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COMPARISON OF THEORETICAL AND MEASURED SURFACE PRESSURES AT M = 1.2 ON THREE BODIES HAVING DIFFERENT WAISTINGS

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SUMMARY

Surface pressures have been calculated at $M = 1.2$ and zero incidence for three bodies, the first with a nose followed by a cylindrical section of constant radius, the second having a waist of circular cross-section and the third having a waist of elliptical cross-section with constant depth. These results have been compared with the surface pressures measured in the 8 ft by 6 ft transonic tunnel at a Reynolds number of approximately 4×10^6 based on body length.

Agreement between theory and experiment is found to be reasonably good except in regions where the flow expands rapidly round discontinuities in the surface shape.

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1 INTRODUCTION

Pressures measured on the surfaces of bodies at $M = 1.2$ (Ref.1) have been used to check the accuracy of two of the theories at present being used in the design of waisted bodies for wing-body combinations at zero incidence. One unwaisted and two waisted bodies have been considered. The unwaisted body and one of the waisted bodies were of circular cross-section, the remaining body having an elliptic cross-section of constant depth over the waisted portion.

The surface pressure distributions on the axially-symmetric bodies have been calculated using the combined quasi cylinder and slender body method of Warren and Fraenkel[2], and the pressure distribution produced by the elliptic waisting of the third body by the method due to Nielsen[3].

2 DESCRIPTION OF BODIES

The three bodies considered are the circular unwaisted body, the circular-waisted body and the elliptic-waisted body of Ref.1; all have identical noses. The bodies and nose are illustrated in Fig.1 together with the system of axes used, and the ordinates for each body are given in Table 1.

The axial position of a surface pressure hole is measured from a plane E normal to the body axis and coincident with the beginning of the waisting. This plane makes a convenient origin for calculation and illustration purposes.

The experimental surface pressure holes are situated on the starboard upper quadrant along four lines given by $y = 0$ (the body top), $y = r_o/3$, $y = 2r_o/3$ and $y = r_o$ (the body side). Their axial positions are indicated in Figs.2 and 3.

3 CALCULATION METHODS

The methods used for determining the flow fields are now considered briefly. For this purpose it is convenient to employ the cylindrical polar co-ordinates illustrated in Fig.1.

3.1 Axially symmetric bodies (combined method)

In Ref.2 the quasi-cylinder and slender body theories for the supersonic flow past bodies of revolution are combined, and the velocity distribution on the surface of a body given as:-

$$\frac{v_x(X,r)}{V_o} = -\frac{1}{2\pi\beta r}\int_{\xi=0}^{X} U\left(\frac{X-\xi}{\beta r}\right) d\,S'(\xi) \qquad (1)$$

where $S(X)$ is the cross sectional area of the body, $S'(\xi)$ denotes dS/dX at $X = \xi$, and $U(X)$ is a function defined in Ref.2.

The pressure coefficient anywhere on the surface of the body can now be calculated using the quadratic approximation[4] to Bernoulli's equation in the form

$$C_p = -\frac{2v_x(X,r)}{V_o} - \left(\frac{dr}{dX}\right)^2 . \qquad (2)$$

3.2 Elliptic waisted body (Nielsen's method)

In Ref.3 Nielsen presents tables of characteristic functions (W functions) that are useful in problems of supersonic flow involving aerodynamic shapes which are quasi-cylinders of nearly circular cross-section. In an illustrative example he considers an infinitely long surface which lies everywhere near to the cylinder $r = R$ and which deviates only slightly in streamwise slope from the stream direction. He expresses the streamwise slope as a Fourier series, whose coefficients are functions of X:-

$$\frac{dr}{dX} = \sum_{m=0}^{\infty} f_m(X) \cos m\theta \tag{3}$$

and shows that the pressure coefficient anywhere in the external field is:-

$$C_p = \frac{2}{\beta} \sum_{m=0}^{\infty} \cos m\theta \left\{ \frac{f_m[X - \beta R(r/R - 1)]}{\sqrt{\frac{r}{R}}} - \frac{1}{\beta R} \int_0^{X-\beta R(r/R-1)} f_m(\xi)\, W_m\left[\frac{X}{\beta R} - \left(\frac{r}{R} - 1\right) - \frac{\xi}{\beta R}, \frac{r}{R}\right] d\xi \right\} \quad \ldots.(4)$$

where $\beta = \sqrt{M^2 - 1}$ and the functions $W_m(x,r)$ are tabulated.

For a body having elliptic cross-sections of small eccentricity and constant depth, the radius vector is given by the approximate equation:-

$$r = R - f(X) - f(X) \cos 2\theta \, . \tag{5}$$

In the horizontal plane of the elliptic waisted body, $\theta = 0$ we have:-

$$2f(X) = R - r_o \tag{6}$$

and the radius vector differs from the basic radius by $2f(X)$.

By differentiating equation (5) with respect to X and comparing with equation (3) we see that only the $m = 0$ and $m = 2$ terms need be retained in equation (4) and

$$f_o(X) = - f'(X) \tag{7}$$

$$f_2(X) = - f'(X) \, . \tag{8}$$

By differentiating equation (6) or substituting equations (7) and (8) into equation (3) it is seen that

$$f'(X) = - \frac{1}{2} \frac{dr_o}{dX} \, .$$

The pressure distribution at $r/R = 1$ (the surface of the quasi-cylindrical body) due to body waisting can be calculated using:-

$$C_p = \frac{2}{\beta} \sum_{m=0}^{2} \cos m\theta \left\{ f_m(X) - \frac{1}{\beta R} \int_0^X f_m(\xi)\, W_m \left[\frac{X-\xi}{\beta R}, 1 \right] d\xi \right\} . \qquad (10)$$

4 RESULTS AND DISCUSSION

The calculated pressure distributions and the measured surface pressures over the three bodies are presented in Figs.2 and 3. In the presentation use has been made of the rectangular Cartesian axes shown in Fig.1 and, following the example of Ref.1, the axial position of a point on the body surface has been expressed non-dimensionally as the number of root chord lengths aft of the origin E. Negative values of the axial position correspond to points forward of E.

The calculated pressure distribution for the nose is the same for all three bodies and is shown in Fig.2. The calculated pressure distributions for the circular-unwaisted body and the circular-waisted body are also given over the range of positive values of axial position considered. Apart from the discontinuity at the origin for the circular-waisted body, the "combined" method of Warren and Fraenkel predicts the trend of the measured surface pressures everywhere with reasonable accuracy.

The calculated pressure distribution due to body waisting on the elliptic-waisted body, along the four lines $y = 0$, $r_o/3$, $2r_o/3$ and r_o previously mentioned in Section 2, are presented in Fig.3. Before a comparison can be made with measured surface pressures a small correction must be made for the effect of the nose. This correction has been accomplished by deducting from the measured surface pressures over the elliptic-waisted body the corresponding surface pressures for the circular-unwaisted body, the difference being due to the body waisting only.

Agreemeht between theory and experiment is again reasonably good except in the region where the flow expands rapidly round the discontinuity in the body-side shape. Tests at higher Reynolds numbers and with varying boundary layer conditions ahead of the discontinuity will be necessary before it is possible to decide whether the theoretical methods need to be improved for this region.

It is clear from the results given in this Note that the accuracy of the methods considered is adequate for preliminary design purposes, though the effects of discontinuities in surface shape on the local flow need further examination.

5 CONCLUSIONS

Surface pressures have been calculated at $M = 1.2$ and zero incidence for three bodies, and the results have been compared with the surface pressures measured in the 8 ft x 6 ft transonic wind tunnel at a Reynolds number of approximately 4×10^6 based on body length.

Agreement between theory and experiment is found to be reasonably good except in regions where the flow expands rapidly round discontinuities in surface shape.

LIST OF SYMBOLS

C_p	$= \frac{p - p_\infty}{q}$ pressure coefficient
p	local static pressure
p_∞	free stream static pressure
q	free-stream dynamic pressure
M	free-stream Mach number
β	$\sqrt{M^2 - 1}$
v_x	velocity increment in X-direction
V_o	free stream velocity
r	local body radius, radius vector
r_o	value of the radius vector in the horizontal centre plane
(X,y,z)	rectangular co-ordinates
(X,r,θ)	cylindrical polar co-ordinates
$S(X)$	body cross sectional area
ξ	dummy variable of integration
U	function used in equation (1), see Ref.2
$f_m(X)$	velocity amplitude function for cos $m\theta$ distortions
R	basic body radius
ℓ	length of model, equal to 38.581 inches
$2f(X)$	amount the radius vector differs from the basic body radius in the horizontal centre plane
$W_m(X,r)$	characteristic function of order m, see Ref.3
c	root chord, equal to 7.5 inches

LIST OF REFERENCES

Ref. No.	Author	Title, etc.
1	Shaw, M. M.	Pressure measurements at transonic speeds on four bodies and four 55° swept wing body combinations. A.R.C. 21,635. July, 1959.

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Ref. No.	Author	Title, etc.
2	Warren, C. H. E. Fraenkel, L. E.	A combination of the quasi-cylinder and slender body theories. Journal of the Royal Aeronautical Society Vol.59, p.305. April 1955.
3	Nielsen, J. N.	Tables of characteristic functions for solving boundary-value problems of the wave equation with application to supersonic interference. N.A.C.A. Tech. Note No. 3873. Feb. 1957.
4	Ward, G. N.	Linearized theory of steady high-speed flow. Cambridge University Press 1955.

TABLE 1

Body Shapes

Circular unwaisted body		Circular waisted body		Elliptic waisted body	
Distance aft of "E" X (ins.)	Radius r(ins.)	Distance aft of "E" X (ins.)	Radius r(ins.)	Distance aft of "E" X (ins.)	Radius vector r_o(ins.)
-18.942	0	-18.942	0	-18.942	0
-17.142	0.360	-17.142	0.360	-17.142	0.360
-15.342	0.595	-15.342	0.595	-15.342	0.595
-13.542	0.790	-13.542	0.790	-13.542	0.790
-11.742	0.960	-11.742	0.960	-11.742	0.960
-9.942	1.108	-9.942	1.108	-9.942	1.108
-8.142	1.236	-8.142	1.236	-8.142	1.236
-6.342	1.345	-6.342	1.345	-6.342	1.345
-4.542	1.433	-4.542	1.433	-4.542	1.433
-2.142	1.500	-2.142	1.500	-2.142	1.500
"E" 0	1.500	"E" 0	1.500	"E" 0	1.500
0.75	1.500	0.75	1.418	0.75	1.409
1.50	1.500	1.50	1.328	1.50	1.283
2.25	1.500	2.25	1.236	2.25	1.137
3.00	1.500	3.00	1.160	3.00	1.005
3.75	1.500	3.75	1.100	3.75	0.900
4.50	1.500	4.50	1.058	4.50	0.825
5.25	1.500	5.25	1.032	5.25	0.780
6.00	1.500	6.00	1.022	6.00	0.762
6.75	1.500	6.75	1.029	6.75	0.769
7.50	1.500	7.50	1.031	7.50	0.777
8.25	1.500	8.25	1.040	8.25	0.794
9.00	1.500	9.00	1.054	9.00	0.820
9.75	1.500	9.75	1.074	9.75	0.855
10.50	1.500	10.50	1.100	10.50	0.899
11.25	1.500	11.25	1.132	11.25	0.952
12.00	1.500	12.00	1.171	12.00	1.014
13.50	1.500	13.50	1.253	13.50	1.147
15.00	1.500	15.00	1.335	15.00	1.280
16.50	1.500	16.50	1.417	16.50	1.413
18.00	1.500	18.00	1.479	18.00	1.489
19.639	1.500	19.639	1.500	19.639	1.500

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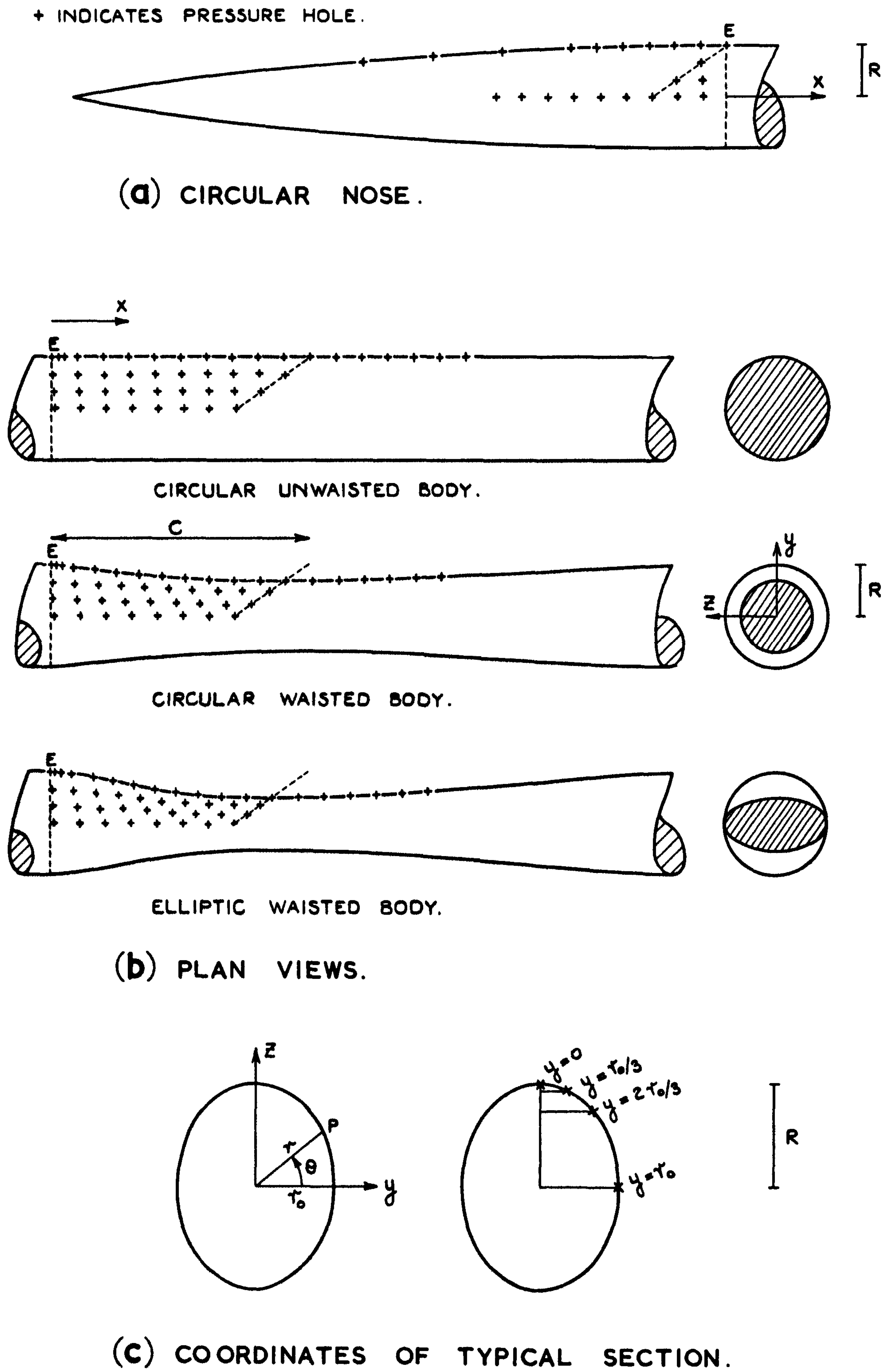


FIG.1. BODY SHAPES AND AXES.

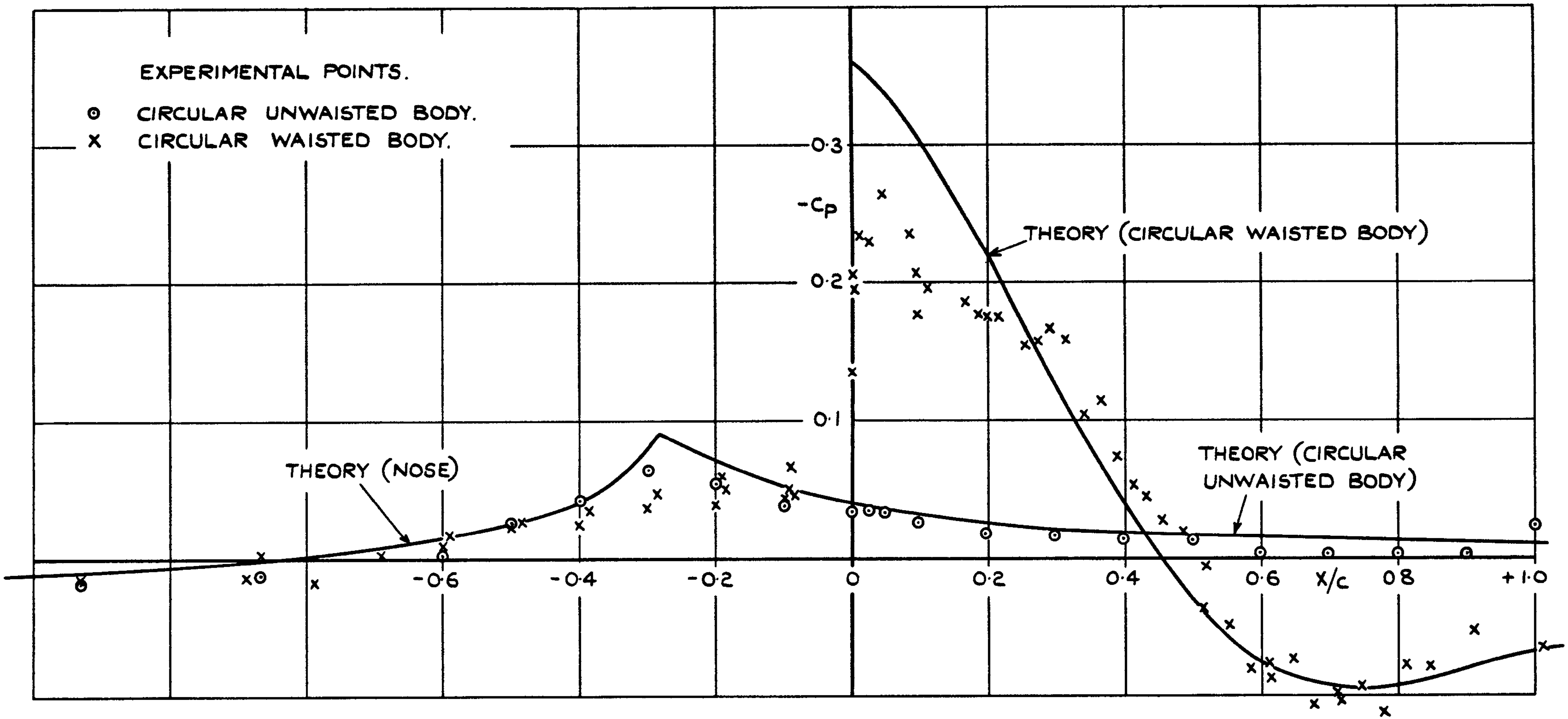
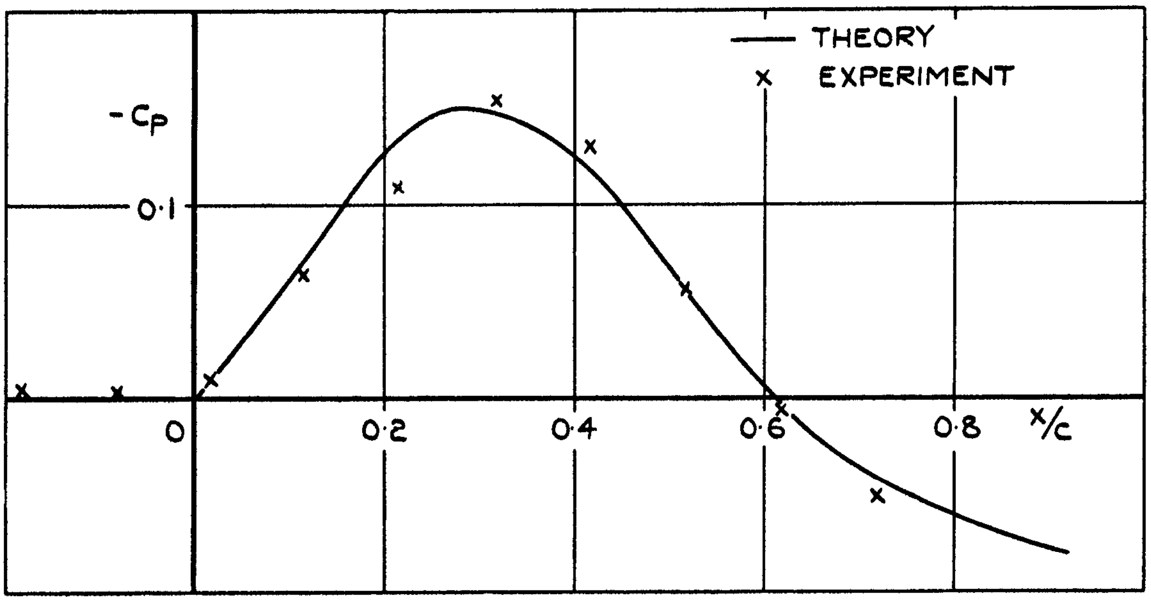
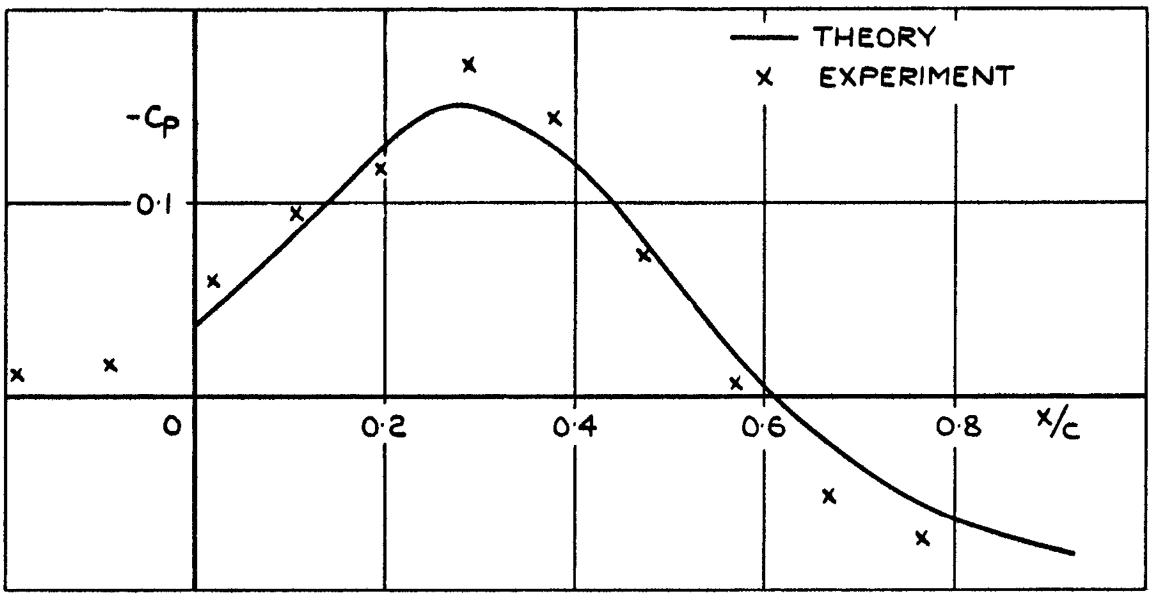


FIG. 2. PRESSURE DISTRIBUTIONS FOR AXIALLY SYMMETRIC BODIES.

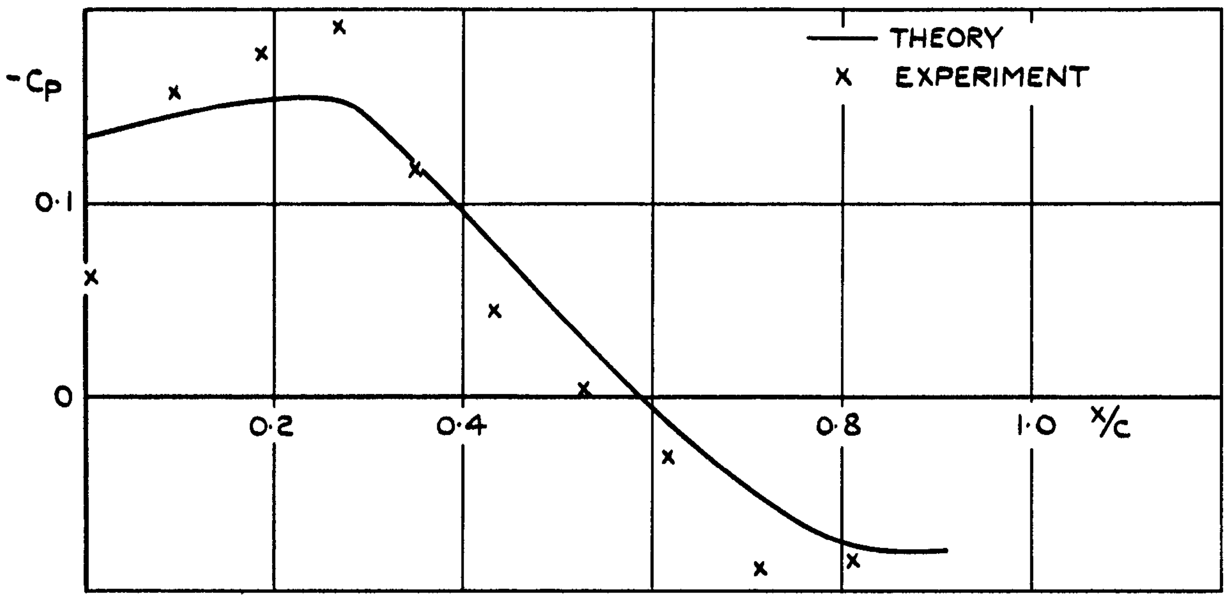


(a) $y = 0$ (BODY TOP)

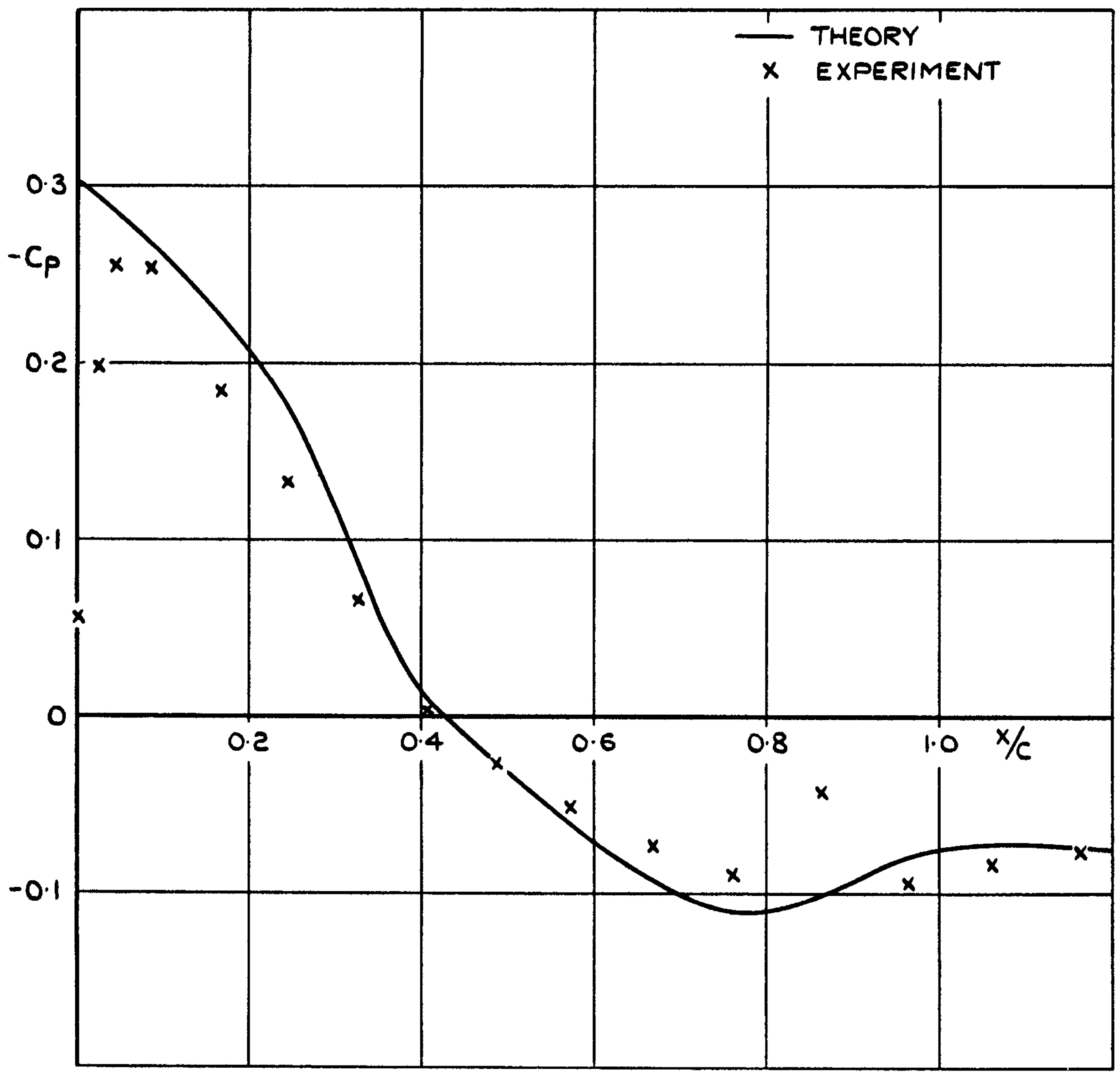


(b) $y = \frac{1}{3} t_0$

FIG. 3. PRESSURE DISTRIBUTION DUE TO WAISTING ON ELLIPTIC WAISTED BODY.



(c) $y = \frac{2}{3} t_o$



(d) $y = t_o$ (BODYSIDE)

FIG. 3 (CONCLUDED)

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