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by

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A COLLECTION OF DATA ON THE LIFT-DEPENDENT DRAG OF UNCAMBERED SLENDER WINGS AT SUPERSONIC SPEEDS

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SUMMARY

Experimental data are presented on the lift-dependent drag of uncambered slender wings at supersonic speeds. It is shown that the values of the overall lift-dependent drag factor K collapse into a single curve, for a wide range of planform shapes and slenderness parameters, if K is plotted against

$$\frac{1}{2p} \cdot \frac{\beta s}{\ell}$$

and the implications of this dependence of K on p are briefly discussed.

Replaces R.A.E. Technical Note No Aero 2696 - A.R.C. 22,371.

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ILLUSTRATION

1 INTRODUCTION

An analysis has been made of recent experimental results on the lift-dependent drag of uncambered slender wings at supersonic speeds, including wings of ogee, delta, gothic and mild gothic planform with various aspect ratios and covering a wide range of the slenderness parameters $\beta s/\ell$. It has been found* that these results, together with those of Cane and Collingbourne for delta wings[6], collapse into a fairly well-defined single curve for all planforms if the overall lift-dependent drag factor K, given by

$$K = \frac{\pi A \left(C_D - C_{D_o}\right)}{C_L^2} \tag{1}$$

is plotted against

$$\frac{1}{2p} \cdot \frac{\beta s}{\ell} \qquad \text{or} \qquad \tfrac{1}{4}\beta A$$

which are of course equivalent, since

$$p = \frac{S}{2s\ell} = \frac{2}{A} \cdot \frac{s}{\ell} \tag{2}$$

where S = planform area

s = maximum semispan

ℓ = length

A = aspect ratio

$\beta = (M^2 - 1)^{\frac{1}{2}}$.

Although applying only to uncambered wings (few data are so far available on wings with representative camber design) this result is of interest in the light of the discussions which have been and are proceeding on the subject of the best choice of the planform shape parameter p for a supersonic transport aircraft.

2 PRESENTATION OF DATA

The values of K derived from the tests, at a lift coefficient of about 0.1, are presented in Fig.1 together with a table showing the source of the data, the type of planform and the values of p, aspect ratio, s/ℓ and centre-line thickness:chord ratio for each wing. In Fig.1 delta wings are represented by filled-in symbols (triangles, squares, circles) and wings with curved leading edges and streamwise tips are represented by unfilled symbols and by crosses etc. Of the latter, the crosses etc. represent gothic-type wings with $p > 0.5$, and the other unfilled symbols denote ogee-type wings with $p \leqslant 0.5$. Also shown are the points from Fig.15 of Cane and Collingbourne's analysis of delta wings in Ref.6; of these the unticked symbols apply to round-nosed sections of thickness:chord ratio between 0.05 and 0.08, and the ticked symbols are for sharp-nosed sections and for thickness:chord ratios of about 0.03. Apart from the round-nosed deltas of Ref.6 all the other wings of Fig.1 have sharp leading edges.

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* The collection and analysis of the results in Fig.1, and the choice of $\frac{1}{2p} \frac{\beta s}{\ell}$ as the collapsing parameter, are due to Dr. J. Weber, R.A.E.

3 RESULTS

It can be seen from Fig.1 that the wings with streamwise tips in particular define a mean curve of K against $\beta s/2p\ell$ or $\beta A/4$ with remarkably small scatter for $\beta s/2p\ell > 0.3$. At lower values of $\beta s/2p\ell$, i.e. in the transonic speed range except for very slender planforms, the curve is less well defined. At these speeds, however, it would not be expected that the results would collapse in terms of this parameter alone.

Under cruising conditions the value of $\beta s/2p\ell$ is unlikely to lie outside the range 0.3-0.8, and Fig.1 shows that over this range of what might be termed the "modified slenderness parameter", the value of K for uncambered wings is given very closely by

$$K = 0.75 + 2.55\left(\frac{1}{2p} \cdot \frac{\beta s}{\ell}\right) \quad . \tag{3}$$

The average value of K for delta wings appears to be somewhat higher than this, the wings with streamwise tips lying generally at the lower edge of the overall scatter band.

4 DISCUSSION

As indicated in Fig.1, the parameter $\beta s/2p\ell$ for wings with curved leading edges is the same as the parameter $\beta A/4$, which for delta wings is equal to $\beta \cot \Lambda$ as used by Cane and Collingbourne in their analysis of delta wings. For wings of similar shape, e.g. in particular for delta wings, this form of the collapsing parameter is suggested by the supersonic similarity laws. For wings of dissimilar shapes, however, e.g. gothic vs. ogee, there is no a priori justification for such a common parameter. The fact that one exists indicates that, at any rate on uncambered wings, changes in loading are taking place which tend to reduce the otherwise favourable effects of reducing the value of p. This is best seen by reference to the usual equation for lift-dependent drag coefficient in terms of vortex and wave drag contributions:-

$$C_{D_i} = \frac{1}{2\pi} C_L^2 \frac{p}{s/\ell}\left[K_V + 2\left(\frac{\beta s}{\ell}\right)^2 K_W\right] \quad . \tag{4}$$

In (4) K_V is the vortex drag factor depending on the spanwise distribution of chord load at the trailing edge, and K_W is the lift-dependent wave drag factor depending on the chordwise distribution of cross load. Compared with equation (1)

$$K = K_V + 2\left(\frac{\beta s}{\ell}\right)^2 K_W \quad . \tag{5}$$

In the absence of changes in K_V and K_W the value of K would depend only on $\beta s/\ell$ and would be independent of changes in p. Decrease of p would then lead to a proportionate decrease of lift-dependent drag coefficient at a given value of C_L. However, Fig.1 shows that in practice on uncambered wings, K increases as p decreases (as represented by equation (3), for instance), showing that on these wings either K_V or K_W or both increase with decrease of p. Qualitatively, this is what would be expected in the absence of special measures such as camber and twist to prevent it. It is known, for instance, that the high-p gothic planform is very nearly that to give, in R.T. Jones' theory, elliptic chord and span loadings and achieves low values of K fairly readily. As the planform is progressively "pinched in" to give first the p = 0.5 delta and then ogees, it can be expected that on uncambered wings both chord and span loadings will depart further and further from the elliptical, with insufficient load carried near the apex and near the tip. One object of the test series planned for cambered wings[8] is to find ways of mitigating these adverse loading changes on low-p planforms. The design calculations have already shown that the lower the value of p, the more difficult it is to achieve a low theoretical value of K_W in particular; there is obviously a limit to the amount of load that the narrow front part of an ogee can be expected to carry. The design problem is complicated by low-speed requirements, since the leading-edge attachment condition leads to increasing spanwise droop as the load near the front is increased in the design condition. This could lead to unfavourable development of the upper surface separated vortex flow at take-off and landing incidences, with splitting of the vortex sheet, loss of non-linear lift and possibly stability and control problems. It is therefore possible that, even with the best possible use of camber, something of the trend noted here for plane wings will persist for cambered wings at low values of p; the forthcoming wind-tunnel programme[8] should show how important this trend is.

Finally, an example will be given of the effect of the trend shown in Fig.1 for uncambered wings. Consider an aircraft having

$$p = 0.5$$
$$s/\ell = 0.21$$
$$M = 2.6$$

so that

$$\frac{1}{2p} \cdot \frac{\beta s}{\ell} = 0.504 \quad .$$

Equation (3) gives for these conditions:

$$K = 0.75 + 1.28 = 2.03$$

so that

$$C_{D_i} = \frac{1}{2\pi} C_L^2 \frac{p}{s/\ell} \times 2.03 = \underline{0.77\ C_L^2} \quad .$$

If now p is reduced to 0.4 at constant M and s/ℓ, we have

$$\frac{1}{2p} \cdot \frac{\beta s}{\ell} = 0.630$$

and

$$K = 0.75 + 1.60 = 2.35$$

giving

$$C_{D_i} = \underline{0.713\ C_L^2} \quad ,$$

a reduction of $7\frac{1}{2}\%$ compared with the original value. If K had remained constant, C_{D_i} would have decreased in proportion to p, giving a reduction of 20% compared with the original value. Thus, because of the variation of K with p found here for plane wings, the reduction in lift-dependent drag due to reducing p from 0.5 to 0.4 is only $37\frac{1}{2}\%$ of what it would have been if K had remained constant. The point should perhaps be made, however, that this smaller figure is still of course an improvement over the original;

the stage has not been reached where decrease of p has an adverse effect on lift-dependent drag even for uncambered wings. It is also worth noting that quite apart from the effects on lift-dependent drag there are other factors, possibly more important, tending to favour lower values of p for supersonic transports. These are:-

(i) the reduction in wing area and hence structure weight due to reducing p at constant s/ℓ,

(ii) a potential reduction in zero-lift wave drag, which would decrease as p^2 if the volume coefficient τ* and the zero-lift wave drag factor both remained unaltered (which in general they would not), and

(iii) the easing of balance problems, for passenger transports at any rate, as p is reduced.

The increase of K with decrease of p found here on uncambered wings is therefore not likely to lead to much greater values of p in practice, particularly in view of the hoped-for lessening of this tendency when full use is made of camber. The chief conclusion to be drawn is that as p is reduced, more and more attention will have to be paid to camber design in order to avoid diminishing returns due to accompanying increases of K.

LIST OF REFERENCES

No.	Author	Title, etc.
1	8' × 8' Tunnel, RAE, Bedford.	Three-component measurements, mainly at M = 2, on shapes produced by empirical modifications to a delta wing of aspect ratio 1.78. Unpublished M.O.A. Report.
2	Squire, L.C. Capp, D.S.	An experimental investigation of the characteristics of an ogee wing from M = 0.4 to M = 1.8. A.R.C. C.P. No.585. August 1959.
3	Clark, R.V.	Aerodynamic design of wind tunnel models of ogee planform.** Handley Page Aero/343, November 1959.
4	Evans, J.Y.G.	A review of work in the 8 ft tunnel at Bedford related to the design of slender wing supersonic aircraft. Unpublished M.O.A. Report.
5	Haines, A.B. Jones, J.C.M.	Further results from tests on a slender delta wing with cambered leading edges in the ARA Transonic Tunnel. ARA Aero. Memo. No.20, April 1959.

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* τ = volume ÷ (wing area)$^{3/2}$.

** Wing design only; test data from recent 8' × 8' tunnel tests not yet published.

LIST OF REFERENCES (Contd.)

No.	Author	Title, etc.
6	Cane, E.G. Collingbourne, J.R.	An analysis of some data on lift-dependent drag at supersonic speeds. Unpublished M.O.A. Report.
7	Squire, L.C.	An experimental investigation at supersonic speeds of the characteristics of two gothic wings, one plane and one cambered. A.R.C. R & M 3211. May 1959.
8	Evans, J.Y.G. Squire, L.C.	Proposed test programme on cambered slender wings in the 3' × 3' and 8' × 8' wind tunnels. Unpublished M.O.A. Report.

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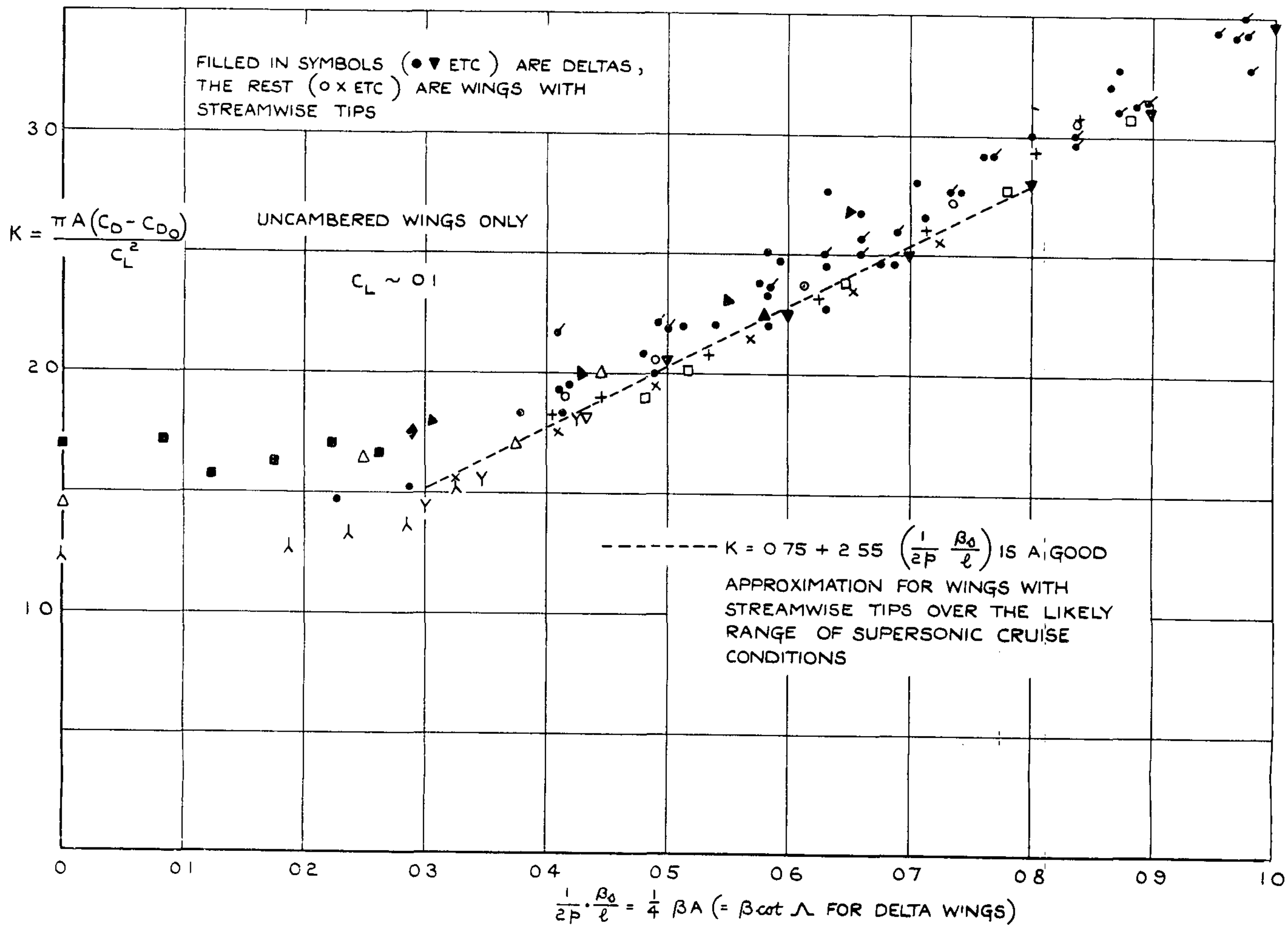

SYMBOL	REFERENCE	WING TYPE	p	ASPECT RATIO	s/ℓ	ROOT t/c
□	REF 1 (8′ × 8′) CONFIG^N 9	OGEE	0.384	1.27	0.244	0.056
○	REF 1 (8′ × 8′) CONFIG^N 7	OGEE	0.409	1.27	0.259	0.056
△	REF 2 (3′ × 3′)	OGEE	0.500	1.20	0.300	0.050
▽	REF 3 (HANDLEY PAGE, 8′ × 8′)	OGEE	0.500	1.00	0.250	0.063
▲	REF 4 (8′ × 8′) WING F	DELTA	0.500	1.33	0.333	0.084
◆	REF 4 (8′ × 8′) WING J	DELTA	0.500	0.67	0.167	0.084
▶	REF 4 (8′ × 8′) WING K	DELTA	0.500	1.00	0.250	0.084
■	REF 5 (ARA)	DELTA	0.500	1.07	0.268	0.100
▼	REF 1 (8′ × 8′) CONFIG^N 1	DELTA	0.500	1.78	0.445	0.084
●	REF 6 (CANE AND COLLINGBOURNE)	DELTAS WITH MODERATE L.E. RADIUS	0.500	0.70 - 4.0	—	0.05 – 0.08
●′	REF 6 (CANE AND COLLINGBOURNE)	DELTAS WITH SMALL OR ZERO L.E. RADIUS	0.500	0.70 - 4.0	—	~ 0.03
+	REF 1 (8′ × 8′) CONFIG^N 2	MILD GOTHICS (DELTAS	0.560	1.39	0.389	0.084
×	REF 1 (8′ × 8′) CONFIG^N 3	WITH ROUNDED TIPS)	0.611	1.09	0.333	0.084
Y	REF 4 (8′ × 8′) WING L	GOTHIC	0.667	1.00	0.333	0.068
⅄	REF 7 (3′ × 3′)	GOTHIC	0.667	0.75	0.250	0.082

FIG.1. OVERALL LIFT-DEPENDENT DRAG FACTOR K vs $\frac{1}{2p}\frac{\beta_0}{\ell}$ FOR UNCAMBERED WINGS.

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