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by

D. Williams, D.Sc., M.I.Mech.E., F.R.Ae.S.

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D. Williams, D.Sc., M.I.Mech.E., F.R.Ae.S.

SUMMARY

The concept of scale-effect introduced in Ref.5 to account for dissimilar critical crack lengths in similar flat sheets is placed in better perspective by relating it to Griffith's well known work on crack propagation. The results of recent experiments are quoted that confirm the reliability of the approach via scale effect and also confirm the semi-empirical formula for relating critical stresses in flat sheets to those in corresponding cylinders.

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1 INTRODUCTION

A great deal of work has been done in recent years in an endeavour to understand what determines critical crack lengths in structural materials. This work has been intensified since the advent of pressure cabins and has therefore largely been concerned with cracks in sheet material. The purpose of this note is to take stock of the situation on the basis of past work and of some recent experimental results and, in particular, to emphasise any results that are likely to be of direct interest to the designer.

2 GRIFFITH'S WORK

Few papers on crack propagation can be read without meeting in the first few paragraphs the name of Griffith with reference to his classic work on crack propagation in brittle sheet material. Griffith considered the effect of introducing a crack across a field of uniform tensile stress in a thin sheet with loaded edges fixed in space, the material of the sheet being brittle and perfectly elastic. In order to appreciate its bearing on later work it is worth examining his general approach to the problem.

To fix ideas consider a large sheet ABB'A'

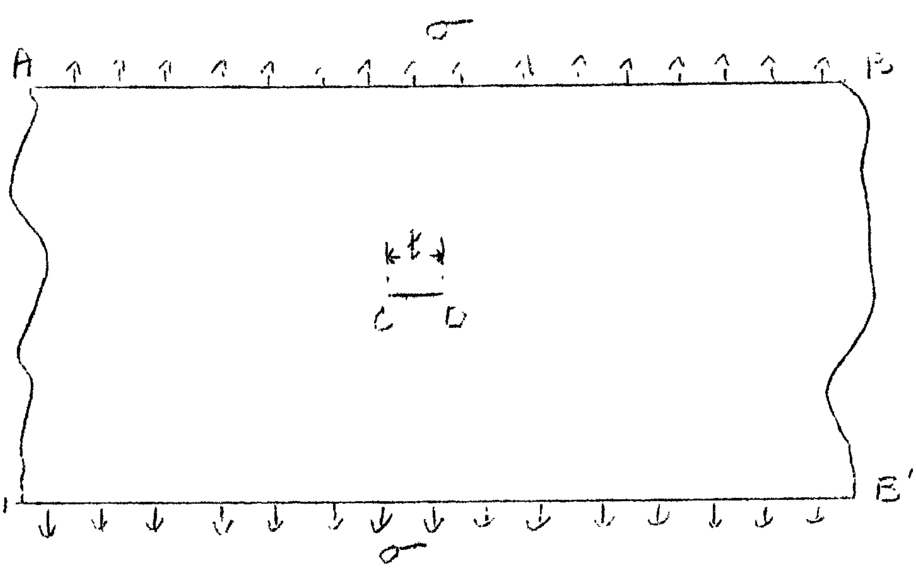

Fig.1

of indefinite width subjected to tensile stresses σ at the fixed boundaries AB, A'B' as shown in Fig.1. If now a slit (or crack) CD is made in the sheet it follows from the structural analogue[2*] of Kelvin's Theorem in dynamics that the strain energy of the sheet must drop. It can also be readily shown[3] that if the amount of the strain energy is U_1 when the crack has length ℓ the rate of increase of strain energy with crack length $dU_1/d\ell$ with boundaries fixed is exactly equal but opposite in sign to the rate of increase of strain energy with boundaries free under the same (constant) applied stress. But if the crack CD is small enough to ensure that the stress disturbance it causes is only local, the strain energy increase with boundaries free can be calculated in terms of the length ℓ of the crack. This was done by Griffith[1] using formulae derived by Inglis[4] for the stresses caused by an elliptical opening

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* The analogue to the theorem states that "If the displacements of an elastic system are fixed the strain energy is least when the constraints are least" (see Ref.2).

in which the major axis is normal to the tensile stress. By taking the limiting case where the ratio of minor to major axis approaches zero Griffith found this increase to be equal to

$$U_1 = \left(\frac{\nu}{E}\right) \pi \left(\frac{\ell}{2}\right)^2 \sigma^2 \qquad (1)$$

for a sheet of unit thickness.

By the above argument, therefore, the rate of increase of strain energy with crack length under fixed boundary conditions is given by

$$\frac{dU_1}{d\ell} = -\frac{\pi \ell \nu \sigma^2}{2E}, \qquad (2)$$

where ν and E represent Poisson's ratio and Young's modulus respectively.

If now the Total Potential Energy of the system were represented by the strain energy U_1 equation (2) would indicate an unstable condition however short the crack length ℓ, since for equilibrium it is necessary that

$$\frac{d\,(T.P.E.)}{d\ell} = 0 \ . \qquad (3)$$

Griffith resolved this difficulty by introducing the notion of energy absorbed by surface tension. By analogy with the behaviour of surface tension in liquids the fresh surfaces formed by the walls of the crack absorb energy per unit area of amount

$$U_2 = 2TA \ , \qquad (4)$$

where T = surface tension per unit length

A = area of fresh surface.

For a sheet of unit thickness the energy absorbed in forming a crack of length ℓ is therefore given by

$$U_2 = 2T\ell \qquad (5)$$

which gives

$$\frac{dU_2}{d\ell} = 2T \ . \qquad (6)$$

Using equation (3), we now have, for equilibrium,

$$\frac{dU}{d\ell} = \frac{d}{d\ell}(U_1 + U_2) = 2T - \frac{\pi \ell \nu \sigma^2}{2E} = 0 \ . \qquad (7)$$

This gives the critical crack length ℓ for any applied stress σ (or vice versa) if the surface tension T is known. For the sake of conciseness the applied stress σ on the gross area at failure will be referred to as the failing stress.

Taking glass as his material, Griffith deduced its surface tension at room temperature by extrapolating from a curve of surface tension against temperature obtained experimentally for comparatively high temperatures, the lowest being as high as 745°C. In this way he obtained a value for T of 0.0031 lb/in. From (7), with $\nu = 0.251$ and $E = 9 \times 10^6$, this gives

$$\sigma\sqrt{\ell} = 375 , \qquad (8)$$

a value only some 10% greater than that found by Griffith experimentally by bursting glass tubes and bulbs under internal pressure. It is noteworthy that in these experiments the value of $\sigma\sqrt{\ell}$ remained, as indicated by (7), fairly constant in spite of a six to one variation of the crack length ℓ.

3 RELEVANCE OF GRIFFITH APPROACH TO CRACK PROPAGATION IN METAL SHEET

If the condition for crack instability were the same for metals as for an almost perfectly brittle material like glass, one should be able to derive the surface tension for metal sheet of any material by inserting the values of the experimentally measurable values of crack length ℓ and failing stress σ in equation (7). Experiment shows for example that a 48 in. wide sheet of aluminium alloy to D.T.D.746 has a critical crack length of 15 in. under an applied tensile stress of 16,000 lb/in^2. Taking $\nu = 0.3$, $E = 10^7$ lb/in^2, and neglecting the fact that the crack would have been greater than 15 in. if the sheet had not been finite, we find the surface tension T to be given by 58 lb/in. The value of the surface tension of aluminium, as deduced from a similar experimental technique to that used by Griffith for glass, is, however of the same order as that for glass and therefore some 20,000 times too small to account for the energy required to satisfy equation (7).

It is clear from this that the rate at which energy is absorbed during crack extension $dU_2/d\ell$ must be due to something very different from surface tension. The only reasonable assumption that can be made is that this energy absorption is due to plastic deformation of the material in the neighbourhood of the crack extremity. Like the energy absorbed by the surface tension over the fresh surfaces created by the crack, energy absorbed by plastic deformation is an irreversible process. Moreover the strain associated with this deformation must locally, i.e. around the crack extremity, reach a value large enough to cause fracture before the crack can extend.

It is not possible in our present state of knowledge to obtain a quantitative expression for the energy absorbed by plastic deformation but experiment shows that, in general, this energy is no longer proportional to the length of the crack as in formula (4) assumed by Griffith for brittle materials. It is also clear that the energy absorbed U_2 cannot be proportional to the square of the crack length because, by equation (7), that would make the failing stress independent of the crack length instead of progressively smaller with increasing crack length as experiment shows. It is impossible for the energy absorbed to be proportional to a power of ℓ greater than 2 for that would make the failing stress increase with crack length. We are justified therefore in writing

$$U_2 = k_1 \ell^n \sigma_u \ , \qquad (9)$$

where k_1 = constant (dimensionally appropriate)

n = index that depends on the sheet material but must be less than 2 } (10)

σ_u = ultimate stress of sheet material = constant for that material.

Instead of (7) we therefore have

$$\frac{dU}{d\ell} = k_1 \sigma_u n \ell^{n-1} - k_2 \sigma^2 \ell = 0 \ , \qquad (10)$$

where k_2 is of course different from the coefficient of $\sigma^2 \ell$ in (7).

The result is that, instead of the Griffith formula for perfectly elastic but brittle materials, i.e.,

$$\sigma \ell^{\frac{1}{2}} = \text{constant} \ , \qquad (11)$$

we now have a new formula

$$\sigma \ell^{(1-n/2)} = \text{constant} \qquad (12)$$

for any one material. To determine the value of the index n it is necessary only to find by experiment the applied failing stress σ for two values of crack length. As shown in para. 4 the curve of σ against ℓ given by (12) is then identical with that derived in Ref.5 on the basis of scale effect.

4 IDENTITY OF SCALE-EFFECT APPROACH WITH THE APPROACH à la GRIFFITH

The basis of the approach used in Ref.5 is that thin sheets of similar shape with similarly shaped holes should fail under the same applied stress. When the hole takes the form of a crack, similitude is only achieved if the crack has a radius of curvature at its extremities proportional to its length. The fact that the sharpness of the crack at its ends is independent of its length must however introduce an element of dissimilarity to specimens of different size but otherwise similar in shape. Tests carried out on such specimens can therefore be expected to exhibit a 'scale effect' which may be different for different materials. For any particular material this scale effect is readily found by comparing the applied stress at failure for two similar sheet specimens (with corresponding cracks) of different size. If the scale effect is constant for any one material it follows that the failing stress for any other size of similar sheet (and crack) can be expressed by a simple formula. Such a formula was derived in Ref.5 where, by comparison with available experimental results, it was shown that the scale effect is indeed approximately constant for any one particular material. It was shown in Ref.5 that if

σ_1 = failing stress for crack length ℓ_1 in sheet 1

σ_2 = failing stress for crack length ℓ_2 in sheet 2

then σ = failing stress for any crack length ℓ

is given by the expression*

$$\frac{\sigma}{\sigma_1} = \frac{\sigma_2}{\sigma_1}\left(\frac{\log \ell/\ell_1}{\log \ell_2/\ell_1}\right) , \qquad (13)$$

on the assumption that the cracks are proportional to the linear dimensions of the sheets and that the sheets are all similar.

Using the energy approach after the manner of Griffith one derives the constant n in (12) by writing

$$\sigma_1^{\,2}\, \ell_1^{\,(2-n)} = \sigma_2^{\,2}\, \ell_2^{\,(2-n)} = \text{constant} , \qquad (14)$$

or

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{\ell_1}{\ell_2}\right)^{(1-n/2)} , \qquad (14a)$$

whence

$$(1 - n/2) = \frac{\log \sigma_2/\sigma_1}{\log \ell_1/\ell_2} . \qquad (15)$$

Thus for any crack length ℓ the corresponding failing stress σ is, by (12) and (15), given by the relation

$$\frac{\sigma}{\sigma_1} = \left(\frac{\ell_1}{\ell}\right)^{\left(\frac{\log \sigma_2/\sigma_1}{\log \ell_1/\ell_2}\right)} \qquad (16)$$

which, by taking the logs. of the r.h.s. of each, is readily seen to be identical with (13) above. [At values of ℓ smaller than that which makes $\sigma = \sigma_{ult}$ the failing stress cannot of course exceed σ_{ult}].

5 COMPARISON OF FAILING STRESSES OF SHEET MATERIAL OF DIFFERENT DEGREES OF DUCTILITY

Experiment shows that the index n in (12) normally lies between 0.9 and 2 for light alloy materials, whence it is clear that the strength of a brittle sheet material, for which n = 1, falls off more rapidly with increasing crack length than that of a ductile material that has a value of n greater than unity. In order to obtain the value of the index n for

* See equation (4) of Ref.5 where only the symbols are different.

certain typical materials one may refer to Table 3 of Ref.5 and to the following Table 1, which shows the results of experiments carried out by the Bristol Aircraft Co. on sheets of various light alloy materials.

TABLE 1

(16 s.w.g. sheet specimens, 30 in. × 10 in. and 90 in. × 30 in.)

Material	σ_{ult}	Crack length / Sheet width %	σ/σ_{ult} 10 in. sheet	σ/σ_{ult} 30 in. sheet	Stress reduction ratio	Average stress reduction ratio	Index n	Criterion (1 - n/2)
L 72	64,200	10	0.80	0.64	0.8	0.78	1.55	0.225
		15	0.71	0.56	0.79			
		20	0.64	0.5	0.78			
		25	0.57	0.44	0.77			
		30	0.51	0.39	0.77			
2024-T3 (U.S.)	68,200	10	0.65	0.58	0.89	0.82	1.64	0.18
		15	0.61	0.53	0.87			
		20	0.57	0.47	0.83			
		25	0.53	0.42	0.79			
		30	0.49	0.37	0.75			
D.T.D. 6463	73,400	10	0.70	0.45	0.64	0.56	0.95	0.525
		15	0.61	0.36	0.59			
		20	0.53	0.29	0.55			
		25	0.46	0.24	0.52			
		30	0.41	0.21	0.51			
L 73	65,500	10	0.75	0.5	0.67	0.61	1.11	0.445
		15	0.66	0.4	0.6			
		20	0.57	0.35	0.61			
		25	0.5	0.3	0.6			
		30	0.44	0.26	0.59			
2024 - T.36 (Brit.)	71,700	10	0.73	0.52	0.71	0.65	1.22	0.39
		15	0.65	0.43	0.66			
		20	0.58	0.37	0.64			
		25	0.52	0.32	0.62			
		30	0.46	0.28	0.61			
2014 - T.6 (U.S.)	68,600	10	0.71	0.5	0.7	0.66	1.25	0.375
		15	0.61	0.4	0.66			
		20	0.53	0.35	0.66			
		25	0.46	0.3	0.65			
		30	0.41	0.27	0.66			
D.T.D. 687	77,500	10	0.69	0.36	0.52	0.53	0.85	0.575
		15	0.57	0.30	0.53			
		20	0.48	0.25	0.52			
		25	0.39	0.21	0.54			
		30	0.33	0.18	0.54			
7075-T6 (Brit.)	76,200	10	0.69	0.40	0.58	0.55	0.91	0.545
		15	0.59	0.32	0.54			
		20	0.49	0.27	0.55			
		25	0.42	0.23	0.55			
		30	0.37	0.20	0.54			
2024-T3 (Brit.)	69,900	10	0.71	0.54	0.76	0.68	1.2	0.4
		15	0.65	0.45	0.69			
		20	0.59	0.39	0.66			
		25	0.53	0.35	0.66			
		30	0.48	0.30	0.63			
D.T.D. 746	64,700	15	0.71	20 in. width: 0.54	0.76	0.8	1.356	0.322
		20	0.56	20 in. width: 0.43	0.77			
		25	0.47	20 in. width: 0.37	0.78			
		30	0.41	20 in. width: 0.33	0.8			
		35	0.37	20 in. width: 0.31	0.83			
		40	0.33	20 in. width: 0.28	0.84			

The number $(2 - n)/2$ shown in the last column of the table can be taken as a criterion of the importance of scale effect on crack propagation. It is unfortunate that its value cannot be quantitatively related as yet to the physical properties of the material, but can only be found experimentally. It is seen from the table that, for example, material 2024-T3 (American specification) is a little less subject to scale effect than D.T.D.746 and both are a good deal less subject than 7075-T6 and D.T.D.687.

From the designer's point of view the need for tests is not a serious handicap. What he wants to be assured of is that the value of the index n, once determined by reference to a simple test on a couple of similar sheets of different size - e.g. linear dimensions in ratio 3:1 - can be regarded as fixed for the particular material. Recent experiments go some way towards providing that assurance as the results next discussed show.

6 CONSTANCY OF THE INDEX n FOR ANY ONE MATERIAL IN FLAT SHEET FORM

The value 1.356 of the index n given above for D.T.D.746 was derived by comparing corresponding crack lengths and failing stresses for two sheets, one 20 in. × 10 in. and the other 40 in. × 20 in. (Ref.5). It is interesting therefore to see whether, by using this value of n, the behaviour of a large sheet of the same material* can be reliably estimated.

Recent experiments at R.A.E.** on a sheet 84 in. × 48 in. showed that the failing stress for a crack 15 in. long was 16,000 lb/in^2. Taking the size ratio in relation to the 20 in. × 10 in. sheet as 48/10 = 4.8 (since the slight discrepancy in the length is of little account), we see that a corresponding crack in the smaller sheet has a length 15/4.8 of 10 in. i.e. 3.12 in., for which, by Table 3 of Ref.5 the failing stress is 26,000 lb/in^2. The corresponding failing stress for the larger sheet is therefore given by

$$\sigma = 26{,}000 \times \left(\frac{1}{4.8}\right)^{(1 - n/2)} \qquad (17)$$

$$= 15{,}500 \text{ lb/in}^2$$

for the value of n (= 1.356) quoted above.
This agrees well with the 16,000 lb/in^2 obtained by direct experiment.

It may be noted here that if, as Wells has suggested in an unpublished work, $\sigma \ell^{1/2}$ is constant for any one material, i.e. if the index n in (12) has unit value for all materials, the value of σ would be given by $26{,}000/\sqrt{4.8} = 11{,}900$ lb/in^2, - a serious underestimate.

7 DEDUCTIONS TO BE MADE FROM POINTS ABOVE CONSIDERED REGARDING CRACK PROPAGATION IN FLAT SHEET

(a) The main deduction to be made from the points already treated is that, for any one sheet material, the drop in failing stress consequent on increasing the length ℓ of a crack in a certain ratio in an infinite expanse

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* The material was D.T.D.546 which however is identical in properties to D.T.D.746 material but is not made to the same close limits of thickness.

** Not yet published.

of sheet - or increasing the size (and crack length) of a finite sheet in the same ratio while preserving similarity - can be calculated from formula (12), i.e. $\sigma \ell^{(1 - n/2)}$ = constant. Where n can never exceed 2 and normally lies between 0.9 and 2 but is constant for any one material.

Increasing the length of cracks in an infinite expanse of sheet has approximately the same effect as increasing the size of cracks in a finite sheet so long as the crack lengths are small compared with the sheet dimensions and so affect the stresses at the edges of the sheet in line with the crack to a negligible degree.

(b) The corresponding formula obtained by Griffith for completely elastic but brittle materials like glass makes n equal to unity in the above formula and so becomes

$$\sigma \ell^{1/2} = \text{constant} .$$

Wells has suggested that the same formula is applicable to other materials such as the various light alloys so long as for each material the appropriate constant is used. While this is very nearly true for certain materials such as 7075-T6 and D.T.D.6463, it is far from true for other materials such as 2024-T3 and D.T.D.746, whose crack resistance is much less subject to scale effect. A further point to note is that, although the above formula is true for glass and very nearly true for the light alloy 7075-T6, the value of the constant, dependent as it is on the surface tension in the former and on plastic flow in the latter, is of a different order of magnitude in the two cases, that for the alloy being about 6000 times greater than that for glass.

(c) From the designer's point of view what needs to be known about each kind of sheet material is

(i) The index n that determines scale effect

(ii) The failing stresses at a series of crack lengths in a standard sheet of fairly large size, say 4 ft wide by 8 ft long.

To obtain (i) a comparison must be made between the measured failing stresses of two similar sheets (with similar crack lengths) as in Tables 1, 2, and 3 of Ref.5, the linear size ratio of the two sheets being fairly large - 3:1 say. A convenient way of doing this is to draw a curve of failing stress against crack length covering the whole range of crack lengths for each of the two sheets. The larger of the two sheets should have the size demanded in (ii) above, so that the data there required is gathered at the same time.

A check on the stress ratios of the two sheets (which should be approximately the same for all similar crack lengths) is provided by comparing them with the stress ratios for pairs of cracks of 3:1 length ratios in the (preferably) larger of the two sheets, so long as such cracks are small compared with the sheet width.

With this data to hand, the critical stress corresponding to any length of crack - or the critical crack length for any failing stress - for any width of sheet (so long as the length is not less than about twice the width) can at once be estimated.

It is to be particularly noted that the ℓ.h.s. of (12) is only constant for crack lengths that are the same fraction of the sheet width (assuming that the length is always at least twice the width). The exception is the case where the cracks concerned are very small compared with the sheet width for this approaches the case of finite cracks in an infinite sheet. Under this condition the ℓ.h.s. of (12) has the same value for all crack lengths.

8 CRITICAL CRACK LENGTHS FOR PRESSURISED CYLINDERS

Before the experimental work of Peters and Kuhn[6] in 1957 it had been implicitly assumed that the relation between failing stress and critical crack length is the same whether the sheet is flat or whether it is wrapped up into a cylinder and the tensile stress induced by internal pressure. This assumption they found, after testing a large number of cylinders, to be wholly unjustified. For example, a cylinder of radius 14.4 in. with a two-inch crack was found to fail at about 2/3 of the failing stress of the corresponding flat sheet, while a cylinder of 3.6 in. radius with the same length of crack failed at only 1/3 of the failing stress of its corresponding flat sheet. An explanation of the cause of this reduction in stress - considered obscure by Peters and Kuhn - was put forward in Ref.5, where also the empirical formula, which they evolved for converting flat sheet failing stresses into their cylindrical sheet counterparts, was rationalised.

The reason for the reduction in strength suffered by the sheet when wrapped up into a cylinder and pressurised is that not only have the ordinary hoop forces to be by-passed round the ends of the crack - to that extent the conditions are no different from those obtaining in the flat sheet under the same tensile forces - but, owing to the radial pressure still maintained at and in the immediate neighbourhood of the crack edges, the extra membrane forces thus developed have also to be by-passed. If a close-fitting rigid frictionless sleeve is imagined to surround the pressurised cylinder so that, when the crack is made, further local radial expansion cannot occur near its edges, the stress concentration at the crack ends will be identical with those in a flat sheet under equal tensile stresses.

On the removal of such a sleeve the cylindrical surface will tend to bulge near the crack under the internal pressure and, because of the thinness of the sheet in relation to the radius of the cylinder this is necessarily resisted by membrane rather than bending stresses in the sheet. Such stresses cannot be developed in the circumferential direction because of the proximity of the free edges of the crack. Over the middle region of the crack pressure is resisted partly by longitudinal tension in the bulged sheet and partly by rate of change of bending shear. In the vicinity of the crack ends, however, local bulging is negligibly small and resistance due to bending of the sheet may be ruled out. The only remaining way in which the skin can resist the internal pressure is by membrane tension in a direction oblique to the crack, so taking advantage of the natural curvature of the cylinder in that direction, small though that may be compared with the curvature in the circumferential direction. It is these heavy tensions in the neighbourhood of the crack ends that cause the increased stress concentration at those points.

The formula for the resultant stress concentration, deduced empirical by Peters and Kuhn and rationalised in Ref.5 has the form

$$\sigma_{cyl.} = \sigma_{f.s.}/(1 + k\ell/r) \tag{18}$$

where $\sigma_{f.s.}$ = failing stress for flat sheet,

ℓ = crack length

r = radius of cylinder

k = (non dimensional) constant, independent of the material.

Thus when r approaches infinity and ℓ is still finite the local pressure approaches zero, the cylinder becomes locally a flat sheet and there is no extra stress. Peters and Kuhn found that k had the same value for 2024-T3 and 7075-T6 material a result one would expect, since any peculiarities in the material is taken account of in the quantity $\sigma_{f.s.}$.

8.1 Apparently anomalous experimental results for glass

However, in order to check the constancy of the coefficient k, they checked it against the experimental results obtained by Griffith[1] on spherical glass bulbs of various diameters. They found that the stress at failure was unaffected by the diameter and concluded that the value of k for glass was zero. It was pointed out in Ref.5 that this conclusion is not justified, the factor responsible for the weakness of the cylinder in comparison with the corresponding flat sheet being in this case absent. What happens is that the local pressure around the crack is resisted by the curvature in the line of the crack as soon as the curvature normal to the crack becomes ineffective.

While this is a satisfactory enough explanation to account for the behaviour of the spherical bulbs it does not account for the fact that Griffith's results with glass tubes indicates that here again the failing stress seems to be independent of the diameter. The explanation this time becomes clear as soon as the wall thickness 0.02 in. of Griffith's tubes is compared with their diameter 0.6 in. Magnified to the size of a pressure cabin the wall thickness of the cabin would be over 2 in. thick. This means that the local pressure force round the $\frac{1}{4}$ in. crack at which the tubes were burst was transmitted by bending rather than membrane action, and therefore was not dependent on the diameter of the tube.

That the absence of such a constraint against local bulging as the close-fitting frictionless sleeve mentioned above is the cause of the greatly increased stress concentration at the crack ends gains confirmation from an experiment carried out by the Douglas Aircraft Co. described by Lock[8] et al in the following words "In one unpublished test by Douglas Aircraft Co., a crack in a cylindrical specimen was covered by a plexiglass sheet to prevent excessive bulging of the crack lips. Without the plexiglass radial support to the edges of the crack a specimen failed at 2247 cycles at a hoop stress of 9130 p.s.i., but with the support a similar specimen at the same stress took 39,875 cycles before failure. This gives further indication that the problem in a cylindrical structure under pressure is very different from a flat tension specimen containing a crack".

9 RECENT EXPERIMENTAL RESULTS AND THEIR RELATION TO FORMULAE (12) AND (18)

Certain experimental results* relating critical crack lengths to nominal applied stress have recently been obtained at R.A.E. for both flat sheets and cylinders in D.T.D.546 material (which, apart from closer tolerances, is identical with D.T.D.746).

These in conjunction with results obtained by the Bristol Aircraft Co.[8] over the period 1956-58 may now be used to check the validity of formulae (12) and (18) above, and also to confirm the theory that the added membrane stresses around the crack edges caused by the local pressure constitute the reason for the weakness of the cylindrical vis à vis the flat sheet.

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* Not so far published.

9.1 Stress at failure of large flat sheet deduced from test results for small sheet

It has already been shown in Section 6 above that the index n deduced from tests on 10 in. and 20 in. wide sheet can be used to give the failing stress for a much wider (48 in.) sheet with acceptable accuracy. The procedure followed may be summarised as follows:-

(i) Express crack length ℓ as a fraction of the sheet width L (in crack direction).

(ii) Read off the failing stress σ of the small (10 in.) basic sheet for the same fraction ℓ/L.

(iii) Use formula (12) to give the required failing stress for the larger sheet.

9.2 Failing stress for cylinders deduced from those for corresponding flat sheets

Before failing stresses for cylinders can be deduced from those for corresponding flat sheets, it is necessary to know the value of the constant k in formula (18). This is an empirical constant to be derived from direct experiment. By using calculated failing stresses for the flat sheets corresponding to the two sizes of cylinder used in their experiments, Peters and Kuhn[6] arrived at the value 4.6 for k. Here one set of experimental results will be used to derive the value of k and it will then be seen if this is consistent with other available results.

The larger of the two Bristol cylinders (D.T.D.746) was 144 in. in diameter, 54 in. long and 0.04 in. wall thickness. At a hoop stress of 13,000 lb/in^2 the critical crack length ℓ was 8.1 in. i.e. 8.1/54 or 0.15 of the length L. From Fig.1 (Ref.5) the failing stress for a similar crack in a 10 in. wide sheet is 45,000 lb/in^2. Using flat-sheet scale-effect as in equation (17) one finds the failing stress σ for a flat sheet 54 in. wide to be

$$= 45{,}000 \left(\frac{10}{54}\right)^{0.322} = 26{,}000 \text{ lb/in}^2 \quad . \qquad (19)$$

To obtain the corresponding stress for the same sheet in cylindrical form, we use formula (18) to give

$$\sigma_{cyl.} = \frac{26{,}000}{1 + (8.1/72)\,k} \quad . \qquad (20)$$

Equating this to the known failing stress 13,000 lb/in^2 for this cylinder gives

$$k = 9^* \quad . \qquad (21)$$

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* This is nearly twice the value (4.6) quoted by Peters and Kuhn[6]. The difference is due to the fact that the figures for flat sheet used by them were calculated and differ widely from the experimentally obtained values quoted in Table 1.of Ref.5. It is also noted that when the crack in a cylinder is greater in length than the cylinder radius as in the curves shown by Peters and Kuhn, the value of k is not likely to be the same as that for cracks that are small compared with the radius.

Having derived the value of the constant k we can now apply it to calculate the failing stress for the smaller Bristol cylinder which has the dimensions - length 48 in., diameter 44 in. sheet thickness 0.012 in., material D.T.D.546. It failed at a stress of 6900 lb/in^2 for a critical crack length of 6 in. i.e. $\frac{1}{8}$ of the cylinder length. Referring again to the basic 10 in. wide sheet we find from Fig.1 (Ref.5) that for a crack $\frac{1}{8}$ of the sheet width $\sigma = 48{,}000$ lb/in^2. Following steps (i), (ii) and (iii) of section 9.1 we find the failing stress for the larger sheet to be

$$\sigma = 48{,}000 \left(\frac{10}{48}\right)^{0.322} = 29{,}000 \text{ lb/in}^2 . \qquad (22)$$

From formula (18) the corresponding failing stress for the cylindrical sheet is therefore

$$\sigma_{cyl} = \frac{29{,}000}{1 + k\ell/r} = \frac{29{,}000}{1 + (9 \times 6/22)} = 8400 \text{ lb/in}^2 \qquad (23)$$

which compares with the measured stress of 6900 lb/in^2.

With the same value of k, formula (21) is next applied to the large (pressure-cabin size) R.A.E. cylinder whose dimensions are - length 48 in. diameter 144 in., sheet thickness 0.04*. At an applied stress of 16,000 lb/in^2 the critical crack length was found to be 6 in., i.e. $\frac{1}{8}$ of the cylinder length.

For this ratio of ℓ/L the failing stress for the 10 in. sheet as already found above is 48,000 lb/in^2, and the scaled-up value, by (22), is 29,000 lb/in^2. From formula (18) therefore

$$\sigma_{cyl} = \frac{\sigma_{f.s.}}{1 + 9\ (\ell/r)} = \frac{29{,}000}{1 + \left(9 \times \frac{6}{72}\right)} = 16{,}500 \text{ lb/in}^2 \qquad (24)$$

which is in good agreement with the stress of 16,000 found by experiment.

9.3 Added membrane stresses in cylindrical sheet caused by local pressure in region of crack edges

The radical change that takes place in the membrane-stress distribution around a crack in a flat sheet under tensile stress, when the same sheet is rolled up into a cylinder and the tensile stress is induced by internal pressure, is strikingly illustrated in Figs.2 and 3. These show photographs of brittle-lacquer patterns obtained in the course of the Bristol[8] experiments. Fig.2 shows the pattern obtained around a 6 in. crack in a flat sheet 20 in. (in crack direction) × 40 in. × 0.04 in. under an applied stress of 14,000 lb/in^2. Fig.3 shows the pattern around a crack 9 in. long in a cylinder 44 in. diameter 35 in. long with the same sheet thickness of 0.04 in. under a nominal hoop stress of 4500 lb/in^2. These dimensions are quoted only

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* Cylinders with sheet thicknesses varying between 0.028 in. and 0.064 in. were tested but without significant variation in critical crack length: in all cases the longitudinal stress was zero. (Subsequent experiments with full longitudinal tension present have shown that the critical crack length is longer by about 10% to 15% in the case of the 22 gauge sheet; for heavier gauges (not yet tested) the increase is likely to be less).

as incidental information because quantitative comparison of stress is here of only secondary interest to a comparison of the two stress distributions. These are seen at a glance to be radically different for the flat sheet and the cylinder. In the flat sheet the cracks in the lacquer appear only around the extremities of the slot, the intervening area of sheet being quite free from cracks. Also the directions of the cracks show that the tensile strain, and hence the membrane stress is nearly normal to the direction of the slot. In marked contrast to this the lacquer cracks in Fig.3 show every evidence of heavy stresses all along and in the immediate neighbourhood of the slot. The directions of the cracks in this region moreover show clearly that these stresses lie parallel to the slot at its edges but that, a little distance away from the edges, the stresses start by being parallel to the slot opposite its mid point and become increasingly normal to the slot as the slot extremities are approached. At these latter points the stresses are therefore in the same direction as those of the flat sheet. In the region beyond the slot extremities, and in line with the slot, the cracks that all lie parallel to the slot for the flat sheet are at right angles to this direction in the cylinder. This suggests that the membrane stresses caused by the local pressure have swamped the flat sheet component of the stress field. This is not surprising in the light of formula (18) which for $k = 9$, $\ell = 9$ in. and $r = 22$ in., gives the failing stress for the cylinder as

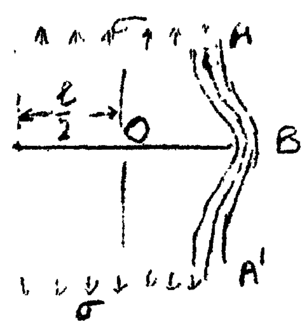

Fig.4

$$\sigma_{cyl} = \sigma_{f.s.}/(1 + k\ell/r)$$

$$= \sigma_{f.s.}/(1 + 9 \times 9/22)$$

$$= \frac{\sigma_{f.s.}}{4.7} = 0.21\ \sigma_{f.s.} \qquad (25)$$

This means that the stress concentration for the cylinder is 1/0.21 or about 5 times that in the flat sheet.

The drastic change above noted in the direction of the principal tensile stress can only be attributed to the heavy longitudinal tensile stresses developed each side of, and parallel to, the slot. In the flat sheet case the only stress along the edge of the slot is a compressive stress equal in magnitude to the applied tensile stress σ. This helps to counteract the inward pull (towards the centre O of the slot) caused by the curved tension lines ABA', the remaining resistance coming from tension in the sheet to the right of B (Fig.4). It is only to be expected therefore that when the usual compression stress along OB is changed into a tension of much greater absolute value the sheet to the right of B is subjected to heavy tension in the line of the slot. This accounts for the new direction of the lacquer cracks in that region.

10 DESIGNING FOR IMPROVED CRACK RESISTANCE IN PRESSURE VESSELS SUCH AS PRESSURE-CABINS

The very considerable reduction in the critical crack length of a pressurised cylindrical sheet below that for the corresponding flat sheet under the same tensile stress is well demonstrated by the recent R.A.E. tests on a large flat sheet and on a sheet of the same width converted into a pressure cylinder without longitudinal tension. For the same tensile stress the critical crack length dropped from 15 in. to 6 in. Another way of presenting this fact is to say that, if there were no drop in stress for a given crack length as a result of converting from flat sheet to cylindrical form, a stress of 28,000 lb/in^2 would be needed for fast propagation of a 6 in. crack, whereas in point of fact the critical stress is only 16,000 lb/in.

It has already been pointed out that the reason for this reduction is the local pressure at and in the immediate neighbourhood of the crack edges. Near the crack ends this pressure has to be equilibrated by membrane action in the sheet because the bending action of the sheet is here very small. Also, because the membrane pull acts obliquely along lines of small curvature, its magnitude is much enhanced. The result is a much increased stress concentration at the crack ends.

One way of avoiding this situation is to provide the necessary bending stiffness that the sheet by itself lacks. Stringers can provide this bending stiffness if suitably located in relation to the crack. The greater the pitch of the stringers the less support they can give to the sheet adjoining the crack edges. The position would clearly be much improved if stringers were, so to speak, continuous circumferentially, in other words if stringers as such were replaced by thin corrugated sheet of approximately equal weight. Failing this, the pitch and gauge thickness of stringers might well be reduced without increasing their overall weight.

That closing up the pitch of stringers improves the crack-resisting properties of the sheet is clearly demonstrated by the Bristol experiments[9] on cylindrical test specimens representing typical pressure-cabin construction. Two of these specimens 12 ft in diameter and 12 ft long were reinforced with frames at 20 in. pitch and stringers at varying pitches, while the third was unreinforced and only 54 in. long. The sheet material was to D.T.D.746. The figures given in the following Table are extracted from the Bristol results.

TABLE 2

Stringer pitch	Nominal hoop stress	Critical crack length
4.4 in.	12,800 lb/in^2	14.15 in.
5.05 in.	13,000 lb/in^2	12.3
7.7	12,800 lb/in^2	11
7.9	13,000 lb/in^2	10.16 in.

Comparing the first and last rows and neglecting the very slight differences in stress, we see that a 55% reduction in stringer pitch from 7.9 in. to 4.4 in., enables a 37% increase in critical crack length to be achieved at the stated stress.

11 STRINGER PITCH IN RELATION TO PITCH OF CIRCUMFERENTIAL FRAMES OR BANDS

With conventional frames at 20 in. pitch the improved crack resistance shown in Table 2 above consequent on closing up the pitch of stringers is worth consideration by the designer. If, however, circumferential bands at 10 in. pitch are used, as has been suggested by the writer[9], it is clear from the table that, at the stated hoop stress, - a figure generally accepted in practice - there is no point in closing up the stringer pitch. This follows from the fact, experimentally demonstrated[10], that in the presence of such bands a crack can never reach a critical length when stringers are used at the conventional pitch of about 6 in. and the nominal hoop stress is in the region of 15,000 lb/in^2.

12 SUMMARY OF MAIN POINTS BROUGHT OUT ABOVE AND CONCLUDING REMARKS

From the foregoing remarks the following points emerge.

(i) The formula connecting critical crack length with applied stress for an elastic but brittle material (such as glass) established by Griffith can with a slight modification be made applicable to ordinary light-alloy sheet material.

The well known Griffith formula states that, for any one material,

$$\sigma \ell^{1/2} = \text{constant} \quad , \qquad (26)$$

where σ is the applied stress and ℓ the critical crack length. The modified formula has the same form except that the index 1/2 is replaced by $(1 - n/2)$ in which n is constant for any one material and normally lies between 1 and 2 but must be less than 2. Thus we may write

$$\sigma \ell^{(1 - n/2)} = \text{constant} \quad . \qquad (27)$$

The value of the constant in the latter formula for metals turns out to be of a different order of magnitude from that in the former for glass, being some 6000 times greater.

(ii) The modified formula has been shown to be identical with that deduced in Ref.5 on the basis of attributing to a scale effect the difference in the failing stresses of sheets of different size but similar in shape and crack length. As suggested in Ref.5, the scale effect is due to lack of similarity in the shape of the crack at its extreme ends, which shape instead of being dimensionally similar is dimensionally identical.

(iii) Before use can be made of the modified formula the index n must be determined by experiment on two similar sheets of different size, a linear ratio of at least 3:1 being advisable. A curve of failing stress versus crack length is drawn for each sheet and the average value of n obtained by comparing different crack lengths. For any one material this index is then known once and for all.

(iv) The modified formula enables one to calculate the failing stress corresponding to any crack length in any size of sheet or conversely the critical crack length for any applied stress. For this purpose the curve of σ against ℓ obtained in (iii) above for the larger of the two sheets is preferably used. Incidentally the linear size of a sheet is nearly enough settled by its width (in the crack direction) alone, so long as the length is at least twice the width.

(v) In comparing the failing stress of a flat sheet with that of the corresponding cylinder, the empirical formula suggested by Peters and Kuhn[7] and rationalised in Ref.5 may be used. It takes the form

$$\sigma_{cyl} = \sigma_{f.s.}/(1 + k\ell/r) \tag{28}$$

where $\sigma_{f.s.}$ = failing stress for the flat sheet,

ℓ = crack length,

r = cylinder radius,

k = constant for all materials and non-dimensional.

The value of k can only be determined reliably by experiment, and in practice need only be considered for cases in which the crack length is not greater than the radius. The value of k derived from recent experiments described above appears to be about 9.

(vi) The crack properties of reinforced flat sheets and cylinders depend on those of the corresponding unreinforced components. To obtain the critical crack length ℓ_o for a given plain flat sheet of width L_o (and length not less than twice the width) under any specified applied stress σ_o we need to refer to the curve relating applied stress σ_1 to critical crack length ℓ_1 for one of the basic sheets of width L_1 mentioned in (iii) above. Formula (27) enables us to write, for similar sheets marked O and 1,

$$\left(\frac{\ell_o}{\ell_1}\right)^{(1 - n/2)} = \frac{\sigma_1}{\sigma_o} , \tag{29}$$

but, as we can only compare similar crack lengths in similar sheets we must write

$$\frac{\ell_o}{\ell_1} = \frac{L_o}{L_1} . \tag{30}$$

From (29) therefore

$$\frac{\sigma_1}{\sigma_o} = \left(\frac{L_o}{L_1}\right)^{(1 - n/2)} \tag{31}$$

which gives the value of σ_1 in terms of known quantities. The critical crack length ℓ_1 corresponding to σ_1 for the basic sheet is now read off from the above-mentioned curve and equation (30) then gives the required crack length ℓ_o.

To obtain the critical crack length ℓ_c for a cylinder of length L_c and radius r subjected to a specified nominal hoop stress σ_c we first consider the corresponding flat sheet of width L_c and, by (31) write

$$\sigma_1 = \sigma_o \left(\frac{L_o}{L_1}\right)^{(1-n/2)} \qquad (32)$$

where $L_o = L_c$. But from (28)

$$\sigma_c = \sigma_o/(1 + k\ell_o/r) \quad , \qquad (33)$$

where

$$\ell_o = \ell_c = \ell_1 \frac{L_c}{L_1} = \text{required crack length.} \qquad (34)$$

Substituting in (32) the value of σ_o given by (33), we obtain the relation

$$\frac{\sigma_1}{\sigma_c\left(1 + \frac{k\ \ell_1 L_c}{r\ L_1}\right)} = \left(\frac{L_o}{L_1}\right)^{(1-n/2)} \qquad (35)$$

or

$$\left(\frac{\sigma_1}{1 + A\ell_1}\right) = B \qquad (36)$$

where

$$A = \frac{k}{r} \cdot \frac{L_c}{L_1} = \text{known constant}$$

and

$$B = \sigma_c \left(\frac{L_o}{L_1}\right)^{(1-n/2)} = \text{known constant.} \qquad (37)$$

From the curve giving σ_1 against ℓ_1 the values of these two quantities that satisfy (36) are readily found by inspection. The critical crack length for the cylinder ℓ_c is then given by (34).

If the critical crack length is specified and the applied stress for the cylinder required the approach is more direct. For ℓ_1 is then obtained directly from (30) on substituting ℓ_c, L_c for ℓ_o, L_o. The stress σ_1 corresponding to crack ℓ_1 is read off the appropriate curve and, on substitution in (31) gives σ_o for the flat sheet L_o. Equation (28) on substitution of σ_o for $\sigma_{f.s.}$ and ℓ_c for ℓ then gives the required applied stress for the cylinder.

(vii) The Bristol experiments[8] show that, for a full size (12 ft dia) pressure cabin with conventional frame spacing (20 in.), a reduction of stringer pitch is a potent factor in increasing the crack resistance of the skin. As Table 1 above shows a 55% pitch reduction results in a 37% increase in critical crack length. As pointed out above, however, this benefit is not worth striving for on the designer's part if the circumferential bands that have for sometime been advocated by the writer [9,10] are used. Such bands at 10 in. pitch preclude the possibility of a crack ever reaching a critical length under the normal hoop stresses used in practice.

(viii) It should be pointed out that the numerical values of the index n for the three materials mentioned in Section 5 are derived from the results of experiments that were not specifically designed for the purpose. It is therefore highly desirable that properly designed experiments should be carried out to check these values and to correct them if necessary. It is equally desirable that the values of the index should be obtained for other light alloy sheet materials and also for steel.

The present paper does not, nor is it intended to, throw a new light on the mechanism of crack propagation in sheet material. More will doubtless be learnt about this as time goes on but it is in the highest degree unlikely that the added knowledge will ever enable the crack-propagation properties of a sheet material to be quantitatively deduced from the mechanical properties of the material alone.

In the case of a perfectly elastic brittle material it is possible to calculate, as Griffith did, both the energy released and the energy absorbed as a result of introducing a crack in the sheet, and hence, for any given crack length, to obtain the stress at which the release and absorption rates are equal. But for conventional structural materials - light alloy or steel - some degree of plastic deformation with, in consequence, a non-linear relation between stress and strain, must always supervene before fracture and this precludes accurate calculation.

In these circumstances it seems reasonable to adopt a semi-empirical approach, such as that described here, by which for each sheet material the crack-propagation properties of flat sheet and pressurised cylinders in that material can be derived from a single basic experiment.

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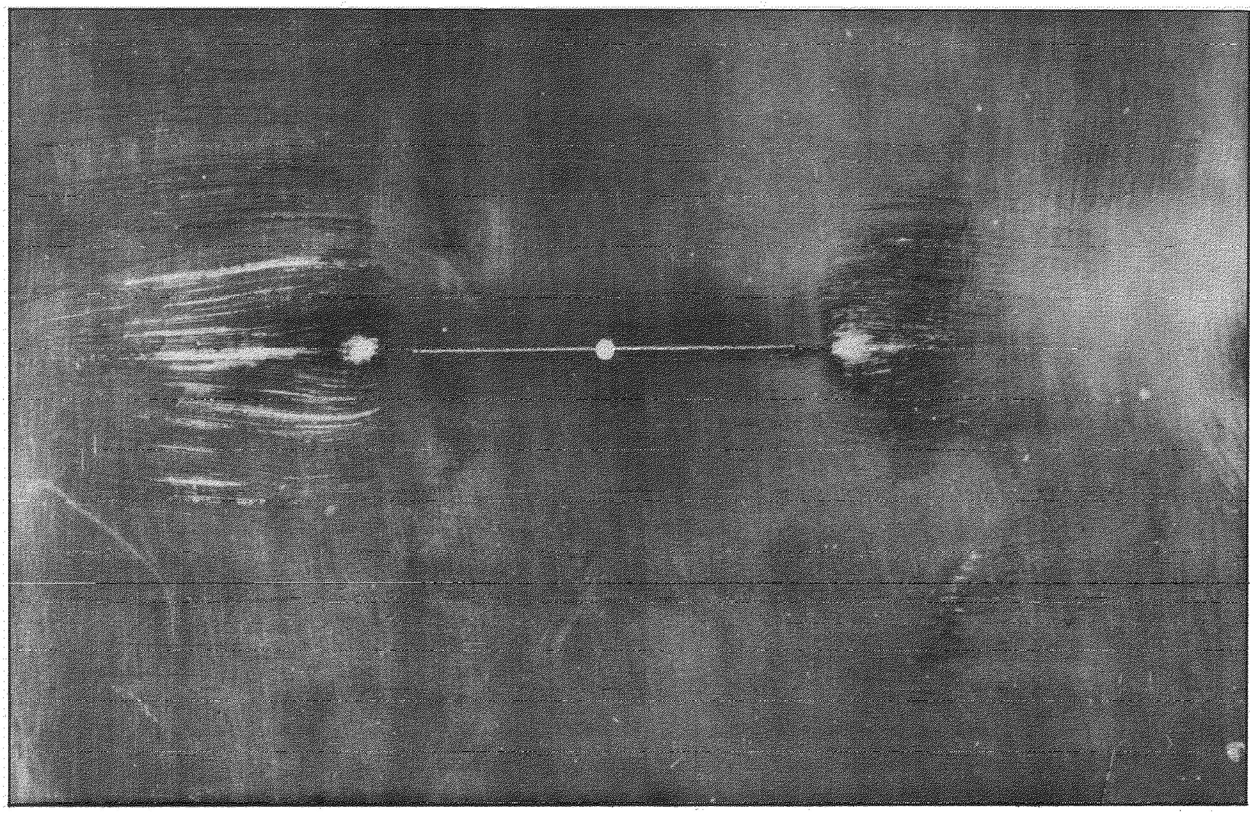

FIG. 2. TEST No.32 - BRITTLE LACQUER CRACK PATTERN AROUND 6" LONG SLOT IN A COUPON SPECIMEN

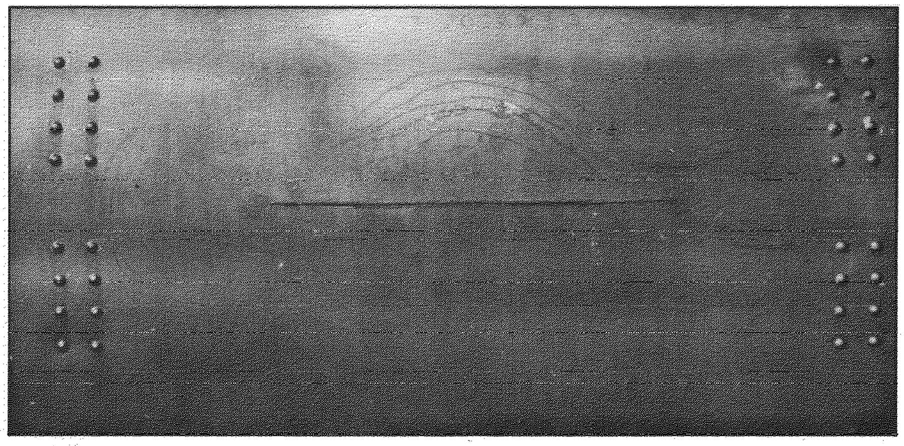

FIG. 3. TEST No.29. BRITTLE LACQUER CRACK PATTERN AROUND SLOT IN PRESSURISED CYLINDER

A.R.C. C.P. No.564

539.219.2 :
620.191.33 :
669.715-415

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