

C.P. No. 453

(19,576)

A.R.C. Technical Report

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The Velocity Distribution in a Turbulent Boundary Layer on a Flat Plate

by

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1959

FOUR SHILLINGS NET

ADDENDA

Before finally assessing the results of the present paper the author feels that the following additional papers should now be studied:-

1. P. Bradshaw and
N. Gregory Calibration of Preston tubes on a
flat plate using measurements of
local skin friction.
A.R.C.20,199. May, 1958.
2. R. N. Cox Wall neighbourhood measurements in
turbulent boundary layers using a hot
wire anemometer.
A.R.C.19,101. February, 1957.
3. D. W. Smith and
J. H. Walker Skin friction measurements in
incompressible flow.
A.R.C.20,316. July, 1958.

ERRATUM

On page 3, Sec. 7(2) read: The uncorrected inner
logarithmic law:-

The Velocity Distribution in a Turbulent
Boundary Layer on a Flat Plate

- By -

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4th October, 1957

SUMMARY

Detailed boundary-layer traverses have been made in a two-dimensional turbulent boundary layer on a flat plate with zero pressure gradient. Transition was promoted by trip wires and by glass-paper strips. A constant Reynolds number per foot of 3.9×10^5 was maintained throughout the experiments.

Assuming that Preston's surface pitot-tube technique for measuring local skin friction, is correct, (see Ref. 1), the measured velocity distributions are compared with established inner laws and velocity defect laws.

Ignoring corrections due to the effects of turbulent fluctuating velocities and the variation of static pressure through the boundary layer, the velocity distribution in the inner part of the boundary layer is found to agree with the inner law proposed by Nikuradse. The above effects are shown, however, to have a significant influence on the determination of velocity in the region where the inner law is assumed to hold. Corrections have been made to some of the measurements and the resulting inner law is found to agree closely with the one proposed by Coles.

1. Introduction

In Ref. 1 it is shown that accurate values of local skin friction may be obtained by Preston's² surface pitot-tube technique. This technique was used in the experimental investigation described in Ref. 1, where the skin friction measurements and detailed boundary-layer traverses were made on a smooth flat plate with zero pressure gradient. Three different transition devices were used. The range of Reynolds numbers in the experiments was $1500 \leq R_0 \leq 4000$.

This paper discusses the results obtained and compares the velocity distributions with established inner laws and velocity defect laws. The influence of the turbulent velocity fluctuations on measurements with a pitot tube, and the effect of variation of static pressure through the boundary layer are also considered; these are shown to be appreciable in the region where the inner law is assumed to hold.

2. Notation

ρ	density of air
μ	viscosity of air
ν	kinematic viscosity of air
τ_o	local skin friction
P_o	static pressure at reference position in working section of tunnel
P	mean static pressure at any position in the boundary layer
U_1	velocity outside boundary layer
u	mean velocity component parallel to wall at any point in the boundary layer
u',v',w'	instantaneous values of the fluctuating velocity components
U_T	$\sqrt{\frac{\tau_o}{\rho}}$
x	distance along flat plate from leading edge
y	vertical distance above surface of plate
δ	boundary-layer thickness
$\theta = \int_0^\delta \frac{u}{U_1} \left(1 - \frac{u}{U_1}\right) dy$	boundary-layer momentum thickness
$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_1}\right) dy$	boundary-layer displacement thickness
$c_f = \frac{\tau_o}{\frac{1}{2}\rho U_1^2}$	local skin-friction coefficient
$R_\theta = \frac{U_1 \rho \theta}{\mu}$	Reynolds number based on θ
$H = \frac{\delta^*}{\theta}$	formparameter

3. Apparatus

The apparatus and measuring technique are described in detail in Ref. 1 and only a brief outline will be given here.

The boundary-layer measurements were carried out on a smooth flat plate which spanned the working section of a wind tunnel. The plate was 6 ft long, 28 in. wide and $\frac{1}{4}$ in. thick and had an elliptical leading edge. Boundary-layer traverses and skin-friction measurements could be made at certain chosen stations along the centre-line of the plate (10, 15, 30, 45 and 60 inches from the leading edge). Static holes were located on a line one inch to the side of the centre-line, (see Fig. 1, Ref. 1).

Transition and artificial thickening of the boundary layer on the plate were accomplished in two ways: by using trip wires and by using a glass-paper strip. Two different trip wires were used. They were held in contact with the surface by a strip of Sellotape and were fixed one inch from the leading edge. Their effective diameters, i.e., the distances from the surface of the plate to the top of the Sellotape were 0.013 in. and 0.022 in. The glass-paper strip was 2.5 in. wide and was cemented on to the plate 0.75 in. from the leading edge. It had a particle size of approximately 0.015 to 0.020 in.

A flattened pitot tube was used for all the velocity measurements. The geometric centre was at a height $y = 0.0028$ in. when the pitot tube rested on the surface. The opening was 0.047 in. wide and 0.002 in. high. The skin friction measurements using round pitot tubes on the surface are described in Ref. 1.

All pressure differences were measured on two sensitive null-reading manometers of the inclined tube type, the accuracy of each individual reading being within 1 per cent.

3.1 Procedure

Preliminary experiments were made to ensure that the pressure distribution over the plate was as nearly uniform as possible. Also, the two-dimensional nature of the flow was checked by measuring profiles at three spanwise positions 45 inches from the leading edge.

Boundary-layer traverses were then made at each of the five stations where the skin friction was measured, i.e., along the centre line of the plate, and with each of the three transition devices.

Considerable difficulty was experienced in repeating sets of results and over a period of several days, with a given transition device, the boundary layer development and the distribution of skin friction were found to change to a marked degree. Reasons for these changes are suggested and discussed in Ref. 1. However, by constant checking and by repeating measurements, five complete sets of results were obtained, two with the 0.013 in. diameter transition wire, one with the 0.022 in. diameter wire and two with the glass paper strips.

All the experiments were made at a constant Reynolds number per ft (3.9×10^5). Adjustments were made to the free-stream velocity, before each measurement to take account of changing temperature and pressure.

3.2 Corrections and precautions

The corrections applied and precautions taken were as follows:-

- (i) A correction was applied for the effect of the pitot tube on the pressure measured at the static hole.

(ii)/

- (ii) The measured values of "y" in all the pitot traverses were corrected to allow for the displacement of the effective centre of the flattened pitot tube. This displacement was taken from the work of Young and Maas³ as $0.24h_p$, where h_p is the external height of the mouth of the pitot tube.
- (iii) Care was taken to determine, as closely as possible, the micrometer reading when the pitot tube just rested on the surface (i.e., the zero). In each traverse this was estimated to within 0.0005 in.
- (iv) The value of y when the pitot tube was moved away from the surface was checked at suitable intervals by using slip gauges.

Corrections were not applied for the effects of the turbulent fluctuating velocities on the traversing pitot tube readings and the effect of the variation of static pressure through the boundary layer on the determination of the ratio u/U_1 . These effects are discussed later in the paper.

It may be noted here that the Reynolds numbers of the pitot tubes were not low enough to introduce any appreciable viscous effects⁴.

4. The Logarithmic Inner Law for Flat Plates and Pipes

In Fig. 1 the inner parts of the velocity profiles are plotted in the form

$$\frac{u}{U_T} = A \log_{10} \frac{U_T \rho y}{\mu} + B \quad \dots (1)$$

(the value of U_T being found from measurements with a surface pitot tube) and compared with Preston's corrected* results. It must be noted here that this comparison only serves to indicate the general accuracy of the measurements. As Preston's pipe calibration curves were used to obtain the values of skin friction from the surface pitot-tube measurements, and as the existence of a region of local dynamical similarity has been firmly established for the boundary layer on the flat plate (see Ref. 1), the two sets of results should lie on the same curve. The very small amount of scatter in the measured points and their good agreement with the pipe results illustrates the accuracy of the measurements.

In/

* When this comparison was first made it was found that the results for the flat plate were always displaced by approximately $1\frac{1}{4}$ per cent above the pipe measurements made by Preston (Ref. 2). Complete agreement was obtained only when the following corrections were applied to the pipe results:-

- (a) a pitot-tube Reynolds number correction mentioned already in Section 3.2 and discussed in Ref. 4,
- (b) a correction to the measurements at the higher Reynolds numbers due to the acceleration of the flow in the pipe,
- (c) a correction due to the difference in pressure between the atmosphere and the working section of the pipe, which affected the calculation of density.

In Fig. 1 it will be seen that some of the experimental points actually cross the curve $\frac{u}{U_\tau} = \frac{U_\tau \rho y}{\mu}$; this is probably due both to an uncertainty in the Young and Maas (Ref. 3) correction close to the surface, and to the effect of the turbulent fluctuating velocities on the pitot-tube readings. In this particular region this effect is appreciable and, as is shown later, the measured velocities are of the order of 5 per cent greater than the true velocities.

4.1 Comparison with other established logarithmic laws

Fig. 1 shows that the experimental points lie very close to the line

$$\frac{u}{U_\tau} = 5.5 \log_{10} \frac{U_\tau \rho y}{\mu} + 5.8 \quad \dots(2)$$

which was suggested by Nikuradse's measurements, Fig. 2 shows a comparison between this law and the inner laws obtained on smooth surfaces by Coles⁵, Clauser⁶ and Landweber⁷. The lack of good agreement is at once apparent, and if the variations are considered to be due only to errors in determining the skin friction, then the maximum discrepancy is about 6 per cent in U_τ , or 12 per cent in skin friction. A factor contributing to these discrepancies is the lack of precise information on the displacement of the effective centres of the pitot tubes; some experimenters have made approximate corrections while others have neglected it. However, the major source of error is likely to be in the measurement of skin friction.

4.2 The "true" inner law

It is believed that these measurements give a reasonably accurate inner law. However there are still some uncertainties which need to be clarified before a very accurate form of the law can be derived. In particular, an accurate determination should be made of the displacement of the effective centre of a flattened pitot tube in a boundary layer. Also the measurements must be corrected for the effects of both the turbulent fluctuating velocities and the variation of static pressure through the boundary layer.

To illustrate the importance of these latter effects on the inner law, their influence has been considered in detail in the Appendix. There it is shown that the ratio of the true mean velocity u to the measured velocity in the boundary layer u_m (obtained from a pitot static-hole combination) is given by

$$\frac{u}{u_m} = \sqrt{1 - \frac{\overline{u'^2} + \overline{w'^2} - \overline{v'^2}}{u_1^2} \times \frac{U_1^2}{u_m^2}},$$

for zero pressure gradient. This correction factor has been evaluated from data given in Ref. 9 and its variation with y/δ shows that the measured value of u/U_1 will be too high by $\frac{1}{2}$ to 3 per cent in the region where the inner law can be expected to hold. The effect on the inner law is therefore quite significant, as can be seen from Fig. 3 which compares a set of corrected results with the original curve. The corrected results shown in Fig. 3 are in good agreement with the inner law derived by Coles.

5. Comparison between the Present Results and Accepted Velocity Defect Laws

The velocity defect law for a turbulent boundary layer may be written in the form

$$\frac{U_1 - u}{U_\tau} = f(y/\delta). \quad \dots(3)$$

This law is valid for all but the inner region of the boundary layer, and for a fully developed* layer with zero pressure gradient has been found to be independent of both Reynolds number and surface roughness. (If the pressure gradient is not zero, a universal curve will be obtained only for a fully developed turbulent boundary layer of the equilibrium type defined by Clauser in Ref. 6.)

The profiles obtained in the present investigation have been plotted in the form given by Eq. (3), first to find where the boundary layer became fully developed, and then to compare the fully developed profiles with the results of other investigators.

In Fig. 4 a typical set of profiles is plotted in the form given by Eq. (3). Immediately it can be seen that the boundary layer is not fully developed over the whole plate, and in fact, it appears to grow for almost 30 inches down the plate before this state is reached. The points obtained at the 30, 45 and 60 inch stations, however, lie closely on a single curve. Similar results were obtained for the other sets of profiles although in some it was noticed that the boundary layer was not fully developed even at the 30 inch station. For all the results the 45 inch and 60 inch profiles showed quite good agreement and hence only these were used in making a comparison with other proposed forms of the velocity defect law.

When making this comparison it was found that, although each set of results was consistent, there was a definite displacement between the mean profiles obtained with the two different types of transition device. The profiles were therefore separated in this way and are shown in Figs. 5 and 6, where they are compared again with the results of Coles, Clauser and Landweber. From these figures it will be seen that not only is there a quite marked difference between the transition wire and glass paper results, but also there is a wide divergence between the curves obtained by the other experimenters.

The discrepancies between these earlier results are, of course, again partly due to inaccuracies in the determination of U_τ but they are also due to the different methods adopted by each author for the determination of the boundary layer thickness. The method adopted for the present results was to plot the outer part of each profile in the form $\log_{10} y$ versus $\log_{10} \frac{u}{U_1}$. The curve through the points up to a value/

*The rates at which the two usually accepted parts of the boundary layer respond to a given disturbance differ considerably. While the inner part adapts itself almost instantaneously to changes in environment these changes affect the outer part very slowly. Consequently, although the skin friction and the inner part of the boundary layer profile change sharply downstream of a transition device, the adjustment of the outer part of the profile is a gradual process. Hence the boundary layer must progress a certain distance downstream of a transition device before it can assume a fully developed state.

value of $u/U_1 = 0.997$ was extrapolated to the y axis and the value of y there taken as the boundary layer thickness. The method used by Coles was to plot $\left(1 - \frac{u}{U_1}\right)^{2/3}$ against y and fit a straight line to the outer part of the profile. The intercept on the y axis then defined the boundary layer thickness. This method was found to give values of δ slightly less than the logarithmic plotting referred to above and considerably less than the values obtained from either the tabulated boundary layer characteristics given by Landweber⁷ or the formula used by Hama¹⁰,

$$\delta = 0.30\delta^* \sqrt{\frac{2}{c_f}}$$

As implied in Ref. 10, this is derived from Clauser's form of the velocity defect law (assuming, of course, that the law applies over the complete boundary layer thickness).

Errors in boundary layer thickness, however, cannot account for the differences in the mean curves between the results obtained with the glass paper strips and those obtained with the transition wires, because the method used to find δ was the same in both cases. The differences must mean either that the boundary layer is not fully developed even at the 45 in. and 60 in. stations, or that the structure of the turbulent boundary layer is influenced by the type of disturbance introduced to cause transition.

It might be noted here that the different state of the boundary layers obtained with the two types of transition device, together with the different methods of obtaining the boundary layer thickness must account for the apparent inconsistency between the results shown in Figs. 5 and 6 and those in Fig. 2, where various inner laws were compared. From the latter it would appear that the skin friction values obtained by Coles were larger than the corresponding values for the present results, but Figs. 5 and 6 indicate the reverse.

The uncertainty in finding the boundary layer thickness provides a serious limitation to the use of the velocity defect law. Various authors have therefore tried to express the velocity defect in terms of some quantity other than δ , which is more easily defined. This has been done in the following way. If it is assumed that a universal relation of the form given by Eq. (3) holds throughout the whole boundary layer then

$$\int_0^1 f(y/\delta) d(y/\delta) = k = \text{constant.}$$

Hence

$$k = \sqrt{2/c_f} \times \frac{\delta^*}{\delta},$$

$$\text{or } y/\delta = k\sqrt{c_f/2} \times y/\delta^* = k \frac{U_\tau}{U_1} \times y/\delta^*.$$

Therefore Eq. (3) may be written as

$$\frac{U_1 - u}{U_\tau} = f\left(k \frac{U_\tau}{U_1} \frac{y}{\delta^*}\right) \quad \dots(4)$$

It can also be shown¹¹ that if in the turbulent boundary layer there is a region where the inner law and the outer law are both applicable (and there is ample evidence to show that this is so¹²), the equation must have the form

$$\frac{U_1 - u}{U_T} = A \log y/\delta + C,$$

or in view of Eq. (4)

$$\frac{U_1 - u}{U_T} = A \log \frac{U_T y}{U_1 \delta^*} + D. \quad \dots(5)$$

The results shown in Figs. 5 and 6 have been plotted in this way in Figs. 7 and 8 respectively. The displacement between the transition wire and glass paper strip results is still evident, as indeed would be expected even though the inaccuracies in the determination of δ have been eliminated, but the results of Coles and Clauser are now in much better agreement. This emphasises the importance of obtaining at least a common definition of boundary layer thickness.

6. The Variation of the Form Parameter H in a Turbulent Boundary Layer on a Flat Plate

If it is assumed that in a turbulent boundary layer there are two universal relations with the forms

$$\frac{u}{U_T} = f\left(\frac{U_T \rho y}{\mu}\right)$$

and

$$\frac{U_1 - u}{U_T} = F\left(\frac{y}{\delta}\right),$$

and that there is a common region where both are applicable it can be shown (cf. Coles⁵) that H must be a universal function of $c_f/2$ or R_θ . In Fig. 9 values of H found from the present results are plotted against R_θ . Although the results are subject to a fair amount of scatter they are seen to agree (especially in the region $2000 < R_\theta < 4000$) quite well with the results of both Coles and Landweber.

7. Conclusions

The main conclusions may be summarised as follows:-

1. Comparison of some of the "established" inner laws for the turbulent boundary-layer velocity profile, shows a marked divergence between them. If the difference is attributed solely to errors in the determination of local skin friction then the maximum discrepancy in the present comparison is approximately 12 per cent.

2. The inner logarithmic law obtained from the results of the present investigation agrees closely with the original one obtained by Nikuradse, viz.,

$$\frac{u}{U_T} = 5.5 \log_{10} \frac{U_T \rho y}{\mu} + 5.8.$$

3. If corrections are made to take account of the effects of the turbulent velocity fluctuations on the pitot tube readings and the variation of static pressure through the boundary layer, the values of the constants in the above equation are changed. These "corrected" results agree closely with the law proposed by Coles, viz.,

$$\frac{u}{U_T} = 5.75 \log_{10} \frac{U_T \rho y}{\mu} + 5.1.$$

A correction factor has been evaluated and it is shown that the measured velocity will be too high by $\frac{1}{2}$ to $2\frac{1}{2}$ per cent in the region where the logarithmic law is assumed to apply. It is suggested that this correction factor will apply universally for zero pressure gradients, provided the distributions of u'/U_1 , v'/U_1 and w'/U_1 , in the appropriate region of the boundary layer, exhibit longitudinal similarity.

4. Before an effective comparison of turbulent boundary-layer velocity profiles can be made on a velocity-defect basis, either a common definition of the boundary-layer thickness must be adopted or the parameter $\log \left(\frac{U_T}{U_1} y/\delta^* \right)$ must be used.

5. Velocity profiles plotted on a velocity-defect basis indicate that the nature of the transition-promoting device has a small but apparently permanent effect on the shape of the velocity profile. On the other hand, it may be that the apparently fully-developed profiles obtained with the two types of transition device, are different when plotted in the above fashion, simply because equilibrium conditions have not been fully realised. If this is the case then clearly a turbulent boundary layer must be allowed to flow undisturbed for a comparatively large distance (e.g., greater than $70 \times \delta$) before it can be assumed to be fully developed.

6. Both Coles and Landweber show that, if there are universal "inner" and "outer" laws for fully developed turbulent boundary layers, and if these overlap in a common region, the form parameter H can be expressed as a universal function of R_θ . This conclusion is confirmed by the present results, which agree quite well with those proposed by both authors.

Acknowledgment

The above work was supervised by Dr. M. R. Head and Prof. J. H. Preston and their advice and encouragement is gratefully acknowledged.

APPENDIX/

APPENDIX

The Effects of the Turbulent Fluctuating Velocities on
the Measurement of the Mean Velocity in the Boundary Layer

Goldstein¹³ has shown that the mean pressure in a pitot tube in turbulent flow is not $p + \frac{1}{2} \rho u^2$, where u is the mean velocity, but $p + \frac{1}{2} \rho u^2 + \frac{1}{2} \rho \overline{q'^2}$, where $\overline{q'^2}$ is the mean square of the resultant fluctuating velocity. Further, it is known that there is a small variation of static pressure across a boundary layer. Consequently if a pitot tube is used in conjunction with a static hole to measure the dynamic pressure in a boundary layer the measurements will have to be corrected to obtain the true dynamic pressure, based on the mean velocity.

These corrections are generally small enough to be neglected, but it appears likely that they can play a significant part in determining the "true" curve of $\frac{u}{U_T}$ versus $\log \frac{U_T \rho y}{\mu}$. This is simply because the turbulent velocity fluctuations reach their maximum values near the edge of the laminar sub-layer and are therefore comparatively large in the region where this inner law is applicable.

In the investigation described in Ref. 1 the static pressure p_0 was always measured by using a static hole in the surface of the plate, and under these circumstances the measured dynamic pressure $\frac{1}{2} \rho u_m^2$ is given by

$$\frac{1}{2} \rho u_m^2 = H_1 - p_0,$$

where H_1 is equal to the total pressure at a particular height y in the boundary layer and on the streamline considered. Now the correct value of the mean dynamic pressure $\frac{1}{2} \rho u^2$ is given by

$$\frac{1}{2} \rho u^2 = H_1 - p - \frac{1}{2} \rho \overline{q'^2}.$$

Hence

$$\frac{u^2}{u_m^2} = 1 - \frac{\overline{q'^2}}{U_1^2} \times \frac{U_1^2}{u_m^2} + \frac{p_0 - p}{\frac{1}{2} \rho U_1^2} \times \frac{U_1^2}{u_m^2}. \quad \dots(1A)$$

The variation of static pressure through the boundary layer must be known in order to obtain $(p_0 - p)$, and may be found from the second equation of motion. For turbulent boundary layers this can be approximated by

$$\frac{\overline{\partial u'v'}}{\partial x} + \frac{\overline{\partial v'^2}}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial y}.$$

Integrating from 0 to y gives the static pressure at any point y relative to the static pressure at the surface.

$$\text{Thus } - \frac{1}{\rho} (p - p_0) = \int_0^y \frac{\partial}{\partial x} \overline{u'v'} dy + (\overline{v'^2})_y,$$

or/

$$\text{or} \quad \frac{p_0 - p}{\frac{1}{2}\rho U_1^2} = \frac{2}{U_1^2} \int_0^y \frac{\partial}{\partial x} \overline{u'v'} dy + 2 \left(\frac{\overline{v'^2}}{U_1^2} \right)_y,$$

which may be expanded into the form

$$\frac{p_0 - p}{\frac{1}{2}\rho U_1^2} = \frac{2\overline{v'^2}}{U_1^2} + \frac{2\partial}{\partial x} \int_0^y \frac{\overline{u'v'}}{U_1^2} dy + \frac{4}{U_1} \frac{dU_1}{dx} \int_0^y \frac{\overline{u'v'}}{U_1^2} dy. \quad \dots(2A)$$

This can now be substituted into Eq. (1A), which becomes:

$$\frac{u^2}{u_m^2} = 1 - \frac{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}}{U_1^2} \times \frac{U_1^2}{u_m^2} + \frac{2\overline{v'^2}}{U_1^2} \times \frac{U_1^2}{u_m^2} + \frac{2U_1^2}{u_m^2} \frac{\partial}{\partial x} \int_0^y \frac{\overline{u'v'}}{U_1^2} dy + \frac{4U_1^2}{u_m^2} \frac{dU_1}{dx} \int_0^y \frac{\overline{u'v'}}{U_1^2} dy. \quad \dots(3A)$$

Eq. (3A) therefore represents the correction factor to be applied to the values of $\frac{u_m}{U_1}$, calculated from the pitot and static-hole measurements, to obtain the true value of u/U_1 .

For zero pressure gradient Eq. (3A) reduces to

$$\frac{u^2}{u_m^2} = 1 - \frac{\overline{u'^2} + \overline{w'^2} - \overline{v'^2}}{U_1^2} \times \frac{U_1^2}{u_m^2} + \frac{2U_1^2}{u_m^2} \frac{\partial}{\partial x} \int_0^y \frac{\overline{u'v'}}{U_1^2} dy \quad \dots(4A)$$

and the terms appearing on the right-hand side of this equation have been evaluated in the region $0 < y/\delta < 0.2$, using the data published by Klebanoff⁹. The experimental data available, however, were not sufficient to calculate the Reynolds shear-stress term. Its relative magnitude was

determined by assuming first that $\frac{\overline{u'v'}}{U_1^2}$ would be a universal function of $\frac{U_1 \rho y}{\mu}$ and hence, as the variation of U_1 along a surface is comparatively

small when there is zero pressure gradient, that $\frac{\overline{u'v'}}{U_1^2}$ would also exhibit longitudinal similarity in the particular region of the boundary layer considered. However, after evaluating the term

$$\frac{2U_1^2}{u_m^2} \frac{\partial}{\partial x} \int_0^y \frac{\overline{u'v'}}{U_1^2} dy$$

by making this assumption, it was found to provide a negligible contribution to the right-hand side of Eq. (4A) and was therefore neglected. Eq. (4A) consequently reduces to

$$\frac{u^2}{u_m^2} = 1 - \frac{\overline{u'^2} + \overline{v'^2} - \overline{w'^2}}{U_1^2} \times \frac{U_1^2}{u_m^2} \quad \dots(5A)$$

or/

or

$$\frac{u}{u_m} = \sqrt{1 - \frac{\overline{u'^2} + \overline{w'^2} - \overline{v'^2}}{U_1^2}} \times \frac{U_1^2}{u_m^2}$$

Fig. 10 shows the variation of the correction factor u/u_m through the inner part of the boundary layer on the flat plate. It also shows the magnitude of the correction when the variation of the static pressure through the boundary layer is neglected.

From this figure it can be seen that velocities, obtained from pitot-tube and static-hole measurements in a turbulent boundary layer with zero pressure gradient, will be too high by $\frac{1}{2}$ to 3 per cent in the region in which the logarithmic law can be expected to apply. This correction factor will be universal if, over a fairly wide range of Reynolds number, the terms $\frac{\overline{u'^2}}{U_1^2}$ etc., are found to exhibit longitudinal similarity in a turbulent boundary layer.

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<u>No.</u>	<u>Author(s)</u>	<u>Title, etc.</u>
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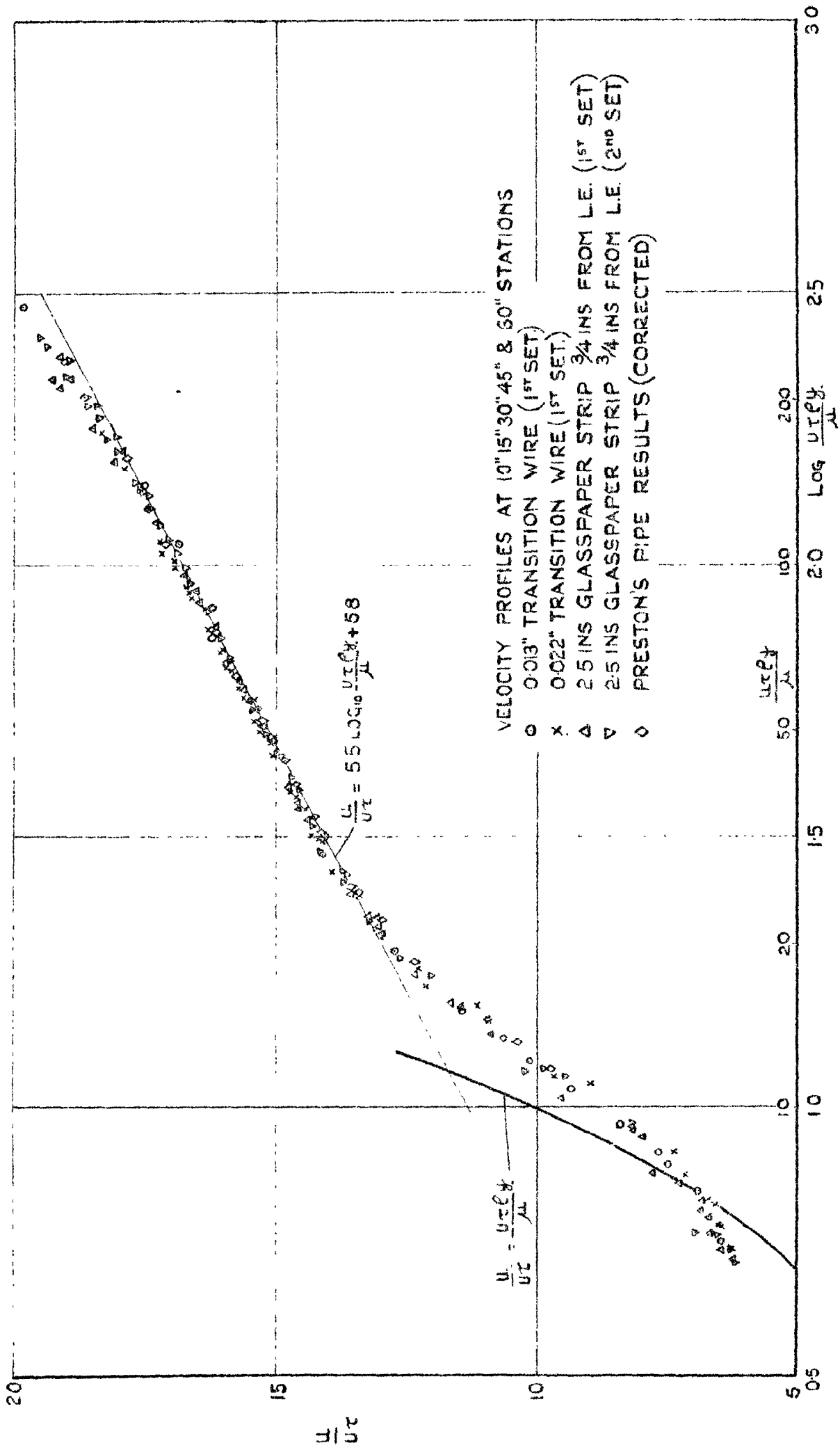


Fig. 1

FIG. 1. UNIVERSAL VELOCITY DISTRIBUTION FOR TURBULENT BOUNDARY LAYERS ON A SMOOTH FLAT PLATE.

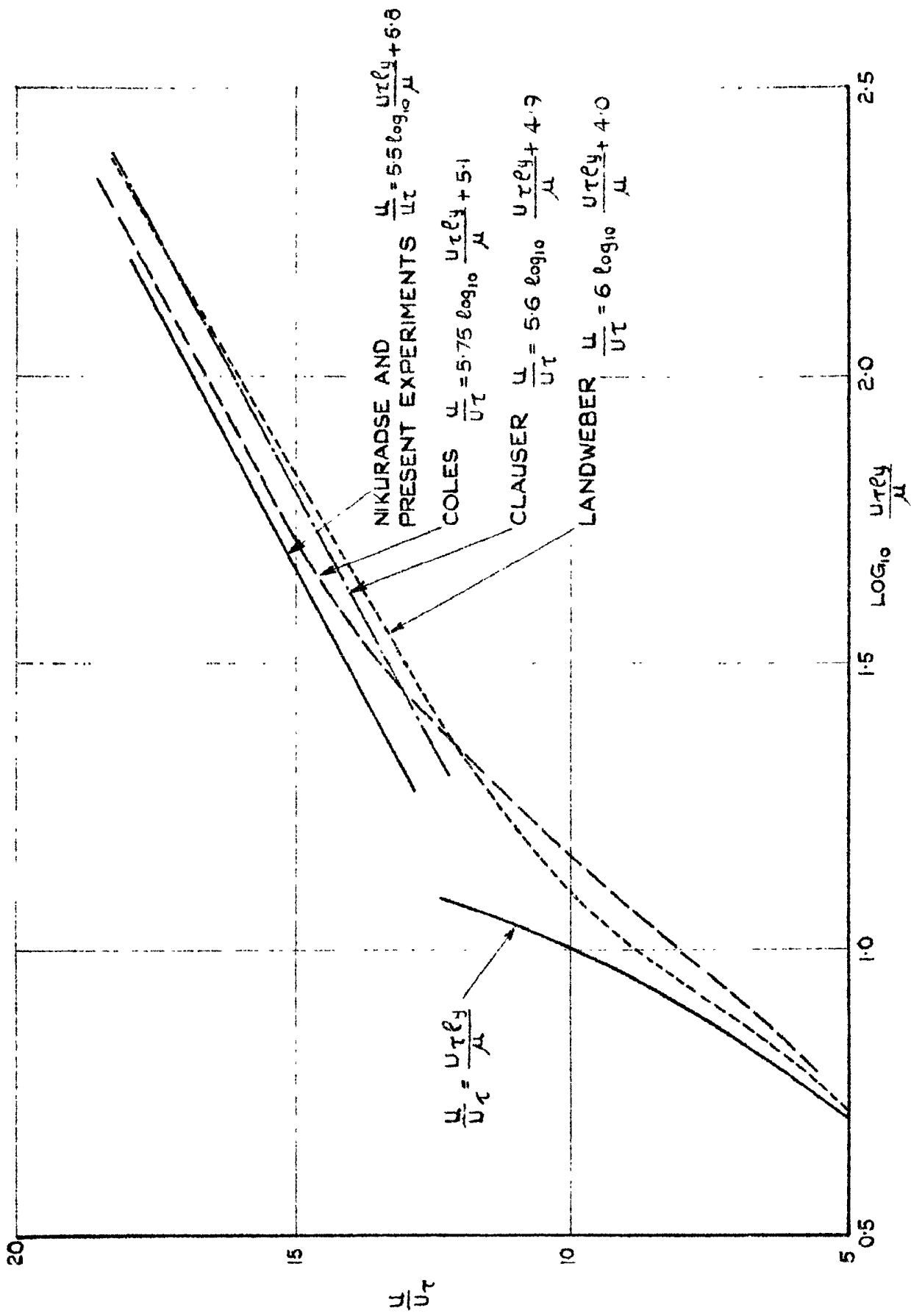


FIG. 2.

FIG. 2. COMPARISON BETWEEN SOME OF THE ACCEPTED LOGARITHMIC LAWS FOR $\frac{u}{u\tau}$ THE INNER PART OF TURBULENT BOUNDARY LAYER VELOCITY PROFILE. 2

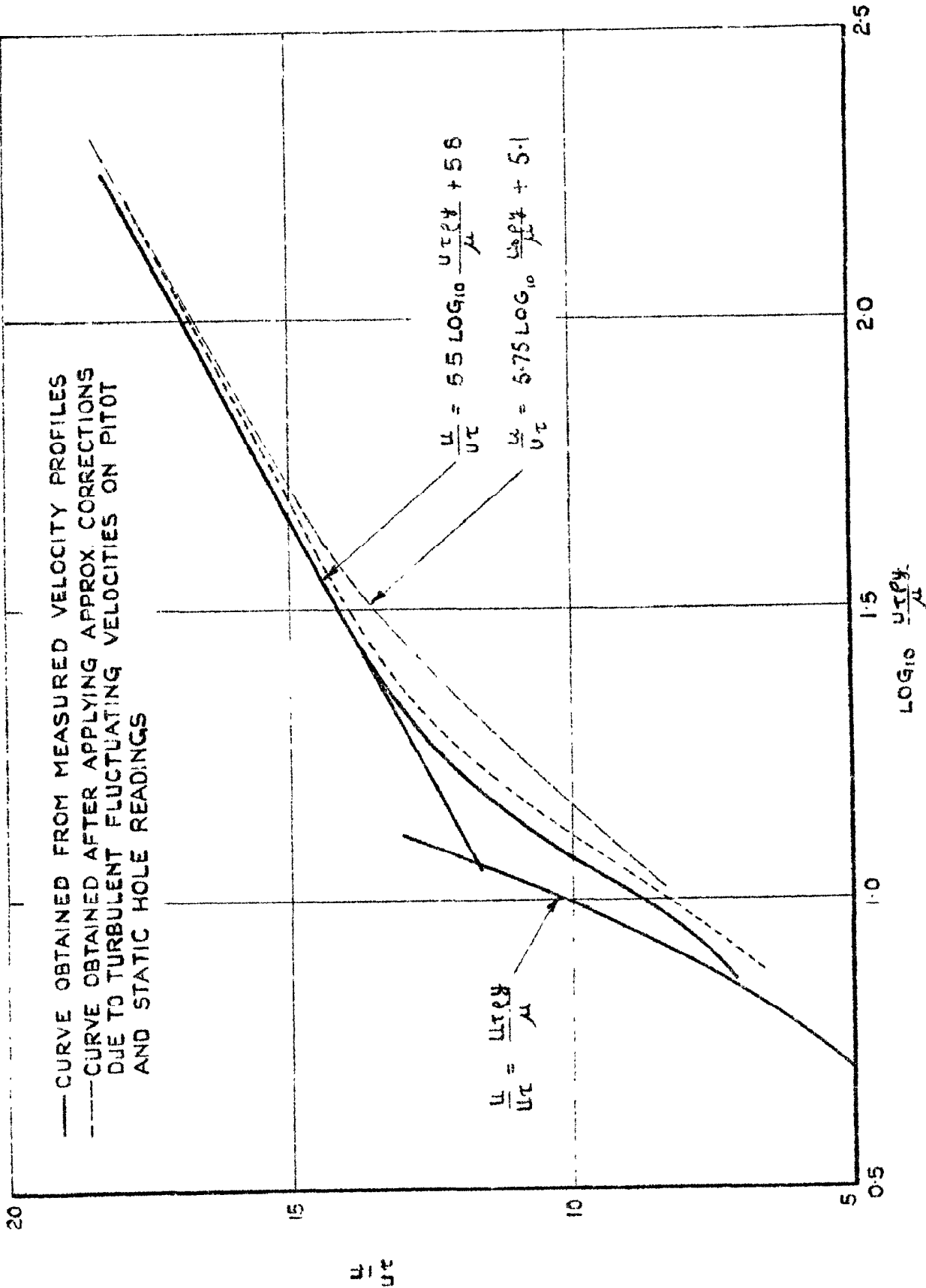


FIG. 3.

FIG. 3.

FIG. 3. THE INFLUENCE OF THE TURBULENT FLUCTUATING VELOCITIES ON THE "INNER LAW" OBTAINED BY A PITOT TRAVERSE.

Fig. 4.

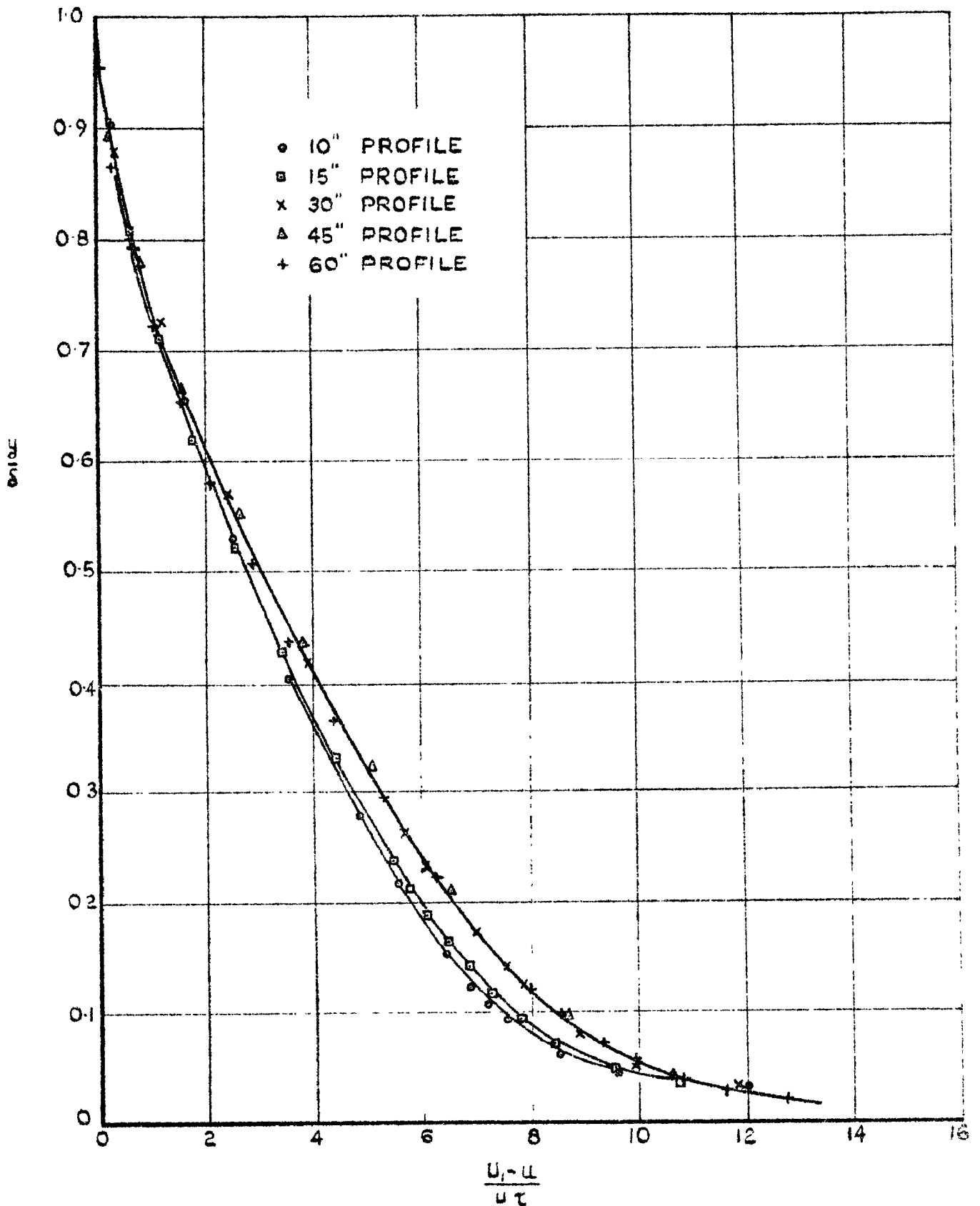


FIG. 4. TURBULENT BOUNDARY LAYER
 PROFILES FOR 2.5 INS. GLASSPAPER STRIP
 0.75 INS BACK FROM LEADING EDGE.
 (2nd. SET OF MEASUREMENTS.)

Fig. 5.

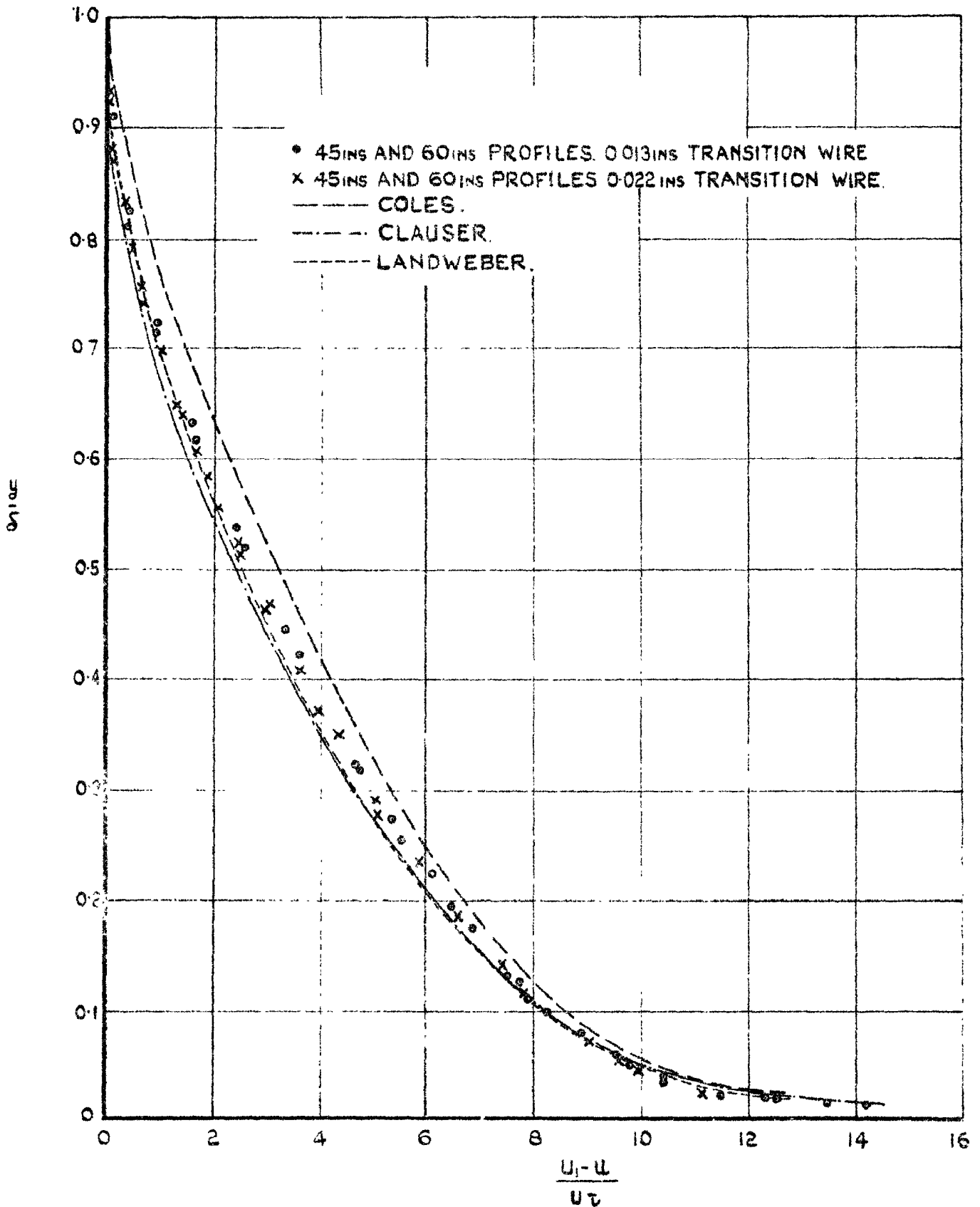


FIG.5. UNIVERSAL PLOT OF TURBULENT BOUNDARY LAYER PROFILES FROM TRANSITION WIRE DATA.

FIG. 6.

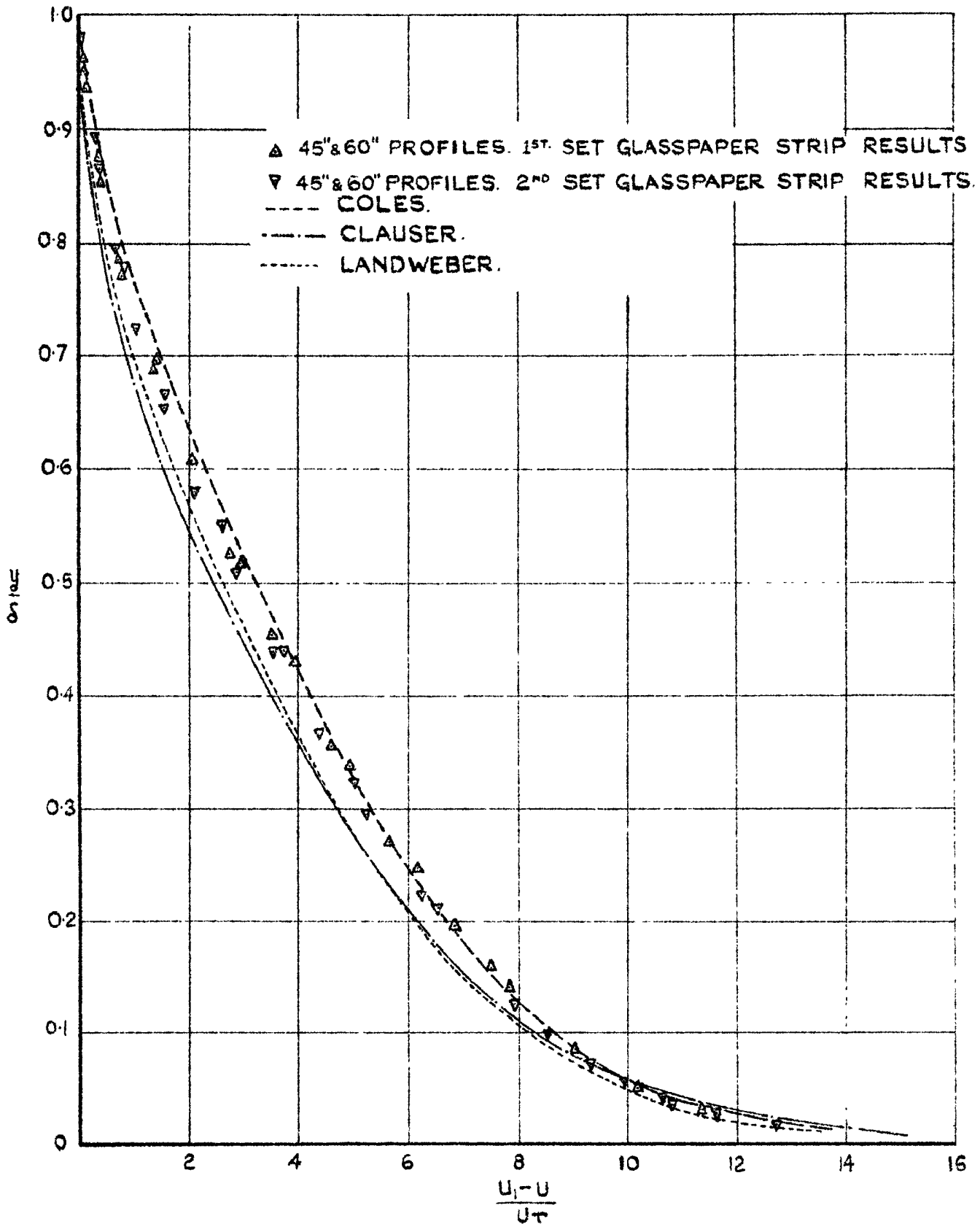


FIG. 6. UNIVERSAL PLOT OF TURBULENT BOUNDARY LAYER PROFILES FROM 2.5 INS. GLASSPAPER STRIP DATA.

Fig. 7.

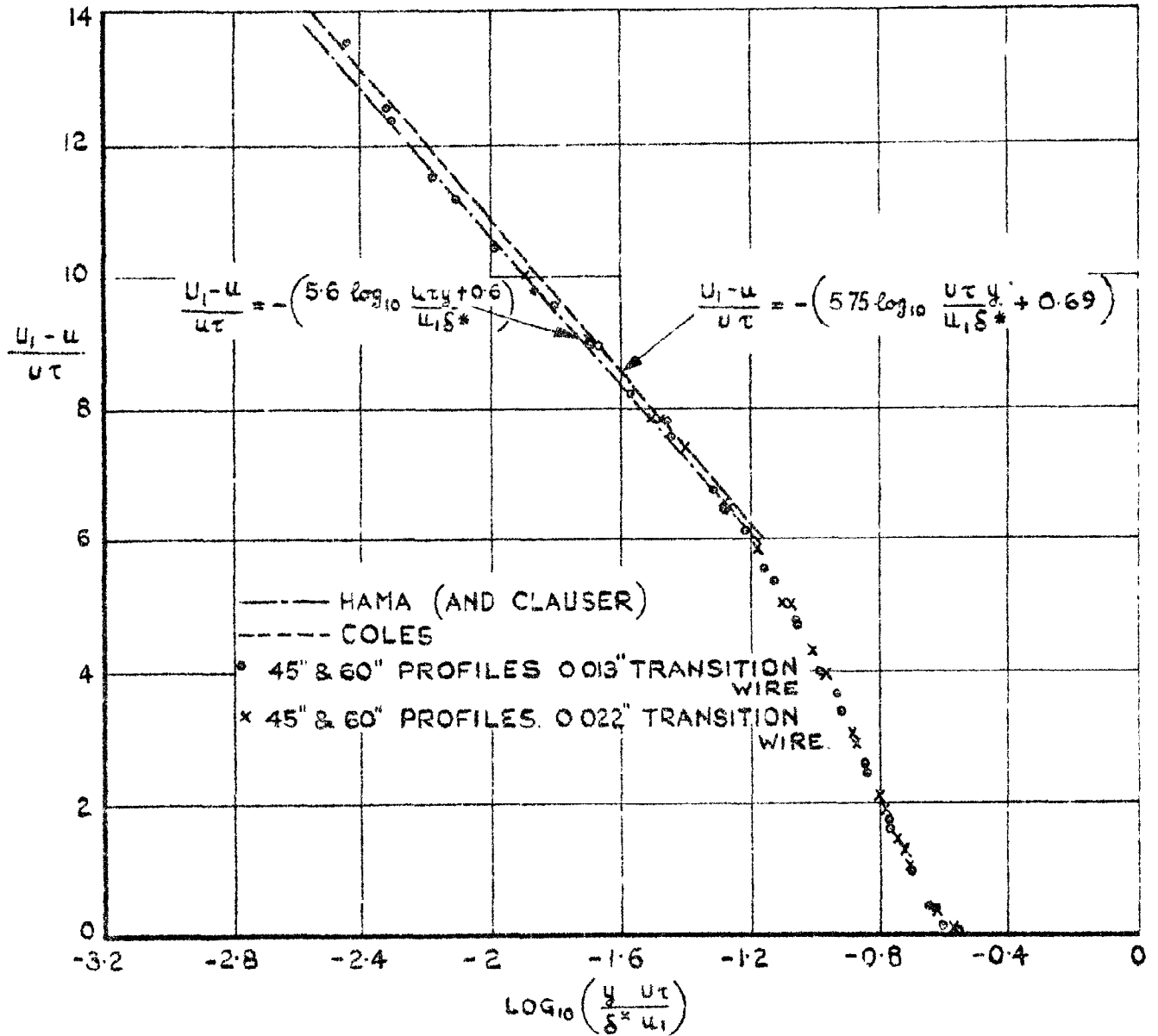


FIG. 7. COMPARISON BETWEEN VELOCITY DEFECT LAWS PROPOSED BY HAMA & COLES AND RESULTS OBTAINED FROM THE TRANSITION WIRE DATA.

FIG. 8

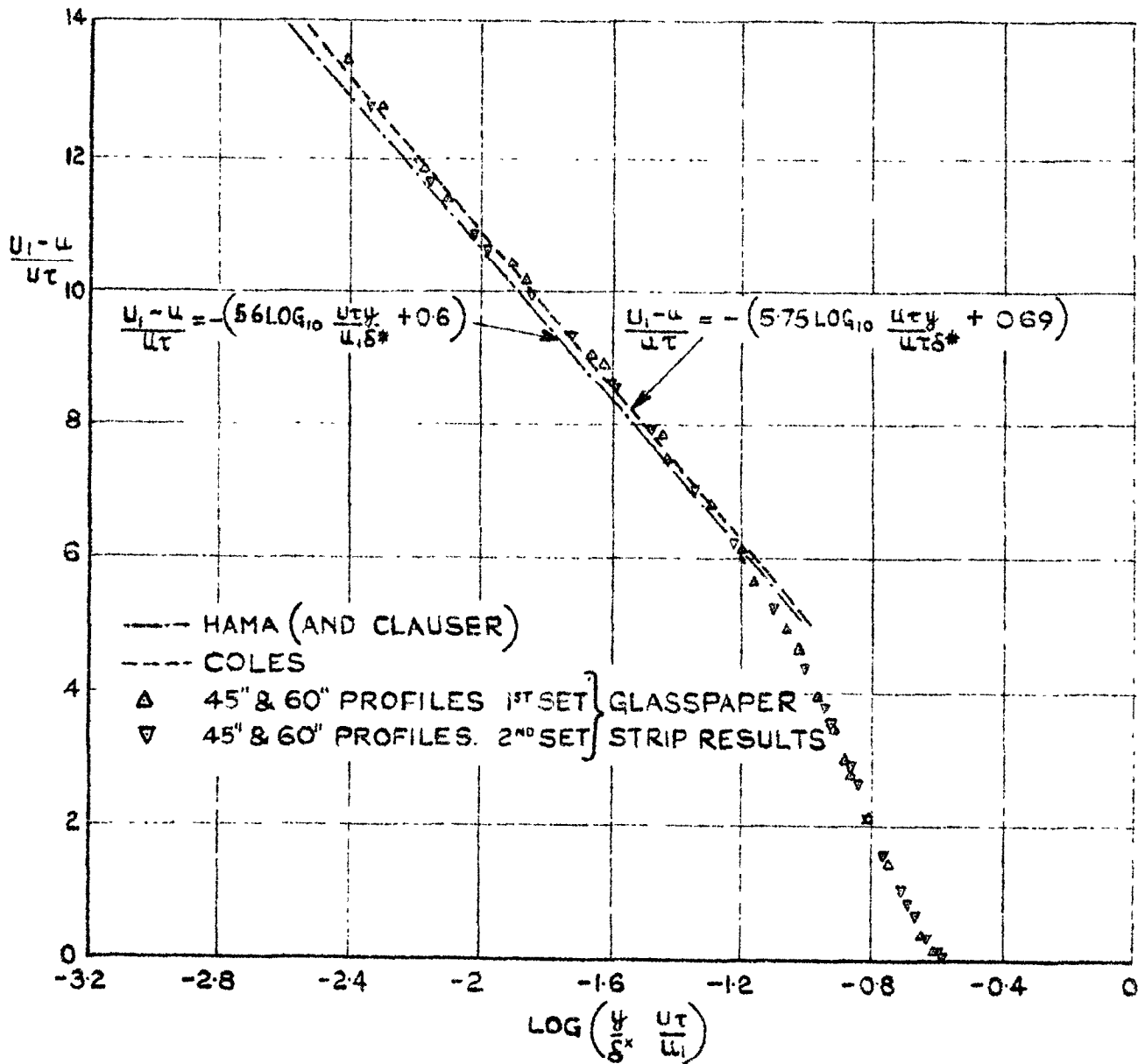


FIG. 8. COMPARISON BETWEEN VELOCITY DEFECT LAWS PROPOSED BY HAMA AND COLES, AND RESULTS OBTAINED FROM 2.5 IN. GLASSPAPER STRIP DATA.

Fig. 9.

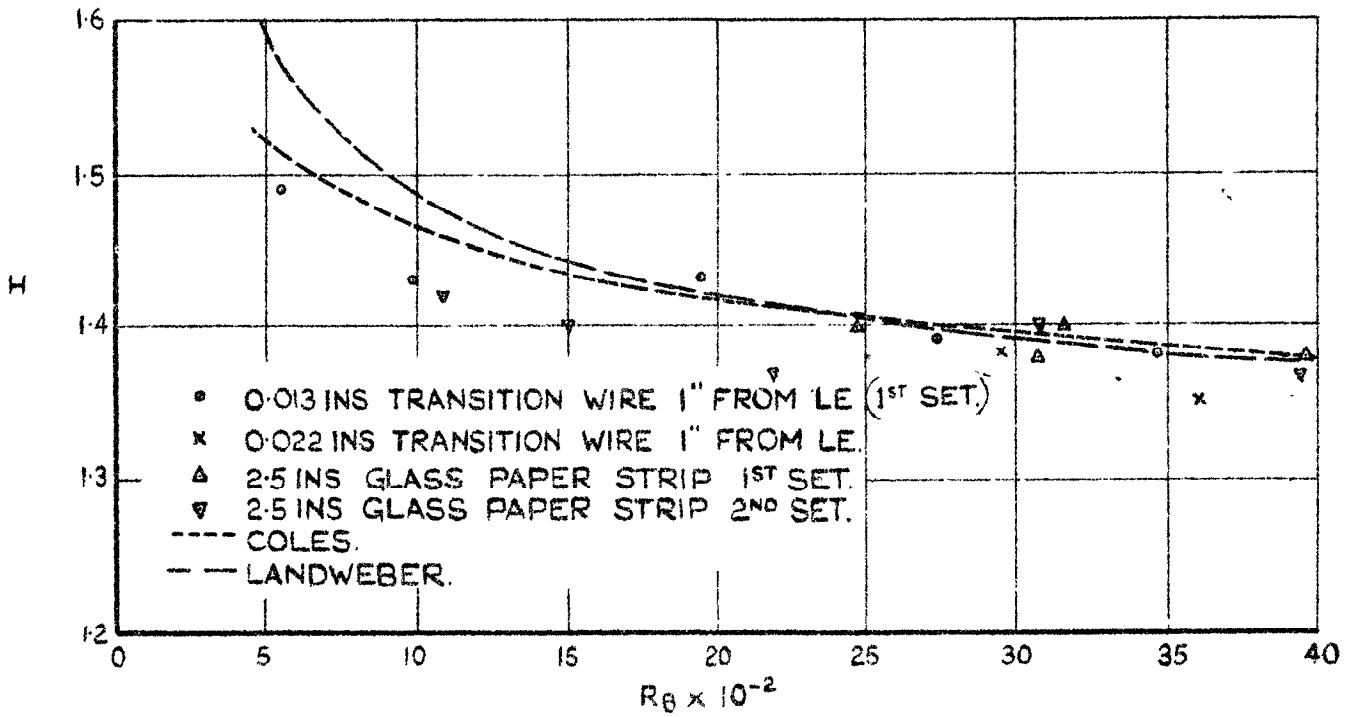


FIG. 9. PROFILE PARAMETER H AS A FUNCTION OF REYNOLDS NUMBER R_0 .

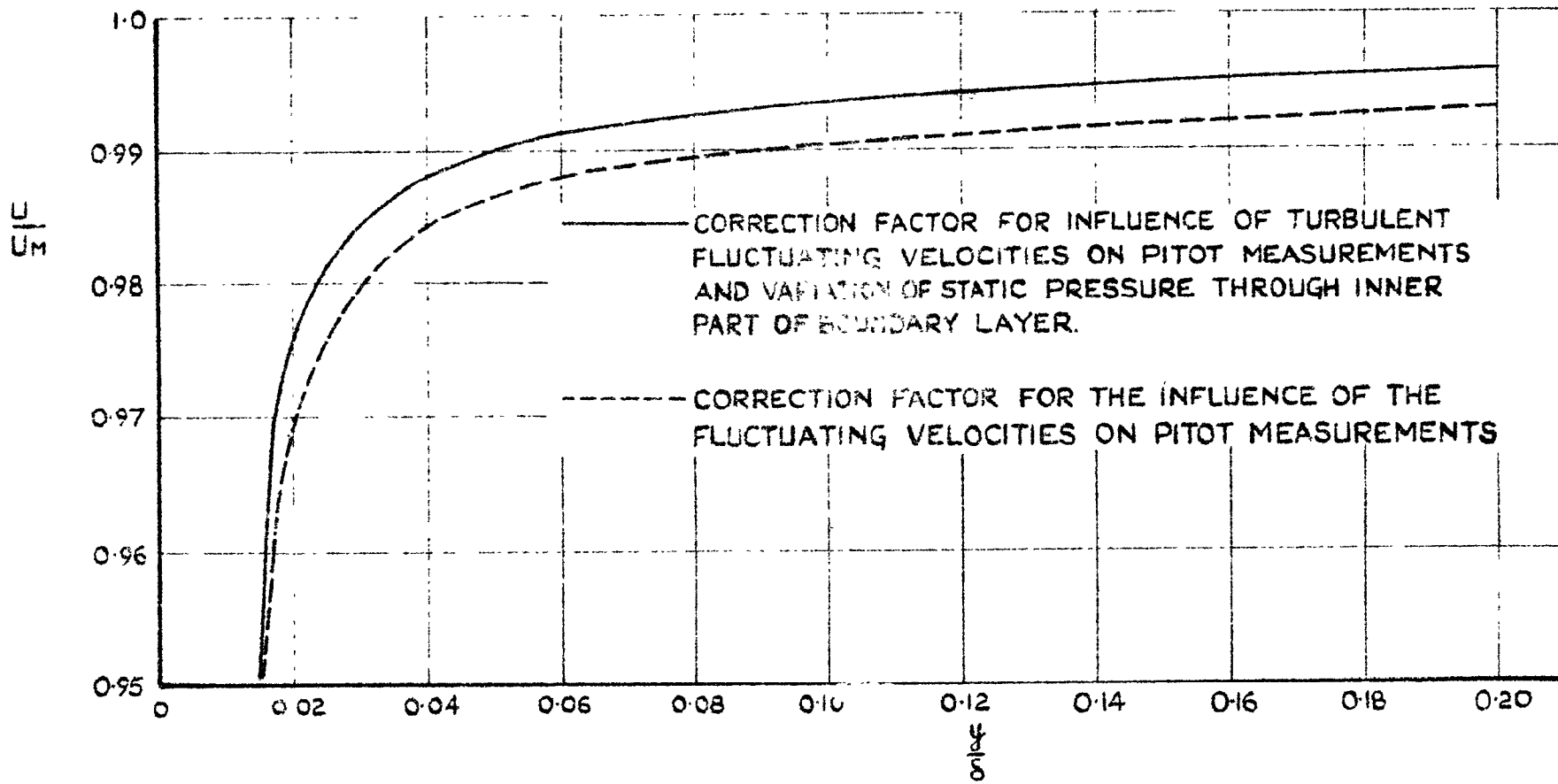


Fig. 10

FIG. 10. CORRECTION FACTORS FOR THE INFLUENCE OF TURBULENT FLUCTUATING VELOCITIES ON PITOT MEASUREMENTS AND FOR THE VARIATION OF STATIC PRESSURE THROUGH THE INNER PART OF BOUNDARY LAYER.

FIG. 10.

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