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The Internal Damping Due to Structural Joints and Techniques for General Damping Measurement

by

D. J. Mead, D.C.Ae.,
Department of Aeronautical Engineering,
University of Southampton

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Corrigenda and Addenda

Pages 5 and 6. Attention has been drawn to the ambiguity of the term "$d\dot{x}$" (in equation 2.1) and "$d\omega$" appearing in equation 2.2 and line 2 of page 6. These terms are the products of d (the viscous damping coefficient) and $\dot{x}$ and ω respectively. They are <u>not</u> the differentials of these quantities, with which the symbols may be confused.

Page 10. The first term of equation 3.3 should be divided by 2, i.e. it should be

$$\tfrac{1}{2}.k_p k_v \Sigma n.\omega.P_n y_n \sin \varepsilon_n$$

and similarly in the expression following "i.e. the D.C. component is ... ".

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SUMMARY

This paper discusses in qualitative terms the source of damping in rivetted or bolted joints which are subjected to harmonically varying loads. Owing to the complicated nature of the source, an analytical determination of the damping properties of a joint is not deemed feasible, but from characteristics of the joint which may be obtained experimentally, a method is suggested of predicting the structural damping coefficient of a simple structure, vibrating in a known mode and at a known frequency.

Several techniques for the measurement of structural damping are described, and their applicability to different cases is considered. Some new methods are suggested, which will be of use in a specialized programme of research into joint damping.

Three of the methods have been used in an experiment to determine the damping properties of simple joints in the bending flanges of a vibrating box beam. Over the limited range of load amplitude covered in the experiments, the energy loss in the joints at about 40 c.p.s. was found to be proportional to the joint load raised to the power of approximately 2.7. This energy loss was found to vary almost in proportion to the frequency of the oscillating load on the joint, but the possible errors involved in changing the frequency throw doubt on this result. Suggestions are made for the reduction of these errors.

Of the methods of damping measurement described and used, the most satisfactory is considered to be the method of measuring the energy input into the structure to maintain steady vibration. This method is least susceptible to errors due to harmonic distortion, may be used on systems with non-linear damping and gives results more quickly and accurately than the other methods.

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1./

1. Introduction

The existence of damping forces within an oscillating aircraft structure has become increasingly important within recent years. Formerly, it had been possible to ignore their existence since in the type of problem encountered they could be shown to have the effect of a small safety factor e.g., raising critical flutter speeds, or limiting vibration amplitudes. More recently it has been shown that in certain flutter problems the inclusion of the effects of structural damping can reduce the critical speeds, and in the problem of aircraft ground resonance tests it is now well known that the structural damping destroys the purity of the higher modes of vibration. More recently still, the fluctuating jet efflux pressures acting on the adjacent structure have caused serious fatigue trouble. This problem is primarily one of a resonant condition of the structure being maintained by the jet pressures and the stress amplitudes set up are therefore directly related to the damping of the structural modes.

An investigation into the nature, cause, and effect of the structural damping forces is clearly warranted. By 'nature' is implied the variation with amplitude and velocity of distortion. In other words it is required to find the precise mathematical form in which the damping forces are to be included in the equations of motion of the distorting structure. Some early work along these lines gave rise to the "complex stiffness" concept of structural damping or "hysteretic damping" in which the damping force in harmonic motion is equivalent to a force proportional to the displacement but in phase with the velocity. Thus, if the displacement is q, the elastic restoring force is $c.q$, and the hysteretic damping force is $i.hq$, the total "stiffness" coefficient being the complex quantity $(c + ih)$. Subsequent work on a full-scale aeroplane wing[1] has shown a marked non-linearity in the damping, which increases with amplitude. Further work in the U.S.A. on a built-up box beam[2] was of a rather special nature, not directly applicable to aircraft structures, but the non-linearity of the damping (which had been predicted) was again demonstrated by experiment. This latter experiment, however, was not on a vibrating specimen, and the variation of damping with frequency was not investigated. There appears to be a complete absence of <u>fundamental</u> work under oscillating conditions on the simple source of structural damping - viz., a riveted joint. For this reason, the work at the University of Southampton is proceeding. It would be very desirable to study the nature of the damping under any dynamic condition, and not just under harmonic motion. If the damping is non-linear, it is not strictly true to represent an arbitrary motion by the superimposed effects of the normal modes of vibration of the structure. Hence, in the interests of general dynamic response problems, the damping under general conditions needs to be studied.

In certain aircraft vibration problems an increase of structural damping has a beneficial effect, e.g., in reducing the vibration and stress amplitudes set up by the jet efflux, and in increasing the critical speeds of certain types of flutter. In the jet efflux problem, where a damping increase is always beneficial, the modes of vibration excited involve distortion of the cross-section of the fuselage, tail-plane and control surfaces. In these modes, considerable increases of damping can be achieved by inserting "damping materials" between the adjacent surfaces of structural joints, and by spraying the surfaces elsewhere with damping compounds. The work in hand along these lines is aimed at determining the damping increase possible, and whether or not it is worth-while, considering the presence of acoustic damping whose magnitude is as yet unknown.

In the flutter problem, Broadbent[3] has investigated the general effect of structural damping on certain types of flutter. In this work, the structural damping was considered to be of the conventional linear viscous type (as distinct from hysteretic type), and 'direct' damping only

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in the constituent modes was included, i.e., 'cross' damping, or damping coupling was ignored. If the effect of the latter is to be investigated also, some idea of its magnitude compared with the direct terms is required. It is very difficult, if possible at all, to obtain this experimentally, but if a method of calculating the structural damping was available, an estimate of the cross-damping could be made for a simple structure. Such a method is presented in §2.2, where it is shown that the basic quantity required in the calculation is the energy dissipated by a single joint under a complete load cycle.

The experimental work necessary for the above investigations requires techniques for measuring the damping of structural components and idealized specimens. In some cases the frequency of vibration of the specimen will be high, and the damping low. In other cases, the damping may be high and the frequency low, but any intermediate combination may exist. Different techniques are required, therefore, for measuring the damping in the different cases. The main purpose of this report is to describe the techniques, both those already used and those proposed, and their applicability to the different cases. The results of the experimental work completed up to now are presented and discussed.

2. The Damping of a Riveted (or Bolted) Joint

2.1 The mechanism of the damping

The adjacent surfaces of two components (say plates) forming a simple joint are compressed one against the other by the bolt tightness or rivet forming. Actual contact between the surfaces will not be uniform, but will take place at the 'high spots' of the surfaces. These high spots (or 'contact areas') are not smooth surfaces on a macroscopic scale but consist of many jagged asperities, the peaks of which provide the contact between the surfaces. The higher the normal pressure, the deeper will the asperities be embedded in the other surface, and the greater will be the amount by which they are deformed.

When a small tangential load is applied to the joint, the load will be transferred from one plate to the other through the static friction over the contact areas and also through shear in the attachment. Elastic displacements will occur due to the elastic stress distribution in the plates (away from the contact areas), in the attachments, and also at the contact asperities. For small loads, it is probable that the proportion carried by the rivet or bolt is small (if anything at all) due to the imperfect connection between the attachment and the surrounding material. Any energy loss involved in a complete load cycle will be due to elastic hysteresis of the material of the joint.

As the tangential load increases, the stresses around the circumference of the contact areas enter the plastic range, and energy is lost in a load cycle due to plastic deformation. The reduced stiffness of the contact areas due to plasticity, together with the generally increased elastic displacements of the joint, probably mean that the proportion of the load on the attachment is increasing.

As the load increases further still, the static friction stress at the circumference of the contact areas is overcome, and the possibility exists of local relative slipping in an annular region around the circumference. This slipping takes place against the dynamic friction, with a consequent increase in energy loss. Further increase of tangential load increases the area of slip, and the energy loss increases rapidly. Rivet deflection from now on is accompanied by slipping.

This mechanism of energy loss has been suggested by Johnson[4] following work on the contact of a hardened steel ball with a hard steel flat plate. In his work, there were no bolts or rivets to restrict displacements after total slipping had occurred, and the above remarks

which/

which include the effects of attachments must be regarded as tentative. However, it is seen that energy is lost in three distinct stages:

(i) In the elastic range, due to elastic hysteresis,

(ii) In the plastic range, due to plastic deformation,

(iii) In the slipping range, due to dynamic friction.

In the first range, the energy loss (or damping) in the joint will be of exactly the same nature as that in a straightforward unjointed test specimen of the joint material under a cyclic load. For mild steel, the energy loss per cycle of stress less than 8000 lb/in^2 is proportional to the stress amplitude raised to the power 2.3, and this energy loss is independent of frequency. This approximates, in some measure, to a system with hysteretic damping, where the energy loss is proportional to the stress amplitude squared, and is also independent of frequency. For other materials, no extensive work has been carried out.

In the second range, the energy loss will depend on the stress distribution over the contact areas, and also on the material stress-strain relationship in the plastic range. Due to the complexity of the stress distribution and nature of the contact areas, a theoretical calculation of the energy loss is not feasible in this range.

In the third range, the energy loss depends on the coefficient of sliding friction, on the normal pressure between the plates, and on the amplitude of relative displacement. For a rivetted joint, it is virtually impossible to measure the second of these factors - or to predict it - so a theoretical calculation is again precluded. The extent to which this third range is entered is still open to question.

2.2 The representation of the damping

The complexity of the source of joint damping obviously means a corresponding complexity of its mathematical representation in the equations of motion of the system in which it is incorporated. If a single degree of freedom only is considered, and the motion is to be harmonic, it is most convenient to use the concept of "equivalent viscous damping", in which the viscous damping coefficient has such a value that the energy dissipated in a harmonic displacement cycle of a certain amplitude and frequency is the same as that of the actual damping system in the same displacement cycle. Alternatively, an equivalent hysteretic damping coefficient may be postulated, defined in analogous terms. With a damping system of the type described in §2.1, the equivalent viscous damping coefficient will clearly be a function of amplitude, and possibly also of frequency. It should be emphasized that the use of equivalent viscous damping is only justifiable in the treatment of systems under a steady harmonic exciting force, in which the harmonic purity of the response is not appreciably affected by the non-linearity of the damping. More work needs to be done on the approximate representation of complicated damping sources in systems subjected to non-harmonic loading.

Suppose, for the moment, that the damping at a joint is purely viscous. The joint may now be representated by a spring of rate c (representing the elastic flexibility of the rivet) in parallel with which is a damper of rate d. The mass of the joint may be ignored in this instance. Suppose now that the joint is subjected to a harmonic load, $P_o \sin \omega t$. Then, denoting the relative deflection of the joint plates by x, the equation of motion may be written

$$d\dot{x} + cx = P_o \sin \omega t \qquad \ldots(2.1)$$

where/

where $$x = x_0 \sin(\omega t + \epsilon) = \frac{P_0 \sin(\omega t+\epsilon)}{\sqrt{c^2 + d^2\omega^2}}$$

where $$\epsilon = \tan^{-1} \frac{d\omega}{c}.$$

Now the energy dissipated per cycle, E, is given by $\pi P_0 x_0 \sin\epsilon$, which upon subsitution becomes

$$E = \frac{P_0^2}{c} \frac{\pi \frac{d\omega}{c}}{1 + \left(\frac{d\omega}{c}\right)^2}. \qquad \ldots(2.2)$$

For a viscous damper, the energy dissipation is therefore proportional to $(\text{load})^2$, and if $d\omega/c$ is small, it is also approximately proportional to ω. For a hysteretic damper, the corresponding expression may be obtained by replacing $d\omega$ by the constant h, whence the energy dissipation is seen to be independent of frequency.

In point of fact, neither d nor $d\omega$ will be constant with amplitude, but must take the value of the equivalent viscous (or hysteretic) damping coefficient for the particular load (or displacement) amplitude. We should now therefore write the energy loss in the form

$$E = P_0^2 f(P_0, \omega) \qquad \ldots(2.3)$$

Consider now a structure which contains a number of joints, and which is vibrating in its r^{th} pure normal mode of vibration with a reference point amplitude of q_r. Denote the load coming on to the separate joints by $q_r p_{1r}$, $q_r p_{2r}$, $q_r p_{3r}$ etc., where p_{1r} is the load on joint 1 per unit amplitude in mode r. Suppose the function $f(P_0, \omega)$ is different for each joint, then the energy loss per cycle will be

$$q_r^2 \sum_n p_{nr}^2 f_n(q_r p_{nr}, \omega). \qquad \ldots(2.4)$$

If the equivalent direct damping coefficient corresponding to the generalized co-ordinate q_r is b_{rr}, then the energy loss per cycle is

$$\pi b_{rr} \omega q_r^2. \qquad \ldots(2.5)$$

Whence, equating (2.4) and (2.5),

$$b_{rr} = \frac{1}{\pi\omega} \sum_n p_{nr}^2 f_n(q_r p_{nr}, \omega). \qquad \ldots(2.6)$$

If $f(P_0, \omega)$ is known for each joint of the structure, the equivalent viscous damping coefficient for the mode may be calculated from (2.6).

Suppose now that the joint damping behaves in a purely <u>hysteretic</u> manner. The energy loss given by (2.3) now becomes kP_0^2 where k is a constant for all values of P_0 and ω. Clearly, therefore, k is the energy dissipated by the joint at unit load. The energy dissipated by the whole structure in one cycle of the r^{th} mode (corresponding to equation (2.5)) is now $\pi h_{rr} q_r^2$, whence it is found that

$$h_{rr} = \frac{1}{\pi} \sum_n k_n p_{nr}^2. \qquad \ldots(2.7)$$

Furthermore,/

Furthermore, by using Bishop's "Hysteretic" Dissipation Function (Ref.5) it is readily shown that the damping coupling coefficient between modes r and s is

$$h_{rs} = \frac{1}{\pi} \sum_n k_n p_{n_r} p_{n_s} . \qquad \ldots(2.8)$$

Thus, to arrive at the damping coefficient for a particular mode of vibration, it is necessary only to know the "energy dissipation coefficient", k_n (or $f_n(P_o, \omega)$) for each joint in the structure, together with the joint loads for the mode concerned at unit reference amplitude.

2.3 The effect of the joint flexibility

If a joint in the structure carried only direct bending loads, its flexibility may be represented by a local reduction in EI, the flexural rigidity of the structure. It is more difficult, however, to represent the flexibility of a joint transmitting shear, such as that between a stringer riveted to a skin panel, when the stringer and skin together are vibrating as a beam. (Such a mode of vibration is encountered in the problem of jet-efflux excitation). For the moment, suppose the joint is purely elastic (i.e., no friction or damping) and that the beam is encastré. For the joint, therefore, to transmit shear, there must be a relative displacement between the stringer and skin. At an encastré end, where there may be a large shear on the beam, no relative displacement is possible, and the joint carries no load. Simple beam theory predicts shear loads (and therefore joint loads) which are proportional to the applied shear at all points, including encastré ends.

The elementary beam theory must therefore be modified to take account of the joint flexibility, and the correct equation relating the loading to deflection ($w = EI \frac{d^4y}{dx^4}$ in the elementary theory) may be shown to contain the 6^{th} derivative of y, and the 2nd derivative of w and new constants involving the section dimensions and the joint (or rivet) flexibility. The effect of the flexibility on the joint load, as described above, may be shown to die out exponentially away from the encastré end, and well away from such a constraint, the shear on the joint is the same as that predicted by the elementary beam theory.

When the effects of damping and friction are included, static friction will permit a certain amount of load to be carried by the joint near the encastré end. The energy dissipation per unit length of beam at any point will be of the form $S^2.f(S, \omega)$ where S is the shear per unit length on the joint at that point, as given by the theory outlined above, modified slightly to allow for the presence of static friction. The energy dissipation per unit length clearly, therefore, varies along the length of the beam.

2.4 The quantities to be measured in an experimental investigation

To investigate the joint damping characteristics experimentally, it is apparent that the important quantity required is the energy loss within the joint per cycle of load at different amplitudes. For the case of the type of joint discussed in §2.3, the rate of energy dissipation per unit length of the beam is required. For ad hoc experiments to compare the damping capacity of different types of joint in the same type of structure, vibrating in similar modes, the "equivalent damping ratio" is a suitable basis for comparison (i.e., ratio of equivalent damping coefficient to critical damping coefficient). Likewise, for investigations into the effect of damping compounds sprayed on the vibrating surfaces, the equivalent damping ratio is the most convenient quantity to determine.

In the case of hysteretic damping, the term "damping ratio" has not the physical significance as for viscous damping, since hysteretic damping is really only defined for harmonic motion, and not for aperiodic motion. Its definition must therefore be by analogy with the expression for the damping ratio for viscous damping, thus:

The equation of motion of a viscous damped system under harmonic excitation is:

$$a\ddot{q} + b\dot{q} + cq = P\,e^{i\omega t}$$

and for a hysteretic damped system

$$a\ddot{q} + (c + ih)q = P\,e^{i\omega t}.$$

The latter is equivalent to

$$a\ddot{q} + \frac{h}{\omega}\dot{q} + cq = P\,e^{i\omega t}$$

i.e., the viscous damper, b_e, which is equivalent to the hysteretic damper h, at frequency ω, has a rate $\frac{h}{\omega}$.

Now the critical viscous damping coefficient is $2\sqrt{ac}$ and therefore the viscous damping ratio, $\delta_v = \frac{b}{2\sqrt{ac}} = \frac{b}{2a\,\omega_n}$ where ω_n is the undamped natural frequency of the system.

Since we are most interested in comparing damping ratios at the natural frequencies of systems, we may define the hysteretic damping ratio, δ_H, as the ratio of the equivalent viscous damping coefficient at the natural frequency to the critical viscous damping coefficient for the system.

i.e.,
$$\delta_H = \frac{b_e}{2a\,\omega_n} = \underline{\underline{\frac{h}{2a\,\omega_n^2}}} = \frac{h}{2c}.$$

The complex stiffness, $(c + ih)$ of the hysteretic damped system is often written $c(1 + i\mu)$, i.e., $h/c = \mu$. $\therefore$ $\underline{\underline{\delta_H = \mu/2}}$.

3. Some Techniques of Damping Measurement

In this section, the principles of the techniques are described. The experimental details, showing how these techniques are applied to the measurement of joint damping, are given in section 4.

3.1 The static hysteresis test

For hysteretic damping, it has been shown in §2.2 that the energy dissipated by the damper when subjected to an oscillating load is independent of frequency. This is the same as saying that the area contained within the hysteresis loop of a cyclically loaded joint is the same for all frequencies. Hence, the energy loss per cycle may be obtained at virtually zero frequency, by simply loading the joint by known dead-weights over a complete cycle, accurately measuring the deflection, and hence obtaining a hysteresis loop whose area is equal to the energy loss per cycle.

3.2 The oscillation decay curve method

This is the well-known method in which the system is excited in a pure normal mode of vibration at the corresponding natural frequency. The exciting force is suddenly switched off, or otherwise disengaged from the system, and a record is taken of the ensuing decaying oscillation. Provided no other modes of vibration are excited by the transient effect of cutting off the exciting force, and provided also that the stiffness and damping (hysteretic or viscous) are linear, the oscillation decays exponentially. Plotting against time the logarithm of the amplitude of the decaying oscillation, the slope of the resulting straight line is equal to the damping ratio times the undamped natural frequency of the mode.

For a linear system having several degrees of freedom, there is the possibility that the unwanted modes excited on cutting off the exciting force will seriously impair the "exponential purity" of the decaying waveform. Ellington and McCallion[6] have given a method whereby the frequencies and damping ratios of the modes contributing to the decaying motion can be found from a record of the motion. This method, however, is likely to give large errors in the damping ratio for quite small errors in measurement if the frequencies of two of the constituent modes are close and their dampings small.

When the damping is non-linear, the decaying oscillation will no longer be of a simple exponential form. The plot of the logarithm of the amplitude against time will be a curve, the slope of which will be proportional to the equivalent damping ratio at that amplitude, provided the damping is small. If the damping is large, it is doubtful whether this method will give results of sufficient accuracy.

3.3 The energy input method

The energy input required to maintain a system in a state of steady vibration at constant amplitude is, of course, equal to the energy being dissipated within the system by the damping sources. This energy dissipation has been related to the damping coefficient corresponding to a particular mode in §2.2. Since, in the joint damping investigation, it is the energy dissipation at the joint that is required, it is particularly convenient to use a method of damping measurement which involves the direct measurement of energy input.

If the force exciting the system is $P(t)$, and the velocity of the system at the point of excitation is $\dot{y}$, then the work done in time t_1 is $\int_0^{t_1} P(t)\dot{y}\,dt$. When both the exciting force and the velocity are harmonic, we may write $P(t) = P_1 \cos \omega t$, and $y = y_1 \cos (\omega t + \epsilon)$. The energy input per cycle then becomes -

$$- \pi P_1 y_1 \sin\epsilon. \qquad \ldots(3.1)$$

The measurement of the energy input may therefore be resolved into measuring the amplitude of force and displacement (P_1, y_1) and the phase relationship (ϵ) between them. If the system is being excited near resonance of one mode, and the presence of other modes is inappreciable, then ϵ will be close to 90°, and the energy input per cycle is approximately $\pi . P_1 y_1$. This involves the measurement of two quantities, while for greater accuracy the third quantity, ϵ, is required. ϵ may be difficult to measure if non-linearities in the system introduce harmonics into y or P. In this case the expression for the energy input per cycle of the fundamental frequency component becomes

$$- \pi \sum_{n=1}^{\infty} n . P_n y_n \sin \epsilon_n \qquad \ldots(3.2)$$

where/

where P_n, y_n are the amplitudes of the n^{th} harmonics in P and y, and ϵ_n is the phase relationship between the n^{th} harmonics of P and y. Clearly, therefore, in the presence of distortion, the measurement of energy input by finding the P_n's and y_n's is impracticable.

To avoid this difficulty, the technique may be used of taking electrical signals proportional to and in phase with the force and velocity, multiplying them together, and measuring the D.C. component of the product.

Suppose the signal proportional to the force is

$k_p P(t) = k_p \sum_{n=1}^{\infty} P_n \cos n\omega t$ and the signal proportional to the velocity is

$k_v \dot{y} = k_v \sum_{n=1}^{\infty} y_n n\omega \sin (n\omega t + \epsilon_n)$. Then

$k_p P(t) . k_v \dot{y} = k_p k_v \Sigma\, n.\omega.P_n y_n \sin \epsilon_n + k_p k_v \Sigma$ terms involving

$\sin(2n\omega t + \epsilon_n)$ i.e., the D.C. component is $k_p k_v . \omega \sum_{n=1}^{\infty} n.P_n y_n \sin \epsilon_n$.

$$\ldots (3.3)$$

Since ω is the known fundamental frequency of excitation, and k_p and k_v are also known, the value of $\pi \sum_{n=1} n.P_n.y_n \sin \epsilon_n$ may readily be found. If either P or $\dot{y}$ is free from harmonic distortion, the energy input per cycle reduces to the form of equation (3.1). The distortion of one of them still makes the measurement of $\sin \epsilon$ difficult, but the technique of multiplying the signals still avoids the difficulty.

If this method is to be used to determine the damping coefficient corresponding to a particular mode, it is necessary that that mode only shall be excited, and that no others shall be present in the motion. Denoting the generalized viscous damping coefficient by b_{rr}, which corresponds to the co-ordinate y_r, representing the displacement in the r^{th} mode, the energy dissipated per cycle is $\pi y_r^2 b_{rr} \omega$. Equating this to the energy input measured, b_{rr} is found. If the damping is hysteretic, the energy dissipation per cycle is given instead by $\pi . y_r^2 h_{rr}$, whence h_{rr} may be determined.

3.4 The energy flux method

If a damping system exists within a steadily vibrating beam, there will in general be a flux of energy along the beam from the point of excitation to the points of energy dissipation. When the damping system consists of discrete joints in the beam, the flux of energy will change abruptly at any one of the joints. If this energy change can be measured, the basic quantity required for the joint damping investigation will be obtained. Furthermore, it will permit a detailed check on the "superposition theory" of §2.2, where the total damping and energy loss for a multi-jointed beam is derived from a knowledge of the energy dissipation characteristics of single joints.

Such a flux of energy can only exist when the phase relationship between the displacement of a reference point and the displacement of any other point varies throughout the structure. This is the same as saying that the beam must vibrate in two modal components, one of which is in phase with the reference displacement, and the other in quadrature with it.

Let the in-phase modal displacement amplitude at any point be denoted by v_1 and the quadrature displacement amplitude by v_2. Associated with these displacements are the shear and bending moments S_1, M_1; S_2, M_2 in phase with v_1 and v_2 respectively.

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The resultant displacement, shear and bending moment at any time may therefore be written

$$v_1 \sin \omega t + v_2 \cos \omega t = v$$
$$S_1 \sin \omega t + S_2 \cos \omega t = S \qquad \ldots(3.4)$$
$$M_1 \sin \omega t + M_2 \cos \omega t = M$$

v will be taken as positive upwards, M positive when causing sagging, S positive thus:

Now consider the shear S acting on a section of the beam whose vertical velocity is $\dot{v}$. The work done by S on the section during a complete cycle is

$$+ \int_0^{2\pi/\omega} S.\dot{v}\, dt .$$

Likewise considering the bending moment M moving through an angular displacement at the rate $\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}\right)$, the work done in one complete cycle by M is

$$- \int_0^{2\pi/\omega} M. \frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}\right) dt .$$

Substituting for v, M, and S, the total work done by M and S on the section during one cycle, becomes

$$\pi\left\{ S_2 v_1 - S_1 v_2 - M_2 \frac{dv_1}{dx} + M_1 \frac{dv_2}{dx} \right\}. \qquad \ldots(3.5)$$

The work done on the section is, of course, the "energy flux" across the section.

Consider now the change in this quantity across a joint where energy is being dissipated. Let v_1, v_2, S_1, S_2, M_1, M_2 be the values at $x = x_1$ on one side of the joint, and on the other side at $x_1 + \ell$ (ℓ small), let them be $(v_1 + \delta v_1)$, $(v_2 + \delta v_2)$ etc.

Now the joint represents a discontinuity, and due to the rivet deflection the possibility exists that there will be a small discontinuity in $\frac{dv}{dx}$. Let the total increment of $\frac{dv_1}{dx}$, $\frac{dv_2}{dx}$ over the distance ℓ be $\Delta \frac{dv_1}{dx}$, $\Delta \frac{dv_2}{dx}$. Due to the discontinuity it is <u>not</u> possible to say $\Delta \frac{dv_1}{dx} \doteqdot \ell \left(\frac{d^2 v_1}{dx^2}\right)_{x=x_1}$. Then, to second order,

δv_1/

$$\delta v_1 = \left(\frac{dv_1}{dx} + \tfrac{1}{2}\, \Delta \frac{dv_1}{dx} \right) \ell . \qquad \ldots (3.6)$$

Since the system is in simple harmonic motion

$$\delta S_1 = \omega^2 \times \text{mass of element } \ell \times \text{mean amplitude of element}$$
$$= \omega^2 \times m\ell \times \left(v_1 + \frac{\ell}{2} \frac{dv_1}{dx} \right)$$

and $$\delta M_1 = (S_1 \times \ell) + \left(\delta S_1 \times \frac{\ell}{2} \right)$$

$$= S_1 \ell + \omega^2 m \frac{\ell^2}{2} \left(v_1 + \frac{\ell}{2} \frac{dv_1}{dx} \right)$$

with similar expressions for δS_2 and δM_2.

Substituting $(S_1 + \delta S_1)$, $(v_1 + \delta v_1)$ etc, into (3.5), the expression for the energy flux at $x_1 + \ell$ is obtained. Subtracting the energy flux expression for $x = x_1$ (i.e., equation (3.5) as it stands), the energy loss at the joint is found to be

$$\pi \left\{ \left(M_2 + S_2 \frac{\ell}{2} \right) \Delta \frac{dv_1}{dx} - \left(M_1 + S_1 \frac{\ell}{2} \right) \Delta \frac{dv_2}{dx} \right\} . \qquad \ldots (3.7)$$

But $M_2 + S_2 . \frac{\ell}{2}$, $M_1 + S_1 \frac{\ell}{2}$ are the bending moment components mid-way between x_1 and $x_1 + \ell$. If the terms "quadrature" and "in-phase" are now taken to be with reference to the resultant bending moment at this mid-point, there will be no $\left(M_2 + S_2 \frac{\ell}{2} \right)$, since a harmonically varying quantity has no quadrature component with reference to itself. The energy loss at the joint then becomes

$$- \pi . \left(M + S \frac{\ell}{2} \right) \Delta \frac{dv_2}{dx} . \qquad \ldots (3.8)$$

The measurements required are therefore the bending moment at the joint, and the quadrature component (with respective to this moment) of the change of slope over a small length across the joint.

The expression (3.8) gives the correct energy loss only if the joint is part of the structure carrying bending stress. If, at the same point, there is a joint which carries only vertical shear loads, and dissipates energy as a load cycle is applied, its energy loss must be allowed for by admitting a discontinuity in <u>displacement</u> Δv_1 and Δv_2 at the joint. Equation (3.6) then gives

$$\delta v_1 \doteqdot \Delta v_1 + \left(\frac{dv_1}{dx} + \frac{1}{2} \frac{dv_1}{dx} \right) \ell$$

and the expression for the energy loss becomes

$$- \pi . \bar{M} . \ \Delta \frac{dv_2}{dx} + \pi . \bar{S} \ \Delta v_2 \qquad \ldots (3.9)$$

where/

where $\bar{M}$, $\bar{S}$ are the moment and shear amplitudes respectively mid-way between the two sections; $\frac{dv_2}{dx}$ is as defined above, but Δv_2 is the quadrature component of the displacement discontinuity, taking the shear $\bar{S}$ as the phase reference.

The practical difficulties of measuring Δv_2 on a vibrating beam will rule out this method for measuring the energy loss at a shear joint or a combined bending and shear joint, but it is quite possible that $\Delta \frac{dv_2}{dx}$ will be measurable. Therefore, on a box-beam which has joints only in the walls carrying the bending loads, and which is vibrating as a free-free beam, accurate measurement of the inphase and quadrature components of displacement will enable the bending moment to be calculated (by double integration of the inertia forces) and accurate displacement measurements on either side of the joint will enable $\Delta \frac{dv_2}{dx}$ to be determined. Since there is no shear joint, the displacement v_2 will be continuous and Δv_2 therefore zero. The total energy loss at the joint is then

$$-\pi.\bar{M}.\ \Delta \frac{dv_2}{dx}.$$

It should be emphasized that this technique is only of limited use, as it is only in a research programme of this nature that the energy dissipation in a small part of a beam is required. Obviously, the usefulness of the method depends on the accuracy with which the quadrature components of slope change can be measured. This is discussed in §4.6.

3.5 The frequency response curve method

This well known method applies to a single degree of freedom linear system with viscous or hysteretic damping. The system is excited by a harmonically varying force which may be constant in magnitude, or varying in proportion to frequency or $(\text{frequency})^2$. The amplitude of response is measured at different forcing frequencies in the neighbourhood of resonance, and the frequency response curve drawn. From this curve are found the two frequencies at which the amplitude is $R \times$ the maximum (resonant) amplitude. If the difference of these frequencies is $\delta\omega$, and the natural frequency is ω_n, then the damping ratio is given approximately by

$$\frac{1}{2\sqrt{\frac{1}{R^2} - 1}} \left(\frac{\delta\omega}{\omega_n} \right).$$

This is correct for either δ_V or δ_H, the viscous or hysteretic damping ratios.

The method is limited to linear systems of effectively one degree of freedom. If there are several degrees of freedom, the presence of unwanted modes at the resonant frequency of the desired mode may cause considerable error. The accuracy of the above expression improves with smaller damping, but the experimental difficulties of measuring the amplitude on the steep sides of the resonant peak increase as damping decreases. High stability of frequency is therefore required for the exciting force. As the damping increases, the factor R must be limited to maintain accuracy. If, however, R is taken to be no less than $1/\sqrt{2}$ (which effects a great simplification in the above formula), then the error will be no more than 10% for $\delta = 0.25$.

The/

The advantage of the method is that only ω and amplitude need to be measured, over a small frequency range near the resonant peak.

3.6 The phase change method

When a linear, damped single degree of freedom is excited harmonically, the displacement lags in phase behind the exciting force by the angle ϵ given by

$$\tan \epsilon = \frac{\omega b}{(c - a\omega^2)} \quad \text{(for viscous damping)}$$

$$\text{or } \tan \epsilon = \frac{h}{(c - a\omega^2)} \quad \text{(for hysteretic damping).}$$

On differentiating, it may be shown that for viscous damping in the immediate vicinity of the natural frequency, ω_n,

$$\frac{\partial \epsilon}{\partial \omega} = \frac{2a}{b} = \frac{1}{\delta_v . \omega_n} \qquad \ldots(3.10)$$

and for hysteretic damping

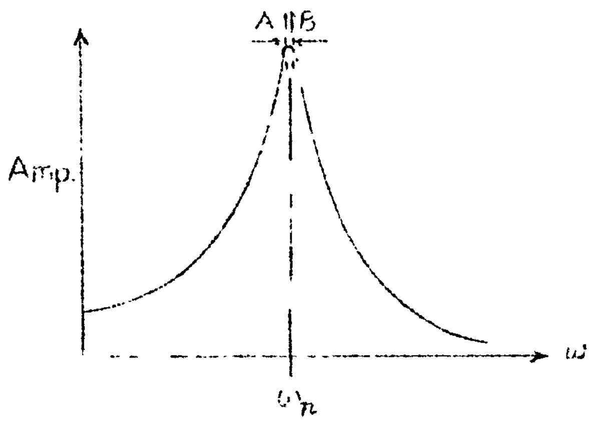
$$\frac{\partial \epsilon}{\partial \omega} = \frac{1}{\delta_H . \omega_n}. \qquad \ldots(3.11)$$

Hence, if the system is excited successively at two different frequencies ω_1, ω_2, both very close to resonance, and the corresponding phase angles ϵ_1 and ϵ_2 are measured, then

$$\frac{\partial \epsilon}{\partial \omega} = \frac{\epsilon_2 - \epsilon_1}{\omega_2 - \omega_1}$$

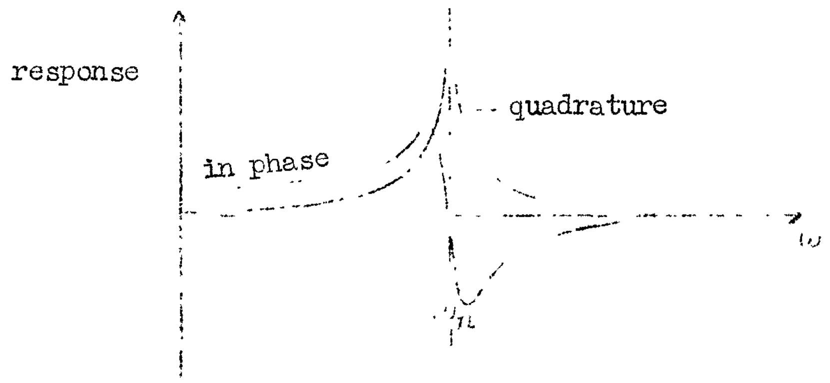
$$\therefore \quad \delta_H \text{ or } \delta_v = \frac{\omega_2 - \omega_1}{\omega_n} \cdot \frac{1}{\epsilon_2 - \epsilon_1}. \qquad \ldots(3.12)$$

ϵ, of course, is changing very rapidly at ω_n when ω_n is small. Once again, therefore, very high stability of frequency is required for the exciting force. Furthermore, the method is accurate only so long as the measurements are taken at frequencies "on the resonant peak", and not at those on the sides, i.e., suppose the peak is thus:



then/

then the measurements must be taken between A and B, when the amplitude is not appreciably different from the maximum amplitude. The reason for this may be seen by considering the in-phase and quadrature components of the response in the neighbourhood of ω_n.



At ω_n, the in-phase component is varying almost linearly with frequency, while the quadrature component is stationary. The rate of change of phase angle at ω_n may be shown to be

$$\frac{\text{Rate of change of in-phase response, with respect to } \omega, \text{ at } \omega_n}{\text{Magnitude of quadrature response at } \omega_n}.$$

This will be almost constant over the narrow frequency range where both numerator and denominator are almost constant, i.e., over the peak of the quadrature curve. (The in-phase response maintains a 'constant' slope over a wider range than this.) Outside this range, $d\epsilon/d\omega$ changes rapidly.

The requirements for this method to be of use are high stability of frequency of the exciting force, the ability to vary the frequency continuously, and accurately to measure the frequency increments.

The advantage of the method is that the damping is measured at virtually constant amplitude, and that only four quantities need to be determined - the two frequencies, and 2 phase angles.

3.7 The self-excited method

If the system is excited electro-magnetically, the exciter can be supplied through a power amplifier from an electrical velocity transducer attached to the system at the point of excitation. If there is no phase shift within the electronic circuits supplying the exciter, the system will execute a self-maintained oscillation at a natural frequency of the system, when the gain of the supply amplifier is at a certain critical value. The theory governing the motion of the system is as follows. Let q_1, q_2, q_3 ...... be generalized normal co-ordinates, representing the displacement of each normal mode of the system at the (single) point of excitation. Suppose the exciting force at this point is $F(t)$. The equations of motion are

$$a_1\ddot{q}_1 + b_1\dot{q}_1 + c_1q_1 = F(t)$$

$$a_2\ddot{q}_2 + b_2\dot{q}_2 + c_2q_2 = F(t).$$

etc. etc.

Structural damping coupling is assumed not to exist, but its effect may easily be included.

The/

The force, $F(t)$ is applied by means of an electro-magnetic exciter whose force output F is proportional to the supply current, I

i.e., $F = f.I$ where f is the force per unit current.

Now the exciter is supplied with a voltage, E_s, proportional to the total velocity at the point of excitation

i.e., $E_s = \bar{k}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \ldots\ldots)$

This voltage is opposed by $I \times$ the impedance of the exciter when clamped (Z_E) and the back E.M.F. due to the motion $(\dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \ldots\ldots)$ of the exciter coil

$$\therefore E_s = I.Z_E + e(\dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \ldots\ldots)$$

where e is the back E.M.F. per unit velocity.

$$\therefore I = \frac{(\bar{k} - e)(\dot{q}_1 + \dot{q}_2 + \ldots\ldots)}{Z_E}$$

$$\text{and } F = \frac{f(\bar{k} - e)}{Z_E}(\dot{q}_1 + \dot{q}_2 + \ldots\ldots) = (k' - e')(\dot{q}_1 + \dot{q}_2 + \ldots)$$

where k' and e' are real if Z_E is purely resistive. If Z_E has an inductive component, its effect may be included by introducing acceleration terms $\ddot{q}_1$, $\ddot{q}_2$ etc. into F. The equations of motion now become

$$a_1\ddot{q}_1 + (b_1 + e' - k')\dot{q}_1 + c_1q_1 + (e' - k')(\dot{q}_2 + \dot{q}_3 + \ldots\ldots) = 0$$

$$a_2\ddot{q}_2 + (b_2 + e' - k')\dot{q}_2 + c_1q_1 + (e' - k')(\dot{q}_1 + \dot{q}_3 + \ldots\ldots) = 0.$$

There is now an electrical damping coupling between the modes, which must be added to the structural damping coupling when the latter is included.

The only variable term in these equations is k', which is proportional to the amplifier gain. For a critical value of k', the above equations will represent simple-harmonic motion, i.e., the system is self-excited.

Now if the damping in the system, represented by b_1, b_2, etc, is small, the damping coupling terms may be neglected in a first approximation. The equations of motion are then of the simple form

$$a_n\ddot{q}_n + (b_n + e' - k')\dot{q}_n + c_nq_n = 0.$$

This represents S.H.M. when k' is equal to the lowest value of $(b_n + e')$, i.e., at this value of k' the effective direct damping in the n^{th} mode will be zero and the system will be self-excited in predominantly the n^{th} mode. If k' exceeds this value, the system will be unstable, since the mode q_n will be negatively damped.

k' and e' are characteristics of the electronic apparatus, and may be determined accurately. Then, for the mode being excited, $b_n = k' - e'$. It is obviously important for accurate determination of b_n that $k' - e'$ should not be the small difference of two large quantities, i.e., the electrical damping effect, e', of the exciter on the structure should not be much larger than the structural damping b_n. If possible, it should be much smaller.

To/

To stabilize the amplitude of the motion of the undamped system, a slight non-linearity is often introduced into the amplifier, such that k' is only equal to $(b_n + e')$ when $\dot{q}_n$ has a certain (pre-determined) amplitude. If $\dot{q}_n$ falls below this value, k' increases, where if $\dot{q}_n$ rises, k' decreases. The effect of this non-linearity on the proposed method of damping measurement needs further investigation, but so long as the oscillation has no appreciable harmonic distortion due to the non-linearity, it would seem that the method should still be satisfactory.

The effect of the damping coupling terms also needs further investigation. The condition for neutral stability of the system is no longer that the total direct damping in one mode should vanish, but the Routhian Test Function method of examining stability should be used.

An alternative method for measuring the damping of a single degree of freedom by this self-excited technique would be to introduce a known phase-shift between the velocity proportional signal and the exciter supply voltage. The natural frequency of the whole system would be measured with zero phase-shift, and again with a small phase-shift of, say 10°. The change in frequency can be shown to be the same as that required in a conventionally excited system to produce the same phase shift, i.e., the method provides a way of determining

$$\frac{d\varepsilon}{d\omega} = \frac{1}{\delta . \omega_n}$$

(see §3.6) δ in this case is the ratio of the total (structural + electrical) damping to the critical damping.

The mode which will be self-excited is usually the fundamental, since higher modes usually have higher damping coefficients. It is conceivable, however, that a higher mode may have the lowest damping, in which case that mode will be excited. This fact shows the limitations of the method for damping measurement as it is not known before the experiment which mode will appear. Other modes can be excited by introducing tuneable filters into the circuit, and hence making k' vary with frequency. The theory then becomes more complex, and needs further examination before the method can be used with certainty for damping measurement.

The method is further restricted in application to systems without close natural frequencies, if single point excitation is to be used.

3.8 Multi-point excitation methods

Most of the methods described above break down when the mode whose damping is required is close in natural frequency to another mode. When exciting with a single point exciter at the desired natural frequency, the energy put into the system goes to sustain motion in both modes, and the response characteristic (displacement and phase) are no longer functions of the mass, damping and stiffness of one mode only.

If just two modes are present in the motion, the addition of one other exciting force of appropriate magnitude and phase can eliminate the presence of one of them. Let two forces $F_1 e^{i\omega t}$, $F_2 e^{i\omega t}$ be acting at points on the structure where the displacements in mode 1 are $\phi_{11} q_1$, $\phi_{12} q_1$ and in mode 2 are $\phi_{21} q_2$, $\phi_{22} q_2$. F_1 and F_2 may be complex, to represent a phase difference between them.

Suppose structural damping coupling exists between the modes. The equations of motion become

$$a_1 \ddot{q}_1 + b_{11} \dot{q}_1 + c_1 q_1 + b_{12} \dot{q}_2 = F_1 \phi_{11} e^{i\omega t} + F_2 \phi_{12} e^{i\omega t}$$

$$a_2 \ddot{q}_2 + b_{22} \dot{q}_2 + c_2 q_2 + b_{21} \dot{q}_1 = F_1 \phi_{21} e^{i\omega t} + F_2 \phi_{22} e^{i\omega t}.$$

The/

The solutions of these equations will be

$$q_1 = \bar{q}_1 e^{i\omega t}, \quad q_2 = \bar{q}_2 e^{i\omega t}$$

where $\bar{q}_1$, $\bar{q}_2$ will in general be complex.

$$\left.\begin{aligned} \therefore\ \bar{q}_1(c_1 - a_1\omega^2 + i\omega b_{11}) + \bar{q}_2\ i\omega b_{12} &= F_1\phi_{11} + F_2\phi_{12} \\ \text{and } \bar{q}_2(c_2 - a_2\omega^2 + i\omega b_{22}) + \bar{q}_1\ i\omega b_{21} &= F_1\phi_{21} + F_2\phi_{22} \end{aligned}\right\} \quad \ldots(3.13)$$

From these equations, it is seen that there is one set of complex values of F_1 and F_2, at each frequency, for which $\bar{q}_2$ is zero, i.e., the second mode will have been eliminated from the motion. In particular, at the natural (undamped) frequency of q_1, if $\bar{q}_2$ is zero,

$$\begin{aligned} \bar{q}_1 . i\omega b_{11} &= F_1\phi_{11} + F_2\phi_{12} \\ \text{and } \bar{q}_1 . i\omega b_{12} &= F_1\phi_{21} + F_2\phi_{22} \end{aligned} \quad \ldots(3.14)$$

If $\bar{q}_1$ is now considered to be purely imaginary, both F_1 and F_2 must be purely real, i.e., F_1 and F_2 are in phase with one another, but in quadrature with q_1. Hence, at the natural frequency of q_1, q_2 may be eliminated by having F_1 and F_2 in phase with one another, and adjusting their relative amplitudes. At any other frequency, q_{12} can only be eliminated by adjusting the relative phase of F_1 to F_2, as well as the relative amplitudes.

The technique of measuring damping in this case is to excite the system with forces in phase with one another, as close to the natural frequency as can be estimated, and vary the force amplitude ratio to minimize phase shift through the structure. Adjust the frequency to bring the amplitude as near as possible into quadrature with the exciting forces, and readjust the force amplitude.

Then, from above, since $i\omega\bar{q}_1 = \dot{q}_{1\,max}$

$$b_{11} = \frac{F_1\phi_{11} + F_2\phi_{12}}{\dot{q}_{1\,max}} = \frac{\text{Total Energy Input (per cycle)}}{\pi\omega q^2_{1\,max}}$$

Thus, the energy input method of damping measurement is applicable. The other methods of damping measurement in which a change of frequency is involved, will not be practicable, since to maintain motion in one mode only, the amplitude ratio and phase relationship of the forces must be varied as the frequency changes. If however, the damping coupling is zero, the force amplitude ratio and phase relationship at the natural frequency can be seen to be the same as that required at any other frequency, for a single mode to be excited. The other methods of damping measurement may then be used.

It would appear from the above equations, that the damping coupling coefficient, b_{12}, may be determined experimentally. For, from equation (3.14)

$$b_{12} = \frac{F_1\phi_{21} + F_2\phi_{22}}{\dot{q}_{1\,max}}$$

where F_1, F_2 are the forces required to eliminate q_2 and to maintain an amplitude of oscillation $q_{1\,max}$ in the mode 1 at its natural frequency. ϕ_{21}, ϕ_{22} are the "displacement factors" for mode 2, as defined. $F_1\phi_{21} + F_2\phi_{22}$ will be, almost certainly, the small difference of large quantities, and practical difficulties of accurate measurement may therefore make it impossible to find b_{12} with any accuracy.

The/

The above theory for the excitation of just one mode out of a possible pair may easily be extended to show that when n modes are liable to be excited, n independent forces need to be used in certain proportions to cause one pure mode to be excited. The direct damping coefficient for each mode is again found by the energy input method, when the mode concerned is being excited at its natural frequency.

4. The Practical Application of the Techniques in a Joint Damping Investigation

4.1 The general nature of the experiments

For the investigation into the damping properties of riveted joints, the joints were incorporated in the walls of a rectangular section hollow extrusion of light alloy. A length of 10 ft was tested as a beam, in which the joints carried bending loads only. The damping of the beam when vibrating in a fundamental mode was measured by the energy input method and oscillation decay curve method, and the energy loss per cycle was also obtained by the static hysteresis method by subjecting the beam when pin-ended to a complete cycle of a concentrated load at its centre.

The experiments were carried out with different numbers of joints, and different numbers of rivets in each joint. Furthermore, in the oscillation decay experiments, masses were added to the beam to vary the natural frequency of the whole system and so to investigate the effect of frequency on the damping.

To allow for the damping of the beam supports, the vibration exciter, the added masses, and all other extraneous effects, identical experiments were carried out on an unjointed beam of the same type. This measured damping was therefore subtracted from the damping of the jointed beam.

The results of the experiments are given in Section 5.

4.2 Description of the specimen tested

The dimensions of the beam cross-section were 4.5 in. × 2 in. × 1/8 in. To make the joints, slots were cut across and through the 4.5 in. top and bottom walls. Joint plates, of 10 S.W.G. aluminium, 4 in. square, were riveted across the slots to form a butt-joint with a single plate per joint. The rivets were 7/32 in. diameter, and snap-headed.

The joints were made four at a time, symmetrically about the centre line. The beam was tested with three different joint arrangements:

(i) 4 joints, each 7.5 in. from the beam centre, and 4 rivets per joint (i.e., 2 rivets on either side of the slot).

(ii) 4 joints each at 7.5 in. from the beam centre and 10 rivets per joint.

(iii) 8 joints, with 4 at 7.5 in. and 4 at 15 in. from the beam centre, and 10 rivets per joint.

In each of the experiments, the beam was supported by knife-edges firmly attached to the beam, resting in V-notches attached to an independent supporting structure. The apices of the knife-edges were in the plane of the beam neutral axis (see Fig. 1).

The masses used to change the beam natural frequency were circular solid cylinders of steel, different thicknesses being used for the different frequencies required. They were attached, one on each of the shorter sides of the beam, by means of 3/8" diameter steel bolts, which passed right through the beam. They were situated at 4 stations along the beam, at 20 in. and 59 in. on either side of the beam centre. (see Fig. 2)

4.3/

4.3 The static hysteresis test

The knife-edge supports were fixed to the ends of the beam, and the V-notches were suspended from inverted channels resting on a rigid steel frame (see Fig.3). At one end, the V notch was rigidly attached to the steel channel, but at the other end was suspended by means of a thin wire, to permit longitudinal contraction of the beam when bent. A channel of the same length of the beam spanned the two inverted supporting channels, and carried a dial-indicator at the centre, measuring the deflection of the centre of the beam relative to its pinned ends. Up and down loads were applied to the beam at its centre on a bolt through the shorter sides of the beam. Up loads were applied through a lever system (see Fig.4), all of whose pivots consisted of hardened steel knife edges to minimize friction effects. Down loads were applied directly on to a loading pan suspended from the bolt. Complete load cycles of amplitude 10, 20, 30, 40, 50, 60 and 70 lb were applied, corresponding to a maximum joint load of about 1800 lb. To prevent the beam from lifting off the supports when the upload was applied, weights were suspended from the knife edges, to hold them down in the V-notches.

The procedure was first to apply rapidly a number of complete load cycles of the required amplitude in order to make the hysteresis loop "close up". Three cycles were then applied while complete records of the deflection cycles were taken. To eliminate the measurement of the energy loss within the dial indicator, its plunger was arranged not to touch the beam as loads were being applied. The plunger was lowered on to the beam as soon as the load increment had been added, and a sensitive electrical contact gave a warning as soon as it just touched the beam.

4.4 The energy input measurement

With the beam supported as for the static hysteresis test, it was excited at the centre by an electro-magnetic vibration exciter at (or near) its fundamental natural frequency. (It is not necessary that the exciting frequency shall be exactly equal to the natural frequency). The exciter connecting rod was attached to a strain gauge force transducer, which was rigidly fixed to the under-surface of the beam (see Fig.5). This force transducer consisted, in effect, of a short cantilever with four strain gauges at its root, two on the "tension" surface, and two on the "compression" surface. These four gauges formed the four arms of a Wheatstone Bridge circuit, and the out-of-balance voltage (or current) in the circuit was therefore proportional to the bending strain in the cantilever, and hence to the shear load applied to the cantilever, i.e., proportional to the oscillating force applied from the exciter rod.

The oscillating deflection of the centre of the beam was measured by a simple velocity transducer, consisting of a rectangular coil (attached to the beam) which moved in the field of a magnet rigidly attached to an independent structure. The voltage induced in the coil was amplified by a D.C. amplifier, whose voltage output was measured on an Avometer. The velocity transducer was calibrated, in situ, at the beam resonant frequency against a calibrated displacement transducer attached to the beam at the same point.

The multiplication of the signals from the velocity and force transducers, necessary for the evaluation of the energy input, was performed in effect by supplying the force transducer with a current proportional to, and in phase with the local velocity. The out-of-balance current then flowing across the strain-gauge bridge consists of a D.C. and A.C. component, the D.C. component being proportional to the energy input per unit time (as shown in Appendix A). The D.C. component may be measured by means of a sensitive galvanometer, having a long time constant compared with the period of the A.C. component. The galvanometer may therefore be connected straight across the Wheatstone Bridge in the usual way. The circuit diagram of the whole arrangement is shown in Fig.6.

The strain gauge bridge contained a variable balancing resistance, for initially balancing the bridge at the zero oscillating load condition. This was necessary as the D.C. amplifier supplying the bridge could 'drift', and give a small D.C. output together with the A.C. output required. The drift, however, was corrected within the amplifier at frequent intervals during any experiment, but the possibility of very small D.C. current outputs could not be altogether eliminated. Had the bridge no balancing resistance, this D.C. would cause a spurious D.C. to flow through the galvanometer.

The experimental procedure was therefore initially to balance the force transducer, and to correct the amplifier for drift. The beam was excited at various amplitudes at the natural frequency of 16.7 c.p.s., and the corresponding D.C. indicated by the galvanometer was recorded. Using the appropriate calibration constants previously obtained for the force and velocity transducers, the energy input per cycle was readily calculated.

4.5 The oscillation decay measurements

The beam was suspended at the nodal points of the modes excited on bungee-elastic cords from an overhead structure, as shown in Figs. 1 and 2. In order to keep the beam at a constant height, the number of cords, and their length, had to be varied as the masses on the beam were changed to vary the fundamental frequency. The vibration exciter was attached at the centre of the beam to the under-surface and excited the beam at its natural frequency in the fundamental mode. The frequencies covered by varying the attached masses were approximately 15, 24, 30 and 42 c.p.s. To record the oscillation decay, a double throw switch included in the supply wire from the power amplifier to the exciter enabled the exciter suddenly to be switched off, and at the same time to be connected to an oscilloscope, after which it acted as a vibration velocity transducer, the trace on the oscilloscope being proportional in length to the peak velocity of the decaying oscillation. This trace was photographed on to a moving film, thereby recording the oscillation decay curve.

The mode of oscillation of the beam was measured by 6 velocity transducers supported on an independent frame around the beam, and connected to the beam at suitable points along the length of the beam (see Fig. 2 where 3 only of the transducers are shown in position). The damping and mass effect of these transducers was considered small enough to have no effect on the mode shape, but measurements showed that they considerably increased the damping. For this reason, they were not attached when the damping measurements were made. When the mode had been measured, the beam suspension points were moved to coincide accurately with the nodal points.

Before measuring the damping of the jointed beam by this means, a careful investigation was made into the principal sources of damping on an identical unjointed beam. The effect of the bungee-elastic suspension was examined by measuring the damping with the suspension points at several different distances from the nodal points. The effect of the attached masses was examined from the point of view of reproducibility of results. The damping was measured with the beam in a given condition, after which all the masses were removed. and then replaced again and the attachment bolts tightened. Another damping measurement was then made. Similarly, the non-electrical damping of the exciter was examined for reproducibility by dismantling and re-assembling, and ensuring that the damping measured under given conditions was the same before and after dismantling. These investigations ensured that the extraneous damping could be reproduced, provided the suspension was always at the nodal points and the mass attachment bolts tightened to the same extent each time.

4.6/

4.6 Energy flux measurement

This method of damping measurement has not yet been used in practice. The following remarks are therefore tentative suggestions as to how the measurements may be made.

The beam may be supported in either the free-free state of §4.5 or in the simply-supported state of §4.3, and excited at a natural frequency. Two pairs of velocity transducers should be attached to the beam, one pair close together along the beam on either side of a joint. The transducers should consist of simple rectangular coils attached rigidly to the beam, moving in the field of magnets attached to an independent frame, and there should be no mechanical connection between the coil and the magnet or magnet support structure. This eliminates mechanical damping arising from the transducers. Electrical damping, due to the metal beam vibrating in the field of the magnet, should be minimized by having the magnet as far away as possible from the beam. These transducers should be accurately calibrated in situ against another calibrated transducer which can be attached to the beam at the same points, and at the same time.

If the joint is formed by a joint plate, 2 strain gauges should be attached to the plate, to record strains proportional to the load in the plate, and therefore proportional to the bending moment in the beam at the joint. The output from these gauges should be calibrated against a known bending moment on the joint by applying a static load to the centre of the beam.

When the beam is vibrating, the output from each velocity transducer in turn should be amplified in the D.C. amplifier (with no phase shift) whose output should then be used to supply the strain gauge bridge formed by the gauges at the joint. The D.C. component of the out-of-balance current flowing across the bridge may be shown to be proportional to the product of the joint bending moment and the quadrature component of displacement at the transducer (i.e., in quadrature with respect to the joint bending moment). The four measurements so obtained, using each transducer, are sufficient (if accurate enough) to evaluate the product (joint bending moment) × (quadrature component of change of slope across the joint), which has been shown to be proportional to the energy dissipation at the joint.

The evaluation of the above product means, in effect, that the measured quadrature displacement curve is being differentiated twice - usually a very inaccurate process on experimental results, unless the results are of very great accuracy.

An alternative method may be devised on recognizing that the expression for the energy dissipation at a joint in equation (3.8) (§3.4) is the same as

$$\pi \text{ (Load on joint)} \times \left(\begin{array}{l} \text{quadrature component of longitudinal} \\ \text{deflection of one side of joint} \\ \text{relative to the other side.} \end{array} \right)$$

If a velocity transducer can be designed to detect this relative movement across the joint, and its output is amplified and supplied to the strain gauge bridge at the joint, then the D.C. component will be directly proportional to the above quantity, and no effective double differentiation is required. The difficulty in designing such a transducer is that it has to detect very small movements, while at the same time it is undergoing large transverse accelerations.

As another alternative method, velocity transducers distributed along a free-free beam may be used to detect the in-phase and quadrature displacements along the beam. If the amplified output of one of these transducers is supplied to a strain-gauge type displacement transducer

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at any selected reference point, the D.C. component of the resulting out-of-balance current may be shown to be proportional to the sine of the phase angle between the displacement of the two points. This provides an accurate way of measuring small phase angles, from which the quadrature and in-phase components of the motion may be calculated. From these, the corresponding quadrature and in-phase components of shear and bending moment may be calculated. The value of the energy flux along the beam may now be evaluated from equation (3.5) (§3.4). This should be constant (or nearly so) between joints, where no appreciable energy is being dissipated, and should be stepped at the joints across which the energy flux will change suddenly.

4.7 The applicability of the other methods of damping measurement

In the experiments performed to date, the methods already outlined have been satisfactory, and the other methods have not been required. The frequency response curve method, the simplest of these others, was not used as an accurately stepped oscillator was not available for the generation of the precise frequencies required. If higher frequency work is to be undertaken on the beams described, and the damping in the overtone modes is to be investigated, it may be necessary to use the multi-point excitation technique, but details have yet to be worked out.

4.8 The applicability of the damping measurement techniques to high frequency experiments

Besides the investigations into the fundamentals of joint damping, work is proposed on the damping of typical fuselage components in connection with the jet efflux problem. The typical components to be examined are lengths of stringer with skin attached, either riveted reduxed or spot welded. The lengths involved will be of the order of a typical frame spacing. Other components are skin panels, attached to a rectangular frame, to represent a fuselage panel surrounded by stringers and frames. The frequencies involved in these experiments will be in the range 100-400 c.p.s., and possibly higher. Furthermore, the damping in the various modes of a complete model fuselage is also required, and the frequency range will be of the same order.

A static hysteresis test may be carried out on the simpler "beam" type components, but the difficulty involved in using this method to measure the energy loss in a particular prescribed mode of distortion rules it out for general use.

The energy input method may be used, provided the addition of the force transducer does not appreciably affect the natural modes. This is probably impossible to satisfy on the small components to be tested. In this case, the energy input can be determined from a separate measurement of the amplitudes and phase relationships of the force and displacement. The force must be measured by measuring the current through the exciter coil, which should be attached directly to the specimen under test (i.e., no long connecting rods), and should have no mechanical connection to the magnet in which it operates. It must obviously be as light as possible. Alternatively, if another light coil is used as a velocity transducer, the output from this may be multiplied by the signal proportional to the exciter current by using a simple wattmeter, which then records a quantity proportional to the energy input. Measurement of the vibration mode may be difficult, as the transducers must not affect the mode. An adapted gramophone pick-up should be most suitable for this purpose.

The oscillation decay curve method may be difficult to use with confidence, as nearby modes may interfere with the purity of the decay. The method of Ref.6 could be used in this case, but the criticisms raised in §3.2 still apply, and in any case, the computation required to find the damping ratio is very considerable. Some very preliminary

The energy flux method is not applicable when shear distortion is present in the beam deflection. In the stringer-skin "beams", the damping exists by virtue of the shear "deflection" of the skin relative to the stringer, and the differential equation of motion (as mentioned in §2.3) shows that shear distortion, in the usual sense, is present in the transverse displacement of the beam. It may, however, be possible to develop the theory of this method to permit the energy flux to be measured in a beam with this type of "shear" damping, but with no damping due to bending loads.

The frequency response curve method may be used, with the same restrictions as mentioned before, viz., the presence of modes other than that required shall not be appreciable. Very accurate adjustment and measurement of frequency is required in the high frequency, lightly damped modes encountered. The current through the exciter coil (assuming electro-magnetic excitation is used) must be kept constant as the frequency is varied. If no adjustment of the exciter current is made, the damping measured will include the electrical damping of the exciter coil circuit.

The phase angle change method is also useful under the same restrictions as above. The phase angle to be measured is that between the current through the exciter coil (being proportional to the voltage across a resistance in the coil circuit) and the voltage output from a velocity transducer at the same point. If the phase angle between the exciter coil voltage and the velocity transducer voltage is measured, the total (electrical + structural) damping will be found.

The self-excited method is useful when an accurate oscillator is not available to give the very fine frequency control necessary in the previous two methods when lightly damped systems are examined. This technique has not yet been developed at all for use in the current series of experiments, and no further practical details can therefore be given.

The multi-point excitation technique, together with the appropriate energy input method, is envisaged as being used on the model fuselage damping measurements. A large number of modes can here be expected to exist within a fairly narrow frequency band. The exciter coils should preferably be attached directly to the fuselage side and have no mechanical connection to their associated magnets (to eliminate mechanical damping of any sort). The current supply to each coil must be independently adjustable in amplitude and phase, and when exciting at a resonant frequency, care must be taken to ensure that all the exciter currents are in phase (or 180° out of phase). (This will not automatically follow if the supply voltages to all the exciters are in phase). At each exciter position there must be a velocity transducer in order for the energy input at each exciter to be determined. As the modes of vibration of a uniformly stiffened cylinder may be calculated beforehand, and the experiment will be to check the calculated frequencies and to measure the damping, the exciters could be moved in the course of the experiment to be in the optimum position for exciting a given mode.

5. Results of Experiments on Joint Damping

5.1 The static hysteresis test

From plots of the width of the hysteresis loop against load, values of the average energy loss per cycle were obtained for each of the different load amplitudes. This average energy loss has been plotted, in Fig. 8, against the corresponding joint bending moment amplitude, in a manner described in §6.2. A typical plot of the width of the hysteresis loop against load is given in Fig. 7.

No experiment has yet been performed to determine the energy loss of an unjointed beam under the same conditions.

5.2 The energy input experiment

The measured energy input is plotted against $\log_{10}$ (Joint Bending Moment) in Fig. 8. As this measurement has only been made on the 8-jointed beam (10 rivets per joint), and the energy loss is therefore occurring at different bending moment amplitudes at each joint, the bending moment amplitude of the abscissa is the root mean square of the moments at the two different joint stations. (There are four joint stations but they are symmetrically placed about the beam centre.) The reason for this is discussed in the next section, §6.2

No experiment has yet been performed to determine the energy loss of an unjointed beam, under the same conditions.

5.3 The oscillation decay experiment

The analysis of the decay records, and subsequent reduction of results, has been performed as follows.

(a) The film record was projected through a photographic enlarger, and the peak amplitudes at regular intervals were measured. The average logarithmic decrement was calculated from these measurements for each of the intervals. A typical plot of the log. dec. against the corresponding amplitude of the photographic record is shown in Fig. 11. At least two decay records were made for each beam condition, and the logarithmic decrements from each have been superimposed to obtain a mean curve of log. dec. versus amplitude.

(b) The energy loss has been evaluated from the expression

$$\text{Energy Loss per cycle} = \text{Maximum Kinetic Energy during cycle} \times \text{log. dec.}$$

$$= \text{Generalised Mass} \times \text{Peak Amplitude}^2 \times \omega^2 \times \text{log. dec.}$$

The generalised mass was calculated using the mode of vibration measured for the appropriate beam condition and from the known mass distribution of the beam. The frequency, ω, was assumed to be the same as the forced resonant frequency at which the beam was excited.

(c) The energy loss in the joints of the beam was found by subtracting the energy loss of the corresponding unjointed beam (with weights attached) from that of the jointed beam (with weights attached). The details of this procedure are explained and discussed in the next section (§6).

(d) Knowing the mode of vibration, the mass distribution and frequency, the inertia force distribution on the beam was calculated, and thence the bending moment at the joints in each beam condition for different amplitudes.

(e) From these calculations, curves of energy loss per cycle have been plotted in Fig. 9 against the joint bending moment for the beam with 4 joints. (10 rivets per joint.)

For the case of the 8 jointed beam in the highest frequency range (i.e., no weights attached) the energy loss in the inner joints determined from the experiments on the inner joints alone, has been subtracted from the total joint energy loss. This difference should be the energy loss in the 4 outer joints only, and is plotted in Fig. 8 against the bending moment at the outer joint.

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From the curves of Fig. 9, the variation with frequency of the energy loss in the 4 inner joints has been plotted (in Fig. 10), for different joint bending moment amplitudes.

6. Discussion

6.1 Comparison of the three methods of damping measurement used

Of the three methods used, the energy input method was by far the quickest in use, entailing the least computation in determining the actual energy input from the measurements taken. Moreover, it is superior to the oscillation decay method in that one of the extraneous sources of damping - the electrical and mechanical damping of the exciter itself - does not contribute to the energy input measured. The energy measured is purely that which passes between the exciter and the beam. The apparatus required for this method is more elaborate than in the other cases, and has taken longer to develop, but having been developed, it will certainly make possible a much more rapid rate of progress in the work.

The static hysteresis test requires many very accurate measurements of the beam deflection to be made, and the subsequent plotting of the hysteresis loop width involves a good deal of tedious work. Even more tedious is the reduction of the results of the oscillation decay curves, involving the measuring of many trace amplitudes on a projected picture from the film record. These measurements are bound to be inaccurate when the trace amplitude is small and when the rate of decay has fallen off. In contrast to this, the accuracy of the energy input method is little less at the low amplitudes than at the high, using the equipment developed.

6.2 Comparison of the results of the three methods

Before comparing the values of joint energy loss obtained from the three methods, the following points should be noted:

(a) The hysteresis and energy input methods have so far been used only on the 8-jointed beam, on which no weights were attached. The energy loss measured was therefore the total energy loss at all the joints, together with that dissipated by the extraneous damping sources for which no correction has yet been obtained.

(b) The system of loading on the beam in all three cases was different, i.e., a central concentrated load in one case, the inertia forces on a simply supported beam for the energy input case, and the inertia forces on a free-free beam for the decay curve case. The ratio of the bending moment amplitudes at the inner and outer joints will therefore vary in each case. The total joint energy losses may not therefore be compared at, say, equal values of inner joint bending moment or equal values of the mean bending moment (which would be admissible if the inner and outer joint moments always bore the same ratio). Since, for an idealized viscous or hysteretic damping source, the energy loss is proportional to the $(\text{load})^2$, it has been decided to compare the total damping at equal values of the root mean square of the two moments, i.e., $\left(\dfrac{B.M^2_{inner} + B.M^2_{outer}}{2}\right)^{\frac{1}{2}}$. The greatest difference between the two moments never exceeds 15% of the greater, so any error in this assumption should be insignificant.

From Fig. 8, it is seen that there is close agreement between the energy loss measured for the 8-jointed beam by the hysteresis and energy input methods. At the upper end of the bending moment range, the hysteresis test yields results which are about 5% greater than those of the energy input method. It must be remembered that these results include the energy loss at the beam supports. In the hysteresis test, this would derive only from friction at the knife-edges, but in the energy input (vibration) test there would be a certain amount of loss in the steel frame which supported the beam and reacted all the oscillating inertia forces on the beam. The stiffness of the frame was probably insufficient for this purpose, for which it was not originally erected; furthermore, some energy could have been dissipated in the system of weights and cords which held the beam down on to the supports. Although they were suspended from the (nominal) nodes of the vibrating system, the rotation of the beam at the ends could excite transverse vibrations of the cords, and so cause further dissipation.

The energy loss measured for eight joints at 38.3 c.p.s. in the decay curve experiment is of the order of one half of that measured by the other two methods (see Fig.8). This could be accounted for by the losses at the beam supports in these two methods.

Clearly for a reasonable comparison between the methods, it is necessary to measure the energy loss on an unjointed beam in order to correct for the end support effects. Time alone has prevented this.

6.3 The results of the decay curve experiments

The logarithmic plot of the energy loss against bending moment in Figs. 8 and 9 shows a considerable "linear" range for the inner joint energy loss at 40.1 c.p.s. This linearity corresponds to the relationship

$$\text{Energy Loss (in. lb)} = (\text{Bending Moment, lb in.})^{2.77} \times 2.82.10^{-10}.$$

The linear part of the outer joint curve of Fig.8 corresponds to the relationship

$$\text{Energy Loss} = (\text{Bending Moment})^{2.58} \times 1.41.10^{-9}.$$

Although the two constants appear to differ considerably, over the bending moment range where this relationship holds the actual energy losses differ by no more than about 30% of the greater. A mean straight line drawn between the two curves of Fig.8 yields

$$\text{Energy Loss} = (\text{Bending Moment})^{2.61} \times 9.5\ 10^{-10}.$$

All of these results are for the total energy loss in 4 joints. At both extremes of the energy loss curves, there is a departure from linearity. While this may indicate that the simple laws deduced above are true only over a limited range, it should be borne in mind that the accuracy in measuring the logarithmic decrement from the decay curve falls off at the extremes of amplitude - at the large amplitudes, immediately after cutting the excitation, due to transients from other modes distorting the decaying wave-form, and at the small amplitudes due to the difficulty of accurate visual amplitude measurement. These inaccuracies would be avoided by the use of the energy input method.

The energy loss per cycle at other frequencies varies in a very similar way to that at 40.1 c.p.s., but the logarithmic plots (Fig.9) show more curvature than the 40.1 c.p.s. curve. These other curves have been obtained by subtracting the energy loss in the unjointed beam (with weights) from the energy loss in the jointed beam (with weights), the former being of the order of one half the latter, i.e., the joint energy loss was approximately equal to the "extraneous" energy loss. (When no weights were attached, the extraneous energy loss was 10% of the joint energy loss). Considerable inaccuracy could be introduced here for the following reason. The introduction of the joints reduces the stiffness of the beam, lowers its natural frequency and changes the mode of vibration. At the stations where the concentrated masses were attached, the amplitudes for the jointed

beam/

beam were therefore different from the amplitudes of the unjointed beam for a given amplitude at the beam centre. The energy loss of the unjointed beam with weights was primarily due to the weights themselves, i.e., due to the oscillating inertia forces on the weights being transferred by friction forces to the sides of the beam. As the mode and frequency alters due to the addition of the joints, the energy loss due to the weights may be expected to change at the same time. Unless the law is known relating this energy loss to both mode (or amplitude) and frequency, an accurate correction cannot be made to the total energy loss measurement in order to deduce the joint energy loss alone. The law which was used to make the correction was deduced from logarithmic plots of the energy loss in the unjointed beam with weights, which showed that over the greater part of the amplitude range considered, the energy loss was proportional to the cube of the amplitude. There was no significant difference between the energy losses at a given amplitude whether the large or small weights were attached to the beam, i.e., no significant change with frequency. As the produce (weight $\times$ frequency^2) was almost the same for each beam condition, it was assumed that the energy loss was proportional to the cube of the inertia force on the weights. This permitted a simple calculation to determine the 'weight' energy loss in the jointed beam mode at a certain frequency when the weight energy loss was known in the unjointed beam mode, with the same weights, at a slightly different frequency. The validity of the cube law, and the accuracy of the correction is open to doubt, and may lead to considerable inaccuracy in the resulting values for the joint energy loss.

From the curves of Fig. 9, Fig. 10 has been derived, showing the variation of joint energy loss with frequency, for a given bending moment amplitude on the joint. With only 4 points for each curve, there is not really enough information to enable the correct curve to be drawn in, especially since the errors mentioned above will affect the points considerably. The straight lines drawn through the origin do, however, fall remarkably close to three of the points for each value of the bending moment. The points at 23.9 c.p.s. are, however, consistently below the straight lines. While experimental errors, such as misalignment of the vibration exciter, or misalignment of the knife-edge supports can increase the measured damping and energy loss, it is not conceivable that errors such as these could cause a decrease. If the values of 29.4 c.p.s. were consistently high due to such errors, and should in fact fall below the straight lines, (by a considerable amount) and all the other values were correct, the curves to be drawn would be completely different. The fact that there is any frequency dependence at all is surprising in a system of this nature, and casts further doubt upon the accuracy of the corrections made to allow for the energy loss of the weights.

6.4 Suggestions for improving the experimental technique

A considerable reduction in the extraneous energy loss could be achieved by attaching the weights to the top and bottom surfaces of the beam, instead of to the sides. Their inertia forces would then be transferred to the beam as normal pressures, instead of through frictional shear stresses on the sides. Frictional shear stresses will still exist to a very small extent, due to the inertia forces corresponding to the longitudinal accelerations of a point at a distance from the beam neutral axis (i.e., acceleration = angular acceleration of beam $\times$ distance from neutral axis). This acceleration should be negligible compared with the present transverse acceleration experienced, and the corresponding inertia force, shear stress and energy loss very much smaller. Another reduction in extraneous energy loss could be obtained by turning the beam on to its side, exciting it laterally, and supporting it from the nodes by wires. If these supports are not exactly at the nodes, the vertical wires will cause negligible restraint on the beam, and even if vibrating would dissipate very little energy. They should still be adjusted to be as close to the nodes as possible.

The/

The use of the energy input method, as mentioned in §6.1, reduces the extraneous damping measured, and provides quicker and more accurately obtained results over a wider range of amplitudes.

Each of these improvements should enable the series of experiments covering the effect of frequency and joint variation to be repeated accurately and with fewer sources of error and doubt in a few days work, as compared with several months of work hitherto.

6.5 Suggests for extending the range of the investigations

The range of bending moment covered in the experiments was limited by the permissible amplitude of the vibration exciter at the beam centre. This can obviously be overcome by moving the exciter nearer to a node in the beam, and increasing the force from the exciter. A higher frequency range could be covered by reducing the length of the beam in stages, whereas lower ranges could be covered by joining two similar 10 ft beams, with the joint to be tested at the centre.

When much higher frequencies are encountered (e.g., in the investigations into typical fuselage skin-stringer joints) the acoustic radiation damping may constitute a large part of the damping to be measured. This can be minimized by testing the specimen in an evacuated chamber. The acoustic damping is proportional to the density, hence damping measurements at several different densities should permit a linear extrapolation to determine the damping at zero density, i.e., zero acoustic damping.

7. Conclusions

The complexity of the source of damping at structural joints is such that a general analytical approach to determine its magnitude is not possible and an experimental approach must therefore be used. Of the techniques available for the experimental determination of the damping, methods of direct energy input measurement are the most suitable in the presence of non-linear damping and harmonic distortion of the oscillating structure. For linearly damped structures when no harmonic distortion is present, the other methods discussed may be more conveniently applied, depending on the nature of the structure to be tested.

Of the methods of damping measurement which have been used in the investigation described, the energy input method was the quickest to use, and holds the possibility of giving the most accurate results. A comparison of the results from the three methods is not strictly justifiable, owing to the differing conditions of the specimen when tested by the different methods. The conditions were most similar for the static hysteresis test and the energy input measurement, and considering the differences that existed, the results from these two experiments were in good agreement

In the investigations into the energy loss in riveted joints subjected to harmonic loads, it was found that over a limited range and at a certain frequency, the energy loss was proportional to the load amplitude raised to a power of approximately 2.7. This implies that the equivalent viscous (or hysteretic) damping coefficient of the system in the system in which the joints are incorporated is proportional to (amplitude of motion)$^{0.7}$. The investigation into the effect of frequency on the energy loss for a given load amplitude indicated that the energy loss was proportional to frequency, but the inaccuracies inherent in the experiment were such as to cast serious doubt on this indication. With certain small modifications to the specimen tested, the effect of frequency could be accurately investigated.

List/

List of Symbols

a, a_r	is a generalized inertia coefficient
b_{rs}	is a generalized viscous damping coefficient
b_e	is the viscous damping coefficient equivalent to the hysteretic damping coefficient h at a given frequency
c, c_r	is a stiffness coefficient
d	is the rate of the equivalent viscous damper of a joint
E	is the energy dissipated per cycle
E_s	is the voltage supply to the vibration exciter
e	is the back e.m.f. from the exciter coil due to its motion
e'	is $e \times f/Z_E$
f	is the exciter force per unit supply current
F_1	is the exciter force amplitude at station 1 of the structure
F_2	" " " " " " " 2 " " "
h, h_{rr}, h_{rs}	is a hysteretic damping coefficient
I	is the supply current to the vibration exciter
k	is the energy dissipated by a joint at unit load
$\bar{k}$	is the exciter supply voltage per unit velocity at exciter, in a self-excited system
k'	is $\bar{k} \times f/Z_E$
k_p	is the voltage (or current) per unit load on the load transducer
k_v	is the voltage (or current) per unit velocity of the velocity transducer
ℓ	is the distance between two stations on either side of a joint
m	is the mass per unit length of the beam
M	is a bending moment
M_1, M_2	are the in-phase and quadrature components of bending moment
P_o	is the amplitude of a harmonic load
P_n	is the amplitude of the n^{th} harmonic component of the applied load
$P(t)$	is the load applied to a system
q	is a generalized co-ordinate
$\bar{q}_1, \bar{q}_2$	are the complex harmonic amplitudes of the q's of two modes

R/

R	is the ratio of amplitude at frequency ω to the peak amplitude
$\bar{S}$	is the shear per unit length on a lengthwise joint
S	is the shear force on a beam cross-section
S_1, S_2	are the in-phase and quadrature components of shear force
v	is the transverse displacement amplitude of the beam
v_1, v_2	are the in-phase and quadrature components of v
x	is the relative displacement of a pair of joint plates, or the spanwise distance along the beam
x_0	is the amplitude of x
y, y_1, y_2	are the displacements of a system at the points of excitation
Z_E	is the impedance of the exciter coil, when clamped
ω	is the frequency of excitation
ω_n	is the undamped natural frequency of a system
ϵ	is the phase lead of displacement relative to exciting force
ϵ_n	is the phase lead of the n^{th} harmonic component of displacement relative to the n^{th} harmonic component of exciting force
ϵ_1, ϵ_2	are the phase angles at ω_1, ω_2 close to resonance (§3.6)
δ_V, δ_H	are the viscous and hysteretic damping ratios
ϕ_{rn}	is the displacement at point n per unit reference displacement in mode r.

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APPENDIX A

The Measurement of Energy Input using a Strain-Gauge Force Transducer

The out-of-balance current, i_G flowing through the galvanometer in a strain gauge bridge is proportional to the supply current, i_s, × the resistance change of the strain gauges. Since the resistance change is proportional to the load P being applied to the transducer, we may write

$$i_G = k_p'.i_s.P$$

where k_p' is the galvanometer current per unit supply current at unit load. Now suppose the supply current is proportional to, and in phase with the velocity of the vibrating structure at the point of application of the exciting force $P = P_o \sin \omega t$.

$$\text{i.e.,} \quad i_s = k_v'.\dot{y} \quad \text{where} \quad y = y_o \sin(\omega t + \epsilon), \quad \text{the local displacement}$$

$$\therefore \quad i_G = k_p' k_v' \omega y_o P_o \sin \omega t . \cos \omega t + \epsilon$$

$$= k_p' k_v' \frac{\omega}{2} P_o y_o \{\sin(2\omega t + \epsilon) - \sin \epsilon\}.$$

The D.C. component of this may be written

$$i_{D.C.} = \left(- k_p' k_v' \frac{\omega}{2\pi}\right) \pi P_o y_o \sin \epsilon$$

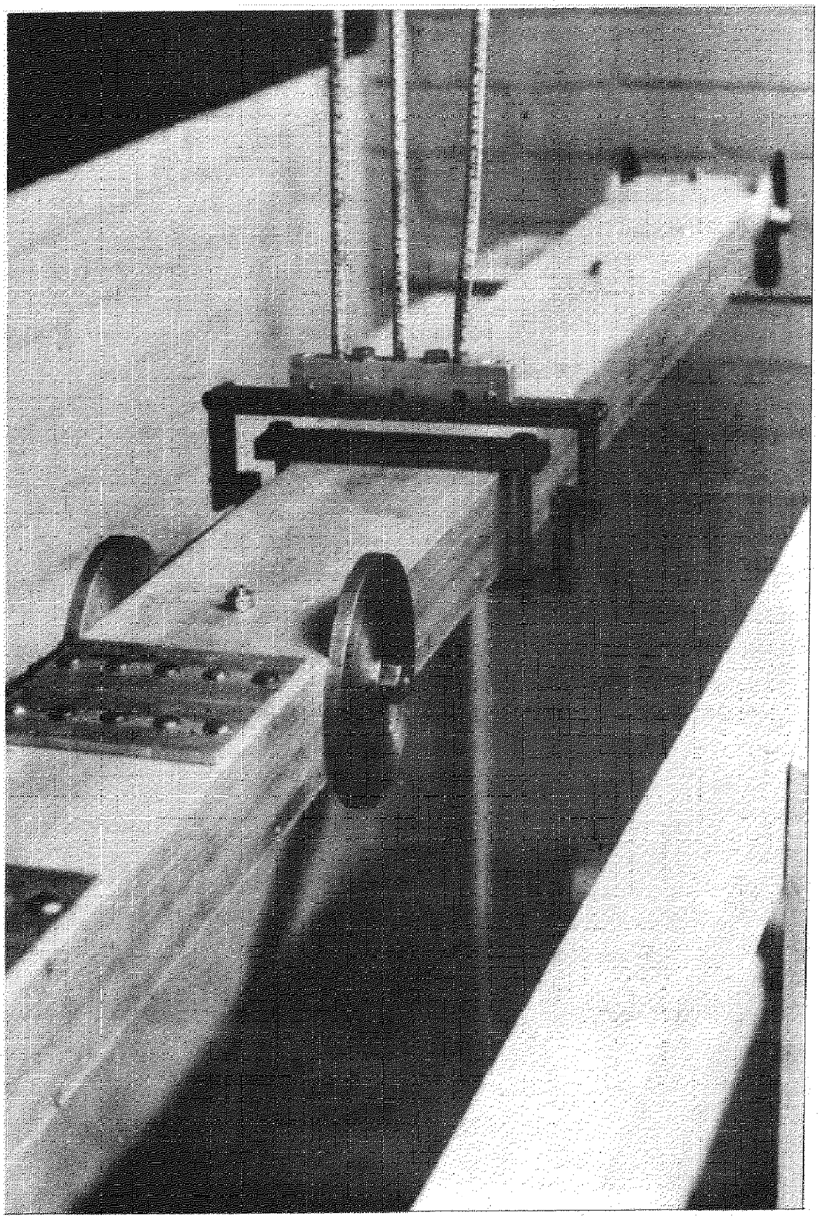
$$= \left(- k_p' k_v' \frac{1}{T}\right) \times \text{Energy input per cycle}$$

where T is the period corresponding to a frequency of ω

$$\therefore \quad \underline{i_{D.C.} = (- k_p' k_v') \times \text{Energy input per unit time.}}$$

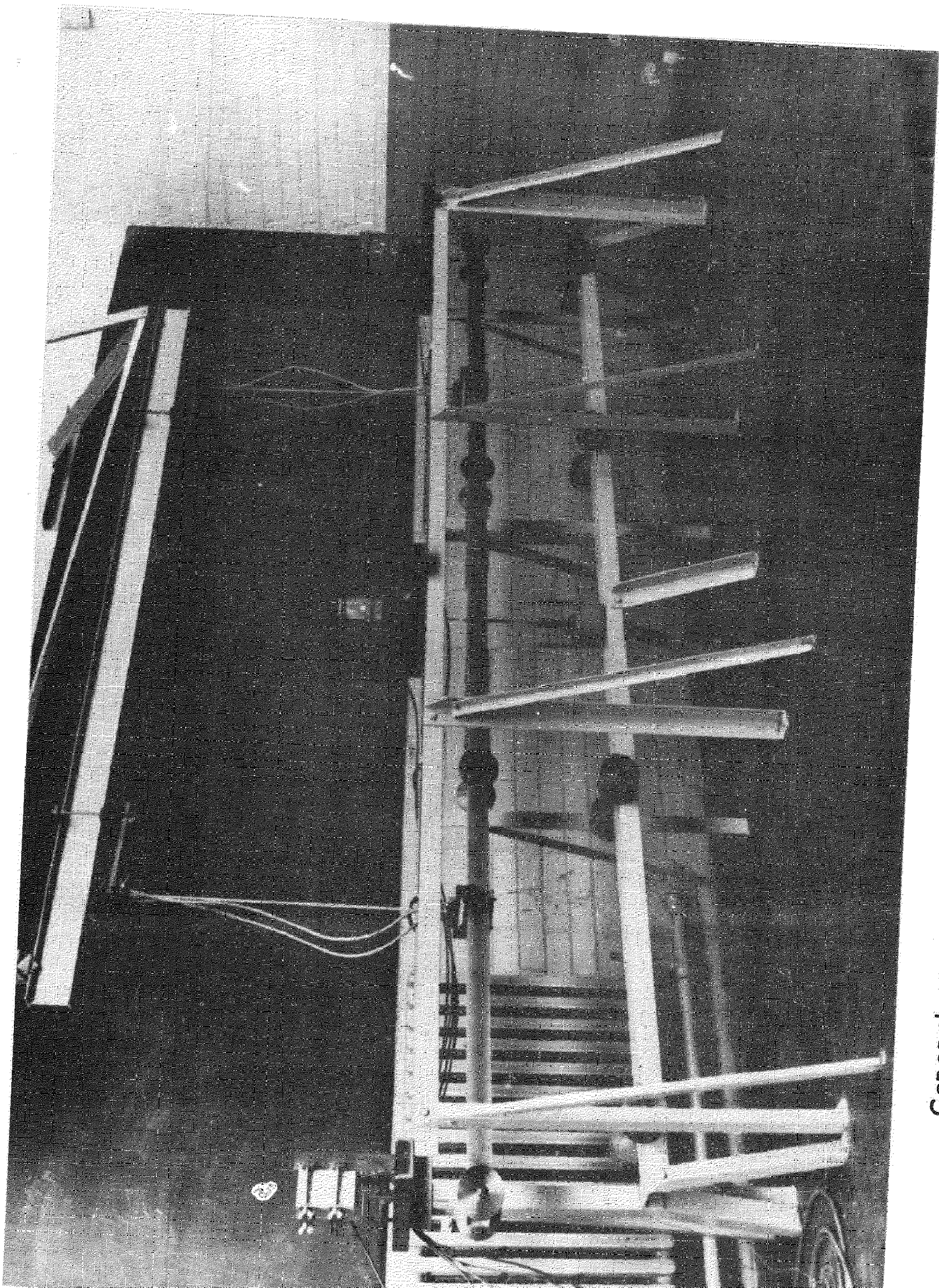
If the velocity and/or force contain harmonics, it is readily shown that $i_{D.C.}$ is <u>still</u> proportional to the total energy input per unit time.

FIG. 1.



Knife-edge support of beam in free-free condition.

FIG. 2.



General arrangement of beam in free-free condition.

FIGS. 3 & 4.

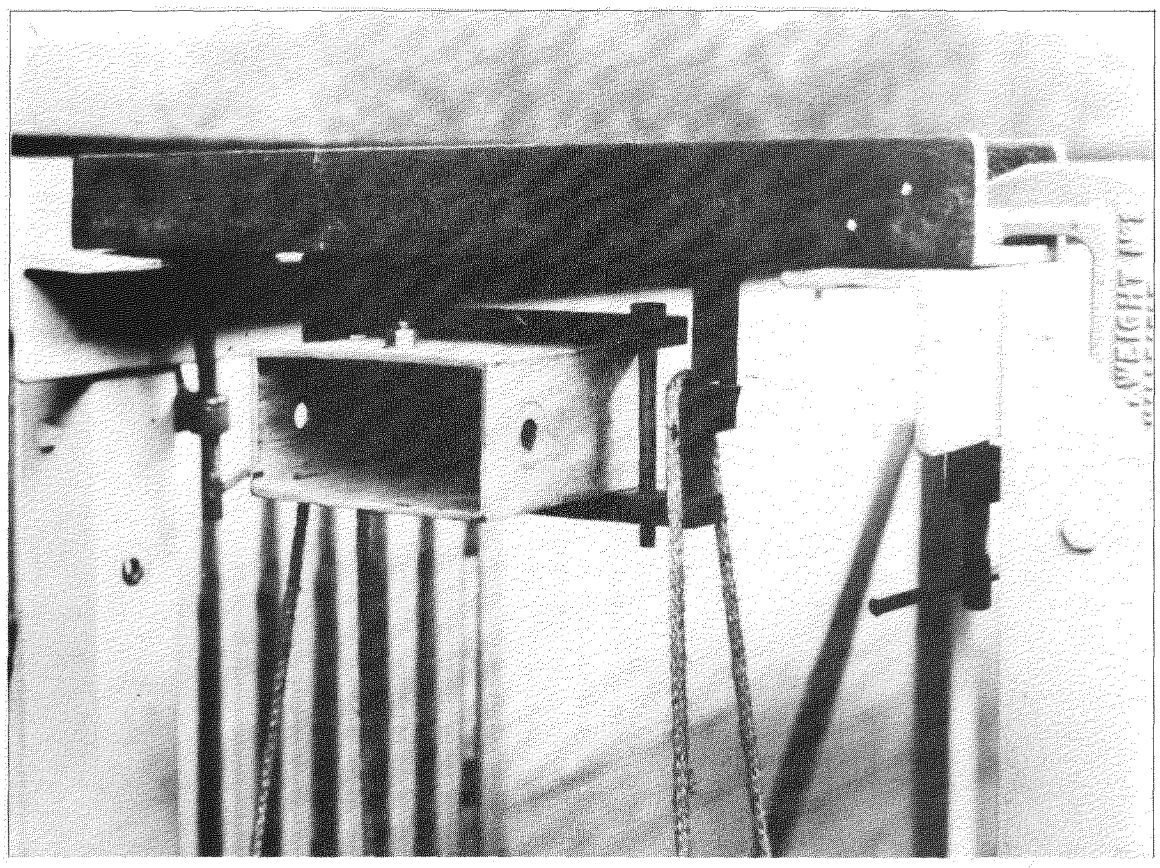


Fig. 3. Knife-edge support of beam in pin-ended condition.

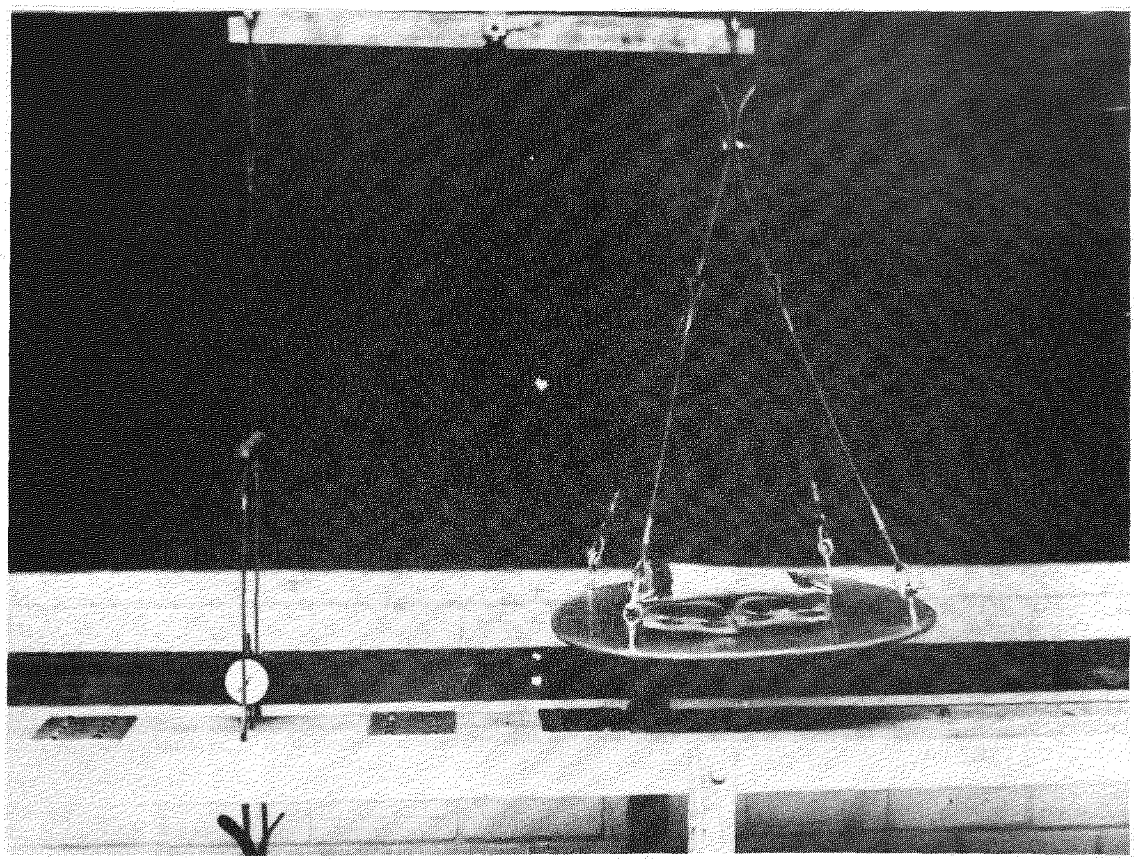
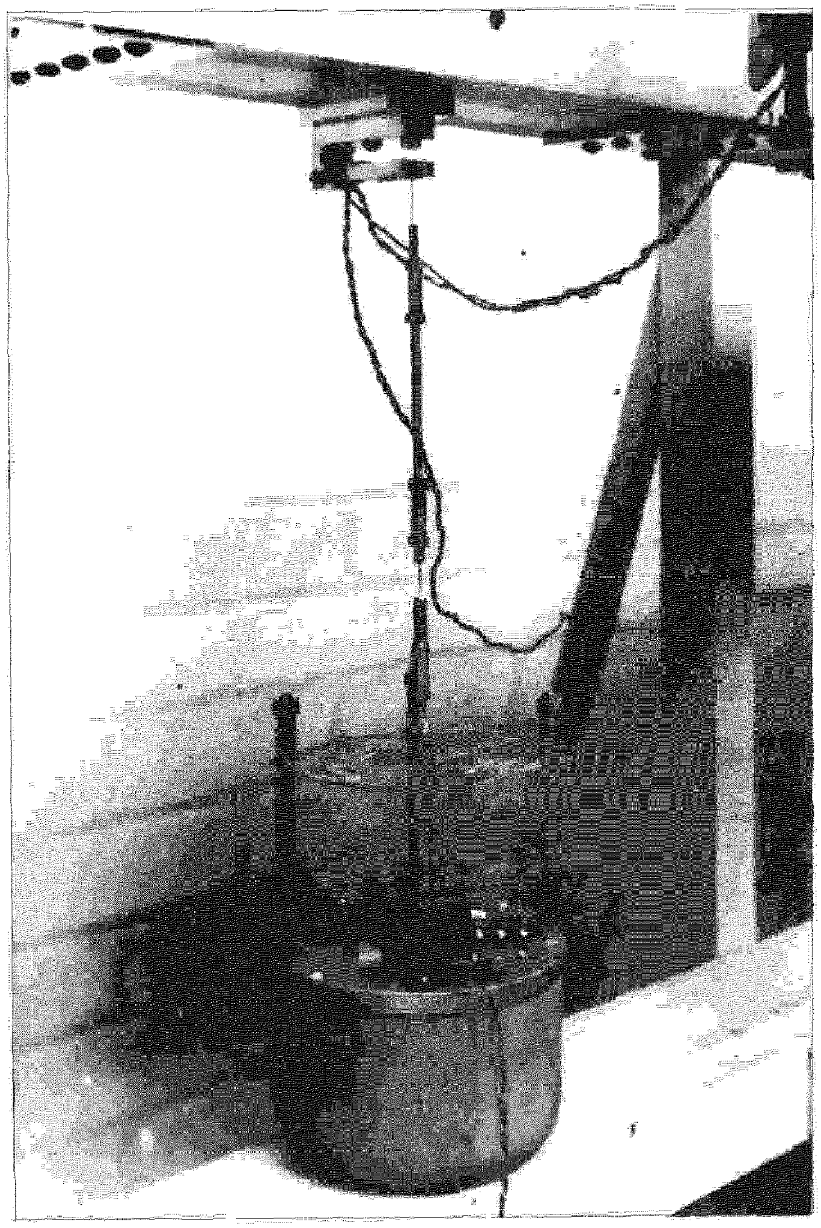


Fig. 4. System of loading the beam in the hysteresis test.

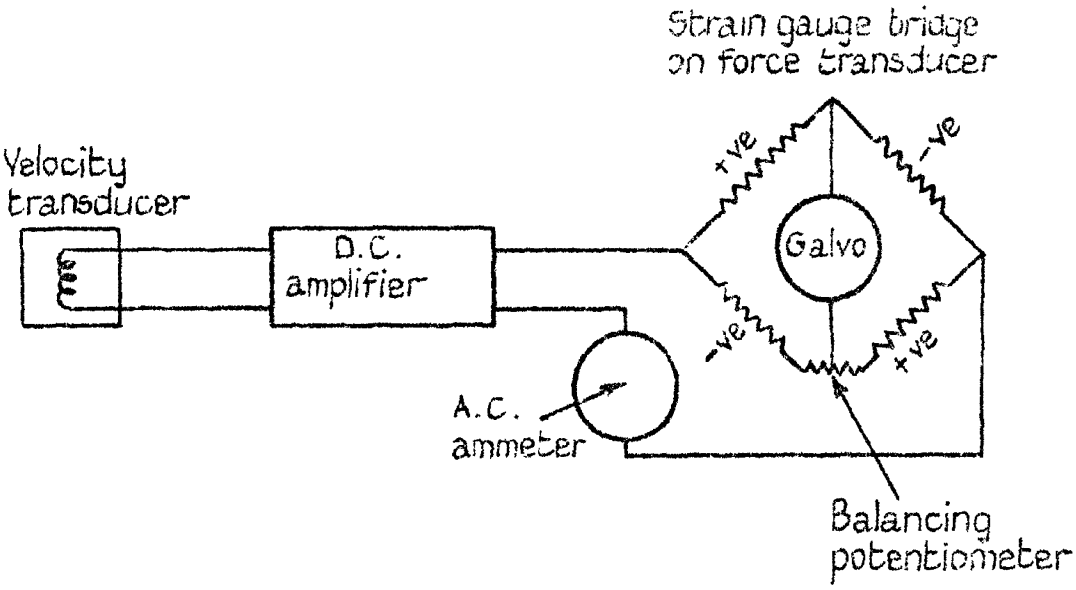
FIG. 5.



The exciter and force transducer for the energy input measurement.

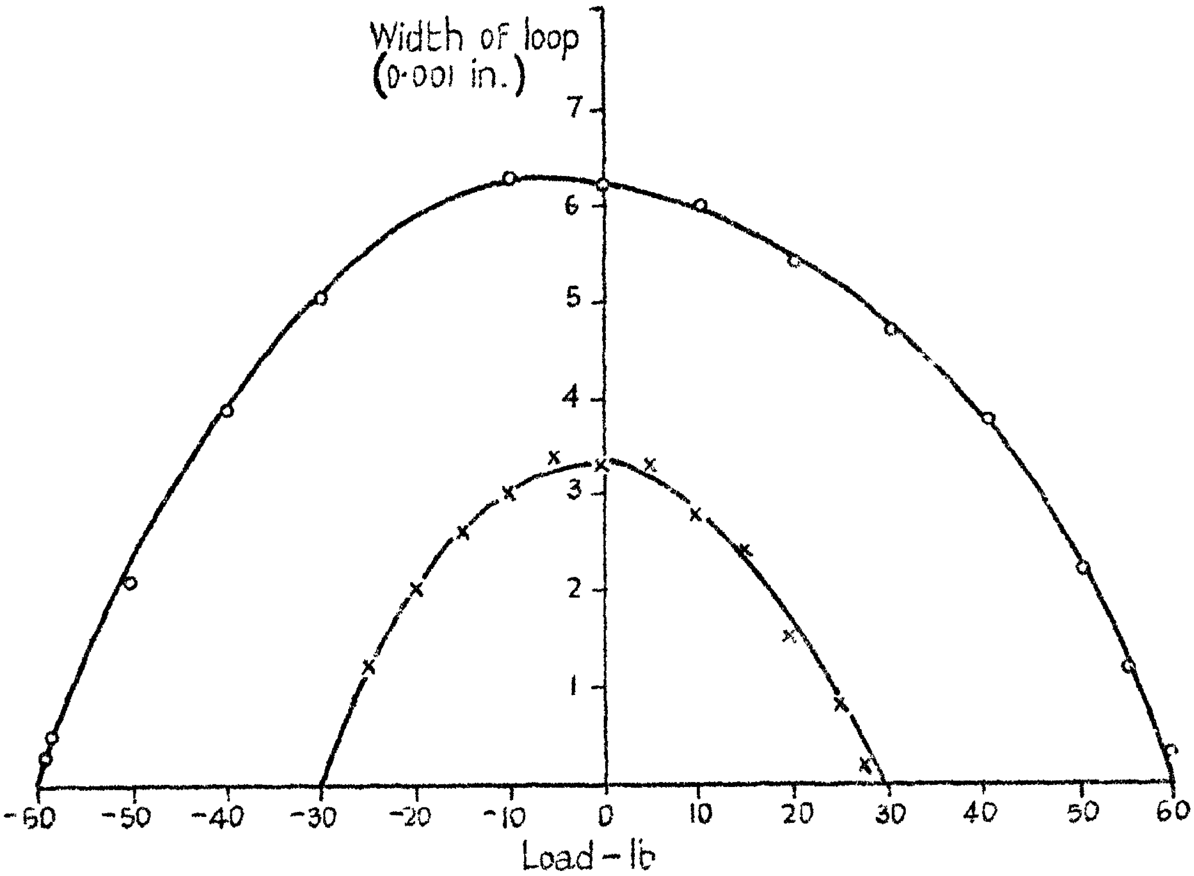
FIGS. 6 & 7.

FIG. 6.



Circuit arrangement of energy input measurement.

FIG. 7.



Width of two typical beam hysteresis loops vs. central concentrated load applied to beam.

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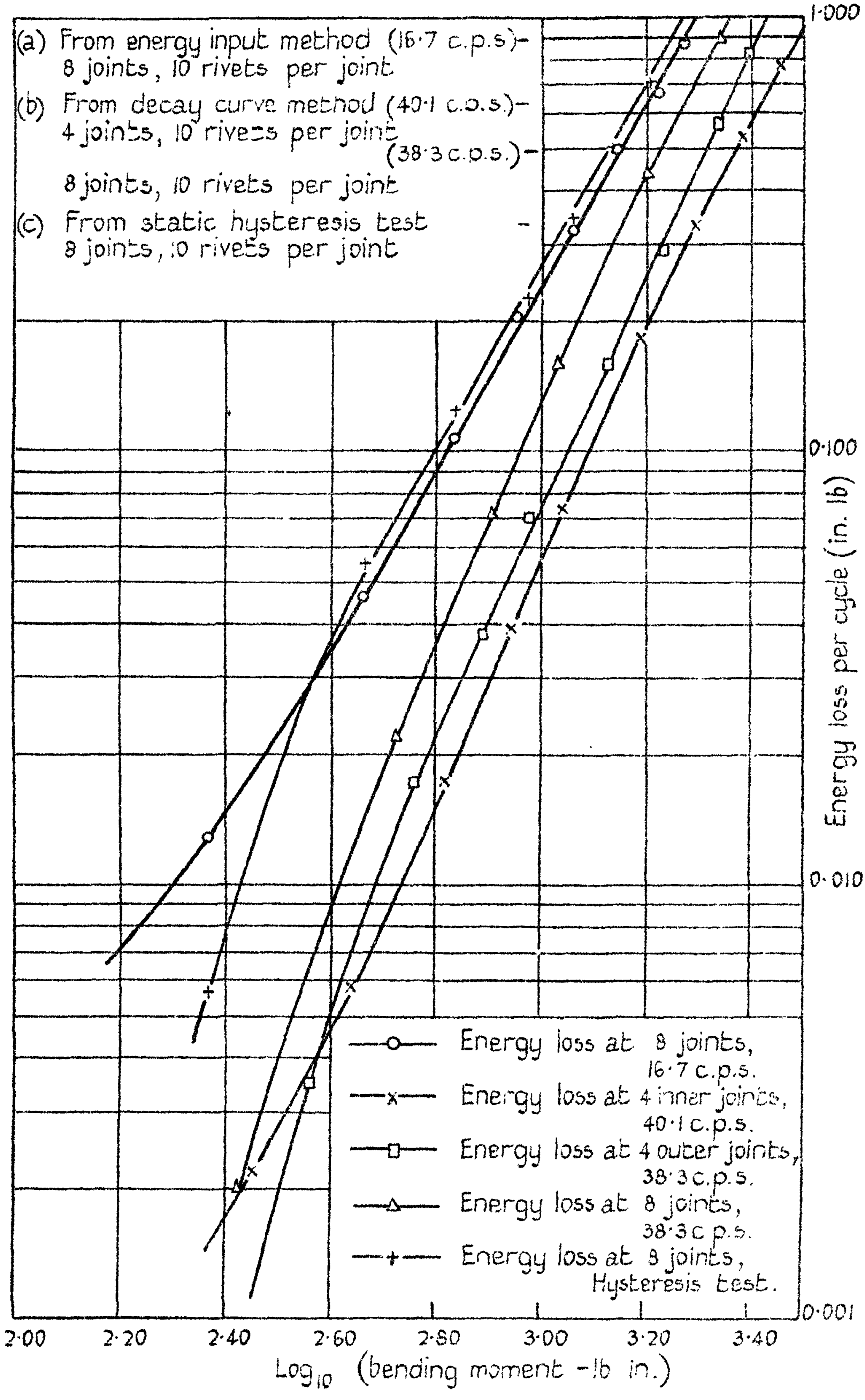
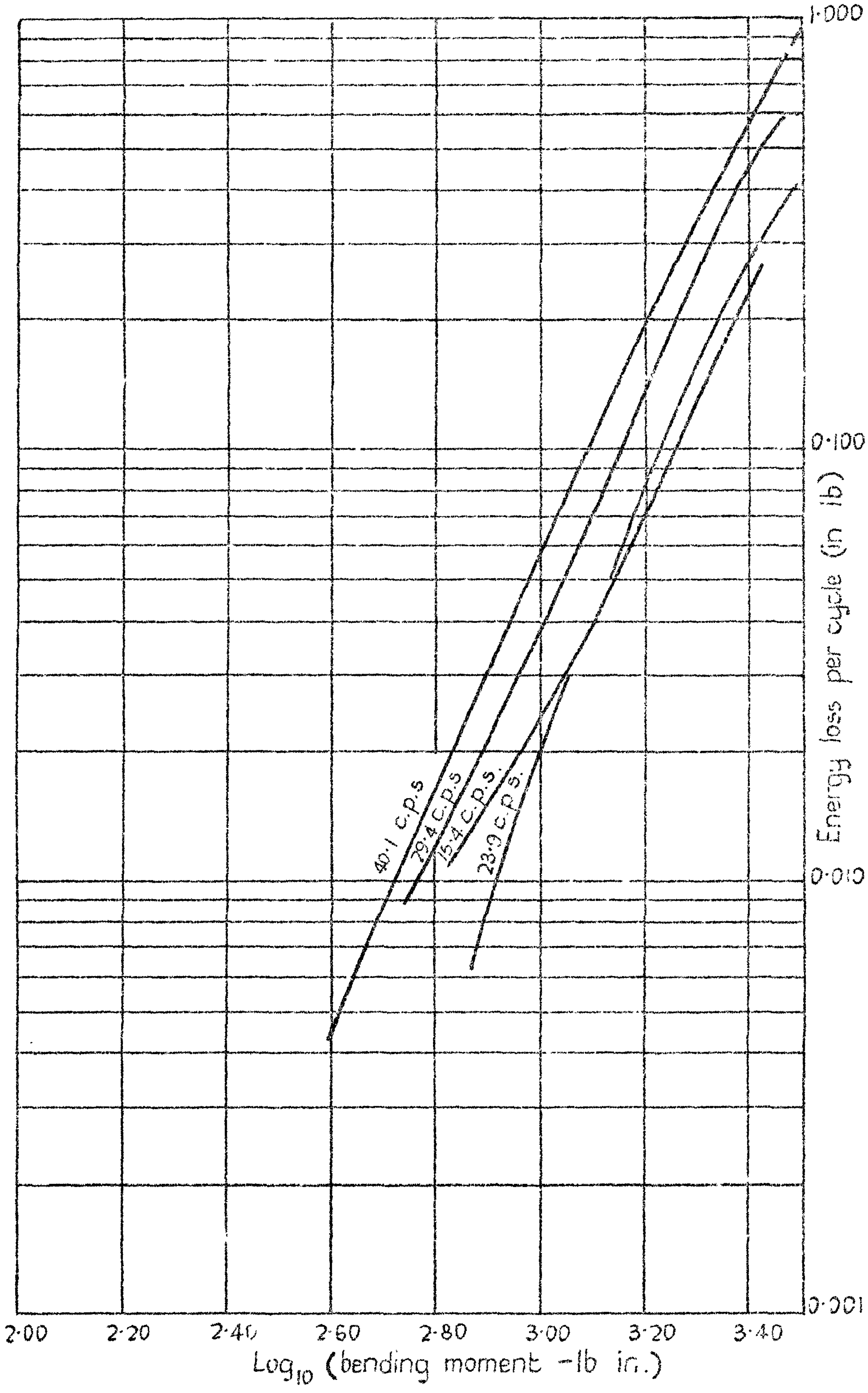
FIG. 8.



Variation of energy loss per cycle vs. joint bending moment amplitude.

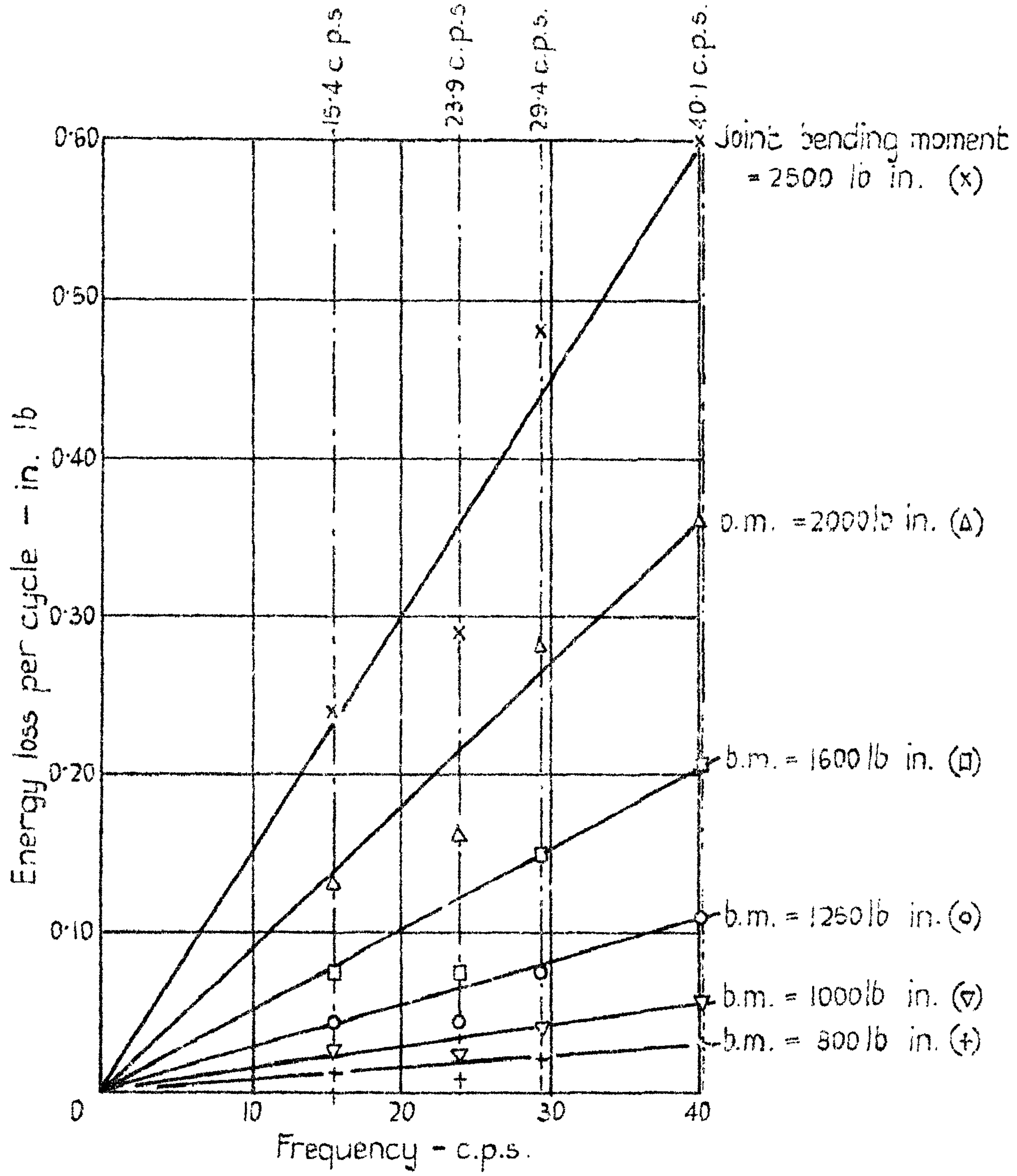
FIG. 9.



Variation of energy loss per cycle in 4 joints with joint bending moment and frequency (10 rivets per joint)

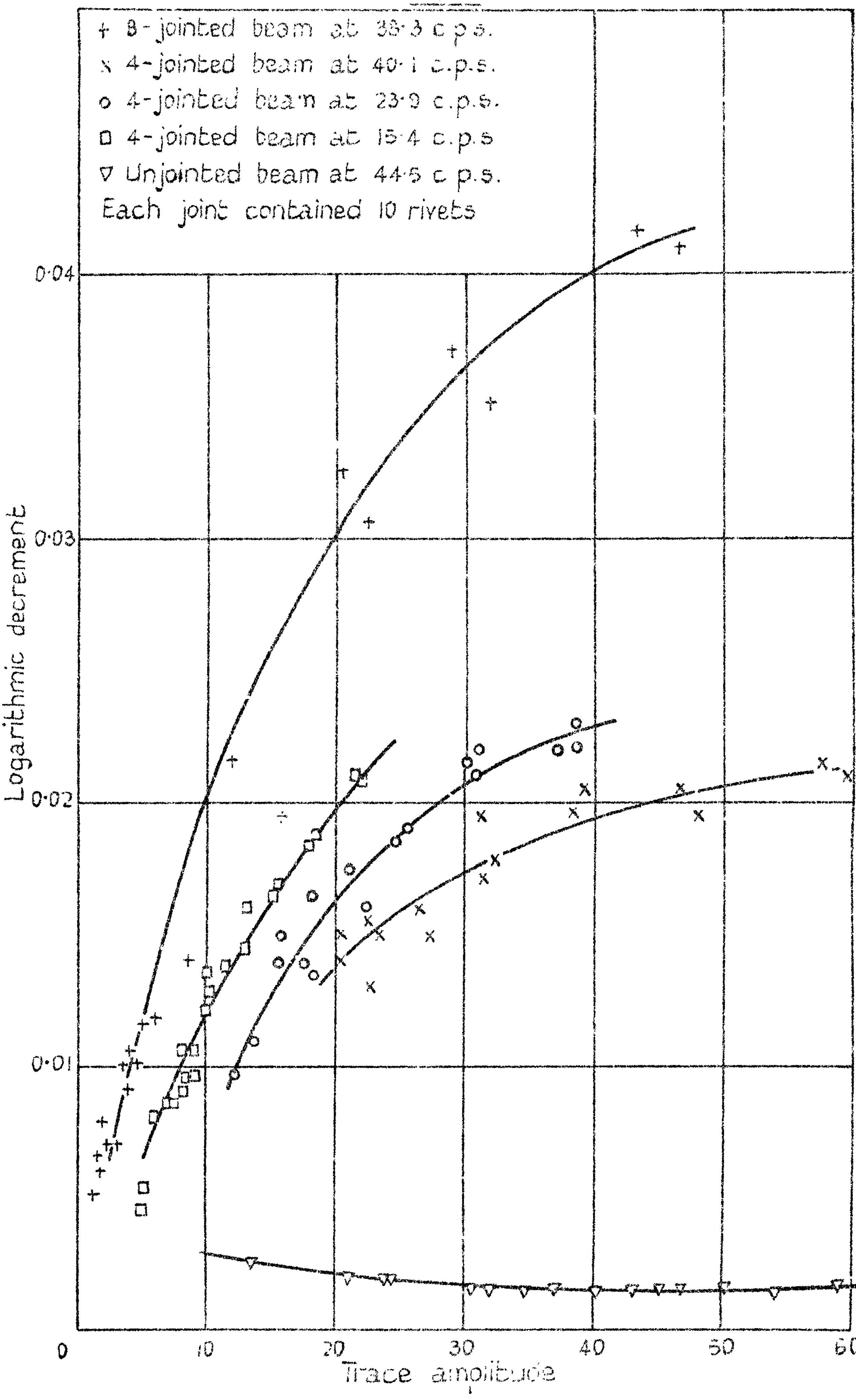
O.T.

FIG. 10.



Energy loss per cycle in 4 joints vs. frequency at different joint bending moment amplitudes

FIG. 11.



Typical plots of logarithmic decrement vs trace amplitude.
(N.B. The trace amplitude is proportional to the peak velocity)

O.T.

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