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Microwave Technique Applied to the Investigation of Ionised Gases in Shock Tubes

By

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SUMMARY

A brief review is given of some techniques which employ the radiation and transmission of millimetre and centimetre wavelength electromagnetic waves. Such techniques may be of use in studies of electron density and ionisation-recombination phenomena which are at present of interest in hypersonic flow.

CONTENTS

1. Introduction

In the laboratory simulation of hypersonic flight, uncertainties sometimes arise because the test gas is dissociated and ionised before it reaches the flow field of the model under test. This factor is of particular significance in the hypersonic shock tunnel, in which a partially dissociated gas in the unexpanded section of the tunnel is cooled in the expansion. The rate at which this gas recombines as it is cooled determines the nature of the flow in front of the bow shock wave on the model. For instance, if the recombination rate is extremely rapid, then it can safely be assumed that equilibrium will be attained at the exit of the nozzle; alternatively, if it is slow, the shock tube does not simulate flight through undissociated air, and the predicted energy transfer from ordered to disordered molecular motion across the shock wave will be considerably in error. A further complication arises in the use of air in such devices as shock tubes since behind the driving shock wave the concentration of NO may reach 10 per cent. The rate at which this is formed is difficult to calculate because the equilibrium concentration does not vary monotonically with temperature but reaches a maximum at about 5000°K. The ionisation potential of NO is only 9.5 electron volts compared with 12.5 electron volts for oxygen the next highest as shown in Table 1 (Ref.1), and hence this is the most copious source of electrons both behind the primary shock wave in a shock tube and in the shock layer surrounding a hypersonic vehicle.

In view of this uncertainty in the actual amount of ionisation present in the working section of a shock tube it seems desirable to make some direct measurements of the ionisation level. It is realised that this may give no indication of atom-atom recombinations which have occurred although the detection of substantial ionisation at the nozzle exit would indicate non-equilibrium flow in the nozzle. The presence of ions may be due to reactions such as:

$$N + O \rightleftharpoons NO^+ + e.$$

The use of nitrogen and oxygen separately would preclude such reactions, and would be a natural first step in the experimental approach to the problem. Tests with monatomic gases such as argon would also be of value since the ionisation level of this gas, 15.76 electron volts is quite close to that of atomic nitrogen and oxygen.

The investigation of ionised gases by the use of microwaves is now well established[2,3] and, although there are some differences in the absolute densities of the gases under consideration, similar techniques should prove useful in the investigations proposed for aerodynamic purposes. Some of the methods which appear to be directly applicable to the measurement of ionisation levels in aerodynamic facilities are described in this paper.

2. Basic Relations for the Microwave Measurement of Conductivity and Electron Density

The general theory for the interaction of free electrons in a vacuum was established by Lorentz and Appleton[3] who also included the effects of magnetic fields. The elementary formulae are presented here for convenience.

2.1 Motion of free electrons in a vacuum

An electron with charge e and mass m when acted upon by an electric field of the form $E_0 \cos \omega t$ moves with an acceleration d^2x/dt^2 where:-

$$m \frac{d^2x}{dt^2} = e\, E_0 \cos \omega t. \qquad \dots(1)$$

Its velocity is
$$\frac{dx}{dt} = \frac{eE_0}{m\omega} \sin \omega t \qquad \ldots(2)$$

and hence the current per electron
$$i = e\frac{dx}{dt} = \frac{e^2E_0}{m\,\omega} \sin \omega t \qquad \ldots(3)$$

and the current density (per cc), where N is the electron density (per cc), is
$$J = Ni = \frac{Ne^2E_0}{m\omega} \sin \omega t. \qquad \ldots(4)$$

Thus the current is out of phase with the applied field, and the conductivity σ has the value
$$\sigma = \frac{J}{E} = \frac{Ne^2}{m\omega}. \qquad \ldots(5)$$

The dielectric constant of the gas is then
$$\epsilon = \epsilon_0 - \frac{4\pi Ne^2}{m\omega^2} \qquad \ldots(6)$$

where ϵ_0 is the dielectric constant of the free space. In air ϵ_0 is taken as unity. At frequencies below ω_p, given by
$$\omega_p^2 = \frac{4\pi Ne^2}{m} \qquad (7)$$

the gas ceases to be transparent and waves launched into it are heavily attenuated[4].

2.2 The effect of collisions of electrons with neutral particles

In general, for the gases and temperatures of aerodynamic interest the gas is only partially ionized, and account must be taken of the effect of collisions on such properties as the conductivity and dielectric constant. The ordered motion of the free electrons under the influence of the electric field $E_0 \cos \omega t$ becomes disordered and some electrons are moved, finally, with the same phase, that is to say their motion is of the form $K \cos \omega t$. Thus the gas dissipates some of the power in ohmic heating, the loss depending on the ratio of the collision frequency ν, of the electrons to the frequency ω, resulting in the real part of the complex conductivity σ_r:
$$\sigma_r = \frac{Ne^2}{m}\,\frac{\nu}{\nu^2 + \omega^2}. \qquad \ldots(8)$$

The collision frequency ν is related to the mean velocity of the particles, their number density n_j and collision cross section Q_j by the following relationship
$$\nu_c = \overline{Ce} \sum_j n_j Q_j \qquad \ldots(9)$$

where/

where $$\overline{Ce} = \sqrt{\frac{8kT}{\pi m}}, \qquad \ldots(10)$$

and k = Boltzmann's constant.

The collisions of electrons with neutral atoms, neutral molecules and ions must be considered in the summation. The unknown or uncertain collision cross sections for electrons with neutrals or ions renders this calculation valueless in the present state of knowledge for measurements either at high density or temperature, i.e., n_j large or $\overline{Ce}$ large, or at low frequency ω. Almost the only possible approach is to select $\omega >> \nu$. At high gas densities this may result in the use of extremely short wavelength microwaves, and ultimately the transference to the far infra-red and the use of optical techniques.

2.3 The measurement of high frequency conductivity

At applied frequencies ω, below the plasma frequency ω_p the wave amplitude will decrease by $1/e$ in a distance d (Ref.4) where

$$d = \frac{c}{\omega_p} \frac{1}{\left(1 - \frac{\omega^2}{\omega_p^2}\right)^{\frac{1}{2}}}, \qquad \ldots(11)$$

and c = wave velocity. Thus by determining the attenuation over a distance it is possible to deduce the plasma frequency and hence N, the number density of free electrons. Assuming that atoms are only singly ionised, which is likely to be the case in shock tubes, the degree of ionisation is thus obtained.

2.4 Experimental techniques for conductivity measurements

In order to obtain experience in gases whose collision cross sections are known, and for which the degree of ionisation may be obtained analytically at any temperature, a programme has been initiated at N.P.L. to determine N for values of $\omega = 10^{10}$ and 4×10^{10} cycles/sec. The apparatus is shown schematically in Fig.1. The attenuation in a path length of 2 inches is compared with a precision variable attenuator. The oscillator frequency is altered until the waveguide horn is matched, so that there are normally no reflections from the interface.

When the ionised gas passes the horns there is some reflection due to the change in dielectric constant, equation (6). It is predicted that attenuations between 5 and 50 decibels will be found in argon for shock Mach numbers between 6 and 10, the upper measurable limit being set by the accuracy and stability of the oscillator output.

Plasma frequencies up to 2×10^{12} c/s can be achieved in the small stainless steel shock tube which has been constructed for these experiments, and this of course demands frequencies well into the infra-red for transmission. The upper limit for frequencies of 10^{10} c/s is an electron density of 10^{12} cm^{-3}. A convenient approximation for the expression

$$\omega_p = \sqrt{\frac{Ne^2}{\pi m}}$$

is $\omega_p = 10^4 \sqrt{N}$ where N is the number of electrons per cc; to be more precise $\omega_p = 8.95 \times 10^3\sqrt{N}$. Thus the range of shock Mach numbers in argon from 6 to 12 covers the range of electron densities from 10^7 cm^{-3} to 6×10^{16} cm^{-3} if the initial pressure of argon in the shock tube does not exceed 10 mm Hg, Fig.2. Values of N as a function of shock Mach number M_s for argon at initial pressures from 10^{-6} to 10^{-1} atmospheres are shown in Fig. 2 (Ref. 9), and where the value of N results in a plasma frequency coinciding with a commercially available source of microwave signals these are also indicated. Two wavelengths in the far infra-red are also shown.

The limits imposed by pressure for the 2 in. shock tube (100 atmospheres) and the 6 in. shock tube (1000 atmospheres) are also shown.

3. Faraday Rotation

One of the principle objections to the direct measurement of conductivity by attenuation measurements is the apparent attenuation which may be caused by reflection of the curved wave fronts from the transmitting horn in the circular symmetry of the shock tube. Microwave lenses can be fitted to the horns to provide plane waves, but there still remains the possibility of internal losses not due to absorption.

3.1 The anisotropy of the gas in the presence of an axial magnetic field

If a linearly polarised TE_{11}* wave is propagated in a circular wave guide, which in the present case is the shock tube, Fig. 3, the plane of polarisation will be rotated by an angle θ radians in the presence of an axial magnetic field H gauss and free electrons of number density N cm^{-3} (Refs. 2 and 5) where

$$\theta = \frac{2.36 \times 10^4}{f^2} \int H\, N\, dx \text{ radians,} \qquad \ldots(12)$$

f = frequency c/s,

and x = path length cm.

For example, with $H = 100$ gauss, $\theta = 2.36$ rad, $x = 10$ cm and $f = 10^{10}$ c/s; N would then be 10^{13} per cc. Should the magnetic field be inclined at an angle ϕ to the direction of propagation of the electromagnetic wave, as may be the case at the end of a field coil, the rotation θ must be modified to

$$\theta = \frac{2.36\ 10^4 \cos\phi}{f^2} \int H\, N\, dx. \qquad \ldots(13)$$

By extending the field coil beyond the length x, it should be possible to maintain the field H at all points perpendicular to the electric vector E. The TE_{11} mode is launched in the waveguide by means of a simple probe extending into the flow, and the rotation is measured by observing the relative amplitude of the signals received at two mutually perpendicular stations, Fig. 4. This relative measurement is unaffected by attenuation, reflection at mismatches in the two detecting probes, provided these have identical sensitivities, a point which can be readily checked.

One limitation to this technique is the maximum frequency ω which may be used to launch TE_{11} wave modes. If frequencies below the critical for the waveguide are used, the wave is not propagated, the cut-off wavelength λ_o is 3.46a, where a = guide radius in cm. For the 2 inch shock tube the cut-off is therefore at 8.78 cm, and if wavelengths shorter than 2a are used then the electric field is not as shown in Fig. 3, but

may/

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*The TE_{11} wave has one electric vector maximum and one magnetic maximum on passing once across the tube.

may be as in Fig. 5, and the interaction would be more complicated than that described by equation (12). This frequency limitation can be quite serious if it desired to propagate low frequency signals in a highly ionised gas, and for the 2 inch shock tube a frequency of 5×10^9 c/s must be employed, limiting the electron density to 2.5×10^{11} cm^{-3}, which is, nevertheless of considerable interest in the N_2 and O_2 cases although rather low for argon.

It is not possible to increase the magnetic field to enhance the rotational effect at low electron densities without producing the additional effect of circular polarization. This phenomenon occurs at magnetic field intensities such that the cyclotron frequency* of the free electrons in the plasma

$$\omega_H = \frac{eH}{mc} \qquad \ldots(14)$$

resonates with the frequency ω of the propagated wave.

The cyclotron frequency is approximately $\omega_H = 1.6 \times 10^7$ H c/s, and hence for frequencies of 10^{10} c/s resonance will occur at 600 gauss, but it was seen that very large rotational effects can be produced with field strengths of only 100 gauss and this phenomenon should not prove a serious limitation provided its existence is realised and a linear relationship $\theta \sim H$ not expected near $\omega_H = \omega$. Linearity appears good up to $\omega_H/\omega = 0.5$. The wave mode TE_{11}, Fig. 3, in general does not represent the true state of affairs in an ionised gas, but provided $(\omega_p/\omega)^2 << 1$ and $(\nu/\omega) < 1$ the simple theory may be used. These restrictions apply equally well to the propagation of any mode in the ionised gas.

4. Alternative Techniques

Although the methods described in sections 2 and 3 are considered most promising, two other interaction phenomena have been used in the past for electron density and temperature measurement.

4.1 Change in resonant frequency of a cavity due to the presence of free electrons

Free electrons in a cavity containing the plasma contribute to an in-phase component of the complex conductivity σ, producing a change in resonant frequency $\Delta\omega$ given (Ref. 2) by

$$\frac{\Delta\omega}{\omega_0} = \frac{Ne^2}{m\omega^2 \epsilon_0} f(r, \theta, z). \qquad \ldots(15)$$

The function $f(r, \theta, z)$ accommodates the variable geometry possible in waveguide cavities, generally cylindrical in the experiments which have been reported[6]. The microwave signal ω may be detected after transmission through the plasma or after reflection. This technique has been applied with considerable success in the case of pulsed discharges in gases, but the necessity of altering the geometry to fit shock tube conditions and the incompatibility of high electron densities, hence high ω_p values, with physically large and manageable containers, hence low ω values, appear to weigh heavily against this method. There does not appear to be any reason why ionisation decay times in nitrogen and oxygen should not be studied in pulsed discharge tubes as an alternative

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*The cyclotron frequency is that at which the electrons will rotate in the presence of a magnetic field H in a cyclotron.

to shock tubes. The recombination of low energy electrons with positive ions in nitrogen has in fact been studied using this technique, and some work has also been done on oxygen[7].

4.2 Noise power radiated by a plasma

It has been found[8] that the radio noise emitted by a plasma is within 3% of that which one would receive from a black body at the electron temperature and a great deal of work has been done on continuous and pulsed gas discharges as calibrating noise sources. The noise power associated with a temperature T and a bandwidth dF is $kT\,dF$, where k is Boltzmann's constant.

As an example with T 10,000°K and dF 10^7 cycles/sec the available noise power is 1.38×10^{-12} watts, a small but measurable noise power when it is realised that a typical receiver would have a minimum detectable noise power of about 10^{-13} watts.

5. Conclusions

It has been shown that useful information on the electron density in a gas may be obtained from microwave absorption and Faraday effects in shock tubes, using techniques which have been found satisfactory in pulsed gas discharges. The possibility of producing electrons in a test gas by means of a discharge and subsequently examining the time scales of recombination should not be overlooked. While the direct absorption measurement would be most useful for electron density measurements across the working section of a shock tube there are inherent matching problems which make this less positive, at first sight, than the Faraday effect.

It is proposed to investigate both the Faraday effect and absorption, and a more detailed analysis will be presented in further papers.

Notation/

Notation

m	mass of electron
e	charge of electron
x	distance
E_o	electric field vector
ω	angular frequency
N	electron density per cc
J	current density
σ	conductivity
ϵ	dielectric constant
$\overline{Ce}$	mean thermal velocity
Q	cross section
n	number of particles
ν	collision frequency
c	velocity of light
M_s	shock Mach number
θ	angular rotation
H	magnetic field strength
f	frequency c/s
ϕ	inclination of magnetic field to direction of propagation of electromagnetic wave

Subscripts

ω:	p	plasma frequency
	H	cyclotron frequency

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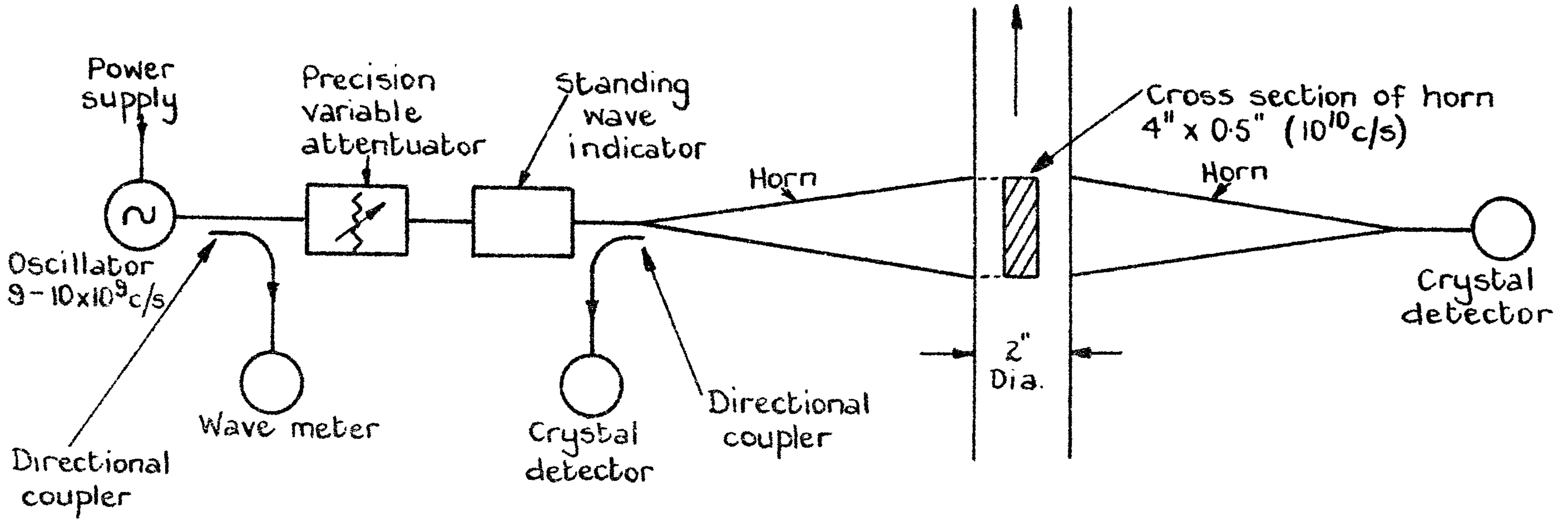
Table 1

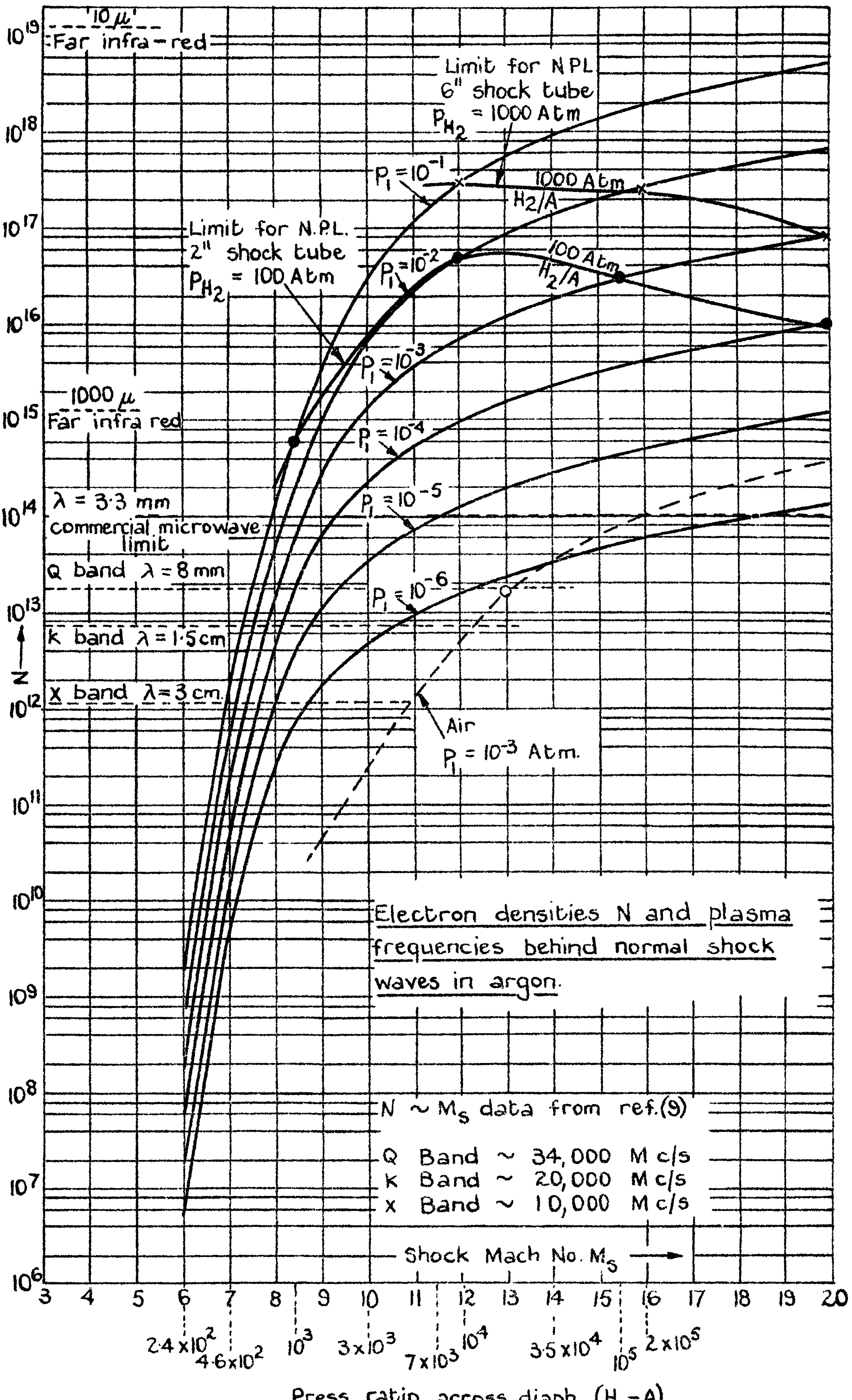
Constituent	Dissociation Potential e.v.	Ionization Potential e.v.
N_2	9.7	15.51
O_2	5.08	12.5
N		14.54
O		13.61
NO	6.48	9.5

FIG. 1.



Attentuation measurements in 2 inch diameter shock tube.

FIG. 2.



Electron densities N and plasma frequencies behind normal shock waves in argon.

Figs (3)(4) & (5)

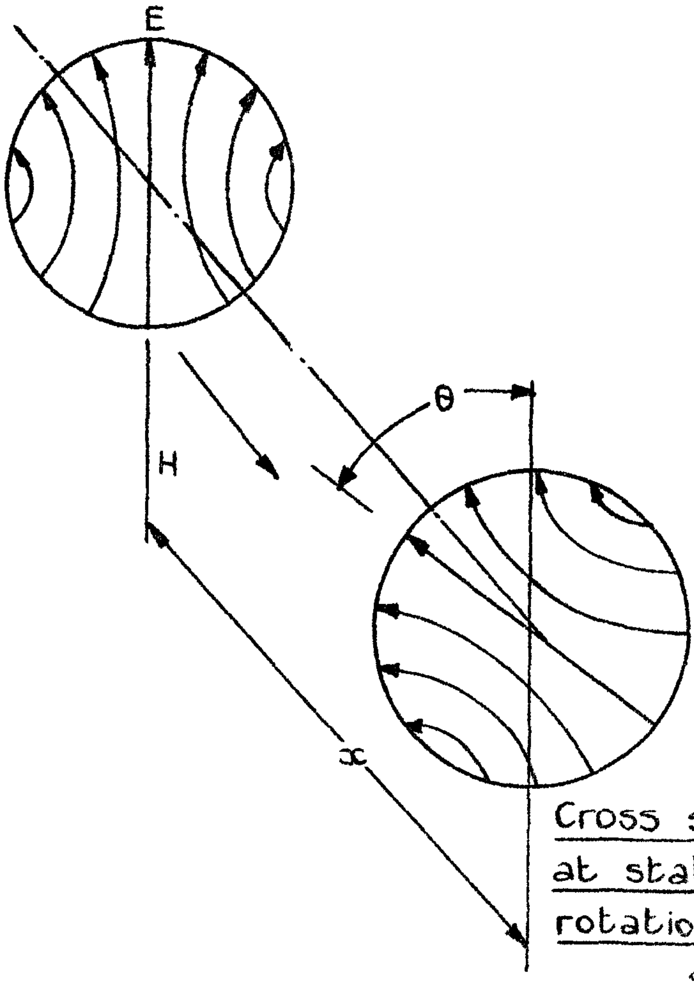


Fig 3.

Cross sections of waveguide at stations x apart showing rotation of TE_{11} wave by angle θ.

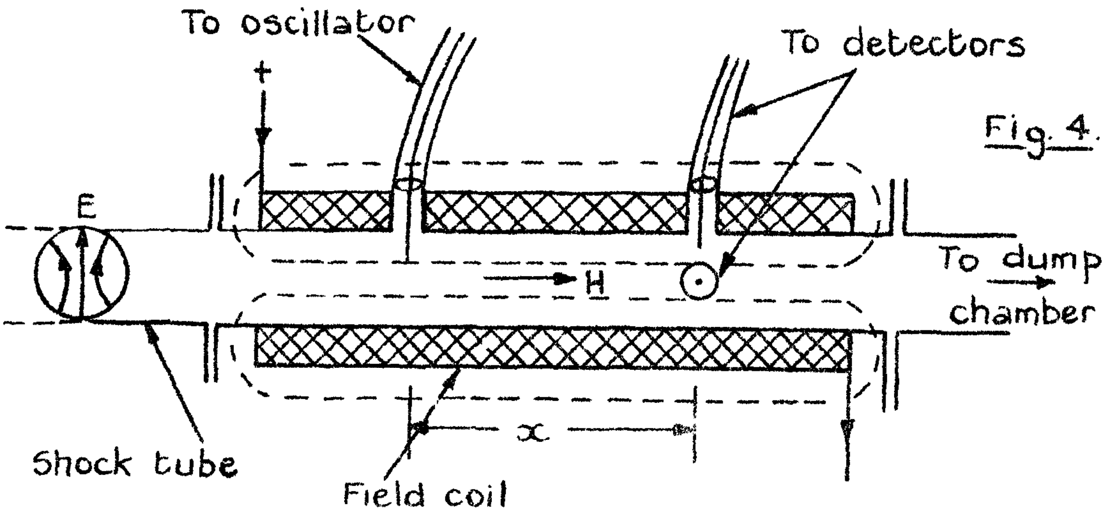


Fig. 4.

Faraday effect magnetic field and probe arrangement.

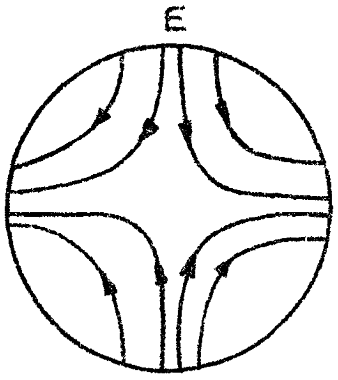


Fig. 5.

Higher modes of oscillation in the shock tube.

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