C.P. No. 557 C.P. No. 557

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Design and Operation of the N.G.T.E. Thermal Shock Analogue

By

C. G. Stanworth and D. S. C. Paine

LONDON: HER MAJESTY'S STATIONERY OFFICE

1959

Price 3s 6d net

U.D.C. No. 539.319:530.17

C.P. No.557

Design and operation of the N.G.T.E. thermal shock analogue

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SUMMARY

A description is given of the N.G.T.E. thermal shock analogue which is suitable for estimating the temperature in a turbine blade section as a function of position and time when the blade is subjected to a step change in gas temperature. The method of operating the analogue and obtaining results has also been described.

The limitations of the analogue have been stated, but they are considered a small penalty in view of the essential simplicity of the design.

Previously issued as N.G.T.E. Memo. No. M.334 - A.R.C.21,912

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1.0 Introduction

Aerodynamic design usually dictates that turbine blade sections have a long thin trailing edge with a comparatively thick centre section. It is then clear that if the temperature of the gas stream, in which the blade is immersed, is suddenly altered due to change in engine operation such as starting, acceleration, deceleration, or flame-out, an uneven and transient temperature distribution will be set up in the section, with the result that high thermal stresses may occur in the material.

A preliminary step in the estimation of the magnitude of thermal stresses in turbine blades is therefore the determination of temperature distributions in blade sections at different times after the application of a sudden change in temperature or thermal shock at the blade boundary. It is to estimate such temperature distributions that the N.G.T.E. thermal shock analogue has been built.

2.0 Theoretical basis

In order to find temperature distributions in turbine blades it is necessary to solve the heat conduction equation in two or three dimensions. The boundary conditions usually associated with such a problem make an exact solution impossible to obtain. In this paper it is assumed that conduction in the spanwise direction can be neglected so that the problem becomes two-dimensional, i.e. the blade is considered to have an infinitely long span and the temperature distribution in any section perpendicular to the span will be the same as in any parallel section at a given instant.

One method[1] of tackling this problem is by a numerical step-by-step process which involves approximating the blade section by a mesh of squares (Figure 1) and considering the heat balance for each square at successive intervals of time. This is effectively using the two-dimensional heat conduction equation in finite difference form.

In the analogue this step-by-step mesh is represented by an electrical network (Figure 2) in which the thermal conductivity and the temperature of the material are simulated by electrical conductance elements and voltages respectively. The capacity of condensers is equivalent to the heat storage capacity of the material. These analogies can be made clearer by setting up and comparing the equations governing the flows of heat and electricity as follows:-

For the thermal case consider a chordwise section of the turbine blade of unit thickness and, in a manner similar to that described above, let this section be approximated by perfectly conducting square prisms of side δx and unit thickness, separated by infinitely thin resistive material such that the thermal resistance between adjacent prisms is the same as between points δx apart in the actual material. Thus, in this simplified thermal problem, heat flow is perpendicular to the sides of the prisms and heat is stored uniformly throughout each prism. By hypothesis no heat flows through the ends of the prisms, since this would be in the spanwise direction. Now consider one of these square prisms, a cross-sectional view of which is shown in Figure 3(a). The numbers on the figure are suffices used to indicate that particular station.

If there is a small change δT_0 in the temperature of the prism, the change of heat storage (with notation as given in Appendix I)

$$= \rho K_m \, . \, (\delta x^2 \, . \, 1) \, . \, \delta T_0 \qquad \dots \quad \dots \quad \dots \qquad (1)$$

In time δt, the heat flow inwards through the faces 1, 2, 3 and 4 of the prism will be

$$= k \cdot \delta t \cdot (\delta x \cdot 1) \sum_{i=1}^{4} \frac{1}{\delta x} (T_i - T_o) \quad \ldots \quad \ldots \quad \ldots \qquad (2)$$

By conservation of heat these quantities must be equal. Hence

$$\frac{\rho K_m \delta x^2}{k} = \frac{1}{\delta T_o/\delta t} \sum_{i=1}^{4} (T_i - T_o) \quad \ldots \quad \ldots \quad \ldots \qquad (3)$$

For the electrical case consider the flow of electricity in a network of equal resistors with junction points connected to earth through equal capacitors (Figure 2) and, in particular, the small portion of this network which is shown in Figure 3(b). Again the numbers are suffices referring to their particular stations.

The change in charge of the condenser C for a change δV_o in applied voltage

$$= C \delta V_o \quad \ldots \quad \ldots \quad \ldots \qquad (4)$$

The resultant charge through the point 0 in time δt

$$= \frac{1}{R} \cdot \delta t \cdot \sum_{i=1}^{4} (V_i - V_o) \quad \ldots \quad \ldots \quad \ldots \qquad (5)$$

and by conservation of charge these quantities must be equal. Hence,

$$C \cdot \delta V_o = \frac{1}{R} \cdot \delta t \cdot \sum_{i=1}^{4} (V_i - V_o)$$

$$\text{or} \quad CR = \frac{1}{\delta V_o/\delta t} \sum_{i=1}^{4} (V_i - V_o) \quad \ldots \quad \ldots \quad \ldots \qquad (6)$$

On comparison of Equations (3) and (6) it is clear that there is an exact analogy between voltage and temperature if

$$\frac{\rho K_m \delta x^2}{k} = CR \quad \ldots \quad \ldots \quad \ldots \qquad (7)$$

If, more generally,

$$\frac{\rho K_m \delta x^2}{k} = \mu CR \quad \ldots \quad \ldots \quad \ldots \qquad (8)$$

where μ is a constant, then

$$\frac{1}{\delta T_o/\delta t} \sum_{i=1}^{4} (T_i - T_o) = \frac{1}{\delta V_o/\delta(\mu t)} \sum_{i=1}^{4} (V_i - V_o) \quad \ldots \quad \ldots \qquad (9)$$

and there is merely a difference of time scales between the thermal problem and its electrical analogue.

Having set up the electrical network so that it is analogous to the thermal properties of the blade material, it is now necessary to simulate the heat transfer coefficient between the blade surface and the gas. This is done by connecting all the edge points of the network to a common conductor through resistances whose values are determined by the desired heat transfer coefficient at the corresponding points on the blade surface. This common conductor is held at a voltage simulating the desired gas temperature.

The equations governing heat flow across the boundary between gas and blade and the equations governing the corresponding flow of electricity may be set up as follows:-

Consider again one of the square prisms, such as shown in Figure 4(a) where one of its neighbouring prisms has been replaced by the gas, g, so that PQ is the boundary between blade and gas.

Then, as before, for a small change δT_o in the temperature of the prism the change in heat storage is given by Equation (1), and the inward heat flow to the prism in time δt is given by

$$k\,\delta t \sum_{i=1}^{3} (T_i - T_o) + h\,\delta x\,\delta t\,(T_g - T_o) \qquad \dots\ \dots\ \dots \quad (10)$$

Expressions (1) and (10) must be equal

$$\therefore \quad \frac{\rho K_m\,\delta x^2}{k} = \frac{\sum_{i=1}^{3} (T_i - T_o)}{\delta T_o/\delta t} + \frac{h\,\delta x\,(T_g - T_o)}{k\ \ \delta T_o/\delta t} \qquad \dots\ \dots \quad (11)$$

For the corresponding electrical case consider the equivalent portion of the network shown in Figure 4(b).

The resultant flow of charge through the point O in time δt

$$= \frac{1}{R}\,\delta t \sum_{i=1}^{3} (V_i - V_o) + \frac{1}{R'}\ \delta t (V_g - V_o) \qquad \dots \quad (12)$$

which, by conservation of charge, must be equal to the change in charge on the condenser C given by Equation (4). Thus

$$\mu CR = \frac{\sum_{i=1}^{3} (V_i - V_o)}{\delta V_o/\delta(\mu t)} + \frac{R}{R'}\ \frac{(V_g - V_o)}{\delta V_o/\delta(\mu t)} \qquad \dots\ \dots \quad (13)$$

But by Equation (8)

$$\frac{\rho K_m\,\delta x^2}{k} = \mu CR$$

and if we choose $R' = \frac{Rk}{h\delta x}$ $\qquad \dots\ \dots\ \dots \quad (14)$

the analogy between temperature and voltage holds at the boundary with the same relationship between time scales as before. In practice R' is a variable resistance which can be preset over a wide range.

In the above equations it has been assumed that the properties of the blade material and the boundary heat transfer coefficient are independent of temperature. Any attempt to modify the analogue to take account of variable material properties would greatly increase its complexity.

3.0 Construction

An analogue has been constructed consisting of a uniform electrical network containing 600 junction points. The connection of the four resistors and one capacitor at each of these junction points is made via a 7-pin socket, a B7G miniature valve base. Four of the pins are connected to the resistors, one to the capacitor, the remaining two being left unconnected. Each junction point is ineffective until the appropriate shorting plug is inserted. In this way it is possible to 'peg out' the shape of the blade on a socket panel containing all the miniature valve bases. Figure 5 shows the general arrangement.

The 600 network points are arranged in 20 sections of five by six points, which are connected together by wander plugs and sockets. By means of this arrangement it is possible to alter the position of each of the sections relative to the others so that greater flexibility may be achieved in obtaining a wide range of blade shapes.

One hundred and fifty shorting plugs are provided to peg out the boundary of the blade section. These are connected via variable resistors to a common conductor which receives a step change of voltage from a dry cell battery (normally 50 volts) to simulate a step change in temperature at the boundary of the blade. The leads from these shorting plugs are stored above and below the socket panel, and the control knobs for the variable resistors are situated under perspex covers at the base of the socket panel.

A Wheatstone bridge circuit has been built into the analogue to enable the boundary resistors, which are 2.5 MΩ potentiometers, to be set to the required values. The bridge consists of two stable resistances of 100 KΩ and a variable calibrated resistance of 2.5 MΩ. The fourth arm is occupied by the resistance being set. A 10 μ A centre-zero galvanometer is used to indicate the bridge balance.

Component values have been chosen so that as wide a range of blade shapes and conditions as possible may be investigated and so that pick-up of stray magnetic fields may be avoided. The equal resistors between the junction points of the network are 10 KΩ (±1 per cent) and the capacitors, which were selected to have high leakage resistance, are 1 μ F (±1 per cent). Thus the value of CR is 0.01 and when values of the blade material properties are known the time scale factor μ can easily be found from Equation (8).

Monitoring of the varying voltages at the junction points of the network is carried out by connecting the relevant shorting plugs to a 12-channel galvanometer recorder. The galvanometers in use have a sensitivity of 2 μ A/cm, and resistances of 4.7 MΩ are connected in series with the galvanometer inputs to give the desired voltage sensitivity. The currents in the leads to the galvanometer recorder cause a small error in the voltages being measured. The error is greatest when the measurements are being made in a region adjacent to a small number of boundary points, such as at the extreme edges of a thin blade profile. The error at the leading and trailing edges of a typical turbine blade simulated on the analogue was about 1 per cent of the step change in temperature. To keep this error to a minimum the 12 junction points at which the voltages are being measured should be as widely spaced throughout the blade section as possible.

As an addition to the original conception of the analogue a further 100 boundary resistors, leads and shorting plugs are provided. These are arranged in ten groups of ten at the top of the analogue (Figure 5). The common conductors for each group may be connected to separate sources of potential, thus providing the ability to simulate different gas flow temperatures over several boundaries as in internally cooled turbine blades. In this case the steady state temperature distributions before and after a transient are non-uniform and these may similarly be obtained by measuring the steady state voltages on the analogue network.

4.0 Method of operating

The planning of an exercise to be carried out on the thermal shock analogue involves firstly fitting the analogue mesh to the given blade section so that as close an approximation as possible may be achieved by having the maximum density of mesh points. The side length δx of the square prisms by which the blade section is approximated can then be calculated and the blade shape pegged out on the socket panel. Having selected the blade material density, specific heat and thermal conductivity, and knowing the distribution of heat transfer coefficient round the blade, the values of the boundary resistances can be calculated and the variable resistors set accordingly. Suitable adjustments must be made to the value of the boundary resistances where the blade profile makes an angle approaching 45° with the horizontal or vertical because in these regions the boundary points of the network simulate larger portions of the blade profile than δx. For later use the time scale factor μ can also be calculated. An example of the above calculations is given for a specific blade in Appendix II.

The next operations are to apply a step change in voltage to the common conductor which is connected to the boundary resistors, and to record photographically, by means of the 12-channel galvanometer recorder, the varying voltages at the junction points of the network. Records are taken from shortly before the step change is applied until the end of the period under investigation. Another (short) recording is made after an interval of about 30 seconds by which time the voltages have become sensibly steady. A typical set of traces is shown in Figure 6; the superimposed time scale is "thermal" time. To avoid confusion some of the traces are made to approach the bottom edge of the film and the remainder the top edge. This sequence of operations is usually carried out several times since only 12 junction points of the network can be monitored by the galvanometer recorder at the same time.

The last step is to convert voltages recorded as traces to temperatures. The manner in which this is carried out is made clear by referring to Figure 7. The step change takes place at D and the blade temperature has become steady at H. (The steady state temperature over the blade section is assumed to be uniform.) The temperature is required at the point G, 3 seconds, say, after the step change. The line ABC (produced by the recorder) is used as a datum from which all measurements are taken. The distances AD, BG and CH are measured and then distances EG and FH calculated. Since the blade temperature is supposed to have become steady at the point H, the distance FH represents the total change in temperature of the junction point under consideration. The distance GE represents that fraction of the total temperature change which has occurred after 3 seconds. Thus, if the total change in temperature is assumed to be from 1000°C to 0°C, the temperature after 3 seconds is given by

$$1000 - \left[\frac{EG}{FH} \times 1000\right]$$

$$= 1000 \left[1 - \frac{EG}{FH}\right]$$

When marking out the horizontal time scale on the recorded traces, the differing rates of progress of the thermal problem and its electrical analogue have to be remembered, and the scale factor μ, previously calculated, taken into consideration.

5.0 Performance

The performance of the analogue has been assessed by using it to solve a simple problem in linear heat flow to which there is a theoretical solution.

The problem chosen was the linear flow of heat between the parallel faces of an infinite slab having finite thickness. Thus the whole of the analogue network arranged 12 points along the ends by 50 points along the sides was used to represent a section through the slab at right angles to its parallel faces. An equivalent step change in temperature from 1000°C down to 0°C was applied to the boundary points along the ends of the analogue network. The sides represented perfect insulators.

The values of the constants employed in this exercise were chosen so that a suitable time scale factor and a convenient temperature drop at the centre of the slab could be obtained.

The temperatures recorded at the ends and centre of the network corresponding to the surfaces and centre of the slab 12 seconds after the application of the step change in temperature are shown in Table I.

	Left hand surface temperatures °C	Centre slab temperatures °C	Right hand surface temperatures °C
	448	635	458
	450	639	456
	456	645	450
	465	646	464
	455	643	461
	451	635	458
	462	655	468
	469	650	464
	460	641	450
	455	644	450
	459	652	456
	461	643	459
Theoretical	460	640	460

For this particular exercise temperatures recorded at junction points along lines parallel to the ends of the rectangle should be the same, but due to several sources of small error such as component tolerances and trace reading inaccuracies the analogue results naturally show some scatter. From the left hand surface temperatures it may be seen that the difference between the maximum and minimum temperatures is 21°C (i.e. 2 per cent of the step change in temperature) and the mean temperature is 458°C. At the centre of the slab the difference between maximum and minimum temperature is 20°C (i.e. 2 per cent of the step change in temperature) and the mean temperature is 644°C. Theoretical estimates^2 of the temperatures at the surface and centre of the slab are 460°C and 640°C respectively, which show close agreement with the analogue results. It is to be expected that correlation between analogue results and theoretical results would not be so close for sections with curved boundaries.

6.0 Limitations

The analogue has certain limitations, which could be overcome only by increasing its complexity. These limitations are as follows:-

(i) Heat transfer coefficient cannot vary with time.

(ii) Spanwise conduction of heat along the blade is neglected.

(iii) The specific heat and thermal conductivity of the blade material must be considered constant and independent of temperature over the range investigated.

(iv) Thermal expansion is neglected.

(v) The steady state temperature of the blade section is uniform for uncooled blades.

(vi) Only step changes in gas temperature can be applied.

(vii) Square mesh simulation of a continuous medium with smooth boundaries.

The relative importance of these limitations alters according to the type of problem under investigation but, in any circumstances, they are considered a small penalty if one keeps in mind the simplicity of the analogue, particularly the fact that the analogy is a precise one and that calibration is unnecessary.

7.0 Conclusions

A description is given of the N.G.T.E. thermal shock analogue which is suitable for estimating the temperature in a turbine blade section as a function of position and time when the blade is subjected to a step change in gas temperature. The method of operating the analogue and obtaining results has also been described.

The limitations of the analogue have been stated but they are considered a small penalty in view of the essential simplicity of the design.

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No.	Author(s)	Title, etc.
1	Max Jakob	Heat transfer. Vol. I Chapman and Hall. 1949.
2	M. Fishenden O. A. Saunders	An introduction to heat transfer. Oxford, 1950.

APPENDIX I

List of Symbols

C	capacity	farad
K_m	specific heat	C.h.u./lb °C
R	network resistance	ohm
R'	boundary resistance	ohm
T	temperature	°C
V	voltage	volt
h	heat transfer coefficient	C.h.u./sq.ft sec °C
k	thermal conductivity	C.h.u./ft sec °C
t	time	sec
δx	mesh spacing	ft
ρ	density	lb/cu.ft
μ	time scale factor	

Suffices

o, 1, 2, 3, 4	stations in the blade
g	gas
i	general station

APPENDIX II

Preliminary calculations for a given exercise

To clarify the method of operation given in Section 4.0, the preliminary calculations are carried out below for a given blade section.

The details of the blade section and material properties are assumed to be known and are as follows:-

Section: 2 in. chord, camber angle 90°, leading edge radius 0.0375 in., trailing edge radius 0.020 in., thickness chord ratio 12.5 per cent, circular arc boundaries.

Material properties: Density $\rho = 515$ lb/cu.ft, specific heat $K_m = 0.13$ C.h.u./lb °C, thermal conductivity $k = 0.00443$ C.h.u./ft sec °C, heat transfer coefficient $h = 0.04$ C.h.u./sq.ft sec °C (assumed constant round blade boundary in the present instance).

For a blade of this shape it can be seen that the best possible arrangement of the network is in rectangular form, 50 points along the sides by 12 points along the ends. Since 50 junction points will represent the 2 in. chord, the mesh spacing

$$\delta x = \frac{2}{50}\text{ in.} = 3.333 \times 10^{-3}\text{ ft}$$

The value of the boundary resistance is

$$R' = \frac{Rk}{h\delta x} = \frac{10^4 \times 0.00443}{0.04 \times 3.333 \times 10^{-3}} = 3.34 \times 10^5\,\Omega$$

This is the value of the boundary resistance where the blade boundary is parallel to one side of the mesh. Where the boundary crosses the mesh diagonally, the resistance should have the value, given by

$$\frac{1}{R'} = \frac{1}{Rk} \cdot h\delta s$$

where δs is the length of boundary associated with that particular point. For the whole profile

$$\Sigma \frac{1}{R'} = \frac{1}{Rk}\Sigma\, h\delta s$$

In this way it can be checked that the total boundary conductance is simulating correctly the heat transfer to the surface.

The value of the time scale factor, μ, is from Equation (8)

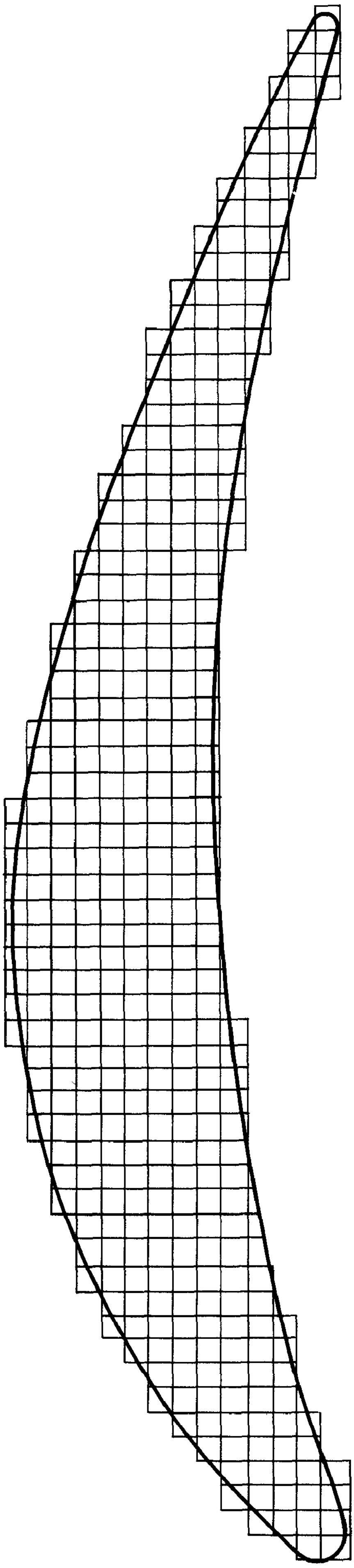
$$\mu = \frac{\rho K_m\,\delta x^2}{kCR} = \frac{515 \times 0.13 \times (3.333 \times 10^{-3})^2}{0.00443 \times 10^{-6} \times 10^4}$$

$$= 16.75$$

From the above calculations it can be seen that, if a second exercise were undertaken with the same blade shape but with alterations in the size and material properties of the blade, only a change in the time scale factor and the values of the boundary resistors would be involved.

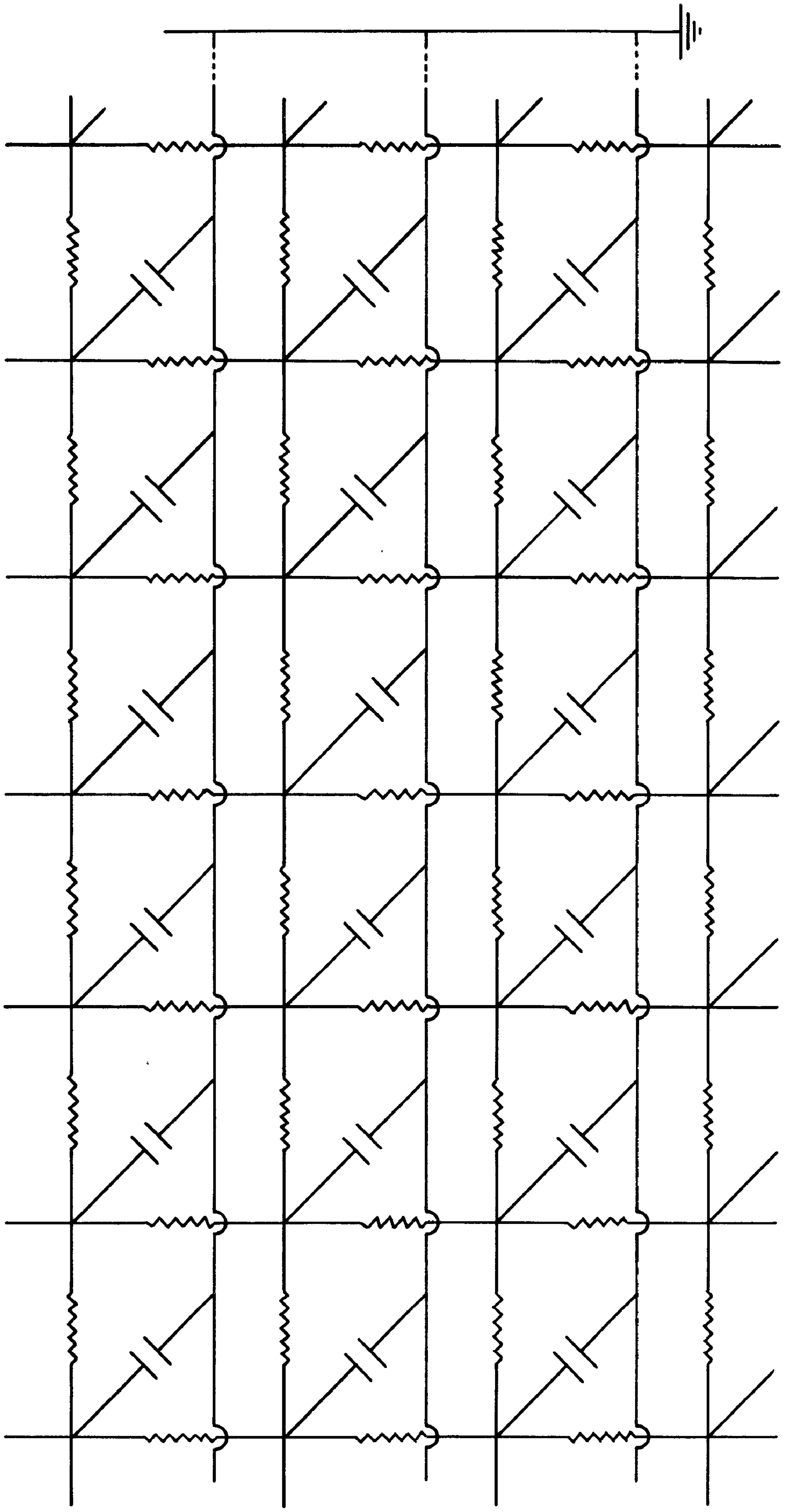
The operation continues with the application of a step change in voltage to the common conductor which is connected to the boundary resistors and then as in Section 4.0.

FIG. 1



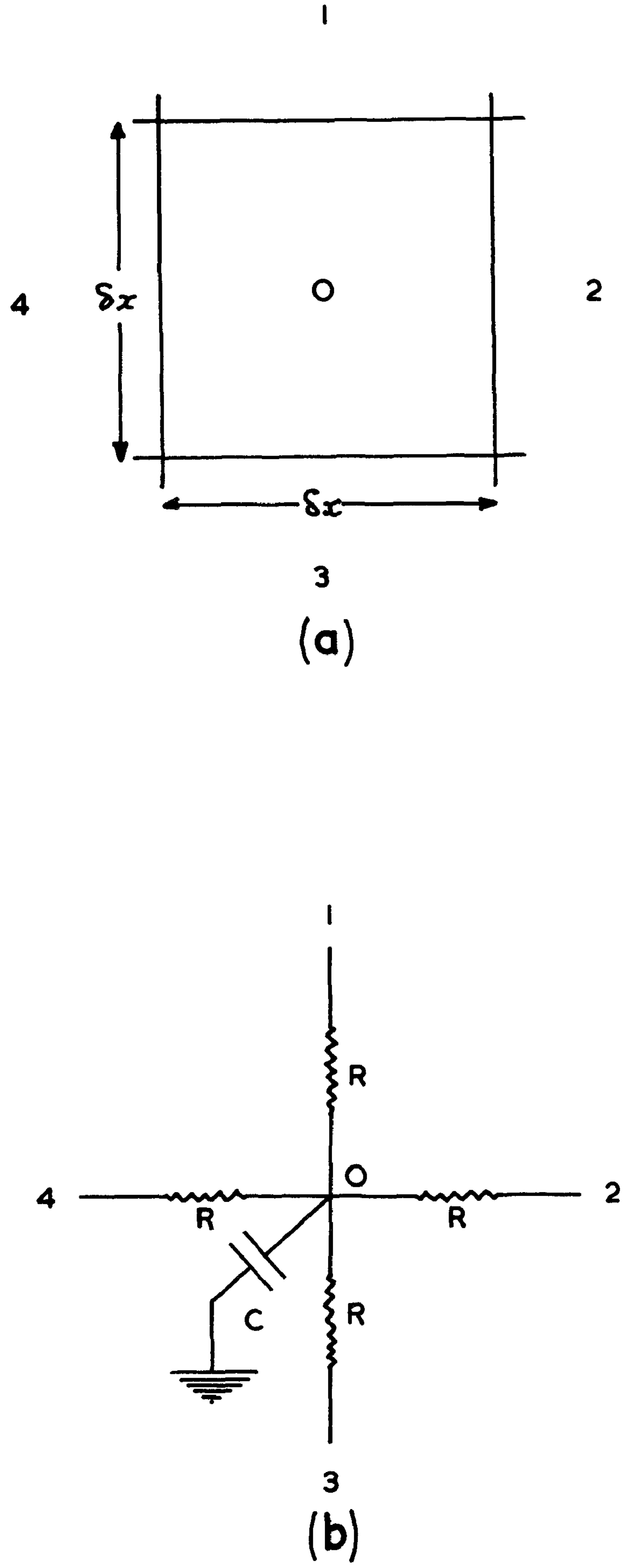
SQUARE MESH APPROXIMATING BLADE SHAPE.

FIG. 2



ELECTRICAL NETWORK IN THERMAL SHOCK ANALOGUE.

FIG 3

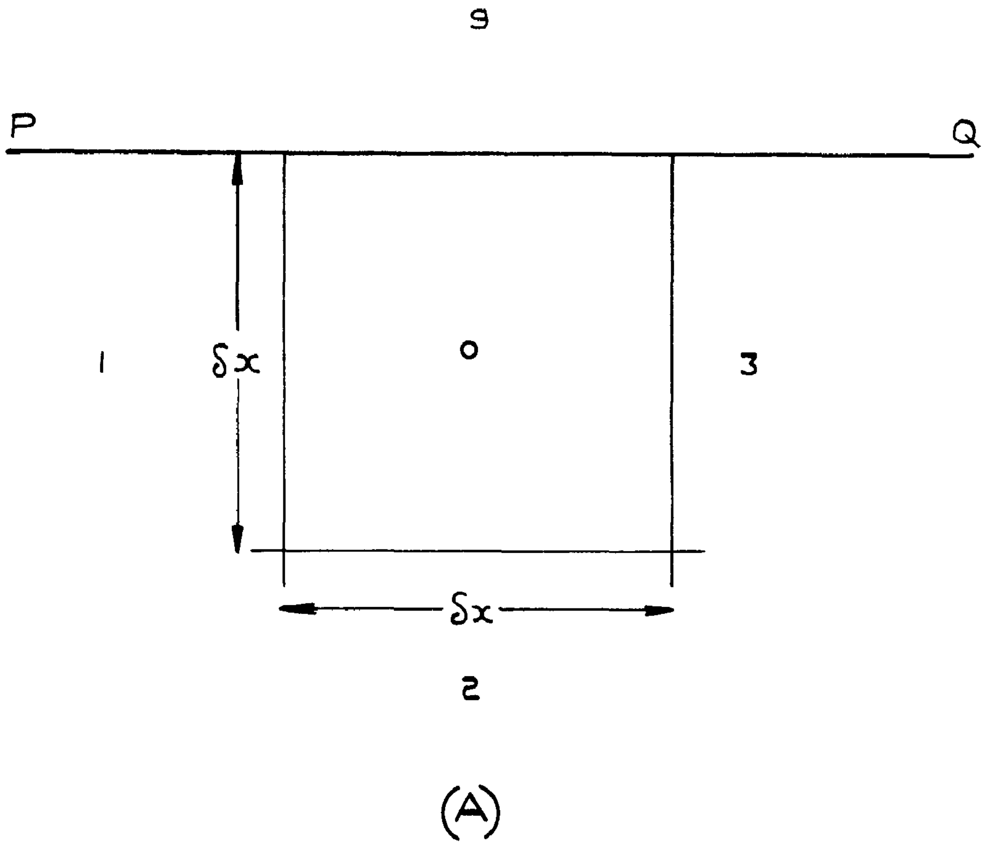


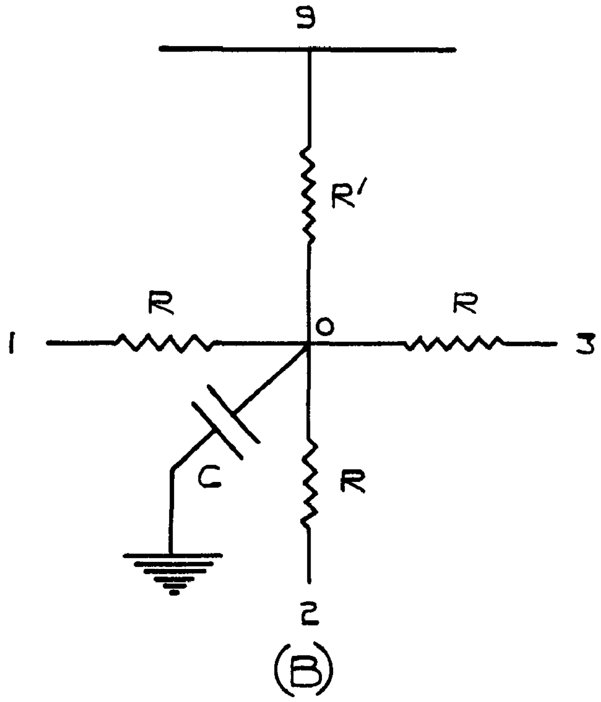
THE BASIC ANALOGUE

(a) THERMAL PROBLEM

(b) ELECTRICAL ANALOGUE

FIG. 4

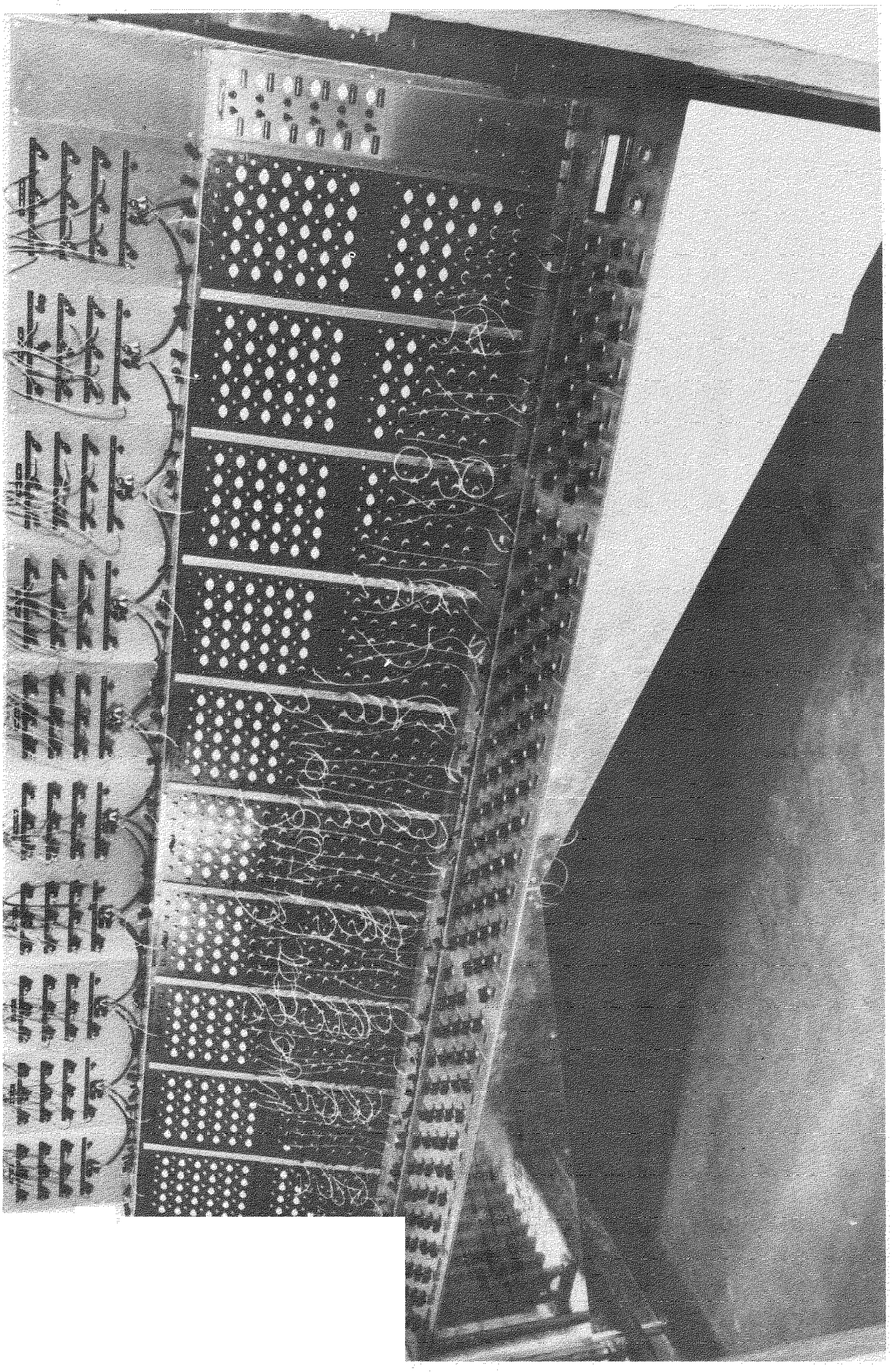




THE ANALOGUE BOUNDARY.

(A) THERMAL BOUNDARY.
(B) ELECTRICAL BOUNDARY.

FIG. 5.



THE N.G.T.E. THERMAL SHOCK

FIG. 6

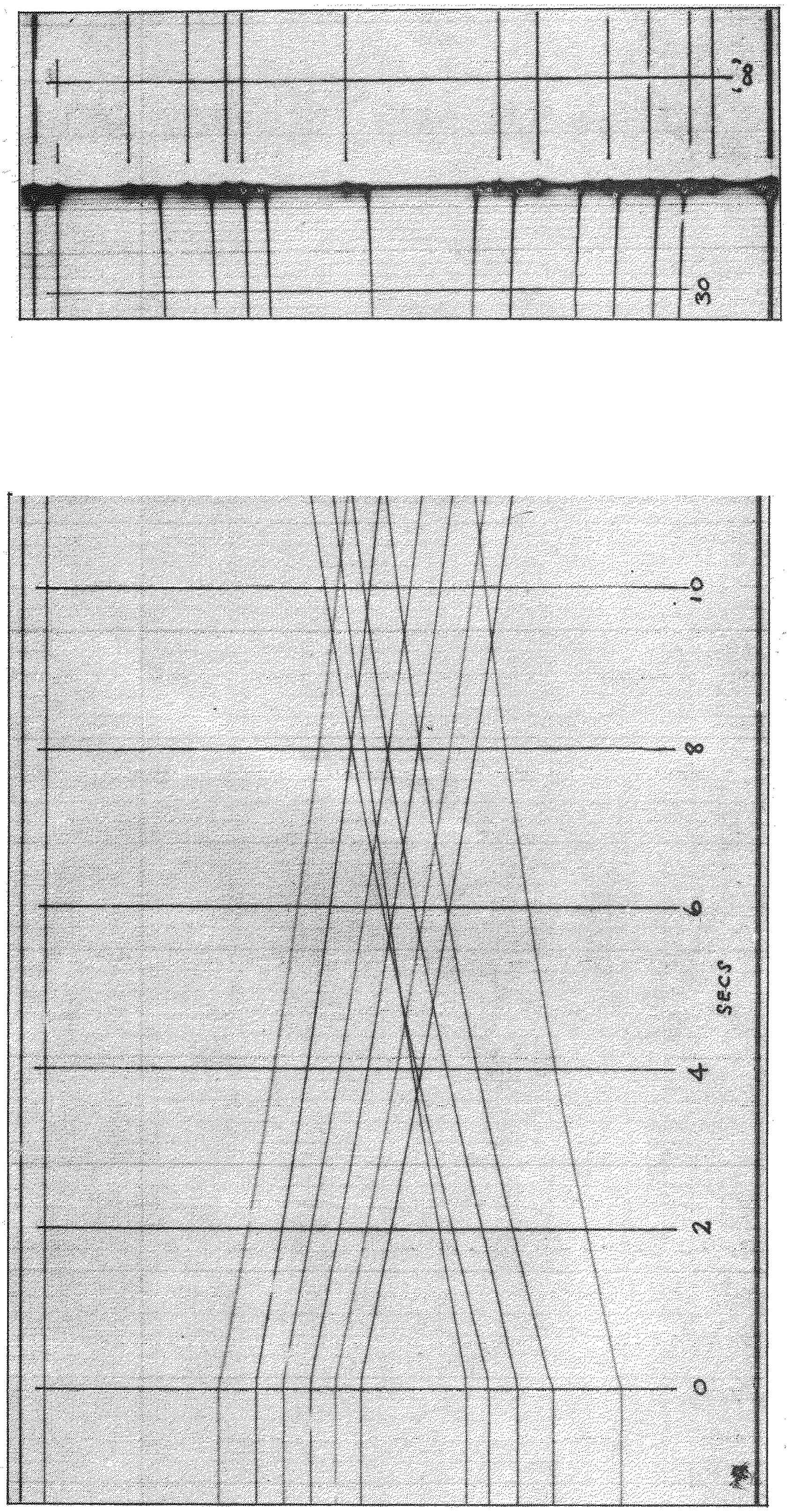


FIG. 7

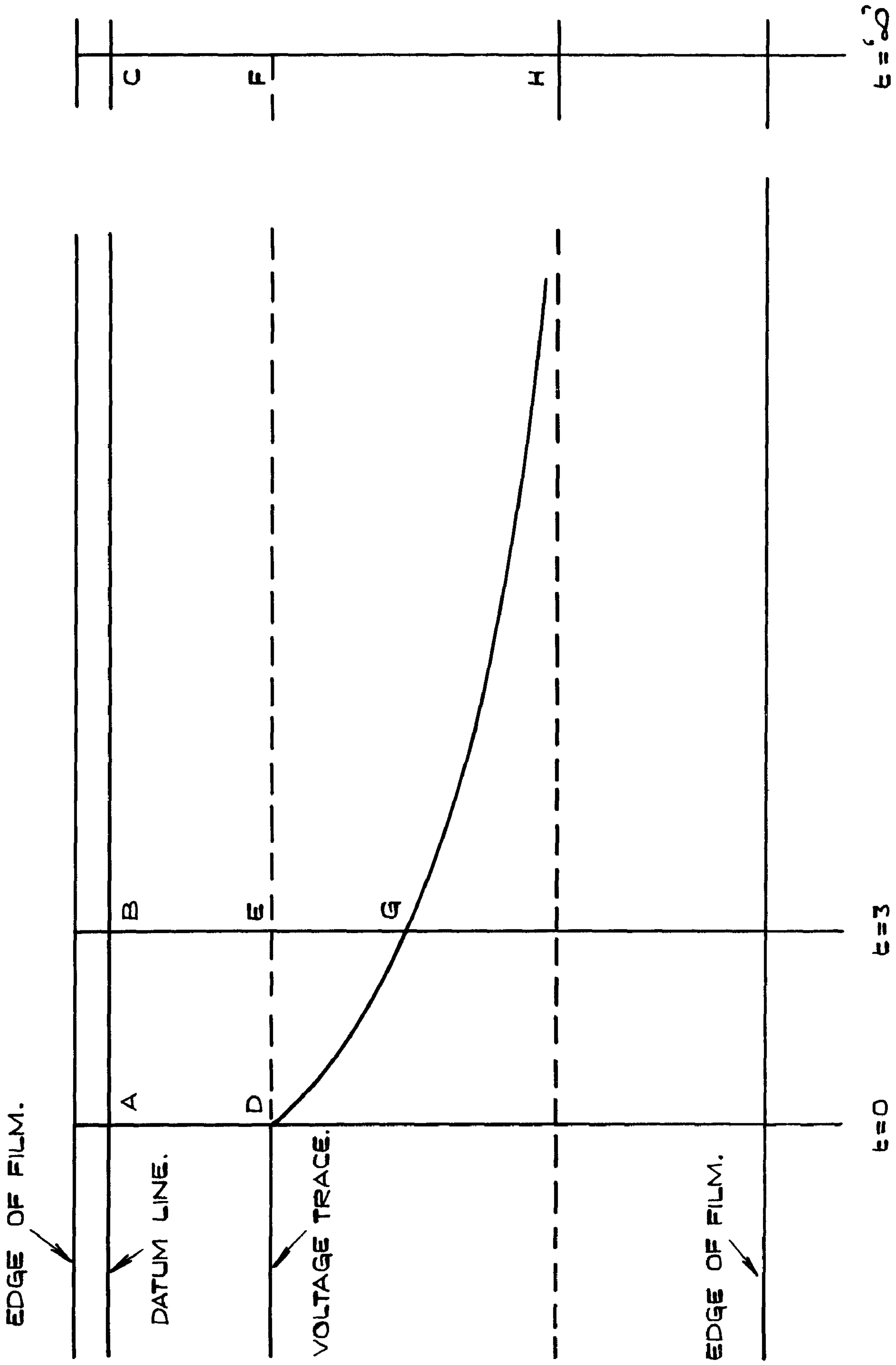


DIAGRAM FOR CONVERTING VOLTAGES RECORDED AS TRACES TO TEMPERATURES.

A.R.C. C.P. No. 557 539.319:530.17
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Printed and published by
HER MAJESTY'S STATIONERY OFFICE

To be purchased from
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Printed in England

S.O. Code No. 23-9012-57