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# Calculation of the Damping for Rolling Oscillations of a Swept Wing

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Calculation of the Damping for Rolling  
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16th October, 1950.

Summary

The damping derivative  $\lambda\dot{\phi}$  for rolling oscillations of a swept and an unswept wing, each of aspect ratio 6, is calculated for a range of frequency parameter values. The theory used is outlined in Ref. 1, and is not described in detail in the present note.

Good agreement with experiment (2) is obtained for the larger values of the frequency parameter. For the lower values the results diverge slightly; this difference may be partly due to the approximations made in the calculation, and partly due to the greater importance of tunnel wall interference effects as the frequency parameter tends to zero (3,4).

1. Introduction

In view of the present day interest in swept wings, the development of methods of calculating the aerodynamic forces, on wings of such plan form, is of considerable importance. Various methods have been suggested for dealing with the case of steady motion, but apart from the theory proposed in Ref. 1, the writer knows of no relatively quick method of calculating flutter derivatives which can be expected to give sufficient accuracy when applied to swept wings. The method of Ref. 1 is used here with several simplifications, to calculate the aerodynamic damping for rolling oscillations of small amplitude, for a swept and an unswept wing. Both wings are of aspect ratio 6, and the swept wing has a constant chord and  $41.5^\circ$  sweepback. Measurements of the damping have been made for these particular wings (2), and the results are compared with the calculated values in Fig. 3.

As yet, there is little experimental evidence available for flutter derivatives for swept wings, with which to compare calculated values. But measurements of the aerodynamic damping for pitching oscillations of a delta wing, are to be carried out at both low and high speeds, and these results when obtained will provide a further check on the theory. Calculations for this particular wing are already in progress for incompressible flow conditions. The present theory ignores compressibility effects, but an attempt will be made later to allow for them.

2. Theory

The theory assumes that the wing is replaced by a thin sheet, the mean of the upper and lower surfaces of the aerofoil, and that the deviations of this sheet from the  $z = 0$  plane are small. (see Fig. 1).

In/

In the mean wing position, the leading and trailing edges lie in the plane  $z = 0$ . In the notation of Ref. 1, any point  $(x,y)$  on the wing can be defined by the chordwise parameter  $\theta$  and the spanwise parameter  $\eta$ , where  $\xi = -\cos \theta$  and  $\eta = y/s$ .

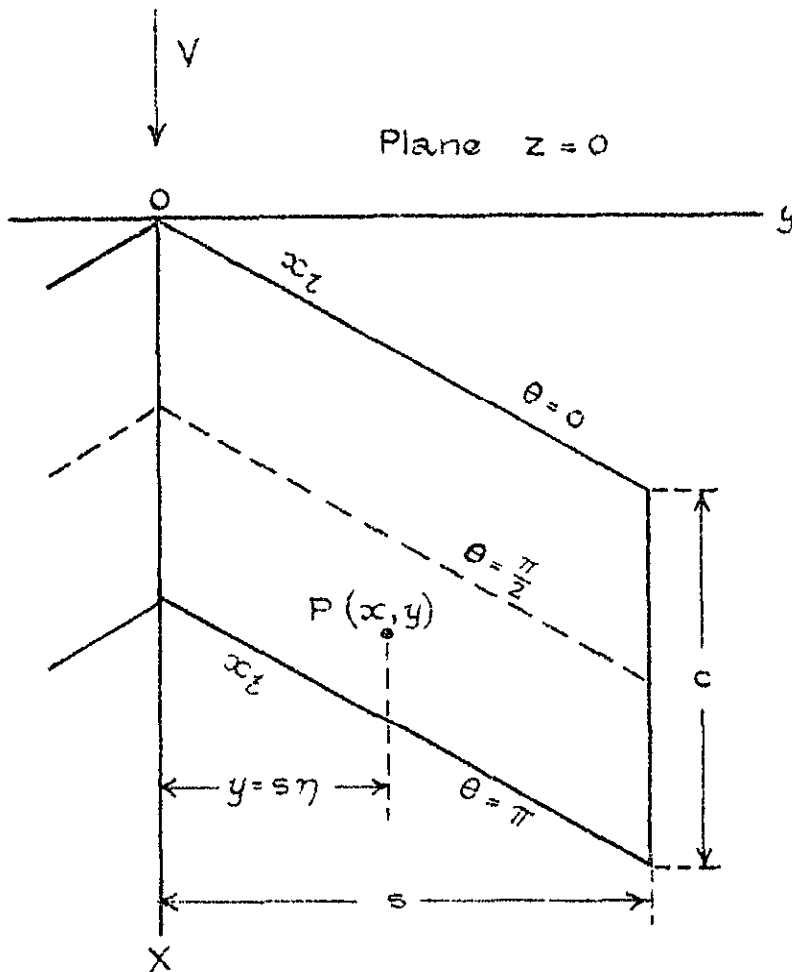


FIG. 1

For simple harmonic oscillations, the normal downward displacement of  $(x,y)$  is defined as  $z = \frac{c}{2} z' e^{i\omega t}$ , where  $\omega/2\pi$  is the frequency of oscillation; and for oscillations in roll  $z' = \frac{2s}{c} \eta \phi'$  where  $\phi'$  is the angular displacement at the wing tip.

As/

As in Ref. 1, it is assumed that the oscillations are of small amplitude, and that the disturbed flow can be reproduced by a distribution of doublets of amplitude\*  $K(x,y)$  over the wing and its wake. The problem is to determine the distribution  $K$ , such that the corresponding normal induced velocity distribution  $W$  will satisfy the tangential flow condition for rolling oscillations. For the present calculation it is assumed that  $K = \sigma V (S'_0 + S''_0) \sum C_{om} A_m$ , the expression being limited to three terms in  $m$ . A symmetrical or an antisymmetrical mode of motion is represented according as  $m$  takes odd or even values. The  $C_{om}$ 's are arbitrary constants, and  $\sigma A_m = \sigma \eta^{m+1} \sqrt{1-\eta^2}$  where  $\eta$  is the spanwise parameter.

The function  $S_0'' = \sin \theta$  where  $\theta$  is the chordwise parameter. The function  $S_0'$  is also dependent on the frequency, where the parameter  $\lambda' = i \rho c / 2V$ , and is given as the integral

$$S_0' = e^{-\lambda' \xi} \int_{-1}^{\xi} \left\{ \operatorname{cosec} \theta - \left[ 1 - C(\lambda') \right] \cot \frac{\theta}{2} \right\} e^{\lambda' \xi} d\xi$$

when  $-1 \leq \xi \leq 1$ , and as  $S_0' = \pi e^{-\lambda' \xi} X_0(\lambda')$  when  $\xi \geq 1$ . The functions  $C(\lambda') = K_1(\lambda') / [K_0(\lambda') + K_1(\lambda')]$  and  $X_0(\lambda') = C(\lambda') I_0(\lambda') + [1 - C(\lambda')] I_1(\lambda')$  are in terms of modified Bessel functions, of first and second kind.

The corresponding downwash  $W$  is given in the form  $W = V \sum C_{om} W_{om}$ , where  $W_{om} = W'_{om} + W''_{om}$ . Since in two dimensions  $\bar{W}'_0 = 0$  and  $\bar{W}''_0 = 1$  at all points on the aerofoil chord, it may be assumed that in three dimensions  $W'_{om}$  is small compared with  $W''_{om}$ . It is also assumed that  $W'_{om}$  and  $W''_{om}$  each have a constant value in the chordwise direction, and that it is only necessary to calculate their values at one point on each chordwise section. Thus, when  $W_{om}$  is known at a number of collocation points  $(x_1, y_1)$  along an axis in spanwise direction, the coefficients  $C_{om}$  can be chosen to satisfy the tangential flow condition

$$\sum C_{om} W_{om} = \lambda' z'$$

When the coefficients  $C_{om}$  are known, the doublet distribution  $K$  and the lift distribution  $\rho V \Gamma$  may be determined. In the notation of Ref. 1,

$$\begin{aligned} \Gamma &= V (\Gamma'_0 + \Gamma''_0) \sum C_{om} A_m \\ \text{where } \Gamma'_0 &= 2 \left\{ \operatorname{cosec} \theta - [1 - C(\lambda')] \cot \frac{\theta}{2} \right\} \\ \Gamma''_0 &= 2 \left\{ \cot \frac{\theta}{2} - \operatorname{cosec} \theta + \lambda' \sin \theta \right\} \end{aligned}$$

### 3. Calculation of the Downwash

It is shown in Ref. 1 that the downwash  $W_{om}$  can be estimated approximately, by replacing the continuous doublet distribution  $\sigma S_0 A_m$  by a finite system of doublet strips.

The doublet<sup>†</sup> distribution  $\sigma S_0''(\theta) A_m(\eta)$  is independent of the frequency and can be replaced by a "126 Falkner lattice", see Ref. 5, of rectangular/

\* For convenience the factor  $e^{i\omega t}$  is omitted throughout and the distributions are referred to in terms of amplitude.

† A doublet distribution of constant strength extending over an area is equivalent to a vortex filament along its boundary.

rectangular vortices. A typical vortex of width  $2s_1$  and constant strength  $cL_0''(n)A_m(\eta_1)$  is located at the spanwise position  $\eta_1$  and the chordwise position  $\xi = \frac{2n-7}{6}$ , where  $n = 1, 2 \dots 6$ . The factors  $L_0''(n)$  are

chosen on a two-dimensional theory basis as indicated in Ref. 1. The downwash induced at a particular point in the plane of a rectangular vortex of unit strength, is obtained from Tables (6) which were constructed for use with the vortex lattice method (5, 7). Using these Tables, the downwash due to a rectangular vortex of strength  $cL_0''(n)A_m(\eta_1)$  is estimated and by summation the total downwash  $W_{om}$  is calculated for any collocation point on the wing.

Since  $W_{om}'$  is small compared with  $W_{om}''$ , its value need not be determined so accurately. For the present calculation it is assumed that sufficient accuracy will be obtained by concentrating the circulation  $cS_0'(\xi)A_m(\eta)$  along a kinked line through the collocation point  $(\xi_1, \eta_1)$  as shown in Fig. 2. This corresponds to the lifting line concept used in steady motion theory.

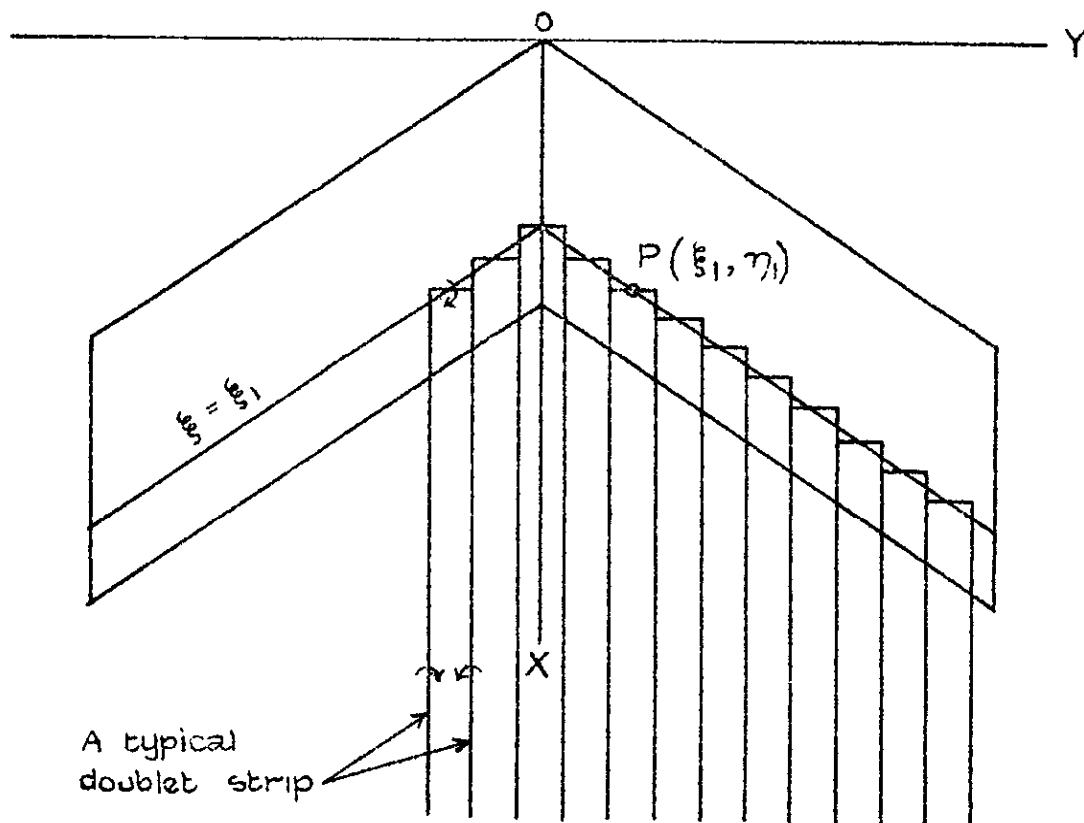


FIG 2.

Thus/

Thus  $cS_0^1 A_m$  is replaced by a distribution of doublet strips of strength  $cS_0^1(\pi) e^{-\lambda'(\xi - \xi_1)} A_m$  extending from the line to infinity. The downwash at  $(\xi_1, \eta_1)$  is calculated by the use of Tables of the downwash distribution  $W_e$  due to a narrow doublet strip  $s_1 e^{-\lambda'\xi}$  having a constant strength spanwise. Values of  $W_e$ , calculated from formulae given in Ref. 1, have been tabulated by Mathematics Division, N.P.L. These tables have not been published as they proved very difficult to use, and more convenient tables are at present being prepared for use in future calculations.

For the present solutions collocation points were taken at the spanwise positions  $\eta = 0.2, 0.6$  and  $0.8$ ; they were placed on the  $\frac{1}{2}$  chord for the rectangular wing and on the  $\frac{5}{6}$  chord for the swept wing.

#### 4. Aerodynamic Derivative Coefficients

The fundamental aerodynamic derivative coefficients are defined for wings of any plan form in Ref. 8 and are derived from the lift distribution  $\rho V \Gamma$ . For a constant chord wing describing oscillations in roll, the stiffness derivative  $\lambda \phi$  and the damping derivative  $\lambda \dot{\phi}$  are expressed as

$$\frac{1}{3} \left( \lambda \phi - \omega^2 \lambda \ddot{\phi} + i \omega \lambda \dot{\phi} \right) \phi^1 = (L_1 + i L_2) \phi^1 = \frac{-L'}{\rho V^2 b^3}$$

where  $L'$  is the amplitude of the rolling moment at the wing tip and  $\omega =$  frequency parameter  $= pc/V$ . The aerodynamic inertia coefficient  $\lambda \ddot{\phi}$  is obtained from a solution for  $\omega \rightarrow \infty$  and is defined as

$$-\frac{1}{3} \lambda \ddot{\phi} = \lim_{\omega \rightarrow \infty} \left( \frac{L_1}{\omega^2} \right) = \frac{\bar{L}_1}{\omega^2}$$

The derivatives  $\lambda \dot{\phi}$  and  $\lambda \ddot{\phi}$  are calculated for values of  $\omega$  to a maximum value 2. The values are tabulated in Table 1 for the swept and unswept wings for both symmetrical (S) and antisymmetrical (A-S) modes of motion. The values are plotted against  $\omega$  in Figs 3 and 4. The experimental results recently obtained for both wings<sup>(2)</sup> are graphed for comparison, and the values obtained for the rectangular wing by W. P. Jones<sup>(9)</sup> are also included.

##### (a) Damping Derivative $\lambda \dot{\phi}$ (Fig. 3)

For  $\omega > 0.5$  there is good agreement between the calculated values of  $\lambda \dot{\phi}$ , for both S and A-S modes, and the experimental results which correspond to a S mode of motion. For  $\omega < 0.5$ , the S and A-S theoretical values diverge so that at  $\omega = 0.13$  there is a difference of 10%. While the S values agree with the experimental results for the rectangular wing, the A-S values agree for the swept wing. It is thought that the latter effect is partly due to the difference in chordwise positioning of the collocation points, see §3 above. The present values also compare well with those of Ref. 9, the S values being slightly higher and nearer the experimental curve.

Both theoretical and experimental results indicate a decrease in  $\lambda \dot{\phi}$  as the sweepback increases. The decrease for the wings considered here, is of the order of 10-15% for a sweepback of  $41.3^\circ$ .

The/

The calculation indicates that the solutions used are not very sensitive for  $\omega > 0.5$ , but for the lower values the approximations assumed may have resulted in a loss of accuracy. The question of convergence and accuracy of solutions used, in applying the theory of Ref. 1, will be considered in future calculations.

Wind tunnel interference effects may also contribute to the difference in the results for  $\omega < 0.5$ . As shown in Ref. 3 and 4, these effects are important at low values of the frequency parameter, and the correction required to allow for these effects increases as  $\omega$  decreases.

(b) Stiffness Derivative  $\lambda\phi$  (Fig 4.)

Although the calculation was primarily undertaken to determine  $\lambda\dot{\phi}$ , values of  $\lambda\phi$  are included to compare with the experimental results. The values obtained, differ considerably from those of experiment. They indicate an increase in  $\lambda\phi$  with increase in sweepback in contrast to the experimental evidence of a decrease. It is difficult however, to obtain reliable results theoretically, since  $\lambda\phi$  depends on the small difference between the two numbers  $L_1$  and  $\bar{L}_1$ , and in the present approximate solutions any inaccuracies will be magnified. Results for the rectangular wing, obtained by W. P. Jones<sup>(9)</sup> also show a wide variation between the values for the S and A-S modes; but the S values are in rough agreement with experiment. In flutter calculations, however, this particular derivative is not of great importance and it is often neglected altogether.

5. Concluding remarks

The solutions used in this note appear to be satisfactory for the calculation of the damping for rolling oscillations of wings of aspect ratio 6. But, it is realised that the method, in the simplified form used here, would probably not be sufficiently reliable if used for the calculation of torsional derivatives or when applied to wings of low aspect ratio. Calculations of the derivatives for a delta wing of aspect ratio 3 are now in progress using the full scheme of Ref. 1, and the results, in conjunction with measured values, will provide more information on the application of the method and a further check on its accuracy.

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Derivative/

Derivative Coefficient for Rolling Oscillations

Table 1a.

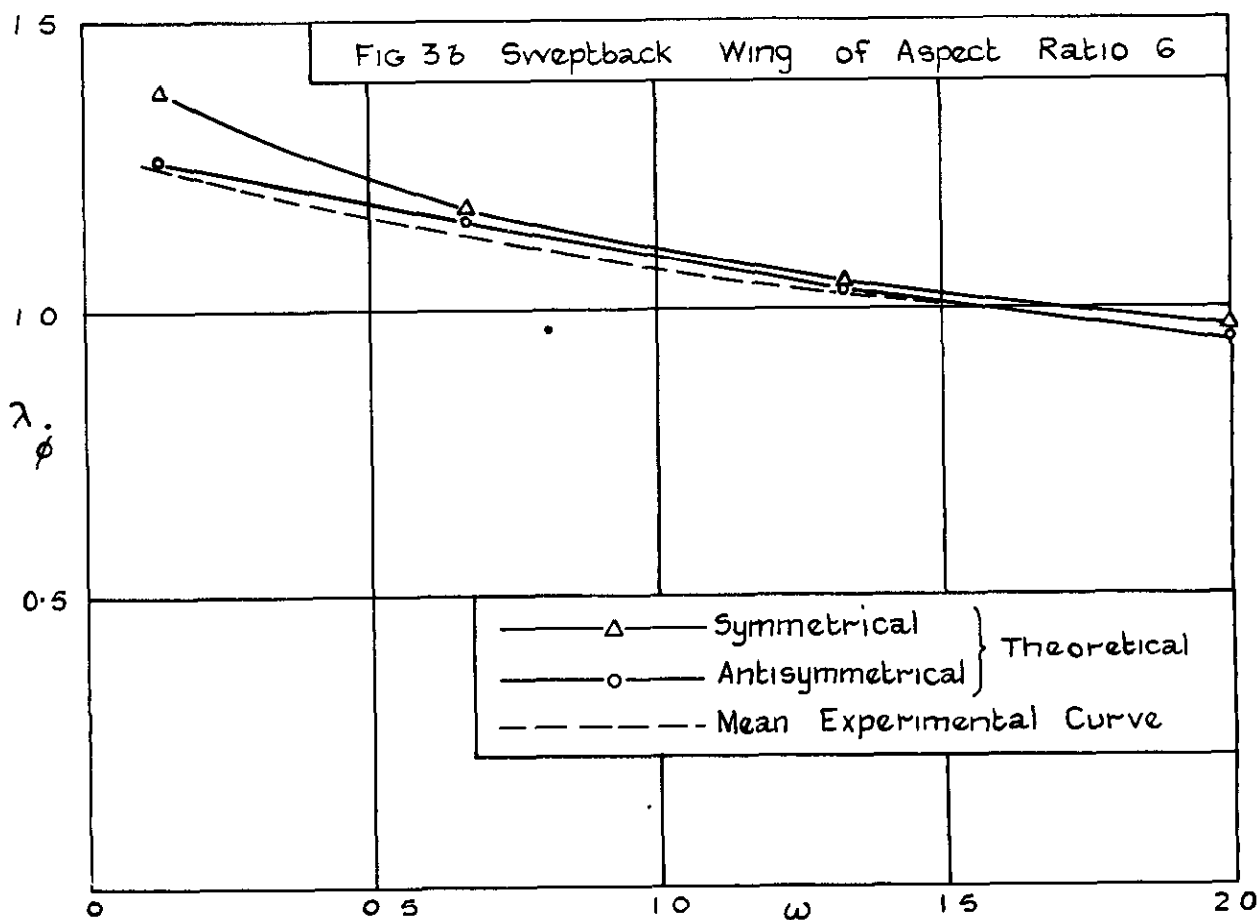
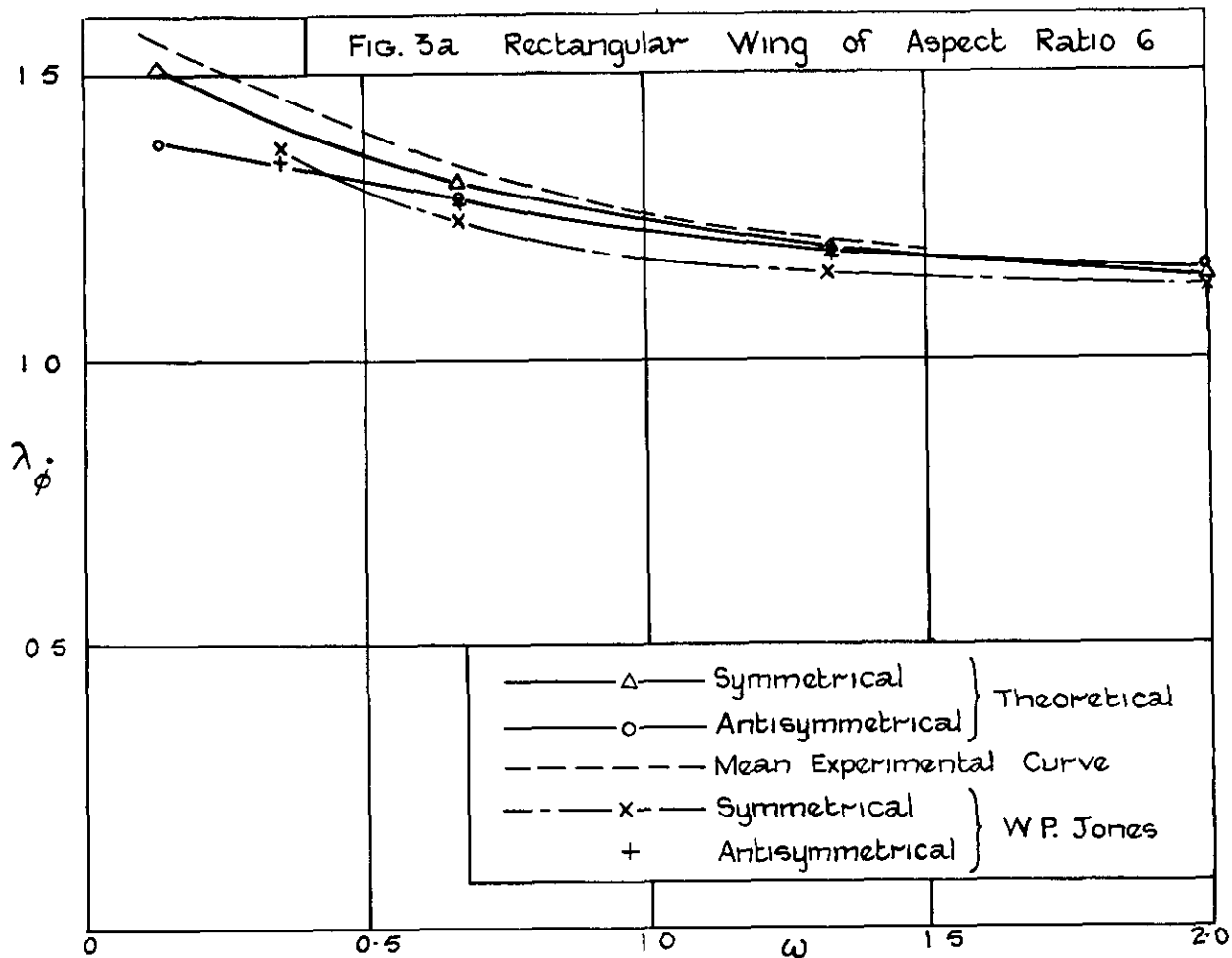
Rectangular Wing of Aspect Ratio 6

$\omega$	$\lambda_{\phi}$	$\lambda_{\dot{\phi}}$
symmetrical		
4/30	0.013	1.510
2/3	0.132	1.311
4/3	0.228	1.193
2	0.266	1.141
antisymmetrical		
4/30	0.006	1.380
2/3	0.090	1.282
4/3	0.184	1.186
2	0.274	1.151

Table 1b.

Sweptback Wing of Aspect Ratio 6

$\omega$	$\lambda_{\phi}$	$\lambda_{\dot{\phi}}$
symmetrical		
4/30	0.015	1.381
2/3	0.173	1.172
4/3	0.381	1.040
2	0.603	0.967
antisymmetrical		
4/30	0.007	1.255
2/3	0.139	1.152
4/3	0.367	1.026
2	0.603	0.942



Damping Derivative Coefficient  $\lambda_{\dot{\phi}}$  for Rolling Oscillations

FIG 4a Rectangular Wing of Aspect Ratio 6

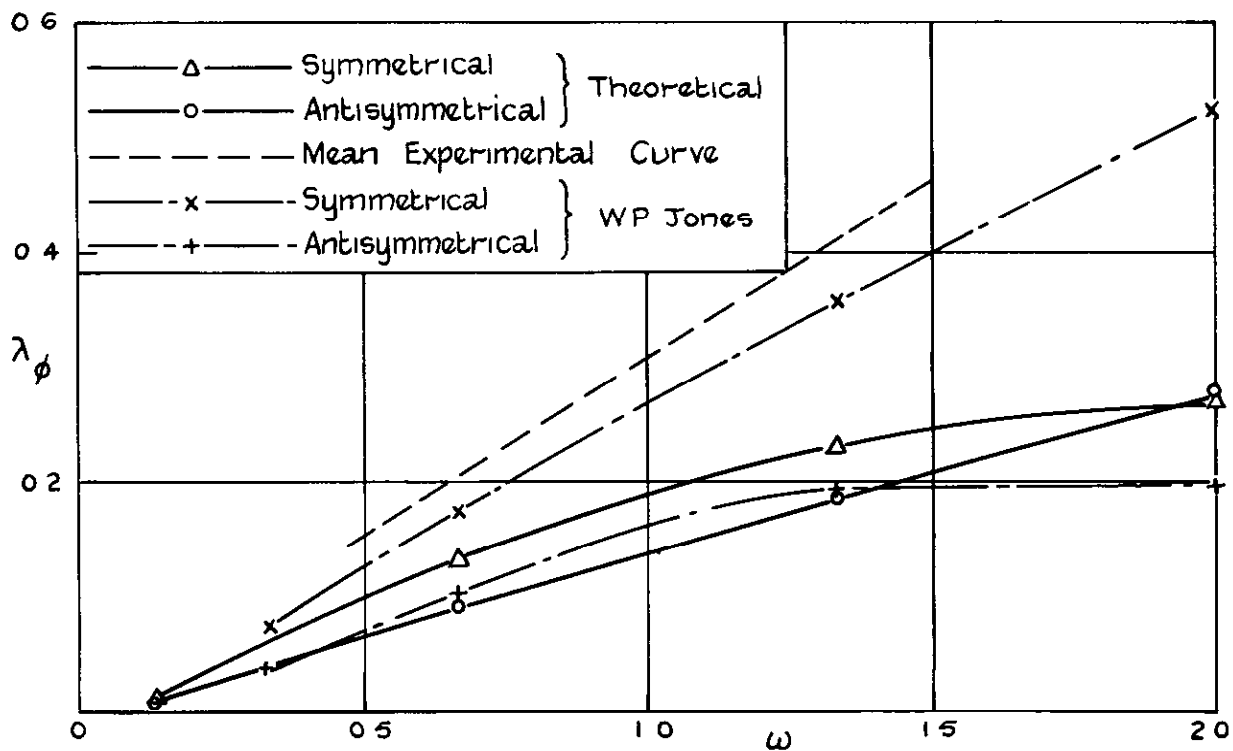
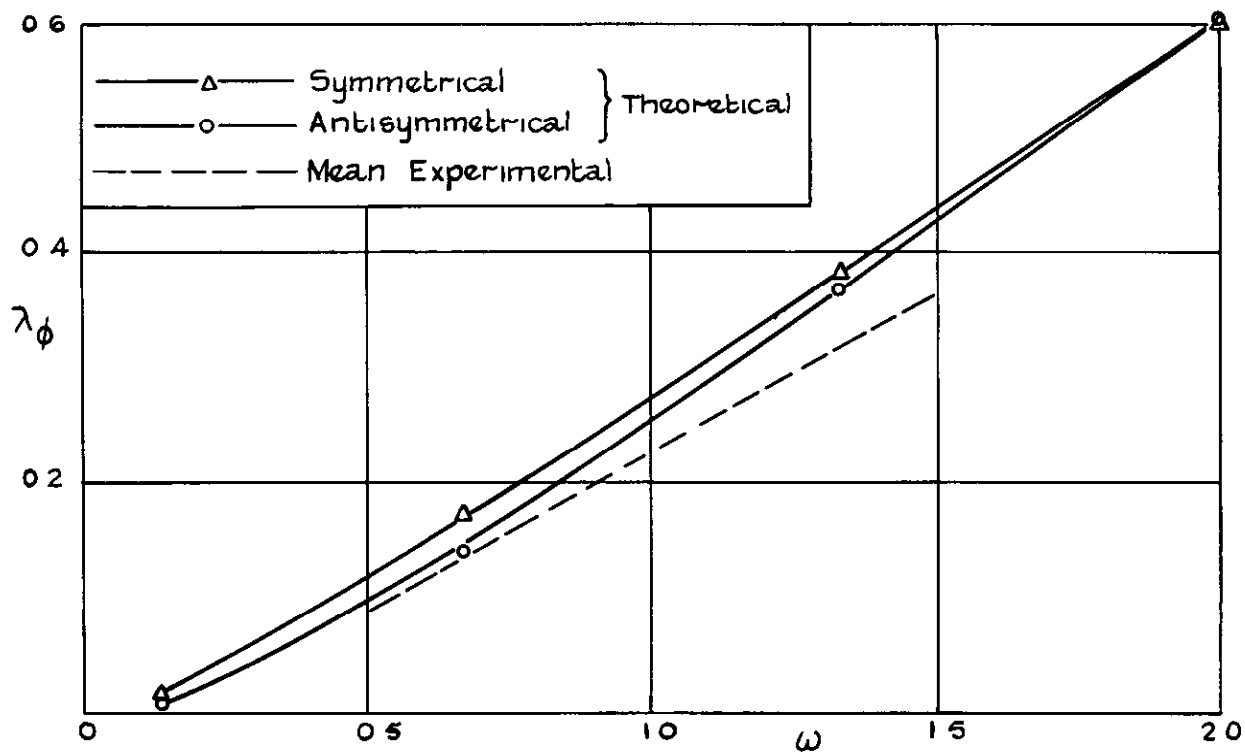


FIG 4b Sweptback Wing of Aspect Ratio 6



Stiffness Derivative Coefficient  $\lambda_\phi$  for Rolling Oscillations



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