C.P. No. 441
(20,125)
A.R.C. Technical Report

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One-dimensional Treatment of Weak Disturbances of a Shockwave

by

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1959

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C.P. No.441

One-Dimensional Treatment of Weak Disturbances of a Shockwave

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Communicated by Professor E. J. Richards

21st April, 1958

SUMMARY

A shockwave enters a region of fluid initially at rest; the shockwave motion is disturbed by interaction with soundwaves or temperature fluctuations. It is the resultant soundwaves and temperature changes behind the shock which are discussed. The Appendix outlines a corrected version of an earlier treatment which considered an initially stationary shockwave. Numerical values of the interaction coefficients up to $M_1 = 5$ are given.

Introduction

Consider the one-dimensional case of a normal shockwave propagating into a fluid in which there is a temperature variation, say an increase. Its propagation velocity will increase due to the higher speed of sound, likewise the fluid velocity behind the shockwave. It follows that if the total energy is to remain unchanged, then the pressure ratio must decrease. This can only be achieved by a sound wave having a pressure decrement travelling back from the shockwave, and this looks in part after the question of continuity of the velocity behind the shock. The variation of shock strength affects the entropy change across the shockwave, subtracting from the increase initially present in the flow.

This is the most simple form of disturbance that can arise. The other form of disturbance is by soundwaves, where instead of an entropy change, there is a change of velocity together with an adiabatic pressure fluctuation. In any case, the result of a disturbance is, firstly, a soundwave proceeding back from the shockwave, and secondly, an entropy change in the flow behind the shockwave (a "temperature wave").

In an earlier note[1] this problem was considered by taking the reference axis fixed with the shockwave, and perturbing the equations of continuity momentum and energy. Alternatively one may consider the shockwave propagating into an initially still fluid, perturbing instead the solutions to the basic equations, which is the approach followed here. This is a special case of a more general investigation, and the author noticed that a discrepancy existed between the numerical results of the two methods, which was traced to the inadvertent omission of a term in the energy equation of the earlier method. This note consequently presents the alternative approach, with a brief outline of the former method (duly corrected) in the Appendix, with revised numerical values. The comparison of the details of the two methods is interesting, as is also that with the earlier paper by Burgers[2]. Reference should be made to the two preceding papers for a fuller discussion of the phenomena.

Notation/

Notation

(in order of appearance)

v_n	fluid velocity in region 'n'
v_{nm}	$v_n - v_m$
p_n	pressure in region 'n'
p_{nm}	$p_n - p_m$
z	p_2/p_1
z'	p_5/p_6
v_s	initial shock velocity
v_s'	final shock velocity
ρ	gas density
c	speed of sound
γ	ratio of specific heats
λ^2	$(\gamma - 1)/(\gamma + 1)$
T_n	temperature in region 'n'
T_{nm}	$T_n - T_m$
C_p	specific heat at constant pressure
$\mathcal{T}$	non-dimensional coefficient, see text
$\dot{\xi}$	change of shockwave velocity
$\mathcal{R}$	non-dimensional coefficient, see text
M_2'	v_{21}/c_2
X	$\dfrac{z}{z-1} - \dfrac{1}{2}\,\dfrac{z}{\lambda^2+z}$
Y	$\dfrac{\gamma}{\gamma-1}\left(\dfrac{1+2\lambda^2 z}{1+\lambda^2 z} - \dfrac{z}{\lambda^2+z}\right) - 1$
Z	$\dfrac{\gamma M_1}{2}\,\dfrac{c_1^2}{c_2^2}\,\dfrac{z}{\lambda^2+z}$

Analysis/

Analysis

As the problem is made linear by considering only small disturbances, it is convenient to introduce all forms of disturbance together, as illustrated in Fig. 1.

From ordinary shockwave theory one obtains

$$v_{21} = (z-1)c_1\left(\frac{1-\lambda^2}{\gamma}\right)^{\frac{1}{2}} \frac{1}{(\lambda^2+z)^{\frac{1}{2}}}, \quad v_{56} = (z'-1)c_6\left(\frac{1-\lambda^2}{\gamma}\right)^{\frac{1}{2}} \frac{1}{(\lambda^2+z)^{\frac{1}{2}}},$$

$$v_S = c_1\left(\frac{\lambda^2+z}{\lambda^2+1}\right)^{\frac{1}{2}}, \quad v_S' = v_5 + c_5\left(\frac{\lambda^2+z}{\lambda^2+1}\right)^{\frac{1}{2}}.$$

Acoustical theory gives

$$p_{32} = \rho_2 c_2 v_{32}, \quad p_{43} = -\rho_2 c_2 v_{43}$$

and

$$p_{81} = \rho_1 c_1 v_{81}, \quad p_{67} = -\rho_1 c_1 v_{67},$$

while the temperature discontinuities have

$$p_{54} = 0 = p_{78}, \quad v_{54} = 0 = v_{78}.$$

As the disturbances are small, one obtains expressions like

$$z' = z\left(1 + \frac{p_{32}+p_{43}}{p_2} - \frac{p_{81}+p_{67}}{p_1}\right),$$

$$c_6 = c_1\left(1 + \frac{\gamma-1}{2\gamma}\,\frac{p_{81}+p_{67}}{p_1} + \frac{1}{2}\,\frac{T_{78}}{T_1}\right),$$

and
$$v_{56} = v_{21}\left[1 + \left(\frac{p_{32}+p_{43}}{p_2}\right)X - \left(\frac{p_{81}+p_{67}}{p_1}\right)\left(X - \frac{\gamma-1}{\gamma}\right) + \frac{T_{78}}{2T_1}\right].$$

Substituting into the identity

$$v_{12} + v_{23} + v_{34} + v_{45} + v_{56} + v_{67} + v_{78} + v_{81} = 0$$

one can, after some rearrangement, obtain

$$\frac{p_{43}}{C_p\rho_2} = {}_S\mathcal{T}_S^{-}\,\frac{p_{81}}{C_p\rho_1} + {}_S\mathcal{T}_S^{+}\,\frac{p_{67}}{C_p\rho_1} + {}_T\mathcal{T}_S\,T_{78} + {}_S\mathcal{R}_S\,\frac{p_{32}}{C_p\rho_2}$$

where the $\mathcal{T}$ and $\mathcal{R}$ are coefficients depending on shock strength only, denoting processes analogous to transmission and reflection respectively, the variables being expressed in terms of temperature rise. The notation is the same as that used previously (hence some apparent inconsistencies), the prefix denoting the cause of the disturbance (S for sound wave, T for temperature), the suffix denoting the resulting quantity referred to. The + and - distinguish between the disturbing soundwaves p_{67} and p_{81} respectively. Thus ${}_S\mathcal{T}_T^{+}$ will refer to the soundwave p_{67} "transmitting" as a temperature wave (T_{54}). The coefficients have the following values

${}_S\mathcal{T}_S^{\pm}$/

$$ {}_S\mathcal{T}_S^{\pm} = \frac{c_2}{c_1}\,\frac{\gamma \dfrac{c_2}{c_1} M_2'\left(X - \dfrac{\gamma-1}{2\gamma}\right) \pm 1}{\gamma M_2' X + 1}, $$

$$ {}_T\mathcal{T}_S = -\frac{\gamma-1}{2}\,\frac{c_2^2}{c_1^2}\,\frac{M_2'}{\gamma M_2' X + 1}, $$

$$ {}_S\mathcal{R}_S = \frac{1-\gamma M_2' X}{1+\gamma M_2' X}, $$

the upper or lower sign being taken in the first as required.

The resultant temperature fluctuation T_{54} is easily obtained in a similar manner, yielding

$$ T_{54} = {}_S\mathcal{T}_T^{-}\frac{p_{81}}{C_p\rho_1} + {}_S\mathcal{T}_T^{+}\frac{p_{67}}{C_p\rho_1} + {}_T\mathcal{T}_T\, T_{78} + {}_S\mathcal{R}_T\frac{p_{32}}{C_p\rho_2}, $$

where

$$ {}_S\mathcal{T}_T^{\pm} = \left({}_S\mathcal{T}_S^{\pm} - \frac{c_2^2}{c_1^2}\right)Y, $$

$$ {}_T\mathcal{T}_T = \frac{c_2^2}{c_1^2} + {}_T\mathcal{T}_S\, Y, $$

$$ {}_S\mathcal{R}_T = (1 + {}_S\mathcal{R}_S)Y. $$

The change of shock velocity is easily found by starting with the expression for $(v_S' - v_S)$,

$$ \dot{\xi} = {}_S\mathcal{T}_{\dot\xi}^{-}\, v_{81} + {}_S\mathcal{T}_{\dot\xi}^{+}\, v_{67} - {}_T\mathcal{T}_{\dot\xi}\, c_1 \frac{T_{78}}{2T_1} + {}_S\mathcal{R}_{\dot\xi}\, v_{32}, $$

where

$$ {}_S\mathcal{T}_{\dot\xi}^{\pm} = 1 \mp \left\{\frac{\gamma-1}{2} M_1 + Z\left({}_S\mathcal{T}_S^{\pm} - \frac{c_2^2}{c_1^2}\right)\right\}, $$

$$ {}_T\mathcal{T}_{\dot\xi} = -\left\{M_1 + \frac{2}{\gamma-1} Z\, {}_T\mathcal{T}_S\right\}, $$

$$ {}_S\mathcal{R}_{\dot\xi} = Z\frac{c_2}{c_1}(1 + {}_S\mathcal{R}_S). $$

The value of the coefficients for a series of Mach numbers are given in the Table, and shown in Fig. 2. A point worth noting is that while the interaction of the shockwave with the temperature wave produces a relatively weak soundwave, the latter may well be large on an acoustical scale. For example a temperature fluctuation of 0.1 °C ahead of a shockwave having $M_1 = 2$ and N.T.P. conditions behind it produces a pressure fluctuation of 120 db above 0.0002 dynes/cm². Another important point is that the reflected sound due to a soundwave catching up a shockwave is very weak, particularly for small shock strengths when the closing velocity is small.

Table/

Table of Interaction Coefficients

M_1	1	2	3	4	5
${}_S\tau_S^+$	1.000	1.345	1.863	2.565	3.439
${}_S\tau_T^+$	0.000	-0.214	-1.036	-2.481	-4.533
${}_S\tau_{\dot\xi}^+$	0.600	0.874	1.029	1.216	1.417
${}_S\tau_S^-$	0.000	0.0975	0.354	0.753	1.300
${}_S\tau_T^-$	0.000	-0.994	-2.952	-5.513	-8.639
${}_S\tau_{\dot\xi}^-$	0.600	0.128	-0.194	-0.458	-0.700
${}_T\tau_S$	0.000	-0.156	-0.335	-0.566	-0.855
${}_T\tau_T$	1.000	1.590	2.235	3.099	4.158
${}_T\tau_{\dot\xi}$	-1.000	-1.376	-1.706	-2.059	-2.434
${}_S\mathcal{R}_S$	0.000	-0.0396	-0.0780	-0.0996	-0.112
${}_S\mathcal{R}_T$	0.000	0.600	1.171	1.507	1.705
${}_S\mathcal{R}_{\dot\xi}$	0.600	0.998	1.164	1.242	1.283

APPENDIX/

APPENDIX

Notation

u	fluid velocity
p_1'	pressure of soundwaves upstream
p_2'	resultant sound wave pressure downstream
p_2''	pressure of soundwave catching up shock
S	entropy, having similar primes and suffices
$a^{\pm}$	$(1 \pm M)^2$
b	$(\gamma - 1)M^2$
$f^{\pm}$	$(\gamma - 1)M^2 \pm 2\gamma M + (\gamma + 3) \pm \frac{2}{M}$
g	$(3 - \gamma) - (\gamma - 1)^2 M^2$
d	$a_2^+ g_2 + b_2 f_2^+$
r	ρ_1/ρ_2

The coefficients a, b, f and g take suffices 1 or 2 according to whether they refer to M_1 or to M_2, e.g., $a_2^- = (1 - M_2)^2$.

In this section the shockwave is taken as the frame of reference. Then equations of continuity, momentum and energy are

$$\rho_1 u_1 = \rho_2 u_2 ,$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

and

$$\frac{u_1^2}{2} + C_p T_1 = \frac{u_2^2}{2} + C_p T_2$$

where the suffices indicate the same flow regions as before, see Fig.3 (where if region 1 were stationary the shock would advance to the left). If disturbances from upstream are denoted by a single dash (e.g., p_1'), that from downstream by a double dash (e.g., p_2''), and the resultant effects downstream by a single dash (e.g., p_2'), $\dot{\xi}$ being the change of shock velocity, then the above equations give

$$\rho_1(u_1'-\dot{\xi}) + \rho_1' u_1 = \rho_2(u_2'+u_2''-\dot{\xi}) + (\rho_2'+\rho_2'')u_1$$

$$p_1' + 2\rho_1 u_1(u_1'-\dot{\xi}) + \rho_1' u_1^2 = p_2' + p_2'' + 2\rho_2 u_2(u_2'+u_2''-\dot{\xi}) + (\rho_2'+\rho_2'')u_2^2 ,$$

$$u_1(u_1'-\dot{\xi}) + C_p T_1' = u_2(u_2'+u_2''-\dot{\xi}) + C_p(T_2'+T_2'').$$

Since the pressure fluctuation completely defines the soundwave, and entropy fluctuation the temperature wave, we add

$$S_1' = C_p \frac{T_1'}{T_1} - R\frac{p_1'}{p_1}, \quad S_2' = C_p \frac{T_2'}{T_2} - R\frac{p_2'}{p_2} ,$$

$$p_1' = \pm \rho_1 c_1 u_1', \quad p_2' = \rho_2 c_2 u_2', \quad p_2'' = -\rho_2 c_2 u_2'',$$

the upper sign for p_1' being taken if the soundwave propagates with the flow direction, and the lower sign if against it (see Fig.3), a notation continued in the superscripts of certain coefficients. After some manipulation the perturbed momentum and energy equations become

$a_1^{\pm}$/

$$a_1^{\pm} p_1' - b_1 \rho_1 T_1 S_1' = a_2^{+} p_2' + a_2^{-} p_2'' - b_2 \rho_2 T_2 S_2',$$

$$f_1^{\pm} \frac{p_1'}{\rho_1} + g_1 T_1 S_1' = f_2^{+} \frac{p_2'}{\rho_2} + f_2^{-} \frac{p_2''}{\rho_2} + g_2 T_2 S_2'.$$

From these the required p_2' and S_2' can easily be found, and then $\dot{\xi}$ from the continuity and momentum equations, corresponding exactly to the earlier ones apart from notational changes:

$$\frac{p_2'}{C_p \rho_2} = {}_S\mathcal{T}_S^{\pm} \frac{p_1'}{C_p \rho_1} + {}_S\mathcal{T}_T \frac{T_1 S_1'}{C_p} + {}_S\mathcal{R}_S \frac{p_2''}{C_p \rho_2},$$

$$\frac{T_2 S_2'}{C_p} = {}_S\mathcal{T}_T^{\pm} \frac{p_1'}{C_p \rho_1} + {}_T\mathcal{T}_T \frac{T_1 S_1'}{C_p} + {}_S\mathcal{R}_T \frac{p_2''}{C_p \rho_2},$$

$$\dot{\xi} = {}_S\mathcal{T}_{\dot{\xi}}^{\pm} u_1' + {}_T\mathcal{T}_{\dot{\xi}} c_1 \frac{S_1'}{2C_p} + {}_S\mathcal{R}_{\dot{\xi}} u_2''.$$

The formulae for the coefficients however take on a different form; in terms of the abbreviations given above they are

$${}_S\mathcal{T}_S^{\pm} = \frac{r a_1^{\pm} g_2 + b_2 f_1^{\pm}}{d}, \quad {}_S\mathcal{T}_T^{\pm} = \frac{a_2^{+} f_1^{\pm} - r a_1^{\pm} f_2^{+}}{d},$$

$${}_S\mathcal{T}_{\dot{\xi}}^{\pm} = \frac{r}{1-r}\left[\pm \frac{\rho_2 c_1}{\rho_1 c_2}\left\{- (\gamma - 1) M_2 \, {}_S\mathcal{T}_T^{\pm} + (1 + M_2 \, {}_S\mathcal{T}_S^{\pm}\right\} - (1 \pm M_1)\right];$$

$${}_T\mathcal{T}_S = \frac{g_1 b_2 - r g_2 b_1}{d}, \quad {}_T\mathcal{T}_T = \frac{a_2^{+} g_1 + r b_1 f_2^{+}}{d},$$

$${}_T\mathcal{T}_{\dot{\xi}} = \frac{2r}{1-r}\left[M_1 - \frac{1}{\gamma - 1} \frac{\rho_2 c_1}{\rho_1 c_2}\left\{(\gamma - 1) M_2 \, {}_T\mathcal{T}_T - (1 + M_2) \, {}_T\mathcal{T}_S\right\}\right];$$

$${}_S\mathcal{R}_S = -\frac{a_2^{-} g_2 + b_2 f_2^{-}}{d}, \quad {}_S\mathcal{R}_T = -\frac{a_2^{+} f_2^{-} - a_2^{-} f_2^{+}}{d},$$

$${}_S\mathcal{R}_{\dot{\xi}} = \frac{1}{1-r}\left[(1 - M_2) - (1 + M_2) \, {}_S\mathcal{R}_S + (\gamma - 1) M_2 \, {}_S\mathcal{R}_T\right].$$

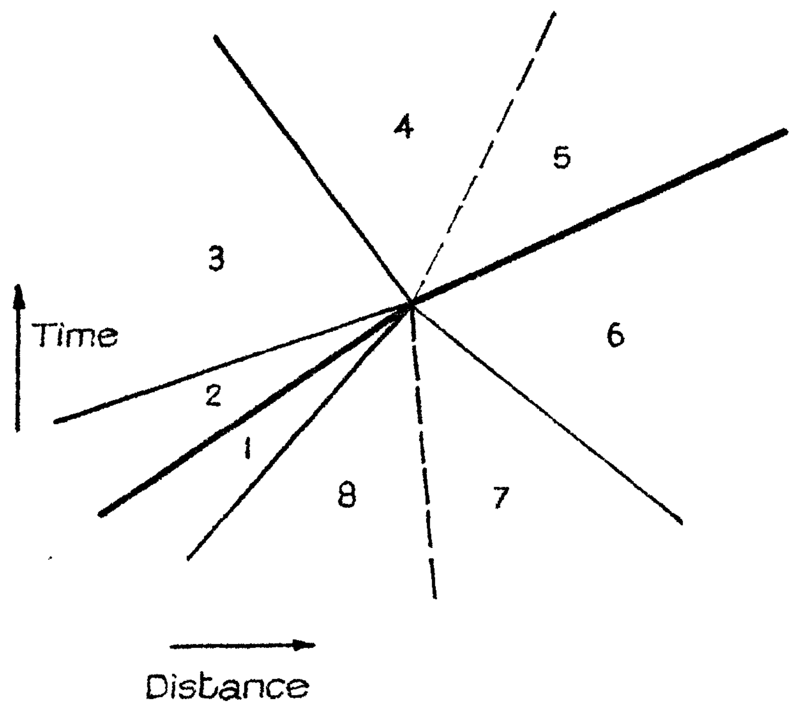
Their numerical values are of course identical with those of the other set. This method has the advantage of preserving a symmetry of the equations in the derivation, but the computational effort required to evaluate the coefficients is noticeably greater.

The author wishes to apologise for any inconvenience arising from the former inaccurate results, to thank Dr. E. M. Kerwin for commenting on the possibility of an error in the earlier note and to thank Miss Nancy Francis of Douglas Aircraft Company, Santa Monica for assistance with the numerical work.

References

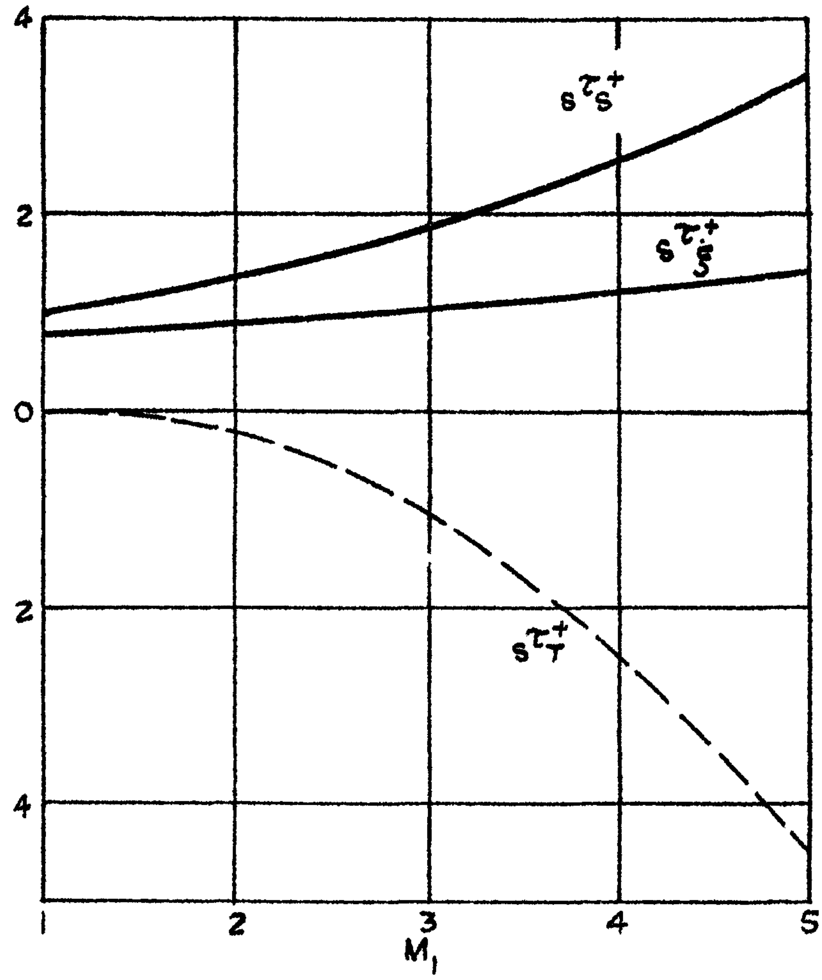
No.	Author(s)	Title, etc.
1	A. Powell	A note on the sound from weak disturbances of a normal shock wave. A.R.C. C.P.194. April, 1954.
2	J. M. Burgers	On the transmission of sound through a shock wave. K. Nederland, Akad. v. Wetenschappen 44, 1946.

Fig. 1.

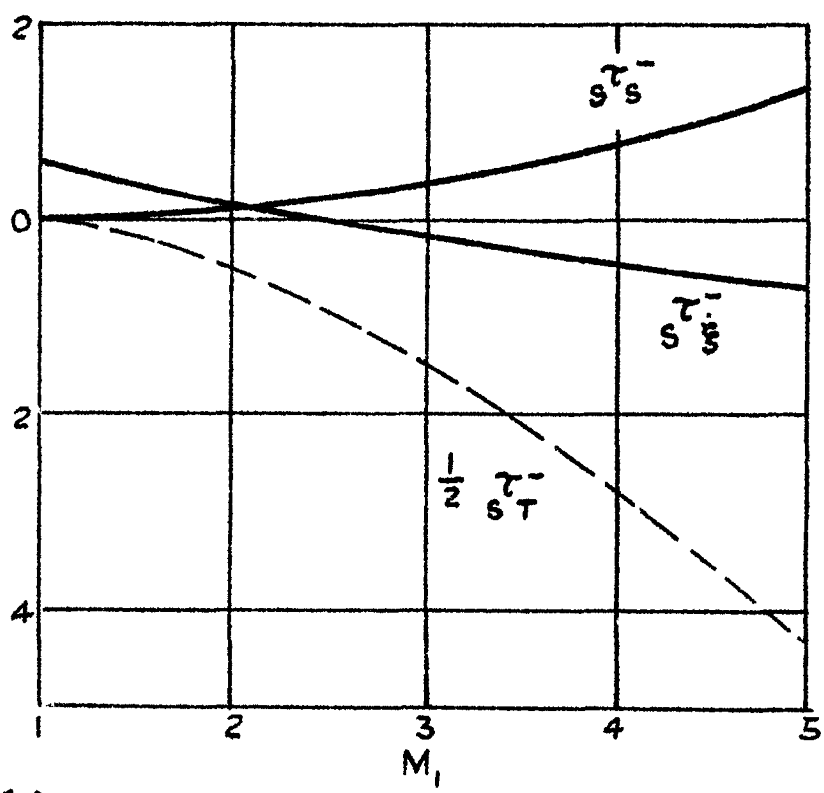


Space-time diagram defining flow regions for disturbed shock wave. Region 1 is to be taken as the undisturbed fluid, with (21) the initial shock-wave front, and (56) the final shock wave. (81) and (67) limit disturbing sound waves overtaken by and colliding with the shock wave, (78) marks a temperature discontinuity in the flow ahead of the shock, while (32) limits sound wave catching up with the shock from behind. As a result of these disturbances are the sound wave (43) and temperature discontinuity (54) —— shock front, —— limit of sound wave, - - - - temperature discontinuity.

FIG. 2 (a-b).



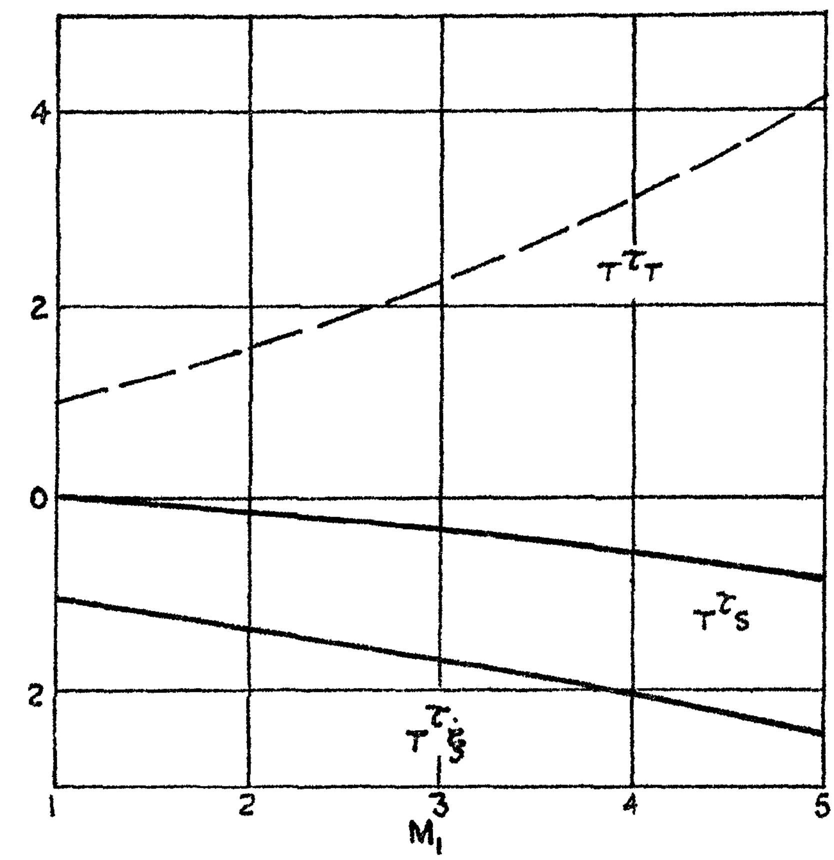
(a) Sound wave colliding with shock wave



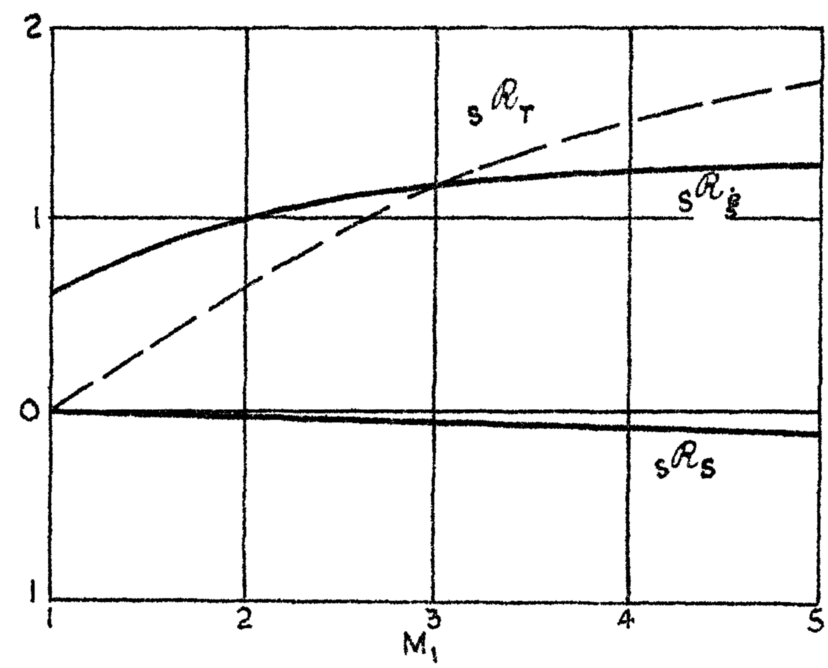
(b) Sound wave caught up by shock wave

Interaction coefficients for disturbances.

FIG. 2. (c & d)



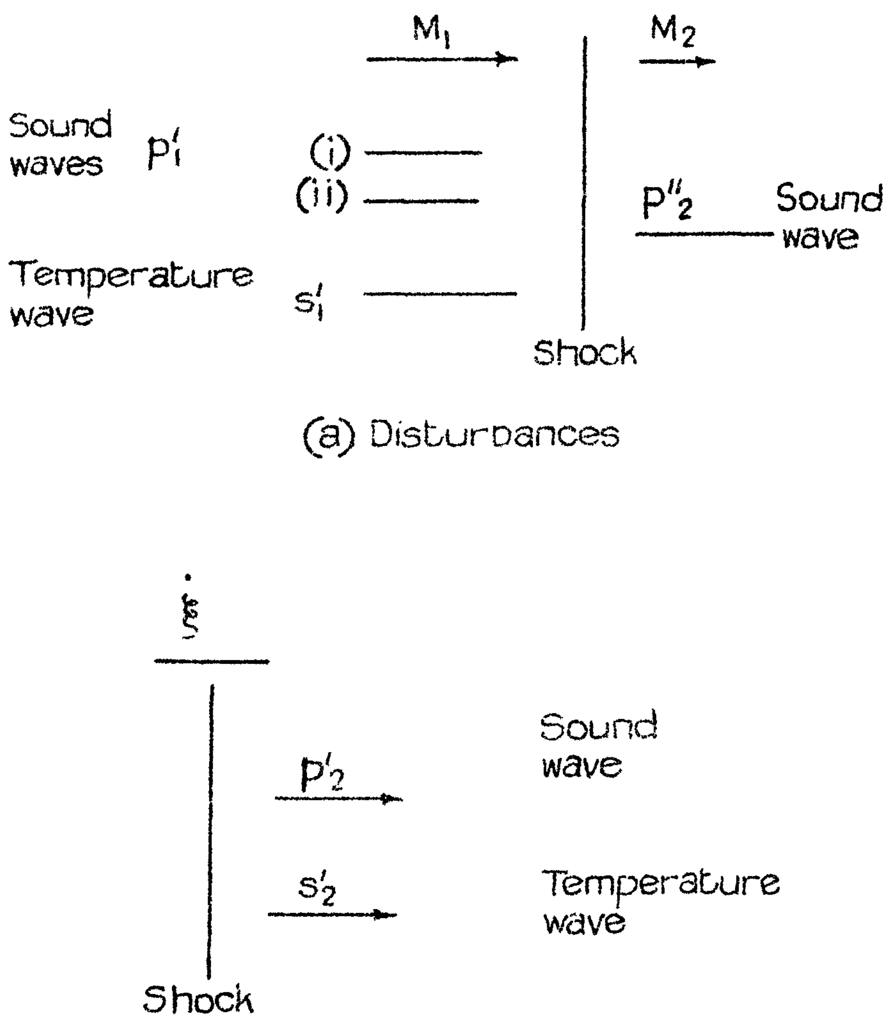
(c) Temperature variation in the flow



(d) Sound wave catching up shock wave

Interaction coefficients for disturbances

Fig. 3.



(a) Disturbances

(b) Resultant perturbation

Diagram illustrating notation used in the Appendix. The sound wave p_1' takes the upper sign, when a choice arises, when moving with the stream (case (i)), and the lower sign when against it (case (ii))

C.P. No. 441
(20,125)
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