ROYAL AIRCRAFT ESTABLISHMENT REDFORD.



MINISTRY OF AVIATION

AERONAUTICAL RESEARCH COUNCIL
CURRENT PAPERS

Pressure Distributions at Zero Lift for Delta Wings with Rhombic Cross Sections

Ьy

E. Eminton

LONDON: HER MAJESTY'S STATIONERY OFFICE

1960

FOUR SHILLINGS NET

U.D.C. No. 533.693.3: 533.6.048.2

October, 1959

PRESSURE DISTRIBUTIONS AT ZERO LIFT FOR DELTA WINGS WITH RHOMBIC CROSS SECTIONS

bу

E. Eminton

SUMMARY

The linearised theory of thin wings is used to calculate pressure distributions over delta wings with rhombic cross sections. A DEUCE programme has been written for the calculation and some of the results are compared with those of slender thin wing theory.

LIST OF CONTENTS

	Page
1 INTRODUCTION	3
2 CALCULATION OF C BY THIN WING THEORY	3
3 WINGS WITH SEXTIC AREA DISTRIBUTIONS	8
4 DISCUSSION	9
LIST OF SYMBOLS	9
LIST OF REFERENCES	10
TABLES 1,2	12
ILLUSTRATIONS - Figs.1-5	-
DETACHABLE ABSTRACT CARDS	
LIST OF TABLES	
Table	
1 C_{p_1} , C_{p_2} , C_{p_3} , C_{p_4} for $\beta s = 0.416$, $y/s = 0.05$, 0.575	12
$c_{p_1}, c_{p_2}, c_{p_3}, c_{p_4}$ for $\beta s = 0.577, \frac{y}{s} = 0.05, 0.575$	12
LIST OF ILLUSTRATIONS	
をできたり出し、水・塩素のできたのは、水・塩素の水・水・塩の水・水・塩の水・水・水・水・水・水・水・水・水・水・水・水・水・水	Fig.
Choice of co-ordinate axes Oxyz	1
Delta planform with Mach lines showing area of integration	2
Pressure distributions for wing I, $S(x) = 0.12 x^2(1-x)$, $\beta s = 0.416$	3(a)
Pressure distributions for wing I, $S(x) = 0.12 x^{2}(1-x)$,)(a)
$\beta s = 0.577$	3(b)
Pressure distributions for wing I, $S(x) = 0.12 x^2(1-x)$, $\beta s = 0.727$	7(.)
Pressure distributions for wing I, $S(x) = 0.12 x^{2}(1-x)$,	3(0)
$\beta s = 0.872$	3(a)
Pressure distributions for wing V, $S(x) = 0.07 x^2(1-x)$ $(4-6x + 4x^2 - x^3)$, $\beta s = 0.416$	
•	4(a)
Pressure distributions for wing V, $S(x) = 0.07 x^2(1-x)$ $(4-6x + 4x^2 - x^3)$, $\beta s = 0.577$	4(b)
Pressure distributions for wing V, $S(x) = 0.07 x^2(1-x)$ $(4-6x+4x^2-x^3)$, $\beta s = 0.727$	+(°)
_	4(c)
Pressure distributions for wing V, $S(x) = 0.07 x^2(1-x)$ $(4-6x + 4x^2 - x^3)$, $\beta s = 0.872$	1 (1)
Pressure distributions for wing II, $S(x) = 0.3 x^2 (1-x)^2$,	4(d)
$\beta s = 0.327$	5(a)
Pressure distributions for wing II, $S(x) = 0.3 x^2 (1-x)^2$,	
$\beta s = 0.416$	5(b)
Pressure distributions for wing II, $S(x) = 0.3 x^2 (1-x)^2$, $\beta s = 0.577$	5(°)
	21-1

1 INTRODUCTION

To evaluate the pressure distribution over the surface of a thin wing at zero lift in a supersonic stream it is usual to make such assumptions anent the thickness of the wing and the Mach number of the stream as allow the potential equation to be linearised together with the boundary conditions that govern its solution. Under these assumptions the pressure is also linearised and may be expressed in the form of a double integral. The ease with which this expression can be evaluated depends on the planform of the wing and its thickness distribution. For a few wings of simple geometry at least one integration can be done analytically, but in general numerical methods are needed and these are complicated by the singular nature of the integrand.

If, however the wing is not only thin but sufficiently slender in planform the expression for the pressure can be reduced to a form which is far more amenable to both analytical and numerical evaluation. To take advantage of this, however, some assessment is needed of the extent to which the extra assumption of slenderness is valid; some comparison of the pressure distributions produced by slender thin wing theory with those produced by thin wing theory is required.

In the 8 ft tunnel at Bedford, a series of tests is in progress on a family of wings which have delta planforms, rhombic cross sections and sextic area distributions. Pressure distributions have been obtained at several Mach numbers for each wing tested. At Bedford too the corresponding slender thin wing theory pressure distributions have been computed. The thin wing theory pressure distributions have been computed at Farnborough using a programme written for DEUCE.

The purpose of this note is simply to present a few of the thin wing theory results in a form which makes simple the calculation of pressure distributions for other wings of the same family, and to use the results to show how the pressure distributions of slender thin wing theory compare with those of thin wing theory for such wings.

2 CALCULATION OF C BY THIN WING THEORY

Consider a thin pointed wing at zero lift in a supersonic stream which is uniform ahead of the disturbance caused by the wing. Let viscosity and heat conduction be neglected. Choose co-ordinate axes Oxyz as shown in Fig.1, with the origin at the apex, Ox along the centre line and Oxy in the plane of the wing. Let the wing be symmetrical about the xy plane and let the function z(x,y) define its upper surface. Let V_0 , M_0 be the velocity and Mach number of the uniform stream and let $M_0^2 - 1 = \beta^2$.

By the linearised theory of thin wings the perturbation velocity potential ϕ at the point x,y,z is

$$\phi(x,y,z) = -\frac{v_0}{\pi} \iint \frac{\partial_z(x,y,t)}{\partial_x t} \frac{dx \cdot dy}{\sqrt{(x-x,t)^2 - \beta^2(y-y,t)^2 - \beta^2z^2}}$$
(1)

where the integration is over that part of the wing for which

 $x-x^{*}>\beta\sqrt{(y-y^{*})^{2}+z^{2}}$. This expression is the solution of a linearised partial differential equation with linearised boundary conditions. The assumptions implicit in its derivation are:

- (1) The local thickness of the wing is small compared with the local span and chord and all tangent planes to the surface are inclined to the uniform stream at angles that are small and not rapidly changing.
- (2) The Mach number of the uniform stream is not too near unity nor too large.

The pressure distribution on the wing surface to the same linear approximation is

$$C_{p}(x,y) = -\frac{2}{V_{o}} \frac{\partial \phi}{\partial x} (x,y,o)$$
 (2)

so the expression we wish to evaluate is

$$C_{p}(x,y) = \frac{2}{\pi} \frac{\partial}{\partial x} \iint \frac{\partial z(x,y')}{\partial x'} \frac{\partial x' \partial y'}{\sqrt{(x-x')^{2} - \beta^{2}(y-y')^{2}}}$$
(3)

where the integration is over that part of the wing for which $x-x^* > \beta |y-y^*|$.

For a wing with delta planform of unit length and semi span s, and with rhombic cross sections

$$z(x,y) = z(x,0)\left(1 - \frac{|y|}{sx}\right), \qquad (4)$$

and

$$\frac{\partial}{\partial x} z(x,y) = g(x) + \frac{|y|}{s} h(x) , \qquad (5)$$

where

$$g(x) = \frac{\partial}{\partial x} z(x,0), \quad h(x) = -\frac{\partial}{\partial x} \frac{z(x,0)}{x}. \tag{6}$$

Whatever the shape of the centre section z(x,o) the integration with respect to y^{\dagger} can be done analytically giving

$$\int \frac{\partial z(x'y')}{\partial x'} \frac{dy'}{\sqrt{(x-x')^2 - \beta^2(y-y')^2}} = \mp \frac{h(x')}{\beta^2 s} \sqrt{(x-x')^2 - \beta^2(y-y')^2}$$

$$- \frac{1}{\beta} (g(x') \pm \frac{y}{s} h(x')) \sin^{-1} \frac{\beta(y-y')}{x-x'}$$

$$+ \text{ an arbitrary function of } x'$$
(7)

the upper sign being required when y' > 0 and the lower when y' < 0.

By looking at Fig.2, which represents a delta wing with a pair of Mach lines defining the area of integration, we can see that for any function f(x,y)

 $\iint f(x,y')dx'dy' \text{ over that part of the wing for which } x-x' > \beta |y-y'|$

$$= \int_{\mathbf{x}^{t} = 0}^{\mathbf{x} - \beta \mathbf{y}} d\mathbf{x}^{t} \left(\int_{\mathbf{y}^{t} = \mathbf{y} - \frac{\mathbf{x} - \mathbf{x}^{t}}{\beta}}^{\mathbf{y} + \frac{\mathbf{x} - \mathbf{x}^{t}}{\beta}} \int_{\mathbf{y}^{t} = 0}^{\mathbf{y} + \frac{\mathbf{x} - \mathbf{x}^{t}}{\beta}} \mathbf{x}^{t} \int_{\mathbf{y}^{t} = \mathbf{y} - \frac{\mathbf{x} - \mathbf{x}^{t}}{\beta}}^{\mathbf{y}^{t} + \frac{\mathbf{x} - \mathbf{x}^{t}}{\beta}} \int_{\mathbf{y}^{t} = 0}^{\mathbf{y}^{t} + \frac{\mathbf{x} - \mathbf{x}^{t}}{\beta}} \mathbf{x}^{t} \int_{\mathbf{y}^{t} = \mathbf{y} - \frac{\mathbf{x} - \mathbf{x}^{t}}{\beta}}^{\mathbf{y}^{t} + \frac{\mathbf{x} - \mathbf{x}^{t}}{\beta}} \int_{\mathbf{y}^{t} = 0}^{\mathbf{y}^{t} + \frac{\mathbf{x} - \mathbf{x}^{t}}{\beta}} \mathbf{x}^{t} \int_{\mathbf{y}^{t} = 0}^{\mathbf{y} + \frac{\mathbf{x} - \mathbf{x}^{t}}{\beta}}^{\mathbf{y}^{t} + \frac{\mathbf{x} - \mathbf{x}^{t}}{\beta}}$$

$$-\int_{1+\beta s}^{x-\beta y} dx^{t} \int_{1+\beta s}^{-sx^{t}} f(x,y^{t})dy^{t} - \int_{1+\beta s}^{x+\beta y} dx^{t} \int_{1+\beta s}^{y+\frac{x-x^{t}}{\beta}} f(x,y^{t})dy^{t}$$

$$x^{t} = 0 \qquad y^{t} = y - \frac{x-x^{t}}{\beta} \qquad x^{t} = 0 \qquad y^{t} = sx^{t}$$
(8)

If we allow (4) to define the function z(x,y) both on and off the wing we can use (7) and (8) to write

$$\begin{split} & - \frac{\pi}{V_0} \ \phi(x,y,o) = \int\limits_{x^*=0}^{x - \beta y} \left(\frac{2h(x^*)}{\beta^2 s} \ \sqrt{(x - x^*)^2 - \beta^2 y^2} + \frac{2yh(x^*)}{\beta s} \ \sin^{-1} \frac{\beta y}{x - x^*} + \frac{\pi g(x^*)}{\beta} \right) dx^* \\ & + \int\limits_{x^*=x - \beta y} \frac{1}{\beta} \left(g(x^*) + \frac{y}{s} h(x^*) \right) \pi \ dx^* \\ & - \int\limits_{x^*=0}^{\frac{x - \beta y}{\beta^2 s}} \left(\frac{h(x^*)}{\beta^2 s} \sqrt{(x - x^*)^2 - \beta^2 (y + sx^*)^2} \right. \\ & - \frac{1}{\beta} \left(g(x^*) - \frac{y}{s} h(x^*) \right) \left(\sin^{-1} \frac{\beta(y + sx^*)}{x - x^*} - \frac{\pi}{2} \right) \right) dx^* \\ & - \int\limits_{x^*=0}^{\frac{x + \beta y}{\beta^2 s}} \left(\frac{h(x^*)}{\beta^2 s} \sqrt{(x - x^*)^2 - \beta^2 (-y + sx^*)^2} \right. \\ & - \frac{1}{\beta} \left(g(x^*) + \frac{y}{s} h(x^*) \right) \left(\sin^{-1} \frac{\beta(-y + sx^*)}{x - x^*} - \frac{\pi}{2} \right) dx^* \end{split}$$

Since the limits in (9) are all continuous functions of x, as are also the integrands and their derivatives between those limits, we may differentiate the integrals with respect to x and obtain

(9)

$$\frac{\pi}{2} C_{p}(x,y) = \frac{\pi}{\beta} \left(g(x) + \frac{y}{s} h(x) \right) + \int_{x^{1}=0}^{x-\beta y} \frac{2h(x^{1})}{\beta^{2}s} \frac{\sqrt{(x-x^{1})^{2}-\beta^{2}y^{2}}}{x-x^{1}} dx^{1}$$

$$- \int_{x^{1}=0}^{\frac{x-\beta y}{\beta^{2}s}} \frac{h(x^{1})}{\sqrt{(x-x^{1})^{2}-\beta^{2}(y+sx^{1})^{2}}} + \left(g(x^{1}) - \frac{y}{s} h(x^{1}) \right) \frac{y+sx^{1}}{(x-x^{1})\sqrt{(x-x^{1})^{2}-\beta^{2}(y+sx^{1})^{2}}} dx^{1}$$

$$- \int_{x^{1}=0}^{\frac{x+\beta y}{1+\beta s}} \frac{h(x^{1})}{\beta^{2}s} \frac{x-x^{1}}{\sqrt{(x-x^{1})^{2}-\beta^{2}(-y+sx^{1})^{2}}} + \left(g(x^{1}) + \frac{y}{s} h(x^{1}) \right) \frac{-y+sx^{1}}{(x-x^{1})\sqrt{(x-x^{1})^{2}-\beta^{2}(-y+sx^{1})^{2}}} dx^{1}$$

$$- \int_{x^{1}=0}^{\infty} \frac{h(x^{1})}{\beta^{2}s} \frac{x-x^{1}}{\sqrt{(x-x^{1})^{2}-\beta^{2}(-y+sx^{1})^{2}}} + \left(g(x^{1}) + \frac{y}{s} h(x^{1}) \right) \frac{-y+sx^{1}}{(x-x^{1})\sqrt{(x-x^{1})^{2}-\beta^{2}(-y+sx^{1})^{2}}} dx^{1}$$

$$- \int_{x^{1}=0}^{\infty} \frac{h(x^{1})}{\beta^{2}s} \frac{x-x^{1}}{\sqrt{(x-x^{1})^{2}-\beta^{2}(-y+sx^{1})^{2}}} + \left(g(x^{1}) + \frac{y}{s} h(x^{1}) \right) \frac{-y+sx^{1}}{(x-x^{1})\sqrt{(x-x^{1})^{2}-\beta^{2}(-y+sx^{1})^{2}}} dx^{1}$$

$$- \int_{x^{1}=0}^{\infty} \frac{h(x^{1})}{\beta^{2}s} \frac{x-x^{1}}{\sqrt{(x-x^{1})^{2}-\beta^{2}(-y+sx^{1})^{2}}} + \left(g(x^{1}) + \frac{y}{s} h(x^{1}) \right) \frac{-y+sx^{1}}{(x-x^{1})\sqrt{(x-x^{1})^{2}-\beta^{2}(-y+sx^{1})^{2}}} dx^{1}$$

$$- \int_{x^{1}=0}^{\infty} \frac{h(x^{1})}{\beta^{2}s} \frac{x-x^{1}}{\sqrt{(x-x^{1})^{2}-\beta^{2}(-y+sx^{1})^{2}}} dx^{1}$$

$$- \int_{x^{1}=0}^{\infty} \frac{h(x^{1})}{\beta^{2}s} \frac{x-x^{1}}{\sqrt{(x-x^{1})^{2}-\beta^{2}(-y+sx^{1})^{2}} dx^{1}$$

$$- \int_{x^{1}=0}^{\infty} \frac{h(x^{1})}{\beta^{2}s} \frac{x-x^{1}}{\sqrt{(x-x^{1})^{2}-\beta^{2}(-y+sx^{1})^{2}}} dx^{1}$$

$$- \int_{x^{1}=0}^{\infty}$$

These last two integrands become infinite at their upper limits in the same way as $^1/\sqrt{x}$ when $x \to 0$. In order to evaluate them numerically therefore we split the range of integration and introduce a transformation to remove the infinity. Writing, for example,

$$\int_{\mathbf{x'}=0}^{\mathbf{x}-\beta y} \frac{h(\mathbf{x'})}{\beta^{2}s} \frac{x-x'}{\sqrt{(x-x')^{2}-\beta^{2}(y+sx')^{2}}} + \left(g(x')-\frac{y}{s}h(x')\right) \frac{y+sx'}{(x-x')\sqrt{(x-x')^{2}-\beta^{2}(y+sx')^{2}}} dx'$$

$$\frac{x-ty/s}{1+t} = \int_{x'=0}^{h(x')} \frac{(h(x'))^{2}}{\beta^{2}s} \sqrt{\frac{x-x'}{(x-x')^{2}-\beta^{2}(y+sx')^{2}}} + \left(g(x') - \frac{y}{s}h(x')\right) \frac{y+sx'}{(x-x')\sqrt{(x-x')^{2}-\beta^{2}(y+sx')^{2}}} dx'$$

$$\sqrt{1-\beta^{2}s^{2}/t^{2}} + \int_{\mathbf{v}=0}^{1} \frac{1}{\beta} \left(h(\mathbf{x}^{!}) \frac{(\mathbf{y}_{/s}+\mathbf{x})}{\sqrt{1-\mathbf{v}^{2}(\beta s+\sqrt{1-\mathbf{v}^{2}})^{2}}} + \left(g(\mathbf{x}^{!}) - \frac{\mathbf{y}}{s} h(\mathbf{x}^{!}) \right) \frac{1}{\beta s+\sqrt{1-\mathbf{v}^{2}}} \right) d\mathbf{v}$$
(11)

where t is an arbitrary constant in the range $\beta s < t < x/\frac{y}{s}$ and $x' = \frac{x\sqrt{1-y^2}-\beta y}{\sqrt{1-y^2}+\beta s}$.

If we treat the other singular integral in the same way, splitting the range at $x' = \frac{x+ty/s}{1+t}$ and writing $x' = \frac{x\sqrt{1-w^2}+\beta y}{\sqrt{1-w^2}+\beta s}$, and also simplify the first integral

$$\frac{\frac{\pi}{2} C_{p}(x,y)}{= \frac{\pi}{\beta} \left(g(x) + \frac{y}{s} h(x) \right) + \int \frac{2h(x^{t})}{\beta^{2}s} \frac{u^{2}du}{u^{2} + \beta^{2}y^{2}}$$

$$\frac{x - ty/s}{-1 + t} \left(\frac{h(x^{t})}{\beta^{2}s} \frac{x - x^{t}}{\sqrt{(x - x^{t})^{2} - \beta^{2}(y + sx^{t})^{2}}} + \left(g(x^{t}) - \frac{y}{s} h(x^{t}) \right) \frac{y + sx^{t}}{(x - x^{t}) \sqrt{(x - x^{t})^{2} - \beta^{2}(y + sx^{t})^{2}}} \right) dx^{t}$$

$$- \int_{x^{t} = 0}^{4 + t} \left(\frac{h(x^{t})}{\beta^{2}s} \frac{x - x^{t}}{\sqrt{(x - x^{t})^{2} - \beta^{2}(-y + sx^{t})^{2}}} + \left(g(x^{t}) + \frac{y}{s} h(x^{t}) \right) \frac{-y + sx^{t}}{(x - x^{t})^{2} - \beta^{2}(-y + sx^{t})^{2}} \right) dx^{t}$$

$$- \int_{x^{t} = 0}^{4 + t} \frac{h(x^{t})}{\beta^{2}s} \frac{(y - y - y - y)}{\sqrt{1 - y^{2}(\beta s + \sqrt{1 - y^{2}})^{2}}} + \left(g(x^{t}) - \frac{y}{s} h(x^{t}) \right) \frac{1}{\beta s + \sqrt{1 - y^{2}}} dx^{t}$$

$$- \int_{x = 0}^{4 + t} \frac{1}{\beta} \left(h(x^{t}) \frac{(y - y - y)}{\sqrt{1 - y^{2}(\beta s + \sqrt{1 - y^{2}})^{2}}} + \left(g(x^{t}) - \frac{y}{s} h(x^{t}) \right) \frac{1}{\beta s + \sqrt{1 - y^{2}}} dx^{t}$$

$$- \int_{x = 0}^{4 + t} \frac{1}{\beta} \left(h(x^{t}) \frac{(-y - y + x)}{\sqrt{1 - y^{2}(\beta s + \sqrt{1 - y^{2}})^{2}}} + \left(g(x^{t}) + \frac{y}{s} h(x^{t}) \right) \frac{1}{\beta s + \sqrt{1 - y^{2}}} dx^{t}$$

$$- \int_{x = 0}^{4 + t} \frac{1}{\beta} \left(h(x^{t}) \frac{(-y - y + x)}{\sqrt{1 - y^{2}(\beta s + \sqrt{1 - y^{2}})^{2}}} + \left(g(x^{t}) + \frac{y}{s} h(x^{t}) \right) \frac{1}{\beta s + \sqrt{1 - y^{2}}} dx^{t}$$

$$- \int_{x = 0}^{4 + t} \frac{1}{\beta} \left(h(x^{t}) \frac{(-y - y + x)}{\sqrt{1 - y^{2}(\beta s + \sqrt{1 - y^{2}})^{2}}} + \left(g(x^{t}) + \frac{y}{s} h(x^{t}) \right) \frac{1}{\beta s + \sqrt{1 - y^{2}}} dx^{t}$$

$$- \int_{x = 0}^{4 + t} \frac{1}{\beta} \left(h(x^{t}) \frac{(-y - y + x)}{\sqrt{1 - y^{2}(\beta s + \sqrt{1 - y^{2}})^{2}}} + \left(g(x^{t}) + \frac{y}{s} h(x^{t}) \right) \frac{1}{\beta s + \sqrt{1 - y^{2}}} dx^{t}$$

$$- \int_{x = 0}^{4 + t} \frac{1}{\beta} \left(h(x^{t}) \frac{(-y - y + x)}{\sqrt{1 - y^{2}(\beta s + \sqrt{1 - y^{2}})^{2}}} + \left(g(x^{t}) + \frac{y}{s} h(x^{t}) \right) \frac{1}{\beta s + \sqrt{1 - y^{2}}} dx^{t}$$

$$- \int_{x = 0}^{4 + t} \frac{1}{\beta} \left(h(x^{t}) \frac{(-y - y + x)}{\sqrt{1 - y^{2}(\beta s + \sqrt{1 - y^{2}})^{2}}} + \left(g(x^{t}) + \frac{y}{s} h(x^{t}) \right) \frac{1}{\beta s + \sqrt{1 - y^{2}}} dx^{t}$$

$$- \int_{x = 0}^{4 + t} \frac{1}{\beta} \left(h(x^{t}) \frac{(-y - y + x)}{\sqrt{1 - y^{2}(\beta s + \sqrt{1 - y^{2}})^{2}}} + \left(g(x^{t}) + \frac{y}{s} h(x^{t}) \right) \frac{1}{\beta s + \sqrt{1 - y^{2}}} dx^{t}$$

$$- \int_{x = 0}^{4 +$$

$$= \frac{x\sqrt{1-v^2} - \beta y}{\sqrt{1-v^2} + \beta s}$$

$$= \frac{x\sqrt{1-w^2+\beta y}}{\sqrt{1-w^2+\beta s}}$$

and t is an arbitrary constant in the range $\beta s < t < x/\frac{y}{s}$. By choosing t so that none of the integrands become too large at their limits the expression for $C_p(x,y)$ in this form can be evaluated quite simply using standard formulae for integration.

There now exists a DEUCE programme to do this, which leaves open the choice of function z(x,o) from which g(x) and h(x) derive. There is freedom also in the choice of the constant t, but a value of 1 was found to be suitable for all calculations so far. The programme uses Weddle's rule over

30 steps (or any multiple of 30), which expresses an integral as a weighted sum of equally spaced function values. Thus we have the means of calculating the pressure distribution over any wing of delta planform, rhombic cross sections and arbitrary centre section shape z(x,0) at supersonic speeds, according to the linearised theory of thin wings.

3 WINGS WITH SEXTIC AREA DISTRIBUTIONS

The cross sectional area S(x) of a wing with delta planform of unit length and semi span s, and with rhombic cross sections is

$$S(x) = 2sxz(x,0), 0 \le x \le 1$$
 (13)

S(x) may therefore be used instead of z(x,o) to define the functions g(x) and h(x) which appear in equation (12) for C_{p} . From (6)

$$g(x) = \frac{1}{s} \frac{\partial}{\partial x} \frac{S(x)}{2x} , h(x) = -\frac{1}{s} \frac{\partial}{\partial x} \frac{S(x)}{2x^2}$$
 (14)

The wings chosen for the series of tests in the 8 ft tunnel at Bedford are members of the family having area distributions of the form

$$S(x) = x^{2}(1-x)(A+Bx+Cx^{2}+Dx^{3})$$
 (15)

Four simple members of this family are

$$S_1(x) = x^2(1-x),$$

 $S_2(x) = x^3(1-x),$
 $S_3(x) = x^4(1-x),$
 $S_L(x) = x^5(1-x).$ (16)

If the pressures corresponding to these wings are c_{p_1} , c_{p_2} , c_{p_3} , c_{p_4} then the pressure on the general wing of the family is

$$C_{p} = AC_{p_{1}} + BC_{p_{2}} + CC_{p_{3}} + DC_{p_{L}}$$
 (17)

From (14) and the form of equation (12) for C_p we can see that for any given S(x), C_p is a function of x, y/s and βs only. If therefore we use DEUCE to work out these four elementary pressure distributions for a range of βs we have the pressure distribution over any wing of the family for a corresponding range of Mach number by simply adding together appropriate multiples of the DEUCE results. As an illustration Tables 1 and 2 give the four elementary pressure distributions for $\beta s = 0.416$ and 0.577 along y/s = 0.05 and 0.575. From these we can deduce the pressure distributions along the same chords for each of the wings in the Bedford tests at two Mach numbers. For those wings which have s = 1/3 these Mach numbers are 1.6 and 2.0

There are three types of wing involved in the tests; they are wings I, II and V of references 2, 3 and 4. The area distributions which define them are

I
$$S(x) = 0.12 x^{2}(1-x)$$

II $S(x) = 0.3 x^{2}(1-x)^{2}$
and $V S(x) = 0.07 x^{2}(1-x)(4-6x+4x^{2}-x^{3})$

which is known as the "Lord V" area distribution. The extent of the computations has been governed by the requirements of the tunnel tests which have dictated both the Mach numbers to be considered and the distribution of points over each wing. These theoretical results were primarily intended to help in the analysis of the tunnel results. They are also given at the end of this note in order to form some comparison between the thin wing theory from which they derive and the results of slender thin wing theory.

4 DISCUSSION

Figs. 3, 4 and 5 show the pressure distributions of thin wing theory and slender thin wing theory for each of the three wings under consideration. The calculations stop short of the leading edge since the thin wing assumptions are not valid there and both theories give logarithmic infinities. A technique for circumventing this difficulty has been devised by Randall⁵. At the trailing edge theory predicts a gradual compression without shocks; the physical interpretation of this result has been discussed by Weber³.

The values of the slenderness parameter β s are rather too high to be realistic; not only is the slenderness assumption violated but thin wing theory itself becomes suspect as β s approaches 1. But they serve to illustrate the nature of the discrepancies between the two theories.

All three wings illustrate how the pressures of slender thin wing theory approximate more closely to those of thin wing theory as β s decreases. On wings I and V slender theory gives smaller suctions towards the trailing edge and hence lower drag but on wing II the situation is reversed. This agrees with the results of reference 4 (in Figs. 7, 8 and 9) which considers the variation of drag with β s.

At all values of β s agreement is better for wing V than it is for the other wings with their larger expansions and compressions. Such a wing, without large pressure gradients, is also favourable to boundary layer development so that the predictions of these inviscid theories are likely to be fulfilled in real flow.

LIST OF SYMBOLS

$C_{p}(x,y)$	pressure	coefficient	at	the	point	х, у
0	_				_	,,,

$$g(x) = \frac{\partial}{\partial x} z(x,0)$$

$$h(x) = -\frac{\partial}{\partial x} \frac{z(x,0)}{x}$$

Mach number of the uniform stream

s semi span of a delta wing of unit length

S(x) cross sectional area at the point x

LIST OF SYMBOLS (Contd.)

t arbitrary constant in the range $\beta s < t < x/\sqrt{s}$ V_o velocity of the uniform stream $= \sqrt{(x-x')^2 - \beta^2 y^2}$ $= \sqrt{(x-x')^2 - \beta^2 (y+sx')^2/(x-x')}$ transformations used in integration $= \sqrt{(x-x')^2 - \beta^2 (-y+sx')^2/(x-x')}$ x,y,zcartestian co-ordinates (see Fig.1) x,y,z'the same when used as running co-ordinates $= \sqrt{M_o^2 - 1}$ $\phi(x,y,z)$ velocity potential at the point x,y,z

LIST OF REFERENCES

Ref. No.	Author	Title, etc.
1	-	Interpolation and allied tables. P.70 H.M.S.O. 1956
2	Lord, W.T. Brebner, G.G.	Supersonic flow past narrow wings with 'similar' cross sections at zero lift. Unpublished 1.0.A. Report.
3	Weber, J.	Slender delta wings with sharp edges at zero lift. ARC 19543, May, 1957.
4	Weber, J.	Some notes about the zero lift wave drag of slender wings with unswept trailing edge. Unpublished M.O.A. Report.
5	Randall, D.G.	An improvement of the velocity distribution predicted by linear theory for wings with straight subsonic leading edges. ARC CP No.418 September, 1958.

 $\frac{\text{TABLE 1}}{\beta s} = 0.416 \quad (s = 1/3, M_o = 1.6)$

у/з	ж	C P1	с _{Р2}	С _р з	C _{P/4}
O _• O5	0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9	0.901 0.560 0.282 0.012 -0.255 -0.522 -0.788 -1.054 -1.319 -1.585	0.218 0.351 0.392 0.341 0.198 -0.035 -0.359 -0.774 -1.280 -1.876	0.035 0.125 0.228 0.303 0.311 0.211 -0.035 -0.469 -1.128 -2.053	0.005 0.037 0.105 0.196 0.275 0.289 0.160 -0.209 -0.937 -2.163
0.575	0.6 0.7 0.8 0.9 1.0	0.284 -0.321 -0.711 -1.047 -1.360	0.463 0.106 -0.280 -0.753 -1.320	0.393 0.257 -0.020 -0.509 -1.252	0.279 0.278 0.120 -0.324 -1.188

 $\frac{\text{TABLE 2}}{\beta s = 0.577}$ (s = 1/3, N_o = 2.0)

y/s	x	C _{P1}	C _{p2}	C _{P3}	C _{P4}
0, 05	0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0	0.778 0.484 0.253 0.031 -0.188 -0.407 -0.625 -0.842 -1.060 -1.278	0.178 0.291 0.328 0.290 0.180 -0.004 -0.260 -0.590 -0.993 -1.469	0.027 0.101 0.185 0.249 0.260 0.187 -0.001 -0.336 -0.849 -1.571	0.004 0.029 0.083 0.157 0.223 0.240 0.147 -0.131 -0.686 -1.628
0.575	0.6 0.7 0.8 0.9 1.0	0.443 -0.144 -0.493 -0.785 -1.051	0.464 0.169 -0.132 -0.503 -0.948	0.331 0.238 0.045 -0.311 -0.865	0.213 0.223 0.124 -0.184 -0.809

	• •			
•			,,	

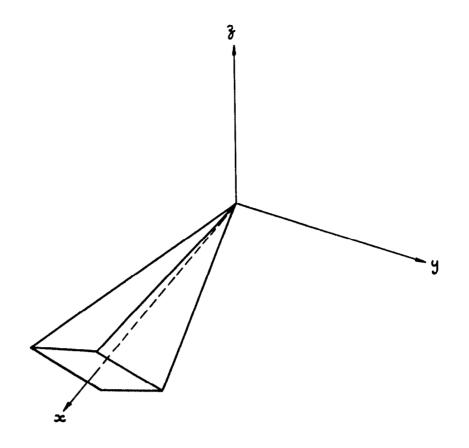


FIG.I CHOICE OF CO-ORDINATE AXES Oxy

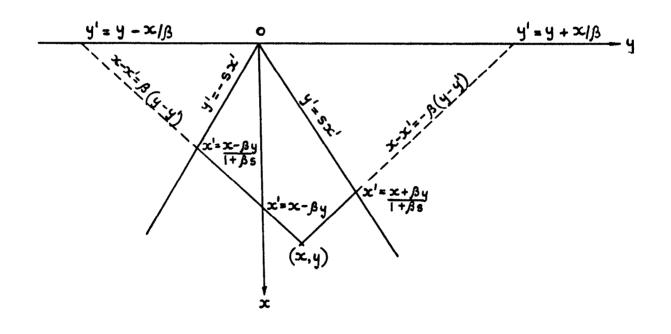


FIG.2 DELTA PLANFORM WITH MACH LINES SHOWING AREA OF INTEGRATION

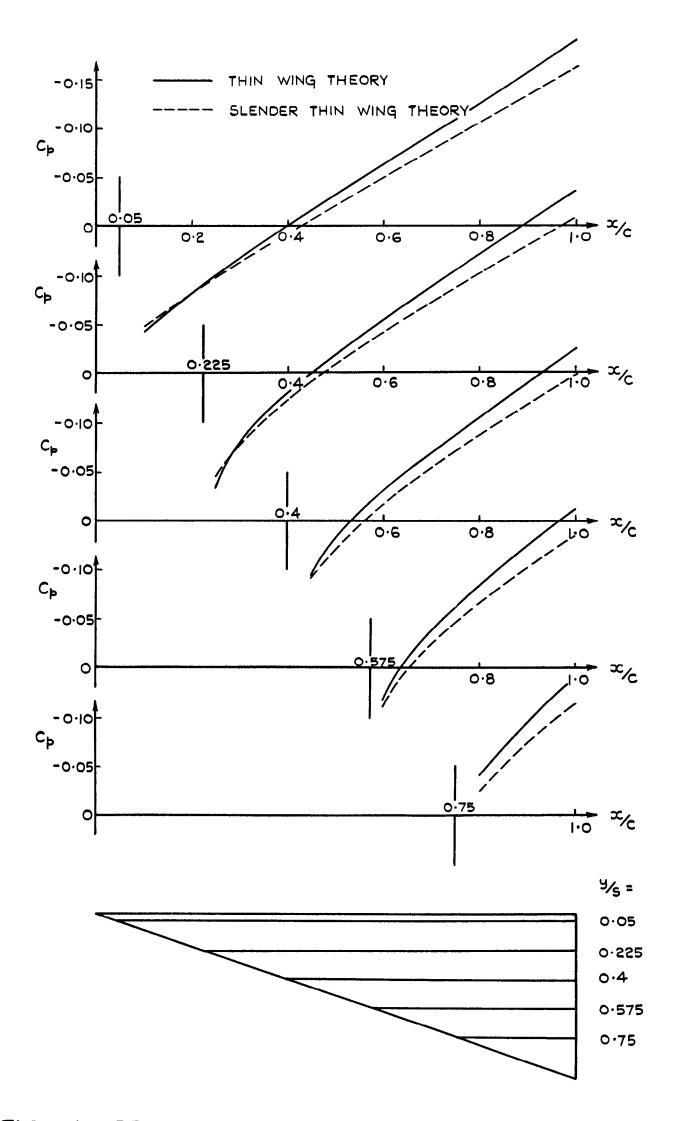


FIG.3(a) PRESSURE DISTRIBUTIONS FOR WING I $S(x) = O \cdot 12 x^2(1-x)$, $\beta s = O \cdot 416$

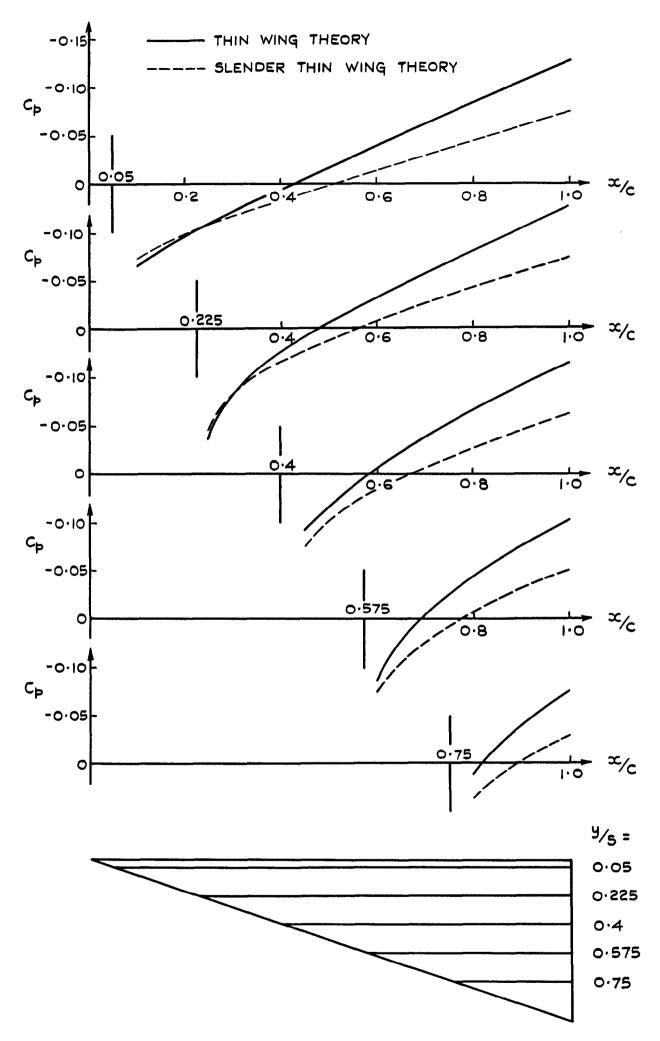


FIG.3(c) PRESSURE DISTRIBUTIONS FOR WING I $S(x) = O \cdot 12 x^2 (1-x)$, $\beta s = O \cdot 727$

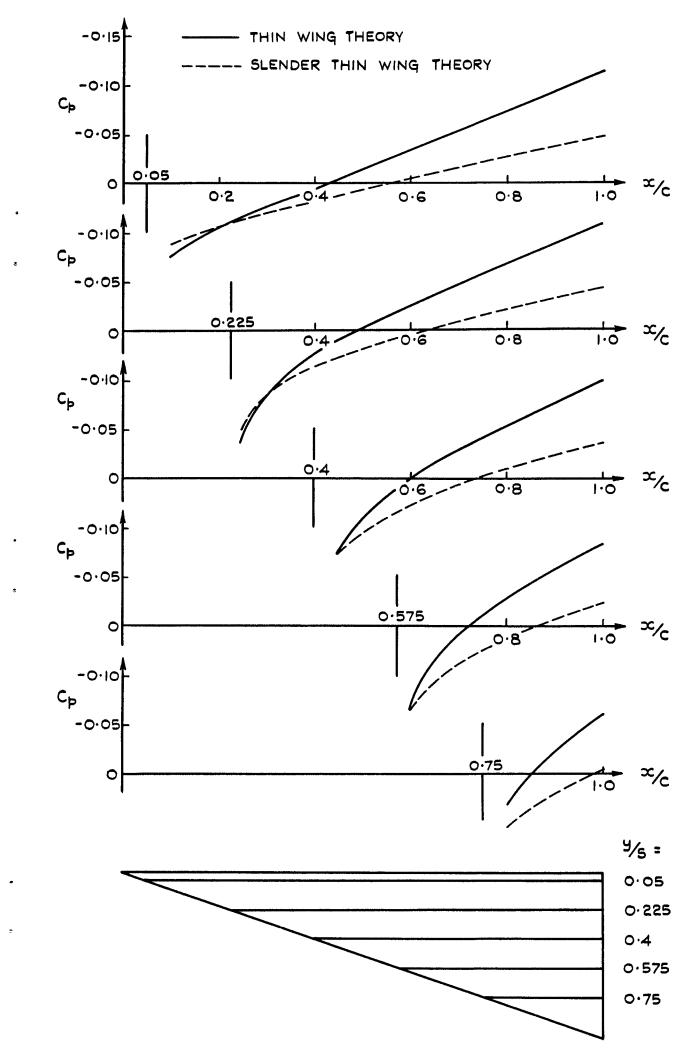


FIG.3(d) PRESSURE DISTRIBUTIONS FOR WING I $S(x) = O \cdot 12 x^{2}(1-x)$, $\beta s = O \cdot 872$

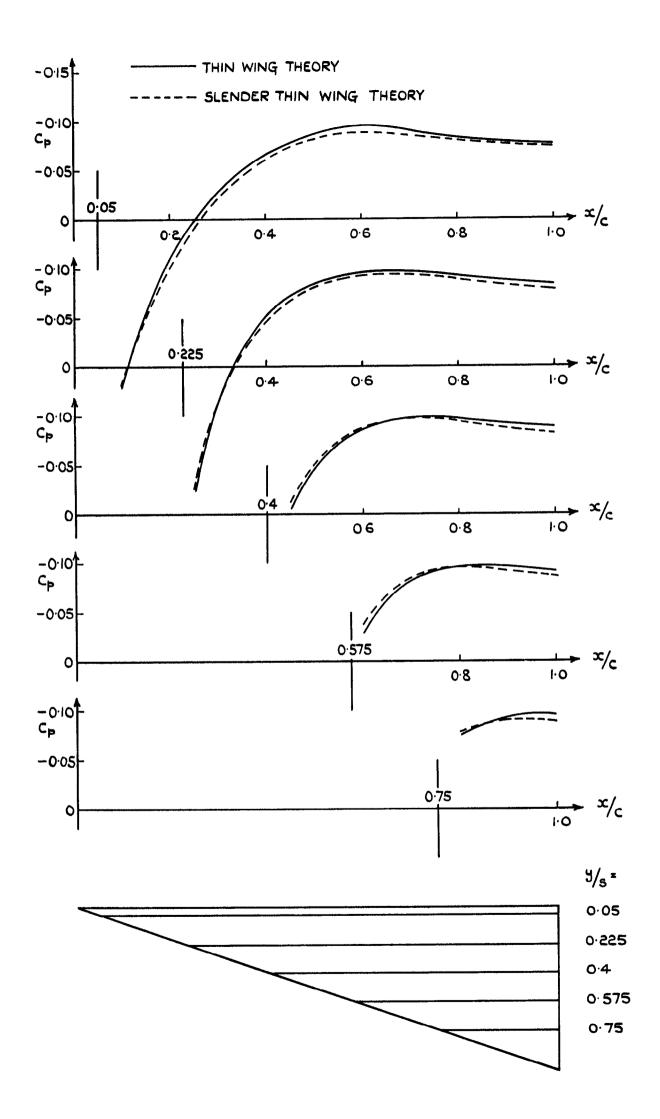


FIG 4(a) PRESSURE DISTRIBUTIONS FOR WING ∇ S (x) = 0.07 $x^2 (1-x) (4-6x+4x^2-x^3)$, β s = 0.416.

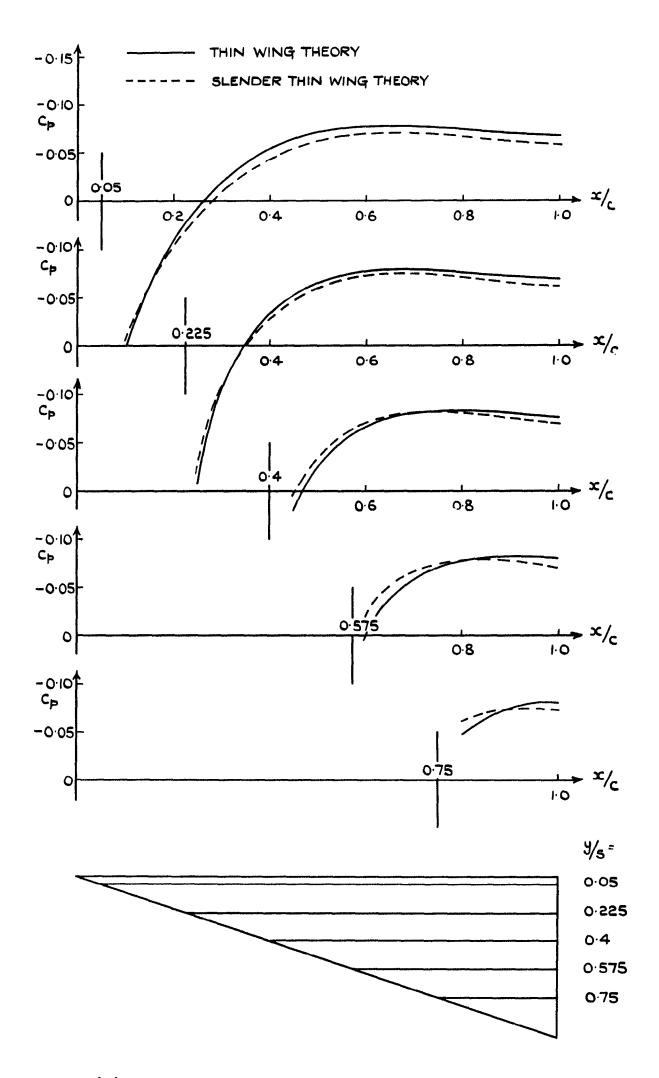


FIG 4(b) PRESSURE DISTRIBUTIONS FOR WING ∇ S(x) = 0.07 x² (1-x) (4-6x+4x²-x³), β s=0.577.

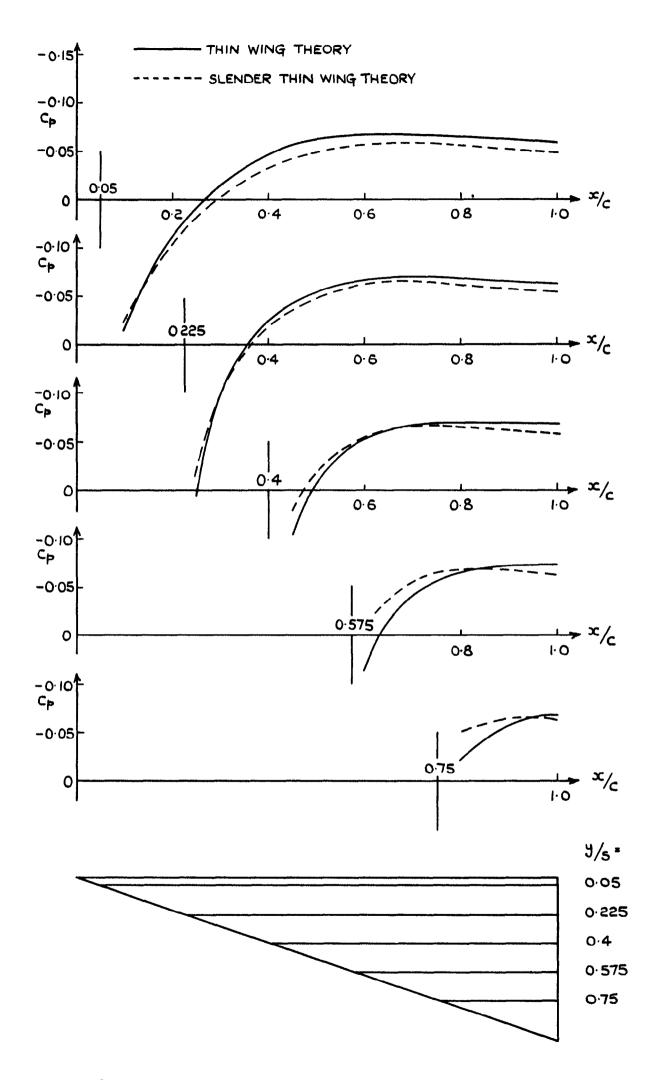


FIG. 4(c) PRESSURE DISTRIBUTIONS FOR WING ∇ S(∞) = 0.07 ∞^2 (1- ∞) (4-6 ∞ + 4 ∞^2 - ∞^3), β s=0.727.

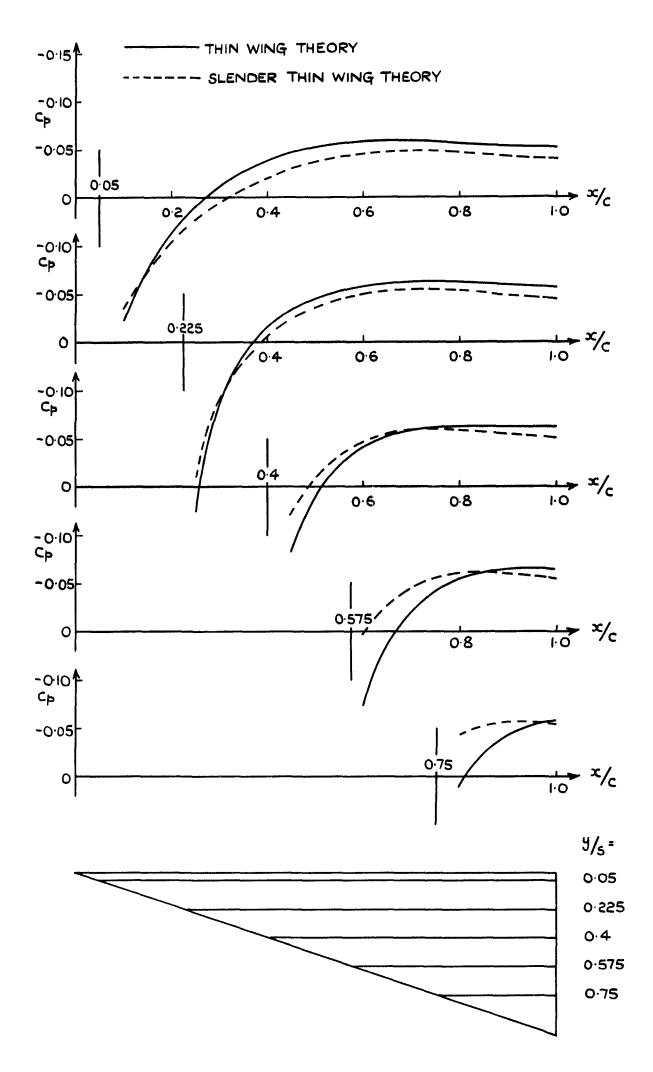


FIG. 4 (d) PRESSURE DISTRIBUTIONS FOR WING $\nabla S(x) = O \cdot O7 x^2 (1-x) (4-6x+4x^2-x^3)$, $\beta s = O \cdot 872$.

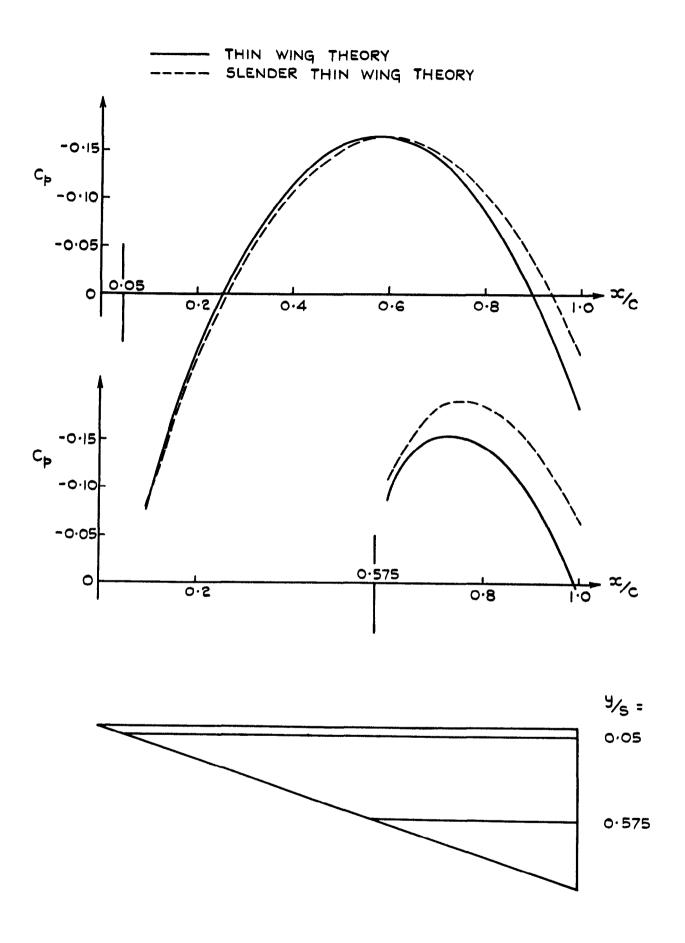


FIG.5(a) PRESSURE DISTRIBUTIONS FOR WING II $S(x) = O \cdot 3 x^2 (i-x)^2$, $\rho s = O \cdot 327$

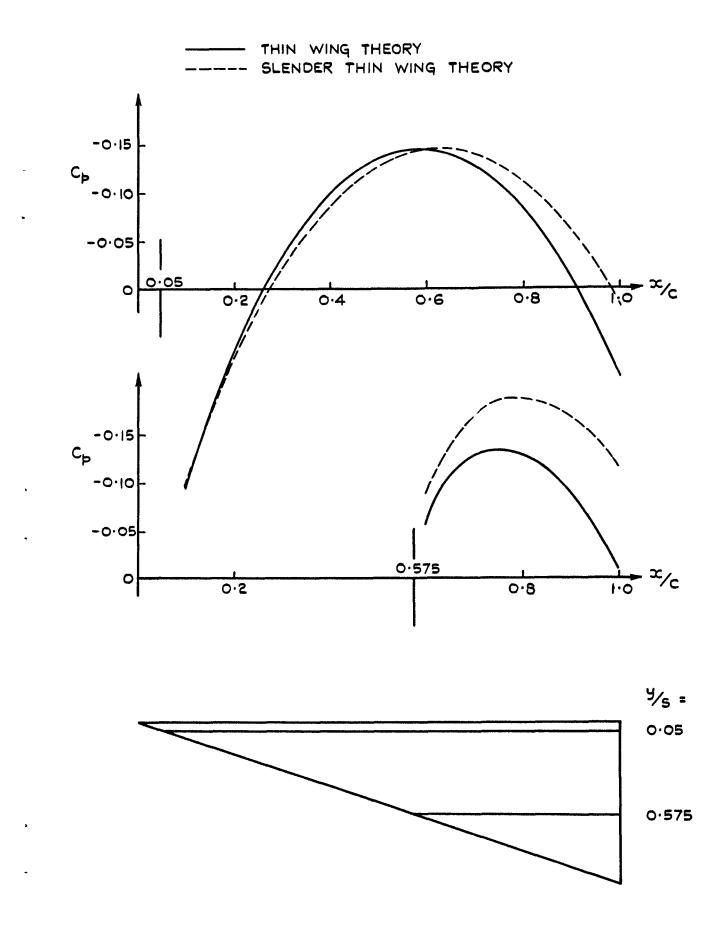


FIG.5(b) PRESSURE DISTRIBUTIONS FOR WING II $S(x) = 0.3 x^2 (1-x)^2$, $\beta s = 0.416$

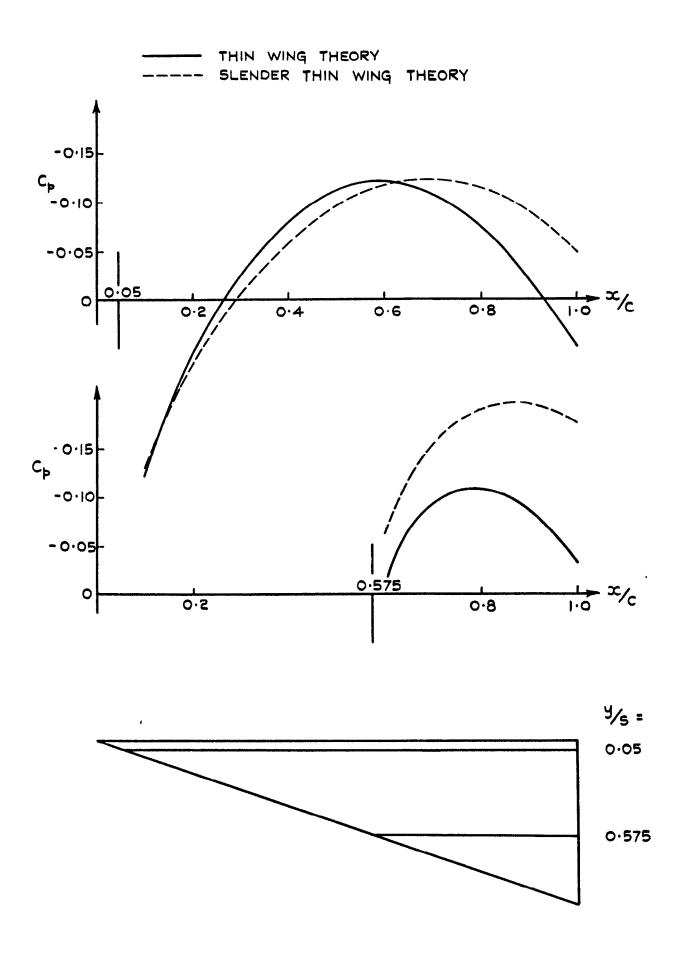


FIG.5(c) PRESSURE DISTRIBUTIONS FOR WING II $S(x) = 0.3 x^2(1-x)^2$, ps = 0.577

C.P. No. 525	533 . 693 . 3: 533.6.048.2	C.F. No. 525	533 , 693 , 3: 533, 6,048, 2
PRESSURE DISTRIBUTIONS AT ZERO LIFT FOR DELTA WINGS WITH RHOMBIC CROSS SECTIONS. Emirton, E. Oct. 1959.	1,2,2,2,3,1 1,2,2,6,4	PRESSURE DISTRIBUTIONS AT ZERO LIFT FOR DELTA WINGS WITH RHOMBIC CROSS SECTIONS. Eminton, E. Oct. 1959.	1,2,2,2,3,1 1,2,2,6,4
The linearised theory of thin wings is used to calc	ulate pressure	The linearised theory of thin wings is used to ca	lculate pressure

The linearised theory of thin wings is used to calculate pressure distributions over delta wings with rhombic cross sections. A DEUCE programme has been written for the calculation and some of the results are compared with those of slender thin wing theory.

The linearised theory of thin wings is used to calculate pressure
distributions over delta wings with rhombic cross sections. A DEUCE
programme has been written for the calculation and some of the results
are compared with those of slender thin wing theory.

C.P. No. 525	533.693.3° 533.6.048.2
PRESSURE DISTRIBUTIONS AT ZERO LIFT FOR DELTA WINGS WITH RHOMBIC-CROSS-SECTIONS. Eminton, E. Oct. 1959.	1,2,2,2,3,1 1,2,2,6,4

The linearised theory of thin wings is used to calculate pressure distributions over delta wings with rhombic cross sections. A DEUCE programme has been written for the calculation and some of the results are compared with those of slender thin wing theory.

© Crown Copyright 1960 Published by HER MAJESTY'S STATIONERY OFFICE

To be purchased from
York House, Kingsway, London w.c.2
423 Oxford Street, London w.1
13a Castle Street, Edinburgh 2
109 St. Mary Street, Cardiff
39 King Street, Manchester 2
50 Fairfax Street, Bristol 1
2 Edmund Street, Birmingham 3
80 Chichester Street, Belfast 1
or through any bookseller

Printed in England