

Downstream of station (3) a mixing process is assumed whereby the wake type flow is smoothed until uniformity is achieved at station (2).

Finally, a single force F is assumed to act on the arc in a direction opposite to v_1, where

$$\underline{F} = \underline{F_\theta} + \underline{F_L} .$$

3 ANALYSIS

The procedure adopted is firstly to write down the conditions at station (3) in terms of conditions at station (1) and an arbitrary division of the flow between the wake and the undisturbed region treating F and Q as known quantities, then to apply the conservation of momentum principle to the peripheral component of momentum whilst passing through the mixing zone between stations (3) and (2). It is thus shown that swirl is produced and after converting from v to u the same formula results as before, i.e. equation (6).

To proceed further it is necessary to solve simultaneously the continuity, momentum and energy equations. This is because there is a pressure change in the mixing region which directly effects the axial momentum. This was done in the main text. Here the conservation equations are written firstly in terms of v equating conditions in region (3) to region (2). These are then transformed to u values and the original equations produced.

We thus show how v_{s2} depends primarily on F and u_2 primarily on Q and F through the continuity, momentum and energy equations. In general v_2 will not be in the direction of v_1.

At station (3) the total mass flow per unit time $= A\rho_1 u_1$ (20)

Assume the wake flow $= \frac{1}{n} A\rho_1 u_1$ (21)

and the unaffected flow $= \frac{n-1}{n} A\rho_1 u_1$. (22)

The momentum equation applied to the wake flow then gives

$$F = \frac{1}{n} A\rho_1 u_1 (v_1 - v_{w3}) \tag{23}$$

and

$$v_{w3} = v_1 - \frac{Fn}{A\rho_1 u_1} . \tag{24}$$

Rate of momentum in wake flow passing station (3) $= \frac{1}{n} A\rho_1 u_1 \left(v_1 - \frac{Fn}{A\rho_1 u_1} \right)$

and rate of momentum in undisturbed flow passing station (3) $= \frac{n-1}{n} A\rho_1 u_1 v_1$.

Finally, considering just the wake flow, the conservation of energy between stations (1) and (2) gives

$$Q = \frac{1}{n} A\rho_1 u_1 \left\{ C_p (T_{w3} - T_1) + \frac{v_{w3}^2}{2} - \frac{v_1^2}{2} \right\} \quad \text{N.B. } P_1 = P_{w3}$$

or

$$C_p T_{w3} = \frac{nQ}{A\rho_1 u_1} + C_p T_1 - \frac{v_{w3}^2}{2} + \frac{v_1^2}{2} . \tag{25}$$

3.1 Momentum in circumferential direction

We can now equate the momentum at station (3) to momentum at station (2) taking components in the plane of the arc.

$$\frac{1}{n} A\rho_1 u_1 v_{w3} \sin a + \left(\frac{n-1}{n}\right) A\rho_1 u_1 v_3 \sin a = A\rho_1 u_1 v_2 \sin b$$

or

$$\frac{1}{n}\left(v_1 - \frac{Fn}{A\rho_1 u_1}\right) \sin a + \left(\frac{n-1}{n}\right) v_1 \sin a = v_2 \sin b$$

i.e.

$$v_1 \sin a - \frac{F}{A\rho_1 u_1} \sin a = v_2 \sin b . \tag{26}$$

From Fig.4

$$v_1 \sin a = u_{arc}$$

$$v_2 \sin b = v_{s2}$$

and

$$u_{s2} = u_{arc} - v_{s2}$$

i.e.

$$v_2 \sin b = u_{arc} - u_{s2} .$$

Substituting in equation (26) gives

$$u_{arc} - \frac{F}{A\rho_1 u_1} \sin a = u_{arc} - u_{s2}$$

or

$$u_{s2} = \frac{F_\theta}{A\rho_1 u_1} \quad \text{as before .} \tag{6}$$

Clearly $u_{s2} = C$ only when $F_\theta = 0$ i.e. $v_{s2} = u_{arc}$ when $F_\theta = 0$. Also $v_{s2} < u_{arc}$ when F_θ finite +ve and angle a = b when

$$\frac{u_1}{u_{arc}} = \frac{u_2}{v_{s2}} . \tag{27}$$

3.2 Momentum in longitudinal direction

Next, equating the axial components of momentum at stations (3) and (2) and writing in terms of velocities relative to the arc, gives

$$\left\{\frac{1}{n} A\rho_1 u_1 \left(v_1 - \frac{Fn}{A\rho_1 u_1}\right) + \left(\frac{n-1}{n}\right) A\rho_1 u_1 \; v_1\right\} \cos a - A\rho_1 u_1 \; v_2 \cos b = (P_2 - P_1)A \tag{28}$$

i.e.

$$\left[\rho_1 u_1 \left\{\frac{v_1}{n} + \left(\frac{n-1}{n}\right) v_1\right\} - \frac{F}{A}\right] \cos a - \rho_1 u_1 \; v_2 \cos b = P_2 - P_1$$

or

$$\rho_1 u_1 (u_1 - u_2) - \frac{F_L}{A} = P_2 - P_1 \tag{29}$$

which may be rewritten as before

$$\rho_1 u_1 (u_2 - u_1) = P_1 - P_2 - \frac{F_L}{A} . \tag{2a}$$

3.3 Energy equation

Similarly, we may equate the energy at station (3) to the energy at station (2).

$$\frac{1}{n} A\rho_1 u_1 \left\{C_p\ T_{w3} + \frac{v_{w3}^2}{2}\right\} + \left(\frac{n-1}{n}\right) A\rho_1 u_1 \left\{C_p\ T_1 + \frac{v_1^2}{2}\right\} = A\rho_1 u_1 \left\{C_p\ T_2 + \frac{v_2^2}{2}\right\}. \qquad \ldots (30)$$

Eliminating $C_p\ T_{w3}$ by using equation (25) yields

$$\frac{1}{n}\left(\frac{nQ}{A\rho_1 u_1} + C_p\ T_1 + \frac{v_1^2}{2}\right) + \left(\frac{n-1}{n}\right)\left(C_p\ T_1 + \frac{v_1^2}{2}\right) = C_p\ T_2 + \frac{v_2^2}{2}$$

or

$$\frac{Q}{A\rho_1 u_1} + C_p\ T_1 + \frac{v_1^2}{2} = C_p\ T_2 + \frac{v_2^2}{2}. \qquad (31)$$

This equation is clearly the equation obtained if the energy at station (1) is equated to that at station (2) allowing for the heat added and using axes fixed relative to the arc.

Equation (31) may be rewritten

$$\frac{Q}{A\rho_1 u_1} + C_p\ T_1 + \frac{u_1^2 + u_{arc}^2}{2} = C_p\ T_2 + \frac{u_2^2 + v_{s2}^2}{2}.$$

Fig.4 shows that

$$v_{s2} = u_{arc} - u_{s2}$$

so that

$$\frac{Q}{A\rho_1 u_1} + C_p\ T_1 + \frac{u_1^2}{2} + \frac{u_{arc}^2}{2} = C_p\ T_2 + \frac{u_2^2}{2} + \tfrac{1}{2}(u_{arc} - u_{s2})^2$$

$$= C_p\ T_2 + \frac{u_2^2}{2} + \frac{u_{arc}^2}{2} - u_{arc}\ u_{s2} + \frac{u_{s2}^2}{2}.$$

Substituting for u_{s2} from equation (6) gives

$$\frac{Q}{A\rho_1 u_1} + C_p\ T_1 + \frac{u_1^2}{2} = C_p\ T_2 + \frac{u_2^2}{2} - \frac{F_\theta}{A\rho_1 u_1} u_{arc} + \frac{u_{s2}^2}{2},$$

which may be rewritten as

$$\frac{u_1^2}{2} + C_p T_1 + \frac{Q}{A\rho_1 u_1} + \frac{F_\theta u_{arc}}{A\rho_1 u_1} = \frac{u_2^2}{2} + \frac{u_{s2}^2}{2} + C_p T_2 , \tag{3}$$

which is the original equation (3).

4 CONCLUSIONS

We thus see that results achieved using the above model for conditions at station (3) will be consistent with the main text, which considered only stations (1) and (2), in as much as the same momentum and energy equations are obtained.

The immediately obvious condition that the arc shall have a discrete wake at station (3) is that

$$w_3 \cos a < 2\pi r \tag{32}$$

where w_3 is the width of the arc wake at station (3) measured normal to v_1. However, inspection of the continuity equation reveals that the supposition of a discrete wake and an unaffected flow region demands an increase in cross-section at station (3). The actual value depending upon F, Q and n. In a practical heater there will probably be a constant or decreasing area and we may then deduce that there must be interference between the flow patterns about adjacent arcs in the cascade. Although there will be only one arc in a heater, this nevertheless means that suitable allowance for this "cascade" interference must be made when applying results from, say a single arc travelling along straight rails, to the case of an arc rotating in an annular gap.

The achievement of a heater suitable for wind-tunnel purposes will be critically dependent upon the mixing process between stations (3) and (2), because it is this which will produce the uniform conditions required at station (2). It is shown above that in general the flow turns between stations (3) and (2) and, in as much as the wakes can be followed through the mixing process, they will be found to turn also.

Electrically, the significance of this investigation is that, an arc with a discrete wake will be moving into unheated air and then can be expected to have a higher column voltage gradient than a similar arc moving into its own wake. It would appear reasonable to expect that in an arc chamber the voltage gradient would change abruptly at a particular throughput where the wake became discrete.

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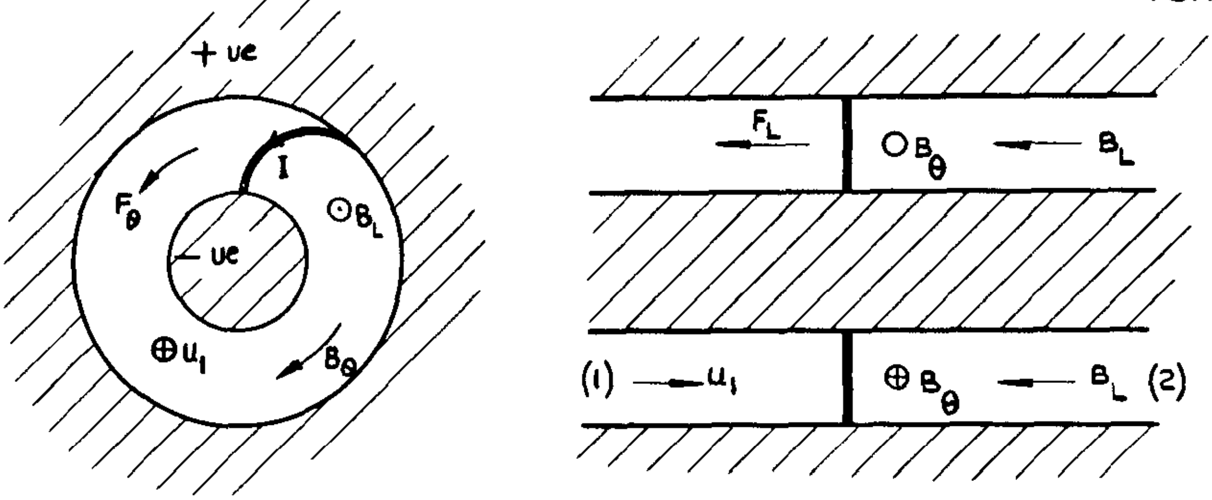

FIG.1. ANNULAR ARC HEATER.

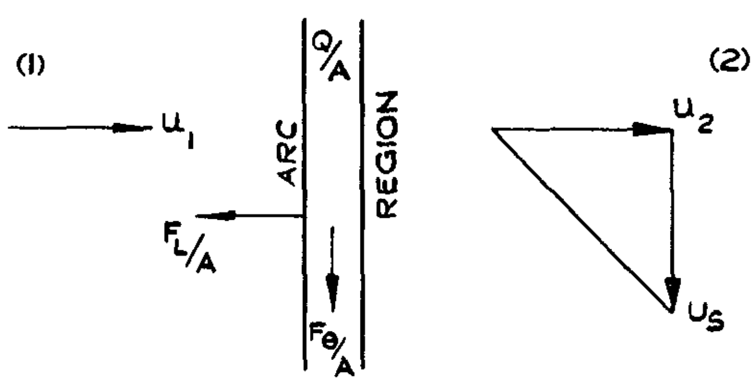

FIG. 2. FLOW MODEL.

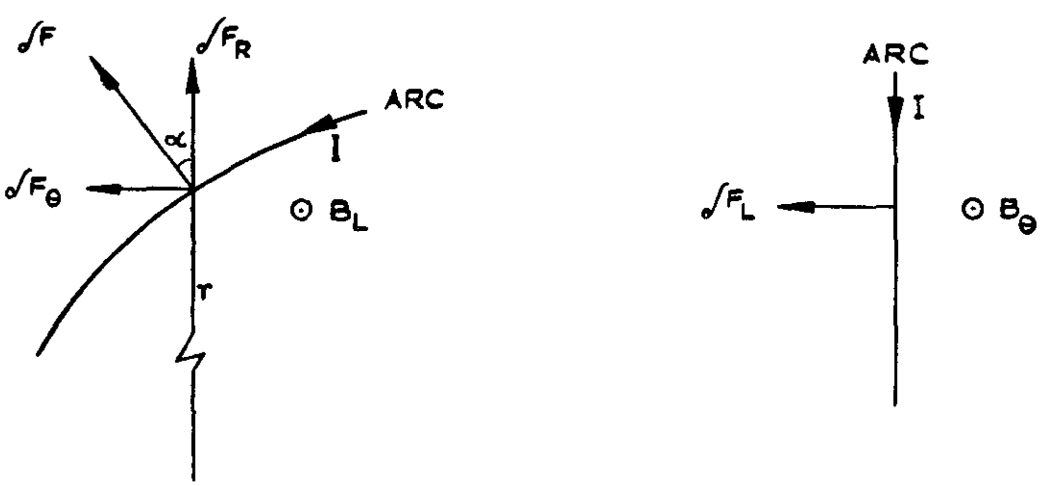

FIG. 3. FORCES ON ARC.

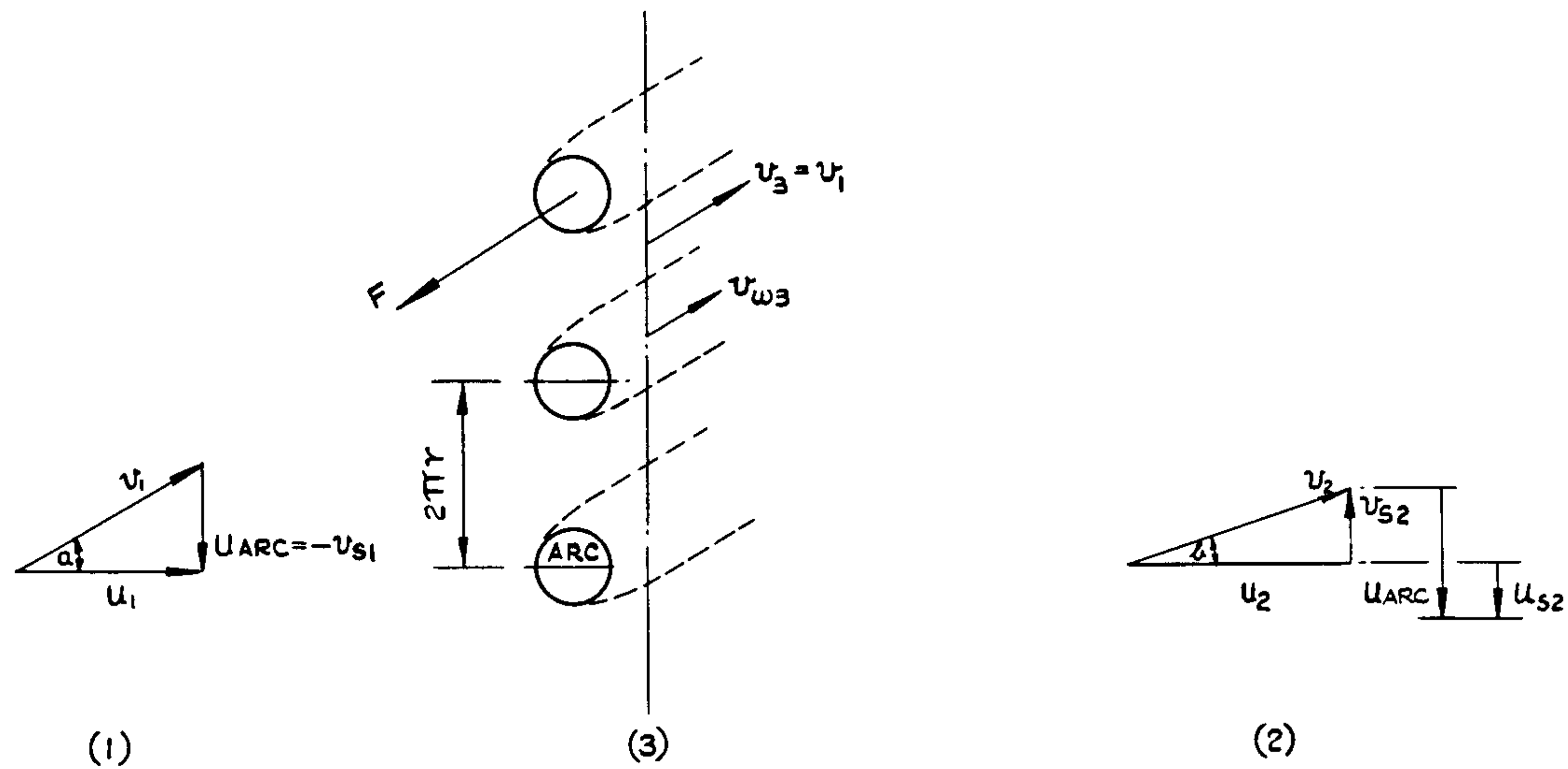

FIG. 4. FLOW MODEL WITH ARC CONSIDERED STATIONARY & HAVING DISCRETE WAKE.

FIG. 4.

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Account is taken in th
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It is shown that the simple annular type of heater will probably produce excessive swirl in the outlet air stream. Alternatives are suggested.

Account is taken in the theory of both the heat addition and the forces produced by the interaction of magnetic fields with the arc current.

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