

C.P. No. 549

C.P. No. 549



MINISTRY OF AVIATION  
AERONAUTICAL RESEARCH COUNCIL  
CURRENT PAPERS

A Force-Displacement Indicator for a Drag Balance

By

*R.E. Franklin*

*Oxford University Engineering Laboratory*

LONDON: HER MAJESTY'S STATIONERY OFFICE

1961

PRICE 2s 6d NET

## A Force-Displacement Indicator for a Drag Balance

- By -

R. E. Franklin,

Oxford University Engineering Laboratory

July, 1960Abstract

The paper describes a null-reading weighing element for a drag balance. Basically the device is a Kelvin Current Balance modified to be linear, to have adjustable sensitivity and to indicate displacement. The drag is measured by reducing the indicated displacement to zero.

1. Introduction

One of the problems currently under investigation at the Oxford University Engineering Laboratory is that of the measurement of skin-friction in turbulent boundary layers. The flow configuration chosen for this study is axial flow in an annular passage of circular cross-section and it is proposed to attempt to measure the skin-friction on the inner, convex surface of the annulus by the floating element technique.

The dimensions of the annulus are 6 in. O.D. and 4 in. I.D. and a diagrammatic sketch of the balance to be used is given in Fig. 1. The floating element is to be mounted on small, very flexible springs and measurement of the force required to push the element back to its central position will give the skin-friction.

Two measuring devices are required for this balance; a weighing device to return the floating element to its central position and to indicate the force required, and a sensing device to detect when the floating element is at its central position. Since the pressure in the tunnel will be below atmospheric pressure an electrical method is to be preferred for both measurements, for then sealing problems are not introduced. If electrical methods are to be used then both devices are best situated downstream of the test section, for the upstream settling length required to establish fully developed flow in the annulus being long, electrical leads to a device upstream of the test section would need to be very long if the flow were not to be disturbed.

The design requirements for the weighing device are that it should be capable of measuring forces from zero to 0.1 lb with an accuracy of 0.0001 lb, that it should be small enough to fit easily into a tube about 3 in. dia., and that the part attached to the floating element should be light. The allowable

movement/

movement of the floating element is to be about 0.005 in. and so the displacement device should be able to detect movements of about 0.0005 in. with ease if the element is to be centralised accurately. The displacement device should also be light and compact.

Several possible schemes for both devices were examined and finally, for reasons which will appear later, a type of instrument based on the Kelvin Current Balance was chosen. Weighing devices of this sort are in use on several wind tunnels in this country, but it has been the experience of the author that the literature dealing with their design is very limited. For this reason, and also because it is felt that in the present design some useful modifications have been made, this report presents an account of the design and calibration of the prototype of the design to be used in the annular tunnel.

## 2. Preliminary Considerations

The Kelvin Current Balance was originally proposed as a method of making absolute measurements of electric current. From the diagrammatic sketch of Fig. 2 it will be seen that the balance consists basically of two sets of three co-axial coils connected in series. In each set the two outer coils are fixed and the inner is one attached to the arm of the balance. The coils are wound in such a way that the centre coil is attracted to one or other of the fixed coils, the direction in which it moves depending upon the direction of the current. When deflected, the balance is returned to its equilibrium position by moving a weight out along the balance arm and the position of the weight for equilibrium gives the current.

An analysis which results in an expression for the force between two co-axial circular coils of radius  $r$  and  $R$  carrying currents  $i$  and  $I$  and separated by a distance  $x$  is possible but rather complicated<sup>1</sup>. If, however, one coil is assumed to be very small compared to the other, the analysis is very simple and results in the expression

$$F = \frac{3}{2} \cdot \eta_0 \cdot \frac{\pi R^2 r^2}{(R^2 + x^2)^{5/2}} \cdot x \cdot I \cdot i. \quad \dots(2.1)$$

We will take this as an approximate formula for the present situation. If we consider the case of three coils where the outer coils are the large coils and have  $N$  turns, the inner coil having  $n$ , we get the approximate result

$$F = \frac{3}{2} \cdot \eta_0 \cdot \frac{\pi R^2 r^2}{(R^2 + x^2)^{5/2}} \cdot x \cdot N \cdot I \cdot n. \quad \dots(2.2)$$

The maximum force occurs when  $x = \frac{1}{2}R$  and usually the centre coil is slightly smaller than the outer ones. If we put  $r = 0.6R$  then equation (2.2) becomes (with  $\eta_0 = 4\pi \cdot 10^{-7}$ )

$$F = \frac{2.75}{10^7} \cdot N \cdot I \cdot n. \quad \dots(2.3)$$

In the Kelvin Current Balance the coils are connected in series and so  $I = i$  and

$$F = \frac{2.75}{10^7} \cdot N \cdot n \cdot I^2. \quad \dots(2.4)$$

It is seen immediately that in this form the device is non-linear. This is an undesirable characteristic and in the present design has been overcome by providing separate sources for the inner and outer coils. Then by keeping the current  $I$  in the outer coils constant and varying the current  $i$  in the inner coil, the device becomes linear.

This modification has a further advantage. For a given value of  $I$  the device has a certain sensitivity  $di/dF$ . But the value of this sensitivity is controlled by the value chosen for  $I$ . Thus we have a method for varying the sensitivity. This is important because when measuring very light loads we can use a small value for  $I$  thus obtaining a high sensitivity, and when measuring heavy loads we may use a higher value of  $I$  and a reduced sensitivity. In this way accuracy may be maintained over the whole range of the device.

After it had been decided to separate the inner and outer coils in this way it was realised that the instrument was now not only capable of weighing but also of indicating displacement. The three coils will now behave as a differential transformer, for if an alternating voltage is applied to the centre coil the windings of the two outer coils are such that the voltages induced in them will be in opposite directions. If the central coil is exactly mid-way between the two outer coils, the voltages induced in them will cancel. However, when the centre coil is displaced the induced voltage in one of the outer coils increases and in the other it decreases. An error signal may therefore be detected which will only disappear when the centre coil is returned to the mid-position.

The fact that these preliminary considerations suggested that it might be possible to combine the weighing device and displacement indicator in one unit, together with the advantages that the combined unit would be linear and have an adjustable sensitivity led to a decision to construct the prototype described in the following sections.

### 3. Details of the Design

The first thing to be estimated is the number of ampere-turns required on the coils. This we may do using equation (2.2). The maximum force to be measured is 0.1 lb and so we have

$$0.1 = \frac{2.75}{10^7} N I i$$

or

$$N I i = \frac{10^6}{2.75} = 0.3635 \cdot 10^6. \quad \dots(3.1)$$

In the present design the inner and outer diameters and the thickness of the formers on which the coils are wound were restricted by the dimensions of the apparatus. Further, the centre coil had to be kept fairly light so that the load on the suspension of the floating element should not be too great, which suggested that the number of turns on the central coil should be made much lower than that on the outer coils. Finally it was thought that the current in the outer coils might be left running for some time whereas the centre coil would only be switched in when a measurement was to be made; the current in the outer coils should then be much smaller than the currents used in the centre coil.

After some thought, the following values were chosen

$$I = 0.5^A$$

$$N = 3000$$

Substitution of these values in (3.1) gives  $n = 48.7$ . Since the above analysis is only approximate it was decided to increase the number of turns on the centre coil to 150.

At this point in the design some consideration was given to the question of the temperature rise of the coils. The heating effect was important in view of the fact that cooling would be difficult and any heat conducted to the main body of the balance might cause distortion. It was thought that a better choice of wire gauge, current and number of turns might be made. An elementary, but nevertheless interesting result appeared.

The system is linear and so the force is proportional to the ampere-turns

$$F = kNI$$

$$\text{or } I = \frac{F}{kN}$$

Now the resistance of the coil is proportional to  $ND/d^2$  where  $D$  is the mean diameter of the coil and  $d$  the diameter of the wire. Thus the power developed in the coil is

$$I^2 R \propto \frac{F^2}{k^2 N^2} \frac{ND}{d^2} = \frac{F^2 D}{k^2 N d^2} \text{ Btu/min.}$$

If all the heat goes into raising the temperature of the wire, the heat required per degree temperature rise is proportional to the volume of wire

$$\therefore \text{Heat required} \propto N D d^2 \text{ Btu/}^\circ\text{F.}$$

Thus the rate of temperature rise is

$$\frac{dT}{dt} \propto \frac{F^2 D}{k^2 N^2 d^4} \text{ }^\circ\text{F/min.}$$

But in this design the cross-sectional area of the coils is limited. This area is

$$A \propto N d^2$$

$$\text{or } N \propto \frac{A}{d^2}$$

Thus we find

$$\frac{dT}{dt} \propto \frac{F^2}{k^2 d^4} \frac{d^4}{A^2} = \frac{F^2}{k^2 A^2} \text{ }^\circ\text{F/min}$$

and so if a certain force is to be developed and the area of the coil is restricted it is not possible to reduce the heating effect in design unless a different wire material is used.

If copper wire is to be used then the above calculation results in the expression

$$\frac{dT}{dt} = \frac{1.167}{10^6} \frac{F^2}{k^2 A^2} \text{ }^\circ\text{F/min}$$

where  $k$  is as defined above.

#### 4. Circuit Connections

When the prototype had been constructed it was first connected up as a D.C. Kelvin Current Balance, the movement of the centre coil being observed through a microscope. When it was clear that the weighing range was adequate the D.C. circuit was disconnected.

The prototype was next connected as a differential transformer in order to examine the displacement sensitivity. For these preliminary tests the centre coil was displaced using a depth micrometer. During the tests the values of the various elements were decided and the best operating frequency found.

When both these preliminary tests were completed and the prototype found to be satisfactory the two circuits were combined. After a few modifications the final circuit was as shown in Fig. 4.

#### 5. Calibration

For the present application the displacement sensitivity is as important as the weighing sensitivity. This is so because the floating element has to be returned accurately to its central position so that the leakage gaps on the upstream and downstream sides are equal. The minimum gap to be used has been set at 0.005 in. and so we need to be able to measure to at least 0.0005 in. The calibration runs were therefore divided into two sets; displacement calibration and weighing calibration. These will be described separately.

##### (i) Displacement sensitivity

To measure the position of the centre coil a voltage of 1V at 2000 cps is applied to the centre coil and the difference between the two voltages induced in the fixed coils is measured. Both voltages are measured with valve-voltmeters. Ideally, when the centre coil (referred to from now on as the primary) is mid-way between the two outer coils (to be referred to as the secondary) the A.C. voltage measured in the secondary circuit should be zero. Owing to stray effects it is not zero but of the order of 10 mV.

To calibrate for displacement sensitivity the primary coil was pushed from its central position by means of a horizontally mounted depth micrometer reading in thousandths of an inch.

The results of the calibration are given in Fig. 5 from which it will be seen that the secondary voltage is a minimum when the coil is in its (magnetic) central position, and increases as the coil is moved to either side, becoming approximately linear with a slope of 2.7 mV per thou.

Near the magnetic central position the sensitivity is low and so the primary coil is adjusted to be about 0.006 in. to one side. The sensitivity of 2.7 mV per thou. is then realised in one direction and it is possible to return the coil to its original position with an accuracy of about  $\pm 0.0001$  in. This is considered satisfactory.

##### (ii) Weighing sensitivity

These calibrations were carried out by attaching a very light thread to the spindle of the primary coil. The thread was then taken over a pulley wheel and a light scale pan attached to the end.

The operating procedure is as follows:

The switch is first put to the "set-zero" position and the thread taken off the spindle. There is then no load on the unit and the primary voltage is adjusted to 1V at 2000 cps and the zero secondary reading taken.

The thread is then attached to the spindle and a weight put in the pan. After moving the switch to the "weigh" position the rheostat in the primary circuit is adjusted until the A.C. secondary voltage is brought back to its zero reading. The ammeter in the primary circuit then gives the appropriate current.

The results of several calibrations for different values of secondary D.C. current are given in Fig. 6 for the range 0-10 gm. All the curves should pass through the same point for zero primary D.C. current; this point should give the weight of the scale pan which is 0.78 gm. The fact that they do not is due to friction in the pulley used for the calibration.

Calibrations for the range 0-40 gm. are given in Fig. 7.

## 6. Conclusions

The results presented in Figs. 5, 6 and 7 show that it is possible to build a combined force-displacement indicator based on the Kelvin Current Balance with satisfactory sensitivity for both measurements. The difficulties with the zero readings in Fig. 6 are due to the friction in the pulley and it is felt that with a pivot based on the crossed-springs principle the difficulty would disappear and it would be possible to weigh with an accuracy of 1%.

The modifications made in the present design open up several possibilities, the most important of which is that it might be possible, with further development, to make the device fully automatic. The error signal which appears in the secondary circuit when the load comes on might be used to operate a servo-mechanism which would adjust the rheostat in the primary until the error disappears.

## 7. Acknowledgements

This research has been made possible by a grant in aid from D.S.I.R.

The author also wishes to express his thanks to Prof. Thom, who first suggested this research topic and who has given advice and encouragement throughout, and also to K. R. McLachlan of Southampton University for several helpful discussions.

---

## References

1. Page and Adams "Principles of Electricity", Chapman and Hall, Ltd.
-

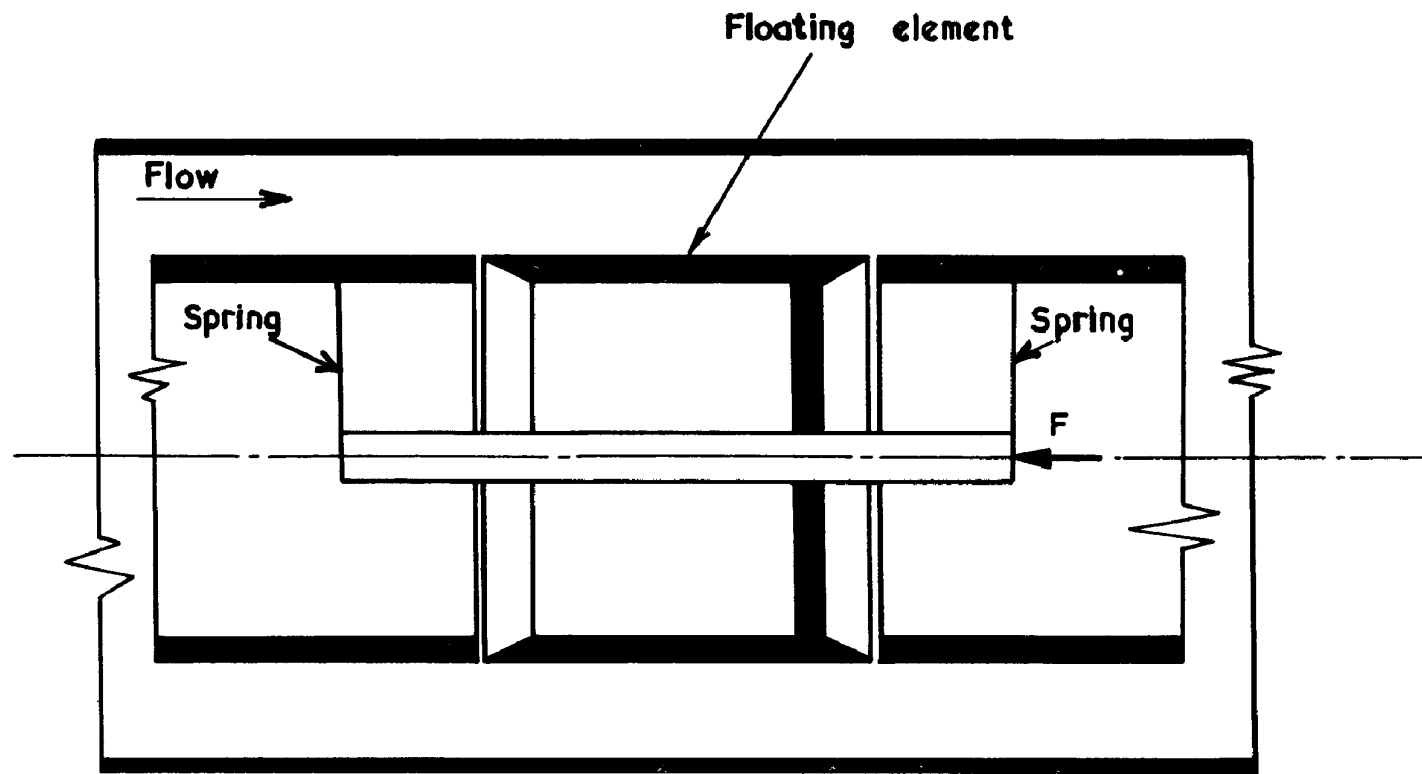


Diagram of skin - friction drag balance

FIG.1.



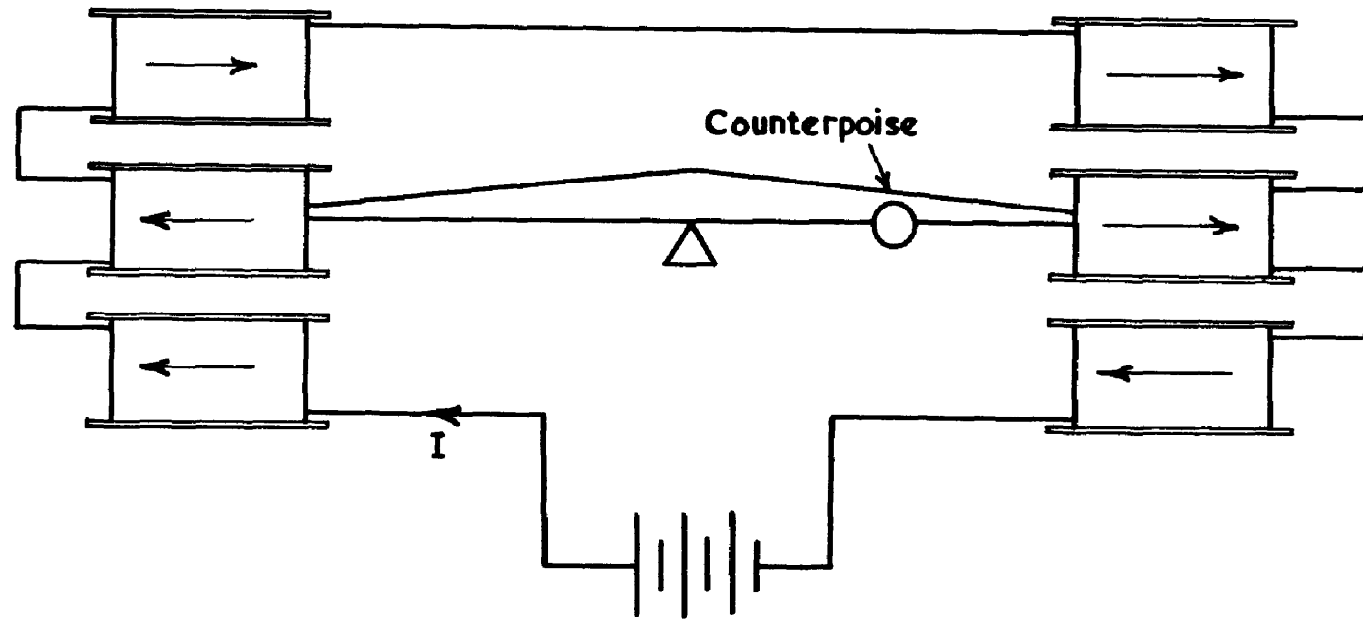
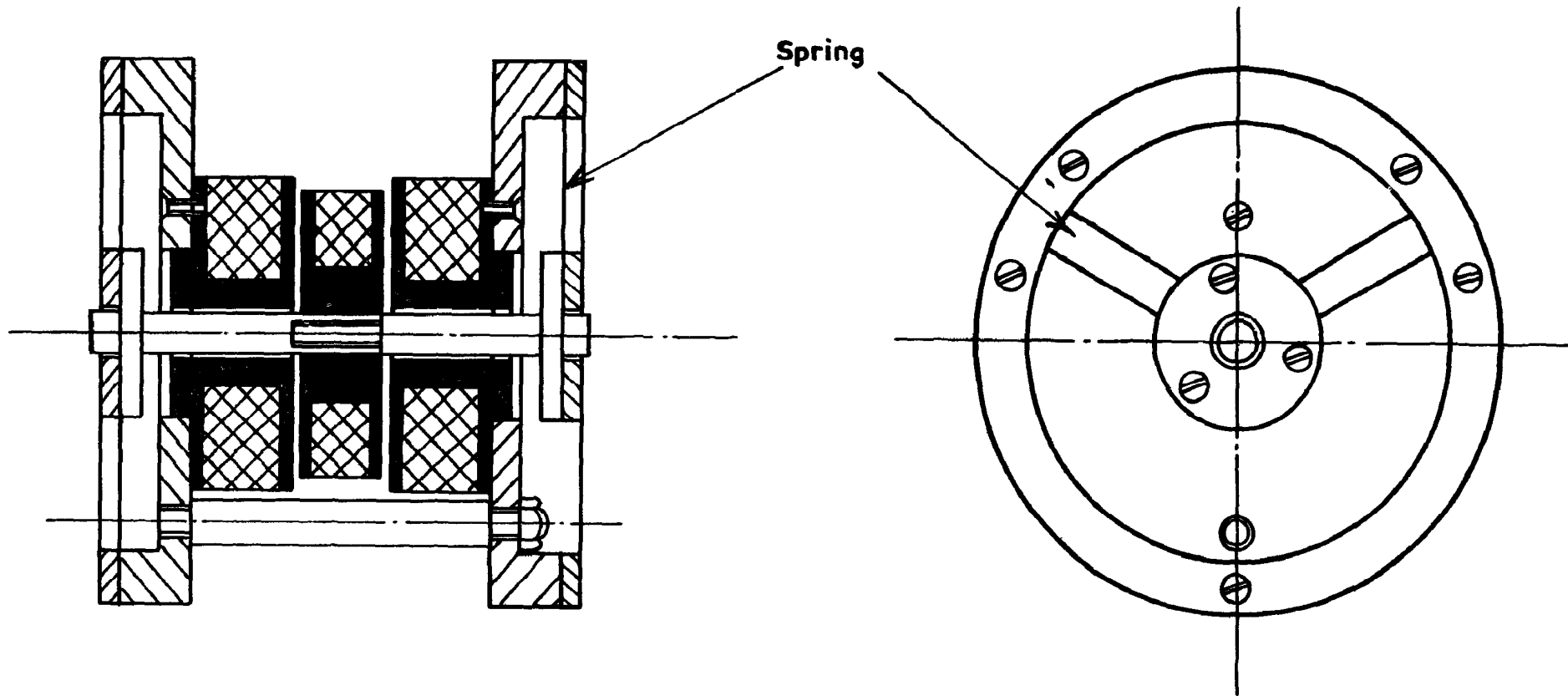


FIG. 2

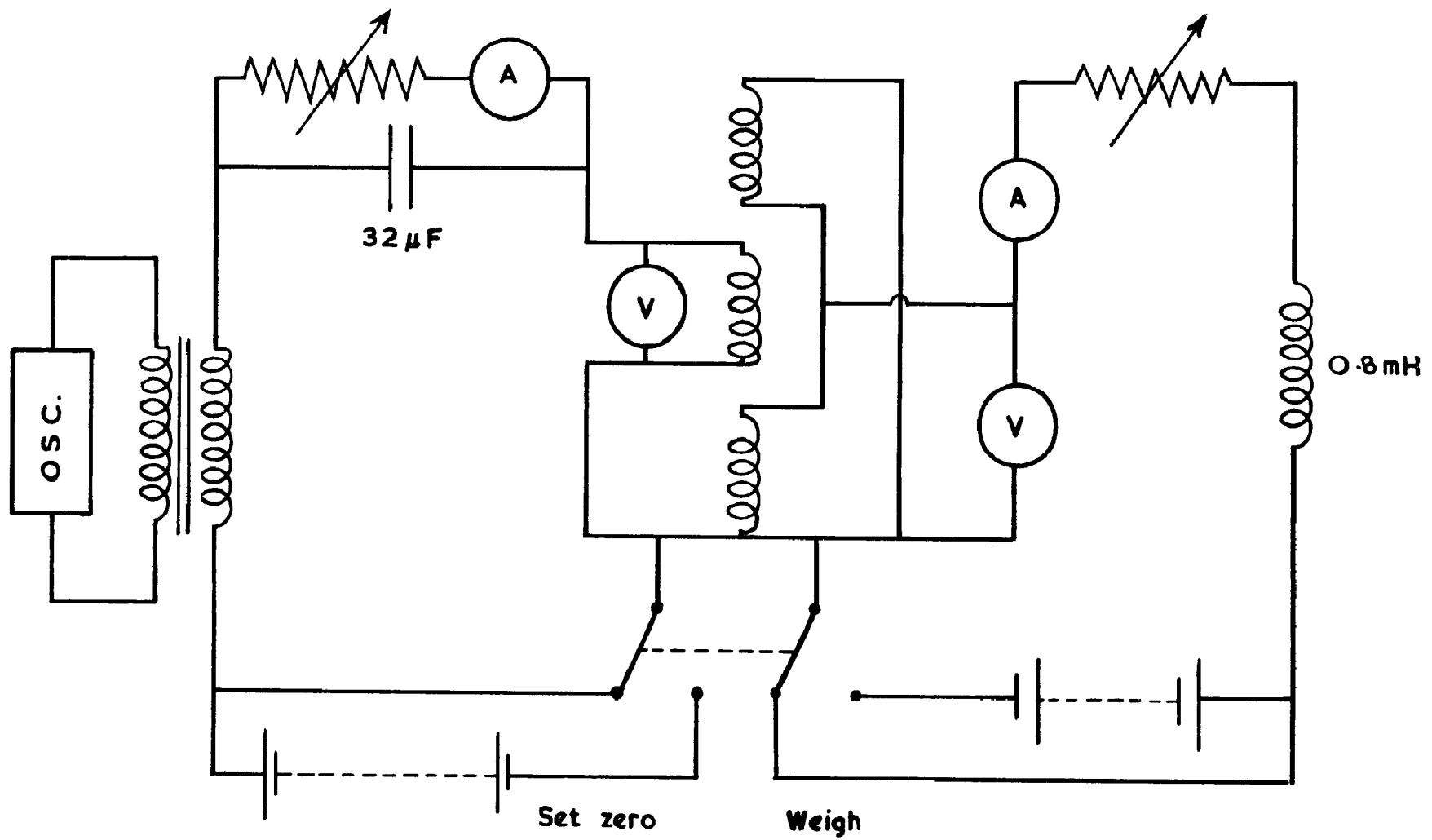
Arrows on coils show relative directions of currents

Diagram of Kelvin current balance



Force - displacement indicator. General arrangement

FIG. 3.



Circuit diagram.

FIG. 4.

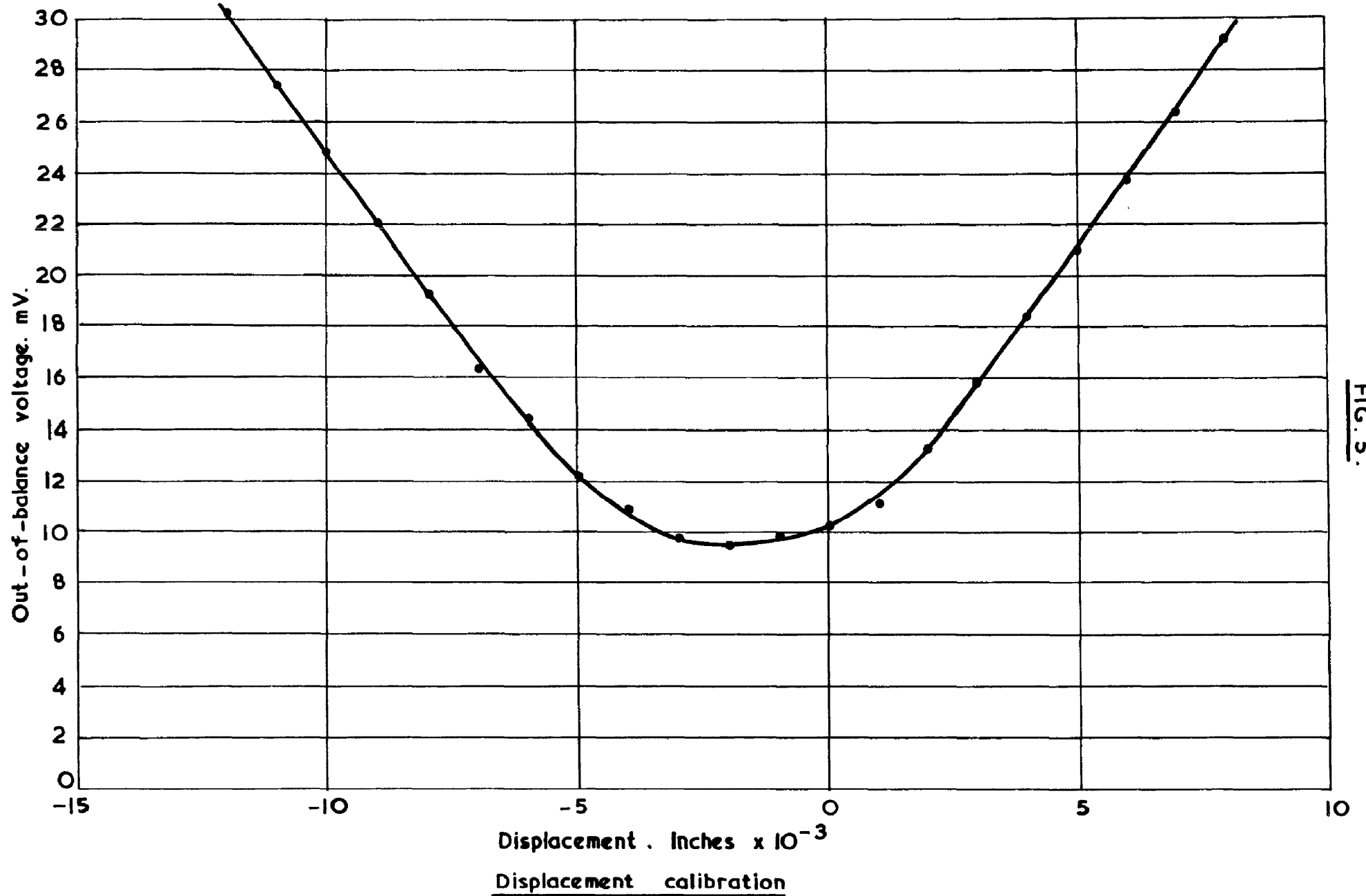
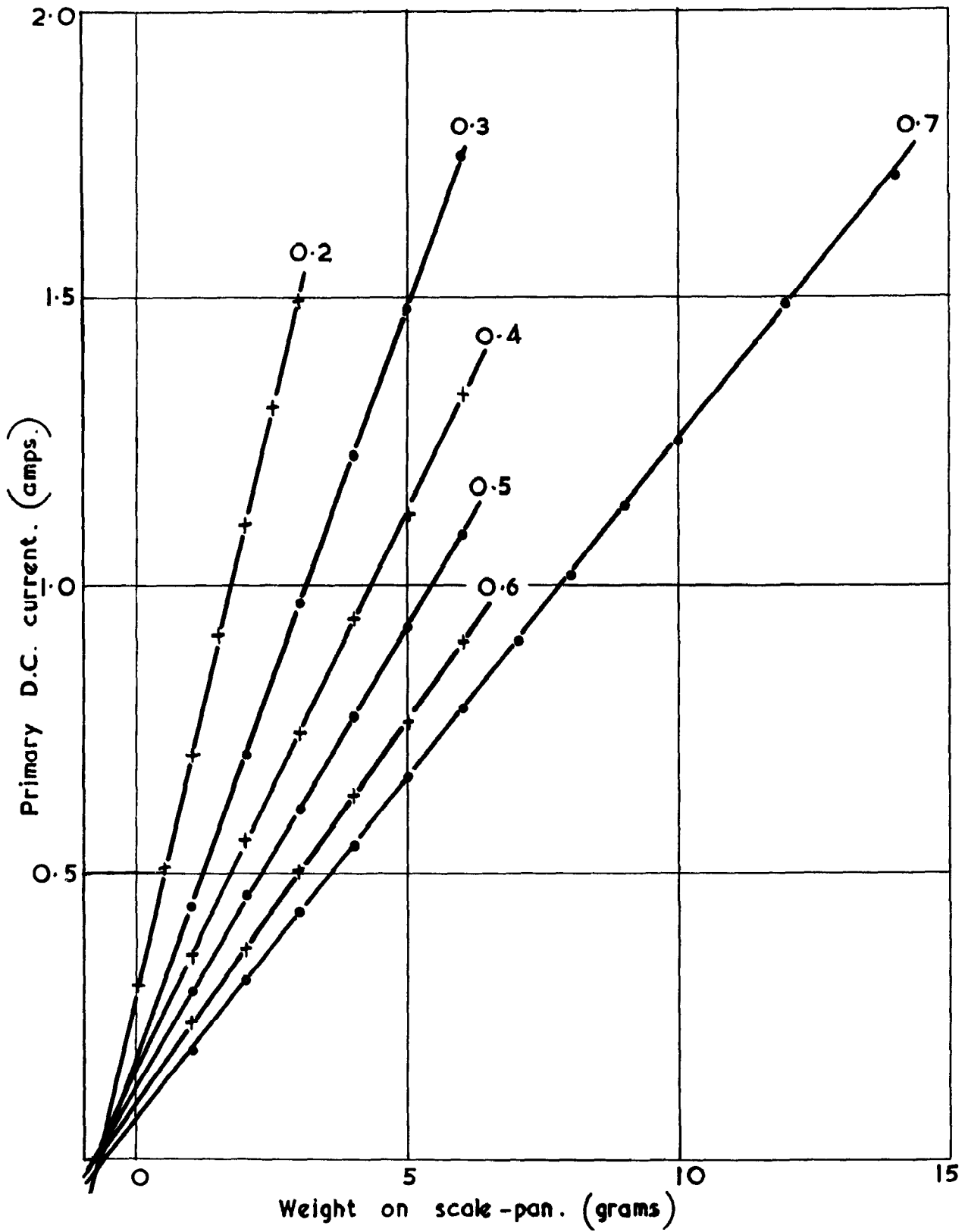


FIG. 5.

FIG. 6.



N.B. Numbers on curves give secondary D.C. current in amps.

Weighting sensitivity 0-10 grams

DS 92863/1 W/61 K4 7/61 CL

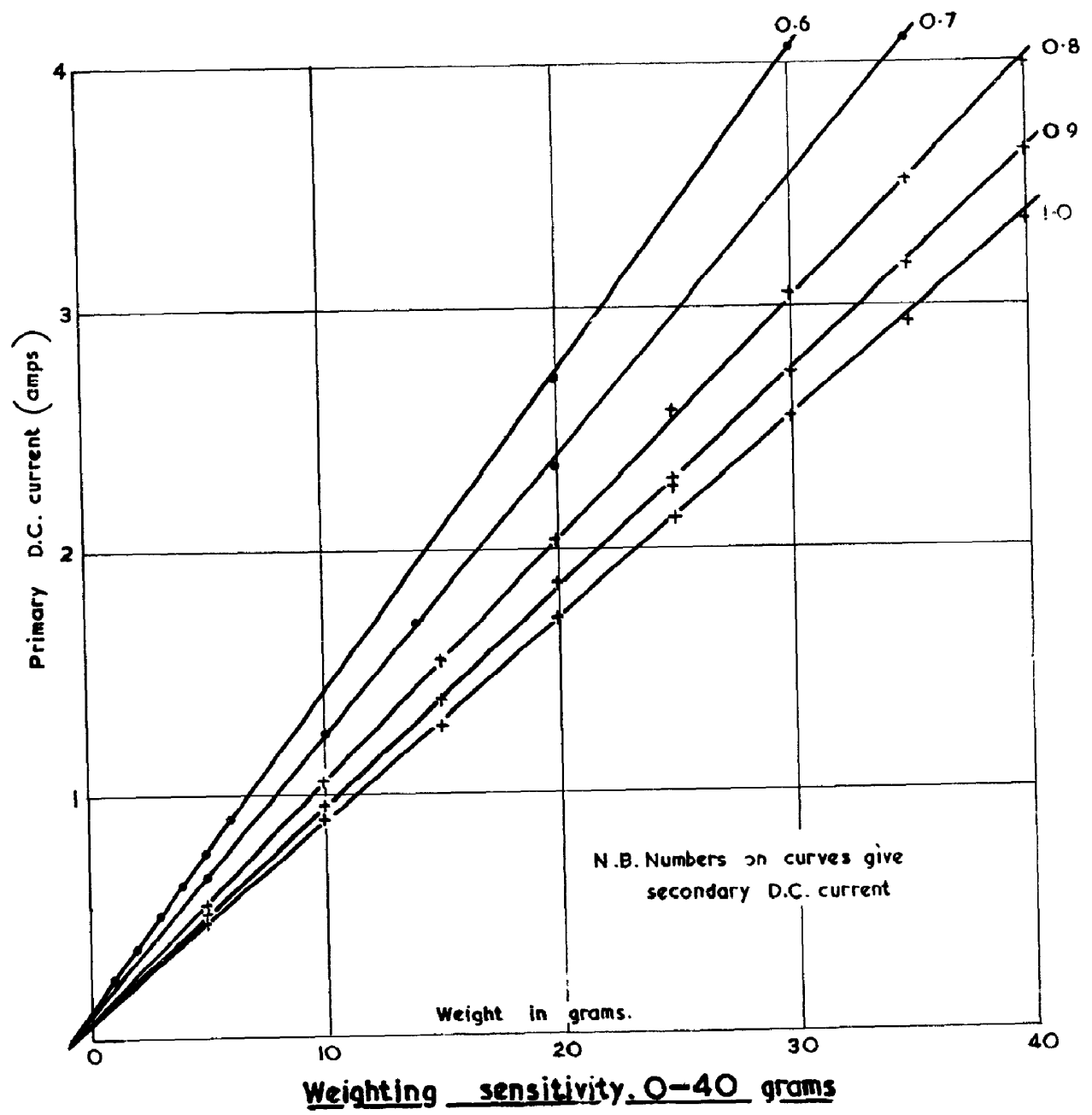


FIG. 7.

A.R.C. C.P.No. 549

July 1960

Franklin, R. E. Oxford University Engineering Lab.

A FORCE-DISPLACEMENT INDICATOR FOR A DRAG BALANCE

The paper describes a null-reading weighing element for a drag balance. Basically the device is a Kelvin Current Balance modified to be linear, to have adjustable sensitivity and to indicate displacement. The drag is measured by reducing the indicated displacement to zero.

A.R.C. C.P.No. 549

July 1960

Franklin, R. E. Oxford University Engineering Lab.

A FORCE-DISPLACEMENT INDICATOR FOR A DRAG BALANCE

The paper describes a null-reading weighing element for a drag balance. Basically the device is a Kelvin Current Balance modified to be linear, to have adjustable sensitivity and to indicate displacement. The drag is measured by reducing the indicated displacement to zero.

A.R.C. C.P.No. 549

July 1960

Franklin, R. E. Oxford University Engineering Lab.

A FORCE-DISPLACEMENT INDICATOR FOR A DRAG BALANCE

The paper describes a null-reading weighing element for a drag balance. Basically the device is a Kelvin Current Balance modified to be linear, to have adjustable sensitivity and to indicate displacement. The drag is measured by reducing the indicated displacement to zero.

© *Crown copyright* 1961

Printed and published by  
HER MAJESTY'S STATIONERY OFFICE

To be purchased from  
York House, Kingsway, London W.C.2  
423 Oxford Street, London W.1  
13A Castle Street, Edinburgh 2  
109 St. Mary Street, Cardiff  
39 King Street, Manchester 2  
50 Fairfax Street, Bristol 1  
2 Edmund Street, Birmingham 3  
80 Chichester Street, Belfast 1  
or through any bookseller

*Printed in England*