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Experimental Study of the Flow over a Particular Afterbody Shape having a Near-Sonic Ridge Line

by

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EXPERIMENTAL STUDY OF THE FLOW OVER A PARTICULAR AFTERBODY SHAPE HAVING A NEAR-SONIC RIDGE LINE

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D. Treadgold

SUMMARY

A study was made, over the Mach number range from 1.42 to 1.92, of the flow over the rear part of one surface of a wing, representing a reversed wedge afterbody with an unswept trailing edge and a sonic ridge line at a Mach number of 1.51.

Pressures were measured and oil flow studies made. The results showed good agreement with exact inviscid flow theory in the expansion region in the vicinity of the ridge line. In the recompression region downstream, separation occurred and the flow was no longer conical so that no satisfactory theoretical estimates could be made.

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1 INTRODUCTION

Many designs so far investigated in the search for efficient slender aerodynamic shapes for supersonic aircraft are so shaped that they are within the scope of existing design methods. The class of bodies therefore has been restricted to those for which

(a) disturbances are small so that the linearised equations of motion may be used and implying no strong shock waves on the body surface; and for which

(b) separation lines and shock waves originate from the trailing edge at some design lift coefficient.

For such cases thin-wing theory or slender thin-wing theory can be applied[1] successfully to give a reasonably correct pressure distribution, as well as drag estimates, for any given shape. In particular the drag can be estimated by the theories given by Ward[2] and Lighthill[3] if the appropriate slenderness assumptions are made. It may be noted that the actual wave drag values, at supersonic Mach numbers, may be well below those of the corresponding optimum body of revolution of the same length and volume.

Griffith[6], however, has suggested a slender forebody but short afterbody for which the above treatment is not adequate. He proposes an afterbody consisting of a large number of facets bounded by near sonic ridge lines which in the limit defines a smooth cylindrical surface.

If linearised theory is applied to such a shape it is found that the estimated drag is very much greater than that of the corresponding body of revolution of the same length and area distribution. However a much lower drag is obtained when a less crude method of calculation is employed. The reason for this may readily be seen by considering the pressure distribution in the vicinity of a near sonic ridge line. On the basis of linearised theory one finds a suction that tends to infinity as $1/\sqrt{n}$, where n is the distance normal to the ridge line, when the ridge line is sonic. This contrasts with the region of uniform finite suction of exact inviscid theory which corresponds to a Prandtl Meyer expansion at the ridge line. Unfortunately, no exact inviscid theory exists for determining the pressures following the expansion. Since this expansion directs the flow towards the plane of symmetry there must follow a recompression process, possibly involving a shock wave, so that the flow is turned into the plane of symmetry. Fowell[4] has suggested a criterion for the maximum deflection angle δ_c in the plane of symmetry for which a continuous recompression process is possible*. This criterion for shock-free flow is given by

$$\sin\left[\left(\frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{2}}\left(\nu_2+\frac{\pi}{2}-\mu_2\right)\right]\cos\mu_2 < M_1\cos\varphi\cot\Lambda\left[M_1^2\sin^2\varphi+\frac{2}{\gamma-1}\right]^{-\frac{1}{2}}$$

and

$$\nu_2 = \nu_1 + \sin^{-1}(\delta_c \operatorname{cosec}\varphi)$$

---

* Such a flow field would have wholly unfavourable pressure gradients and therefore it is improbable that such flow field can be realised in a viscous fluid.

where M_1 = Mach number ahead of the ridge line

φ = angle between the ridge line and the stream ahead of the ridge

ν_1 = Prandtl Meyer angle corresponding to M_1

μ_2 = Mach angle corresponding to ν_2

Λ = Sweepback of the ridge line

This limiting boundary is shown in Fig.1 for various sweepback angles of the ridge-line, Bulakh[5] similarly suggests the existence of a shock wave although he criticizes Fowell's criterion. The strength of the shockwave cannot be found theoretically although an approximate method has been suggested. This is to assume locally a plane shock wave normal to the surface of the wing such that the uniform stream following it is parallel to the plane of symmetry. Assuming no flow separation is encountered then such a plane wave would be further outboard and stronger than the true conical shock surface. The drag associated with a pressure distribution calculated by this approximate method can be seen to depend on the extent of the afterbody surface influenced by the expansion region. Griffith[6], therefore, points out the advantages to be gained by keeping this region as small as possible by employing near-sonic ridge lines.

As mentioned above no theory is, as yet, available to estimate, with any real certainty, the pressure distribution if a shock wave exists, or even to locate the position of the shock on the surface. Since there appears to be large differences in the drags as estimated by the linearised theory and by the approximate flow structure outlined above it was decided to investigate the problem experimentally. The afterbody studied has been taken in its simplest form to consist of a single plane surface.

A pressure plotting model was designed to check the ideas of Refs. 4,5 and 6. In order to ensure the existence of a shock wave the angles of the surfaces were chosen so that Fowell's critical angle δ_c was exceeded. Provision was made to vary the upstream Mach number normal to the ridge line from 0.95 to 1.28 by choice of tunnel liners and by inclining the model to the tunnel airstream.

2 DESCRIPTION OF THE MODEL AND TESTS

A drawing of the model is given in Fig.2 and its installation in the tunnel is shown in Fig.3. The forebody has plane surfaces and is designed to provide an approximately uniform Mach number distribution and boundary layer thickness at the ridge line for the stations where pressures were measured. The plane surface forming the afterbody makes an angle of 23° to the planes of the forebody, the ridge lines formed at the intersection with these two planes were designed to be sonic at Mach number M_1 of 1.51. This Mach number M_1 ahead of the ridge line was varied using M_0 = 1.51 and 1.87 liners. Intermediate Mach numbers were obtained by adjusting the incidence of the model. The uniformity of the stream ahead of the ridge line, at the various incidences, α, can be assessed from Fig.4 and from the oil flow photographs Figs.9a-e. The influence of the disturbances from the side edges of the model have, of course, to be ignored.

The static pressures were measured at 22 points situated at three streamwise stations 0.8, 1.4 and 2.0 in. downstream of the apex of the afterbody. The spanwise location of the holes is given in Fig.2. Pressures were recorded on a multibank mercury manometer which was read visually. The incidence of the model was measured with a telescope fitted with a protractor eyepiece.

Oil flow studies were made at M = 1.51 using titanium oxide in oil. Photographs could only be taken of the surface flow patterns after the removal of the model from the tunnel but, fortunately, only in certain small regions was the flow pattern disturbed by the tunnel shock wave on shutting down the tunnel. An attempt to locate the shock waves in the flow field was made using a conical shadowgraph focused at the apex of the afterbody, but attempts to photograph the image proved unsuccessful because of the awkward position and poor definition of the image. Some of the pressure measurements were repeated with transition fixed ahead of the ridge line using roughness bands of carborundum.

The tests were made in the No.8 (9 in. × 9 in.) supersonic wind tunnel at the R.A.E. Farnborough. The mean Reynolds number was about 0.4×10^6 per in.

3 DISCUSSION OF THE RESULTS

The measured spanwise pressure distributions are given in Figs.6a-n for Mach numbers from 1.42 to 1.91. The pressure coefficients are based on q, the kinetic pressure ahead of the ridge line. In calculating these and the Mach number M_1 the working section value of the stagnation pressure H_0 has been used since, over the range of inclination of the model and the range of Mach number, this procedure involves an error of less than 0.01 in Mach number and less than 2% in the pressure coefficient.

Estimated pressure distributions have been included in Fig.6 where possible. These have been made using the approximate method described in section 1 and illustrated in Fig.7(b). This method is clearly better than the linearised theory as can be seen from the typical example shown in Fig.8. This is not surprising since the pressure coefficients are of the order of 0.5 and therefore the associated disturbances are large and can hardly be expected to conform with the restrictions of a small perturbation theory.

Photographs of the surface streamlines are shown for a number of cases in Figs.9(a)-(e). The interpretation of these results and of the pressure distributions is given below.

3.1 Region of Prandtl-Meyer expansion

The pressures measured at the points 8 and 20 (see Fig.2) situated just downstream of the ridge line are shown in Fig.5. These points are sufficiently near the ridge line to be in a region where one would expect to have uniform flow following a Prandtl-Meyer expansion.

For Mach numbers M_1 ahead of the ridge line greater than 1.51 (i.e. supersonic velocity component normal to the ridge line) it is possible to calculate exactly the pressure by simply resolving the velocities along and normal to the ridge line. The Prandtl-Meyer relations are then applied to the normal component of velocity whilst the tangential component is conserved. It can be seen from Fig.5 that there is a remarkably close agreement between the experimental and the calculated values. This agreement is noteworthy since both the oil flow studies shown in Figs.9(a)-(e), and the spanwise pressure distributions shown in Fig.6, indicate a considerable upstream influence of the ridge line. It would appear from the pressure distributions that the boundary layer thickness must start to decrease at a distance of the order of ten boundary layer thicknesses in advance of the ridge line. It should be noted, however, that the pressure field in this expansion region is wholly favourable so that viscous effects are not normally expected to be large.

The variation of the ratio of static pressures before and after the expansion is also shown in Fig.5. It is interesting to note that this ratio is sensibly independent of M_1 over the range of the test conditions; even for subsonic Mach numbers normal to the ridge line where the Prandtl-Meyer relations cannot be applied.

The calculated flow directions are shown superimposed on the oil flow pictures in Figs.9(d) and (e). It can be seen that the surface streamlines after the Prandtl-Meyer expansion are more inclined to the undisturbed stream before the ridge than the external streamlines. These flow patterns correspond closely to patterns of limiting suggested streamlines for either wholly favourable or wholly unfavourable pressure fields by Maskell and Weber[1].

3.2 Recompression region

As we have mentioned earlier, the flow following the Prandtl-Meyer expansion is directed towards the plane of symmetry, so that by some mechanism the flow has to be turned back parallel to the plane of symmetry.

For the large deflection angle considered in this experiment no gradual compression is possible so that the flow structure must embody strong shocks. The case of $M = 1.53$ is considered in detail in Fig.7. If one considers the flow to be turned by a single plane shock normal to the surface as suggested by Fowell[4] and Griffith[6], instead of by the conical shock that would exist in an inviscid fluid, then its position would be as shown in Fig.7(b). As mentioned in section 1 such a plane shock would be stronger and further outboard than the trace of the true inviscid conical shock on the surface. However comparison of Fig.7(a) and (b) clearly shows the compression to be taking place outboard of the estimated plane shock position. There does not appear to be any indication of a sudden compression although there are not sufficient pressure points to define, with certainty, the pressure distribution at the commencement of compression. However, it does in fact appear that the pressure distribution is not unlike that given by linearised theory inboard of the spanwise station at which this equals the Prandtl-Meyer value (Fig.8), although this result may be purely fortuitous.

The explanation of the premature compression is shown by the oil flow patterns given in Fig.9(d). Here a separation line is clearly visible. A flow structure consistent with the measured results is therefore likely to be of the form shown in Fig.7(c). The existence of the shock wave in the external stream was verified by using a conical form of shadowgraph focused at the apex of the ridge. The quality of the image produced by the system was not high and, possibly because of the non-conical nature of the separation, no detail of the conditions at the foot of the wave could be observed.

The separation line shown in Figs.9(a)-(e) is seen to be far from straight. This departure of the flow from a conical form is to be expected since the separation point will depend on the boundary layer conditions. These in turn depend on a characteristic length which, in the case of this experiment, is the length of the forebody ahead of the ridge line. It is likely that the flow will become conical as the distance from the ridge line become large compared with the forebody length.

The effect of artificially thickening the boundary by placing roughness strips at the leading edge of the forebody is shown in Figs.10(a) and (b).

The results are much the same for both subsonic and supersonic ridge line conditions. The thicker boundary layer causes an earlier expansion and compression.

3.3 Comparison of overall pressure drag with estimated values

Since the flow is not conical it is not possible, with the few pressure measurements made, to give accurate comparison of the pressure drag of the afterbody with the various estimates. If, however, we examine Fig.8 we can readily see that linear theory will give an over estimate of the drag. Some idea of the magnitude may be seen by considering the drag coefficients for the station x = 2.0 in. defined as

$$C_d = \tan \delta \int_0^1 C_p \, d\, y/s \quad .$$

The values of C_d obtained by the various methods are given in the table below for two typical cases

Values of C_d

Mach number	Subsonic 1.45	Supersonic 1.71
Experiment	0.095	0.075
Linear theory	0.180	0.099
Linear theory with a vacuum as limiting suction	0.107	0.090
Prandtl-Meyer expansion and a plane shock wave	-	0.062
Linear theory with Prandtl-Meyer expansion value limiting the suction	-	0.073

From the table it is seen that for the case of a subsonic ridge line linear theory overestimates the drag by a factor of nearly two. This discrepancy is reduced somewhat if the minimum pressure coefficient is limited to $- ^2/_\gamma M^2$ (i.e. a perfect vacuum). The use of the Prandtl-Meyer expansion followed by a plane shock leads to an underestimate of the drag. The best estimate is however to be obtained by using linear theory pressure distribution but limiting the suction to the value given by the Prandtl-Meyer expansion.

Overall drags for slender configurations having an afterbody similar to the ones of these tests have been measured in free flight. These results, which are reported separately, show drags which are similarly lower than the linear theory estimates.

3.4 Further investigations to be made

The present paper has dealt with conditions following a discontinuity in surface slope which was sufficiently large, on Fowell's criterion, to produce a shock wave. It would, however, be of great interest to discover conditions following a much smaller change in surface slope. Some information on these conditions will be provided by tests to be made in the 8 ft × 8 ft Supersonic Tunnel at R.A.E. Bedford on a model designed by Dr. A.A.Griffiths.

This model is designed by the method described in Ref.6 with an afterbody without discontinuities in surface slope.

4 CONCLUSIONS

For the short afterbody simulated by these present tests the following conclusions were reached.

(1) The measured pressures in the neighbourhood of the ridge line agree well with the predictions based on the exact Prandtl-Meyer relations applied to the normal component of velocity.

(2) The expansion around the ridge line is detectable ahead of the ridge at a distance of the order of ten times the estimated boundary layer thickness.

(3) There is considerable departure from a conical flow structure in the recompression région downstream of the expansion due to viscous separation.

(4) Linear theory is shown to overestimate the drag of the afterbody whilst the approximate method assuming a plane shock wave underestimates the drag.

LIST OF SYMBOLS

C_d	drag coefficient for a spanwise strip = $\tan\delta \int_0^1 C_p \, d\left(\frac{y}{s}\right)$
C_p	pressure coefficient = $\frac{p-p_1}{q_1}$
H	stagnation pressure
M	Mach number
n	distance along the surface normal to the ridge line
p	static pressure
q	kinetic pressure
s	local semi-span of the afterbody
x,y,z	rectangular co-ordinates shown in Fig.2
γ	ratio of the specific heats of air
δ	deflection angle at the ridge line in the plane of symmetry
δ_c	critical angle δ (Fowell's criterion see section 1)
Λ	sweepback angle of the ridge line (in the plane of the afterbody)
μ	Mach angle
ν	Prandtl-Meyer angle
φ	angle between the ridge line and the x axis

LIST OF SYMBOLS (Contd)

Subscripts

o	Conditions in the tunnel working section
1	Conditions just ahead of the ridge line
2	Conditions downstream of the ridge line

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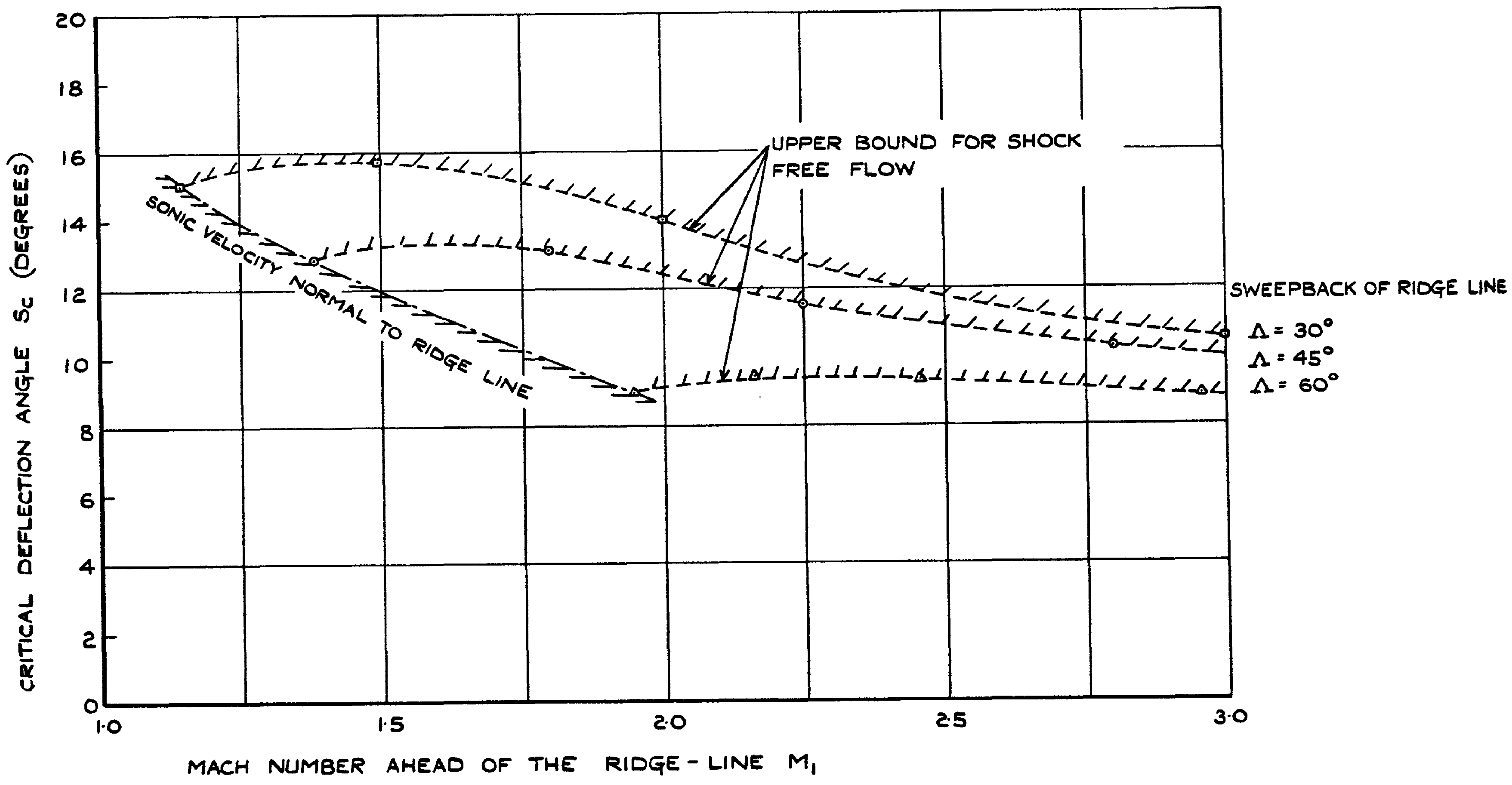
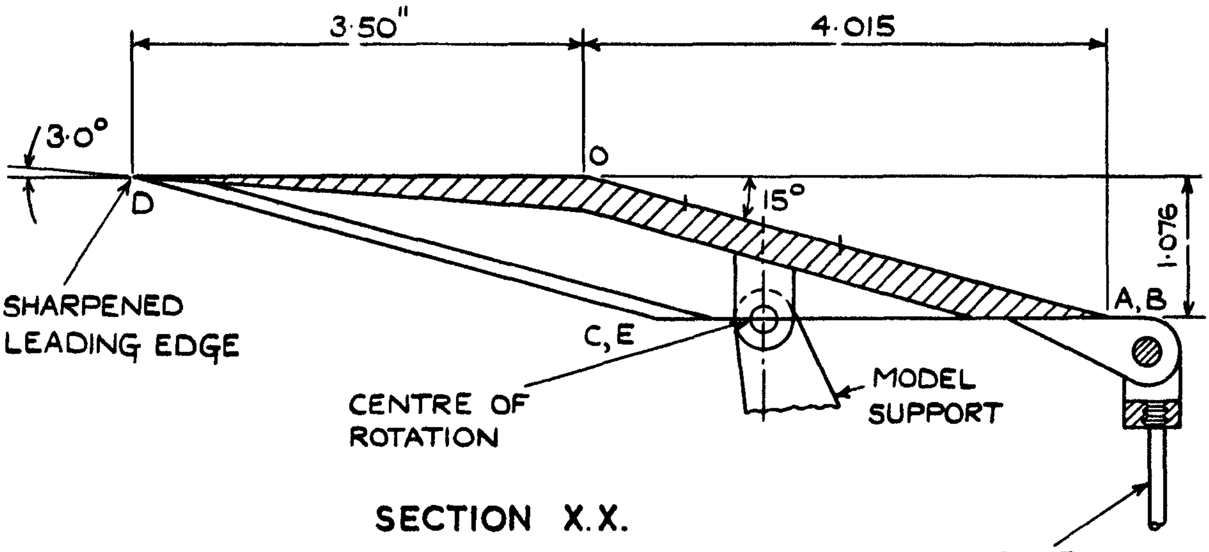
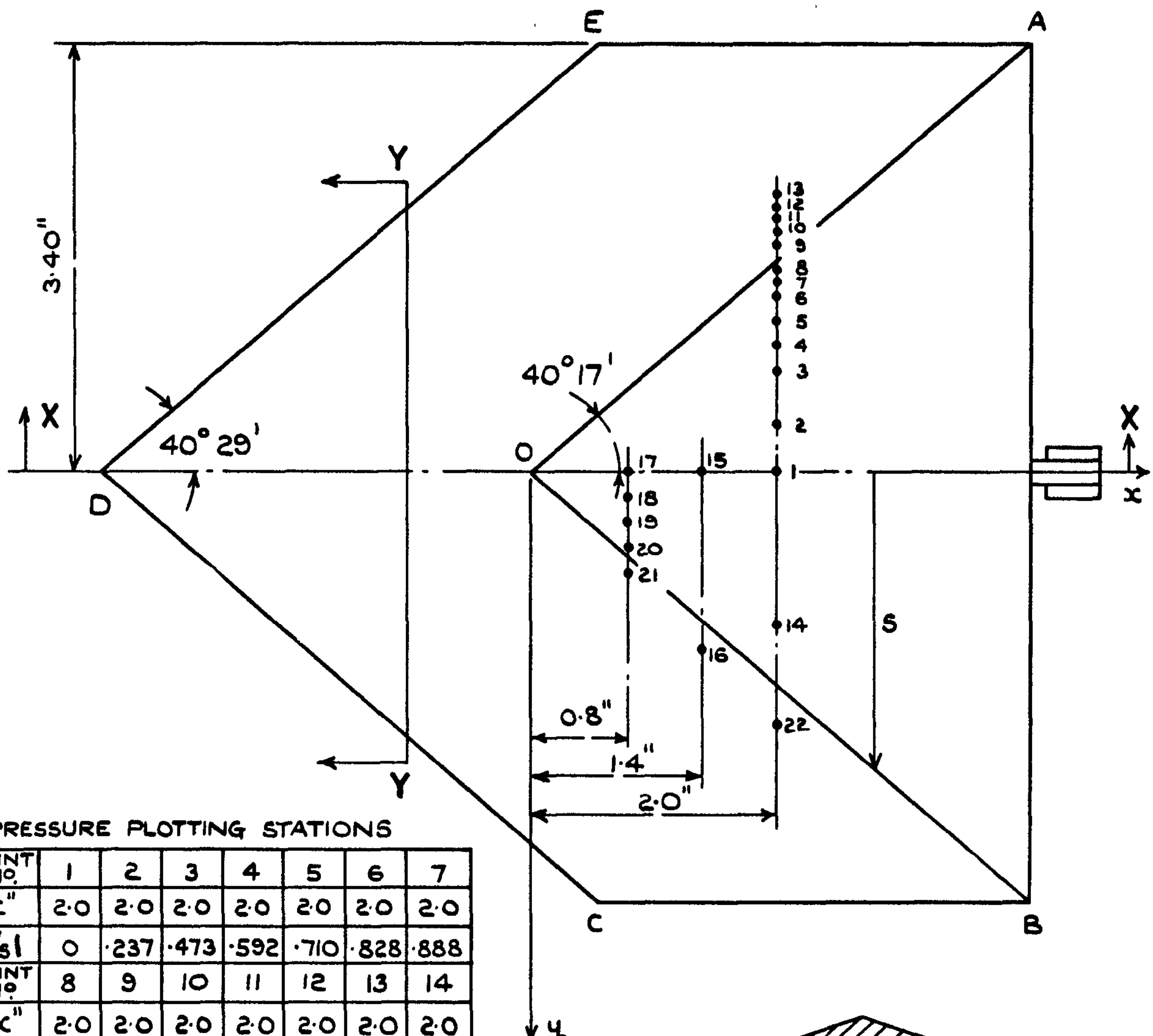


FIG.1. FOWELL'S CRITERION FOR SHOCK-FREE FLOW DOWNSTREAM OF A RIDGE LINE.

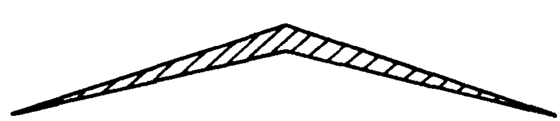


SECTION X.X.



PRESSURE PLOTTING STATIONS

POINT No.	1	2	3	4	5	6	7	
x"	2·0	2·0	2·0	2·0	2·0	2·0	2·0	
|y/s|	0	·237	·473	·592	·710	·828	·888	
POINT No.	8	9	10	11	12	13	14	
x"	2·0	2·0	2·0	2·0	2·0	2·0	2·0	
|y/s|	·947	1·065	1·124	1·183	1·243	1·302	·710	
POINT No.	15	16	17	18	19	20	21	22
x"	1·4	1·4	0·8	0·8	0·8	0·8	0·8	2·0
|y/s|	0	1·183	0	299	·597	·896	1·194	1·186



SECTION Y.Y.

FIG. 2. SKETCH OF PRESSURE PLOTTING MODEL.

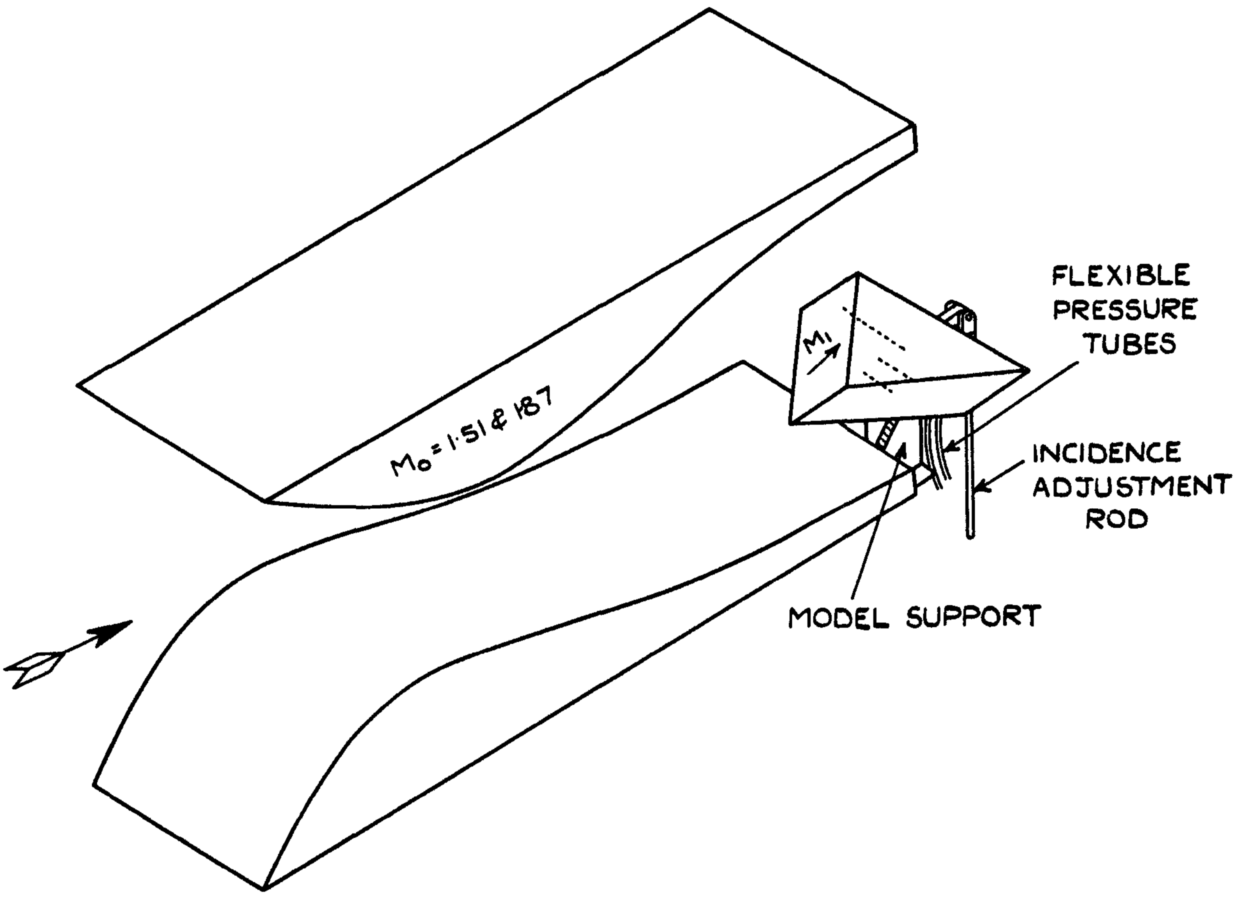


FIG. 3. INSTALLATION OF THE MODEL IN No. 8 (9" x 9") SUPERSONIC TUNNEL.

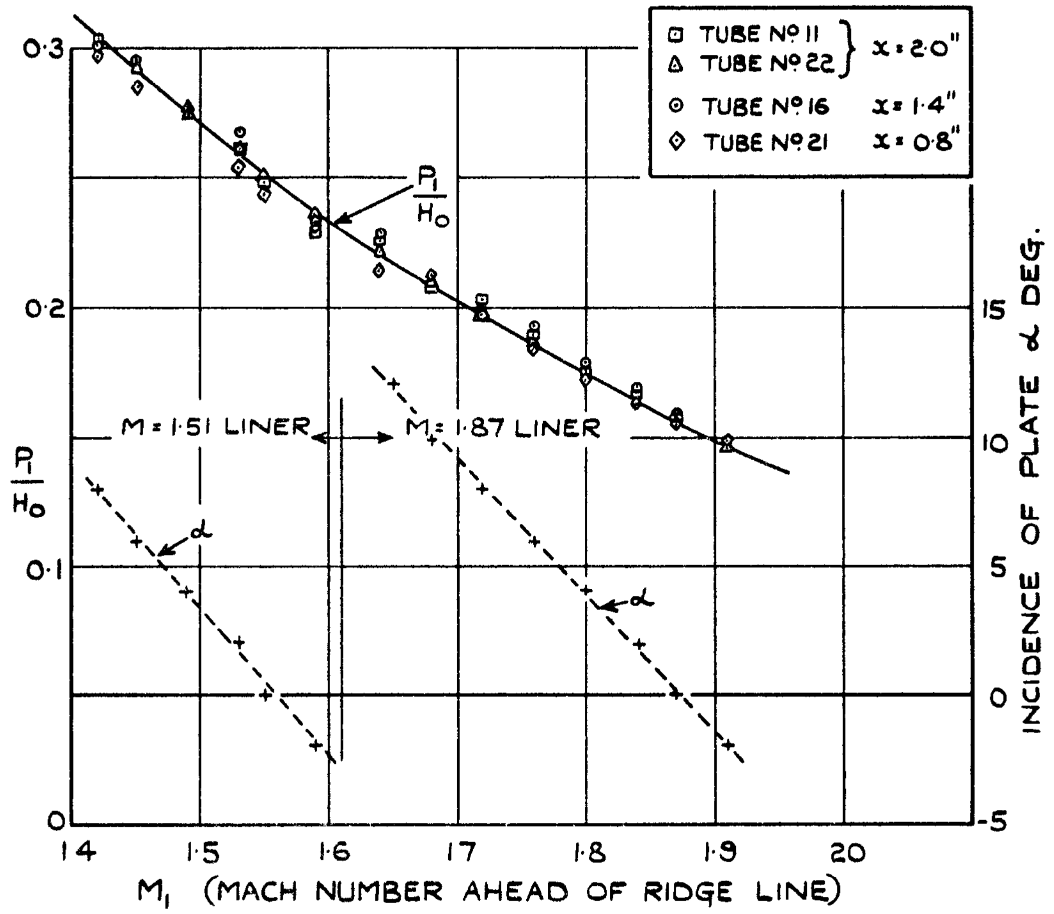


FIG. 4. VARIATION OF STATIC PRESSURE AHEAD OF THE RIDGE LINE.

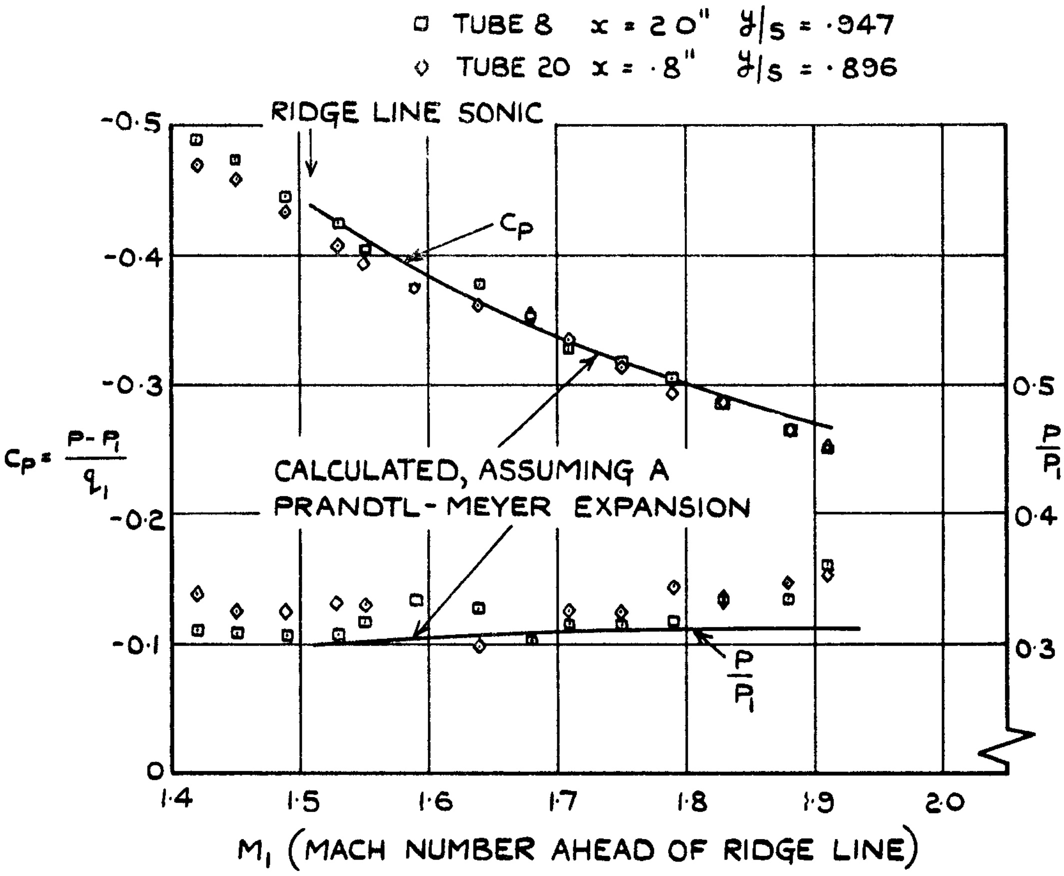


FIG. 5. PRESSURE JUST DOWNSTREAM OF THE RIDGE LINE.

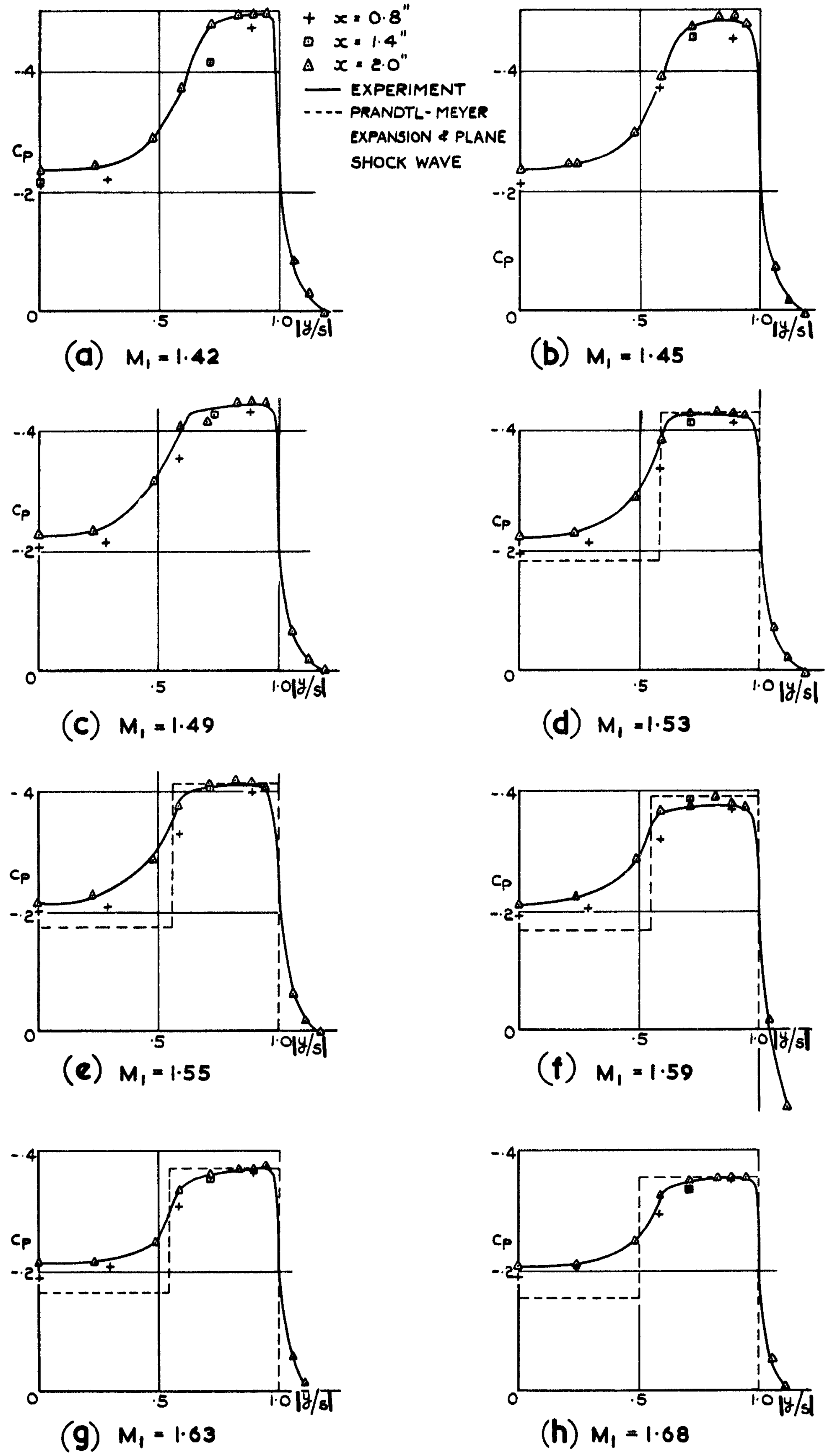


FIG. 6. MEASURED SPANWISE PRESSURE DISTRIBUTIONS.

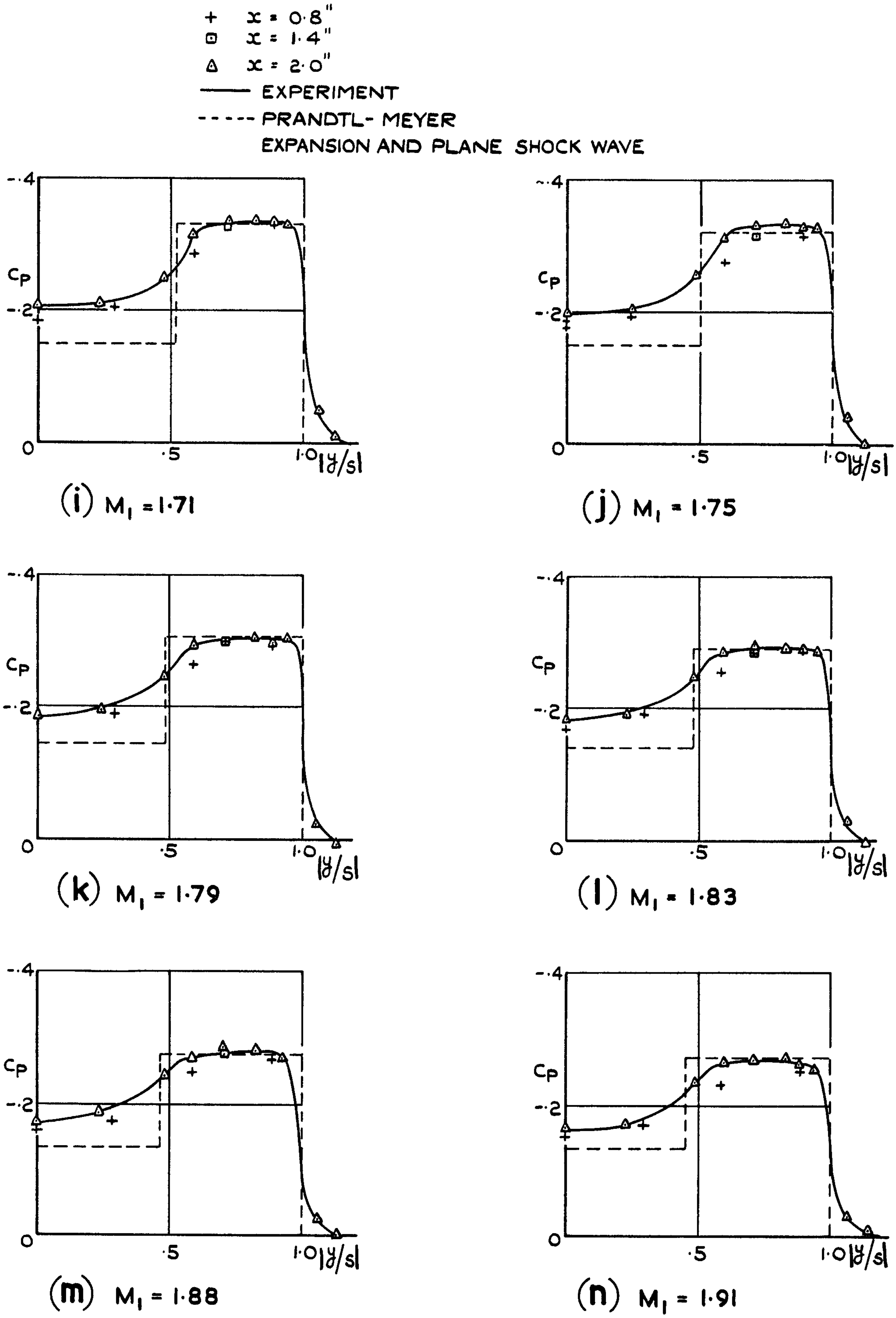
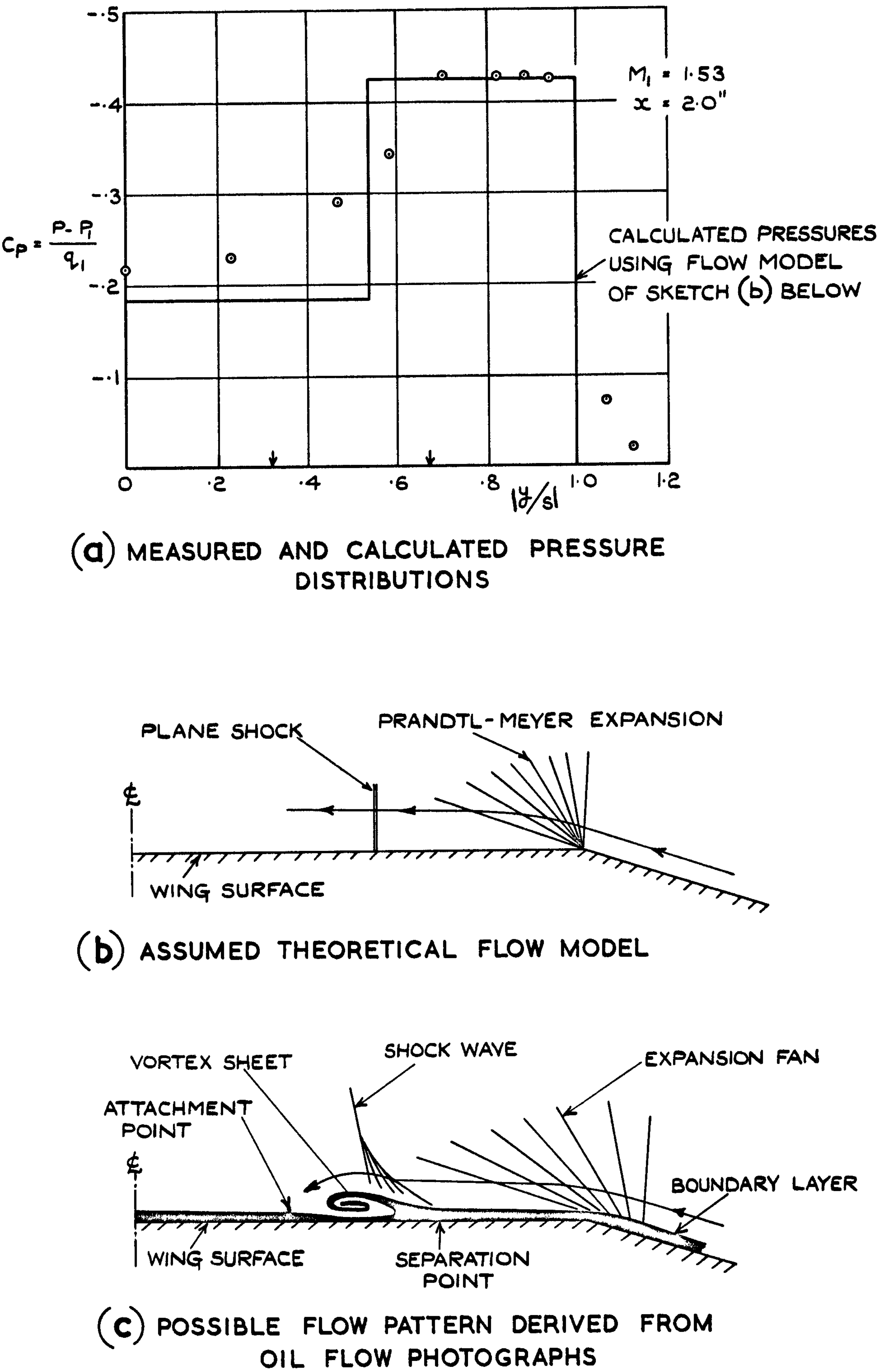


FIG. 6. (CONTD) MEASURED SPANWISE PRESSURE DISTRIBUTIONS.

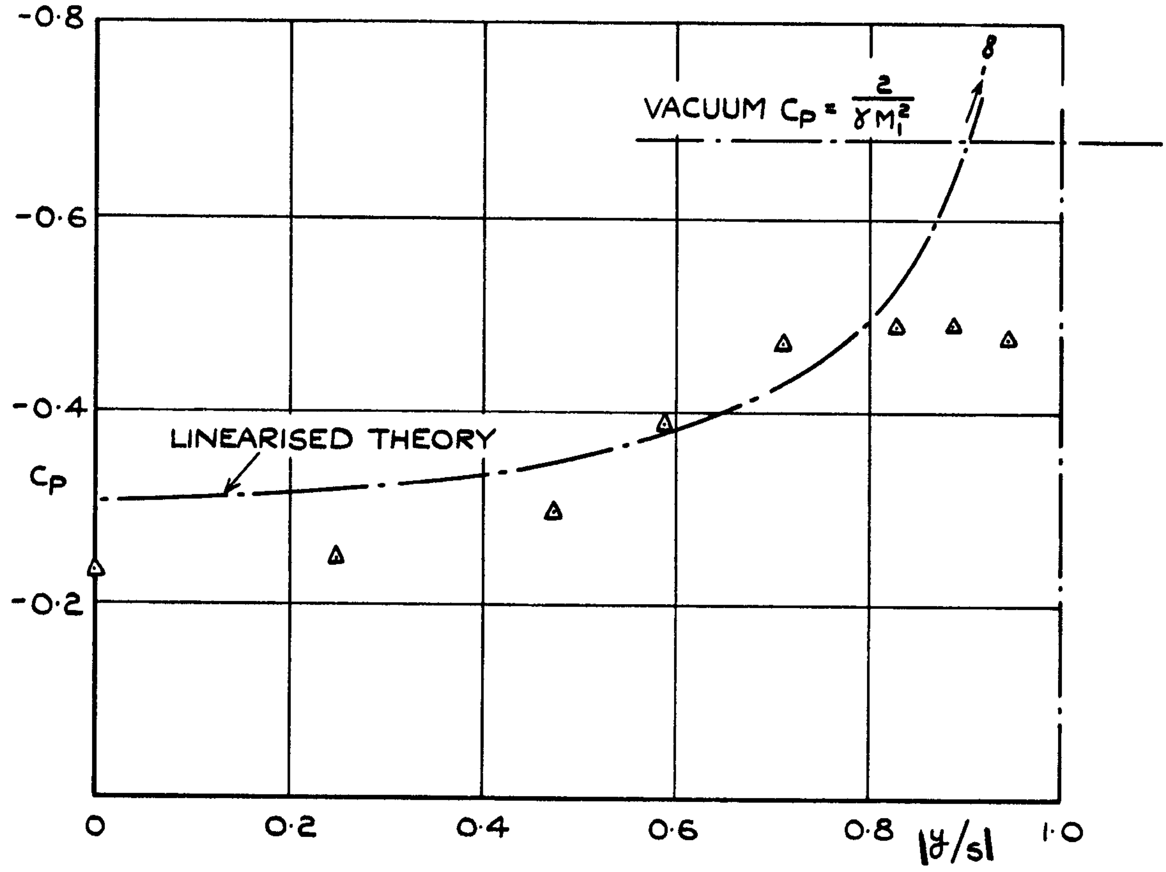


(a) MEASURED AND CALCULATED PRESSURE DISTRIBUTIONS

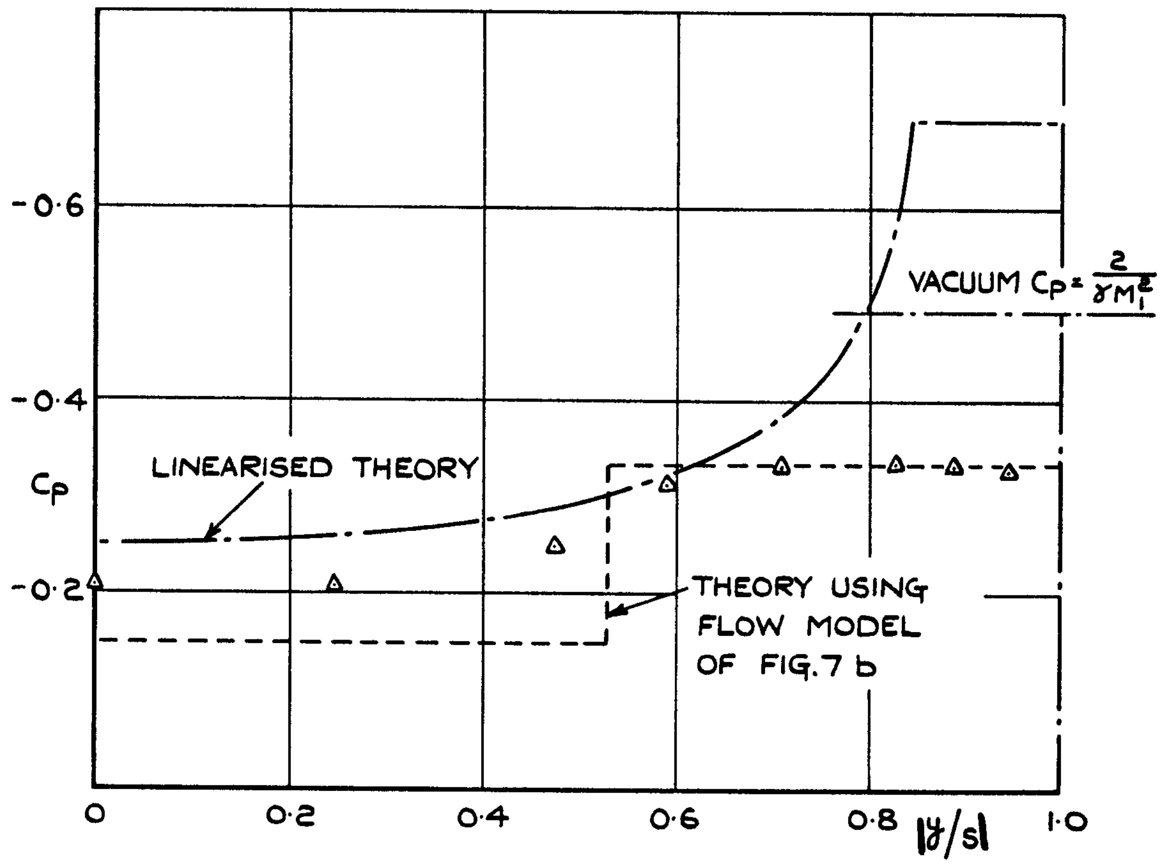
(b) ASSUMED THEORETICAL FLOW MODEL

(c) POSSIBLE FLOW PATTERN DERIVED FROM OIL FLOW PHOTOGRAPHS

FIG. 7. COMPARISON OF EXPERIMENTAL FLOW CHARACTERISTICS AT M = 1·53 WITH THEORETICAL FLOW ASSUMED.



(a) $M_1 = 1·45$ (SUBSONIC RIDGE LINE)



(b) $M_1 = 1·71$ (SUPERSONIC RIDGE LINE)

FIG. 8. COMPARISON OF TYPICAL MEASURED RESULTS WITH LINEARISED THEORY.

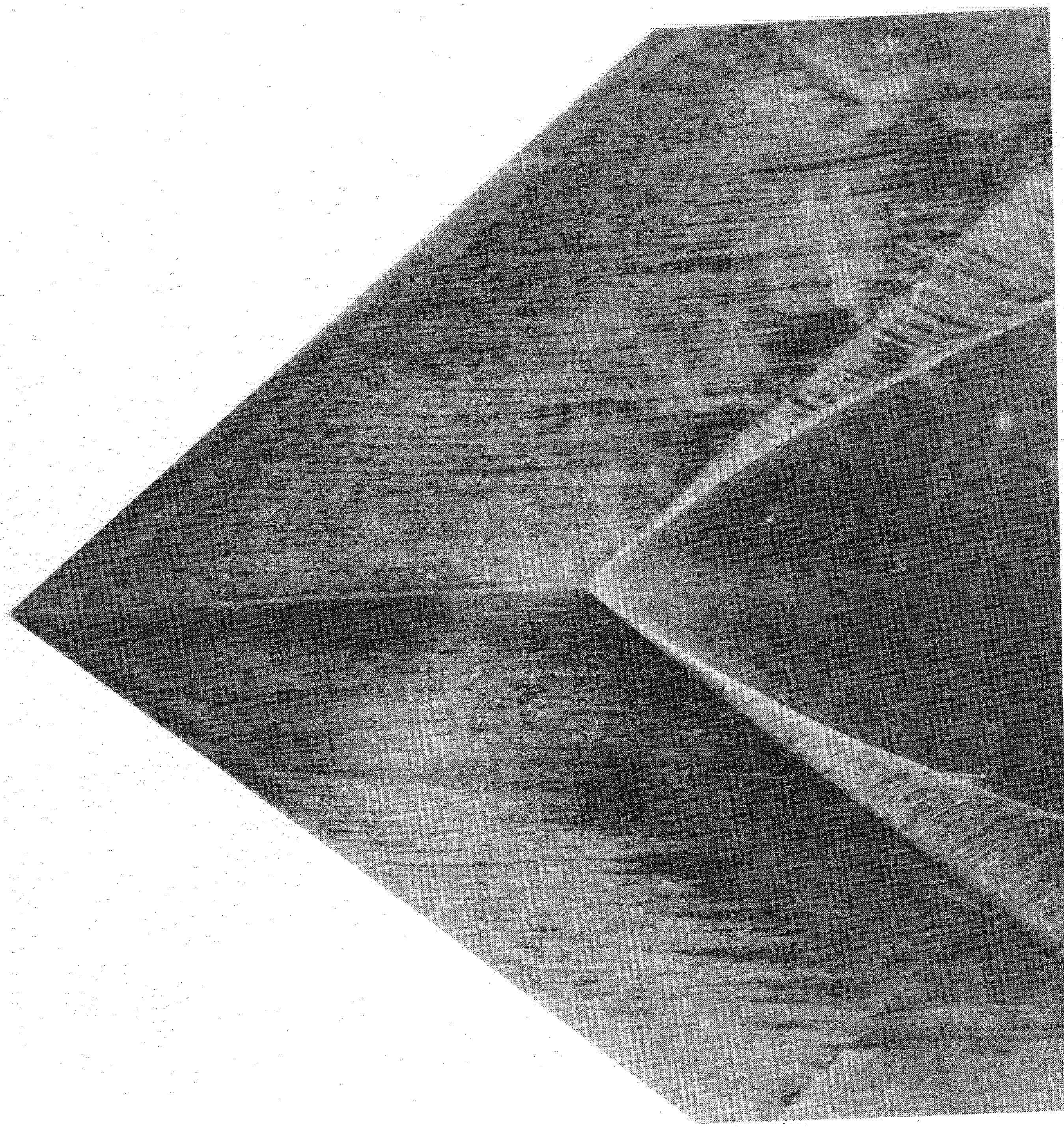


FIG. 9a. OIL FLOW STUDIES. $M_1 = 1 \cdot 42$

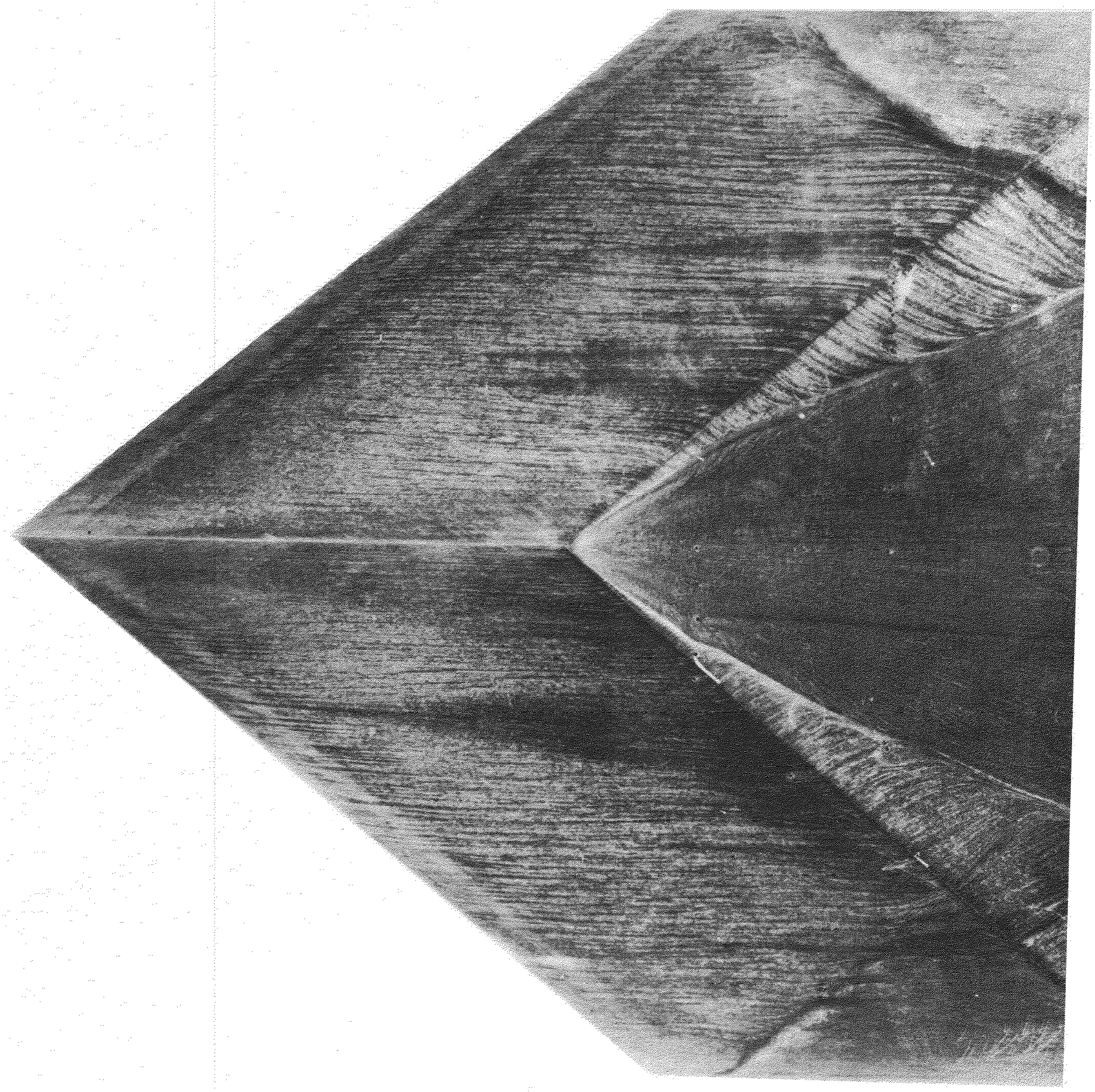


FIG. 9b. OIL FLOW STUDIES. $M_1 = 1 \cdot 45$

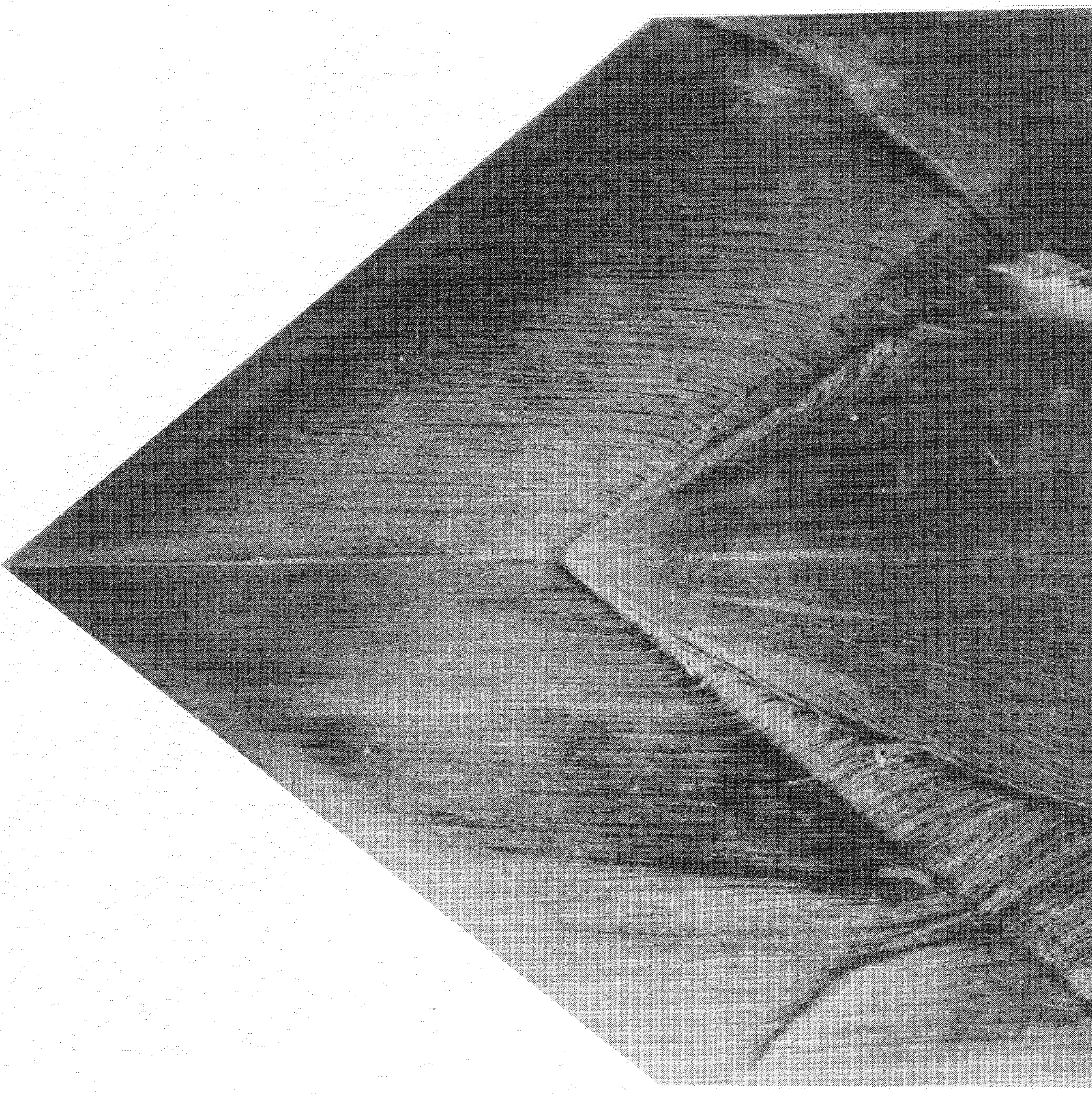


FIG. 9c. OIL FLOW STUDIES. $M_1 = 1 \cdot 49$

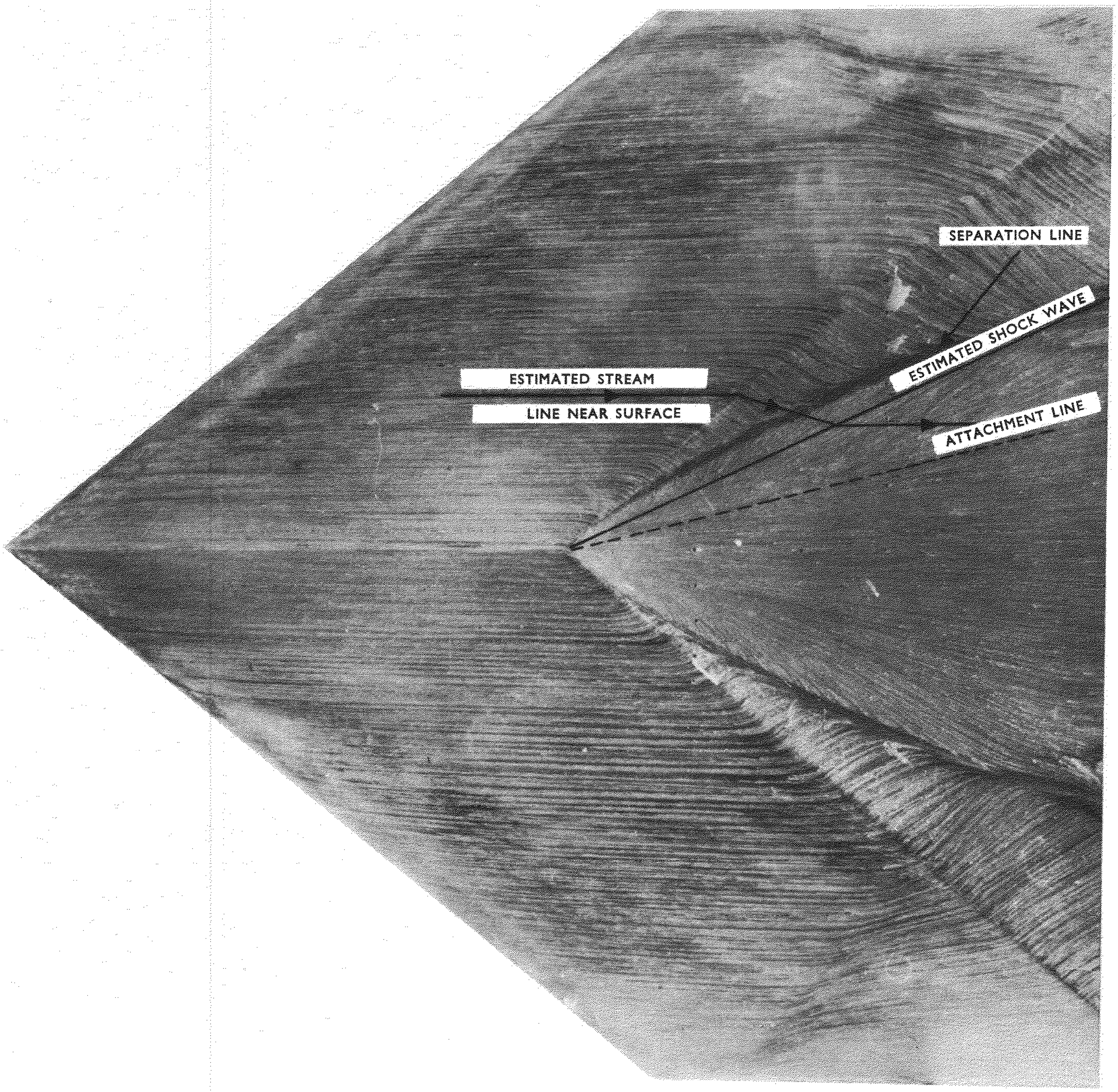


FIG. 9d. OIL FLOW STUDIES. $M_1 = 1 \cdot 53$

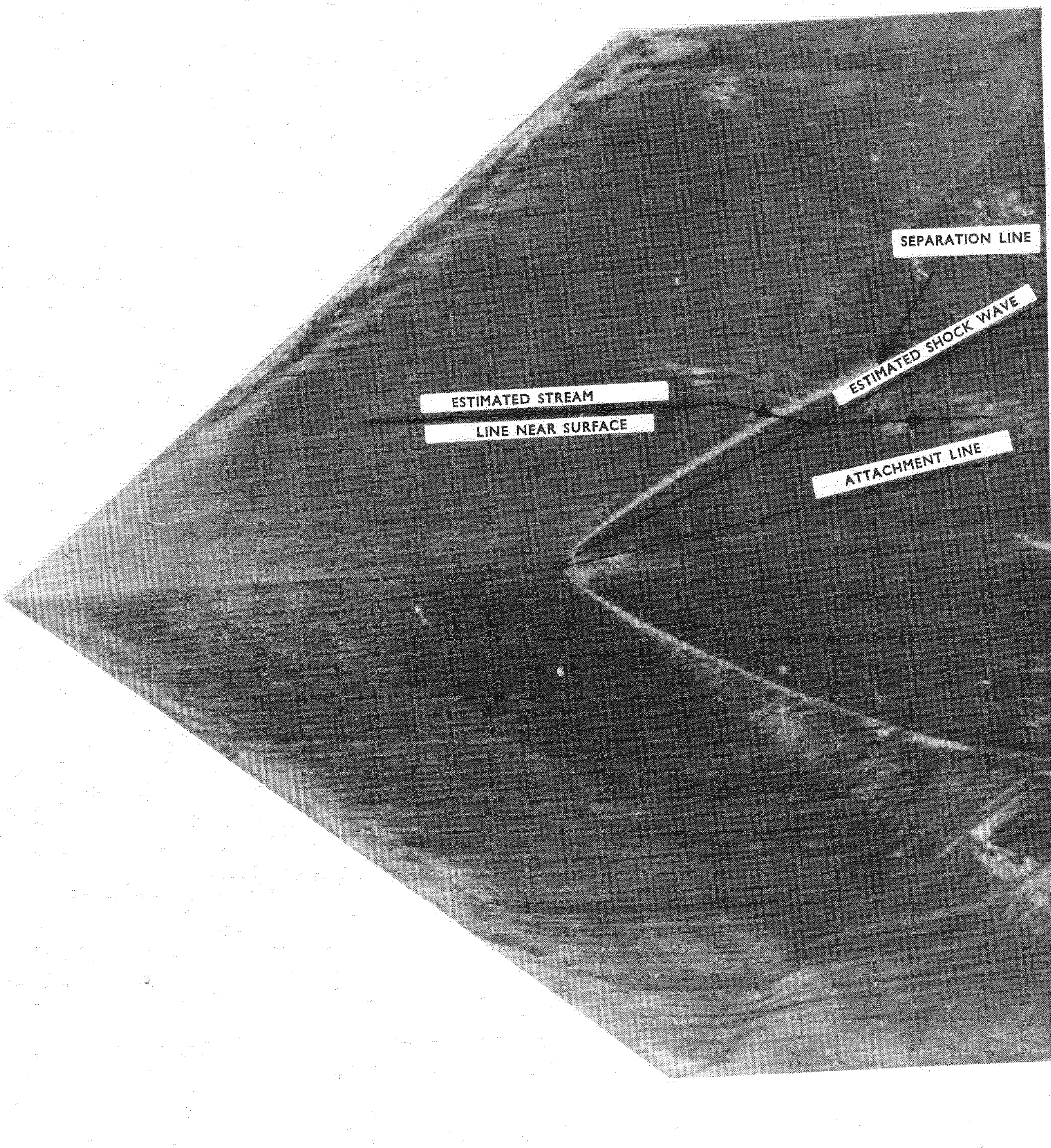
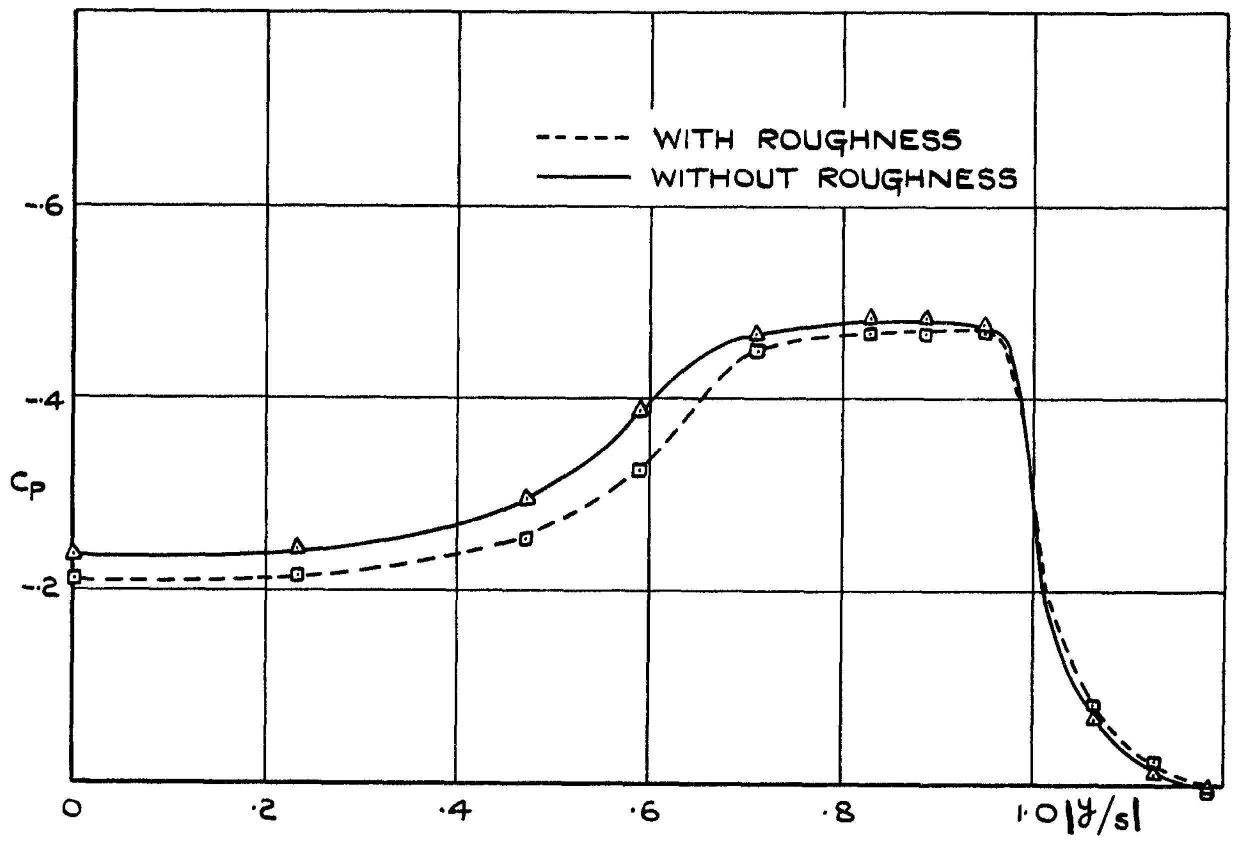
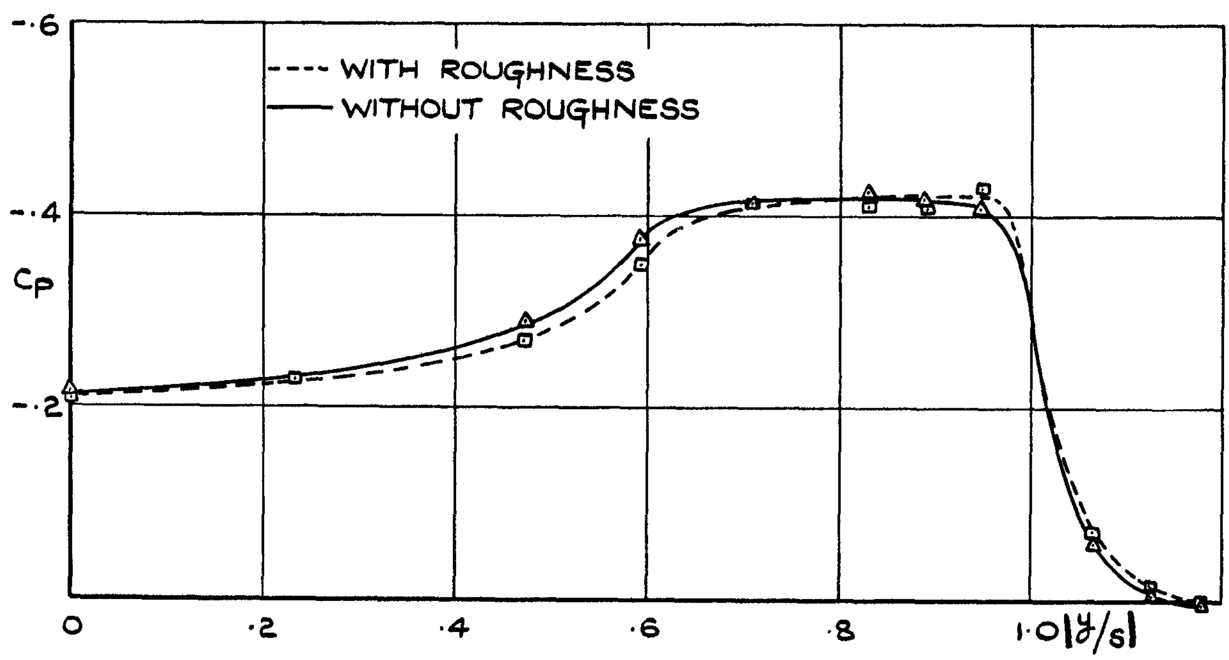


FIG. 9e. OIL FLOW STUDIES. $M_1 = 1\cdot55$



(a) $M_1 = 1{\cdot}45$



(b) $M_1 = 1{\cdot}55$

FIG.10. EFFECTS OF ADDING ROUGHNESS STRIPS AT THE LEADING EDGE OF THE MODEL.

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EXPERIMENTAL STUDY OF THE FLOW OVER A PARTICULAR AFTERBODY SHAPE HAVING A NEAR-SONIC RIDGE LINE. Treadgold, D. October, 1960.

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Pressures were measured and oil flow studies made. The results showed good agreement with exact inviscid flow theory in the expansion region in the vicinity of the ridge line. In the recompression region downstream, separation occurred and the flow was no longer conical so that no satisfactory theoretical estimates could be made.

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