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The Damping of Structural Vibrations

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Contents

1. Introduction

1.1 The application of layers of heavily-damped materials to metal panels to deaden them acoustically is a practice of long standing in the motor-car industry, and in the last decade particularly the use of such damping media has spread rapidly with the increasing urgency of problems of vibration, noise and metal fatigue, above all in the aeronautical field.

The most comprehensive attempt to develop a suitable material for application as a uniform layer to a metal sheet was that of Oberst and his co-workers[1,2,3]. They appear to have been the first to point out publicly that such a material needed to have not only a high damping but also a high elastic modulus. For if the damping material is mechanically weak compared with the panel, then by definition only a small proportion of the total

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elastic energy of deformation is stored within the material. The fraction of this which is dissipated by the material can then be barely significant compared with the total energy. For this reason a layer of, say, cloth firmly cemented to a panel adds very little damping.

Oberst developed a filled high-polymer material having the desirable high modulus and damping. The commercial form of this now available in Great Britain, termed Aquaplas[4], appears to be the best available material for straightforward damping layers. It is normally sprayed on to panels, etc., in semi-liquid form, though it can be trowelled on or first set into sheets and cemented on. This material forms a convenient standard of reference for the judgement of other damping treatments.

The present work represents an attempt to see whether behaviour more advantageous than that of a simple damping layer can be obtained from a layer backed by an additional sheet of metal, or from various other composite treatments of similar character. The work has been largely directed towards treatments which can be applied to an existing structure to modify its properties, and has been less concerned with methods whereby damping could be 'built into' a new structure. This report, which is a final one summarising work carried out under contract to the Ministry of Supply (now Ministry of Aviation) is written with special reference to the damping of aircraft structures, but the techniques discussed have wider application. Two interim reports have been submitted previously, dated August, 1958 and October, 1959. The present report embodies the main results given earlier. By request of the contracting authority, it also contains (Section 1.3) a brief discussion of the part to be played by damping treatments in aircraft and other structures in minimising the effects of noise, vibration and fatigue. This may be omitted by anyone familiar with the problems involved.

1.2 Damping in solids

It is convenient at this point to define the terms which will be used in discussing the mechanical properties of damping layers. For sinusoidal vibrations of sufficiently small amplitude, most solids behave in a substantially linear, viscoelastic manner, and their mechanical properties can be then adequately represented by a so-called complex elastic modulus[5]. Thus, if Young's Modulus is in question, the ratio of complex tensile stress amplitude to complex tensile strain amplitude is equal to the quantity E^x, where

$$E^x = E_1 + jE_2 \qquad \dots (1.1)$$

E_1 is here the real or 'conventional' elastic modulus, while E_2 represents internal damping. E_1 and E_2 are independent of amplitude if this is small but are in general functions of the frequency f or $\omega/2\pi$.

Writing $$E = E_1 + jE_2 = E_1(1 + j\eta_E) \qquad \dots (1.2)$$

defines $\eta_E = E_2/E_1$, the damping factor for tensile strains of the medium. For shearing strains, there is an analogous complex rigidity modulus.

$$G^x = G_1 + jG_2 = G_1(1 + j\eta_G) \qquad \dots (1.3)$$

where η_G is the shear damping coefficient.

When any elastic member is strained, it is often convenient to define its stiffness S^x, i.e., the ratio of the force applied to the displacement produced across it. If the deformation involves strains of purely tensile or shear types, or of other simple definable type, this complex stiffness

$$S = S_1 + jS_2 = S_1(1 + j\eta) \qquad \dots (1.4)$$

has/

has the same damping coefficient $\eta = S_2/S_1$ as that of the relevant tensile (Young's), shear or other modulus.

If such an elastic member is attached to a mass M (Fig.1) to produce a simple one-degree-of-freedom oscillator, of natural angular frequency ω_o, then the ratio of the damping existing at frequency ω_o to that required to damp the system critically is termed the damping ratio δ. δ is in fact equal to one half of the damping factor appropriate to the resonance frequency.

For purely viscous damping, which is easy to analyse theoretically, η is proportional to frequency. This case is however rather rare in practical materials, in which η is usually more nearly independent of frequency. This is often termed hysteretic damping, though in fact its mechanism may still be essentially viscoelastic.

1.3 The effect of damping treatment on structures

The undesirable effects of vibration in structures are manifold, but can all be mitigated to a greater or lesser degree by the addition of absorbers of vibrational energy. The main phenomena of practical concern can be distinguished as follows:

(i) Transmission of vibration along long structural members

In the simple but in practice rather rare case where purely progressive mechanical waves are involved, the wave amplitude in a uniform beam whose elastic properties are linear with respect to amplitude and whose damping factor is η decreases exponentially with distance. For flexural waves such as are usually of interest, it can be shown that the decrease is approximately in the ratio $e^{-\pi\eta_E/2}$ over a distance of one wavelength. Since η_E is very rarely likely to exceed say 0.5, when $e^{-\pi\eta_E/2} = 0.46$ any attempt to localise vibration by damping a structure is likely to be practicable only for frequencies high enough for the flexural wavelength to be considerably smaller than the dimensions of the structure.

(ii) Transmission of sound through panels

Acoustic pressures applied to walls or panels cause them to vibrate, and their motion causes sound waves to be re-radiated on the further side, i.e., the sound is (partially) transmitted. The intensity radiated is a complex function of the system geometry and wavelengths in general. In the simplest case, when the panel vibrates in phase over a region large compared with the wavelengths in air, the intensity is proportional to the square of the panel velocity. This is then the quantity to be minimised. A problem closely related to the foregoing and to case (i) is that of radiation of sound by a panel excited by vibration transmitted to it through a structure from a remote source. It is again necessary to minimise panel velocity in the simple case.

(iii) Fatigue of structural members

In this case which is of pressing importance in the aircraft industry, it is necessary to minimise the stresses present within structural members and at joints, rivet holes, etc.

To examine the effect of damping on the quantities of interest in cases (ii) and (iii), it is necessary to consider a simple model of a structural member. (The discussion which follows is drawn primarily from studies of the excitation of aircraft structures by the random noise fields produced by jet engines, notably by J. W. Miles in the U.S. and members of

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the Aeronautics Department of the Southampton University. A useful and authoritative summary of relevant aspects of this work has been published very recently by D. J. Mead of Southampton[6], and justifies the brevity of the present survey.

Broadly speaking, two types of vibrational resonance may occur in an aircraft structure. The first involves the structure as a whole, or at least large sections of it. The pattern of nodes spreads over large parts of the structure, which behaves as a roughly uniform elastic shell. The second type is characteristically the resonance of a single panel, stringer section or other small localised element of the structure. The principles of response reduction by damping are essentially the same in each case, though the treatments will necessarily be on the more massive scale in the first case. In the case of excitation by noise fields of largely random structure (i.e., having little phase correlation between the pressures at any two points unless these are relatively close) and relatively short wavelengths, the first type of mode may not be of such great practical importance.

Now when a force varying in space and time is applied to the surface of a panel, the response is determined not only by the frequency and panel stiffness and mass, but in a complex way by the spatial distribution of the forces and of the nodal pattern of the manifold modes of resonance of the panel. However for the present purpose it will be sufficient to consider a localised point force acting on a single-degree-of-freedom oscillator.

Such a system, shown in Fig.1, consists of a mass to which the driving force is applied, supported from a rigid foundation by a spring of complex stiffness $(S_1 + jS_2)$ (see Eqn.1.4). It will be supposed for clarity that S_1 and S_2 are substantially independent of frequency $(\omega/2\pi)$, though this is not essential. If the applied force is sinusoidal in time and of amplitude F, the amplitude x of the displacement of the mass, the amplitude V of the velocity of the mass and the amplitude F_t of the force transmitted to the foundation are given by

$$x = \frac{F}{|Z|} \qquad \dots (1.5)$$

$$V = \frac{\omega F}{|Z|} \qquad \dots (1.6)$$

$$F_t = \left|\frac{S_1 + jS_2}{Z}\right| F \qquad \dots (1.7)$$

where

$$Z = (S_1 - \omega^2 M + jS_2) \qquad \dots (1.8)$$

The resonance frequency $\omega_o/2\pi$ is given by

$$\omega_o^2 = S_1/M \qquad \dots (1.9)$$

$(x/F)^2$ is given as a function of frequency for two values of the damping coefficient $\eta = S_2/S_1$ in Fig.2. The maximum values of x/F and of F_t/F occur when $\omega = \omega_o$ and are given by

$$x_o = \frac{F}{S_2} = \frac{F}{\eta S_1} \qquad \dots (1.10)$$

$$F_{to} = \frac{\sqrt{S_1^2 + S_2^2}}{S_2} F = \frac{\sqrt{S_1^2 + S_2^2}}{\eta S_1} F \qquad \dots (1.11)$$

Both/

Both quantities are seen to be inversely proportional to damping factor; x_o is inversely proportional to stiffness, and F_{to} largely independent of it.

The situation is rather different when the driving force has a substantially random waveform instead of a sinusoidal one, so that its Fourier spectrum covers a wide range of frequencies. The broken line in Fig.2 might for instance represent the spectral density curve of exciting power $f(\omega)$ and the response of the system is obtained by integrating the product of the spectral density function and the response function $(x/F)^2$. It is plausible that if the damping is small, the region around resonance will be the only one making a substantial contribution to the integrated product, and in this case the response is essentially an oscillation of the panel at the resonance frequency, with a randomly-varying amplitude. This is borne out by experiments on aircraft panels[7]. The R.M.S. value of the amplitude is given by

$$x_o = \left(\frac{\pi\omega_o f(\omega_o)}{2}\right)^{\frac{1}{2}} \frac{1}{\eta^{\frac{1}{2}} S_1} = \left(\frac{\pi\omega_o f(\omega_o)}{2}\right)^{\frac{1}{2}} \frac{1}{(S_1 S_2)^{\frac{1}{2}}} \quad \ldots (1.12)$$

where $f(\omega_o)$ is the magnitude of the applied spectral density function when $\omega = \omega_o$. Similarly the maximum transmitted force has R.M.S. amplitude

$$F_t = \left(\frac{\pi\omega_o f(\omega_o)}{2}\right)^{\frac{1}{2}} \left(\frac{S_1^2 + S_2^2}{\eta S_1^2}\right)^{\frac{1}{2}} = \left(\frac{\pi\omega_o f(\omega_o)}{2}\right)^{\frac{1}{2}} \left(\frac{S_1^2 + S_2^2}{S_1 S_2}\right)^{\frac{1}{2}} \quad \ldots (1.13)$$

It will be noticed that for sinusoidal excitation, the displacement is inversely proportional to the damping parameters S_2 or η. However, for random excitation, it is the square roots of these quantities which appear in the denominator. This is due to the fact that for the random case, increased damping does not merely reduce the response at the resonance frequency as $1/\eta$ but increases the resonant bandwidth $\eta\omega_o$ and so renders the system susceptible to resonant excitation over a wider frequency range. The factor $\omega_o^{\frac{1}{2}}$ appears in the numerator of Eqns.1.12 and 1.13 for this reason. Thus damping is a less effective variable for random excitation than for sinusoidal. The real stiffness S_1 appears explicitly in Eqn.1.12 and also implicitly in the ω_o term. Using Eqn.1.9 gives $x_o \propto 1/S_1^{\frac{3}{4}} M^{\frac{1}{4}} \eta^{\frac{1}{2}}$. S_1 thus appears to the power $-\frac{3}{4}$. It must be borne in mind of course that the change of resonance frequency due to change of stiffness will in general change $f(\omega_o)$, i.e., will shift the resonance into a region of either more or less powerful excitation, and this effect may override the relatively small changes in S_1 and M_1 brought about by most damping treatments. The changes in $f(\omega_o)$ may be quite rapid as ω_o varies, since a condition of "coincidence" between the trace wavelength of sound waves of the resonant frequency arriving at the panel from a given direction and the flexural wavelength on the panel may be created or removed.

The panel velocity, which was stated to influence sound transmission, is obtained from Eqn.1.12 by multiplying by ω_o and so is proportional to $1/(S_1^{\frac{1}{4}} M^{\frac{1}{2}} \eta^{\frac{1}{2}})$. Damping and mass increase are beneficial, stiffness increase less so. It should be borne in mind however that where passenger comfort in an aircraft is concerned, high-frequency noise makes a particularly important contribution to the total subjective effect, and that many panels may have only high-order resonances in this range. Their corresponding resonant response, which is all that is susceptible to improvement by damping, will then be weak in any case. The <u>mass</u> of the panel tends to be the controlling factor here.

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The last and perhaps the most important quantity to be considered is the stress, both in the panel and its fastenings. Eqns.1.9 and 1.13 for random vibration show F_t, measuring the stress on the fastenings, to be proportional to $(S_1^2 + S_2^2)^{\frac{1}{2}}/\eta^{\frac{1}{2}}\ S_1^{\frac{3}{4}}\ M^{\frac{1}{4}}$, or, since $S_2^2 << S_1^2$ in the range of validity of the equation, to $S_1^{\frac{1}{4}}/\eta^{\frac{1}{2}}\ M^{\frac{1}{4}}$ or $\omega_0^{\frac{1}{2}}/\eta^{\frac{1}{2}}$. Increase of resonance frequency is seen to be somewhat detrimental in this case. In the simple model considered, this quantity is also the stress in the elastic member: however for a beam or panel in flexural vibration, it is closer to reality to consider the maximum surface stress as proportional to the product of the curvature (and hence displacement, Eqn.1.12) and the distance of the surface from the neutral axis of bending. Addition of a damping treatment to one side of a panel will in general increase this distance, and this change will tend to counteract the effect of reduced displacement.

The question now arises of how far damping can usefully be increased to minimise vibration. Fig.2 shows that as η is increased to be of the order of unity the response at resonance becomes comparable with that at much lower frequencies. Now suppose that the power spectrum of the exciting force is substantially flat over the whole frequency range shown in Fig.2. (Some fall-off towards the extremes of the range will not affect the argument appreciably.) Then the R.M.S. displacement is proportional to the square root of the area under the response curves for the various dampings. This quantity is plotted in Fig.3 for the cases where damping factor is (a) independent of and (b) proportional to frequency. In the latter case the curve is exactly proportional to $\eta^{-\frac{1}{2}}$ over the whole range of η. It is seen that the reduction of displacement per unit of added damping falls very much as damping increases, i.e., it is a process of rapidly diminishing returns. The argument presented is too idealised to permit any very safe general conclusion, but so far as it goes it suggests that it will rarely be worthwhile to try to increase the damping factor of a member much above 0.4. Other considerations may of course in any given case dictate a much lower limit.

It is worth bearing in mind here the values of damping which may exist in an untreated structure. This damping arises from (a) acoustic radiation and (b) energy losses by friction at the joints and internally in the metal. Relatively little information exists about these, but likely magnitudes for η from source (a) may be 0.04 - 0.07 for a fuselage vibration and 0.004 - 0.02 for small panels. Losses in the structure probably give values of η of less than 0.02[6]. Cooper[8] and more recently Mentel[9] have discussed the possibility of increasing joint damping by placing highly-damped materials around the edges of the panels.

To sum up: in lightly damped structures subject to random acoustic pressures, the induced vibration is mainly resonant in character, and as such can be reduced, together with all its accompanying troublesome effects, by increased damping. The improvement, other factors remaining constant, is proportional only to $(\eta)^{\frac{1}{2}}$, instead of to η as for sinusoidal excitation (Fig.3). Increase in resonance frequency due to added stiffness is mildly detrimental in that it increases bandwidth of response. It may also have major effects in shifting the resonance to a region of the spectrum where excitation is more or less intense, but this is not a predictable effect in any general case. Increasing the stiffness will usefully reduce panel displacement, though any accompanying shift of the neutral surface of bending away from the panel surface will prevent the panel stresses being reduced in proportion. Increased stiffness will not greatly influence sound transmission or stress at the fastenings of a resonant panel.

1.4 Work in other laboratories

It is apparent that the potentialities of composite damping layers have attracted considerable attention and in the last few years work on them

has/

has been started, probably independently, in several laboratories. These are known to include those of the Aeronautical Engineering Departments of the University of Minnesota (under Prof. B. J. Lazan) and the University of Southampton (under Prof. E. J. Richards), the firm of Bolt, Beranek and Newman Inc., Cambridge, Mass. (E. M. Kerwin and others) and the Acoustics Institute of the U.S.S.R. Academy of Sciences. The basic theory of a composite layer was laid down by Kerwin and his colleagues and is briefly discussed in Section 2. Other work is referred to where relevant throughout the report.

2. Theory of the Backed Damping Layer

When bending waves travel through a three-layer plate in which the central layer is relatively soft, (Fig.4), the nature of the deformation will approximate to one or other of the two types sketched in Fig.5 and termed (5b) pure bending and (5c) shear bending. In the latter case the central layer undergoes a shearing action while the outer layers bend. The theory of bending waves in such a system has been developed by Kerwin and his co-workers[10,11,12], who showed that when the wavelength is long, the deformation is of type (b), and that as the frequency is increased, a steady transition towards type (c) occurs. Thus at low frequency the effective bending stiffness of the plate has the high value appropriate to a thick plate, and decreases with wavelength towards the lower value corresponding essentially to the combined stiffnesses of the two outer layers bending simultaneously and with the same radius of curvature. If the outer layers are of metal, which has low mechanical losses, the damping of the composite plate arises essentially from the shear distortion of the central layer, and in fact passes through a broad maximum in the transition region (Fig.6). The controlling factor is the socalled "shear parameter" g, defined by

$$g = \frac{G_2 \lambda^2}{4\pi^2 E_3 H_3 H_2} \qquad \ldots (2.1)$$

where G_2 = shear modulus of damping layer

H_2 = thickness " " "

E_3 = Young's modulus of backing layer

H_3 = thickness " " "

λ = wavelength of flexural waves in the plate.

We define also

E_1 = Young's modulus of bottom layer (panel)

H_1 = thickness " " "

η_2 = shear damping factor of damping layer.

g is the abscissa of Fig.6. The maximum damping is reached when

$$g = \frac{1}{(1 + \eta_2^2)^{\frac{1}{2}}} \qquad \ldots (2.2)$$

and if it is assumed that $(E_3 H_3 / E_1 H_1)$ is fairly small, then the maximum damping factor of the composite plate is approximately:

$$\eta_{max} = \frac{6 E_3 H_3 H_{30}^2}{E_1 H_1^3} \left(\frac{\eta_2}{1 + \sqrt{1 + \eta_2^2}} \right) \qquad \ldots (2.3)$$

H_{30}/

H_{30} is the distance of the centre of the backing layer from the neutral surface of the composite plate. If, further, the added layers are thin compared with the main panel, so that $H_{30} \doteqdot H_1/2$, and if also $\eta_2^2 << 1$, Eqn.2.3 reduces to

$$\eta_{max} = \frac{3}{4} \cdot \frac{E_3 H_3}{E_1 H_1} \cdot \eta_2 \qquad \dots (2.4)$$

The maximum damping is thus proportional to the damping of the middle layer and to the stiffness of the backing layer relative to that of the main panel.

The shear strain in the damping layer at any point is proportional to the angle of slope of the bent plate at that point. Thus backed damping layers are most effective in those regions on a panel where this slope is greatest, and would for example be relatively ineffective at the centre of a symmetrical oscillating panel. Unbacked layers are in contrast most effective where the panel curvature is greatest.

The theory of Kerwin et al is approximate in that it assumes that the total thickness of the composite system is small compared with the flexural wavelength, and also (though the authors do not state this) that the three segments of a line across the cross-section as in Fig.5c remain straight individually. This will be valid only if the damping layer is fairly soft. Neither of these assumptions is often likely to be serious in practice however, and Kerwin reports good general agreement between theory and experiment.

3. Work at Imperial College

3.1 Experimental measurements

The first apparatus for damping determination used progressive flexural waves excited at one end of a 10 ft steel strip whose other end was embedded in an absorbing material. This was described in the first report (1958). It proved unduly laborious in use however, and most of the measurements to be described were made with a simple resonant strip apparatus described in the second report and since improved by substitution of a small voice coil in a loudspeaker magnet gap for the telephone earpiece driving unit. The basic test strip was of aluminium, 36 - 39 x 1 x 0.125 in. in size, and was excited into its natural resonances over a range of some 20 to 2000 cycles per sec. Damping was determined from either the resonant bandwidth or, at low frequencies, the decay time of free vibrations. Stiffness was determined from resonant frequency. The values quoted for damping represent the excess of the measured damping over that of the bare test strip. The latter was normally small, typically $\eta = 0.001$.

3.2 Simple damping layers

To provide a convenient standard of reference some measurements of damping in strips coated with simple layers of Aquaplas were made. The results are shown in Fig.7, along with a curve for another commercial material of apparently similar nature. ("Supra Thermason", by Supra Chemicals and Paints Ltd., Hainge Road, Tividale, Tipton, Staffs.). The increase of damping with thickness of coating is apparent. This is brought out particularly in Fig.8, where the damping factors of all the treatments examined are plotted against the fractional increase in mass of the system. The damping factor given is the value which is exceeded over a frequency range of one decade.

Now the contribution of a filament in a bent beam to the stiffness or damping increases with its distance from the neutral surface of bending. This being so, a test strip was made up with a rectangular rib of Aquaplas, 7.5 mm wide and 7.3 mm high, running along its centre. It was applied from a bag and nozzle of the type normally used for icing cakes. Its mass was 38% of the strip mass, only slightly greater than that of the flat layer of the top curve in Fig.7. Its damping was considerably higher however, being 0.17 at 21 c/s and 0.38 at 72 c/s. Higher-order resonances were too highly damped to be observable. The bending stiffness of the strip was increased 2.9 times by the rib.

3.3 The backed damping layer

The next experimental results to be described were aimed at confirming the theory of the backed damping layer briefly discussed in Section 2. Figs.9 and 10 show respectively the measured bending stiffness and the damping of a sandwich of 0.08 in. of Aquaplas[4] grade F 102 B, between steel strips each 0.0625 in. thick. The stiffness measurements show clearly the transition in stiffness with increasing frequency, agreeing well with the calculated values. The damping measurements show less satisfactory agreement with theory, being some 2 - 3 times too large. This may be partly due to the stated inadequacy of the theory for rather stiff damping layers. A more probable alternative is that the values of the mechanical constants of Aquaplas used to derive the calculated dampings were inaccurate. A value of 0.2 for η in Aquaplas was obtained from measurements with an unbacked damping layer and used for the computation. More recent measurements however have given values up to 0.38, which would largely remove the discrepancy. It is worth noting that if the bar vibrated in pure bending (Fig.5b) the damping would be more than 100 times less than was measured.

Another set of measurements was made with a layer of fairly soft rubber on the standard 1/8 inch aluminium strip and backed by an 0.022 in. aluminium foil. The material constants of the rubber were obtained by vibrating a small weighted strip of the rubber and observing its longitudinal resonance with a microscope under stroboscopic illumination. The shear constants were then obtained by assuming Poisson's ratio ÷ 0.5.

The results for the backed layer are shown in Fig.11, along with the theoretical curve. A peaked curve is obtained of about the correct height, though at a somewhat higher frequency than predicted.

Most of the curves for backed layers in this and other sections are rather flatter than the theoretical curve of Fig.6. Possible reasons for this are end effects in the test strip (the phenomenon discussed in Section 3.5) and variation of the constants of the damping material with frequency. Whatever the reason, the effect is advantageous in practice in that the damping achieved is less frequency-selective than theory implies.

At present two firms in Great Britain are known to manufacture "damping tapes" - i.e., backed damping layers. Johnson and Johnson Ltd. produce (to special order) their Permacell "Acoustimat", consisting of a fabric strip impregnated with adhesive some 0.010 - 0.012 in. thick and backed by an aluminium foil about 0.005 in. thick. A curve for this tape on a steel strip was given as Fig.7 of the previous report, and other tapes using this damping layer are discussed in Section 3.4. The Minnesota Mining and Manufacturing Co. Ltd. has very recently produced a range (Series 428) of tapes having a thin adhesive layer on aluminium foils of thickness 0.0055, 0.008 and 0.012 in. No tests have yet been made on these tapes but some data supplied by the manufacturer yield points on Fig.8 lying near or rather below the line for the various Permacell tapes.

3.4 The backed layer in practice

Using the formulae of Section 2 for backed layers and those of Oberst[1] for unbacked layers, a theoretical comparison was carried out between the two systems of damping, assuming the most suitable of available materials to be employed in each case, and using the added weight of the damping treatment as a basis of comparison. Some considerations relating to the design of a damping tape are set out in Appendix I. It was concluded (see report No.2) that for the same added weight the backed layer could

give/

give somewhat the greater damping if used in its optimum frequency range. (Fig.6). Kerwin and Ross[13] have also predicted backed layers to be superior, at least for mass increases up to 40% on aluminium panels.

To set against this, the backed layer must in principle be designed for each individual application so that its effective frequency range coincides with that of the vibrations present, and the required increase of damping or of weight is obtained. To do this it is in general necessary to vary thicknesses (and possibly materials) of both backing and damping layers, and so the two layers must be separately available.

A case in which this is possible is that of the Permacell tape (see foregoing Section). The adhesive-impregnated fabric strip which forms the damping layer here is available separately under the name Permacell 45. It is therefore possible to add several layers of this to a panel and cover with a layer of aluminium foil of suitable thickness.

Some experiments were therefore undertaken to examine this possibility. One set of specimens was made up to check that the frequency of maximum loss could be reduced by increasing the thickness of backing and damping layers (Eqns.2.1 and 2.2). This appeared to be the case, but the results were not conclusive quantitatively owing to the very high damping of the thicker treatments which made observation difficult.

Fig.12 shows some further results using superposed layers of the damping material. Treatments were made up consisting of 2 and of 4 layers of adhesive tape backed by aluminium strips 0.013 and 0.021 in. thick respectively. The curves for these are those of Fig.12 marked "comparable single tape". The shapes are rather complex, though still show a generally peaked character.

An alternative way of employing multiple layers which would be more convenient in practice would be to superimpose several damping tapes each with a damping layer and backing of a standard thickness. The curves of Fig.13 show results for up to four superposed layers of damping tape, each consisting of a layer of Permacell 45 and an 0.015 in. aluminium backing. The shift to lower frequencies and higher dampings expected in a single tape of increasing dimensions is in fact evident. In Fig.12 the results for 2 and 4 layers of such damping tape are replotted for comparison with the measurements for comparable 'simple' treatments of similar weight. It is seen that for some 30% mass increase, four distinct damping tapes give about the same maximum damping as one of four times the thickness (though this maximum is somewhat conjectural in each case owing to the difficulty of observing such heavily damped resonances). For 15 - 17% mass increase, the 'simple' tape gives rather more damping than two superposed tapes. The shapes of the damping-frequency curves are in all cases rather complex, but the curves for multiple layers are shifted somewhat to higher frequencies compared with single layers of similar constitution.

The theory of Kerwin et al for single damping tapes has been extended to cover multiple tapes. It is extremely complex and not susceptible to explicit solutions for a general case. A specimen result however is that for two identical superposed tapes thin in comparison with the panel, the damping frequency curve is generally similar to that for a single tape of doubled dimensions, having a virtually identical maximum loss but at a frequency 1.7 to 2 times higher, depending on the tape damping. This is broadly consistent with the experimental results, which did not adequately fulfil the condition that the tapes should be thin. Ungar and Ross[12] have very recently reported making a similar calculation which showed that behaviour of multiple tapes was similar to that of a single tape having the same total metal thickness but the same damping layer as any one of the layers. This agrees substantially with the above result.

As/

As a practical conclusion therefore it seems reasonable to regard multiple layers of a standard tape as sufficiently similar in performance to that of single tapes of greater thickness to justify their considerable advantage in convenience and availability.

It is tempting to try to use the greater stiffness available in a damping material such as Aquaplas to minimise the quantity required and hence the weight. This can in principle be done by having the damping material in the form of small "islands" under the backing instead of a continuous layer. It is doubtful however if the advantage in weight would offset the difficulty of application.

3.5 Cuts in a backed layer (See also Addendum on p.16)

Commercial damping tape with a thin foil backing tends to become somewhat wrinkled during storage and application, an effect which would be expected to reduce the extensional stiffness of the backing on which the maximum damping has been shown to depend (Eqn.2.4). A test was accordingly made but yielded the at first surprising result that wrinkles actually improved the damping on the low-frequency side of the damping peak.

Some systematic measurements were therefore carried out in which an unwrinkled damping tape was weakened by cuts across it dividing it successively into 3, 4, 8, 16 and 32 sections along the 36 in. length of the test strip. The results are shown in Fig.14. The initial finding above is confirmed, in that for a number up to 8 sections (7 cuts) the low-frequency damping increases. For the larger numbers of cuts damping tends to reach a maximum at a frequency which rises as the length between cuts decreases. Marked on the curves for 3, 7, 15, and 31 cuts are the points at which the length between cuts equals one quarter wavelength, in each case fairly near a point of maximum damping. For large numbers of cuts the low-frequency damping decreases again.

The phenomena involved here are not fully elucidated , but the following explanation appears probable:

If one end of a backing layer is pulled parallel to its length, the damping material beneath it near the end is strongly sheared. However, owing to stretching of the backing, this shear decreases away from the end and at a sufficient distance from the end will have fallen to a negligible value. There is a characteristic length L over which the shear decreases and Kerwin[11] has shown that this length is equal to $(H_2 H_3 E_3/G_2)^{\frac{1}{2}}$. This would be of the order of one inch in the present layers. Thus the effect of stresses applied to or removed from the backing at any point is felt only over a distance of the order of L from that point. It follows from Eqn.2.1 that

$$g = \lambda^2/4\pi^2 L^2$$

and the condition for $g \doteqdot 1$ for maximum damping corresponds to $\lambda \doteqdot 2\pi L$.

Now in the low-frequency region where $g >> 1$, the system vibrates in pure bending (Fig.5b) with little shearing of the damping layer. Cutting the tape will allow considerable shear strains to occur in the neighbourhood of the cut, thus adding to the losses which occur on shearing. With sufficient cuts the system probably reaches a state in which shearing predominates, such as is normally only reached at high frequencies ($g < 1$). The damping factor reaches a value comparable with its normal peak value at $g = 1$. However, if the length between cuts becomes comparable with or smaller than L, shearing of the central layer decreases again and the short "islands" of damping tape merely ride to and fro on the vibrating panel without significant deformation.

At higher frequencies $(g < 1)$, shear bending (Fig.5c) is already occurring, and the effect of cuts may well be to release some of the shear strains present. However the change should still not be very marked unless there are several cuts in each wavelength, which implies a very close spacing indeed for high frequencies.

These ideas appear to explain qualitatively the phenomena shown in Fig.14. A detailed theory on these lines is being developed. D. J. Mead at Southampton University, in work as yet unpublished, has also pointed out that cuts in a damping tape can be beneficial.

This effect of cutting the tape has the very valuable result that the damping effect of a backed layer can be rendered substantially equal to its highest possible value over a very much wider range of frequencies than is otherwise the case. This is achieved at no cost in weight or complexity. In fact it actually simplifies the practical problems of application. In damping a panel, it is not only unnecessary but actually disadvantageous to use a carefully-fitted uniform backing sheet. Instead strips of tape from a reel can merely be laid side by side. For vibrations in two dimensions the tapes themselves should be cut across. If less powerful damping is required, the panel can be partially covered by sets of tapes at right angles to each other, the ability to cut them making the crossings no problem.

An additional advantage is that it is permissible to set the frequency of maximum damping near the upper end of the range of frequencies present, rather than near the centre. This allows a thinner and hence lighter damping layer.

Until a quantitative theory is available it is not possible to say what the optimum cut spacing should be in any particular case. At present some four to eight cuts per wavelength in a region some two decades below the normal damping peak seems a not unreasonable figure.

3.6 Deep backing layers

Eqn.2.3 shows that for a given panel and damping material, the maximum damping is proportional to $H_3 H_{30}^2$, and thus increases rapidly with the distance between the centre of the backing layer and the neutral surface. One simple technique for increasing this distance which was tried out was to bend up the edges of the backing layer as shown in Fig.15. Typical measured damping-frequency curves for these forms are shown in Fig.16. The damping layer used was in both cases a single layer of Permacell 45 tape. The curves are more irregular than for plain damping tapes, but show the same features of a broad maximum, and a fall in damping at low frequencies which can be mitigated by cutting the backing at intervals. A very stiff backing would of course not conform to a curved surface such as the side of a fuselage without being cut every few inches, so for ribbed or flanged backings cuts are doubly advantageous. Damping tapes with ribbed backings give a higher damping for a given mass increase, as can be seen from Fig.8, but once again this advantage is only obtained at the expense of an increase in complexity and difficulty of application. Ross, Kerwin and Ungar[12] have proposed the use of a very light, thick backing layer of rigid foamed plastic for the same purpose of increasing H_{30}, and this would provide the feature, useful in some applications, of very good thermal insulation.

3.7 Spaced layers

It was shown in Section 1.3 that so far as minimising panel stress was concerned, increasing the stiffness of the panel could be beneficial in general. In the previous report, a damped channel-section stiffener as in Fig.17 was described and shown theoretically to provide a considerable increase in stiffness along with a damping comparable with that given by a simple Aquaplas layer of the same added weight.

The/

The table below reproduces certain results of this investigation, extended now to show the change in resonance frequency and in panel stress due to the stiffening. Frequency and stress are supposed proportional respectively to $(S/M)^{\frac{1}{2}}$ and to $(S^{-\frac{3}{4}} M^{-\frac{1}{4}} \eta^{-\frac{1}{2}})$ × maximum distance from neutral surface as discussed in Section 1.3. The columns of the table from left to right represent the following ratios:

(1) the thickness of the Aquaplas layer to the panel thickness

(2) the total weight of the treatment to that of the panel

(3) the bending stiffness to that of the panel

(4) the resonance frequency to that of the panel

(5) the peak stress in the panel for a given force after the stiffness is added to that before but <u>neglecting the effect of damping</u>

(6) the damping factor of the structure to that of the Aquaplas itself.

The stiffener considered consists of an aluminium channel of dimensions shown on Fig.17 (cases e and f in the table deviate slightly from these), considered as attached to a strip of aluminium or of steel).

	1	2	3	4	5	6
	Aquaplas thickness ratio	Added weight ratio	Stiffness ratio	Resonance frequency ratio	Panel Stress ratio	Damping ratio
Aluminium on steel panel	a 1.0 b 0.5	0.14 0.10	7.4 5.5	2.5 2.2	0.30 0.35	0.35 0.20
Aluminium on aluminium panel	c 1.0 d 0.5 e 0 f 0.18	0.43 0.28 0.15 0.20	18 12 7.4(6.5) 8.2(7.5)	3.5 3.1 2.5(2.4) 2.6(2.5)	0.20 0.24 0.31 0.29	0.60 0.45 0 (< 0.01) 0.1(0.1-0.2)

The results show that the stiffness and mass increase due to the stiffeners can theoretically reduce panel stresses by some 3 - 5 times in the cases considered, in addition of course to the reduction due to damping.

Cases e and f in the table, representing an aluminium stiffener on a 1/8 in. aluminium strip with dimension ratios closely similar to those of Fig.17, were examined experimentally to check the validity of the theoretical assumption that the system deflects by pure bending. The experimental results are the bracketed quantities in columns 3, 4 and 6. It is seen that the measured increases in stiffness and resonance frequency are not far short of the calculated ones, and the damping increase is fully achieved. An attempt was made to observe the vibration of the strip stroboscopically to see whether any undesirable deformations of the stiffener were occurring (e.g., anticlastic bowing of the surface), but the technique was insufficiently acute to be conclusive. However the degree of agreement with theory can be taken as evidence of the absence of any major spurious modes.

For comparison with the above results, it may be mentioned that the plain Aquaplas layers of Fig.7, representing mass increases of 10% and

34%/

34% respectively, gave the relatively small stiffness ratios (cf., column 3 of the table) of about 1.14 and 1.4, the values being somewhat frequency dependent.

It thus appears that structures such as that described, while rather complex in practice may occasionally find application where the maximum possible mitigation of vibrational stresses or amplitudes is called for.

It may be mentioned that the principle of spacing a damping material away from a vibrating member is being closely studied in other laboratories[12,14,15]. It appears that the material can be used to its maximum possible effect in this way. However the treatments involved must necessarily be somewhat complex and for this reason as well as lack of time this approach has not been followed further in the present investigation. The channel stiffener discussed in Section 3.7 could form a useful starting point for such a study however.

4. Summary and Conclusions

The report describes a study of backed damping layers and related treatments for application to metal beams, panels, etc. to damp out mechanical vibration. Such systems are attracting attention in a number of laboratories throughout the world. They represent a means whereby soft, highly-damped adhesive compounds can be employed as damping agents. The first section includes by request a brief discussion of the rôle of structural vibration and anti-vibration treatments in aircraft structures. The report is couched in terms of the vibration of panels or thin strips. The principles however apply equally to beams, girders and other structural members.

The basic theory of a backed layer or damping tape is briefly summarised in Section 2, and the more detailed design of a tape is discussed in the Appendix. The tape produces maximum damping in a particular frequency range determined by its dimensions and elastic constants. However the peak on a damping-frequency plot is fairly broad, and in particular it has been shown that by cutting the tape across at intervals a much flatter curve still can be obtained. This is achieved with no penalty in weight or cost and actually simplifies application of tapes. In principle it is necessary to vary the thickness of both backing and damping layers to obtain the required peak damping at the required frequency. However if the tape is cut to minimise frequency discrimination and either the area of the panel covered or the number of layers of tape superposed is varied to vary the peak damping, it should be possible to treat a wide range of panels with one or only a few standard patterns of tape. At least one firm now markets such a range in this country.

Fig.8 summarises the results for damping produced by the various treatments tested. Damping tapes gave rather greater damping for a given mass increase than the best materials currently known for simple damping layers. The latter, applied by spraying or spreading, have of course obvious practical advantages for application to curved or uneven surfaces.

The performance of a damping tape can be further improved by effectively thickening the backing for a given weight. One simple means tested of doing this was to turn the edges of the tape up to form flanges. Foamed plastic backings have also been proposed. Other workers have proposed the insertion of shear-stiff spacers between panel and tape to enhance its effects. This somewhat complex treatment has not been examined here, though a similar principle has been employed in a damped channel-section stiffener. This can provide good damping along with a substantial increase of stiffness which is itself beneficial so far as reducing fatigue stresses in the panel is concerned.

Some further, mainly theoretical, study of outstanding points in the work is still in progress, and it is proposed in due course to produce an addendum to the present report covering this.

APPENDIX/

APPENDIX

The Design of a Damping Tape

The wavelength λ of flexural waves of frequency f in a thin strip of thickness H_1, Young's modulus E_1 and density ρ_1 is given by

$$\lambda^2 = \frac{\pi}{3^{\frac{1}{2}}} \frac{H_1}{f} \left(\frac{E_1}{\rho_1} \right)^{\frac{1}{2}} \qquad \text{... (A1)}$$

Assuming that the wavelength in the strip when covered by a damping tape is similar to that without the tape, substitution of this relation into Eqn.2.1 gives the shear parameter g as

$$g = \frac{1}{4.3^{\frac{1}{2}}.\pi} \left(\frac{H_1}{f(H_2 H_3)} \right) \frac{G_2}{E_3} \left(\frac{E_1}{\rho_1} \right)^{\frac{1}{2}} \qquad \text{... (A2)}$$

with symbols as in Section 2. It is convenient to define the density and thickness ratios

$$\alpha = \rho_2/\rho_3 \quad \text{and} \quad \beta = H_2/H_3 \qquad \text{... (A3)}$$

Now suppose for simplicity that the backing material is the same as that of the strip, i.e., $\rho_1 = \rho_3$, $E_1 = E_3$ etc. The fractional weight increase due to the tape is

$$w = \frac{\rho_2 H_2 + \rho_3 H_3}{\rho_1 H_1} \qquad \text{... (A4)}$$

Substituting these quantities in Eqn.A3 gives

$$g = \frac{K}{f} \frac{(1 + \alpha\beta)^2}{\beta w^2} \qquad \text{... (A5)}$$

where

$$K = 0.0462 \cdot \frac{G_2}{H_1 (E_1 \rho_1)^{\frac{1}{2}}} \qquad \text{... (A6)}$$

If the tape is to give maximum damping at this frequency then $g \doteqdot 1$, and solution of Eqn.A5 gives

$$\alpha\beta = \left(\frac{w^2 f}{2K\alpha} - 1 \right) \pm \left[\left(\frac{w^2 f}{2K\alpha} - 1 \right)^2 - 1 \right]^{\frac{1}{2}} \qquad \text{... (A7)}$$

In order for this to be real

$$\alpha K \leqslant \frac{w^2 f}{4} \qquad \text{... (A8)}$$

i.e., for a given frequency, panel and allowable weight increase there is a maximum value of αK and hence of the product $\rho_2 G_2$ of density and shear modulus of the damping layer allowing optimum performance. For the common case of a thin panel, the required value of shear modulus is quite low, comparable with that of a soft rubber for example. Thus damping tapes employ a class of material quite different from that required in unbacked layers, which must have the highest possible modulus consistent with high internal losses.

If/

If $\rho_2 G_2$ lies below the limit set by Eqn.A8, there are two values of $\alpha\beta$, the ratio of damping layer weight to backing layer weight, giving optimum damping. The smaller one will be chosen as it yields the larger optimum damping, and is that corresponding to the negative sign in Eqn.A7. It lies in the range $0 \leqslant \alpha\beta \leqslant 1$. The maximum damping obtained (Eqn.2.4) is then given approximately by

$$\frac{\eta}{\eta_2} = \frac{3}{4}\frac{H_3}{H_1} = \frac{3}{4}\frac{w}{(1+\alpha\beta)} \qquad \text{... (A9)}$$

The above argument applies exactly only to tapes thin compared with the panel and for $\eta_2^2 << 1$. The modifications necessary for a more exact treatment add to the complexity of the equations but do not affect the principles involved.

It has been tacitly supposed throughout that the damping treatment covers the whole panel. If it does not, the damping at any frequency is to a first approximation reduced in proportion to the area covered, provided that the panel continues to vibrate as a whole. If the sections of panel uncovered are large enough to have significant resonances of their own, no such simple ideas can be applied.

Addendum to Section 3.5

The theory referred to at the end of this section has now been worked out and confirms the mechanism outlined. It shows for example that if the damping tape is thin compared with the panel, the optimum length of tape between cuts is about three times the characteristic length L, and therefore is conveniently a property of the tape itself and not dependent on the panel concerned.

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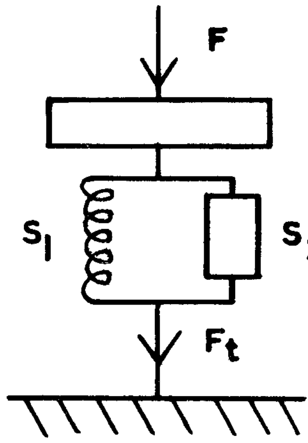
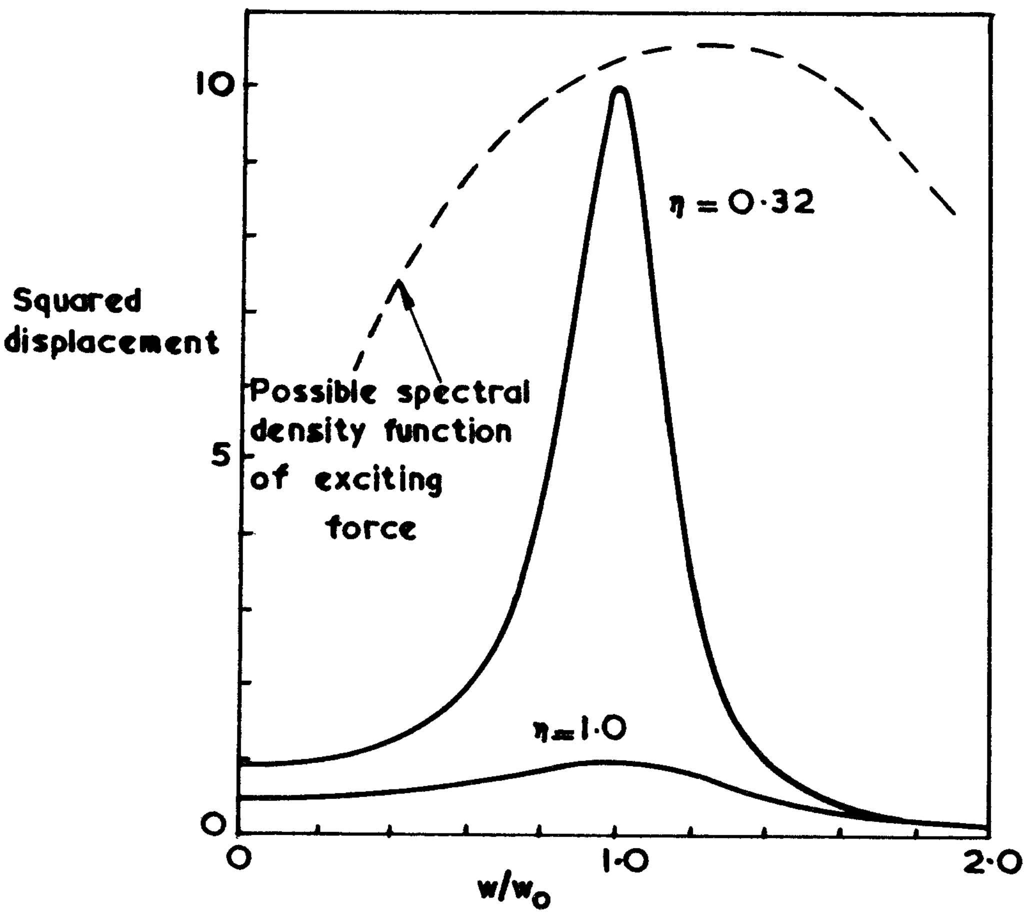
FIGS. 1 & 2.

FIG. 1.



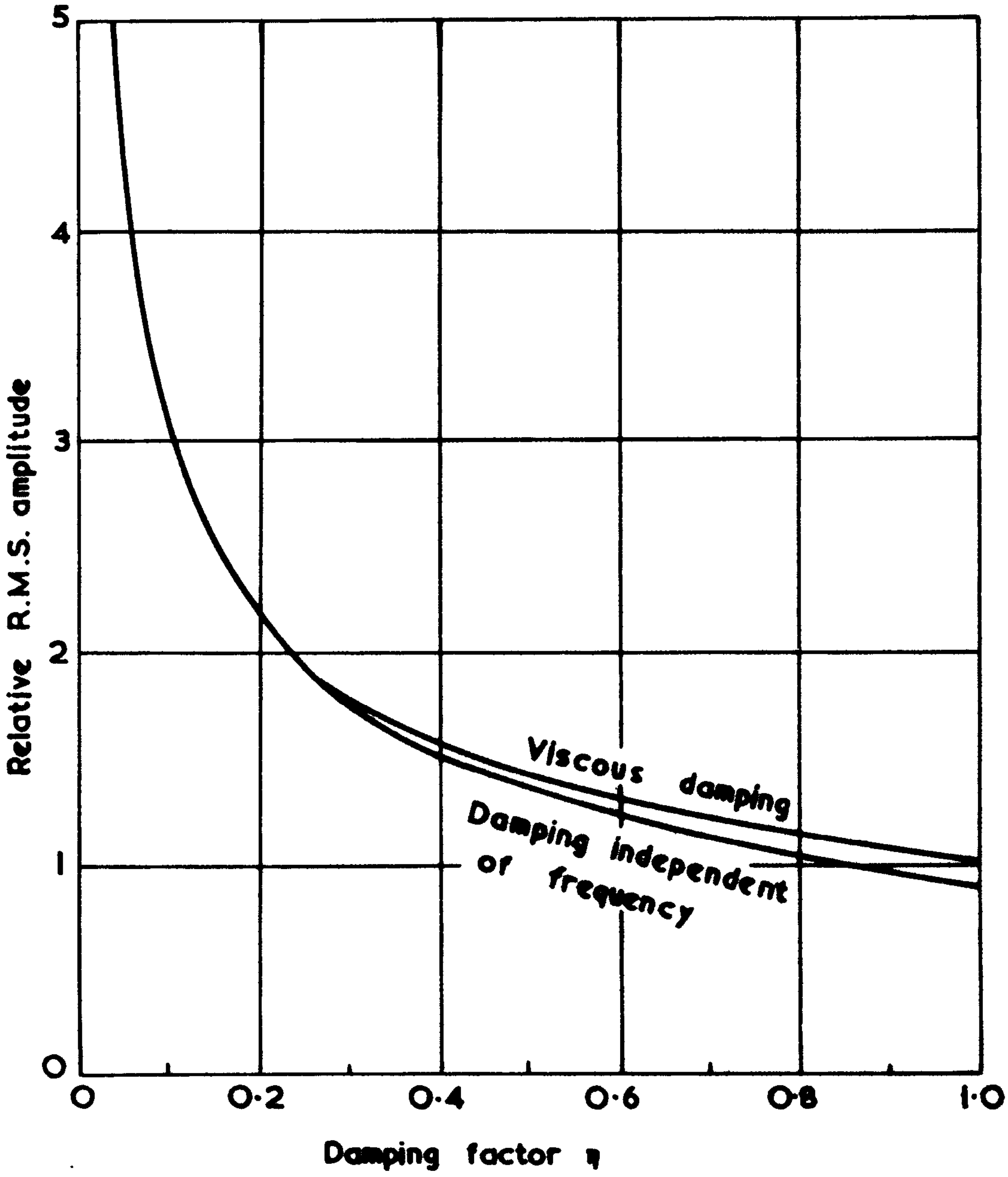
One-degree-of-freedom resonant system.

FIG. 2.



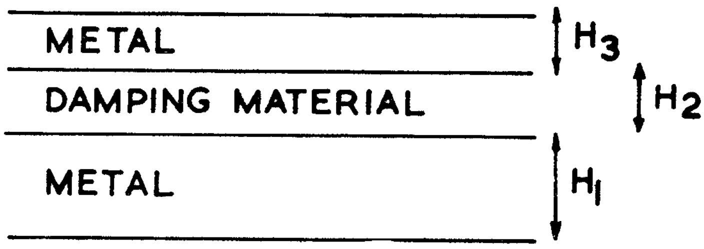
Response function of resonator for a sinusoidal force.

FIG. 3.



Variation of random amplitude with damping factor.

Fig. 4.



Backed damping layer.

Fig. 5.

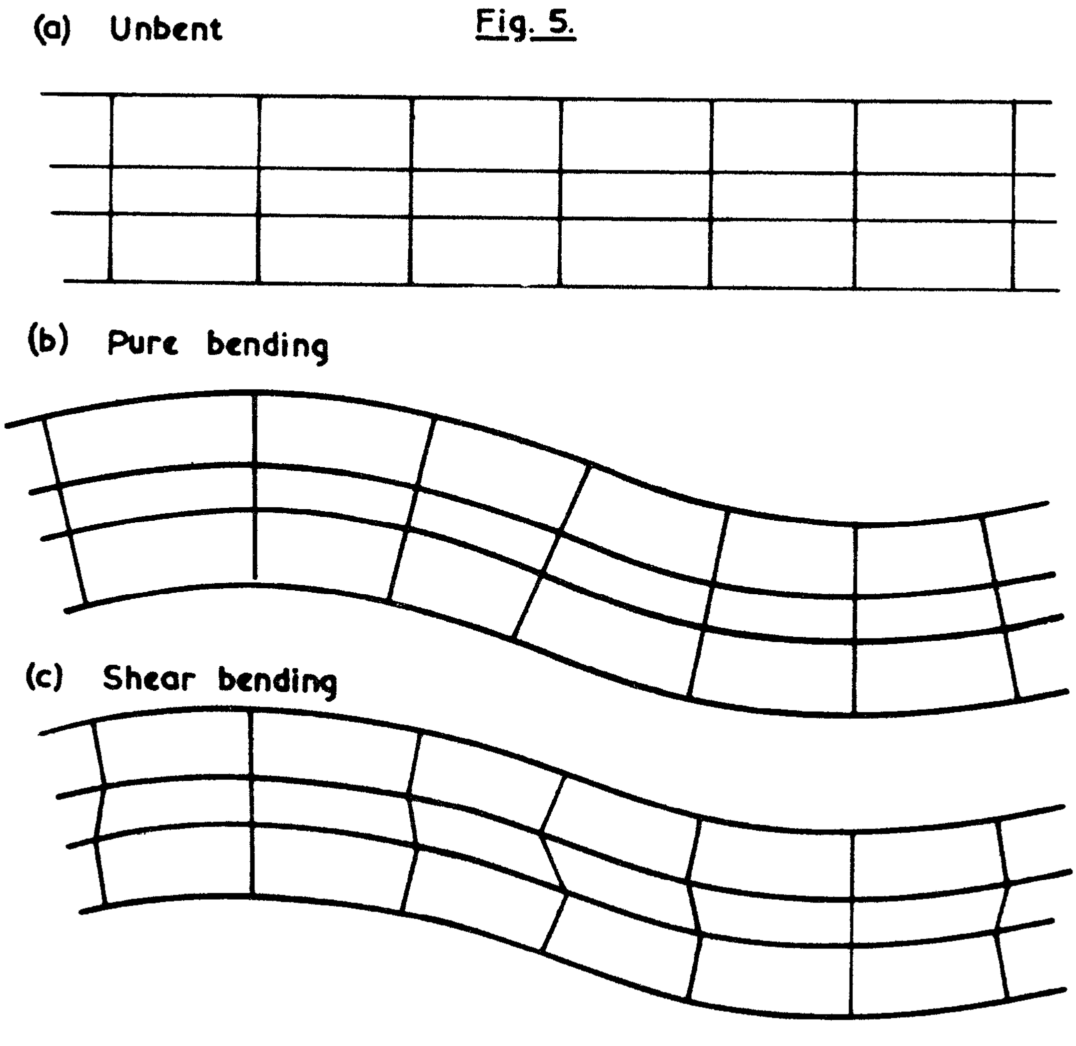
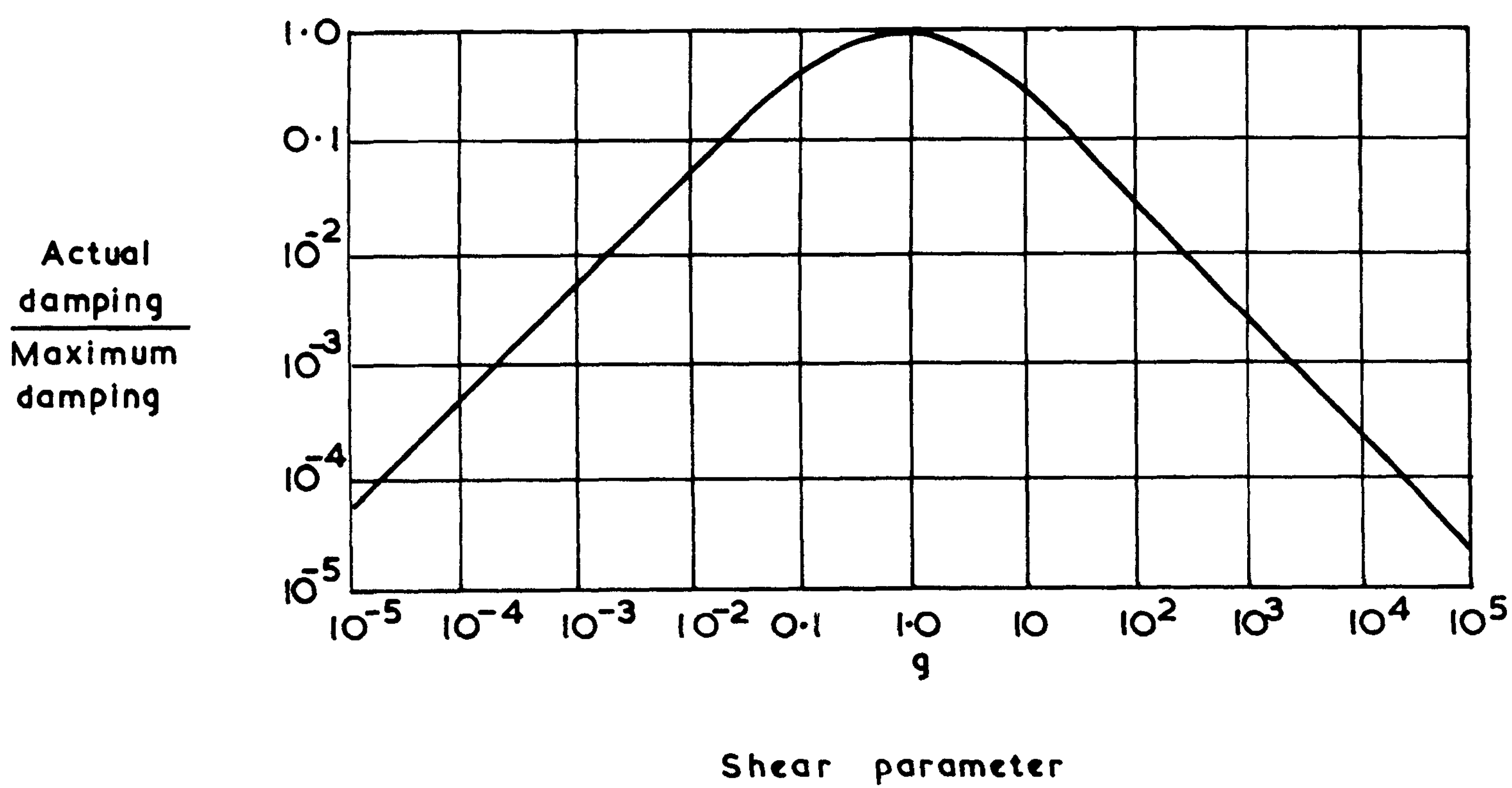
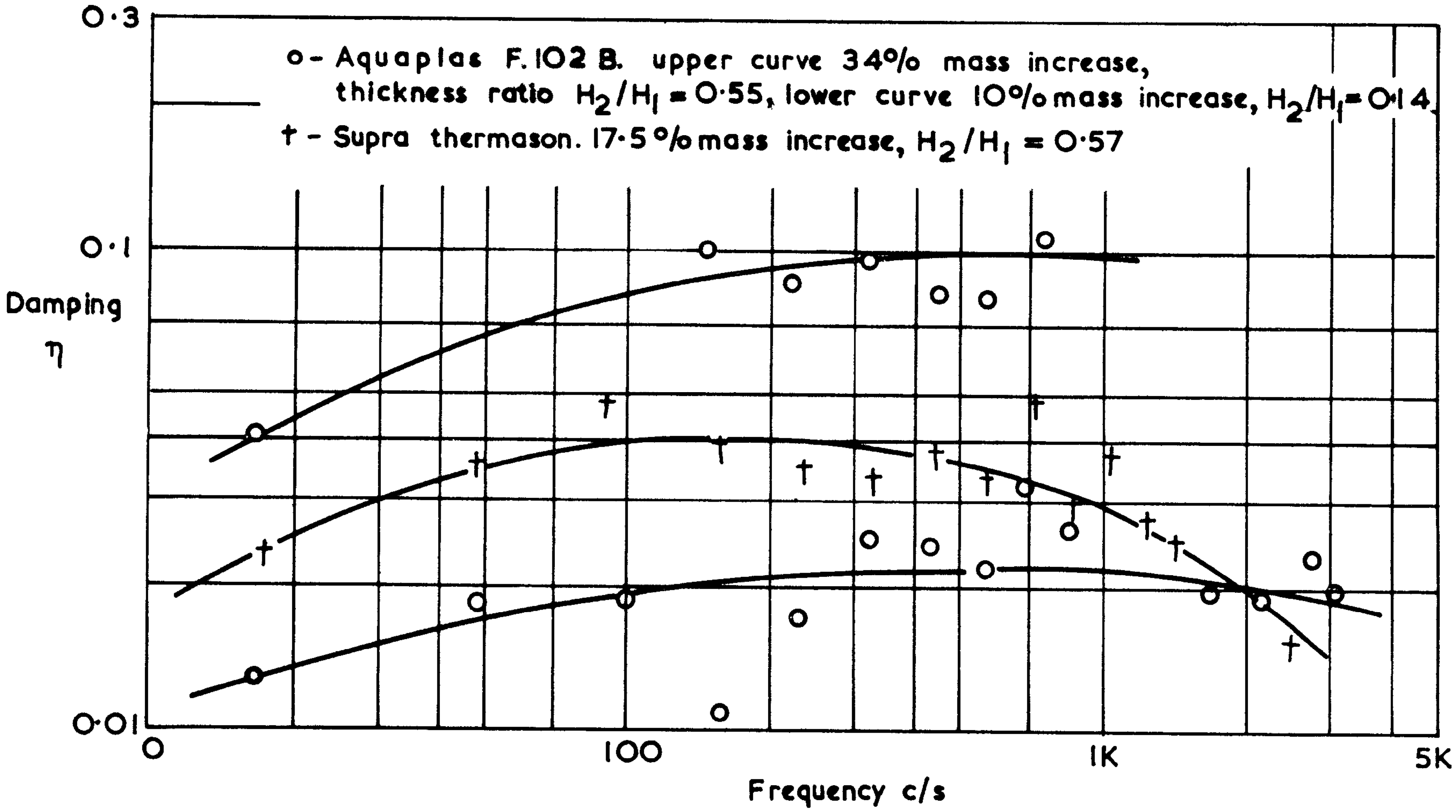


FIG. 6.



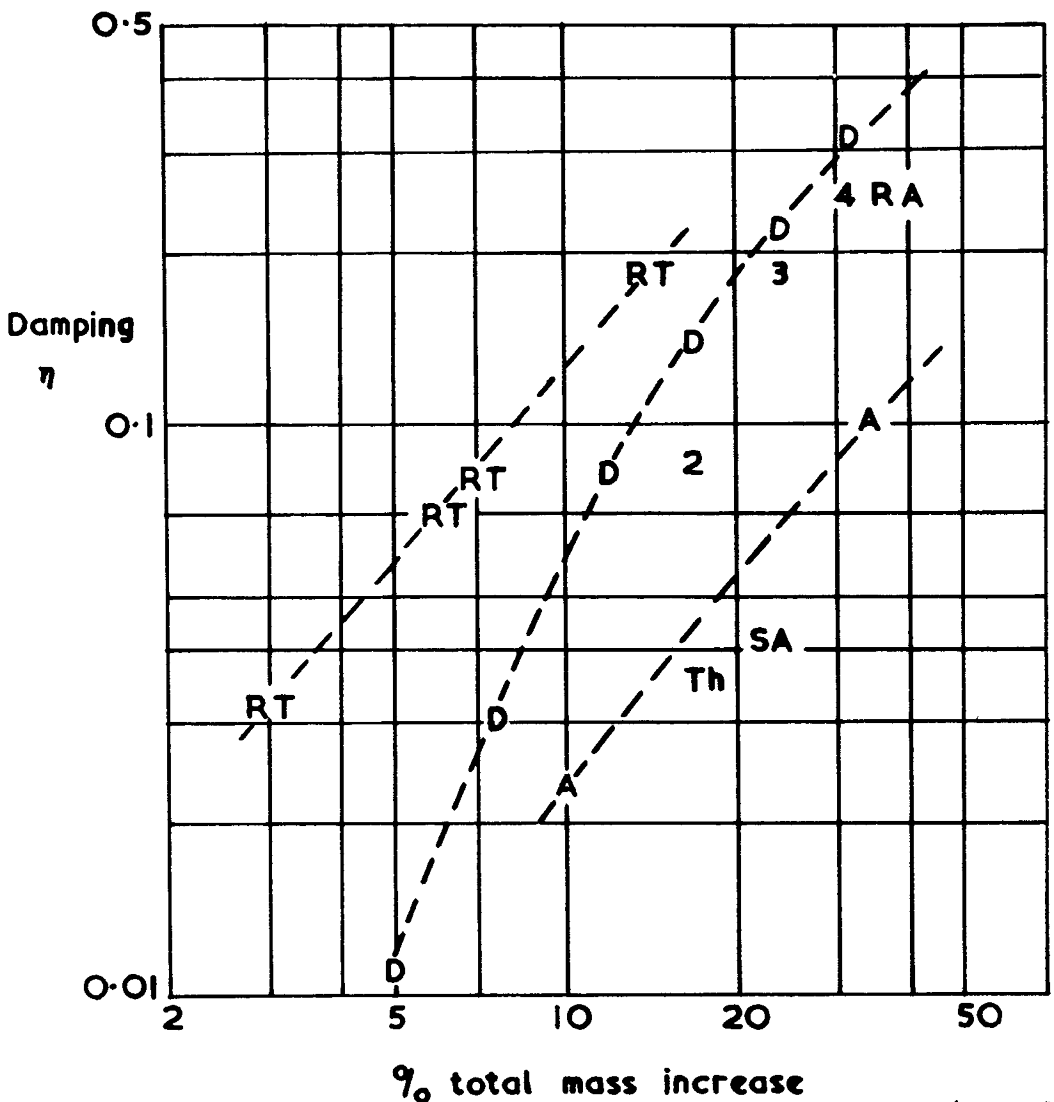
Effect of shear parameter on damping of a backed layer.



Damping of simple layers.

FIG. 7.

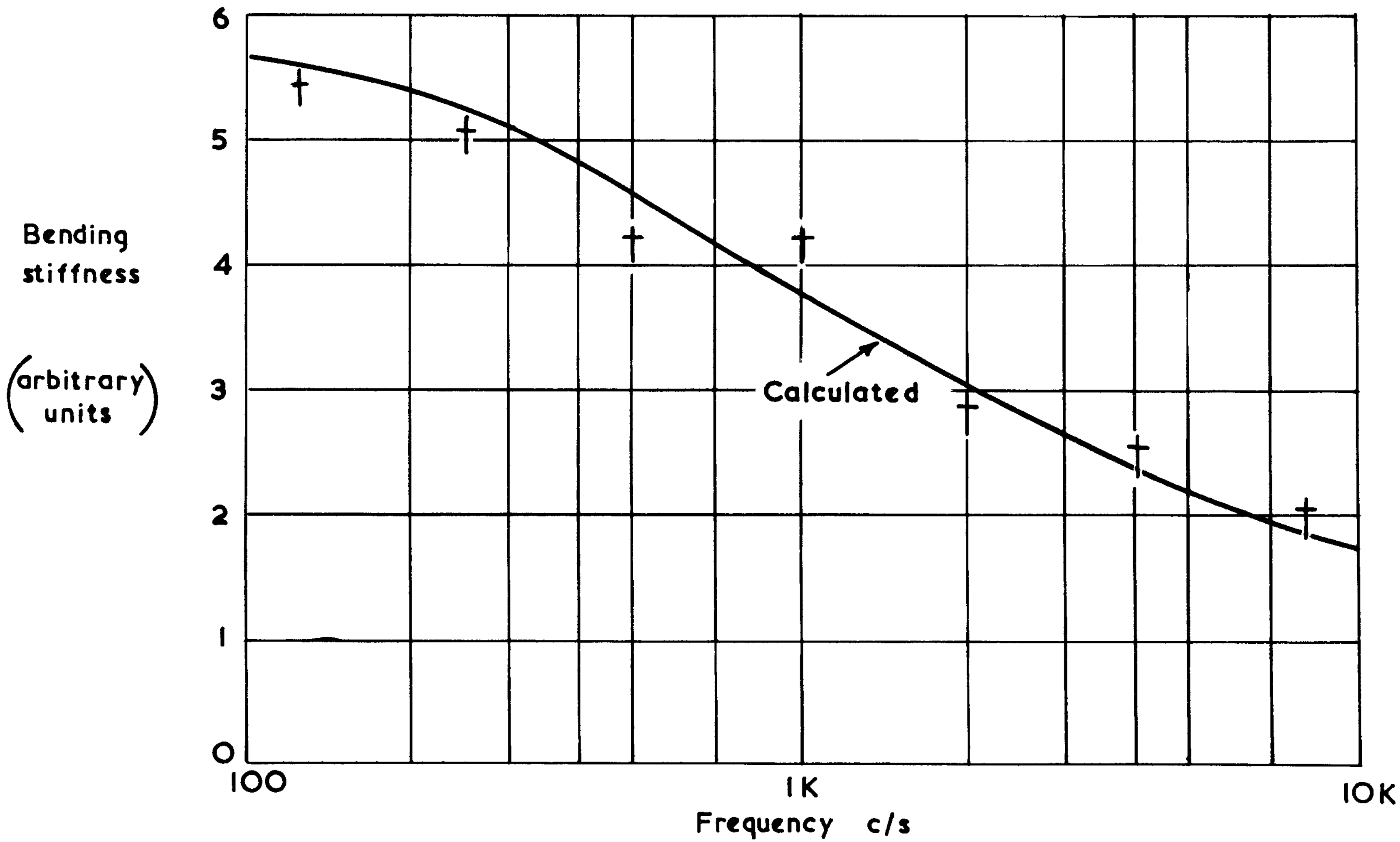
FIG. 8.



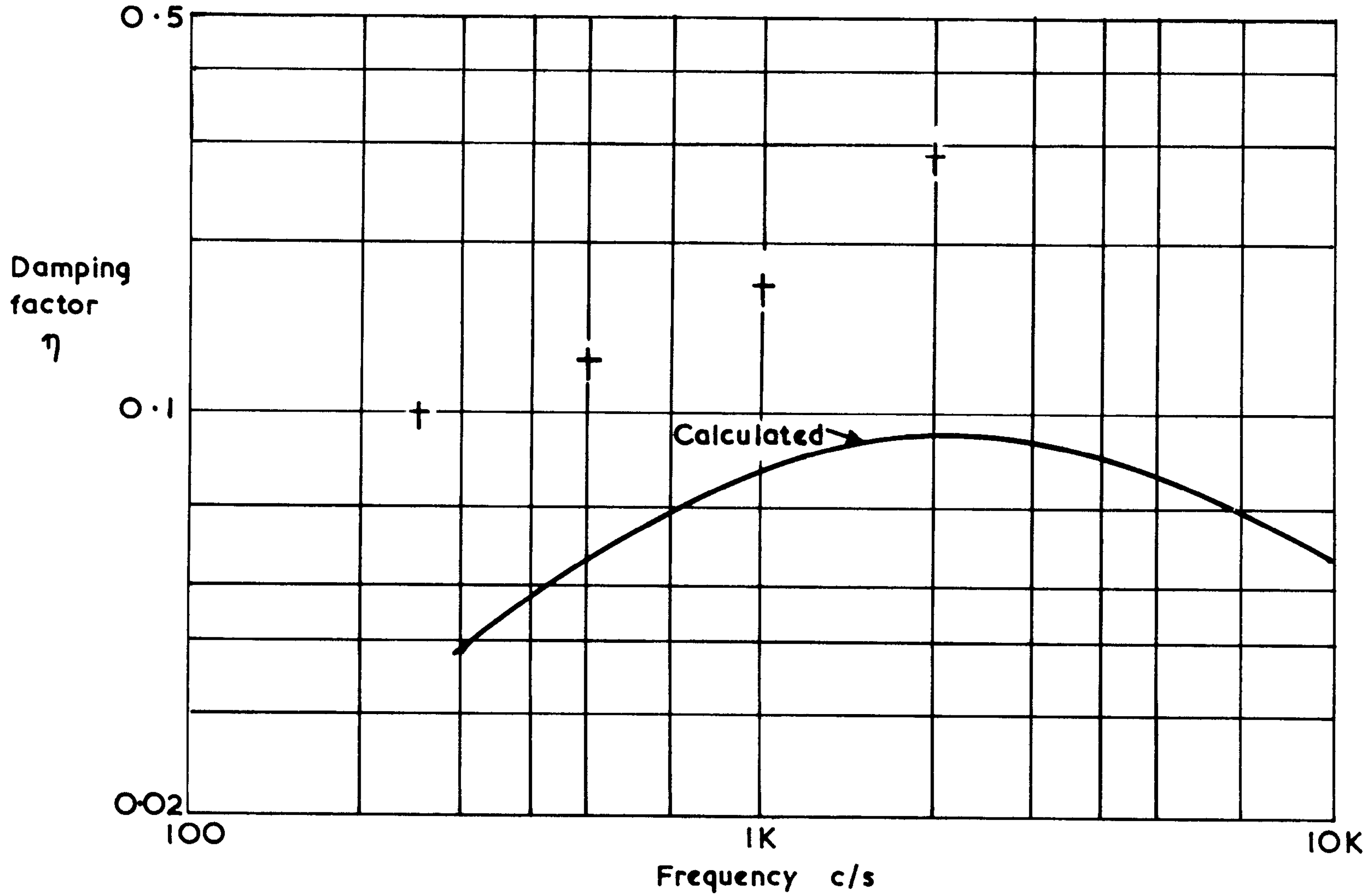
A – Aquaplas F102B Th – Supra Thermason layers (Fig. 7)
SA – Aquaplas on spacer RA – Aquaplas rib.
D – Damping tapes.
2, 3, 4 – Multiple layers of damping tape.
RT – Ribbed damping tapes.

Damping obtained from various damping treatments.

FIG. 9.

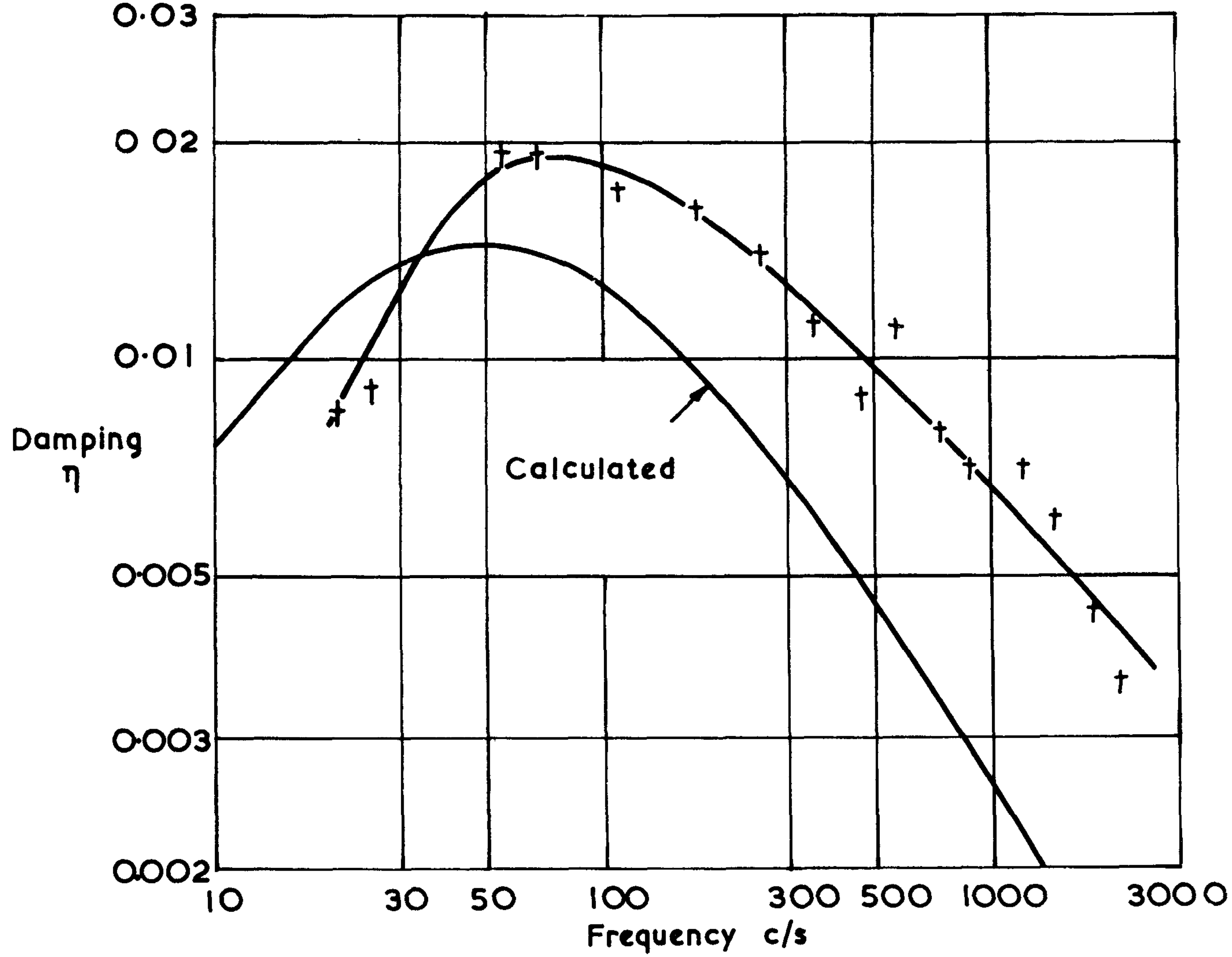


Stiffness of steel-Aquaplas sandwich.



Damping of steel- Aquaplas sandwich

FIG. 10.

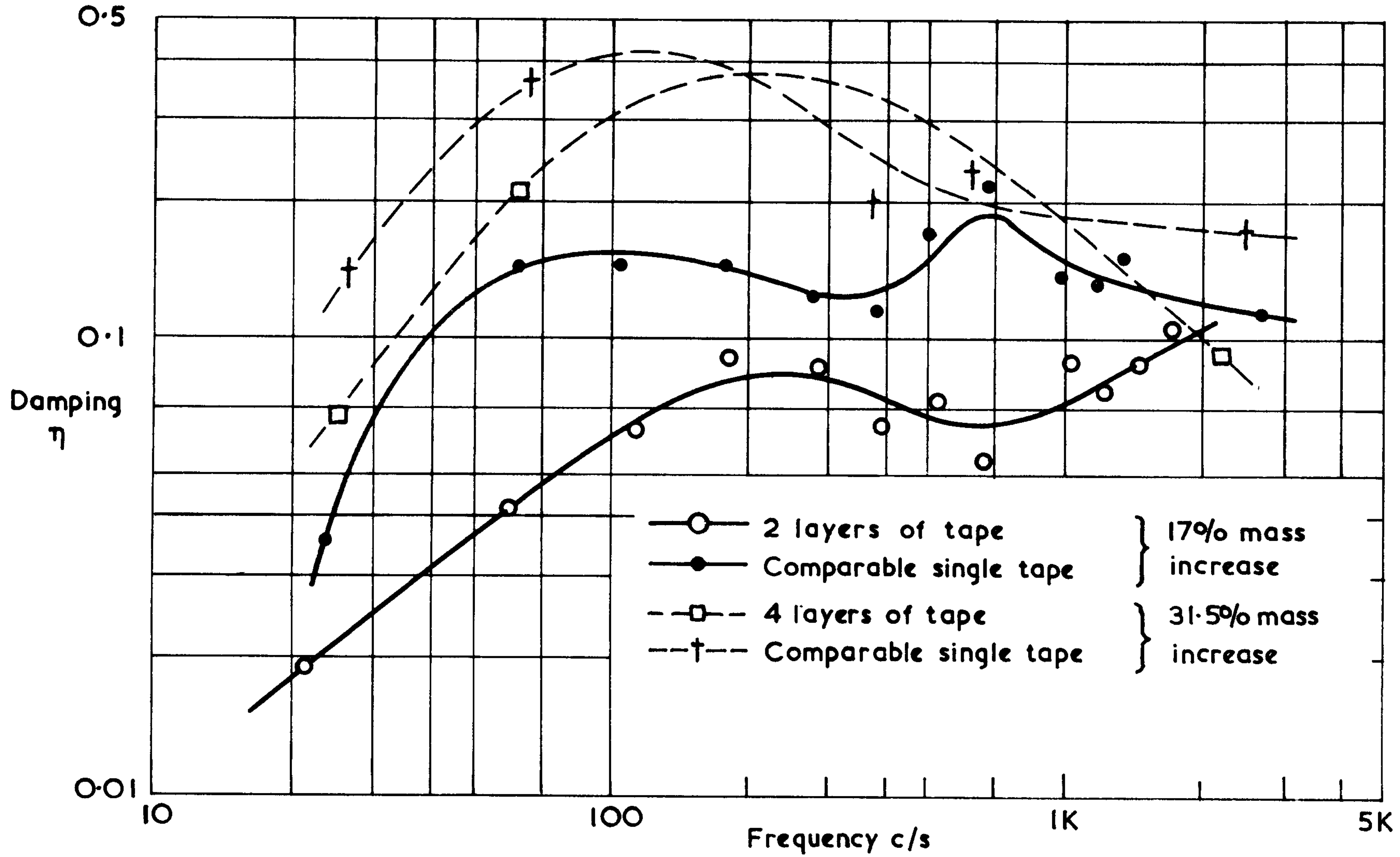


Damping of a backed rubber layer.

FIG. 11.

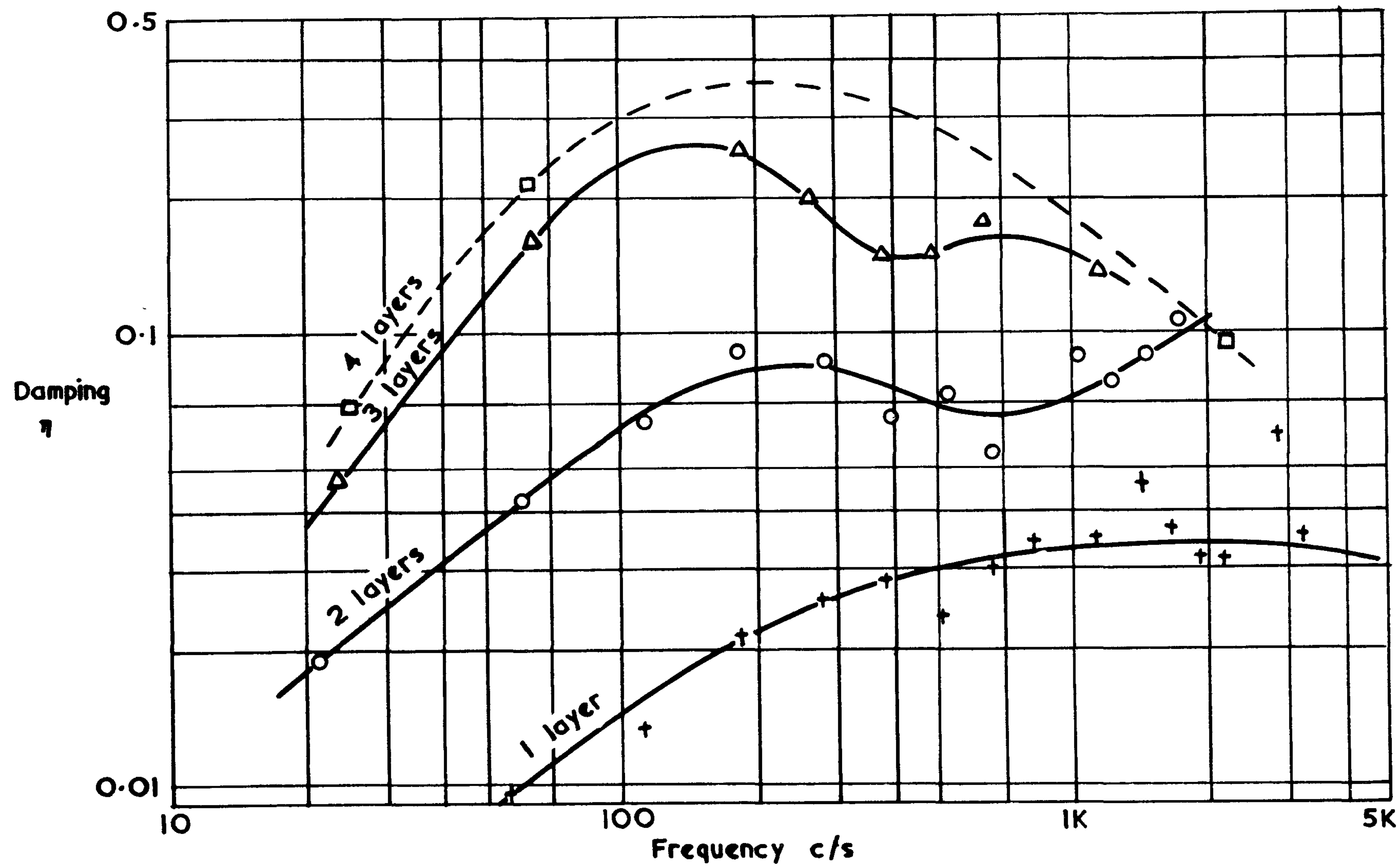
J.a.p.

FIG. 12.



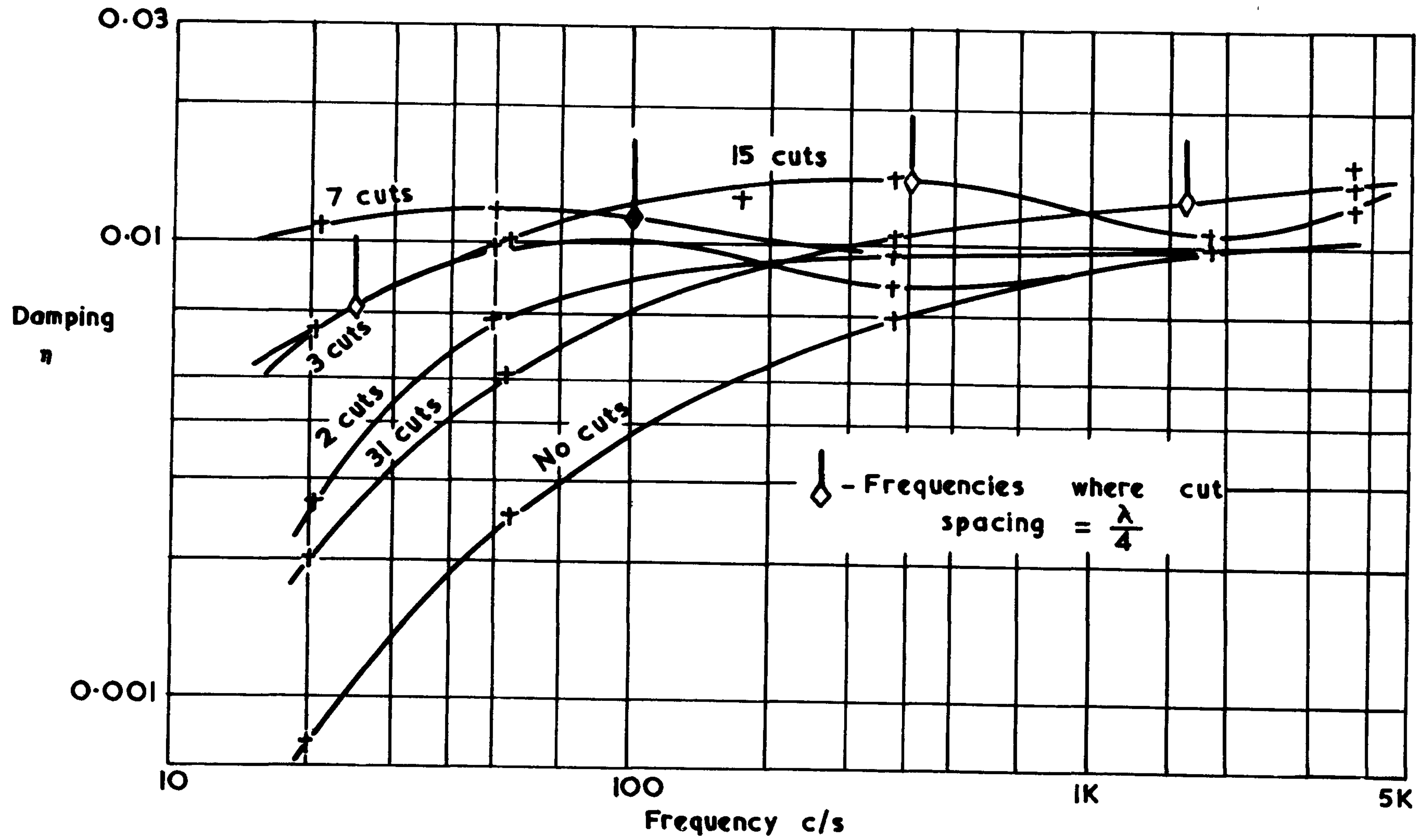
Comparison of multiple and single layers of damping tape.

FIG.13



Multiple layers of damping tape.

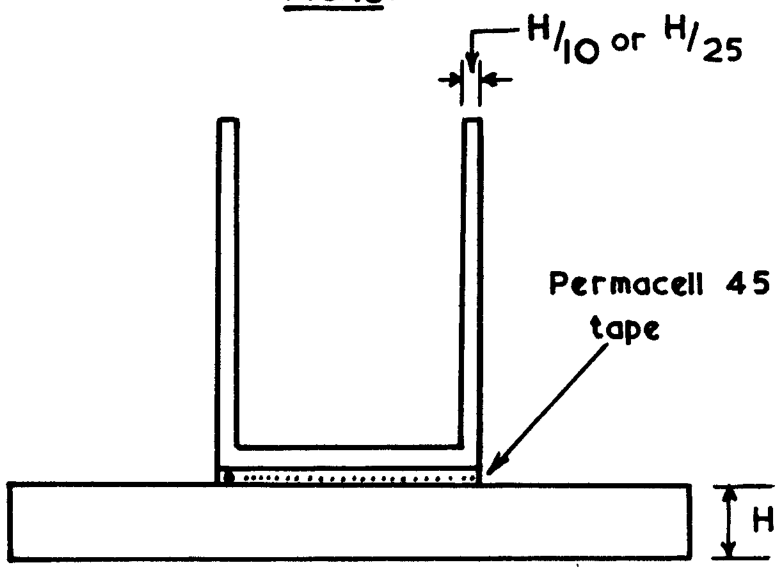
FIG. 14.



Effect of cuts in the backing of a damping tape

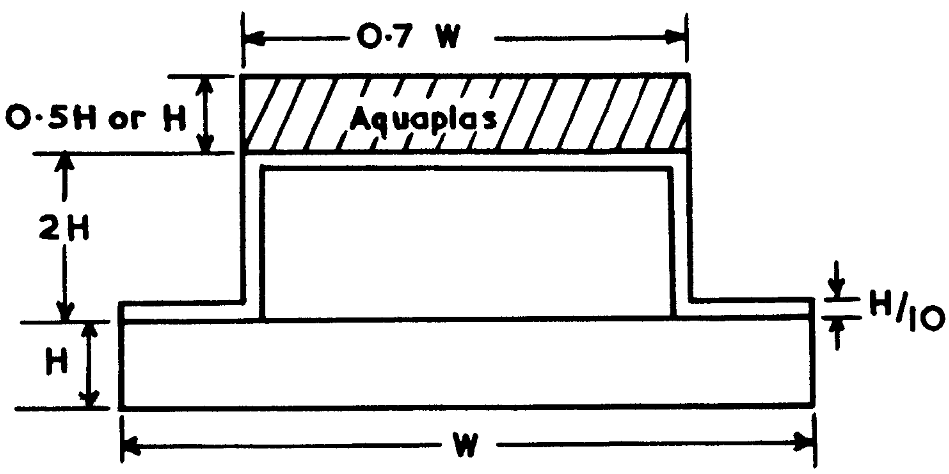
FIGS. 15 & 17.

FIG. 15.



Damping tape with flanged backing.

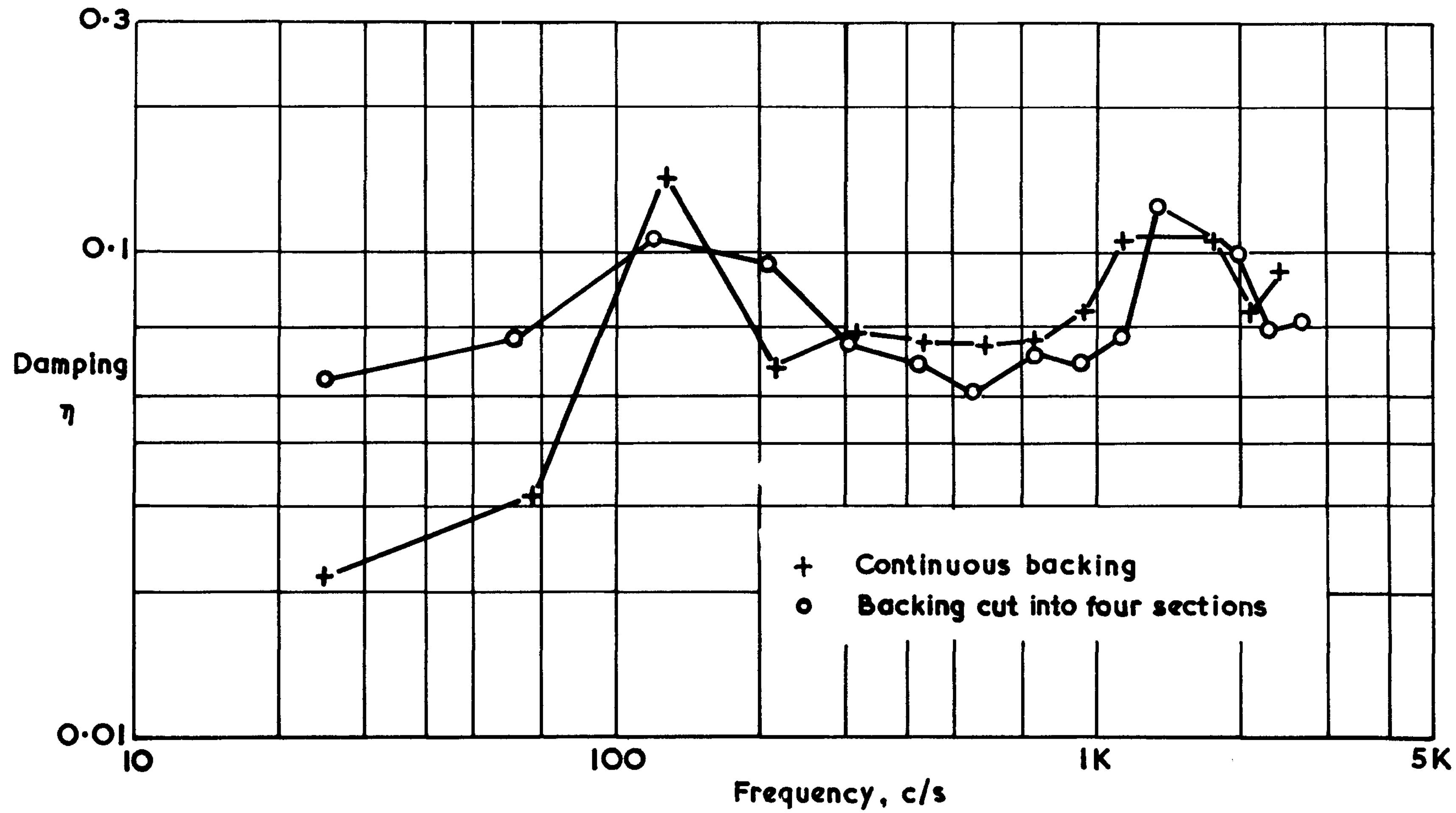
FIG. 17.



Spaced damping layer

Both diagrams are sections perpendicular to the plane of bending.

FIG. 16.



Damping tape with ribbed backing

DS63853/1/Wt58 K4 4/62CL,XL

A.R.C.C.P. No. 596. September, 1960.

Parfitt, G. G. and Lambeth, D. - Imperial College

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