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Power Spectra of the Vertical Component of Atmospheric Turbulence Obtained from Concurrent Measurements on an Aircraft and at Fixed Points

by

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POWER SPECTRA OF THE VERTICAL COMPONENT OF ATMOSPHERIC TURBULENCE OBTAINED FROM CONCURRENT MEASUREMENTS ON AN AIRCRAFT AND AT FIXED POINTS

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SUMMARY

Power spectra of the vertical component of turbulence obtained from concurrent measurements at fixed points on a captive balloon cable and on an aircraft flown in the vicinity of the balloon are compared. The comparison is restricted to a single flight in which runs were made at three heights, corresponding to the heights of observation on the cable. The spectra show good agreement but certain deficiencies in the methods of measurement are disclosed, which limit the extension of this method of comparison.

Replaces R.A.E. Tech Note No. Structures 325 - A.R.C. 24,742.

LIST OF CONTENTS

LIST OF ILLUSTRATIONS

1 INTRODUCTION

The equivalence of power spectra of atmospheric turbulence* measured on an aircraft and at a fixed point is a matter of considerable interest to the aircraft engineer, since, if such equivalence can be established, it greatly enlarges the scope for obtaining low altitude turbulence data for aircraft purposes. The theoretical background to this approach is provided by a paper by G.I. Taylor[1] in which he discusses the connection between the correlation of vertical measurements at two points in space, lying along the path of a turbulent airstream, and the turbulence spectrum at a fixed point in the stream. Taylor shows that the space correlation function and the turbulence spectrum are Fourier transforms of each other. His conclusions have been verified by L.F.G. Simmons[2] in wind tunnel experiments in which turbulence was generated by a grid upstream of the measuring points. There is, however, a considerable step to be taken in applying Taylor's hypothesis to aircraft and fixed point measurements. The turbulence is now no longer generated at a single source but at a series of sources, mechanical or thermal; thus, the homogeneity of the terrain becomes an important factor. In some cases, the turbulence is generated continuously by wind shear between different layers of the atmosphere. In this context, Lin[3] has suggested a limitation at the low frequency end of the spectrum to the range of frequency over which Taylor's hypothesis applies. A further consideration in applying Taylor's hypothesis to aircraft and fixed point measurements is the stationarity of the turbulence with respect to time. Fixed point measurements need to extend over a period of the order of at least one hour to cover the frequency range of interest to the aircraft engineer, and slow changes in meteorological conditions could well occur during a period of this duration which would invalidate the application of the hypothesis.

To date comparisons of tower (or equivalent fixed point) and aircraft gust spectra have been made by R.A. Jones[4] using aircraft measurements made by Crane and Chilton[5]; and by Lappe, Davidson and Notess[6]. The former compares spectra obtained in different parts of the world and aims only at a general comparison of the shapes; within this scope the comparison is quite promising. The comparison made by Lappe, Davidson and Notess is much more comprehensive and is based on concurrent measurements on an aircraft and on each in turn of three towers situated in different types of terrain. Measurements include vertical and horizontal components of turbulence. The results are of considerable interest, although not altogether conclusive, since aircraft measurements were made at heights slightly greater than those of the tower measurements, so that direct comparison is not possible (lowest aircraft measurements at 400 ft, highest tower measurement at 300 ft). Results indicate that at wavelengths of the order of 800 ft or longer, i.e. at the low frequency end of the spectra, the aircraft detects more turbulent energy than does the tower. At smaller wavelengths agreement is good. Differences at the low frequency end of the spectra do not appear to vary consistently, nor can they be simply related to the degree of non-homogeneity of the terrain.

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* The power spectrum of atmospheric turbulence shows the contribution of different frequencies of turbulent airflow to the mean square velocity of the turbulence. The total area under the curve is the mean square velocity.

2 OUTLINE OF EXPERIMENT

This note describes a first attempt in the United Kingdom to obtain concurrent spectra of the vertical component of atmospheric turbulence from aircraft and fixed point measurements. A specially instrumented Canberra was flown at Cardington in the neighbourhood of a captive balloon, to the cable of which special turbulence measuring instrumentation was attached. Results are given here for the first flight made on July 27th, 1960. Measurements were made at three heights, 200, 500 and 1000 ft above ground. The balloon cable measurements cover a period of one hour, during which time the aircraft made several runs past the balloon. Each run was of 2 minutes duration and was made into wind covering a distance of $11\frac{1}{2}$ miles. The period of recording on the balloon cable and the length of the aircraft runs were chosen so that the analysis would cover a range of wavelength from 8000 ft to 50 ft for both aircraft and balloon cable measurements with roughly the same accuracy.

2.1 Instrumentation

Instrumentation for the balloon cable is described by J.I.P. Jones and H.E. Buller[7]. Briefly it consisted of balanced vanes which were faced into wind by a weathercock action. These vanes were hinged about a horizontal axis so that they rotated according to the inclination of the wind. Although the results, therefore, refer strictly to fluctuations of wind inclination they are considered approximately applicable to the vertical component of the turbulent airflow[4].

The sensing of the turbulence from the aircraft was achieved by means of differential pressure measurements on a nose-probe, the forward end of which consisted of a semi-spherical head with orifices at 45° to the longitudinal axis of flight. Differences in pressures at these orifices activated pressure capsules fitted immediately behind the orifices. Corrections had to be made to these probe measurements for the pitching and vertical translation of the aircraft itself which could introduce spurious effects, often of considerable magnitude. This necessitated a high degree of accuracy in the measurements of pitch and acceleration (the latter was integrated to give vertical velocity).

2.2 Topography

The terrain in the vicinity of Cardington consists mainly of flat agricultural land. All runs were made over this type of terrain; the track avoided villages and towns but passed over occasional farm buildings and trees. Thus the terrain could be considered fairly homogeneous for the purposes of the experiment, although more homogeneous terrain could be found, for example, in the Norfolk fens or in the North African desert.

2.3 Meteorological conditions

Details of the meteorological observations are given in Appendix 1. The measurements were made during the late morning of July 27th, 1960. Turbulence was slight to moderate and was probably due primarily to thermal rather than to mechanical causes. The wind was light, 5 to 7 knots, with very little shear from ground to a height of 1000 ft. (Wind measurements were not made above this height.) Instability of the lower atmosphere was indicated by a slightly unstable lapse rate, 3.2°C/1000 ft as measured by the aircraft between 200 and 1000 ft; and by the build-up of cumulus cloud from 3/8 to 6/8 during the course of the experiment. Meteorological conditions did not vary much otherwise; mean wind speed was fairly constant although there was a slight shift in wind direction below 200 ft (of the order of 10°); ground temperature increased by 0.5°C during the period of measurement.

2.4 Data reduction

It is not proposed to discuss this matter in detail. All recordings were obtained originally in continuous trace form and were digitised at discrete intervals of time by punching the height of the trace above the datum on to Hollerith cards. The intervals of time chosen were 1/20th sec for the aircraft and 2.42 sec for the balloon. The power spectrum was obtained from the Fourier transform of the auto-correlation function, the computation being carried out by DEUCE and Mercury. A number of processes were included to refine the accuracy of the power estimates; these included "pre-whitening" and "hanning"[8]. Bartlett's smoothed periodogram[9] was used in the analysis of the balloon measurements. The analysis for both aircraft and balloon was repeated with the digitised readings added in non-overlapping groups of four to obtain more refined data at the low frequency end of the spectra.

3 RESULTS

3.1 Comparison of power spectra

Figs.1, 2 and 3 show the power spectra obtained from the aircraft and balloon cable at heights of 200, 500 and 1000 ft. At wavelengths of a few hundred feet or less, in the part of the spectrum where energy is mainly transported without gain or loss, agreement tends to be quite good, particularly at the greater heights. At longer wavelengths, approaching the energy input end of the spectrum, there appears to be a tendency for the aircraft to detect more turbulent energy than the balloon cable. At very long wavelengths, of the order of 5000 ft, the spectra tend to come together again, but the accuracy of results for such long wavelengths (or low frequencies) is questionable, especially in the case of the aircraft. Thus, despite the homogeneity of the terrain, the aircraft has encountered slightly more turbulent energy by flying over different input sources than is carried in the airstream past the balloon cable. These results are in agreement with those of Lappe, Davidson and Notess, although there appears to be a tendency in the present results for the range in which the aircraft encounters more energy to extend to smaller wavelengths, particularly when the aircraft flies very low down.

So far emphasis has been laid on the differences between aircraft and balloon cable measurements. In point of fact, there are some interesting similarities. In particular, if a comparison is made of the shapes of the spectra at heights of 200 and 1000 ft, both aircraft and balloon cable spectra at 200 ft show a greater tendency to flatten out at the low frequencies than do the spectra at 1000 ft. This implies that the scale of the turbulence is less at 200 ft than at 1000 ft, a result to be expected from earlier tower measurements[11] but not so far confirmed by aircraft measurements. Best fit of the standard expression used in the U.S.A.* is given by L = 400 ft for the 200 ft height spectra and L = 920 ft for the 1000 ft; corresponding root mean square velocities are 3.08 and 3.46 ft/sec respectively.

3.2 Difficulties in methods of deriving gust velocity power spectra

The following drawbacks came to light in the course of the analysis:-

3.2.1 Balloon cable measurements

The instrumentation was not planned for obtaining power spectra over the range of interest to aircraft engineers and the traces contained unwanted high frequencies. The power at these frequencies would have "aliased" or "folded" into the frequency band of interest, causing erroneous results, had not the records been faired out by hand before being read. This fairing by hand is not altogether a satisfactory process however, as it relies too much on individual judgment.

A further difficulty arose with regard to the balloon cable measurements in that the wind was rather light (10 ft/sec), with the result that the vertical gust velocity at times exceeded the horizontal gust velocity causing the vane to come up against the 45° stops. Some of the peak gusts had to be filled in by guesswork. This was only a rare occurrence however, and is not thought likely to affect the derived power spectra significantly. It does, however, point to limitations in the range of turbulence conditions that can be covered by tower or balloon cable measurements. In later tests a further limitation with regard to wind speeds was apparent in that at speeds of 25 ft/sec or greater the records contained spurious signals due to cable oscillation. This limitation would not, however, apply to tower measurements.

3.2.2 Aircraft measurements

The main drawback to this method of obtaining power spectra of the vertical component of turbulence is the difficulty in measuring the aircraft motions with sufficient accuracy at very low frequencies. Very careful instrumentation is required if the vertical component of turbulence derived from the corrected probe readings is not to contain spurious linear or very low frequency trends originating from gyro drift and from errors in the zero datum of the acceleration which accumulate when integrated**. The resulting errors limit the range of wavelength over which the aircraft measurements are valid. Further inaccuracies of a statistical nature arise in the determination

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$$G(\Omega) = w^2 \frac{L}{\pi} \frac{(1 + 3\Omega^2 L^2)}{(1 + \Omega^2 L^2)^2}$$

where w^2 is the mean square gust velocity, L the scale of the turbulence and Ω the space frequency in radians per unit length.

** Similar effects were observed in the analysis of the B-66B results[10].

of the low frequency end of the spectrum due to the shortage of the sample. It is difficult to get a long enough sample of turbulence over homogeneous terrain.

4 CONCLUSIONS

Power spectra of the vertical component of turbulence measured concurrently on an aircraft and on the cable of a captive balloon were found to be in quite good agreement particularly at short wavelengths (less than a few hundred feet). The comparisons refer to a single occasion only when observations were made at heights of 200, 500 and 1000 ft in conditions of light wind and slight instability. At longer wavelengths the aircraft was found to detect slightly more turbulent energy than that indicated by the fixed point measurements.

Both methods have their limitations: the balloon cable measurements can only be made over a limited range of wind speeds; the waveband covered by the aircraft measurements is limited at the low frequency end by inaccuracies in the instrumentation.

5 FURTHER WORK

More comparisons are required to substantiate results obtained so far and to cover different meteorological conditions and terrain. It is planned to extend this work to include concurrent measurements made with the Institute of Meteorology, Darmstadt, Germany, who are proposing a programme of power spectrum measurements in vertical and horizontal planes on a tower at Darmstadt. As their instrumentation is specially designed with power spectral analysis in mind, it is hoped that some of the difficulties inherent in the Cardington balloon cable experiment will be overcome.

LIST OF REFERENCES

No.	Author	Title, etc.
1	Taylor, G.I.	The spectrum of turbulence. Proc. Royal Society, Series A, Vol.164. 1938.
2	Simmons, L.F.G. Salter, C.	An experimental determination of the spectrum of turbulence. Proc. Royal Society, Series A, Vol.165. 1938.
3	Lin, C.C.	On Taylor's hypothesis and the acceleration terms in the Navier-Stokes Equation. Q.A.M. Vol.X. January, 1958.
4	Jones, R.A.	Studies of eddy structure in the first few thousand feet of the atmosphere - A preliminary examination of the spectrum and scale of the vertical component at 2000 ft. M.R.P. 104.4, S.C.111/231. February, 1957.

LIST OF REFERENCES (Contd.)

No.	Author	Title, etc.
5	Crane, H.L. Chilton, R.G.	Measurements of atmospheric turbulence over a wide range of wavelength for one meteorological condition. N.A.C.A. Technical Note No.3702. June, 1956.
6	Lappe, U.O. Davidson, B. Notess, C.B.	Analysis of atmospheric turbulence spectra obtained from concurrent airplane and tower measurements. I.A.S. Report No.59-44. January, 1959.
7	Jones, J.I.P. Butler, H.E.	Studies of eddy structure in the first few thousand feet of the atmosphere - Measurements of the vertical and horizontal components. M.R.P.1038, S.C.111/229. February, 1957.
8	Blackman, R.B. Tukey, J.W.	The measurement of power spectra. Dover Publications, 1958.
9	Priestly, M.B.	The analysis of stationary time series. II Estimation of power spectra. Unpublished M.O.A. Report.
10	Saunders, K.D.	Interim report on the technical analysis of the B-66B low level gust study. Report No. SM-23973. Douglas Aircraft Co. Inc. May, 1960.
11	Panofsky, H.A. Deland, R.J.	One-dimensional spectra of atmospheric turbulence in the lowest 100 metres. Advances in Geophysics, Vol.VI. 1959.

APPENDIX 1

METEOROLOGICAL CONDITIONS ON 27TH JULY, 1960, AT CARDINGTON

Synoptic observations

Time (G.M.T.)	10.00	11.00
Wind	220° 5 kts	200° 5 kts
Cloud Cloud	3/8 cu 2800 ft 7/8 cs 25,000 ft	6/8 cu 2500 ft 7/8 cs 25,000 ft
Ground temp. (dry bulb)	63°F	64°F
Dew point	53°F	55°F

Wind speed (metres/second)

Height / Time (G.M.T.)	2m	4m
10.00-10.15	2.15	2.23
10.15-10.30	2.33	2.52
10.30-10.45	1.97	2.00
10.45-11.00	2.02	2.13

Wind directions (degrees true)

Height / Time (G.M.T.)	200 ft	500 ft	1000 ft
10.00	210	220	240
10.30	235	240	240
11.00	230	235	240

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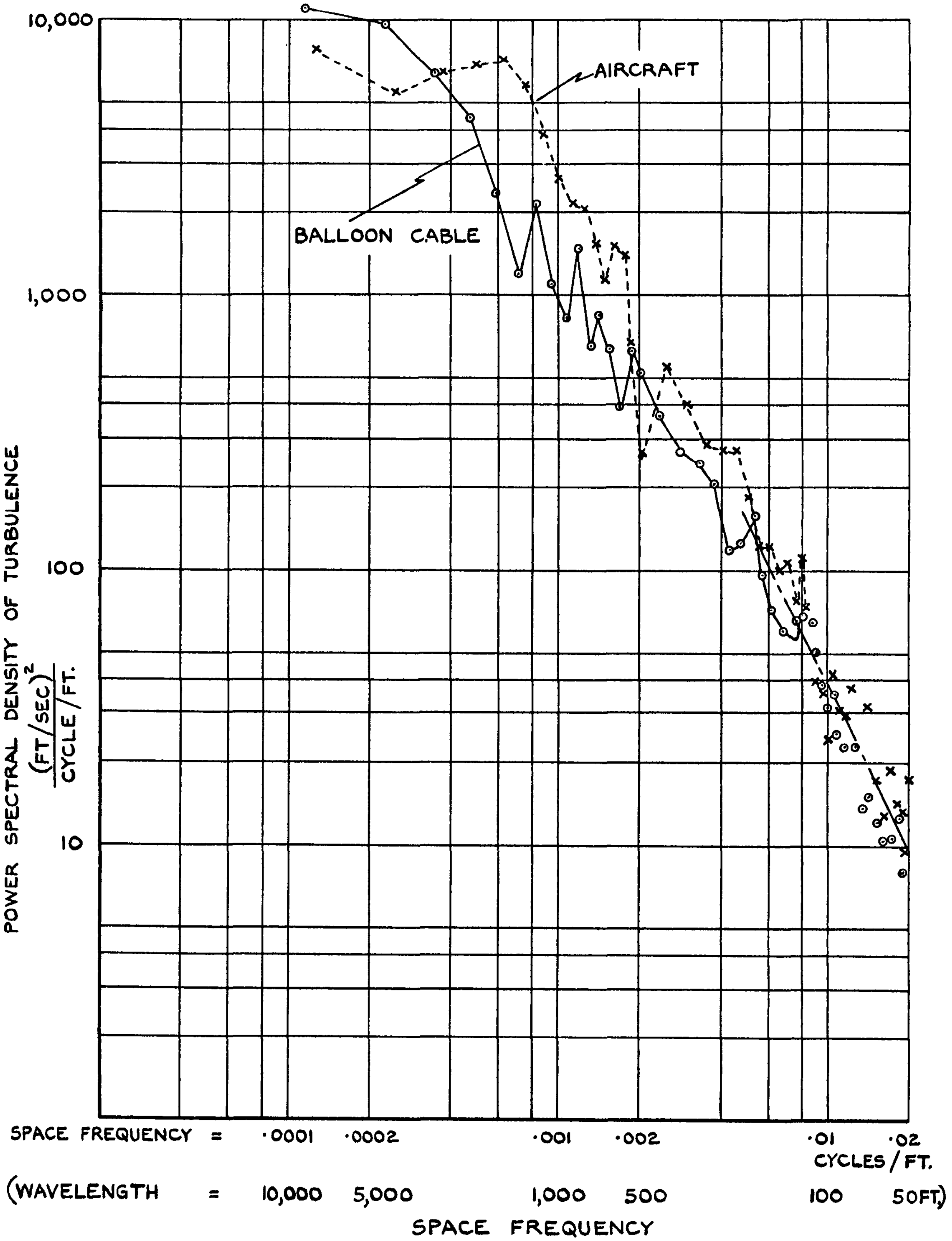

FIG. 1. POWER SPECTRA OF VERTICAL GUST VELOCITY AT 200 FT.

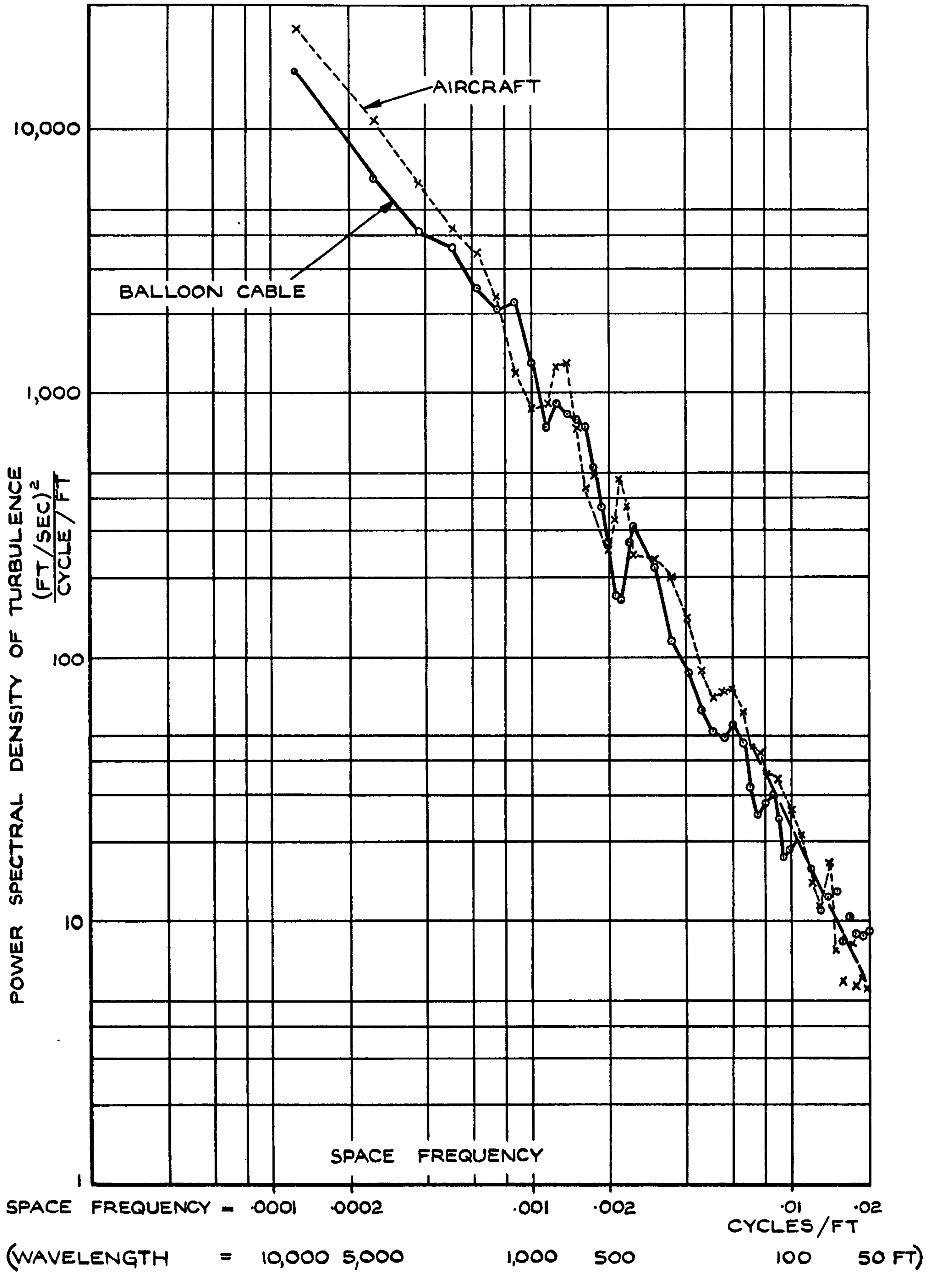

FIG. 2. POWER SPECTRA OF VERTICAL GUST VELOCITY AT 500 FT.

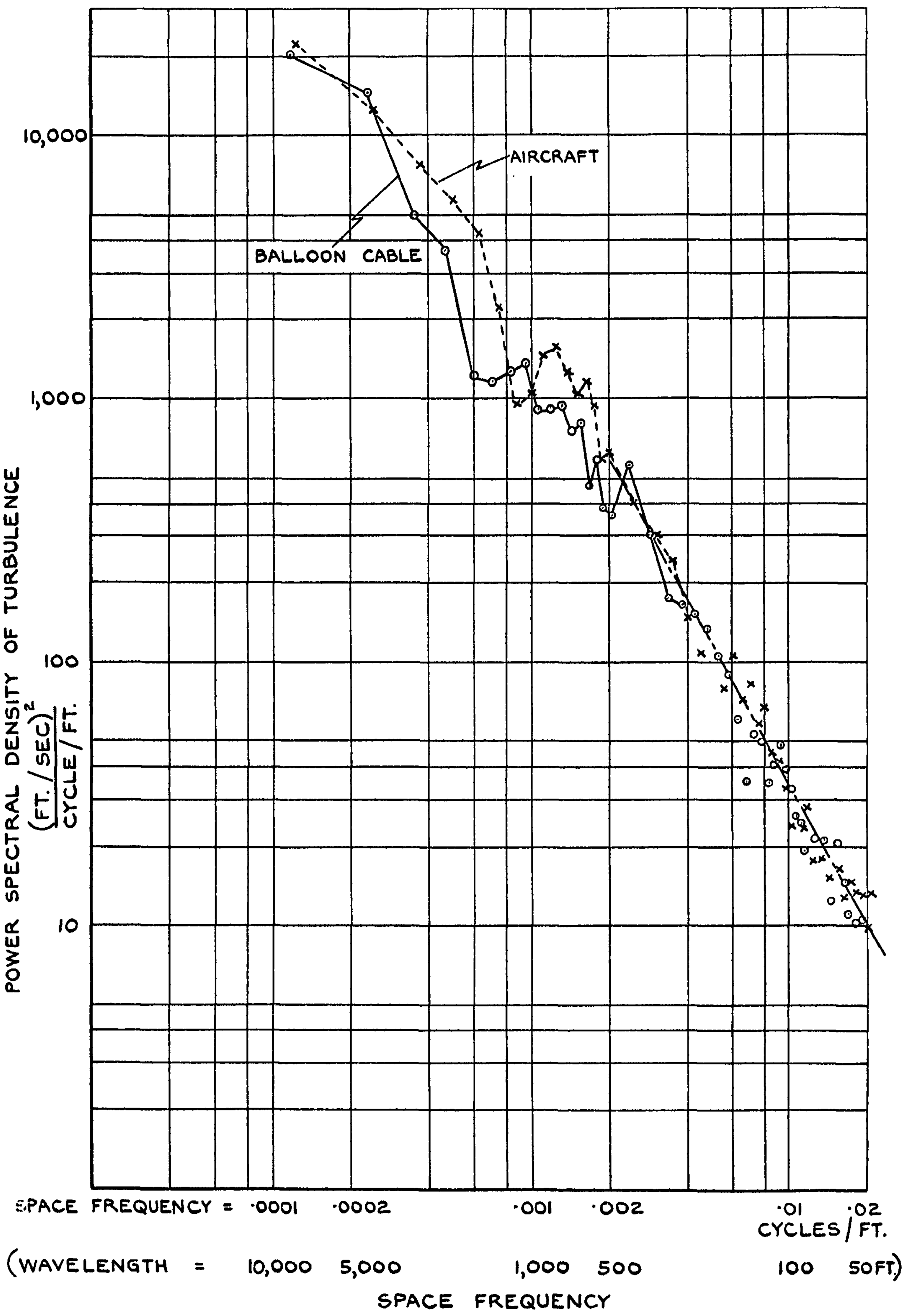

FIG. 3. POWER SPECTRA OF VERTICAL GUST VELOCITY AT 1,000 FT.

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