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Graphical Method of Calculating Performance of Airscrew

By C. N. H. LOCK

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Reports and Memoranda No. 1675

25th October, 1934

Summary.—A rapid method is described of making calculations of airscrew performance by means of charts. The first application is to ordinary strip theory calculations on the basis of the formulae of Ref. 5. Six charts are required for each radius for which the value of thrust grading, etc., are to be derived; of these six, four depend on number of blades but are otherwise universal, since they are independent of shape of blade section, and do not involve the blade width or blade angle explicitly; they are based purely on the application of Prandtl theory to the airscrew and contain no empirical adjustments. The remaining two charts involve the lift and drag curves of the section.

The second application gives a considerable further simplification in that the charts are required for a single standard radius (0.7) only; the thrust coefficient corresponding to a given working condition can then be deduced by a simple operation with three charts while the torque involves three further charts and a simple addition. The accuracy of the second method is increased if the lift and drag charts are deduced by analysis of observations on (model) airscrews, an analysis which can be performed rapidly by means of the remaining four charts; such an analysis of the results of the "wind tunnel tests of high pitch airscrews "⁴ shows that the method will give reasonably consistent results over a range of pitch ratio from 0.3 to 2.5, while there is little doubt that the method will cover the range of blade width likely to occur in practice. Changes of blade section and also of plan form and twist may be included if necessary by modifying the lift and drag curves. The second method has also been remarkably successful in its application to the stalled range of an airscrew, a range in which there is at present no other available method.

It is further suggested that the first method might prove very convenient for analysing wind tunnel tests of model airscrews at high tip speed; the accuracy of application of the second method might be improved by basing the lift curves on full scale values of power, speed and revolutions, combined with an estimate of profile drag.

Note.—For practical use the necessary charts should be plotted on a fairly large scale from the tables given in the report; e.g., Chart 1 should be a transparency measuring at least 22 in. \times 15 in. The small scale charts given in the report are intended for illustration only.

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LIST OF SYMBOLS

(cf. also list of symbols in Ref. 5.)

$$\theta = Blade angle.$$

} Fig. 1. (p. 4). $\alpha =$ Blade incidence.

 $\phi = \theta - \alpha$.

 ϕ_0 §2, Equation 2, and Fig. 1.

 $\beta = \phi - \phi_0$. Fig. 1.

W = Resultant velocity relative to a blade element. Fig. 1.

 $w_1 = \text{Total interference velocity.}$ Fig. 1.

 Ω = Angular velocity, radians per second.

r =Radius of blade element.

R = Tip radius.

x = r/R.

 $\varkappa =$ "Tip loss coefficient" Ref. 5, §2.

 $s = \text{Solidity} = Nc/2\pi r.$

N = Number of blades.

c = Blade chord.

 $k_{\rm L}$, $k_{\rm D}$ = Lift and drag coefficients of blade element (two dimensional flow).

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 $k_{\rm L_0} = k_{\rm L} - sk_{\rm D} \tan \phi$ (above the stalling angle).

§2, Equation (6). Wc

§2, Equation (7). W_c

 $\begin{array}{l} P_1 = `` Induced `' power loss \\ P_2 = Profile drag power loss \end{array} \right\} (Lbs. ft. per second.) \\ \end{array}$

 $\begin{aligned} & k_{\mathbf{P}_{1}} = \mathbf{P}_{1}/2\pi \, \varrho n^{3} \, \mathbf{D}^{5} \\ & k_{\mathbf{P}_{2}} = \mathbf{P}_{2}/2\pi \, \varrho n^{3} \, \mathbf{D}^{5} \end{aligned} \} \ \S{2} \ (9) \ \text{and} \ (12) \ ; \ \S{3.2} \ (5) \ \text{and} \ (6). \end{aligned}$

1. Introduction.—The present report describes a rapid method of carrying out airscrew strip theory calculations by means of suitable charts. The method may be based either on the standard formulae of the "vortex theory" assuming an infinite number of blades (R. & M. 892¹), or on the improved formulae described in R. & M. 1377² and R. & M. 1521³ which include a correction for tip loss.

In carrying out a detailed strip theory calculation on the basis of either assumption by standard methods (as described, e.g., in R. & M. 892¹ or R. & M. 1674⁵) it is assumed that values of the lift and drag coefficients of the section, in two dimensional flow, are known, at a series of standard radii, as well as the chord and blade angle of the sections. It is then possible, for each standard radius and for *assumed values of the blade incidence*, to calculate values of J (= V/nD) and of thrust and torque grading, but it is necessary to cross plot in order to obtain values of thrust and torque grading for given J before drawing and integrating the thrust and torque grading curves.

The charts here described make it possible to determine thrust and torque grading directly for given values of I and so avoid not only the labour of computing the formulae but also the labour of cross plotting. The method is specially convenient when results are required for a single working condition only. There still remains, however, the labour of drawing the thrust and torque grading curves and integrating them graphically. The only way of obtaining further simplification is to confine the calculations to a single standard radius and (assuming the shape of the thrust grading curves, etc., to be the same for all cases) to use a constant integrating factor to determine the area. Considerable success has been attained with the application of this simplification to the airscrews tested in the recent research on high pitch models at the National Physical Laboratory⁴ and the combination of this method with the graphical method is the principal subject of this report. The combination of the two reduces the labour of calculation to such an extent that the performance of a given airscrew at a single value of J can be obtained in a few minutes and its complete performance in considerably under an hour.

2. Basis of Graphical Method.—We proceed to describe the graphical method as applied to calculate the performance of a blade element at a given radius for a given value of J. The method has become possible as a result of neglecting the profile drag in calculating the interference velocity. It has been shown that this omission is fully justified at any rate so long as the blade section is operating below the stalling angle.⁵ The formulae thus simplified are given in Ref. 3, p. 22, and in Ref. 5, §3.

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For the purpose of computing the charts it is convenient to use a slightly modified set of formulae. These may be obtained most simply from the geometry of Fig. 1 together with the fundamental relation

$$w_1 = \frac{sk_L W}{2\varkappa \sin \phi}, \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

which is identical with Ref. 5, p. 4, equation (7). Here $k_{\rm L}$ is the lift coefficient of the blade section in two dimensional flow, s is the solidity ratio at radius r given by

$$s = \frac{Nc}{2\pi r}$$
,

(where N is the number of blades and c is the chord length of the section), and \varkappa is the tip loss coefficient defined in Ref. 5, §2. The formulae may also be easily verified algebraically from those of Ref. 5.





The figure is similar to Fig. 1 of Ref. 5 and represents the various velocity components in a plane at right angles to a blade at radius r.

In calculating the charts it is convenient to use J ($\equiv \pi V/R\Omega$) and β (Fig. 1) as independent variables. Then ϕ_0 is given by the equation

$$\tan \phi_0 = V/r\Omega$$

= J/\pi x , ... (2)

and ϕ is given by

$$\phi = \phi_0 + \beta \quad \dots \quad \dots \quad \dots \quad \dots \quad (3)$$

From equation (1)

$$sk_{\rm L} = 2\varkappa \sin \phi \tan \beta$$
, ... (4)

since

$$w_1/W = \tan \beta$$
,

by the geometry of the figure. Equations (2), (3) and (4) determine $sk_{\rm L}$ as a function of J and β ; or J and ϕ only (for given x) since \varkappa is a known function of ϕ and x (for a given number of blades) by the Tables or Charts of Ref. 5 (Table 1 and Figs. 3, 4, 5 and 6). These equations do not involve s and $k_{\rm L}$ separately nor do they contain the blade angle θ nor the incidence α . It is therefore possible to construct chart 1 (Fig. 2)* consisting of a set of curves of $sk_{\rm L}$ against ϕ for a series of constant values of J.

The essential feature of the method is the use of this chart in conjunction with Chart 2 (Fig. 3) consisting of curves of $sk_{\rm L}$ against α for a series of constant values of s, derived from the lift curve of the blade section in two dimensional flow. Chart 1 is superposed on Chart 2 (one of these two charts must be transparent), the scale of ϕ in Chart 1 being equal but in the opposite sense to the scale of α in Chart 2. The zero of α on Chart 2 is adjusted to coincide with a value of ϕ in Chart 1 equal to the known value of the blade angle θ of the section. The value of ϕ for any point on Chart 1 and the value of α for the corresponding point on Chart 2, then satisfy the geometrical relation (Fig. 1).

Hence the point of intersection of the curve of Chart 1 for a given value of J with the curve of Chart 2 for a given value of s (corresponding to the chord length of the section) determines values of $sk_{\rm L}$ (and $k_{\rm L}$), ϕ and α , which are solutions of equations (2, 3, 4 and 5), for a given value of J; from these the thrust and torque of the blade element can be determined when the blade angle θ , the "solidity" s and the relation between $k_{\rm L}$ and α are known. This result could not be obtained arithmetically for given J except by successive approximation. The dotted curve in Chart 1 shows the position of the curve of $sk_{\rm L}$ against α of Chart 2, for s = 0.07 with the zero adjusted for $\theta = 24.45^{\circ}$ corresponding to an airscrew of P/D 1.0. Its intersection with the curve for (e.g.) J = 0.8, gives :—

$$sk_{
m L}=0.0225$$
 , $\phi=22.45^\circ$, $lpha=2.00^\circ$.

Now that α (or $sk_{\rm L}$) is known it is a simple matter to determine the thrust coefficient of the blade element and the "induced power loss". These may be most conveniently obtained from Charts 3

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^{*} Tables 1 and 2 contain data for constructing the charts (for radius x = 0.7 only); they should of course be drawn on a considerably larger scale.

and 4 (Figs. 4 and 5) consisting of curves of $dk_{\rm T}/d(x^2)$, and $\frac{1}{2}w_{\rm c}$ respectively against $sk_{\rm L}$ for a series of constant values of J. These charts are computed from the following formulae derivable from the geometry of Fig. 1.

$$w_{c} \equiv w_{1} \sec \phi / R\Omega \quad \text{(definition)}$$
$$= \frac{x \sin \beta}{\cos \phi_{0} \cos \phi}, \quad \dots \quad \dots \quad \dots \quad \dots \quad (6)$$

$$W_{c} \equiv W/R\Omega \quad \text{(definition)} \\ = x \cos \beta / \cos \phi_{0} \quad \dots \quad \dots \quad \dots \quad \dots \quad (7)$$

Neglecting the contribution of the profile drag to the thrust (which is justifiable below the stalling angle), the element of thrust on a blade element is given by

$$dT = \varrho c \ dr \cdot W^2 \ k_L \cos \phi$$
,

so that

$$\frac{1}{\varrho (\mathrm{R}\Omega)^2} \cdot \frac{1}{\pi} \cdot \frac{d\mathrm{T}}{d(r^2)} = sk_{\mathrm{L}} \mathrm{W_c}^2 \cos \phi$$

and

$$\frac{dk_{\rm T}}{d(x^2)} = \frac{\pi^3}{4} \cdot sk_{\rm L} \cdot W_{\rm c}^2 \cos \phi \qquad \dots \qquad \dots \qquad (8)$$

Similarly the element of thrust power loss is (Ref. 5, $\S3$ (11))

 $d\mathbf{P}_1 = w_1 \, d\mathbf{T} \, \sec \phi$,

and its coefficient is given by

$$\frac{dk_{\mathbf{P}_{1}}}{d(x^{2})} = \frac{1}{2}w_{c}\frac{dk_{T}}{d(x^{2})}, \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad (9)$$

derivable from Ref. 5, §3, (17) and (33).

It has been found convenient to use the chart of w_c against sk_L (Chart 4, Fig. 5) in combination with equation (9), rather than to construct a chart of $dk_{\mathbf{P}_1}/d(x^2)$, because w_c (as well as $dk_T/d(x^2)$) is nearly linear in sk_L .

In order to obtain the torque and efficiency from the equations

$$k_{\mathbf{Q}} = (\mathbf{J} \ k_{\mathbf{T}}/2\pi) + k_{\mathbf{P}_1} + k_{\mathbf{P}_2} , \qquad \dots \qquad \dots \qquad (10)$$

$$1 - \eta = \frac{k_{\mathbf{P}_1} + k_{\mathbf{P}_2}}{k_{\mathbf{Q}}}, \qquad \dots \qquad \dots \qquad \dots \qquad (11)$$

(Ref. 5, $\S3$, (34) and (35)) it is necessary to obtain the coefficient of profile drag power loss at the blade element. This is given by the equation

$$dk_{\rm P_2}/d(x^2) = (\pi^3/8) \ sk_{\rm D} \ {\rm W_c}^3$$
 , ... (12)

derivable from Ref. 5, §3, (18) and (33). In equation (7) β is seldom greater than 10° and so W_c is practically a function of ϕ_0 or J only (for given x). A table or curve of $\frac{\pi^3}{8}$ W_c³ $\left(= \frac{1}{sk_{\rm D}} \cdot \frac{dk_{\rm P_2}}{d(x^6)} \right)$ as a function of J is therefore prepared by putting $\beta = 6^{\circ}$ say, in equation (5) giving, cos $\beta = 0.995$, (Fig. 6). A value of $sk_{\rm D}$ is determined from the profile drag coefficient of the blade section plotted against α (Chart 6, Fig. 7). If desired a chart of $dk_{\rm P_2}/d(x^2)$ against $sk_{\rm D}$ for given J may be prepared, the curves being straight lines.

An essential feature of the method is that the 3 charts 1, 3 and 4 (with curve 5) are of universal application for a given radius and number of blades, being independent of blade section, chord and blade angle. Charts 2 and 6 on the other hand involve the lift and drag coefficients of the particular blade section, with a constant multiplier for blade width but are not affected by the blade angle or the particular value of J. The combined charts cover all values of θ and s up to the limits of the scales of $sk_{\rm L}$ and ϕ .

It is proposed to calculate sets of Charts 1, 3 and 4 over suitable ranges of $sk_{\rm L}$ for radii x = 0.3, 0.45, 0.6, 0.7, 0.8, 0.9 and 0.95 and for 2, 3 and 4 bladed airscrews. The method will then determine $dk_{\rm T}/d(x^2), dk_{\rm P_1}/d(x^2), dk_{\rm P_2}/d(x^2)$ for a series of even values of J at all the standard radii, and it will then only be necessary to plot these three coefficients against x^2 and integrate graphically in order to obtain $k_{\rm T}$ and $k_{\rm P_1} + k_{\rm P_2}$. (See Ref. 5, Figs. 10, 11 and 12.) The torque coefficient and efficiency are then given by equations (10) and (11).

The effect of compressibility could be included in calculations by this method provided that the values of $k_{\rm L}$ and $k_{\rm D}$ for the sections, considered as functions of the ratio of the velocity to the velocity of sound, were known. Conversely the method would be very convenient for analysing observations of the type made at the R.A.E. on high tip speed airscrews, Ref. 6.

Charts for use by the above method are being calculated at the time of writing but have not yet been used. The method has only been used in conjunction with the further simplification of making calculations at a single radius and using integrating factors. This method will be described in the next section, being to some extent independent of the graphical method and having been used first in point of time.

3. Simplified Method of Calculation.—In order to save part of the labour of making detailed strip theory calculations, attempts have often been made in the past to use calculations at a single standard radius, and to determine the thrust and torque by means of integrating factors. It seemed worth while to attempt this method on the basis of the assumptions of Refs. 2 and 3. The use of charts of the type already described is specially convenient when

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working at a single standard radius. The method has the additional advantage that it can be used either *directly*, to calculate the thrust and torque of the airscrew from the lift and drag curves of the standard section, or *inversely* to deduce lift and drag curves of the standard section from the thrust and torque of the airscrew. The thrust and torque of a typical airscrew might be calculated by the detailed strip theory set forth in §2 and afterwards the inverse method for a standard section employed to deduce lift and drag curves.

It is evident that the success of the present method does not require that the fictitious lift and drag deduced in this way should agree closely with the true lift and drag of the section at the standard radius; it is sufficient if the same lift and drag curves are deduced from an analysis of screws of widely different pitch, solidity and plan form having the same blade section at the standard radius. As a first test of this method it is therefore convenient to use it to analyse the experiments on a family of model airscrews described in Ref. 4.

3.2. Inverse method.—When thrust grading curves are plotted on a basis of radius squared as in Ref. 5, Fig. 10, in accordance with \$2 (8), the curve approximates in form to a semi-circle or semiellipse for which the area would have the value

$$\frac{\pi}{4}$$
 × mid ordinate.

The mid ordinate corresponds to $x^2 = 0.5$ or x = 0.707 and the value x = 0.7 has been chosen for convenience. The procedure is otherwise precisely similar to that used in detailed strip theory calculations except that the formulae

$$k_{\rm T} = \int \{ dk_{\rm T}/d(x^2) \} d(x^2) \qquad \dots \qquad \dots \qquad \dots \qquad (1)$$

$$k_{\rm P_2} = \int \{ dk_{\rm P_2} / d(x^2) \} d(x^2) \quad \dots \quad \dots \quad \dots \quad (3)$$

are replaced by

$$k_{\rm T} = \frac{1}{4}\pi \, dk_{\rm T}/d(x^2) = (\pi^4/16) \, sk_{\rm L} \, {\rm W_c}^2 \cos \phi \, , \qquad \dots \qquad (4)$$

$$k_{\mathbf{P}_{1}} = \frac{1}{4}\pi \ dk_{\mathbf{P}_{1}}/d(x^{2}) = \frac{1}{4}\pi \ dk_{\mathbf{P}_{1}}/d(x^{2})$$
(5)

$$= (\pi^4/32) \ sk_{\rm D} \ W_{\rm c}^{3}. \qquad \dots \qquad \dots \qquad \dots \qquad (6)$$

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The chart of $sk_{\rm L}$ against ϕ for given J (Chart 1, Fig. 2) is constructed for radius 0.7, as well as the chart of $w_{\rm c}$ against $sk_{\rm L}$ (Chart 4); the chart of $dk_{\rm T}/d(x^2)$ and curve of $(1/sk_{\rm D}) \times (dk_{\rm P_2}/d(x^2))$, (Charts 3 and 5) are replaced by a chart of $k_{\rm T}$ against $sk_{\rm L}$ and a curve of $k_{\rm P_2}/sk_{\rm D}$ against J according to equations (4) and (6), (Figs. 4 and 6). The inverse of the method described in §2 is then employed to deduce curves of $k_{\rm L}$ and $k_{\rm D}$ against α from observed performance curves of an airscrew in the form of curves of $k_{\rm T}$ and $k_{\rm Q}$ against J. First read off $sk_{\rm L}$ from Fig. 4 for given J and $k_{\rm T}$. Next read off ϕ from Fig. 2 for given J and $sk_{\rm L}$ and deduce α from § 2(5). To obtain $k_{\rm D}$ determine $w_{\rm c}$ from Chart 4, Fig. 5, from the observed value of J and from the deduced value of $sk_{\rm L}$. Then obtain $k_{\rm P_2}$ from the relation

$$k_{\rm P,s} = k_{\rm Q} - {\rm J} \; k_{\rm T} / 2\pi - {1 \over 2} w_{\rm c} \; k_{\rm T}$$
 ,

using the observed values of $k_{\rm T}$, $k_{\rm Q}$ and J, and deduce $sk_{\rm D}$ from the curve of $k_{\rm P_9}/sk_{\rm D}$ against J in Fig. 5.*

The observed thrust and torque coefficients of the main series of two and four bladed model airscrews of varying pitch⁴ have been analysed in this way, and the values of $k_{\rm L}$ and $k_{\rm D}$ deduced from them are plotted in Figs. 8 and 9 for the two bladers. The results for the four bladers are similar, and the smoothed values for two and four blades as well as the mean of the values for two and four blades are recorded in Table 3.

The extent of the variation of the $k_{\rm L}$ and $k_{\rm D}$ curves with pitch for two bladed airscrews is shown in Figs. 8 and 9 and the results for two bladed and four bladed airscrews were found to be in reasonably good agreement with each other. For a range of pitch from 0.3 to 1.5 the agreement on $k_{\rm L}$ below the stall is good enough for practical purposes; at still higher pitch values there is a small but definite variation with pitch.

The $k_{\rm D}$ points appear to be more scattered than the $k_{\rm L}$ points but for the higher pitched screws the contribution of the drag to the torque is small; in the extreme case of the lowest $k_{\rm D}$ (P/D 2·5) shown in Fig. 9, the drag contributes only $3\frac{1}{2}$ per cent. to the torque observation from which it was deduced. Thus for a screw of this pitch the extreme variations in Fig. 9 would correspond to variations of only about 1 per cent. on torque. It has been verified by comparison with direct strip theory calculations that the tendency for the $k_{\rm L}$ values of the highest pitch airscrews (Fig. 8) to depart from the mean curve is due almost entirely to the assumption of a constant integrating factor, since the detailed strip theory results (Ref. 5) are in good agreement with experiment.

3.3. The Direct Method.—The direct application of the graphical method at a single standard radius now requires little further explanation.[†] The method described in Section 2 is followed in detail except that the chart of $dk_{\rm T}/d(x^2)$ against $sk_{\rm L}$ and the curve of $(1/sk_{\rm D}) \times dk_{\rm P_2}/d(x^2)$ against J (Figs. 4 and 6) are replaced by a chart of $k_{\rm T}$ against $sk_{\rm L}$ and a curve of $k_{\rm P_2}/sk_{\rm D}$ against J respectively according to equations (4) and (6) of §3.2.

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^{*} A detailed example is given in Appendix A.1.

[†] A detail calculation is given in the Appendix A.2.

The most useful course for the practical designer of airscrews who uses a definite set of blade sections will be to compute the thrust and torque of a typical screw (of P/D 1.5 say) by detailed strip theory below the stall (assuming that the lift and drag of the sections is known), and then to employ the *inverse* method of §3.2 to deduce standard $k_{\rm L}$ and $k_{\rm D}$ curves which could be used in conjunction with the *direct* method to calculate the performance of screws over a wide range of pitch and solidity. In the absence of data of this kind (and in any case above the stall) it is suggested that the values of $k_{\rm L}$ and $k_{\rm D}$, given in Table 3, which were deduced from the model airscrew experiments of Ref. 4, should be used for this purpose.

In order to secure the greatest possible economy of time in calculating k_{Q} it would be desirable to construct a chart of :—

$$(\operatorname{J} k_{\mathrm{T}}/2\pi + \frac{1}{2}w_{\mathrm{c}} k_{\mathrm{T}})$$

against sk_L . Then k_{P_2} is deduced from the curve of k_{P_2}/sk_D against J, and k_Q is given by

$$k_{\mathbf{Q}} = (J k_{\mathbf{T}}/2\pi + \frac{1}{2}w_{c} k_{\mathbf{T}}) + k_{\mathbf{P}_{2}} \qquad \dots \qquad (1)$$

It is reasonable to assume that the results are valid over a large range of values of s (solidity ratio) and for a complete range of angles of *pitch* (to the degree of accuracy indicated by Figs. 8 and 9). A change of blade twist or of plan form might affect the lift and drag curves obtained from analysis of airscrew performance in so far as these curves differ from the actual lift and drag curves of the section at radius 0.7 (Fig. 10); the required alteration might be calculated on the basis of detailed strip theory, or a model test of a typical airscrew. The $k_{\rm L}$ and $k_{\rm D}$ values of Table 3 (mean of 2 and 4 blades) are compared in Fig. 10 with the measured lift and drag curves of the section at radius 0.7 (deduced by interpolation from Table 3 of R. & M. 892¹). The lift curves are in good agreement near zero lift but there is an appreciable difference at high lifts and a similar disagreement on drag in the high lift region. The values of maximum lift and minimum drag are however in reasonably good agreement. The effect of change of *blade section* should be allowed for by alteration of the lift and drag curves either directly on the basis of a test of the aerofoil section at a radius 0.7 or of a calculation by the theory of thin aerofoils (Fig. 10) for the lift, and by estimation for the drag.

4. Tables for constructing charts.—Tables 1 and 2 give values of the quantities ϕ , $s_{k_{\rm L}}$, $w_{\rm c}$, $k_{\rm T}$, as functions of J and β , and of $k_{\rm P_2}/sk_{\rm D}$, as a function of J, (for $\beta = 6^{\circ}$ only) for radius x = 0.7. These quantities are directly calculated* by means of formulae (2), (3), (4), (6) and (7) of §2 and (4) and (6) of §3.2, combined with values of \varkappa derived from Ref. 5, Figs. 3, 4, 5 and 6. Of these quantities, ϕ , $w_{\rm c}$, and $k_{\rm P_2}$ are independent of number of blades for given β so that

:

^{*} It is unnecessary to calculate every entry in the table directly, as intermediate values may be filled in by plotting the figures in any column against J.

only $sk_{\rm L}$ and $k_{\rm T}$ have to be repeated when N varies. Results for 2, 3 and 4 blades are given in Table 1 while values of $sk_{\rm L}$ and $k_{\rm T}$ for 6 blades and infinity blades are given separately in Table 2 as they are less often required. Charts 1, 3, 4 and 5 can be constructed by direct plotting from these tables, and it seemed preferable to publish the tables rather than to attempt to reproduce the charts themselves on a sufficiently large scale.

5. Further Simplification for the Torque.—In so far as the contribution of the drag to the torque is fairly small below the stall, especially for high pitch airscrews, it might be worth while to assume the drag coefficient to be independent of incidence; on this assumption k_{Q} is a function of sk_{L} , J and s only, which varies only slowly with s so that a limited number of charts for different values of s would determine values of k_{0} directly without the need of using Figs. 6 and 7 and equation (10) of $\S 2$ or (1) of $\S 3.3$. This further simplification suggests the possibility of using the method to analyse full scale airscrew data in which the forward speed, rotational speed and engine power only are known. The best way to do this would be to estimate the profile drag power from curves such as Figs. 2–7 already available, and then to use the observed torque power less profile drag power, to calculate a point on a curve of $sk_{\rm L}$ against α , precisely as was done in $\S3.2$ on the basis of observed thrust coefficient.

6. Stalled Range.—The simplified method of \$3 has been applied successfully to a stalled airscrew, with the single modification that it has been necessary to replace equation \$2 (8) or \$3.2 (4) by the more exact equations

$$\frac{dk_{\rm T}/d(x^2) = (\pi^3/4) \ s \ W_{\rm c}^2 \ (k_{\rm L} \cos \phi - k_{\rm D} \sin \phi)}{k_{\rm T} = (\pi^4/16) \ s \ W_{\rm c}^2 \ (k_{\rm L} \cos \phi - k_{\rm D} \sin \phi)} \qquad \dots \qquad (1)$$

(Ref. 5, p. 8, equation 24). According to this equation the method of analysis described in §3.2 determines the value of

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$$sk_{L_0} = sk_L - sk_D \tan \phi$$
 (definition of k_{L_0}) .. (2)

and also values of $sk_{\rm D}$ and of ϕ , so that the true value of $sk_{\rm L}$ can at once be calculated. In Fig. 8, all points for which α is greater than 10° include this correction and it is evident that results above the stall deduced from airscrews of varying pitch are reasonably consistent. In the direct application of the method the value of ϕ is at first unknown, and it would be necessary to proceed by successive approximation. The method is described in detail in Appendix A.3 which shows that the additional labour required is quite small.

It is important to emphasize that beyond the stall the results of detailed strip theory calculations based on aerofoil data at present available have been found to be unreliable, and that it is essential to use the lift and drag coefficients for the standard sections (such as those of Table 3) derived from tests of actual airscrews. 7. Conclusions.—The simplifications of strip theory calculations described in the present report have arisen naturally from two modifications of the standard formulae as described in R. & M. 1377².

(1) The omission of the profile drag coefficient in calculating the interference velocity; this made the graphical method possible.

(2) The use of thrust grading curves plotted on a base of the square of the radius instead of the first power. This suggested the possibility of obtaining reasonably accurate results by making calculations at a radius 0.7 and using integrating factors.

A very rapid method of computing airscrew performance has been developed which gives good accuracy up to P/D 1.0 or 1.5 and fair accuracy up to P/D 2.5.

It would be possible to include the effect of compressibility of the air in calculations by the approximate method of §3 but the accuracy obtainable is doubtful because the effect falls off rapidly on proceeding inwards from the blade tip. The possible accuracy could be determined by comparison with detailed strip theory calculations in which the effect of compressibility was included; for this purpose the graphical method as described in §2 would be convenient.

APPENDIX

A.1. Detailed calculation by inverse method (§3.2).—Analysis of observed performance on a model airscrew. Airscrew 2 blades, standard section and width, P/D 1.5 (normal blade angles). Observed data (from Ref. 4, Table 5, p. 18).

			1	1		ļ				1
T	0.19	0.30	0.40	0.60	0.80	1.00	$1 \cdot 20$	1.40	1.60	1.76
Řт	0.1310	0.1330	0.1340	0.1330	0.1265	0.1085	0.0855	0.0590	0.0285	ĥ.
ko	0.0244	0.0227	0.0218	0.0214	0.0220	0.0213_{-}	0.0189-	0.0149-	0.0091-	0.0030
		• • • • • •	0410	0 0		0 0 105	0.0005	0 01105	0 00015	0.0000

Blade angle $\theta = 34^{\circ} \, 19'$, s = 0.0705, at x = 0.7.

Details of calculation for J = 0.30.

From Chart 3, for $k_{\rm T} = 0.1310$, J = 0.30, read off $sk_{\rm L_0} = 0.0457$.

From Chart 1, for $sk_{L_0} = 0.0457$, J = 0.30, read off $\phi = 14^{\circ} 6'$.

Then $\alpha = \theta - \phi = 20^{\circ} 13'$.

From observed values, work out $k_{\rm Q} - \frac{Jk_{\rm T}}{2\pi}$ (total power loss) = 0.0163₅.

From Chart 4, for $sk_{\rm L_0} = 0.0457$, J = 0.30, read off $w_{\rm c} = 0.0799$ and calculate $k_{\rm P_1} = \frac{1}{2}w_{\rm c} k_{\rm T} = 0.0053$.

Then $k_{\mathbf{P}_2} = k_{\mathbf{Q}} - Jk_{\mathbf{T}}/2\pi - k_{\mathbf{P}_1} = 0.0110_5$.

From curve of Chart 5, $k_{P_2}/sk_D = 1.054$, so that $sk_D = 0.01048$, $sk_D \tan \phi = 0.00263$, sk_L (accepted value) = $sk_{L_0} + sk_D \tan \phi^* = 0.0483$.

Hence for $\alpha = 20^{\circ} 13'$, $k_{\rm L} = 0.686$, $k_{\rm D} = 0.149$.

The complete results are given in the following Table and plotted in Figs. 8 and 9.

J	0.19	0.30	0.40	0.60	0.80	1.00	$1 \cdot 20$	$1 \cdot 40$	$1 \cdot 60$	1.76
α ==	$22^{\circ} 14'$	$20^{\circ} 13'$	17° 56'	$14^{\circ} 4'$	9° 59'	6° 20'	3° 9′	0° 7′	-2° 29′	$-4^{\circ} 27'$
$k_{\mathbf{L}} \sim$	0.687	0.686	0.683	0.651	0.587	0.486	0.369	0.242	0.111	0
$k_{\mathbf{D}} = $	0.201	0.149	$0 \cdot 109$	0.051	0.022	0.011_{5}	0.007	0.007_{5}	0.012	0.020

A.2. Detailed calculation by direct method (§3.3) (below the stall).— Calculation of performance of an airscrew having 3 blades, solidity 0.100, P/D 1.1. The data used are the values of $k_{\rm L}$ and $k_{\rm D}$ given in Table 3 derived from analysis of the model tests of Ref. 4. Results therefore apply strictly to a screw having the same section and plan form as those models.

Blade angle at 0.7 is 0.7 π tan $\theta = 1.1$, $\theta = 26.6^{\circ}$.

Details of the calculation for J = 0.8. As explained in Section 3.3 the transparent Chart 1 (for 3 blades x = 0.7) of sk_L plotted against ϕ is superposed on Chart 2 (or the single curve of sk_L against α for s = 0.1) so that the axis of ϕ in Chart 1 coincides with the axis of α of Chart 2 while the zero of the scale of Chart 2 coincides with the point $\phi = 26.6^{\circ}$, equal to the known value of θ . Then the value of sk_L for any value of J may be read off without shifting the charts. For J = 0.8 the point of intersection gives $sk_L = 0.0370$, $\phi = 23.4^{\circ}$, $\alpha = 3.2^{\circ}$; these values satisfy the relation $\phi = \theta - \alpha$. Next the value of $k_T = 0.1115$ is read from Chart 3 for J = 0.8, $sk_L = 0.0370$.

^{*} This term may be omitted below the stall, i.e., for values of α less than 10°.

From Chart 4 for $sk_{\rm L} = 0.0370$, J = 0.8, read of $f_2^1 w_{\rm c} = 0.0232$ and so $k_{\rm P_1} = \frac{1}{2} w_{\rm c} k_{\rm T} = 0.0026$.

From Table 1 or Chart 5, for J = 0.8, $k_{P_0}/sk_D = 1.240$.

From Chart 6 for $\alpha = 3 \cdot 2^{\circ}$, $sk_{\rm D} = 0 \cdot 0009$ and so $k_{\rm P_2} = 0 \cdot 0011$.

Finally $k_{\mathbf{Q}} = \int k_{\mathbf{T}}/2\pi + k_{\mathbf{P}_1} + k_{\mathbf{P}_2} = 0.0146 + 0.0026 + 0.0011 = 0.0183.$

The calculation may be slightly simplified by using a chart of $(J k_T/2\pi + k_{P_1})$ against sk_L in place of Chart 4 (w_c against sk_L).

Results for the complete working range are given in the following Table.

J	$1 \cdot 3$	1.2	1 · 1	1.0	0.8	0.6	0.4	$0\cdot 2$	0
sk L	0.0033	0.0105	0.0178	0.0241	0.0370	0.0490	0.0587	0.0653	0.0665
ϕ	$30 \cdot 9$	$29 \cdot 5$	$28 \cdot 0$	26.5	23.4	19.9	16.6	13.6	10.6
α	 4·3	-2.9	<u>-1·4</u>	-0·1	3.2	6.7	10.0	13.0	16.0
$sk_{\mathbf{D}}$	0.0020	0 •0013	0.0010	0.0009	0.009	0.0015	0.0025	0.0045	0.0085
$k_{\mathbf{T}}$	0.0115	0.0340	0.0590	0.0780	0.1115	0.1455	0 · 1705	0 · 1872	0.1875
k q	0.0056	0.0087	0.0124	0.0148	0.0183	0.0202	0.0205	0.0204	0.0209

For J = 0.2 and 0 the calculations have been repeated by the method of the next section A.3 appropriate to the range beyond the stall; the modified results are

J	sk _L	sk _{I,0}	φ	α	sk _D	k_{T}	kq
$0\cdot 2$ 0	0·0654 0·0666	0.0643 0.0650	$13 \cdot 4^{\circ}$ $10 \cdot 4^{\circ}$	13·2° 16·2°	0.0048 0.0088	0 · 1840 0 · 1845	$\begin{array}{c} 0 \cdot 0205 \\ 0 \cdot 0209 \end{array}$

The effect of the correction for stalling is evidently small for a screw of this pitch value.

A.3. Direct calculation above the stall.—Owing to the necessity of taking account of the term $sk_D \tan \phi$ in §6 (2) it is necessary to proceed by successive approximation, but the convergence of the process will be found to be so rapid that very little extra labour is required.

Beyond the stall, $sk_{\rm L}$ is a function of α (Chart 2) which varies slowly with α while $sk_{\rm L_0}$ is a function of ϕ (Chart 1) such that ϕ varies slowly with $sk_{\rm L_0}$; $sk_{\rm L}$ and $sk_{\rm L_0}$ satisfy the relation §6 (2) :---

$$sk_{L_0} = sk_L - sk_D \tan \phi$$

These considerations suggest the process illustrated by the following example. The essential point is that (after the first step) Chart 1 is used to deduce ϕ from sk_{L_0} and Chart 2 is used to deduce $\underline{sk_L}$ from α throughout the process.

Airscrew:—standard 2 blader, P/D 1.8; $\theta = 39.33^{\circ}$ and s = 0.0705 at x = 0.7. The mean values of $k_{\rm L}$ and $k_{\rm D}$ from Table 3 are used in constructing Charts 2 and 6. The numerical values refer to J = 0.3.

First Approximation, determine ϕ (14.4°), α (24.93°) and $\underline{sk_{L}}$ (0.0487), (not $sk_{L_{0}}$) by the use of Charts 1 and 2 exactly as described in A.2; deduce sk_{D} (0.0173) from α by Chart 6 and obtain sk_{D} tan ϕ (0.0044). Then:

Second Approximation

 $sk_{L_0} = sk_L - sk_D \tan \phi = 0.0443.$ Deduce $\phi (14.0^\circ)$ from sk_{L_0} using Chart 1. Deduce $\alpha (25.33^\circ)$ from the relation $\alpha = \theta - \phi$. Deduce $sk_D (0.0179)$ from α using Chart 6. $sk_D \tan \phi = 0.0044_5$ $sk_L (0.0485)$ from α using Chart 2.

Third approximation.

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05 n $sk_{\mathbf{L}\mathbf{0}} = sk_{\mathbf{L}} - sk_{\mathbf{D}} \tan \phi = 0.0440_5.$

A repetition of the process then gives $\phi = 13.95^{\circ}$, $\alpha = 25.38^{\circ}$, $sk_{\rm D} = 0.0179_{\rm s}$, $sk_{\rm L} \tan \phi = 0.00445$, $sk_{\rm L} = 0.0485$.

The subsequent operations are precisely the same as in A.2. using sk_{L_0} and sk_{D} .

It is evident that the values given by the second approximation $(sk_{L_0} = 0.0440_5)$, $sk_D = 0.0179$ are accurate enough for all purposes, while for most purposes the first approximation to sk_{L_0} (0.0443) is sufficiently accurate. The complete results of the calculation for a series of values of J are given in the following Table.

J	0.3	$0\cdot 4$	0.6	0.8	1.0
ϕ	$14 \cdot 0^{\circ}$	16·0°	$20 \cdot 2^{\circ}$	24 · 7°	28.6°
α	25·33°	23·33°	19·13°	$14 \cdot 63^{\circ}$	10·73°
skl ₀	0.0440	0.0438	$0 \cdot 0440$	$0 \cdot 0444$	0.0415
$sk_{\mathbf{L}}$	0.0485	0.0481	0.0475	0.0465	0.0426
sk _D	0.0179	0.0151	0.0096	$0 \cdot 0046$	0.0021
$k_{\mathbf{T}}$	0.1280	0.1282	0 · 1313	0.1355	$0 \cdot 1305$
kQ	0.0318	0.0290	$0 \cdot 0279$	0.0274	0.0280
	1				

s. Also of		3lades	Lγ	$0 \cdot 0071$	0.0168	0.0262	0.0350	0.0443	0.0530	0.0604	0 • 0677
t of blace		4	Sk _L	0.0024	0.0056 0.0056 0.0071	0.0088	0.0104 0.0116 0.0132	0.0149	0.0173	0.0196	0.020/0.0216
y number		ades	ΤĄ	$0 \cdot 0071$	0.0168	$0 \cdot 0257$	$0 \cdot 0340$	$0 \cdot 0429$	0.0508	$0 \cdot 0576$	0.0637
··7 and an		3 Bl	Чłs	0.0024	$0.0071 \\ 0.0071 \\ 0.0071$	0.0086	$0.0101 \\ 0.0115 \\ 0.0129$	0.0142	0.0167	0.0187	0.0204
for $x = 0$	2	ades	μtr	0.071	0.0121 0.0168 0.0214	0.0253	0.0330 0.0365 0.0365	0.0396	0.0420 0.0452 0.0476	0.0497	0.0540
I and β blades.		2 Bl	rf <i>h</i> s	0.0024	0.0056 0.0056 0.0071	0.0085	0.0110 0.0121	0.0131	$0.0140 0 \cdot 0148 0 \cdot 0155$	0.0161	0.0173
values of 2, 3 and 4			ŵe	0.0244	0.0245 0.0245 0.0246	0.0247	$0.0250 \\ 0.0250 \\ 0.0252$	0.0254	0.023/0.0259 0.0262	0.0265	$0.0209 \\ 0.0272 \\ 0.076$
° for given and $k_{\rm T}$ for			~ @.	5° 10′ 5° 10′ 5° 10′	5 24 5 36 5 4	7° 12' oc 90'	9° 46′ 9° 46′ 11° 3′	12° 18' 19° 91'	13 34 14° 48' 16° 3'	17° 15′ 10° 07′	10° 20' 19° 39' 90° 49'
ϕ and w ulso of $sk_{ m L}$	(9°)		$\frac{k_{\rm P_2}}{sk_{\rm D}}$	$1 \cdot 026$	$1 \cdot 030$	$1 \cdot () \frac{1}{2}$	1.054	$1 \cdot 074$	1 · 106	1.148	1 • 190
f values of $\beta = 6^{\circ}$; a	°.O		ϕ_0	0 1° 10 10	3° 54 3° 54 3° 54	5° 12′ 8° 00′	9° 46 9° 33	10° 18' 11° 24'	11 12° 48' 14° 3	15° 15'	17° 39' 17° 39' 18° 49'
Table of $k_{\mathbf{P}_2}/sk_{\mathrm{D}}$ for	Br.	No. of blades	, <u> </u>	0.05	0.12 0.12	0.2 0.05	$0.35 \\ 0.35 \\ 0.35$	1 + · 0	$\begin{array}{c} 0.55\\ 0.55\\ \end{array}$	9·0	60.0 7.0 27.0

TABLE 1

0.0742 0.0809	$\begin{array}{c} 0.0867\\ 0.0909\\ 0.0955_{5}\\ 0.0997\end{array}$	$\begin{array}{c} 0\cdot 1041\\ 0\cdot 1086_5\\ 0\cdot 1130\\ 0\cdot 1172\end{array}$	0.1214 0.1250 0.1296 0.1333	$\begin{array}{c} 0 \cdot 1374 \\ 0 \cdot 1411 \\ 0 \cdot 1452 \\ 0 \cdot 1492 \end{array}$	$\begin{array}{c} 0\cdot 1539\\ 0\cdot 1575\\ 0\cdot 1608\\ 0\cdot 1646\\ 0\cdot 1693\end{array}$
$\begin{array}{c} 0{\cdot}0234\ 0{\cdot}0243\ 0{\cdot}0243\ 0{\cdot}0259\ 0{\cdot}0259\end{array}$	$\begin{array}{c} 0\cdot0265\ 0\cdot0277\ 0\cdot0287\ 0\cdot0287\ 0\cdot0295\end{array}$	$\begin{array}{c} 0 \cdot 0301 \\ 0 \cdot 0308 \\ 0 \cdot 0314 \\ 0 \cdot 0319 \end{array}$	$\begin{array}{c} 0\cdot0.324\ 0\cdot0.327\ 0\cdot0.331\ 0\cdot0.335\ 0\cdot0.335\end{array}$	$\begin{array}{c} 0\cdot0337\ 0\cdot0340\ 0\cdot0343\ 0\cdot0345\ 0\cdot0345 \end{array}$	$\begin{array}{c} 0\cdot 0347\ 0\cdot 0349\ 0\cdot 0349\ 0\cdot 0350\ 0\cdot 0352\ 0\cdot 0352\ 0\cdot 0354\end{array}$
0.0687 0.0732	$\begin{array}{c} 0 \cdot 0771 \\ 0 \cdot 0807 \\ 0 \cdot 0843 \\ 0 \cdot 0879 \end{array}$	$\begin{array}{c} 0\cdot0913\\ 0\cdot0946\\ 0\cdot0982\\ 0\cdot1017\end{array}$	$0 \cdot 1047$ $0 \cdot 1078$ $0 \cdot 1113$ $0 \cdot 1113$	$\begin{array}{c} 0 \cdot 1176 \\ 0 \cdot 1206 \\ 0 \cdot 1238 \\ 0 \cdot 1271 \end{array}$	$\begin{array}{c} 0\cdot 1301\\ 0\cdot 1333\\ 0\cdot 1333\\ 0\cdot 1363\\ 0\cdot 1394\\ 0\cdot 1428\end{array}$
$\begin{array}{c} 0 \cdot 0217 \\ 0 \cdot 0223 \\ 0 \cdot 0229 \\ 0 \cdot 0235 \end{array}$	$0.0239 \\ 0.0247 \\ 0.0254 \\ 0.0261 \\ 0.0261$	$\begin{array}{c} 0 \cdot 0265 \\ 0 \cdot 0270 \\ 0 \cdot 0274 \\ 0 \cdot 0278 \end{array}$	$\begin{array}{c} 0 \cdot 0280 \\ 0 \cdot 0283 \\ 0 \cdot 0283 \\ 0 \cdot 0283 \\ 0 \cdot 0288 \end{array}$	$0 \cdot 0290 \\ 0 \cdot 0291 \\ 0 \cdot 0293 \\ 0 \cdot 0294 $	0.0296 0.0297 0.0298 0.0299 0.0299
$\begin{array}{c} 0\cdot0572\\ 0\cdot0588\\ 0\cdot0603\\ 0\cdot0614\end{array}$	$\begin{array}{c} 0 \cdot 0623 \\ 0 \cdot 0635 \\ 0 \cdot 0655 \\ 0 \cdot 0677 \end{array}$	$\begin{array}{c} 0 \cdot 0696 \\ 0 \cdot 0720 \\ 0 \cdot 0744 \\ 0 \cdot 0768 \end{array}$	$\begin{array}{c} 0 \cdot 0791 \\ 0 \cdot 0811 \\ 0 \cdot 0838 \\ 0 \cdot 0858 \end{array}$	$\begin{array}{c} 0\cdot0.882\ 0\cdot0.906\ 0\cdot0.930\ 0\cdot0.930\ 0\cdot0.954 \end{array}$	$\begin{array}{c} 0 \cdot 0975 \\ 0 \cdot 1001 \\ 0 \cdot 1015 \\ 0 \cdot 1039 \\ 0 \cdot 1064 \end{array}$
$\begin{array}{c} 0 \cdot 0181 \\ 0 \cdot 0184 \\ 0 \cdot 0187 \\ 0 \cdot 0190 \end{array}$	$\begin{array}{c} 0 \cdot 0191 \\ 0 \cdot 0194 \\ 0 \cdot 0199 \\ 0 \cdot 0200 \end{array}$	$\begin{array}{c} 0\cdot0202\\ 0\cdot0205\\ 0\cdot0207\\ 0\cdot0210\end{array}$	$\begin{array}{c} 0\cdot 0212\\ 0\cdot 0213\\ 0\cdot 0215\\ 0\cdot 0215\end{array}$	$\begin{array}{c} 0{\cdot}0217\ 0{\cdot}0219\ 0{\cdot}0219\ 0{\cdot}0220\ 0{\cdot}0221\ 0{\cdot}0221\end{array}$	$\begin{array}{c} 0\cdot0221\ 0\cdot0222\ 0\cdot0222\ 0\cdot02223\ 0\cdot02223\ 0\cdot02223\ 0\cdot0223\end{array}$
$0.0280 \\ 0.0285 \\ 0.0290 \\ 0.0294$	$\begin{array}{c} 0 \cdot 0300 \\ 0 \cdot 0311 \\ 0 \cdot 0323 \\ 0 \cdot 0337 \end{array}$	0.0351 0.0367 0.0384 0.0384 0.0401	0.0420 0.0440 0.0461 0.0481	$\begin{array}{c} 0.0507 \\ 0.0531 \\ 0.0557 \\ 0.0584 \end{array}$	$\begin{array}{c} 0 \cdot 0611 \\ 0 \cdot 0641 \\ 0 \cdot 0670 \\ 0 \cdot 0705 \\ 0 \cdot 0734 \end{array}$
$\begin{array}{c} 21^{\circ} \\ 23^{\circ} \\ 24^{\circ} \\ 15^{\circ} \\ 25^{\circ} \\ 20^{\circ} \end{array}$	26° 27′ 28° 34′ 30° 37′ 32° 36′	$\begin{array}{c} 34 \\ 36 \\ 36 \\ 38 \\ 39 \\ 42 \\ \end{array}$	41° 19' 42° 50' 44° 17' 45° 41'	$\begin{array}{c} 47^{\circ} & 0' \\ 48^{\circ} & 17' \\ 49^{\circ} & 31' \\ 50^{\circ} & 40' \end{array}$	51° 46' 52° 51' 54° 49' 55° 44'
1 · 240 1 · 294	$\begin{array}{c} 1\cdot 362 \\ 1\cdot 438 \\ 1\cdot 522 \\ 1\cdot 614 \end{array}$	1.710 1.820 1.936 2.062	$2 \cdot 210$ $2 \cdot 364$ $2 \cdot 536$ $2 \cdot 724$	$\begin{array}{c} 2\cdot 910\\ 3\cdot 108\\ 3\cdot 328\\ 3\cdot 566\end{array}$	$3 \cdot 826$ $4 \cdot 094$ $4 \cdot 368$ $4 \cdot 662$ $4 \cdot 964$
$\begin{array}{c} 19^{\circ} & 59^{\circ} \\ 21^{\circ} & 7^{\circ} \\ 22^{\circ} & 15^{\circ} \\ 23^{\circ} & 20^{\circ} \end{array}$	$\begin{array}{c} 24^{\circ} & 27'\\ 26^{\circ} & 34'\\ 28^{\circ} & 37'\\ 30^{\circ} & 36' \end{array}$	$\begin{array}{c} 32^{\circ} & 29^{\circ} \\ 34^{\circ} & 18^{\circ} \\ 36^{\circ} & 3^{\circ} \\ 37^{\circ} & 42^{\circ} \end{array}$	$39^{\circ} 19'$ $40^{\circ} 50'$ $42^{\circ} 17'$ $43^{\circ} 41'$	45° 0′ 46° 17′ 47° 31′ 48° 40′	49° 46′ 50° 51′ 52° 49′ 53° 44′
$\begin{array}{c} 0.8 \\ 0.9 \\ 0.9 \\ 0.9 \end{array}$	$\dot{0}$	+ ič & i	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	00000 00 + 10	01010100 010100 010100

 $\frac{1}{20^{\circ}}$, $\frac{1}{49}$, 0.0276 0.0111

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 $\frac{17^{\circ}}{18^{\circ}}$ $\frac{39'}{49'}$

1continued
TABLE

 $\beta = 4^{\circ}$

ades	ЪТ	$0 \cdot 0283$	$0 \cdot 0465$	$0 \cdot 0652$	0.0830	$0 \cdot 0989$	0.1151	$0 \cdot 1283$	$0 \cdot 1400$	$0 \cdot 1533$	$0 \cdot 1638$
4 Bl	skL	0.0096	0.0157	0.0221	$0.0249 \\ 0.0280 \\ 0.0280$	0.0310 0.0333 0.0360	0.0384	0.0407 0.0427	0.0440	0.0482 0.0499	0.0528
ades	$\mathbf{L} \boldsymbol{y}$	$0 \cdot 0283$	$0 \cdot 0463$	$0 \cdot 0640$	0.0809	0.0955	$0 \cdot 1101$	$0 \cdot 1210$	$0 \cdot 1309$	0 • 1397	0.1490
3 Bl	skı	0.0096	0.0129 0.0157	$0.0189 \\ 0.0217$	$0.0246 \\ 0.0274$	0.0298 0.0323 0.0346	0.0368	0.0387 0.0403	0.0418 0.0432	$0.0444 \\ 0.0456$	0.0468 0.0477
ades	Тų	0.0283	0.0381 0.0463	0.0545 0.0619	0.0689 0.0752	$0.0810 \\ 0.0858 \\ 0.0907$	0.0951	$\begin{array}{c} 0\cdot 0986\\ 0\cdot 1023\end{array}$	$\begin{array}{c} 0\cdot 1060\\ 0\cdot 1093\end{array}$	$0 \cdot 1120 \\ 0 \cdot 1145$	0.1163 0.1184
2 B1	Ч	0 • 0096	0.0129 0.0157	0.0185 0.0210	0.0233	0.0290 0.0290 0.0305	0.0319	0.0330 0.0330	0.0351 0.0361	0.0368 0.0374	0.0378 0.0382
	26'c	0670.0	0.0491	0.0497	0.0500	0.0504 0.0508 0.0512 0.0512	0100.0	0.0530	0.0544 0.0552	0.0560	0.0579 0.0589
	Þ	0	5° 18′	0° 30 7° 54' 9° 12'	10° 29'	$11^{\circ} + 46'$ $13^{\circ} - 3'$ $14^{\circ} - 18'$	15 34	16° 48′ 18° 33′	$ \begin{array}{c} 1.9 \\ 20^{\circ} 27' \\ 21^{\circ} 39' \end{array} $	22° 49' 00° 60'	25° 7′ 26° 15′
			$0 \\ 0.05$	$\begin{array}{c} 0.1\\ 0.15\\ 0.9\end{array}$	0.25	$\begin{array}{c} 0\cdot 3\\ 0\cdot 35\\ 0\cdot 4\end{array}$	0.45	0.5 0.55	$\begin{array}{c} 0.6\\ 0.65\\ 0.7\end{array}$	0.75	0.85 0.985

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0.95

$0.1742 \\ 0.1834$	0.1915	$0 \cdot 1996$	$0 \cdot 2063$	0.2149	0.2230	0.2306	0.2378	0.2453	0.2520	0.2592	0.2659	0.2735	$0 \cdot 2807$	0.2875	0.2952	0.3020	0.3094	0.3159	0.3228
$0.0541 \\ 0.0554 \\ 0.0574$	0.0591	0.0606	0.0618	0.0630	0.0641	0.0651	0.0660	0.0667	0.0673	0.0678	0.0684	0.0688	0.0693	0.0698	$0 \cdot 0702$	0.0705	0.0709	0.0711	0.0714
$0.1562 \\ 0.1622$	0.1687	$0 \cdot 1747$	0.1800	0.1870	$0 \cdot 1932$	$0 \cdot 1988$	0.2044	0.2106	0.2156	0.2213	0.2273	$0 \cdot 2332$	$0 \cdot 2390$	0.2438	0.2496	0.2555	0.2613	$0\cdot 2662$	0.2717
0.0486 0.0493 0.0509	0.0522	0.0532	0.0541	0.0550	0.0557	0.0563	0.0569	0.0574	0.0578	0.0581	0.0586	0.0589	0.0592	0.0594	0.0595	0.0598	0.0600	0.0601	0.0603
0.1201 0.1220 0.1263	0.1296	0.1335	0.1372	0.1414	$0 \cdot 1460$	$0 \cdot 1498$	$0 \cdot 1542$	$0 \cdot 1583$	0.1621	0.1661	0.1705	$0 \cdot 1743$	0.1785	$0 \cdot 1827$	0.1867	0.1907	0.1944	$0 \cdot 1989$	0.2027
0.0386 0.0390 0.0396	0.0401	$0 \cdot 0407$	0.0411	0.0416	0.0421	0.0425	$0 \cdot 0428$	$0 \cdot 0432$	0.0435	0.0437	0.0440	$0 \cdot 0442$	0.0443	0.0445	0.0446	0.0447	0.0447	0.0448	0.0449
$0.0599 \\ 0.0611 \\ 0.0634$	0.0661	$0 \cdot 0690$	0.0722	0.0754	0.0790	0.0827	0.0868	0.0911	0.0956	0.1004	$0 \cdot 1053$	0.1106	0.1163	0.1220	$0 \cdot 1280$	0.1345	0.1408	$0 \cdot 1477$	0.1547
27° 20' 28^{\circ} 27' 30^{\circ} 31'	32° 37′	34° 36'	36°29′	$38^\circ 18'$	40° 3′	41° 42′	43° 19′	44° 50'	46° 17′	47° 41'	49° 0'	50° 17'	51° 31′	52° 40′	52° 46'	54° 57'	55° 50'	56° 49′	57° 44'
0.95	1.2	10.	1.4	1.5	1.6	1.7	1.8	6.1	2.0	5.1	5.2	5.3	2.4	2.5	5. <u>6</u>	2.7	2.8	2.9	3.0

26° 15′ 0.0589 0.0382

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 $6 \cdot 0$

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TABLE

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 $\beta = 6^{\circ}$

			2 BI	lades	3 Bl	ades	4 Bl	ades
ſ	Þ	wc	Т¥S	Lų	Т¥s	k_{T}	skt	k_{T}
0	00	0.0736	0.0216	0.0638	0.0217	$0 \cdot 0638$	$0 \cdot 0217$	0.0638
0.05	7° 18' 0° 96'	0.0738	0.0258	0.0753	0.0263	0.0901	0.0268 0.0311	0.0910
0.15	00 00 00 54'	0.0744	0.0333	0.0969	0.0341		0.0348	
0.2	11° 12′	0.0749	0.0368	0.1071	0.0392	0.1140	0.0400	$0 \cdot 1168$
$0\cdot 25$	12° 29′	0.0754	$0 \cdot 0399$	$0 \cdot 1162$	$0 \cdot 0432$		$0 \cdot 0445$	
0.3	13° 46′	0.0760	0.0426	0.1241	0.0473	0.1378	$() \cdot 0486$	0.1418
0.35	15° 3′	$0 \cdot 0767$	0.0451	0.1300	0.0508		0.0525	
0.4	16° 18′	$0 \cdot 0775$	$0 \cdot 0471$	$0 \cdot 1375$	0.0541	$0 \cdot 1578$	0.0563	$0 \cdot 1649$
0.45	17° 34′	0.0783	0.0491	$0 \cdot 1438$	0.0572		0.0600	
0.5	18° 48′	0.0793	0.0507	$0 \cdot 1485$	$0 \cdot 0597$	$0 \cdot 1748$	$0 \cdot 0634$	$0 \cdot 1861$
0.55	$20^{\circ} 3'$	0.0803	0.0523	$0 \cdot 1537$	0.0622		0.0659	
0.6	21° 15'	0.0814	0.0538	0.1582	0.0643	$0 \cdot 1892$	0.0691	$0\cdot 2040$
0.65	$22^{\circ} 27'$	$0 \cdot 0825$	0.0549	$0 \cdot 1623$	$0 \cdot 0663$		$0 \cdot 0717$	
$\overline{0.7}$	$23^{\circ} 39'$	0.0838	0.0559	$0 \cdot 1660$	0.0681	0.2060	$0 \cdot 0745$	$0 \cdot 2220$
0.75	24° 49′	$0 \cdot 0852$	0.0566	0.1690	$0 \cdot 0699$		$0 \cdot 0765$	
0.8 0	25° 59'	0.0866	0.0575	$0 \cdot 1722$	$0 \cdot 0714$	0.2138	0.0789	0.2371
0.85	27° 7'	0.0871	0.0580	0.1743	0.0729		0.0811	
6.0	28° 15'	0.0897	0.0584	$0 \cdot 1766$	0.0741	0.2240	0.0827	0.2509

$\begin{array}{c} 0\cdot 2649\\ 0\cdot 2780\\ 0\cdot 2890\\ 0\cdot 3000\end{array}$	$\begin{array}{c} 0\cdot 3106\\ 0\cdot 320\\ 0\cdot 331\\ 0\cdot 341\\ 0\cdot 350\end{array}$	$\begin{array}{c} 0 \cdot 360 \\ 0 \cdot 369 \\ 0 \cdot 379 \\ 0 \cdot 389 \\ 0 \cdot 398 \end{array}$	0.407 0.416 0.425 0.434 0.443	$\begin{array}{c} 0\cdot 452\\ 0\cdot 461\end{array}$
$\begin{array}{c} 0{\cdot}0845\ 0{\cdot}0862\ 0{\cdot}0862\ 0{\cdot}0894\ 0{\cdot}0911\ 0{\cdot}0931\ 0{\cdot}0931\end{array}$	$\begin{array}{c} 0 \cdot 0948 \\ 0 \cdot 0966 \\ 0 \cdot 0981 \\ 0 \cdot 0994 \\ 0 \cdot 1005 \end{array}$	$\begin{array}{c} 0\cdot 1016\\ 0\cdot 1024\\ 0\cdot 1032\\ 0\cdot 1037\\ 0\cdot 1048\end{array}$	$\begin{array}{c} 0 \cdot 1053 \\ 0 \cdot 1058 \\ 0 \cdot 1058 \\ 0 \cdot 1064 \\ 0 \cdot 1071 \\ 0 \cdot 1074 \end{array}$	$\begin{array}{c} 0\cdot 1078\\ 0\cdot 1082 \end{array}$
$0.2343 \\ 0.2433 \\ 0.2512 \\ 0.2598$	$\begin{array}{c} 0\cdot 2677 \\ 0\cdot 2767 \\ 0\cdot 2767 \\ 0\cdot 2842 \\ 0\cdot 2918 \\ 0\cdot 2990 \end{array}$	$\begin{array}{c} 0\cdot 3067\ 0\cdot 3144\ 0\cdot 3219\ 0\cdot 3298\ 0\cdot 3298\ 0\cdot 3369\end{array}$	$\begin{array}{c} 0\cdot 3441\\ 0\cdot 3515\\ 0\cdot 3588\\ 0\cdot 3588\\ 0\cdot 3660\\ 0\cdot 3739\end{array}$	$0.3807 \\ 0.3875$
$\begin{array}{c} 0.0752\\ 0.0765\\ 0.0785\\ 0.0800\\ 0.0814\end{array}$	$\begin{array}{c} 0 \cdot 0828 \\ 0 \cdot 0838 \\ 0 \cdot 0838 \\ 0 \cdot 0848 \\ 0 \cdot 0856 \\ 0 \cdot 0864 \end{array}$	$\begin{array}{c} 0\cdot 0870\ 0\cdot 0877\ 0\cdot 0877\ 0\cdot 0883\ 0\cdot 0888\ 0\cdot 0888\ 0\cdot 0888\ 0\cdot 0891\end{array}$	$\begin{array}{c} 0 \cdot 0895 \\ 0 \cdot 0898 \\ 0 \cdot 0902 \\ 0 \cdot 0905 \\ 0 \cdot 0908 \end{array}$	$\begin{array}{c} 0\cdot 0910\\ 0\cdot 0912 \end{array}$
$\begin{array}{c} 0.1791\\ 0.1821\\ 0.1821\\ 0.1863\\ 0.1917\\ 0.1972\end{array}$	$\begin{array}{c} 0\cdot 2027 \\ 0\cdot 2082 \\ 0\cdot 2136 \\ 0\cdot 2193 \\ 0\cdot 2252 \end{array}$	$\begin{array}{c} 0\cdot2300\ 0\cdot2363\ 0\cdot2363\ 0\cdot2410\ 0\cdot2457\ 0\cdot2512\end{array}$	$0.2567 \\ 0.2615 \\ 0.2670 \\ 0.2725 \\ 0.2767 \\ 0$	0.2816 0.2863
0.0589 0.0594 0.0602 0.0611 0.0618	0.0627 0.0633 0.0639 0.0645 0.0651	$\begin{array}{c} 0\cdot 0655\\ 0\cdot 0659\\ 0\cdot 0662\\ 0\cdot 0664\\ 0\cdot 0664\\ 0\cdot 0668\end{array}$	$\begin{array}{c} 0.0670\\ 0.0671\\ 0.0673\\ 0.0673\\ 0.0674\\ 0.0675\end{array}$	$\begin{array}{c} 0\cdot 0675\\ 0\cdot 0676\end{array}$
$\begin{array}{c} 0.0915\\ 0.0932\\ 0.0970\\ 0.1013\\ 0.1059\end{array}$	0.1108 0.1161 0.1218 0.1279 0.1345	$\begin{array}{c} 0\cdot 1413\\ 0\cdot 1486\\ 0\cdot 1563\\ 0\cdot 1563\\ 0\cdot 1644\\ 0\cdot 1730\end{array}$	$\begin{array}{c} 0\cdot 1822\\ 0\cdot 1915\\ 0\cdot 2013\\ 0\cdot 2119\\ 0\cdot 2224\end{array}$	$\begin{array}{c} 0\cdot 2337\\ 0\cdot 2454\end{array}$
29°20' 30°27' 34' 36°36'	$\begin{array}{c} 38^\circ & 29' \\ 40^\circ & 18' \\ 42^\circ & 3' \\ 45^\circ & 19' \end{array}$	46° 50' 48° 17' 49° 41' 51° 52° 17'	53° 31′ 54° 40′ 55° 46′ 56° 51′ 57° 50′	58° 49′ 59° 44′
0.95 1.2	470.078	55570 35570 35570 35570	00000 70000 70000	2.9 3.0

TABLE 1-continued

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ades	\mathbf{r}^{k}	0.1126	0.1787	$0 \cdot 2022$	$0\cdot 2375$	$0 \cdot 2626$	$0 \cdot 2840$	$0 \cdot 3038$	0.3215	0.3370
4 Bl	skr	$0.0382 \\ 0.0451 \\ 0.0511$	$0.0570 \\ 0.0622$	$0.0681 \\ 0.0727$	$0.0782 \\ 0.0829 \\ 0.0871$	0.0913	0.0948	$0.1017 \\ 0.1047$	$\begin{array}{c} 0\cdot 1078\\ 0\cdot 1101_{5}\end{array}$	0.1125 0.1149
ades	μŢ	0.1113	0.1434 0.1736	0.2012	0.2250	0.2453	0.2610	$0 \cdot 2747$	0.2883	0.2999
3 Bl	Tys	$0.0379 \\ 0.0444$	$0.0499 \\ 0.0555 \\ 0.0607$	$0.0660 \\ 0.0705$	$0.0749 \\ 0.0788 \\ 0.0823$	0.0853	$0.0880 \\ 0.0905$	$0.0927 \\ 0.0946$	$0.0968 \\ 0.0988$	$\begin{array}{c} 0\cdot 1004 \\ 0\cdot 1019 \end{array}$
des	ц	$0.1086 \\ 0.1218$	$0.1357 \\ 0.1482 \\ 0.1588$	$0 \cdot 1675$ $0 \cdot 1768$	0.1845 0.1917 0.1978	$0\cdot 2038$	$\begin{array}{c} 0\cdot 2100\\ 0\cdot 2136\end{array}$	$\begin{array}{c} 0\cdot 2182\\ 0\cdot 2214\end{array}$	$0\cdot 2242 \\ 0\cdot 2270$	$0.2306 \\ 0.2330$
2 Bla	Тұs	$\begin{array}{c} 0{\color{red}{\cdot}}0376 \\ 0{\color{red}{\cdot}}0424 \end{array}$	$0.0472 \\ 0.0516 \\ 0.0555$	0.0587 0.0620	0.0647 0.0672 0.0693	0.0712	0.0730 0.0743	0.0756 0.0766	$0.0772 \\ 0.0780$	0.0787 0.0794
	ac B	$0.0984 \\ 0.0988$	$\begin{array}{c} 0\cdot0992\ 0\cdot0998\ 0\cdot1005 \end{array}$	0.1013 0.1022	0.1032 0.1043 0.1056	$0 \cdot 1069$	$0 \cdot 1084$ $0 \cdot 1099$	$0.1116 \\ 0.1134$	$0.1154 \\ 0.1174$	0.1196 0.1219
	¢	8° 9° 18′	10° 36′ 11° 54′ 13° 12′	14° 29' 15° 46'	17° ^{±0} 18° 18′ 19° 34′	20° 48'	22° 3′ 23° 15′	$\frac{24^{\circ}}{25^{\circ}}$ $\frac{27^{\circ}}{39^{\circ}}$	26° 49′ 97° 59′	29° 15′ 30° 15′
	ſ	0 0.05	$\begin{array}{c} 0 \cdot 1 \\ 0 \cdot 15 \\ 0 \cdot 2 \end{array}$	$\begin{array}{c} 0.25\\ 0.25\\ 0.2\end{array}$	0.35	0.5	0.55	0.65	0.75 0.8	0.85

	0.3506	0.3650	0.3791	0.3919	0.4031	0.4163	0.4288	0.4402	0.4523												
0.1166	0.1183	$0 \cdot 1217$	$0 \cdot 1247$	0.1271	$0 \cdot 1292$	0.1314	0.1334	0.1349	0.1365												
	0.3113	0.3221	0.3306																		
$0 \cdot 1033$	$0 \cdot 1046$	0.1070	$0 \cdot 1088$	<u></u>																	
0.2362	0.2392	0.2452	0.2507	0.2583	$0 \cdot 2633$	0.2707	0.2767	0.2833	0.2902	0.2956	0.3017	0.3077	0.3137	0.3198	0.3253	0.3306	0.3356	0.3423	0.3472	$0.3520 \\ 0.3575$	
0.0798	0.0805	0.0816	0.0826	0.0839	0.0847	0.0857	0.0864	0.0871	0.0878	0.0882	0.0887	0.0891	0.0895	0.0897	0.0900	0.0901	0.0902	0.0904	0.0905	0.0906	
$0 \cdot 1243$	0.1269	0.1323	0.1383	0.1449	0.1519	0.1595	$0 \cdot 1677$	$0 \cdot 1763$	$0 \cdot 1858$	$0 \cdot 1957$	0.2061	0.2173	0.2290	0.2415	0.2548	0.2685	0.2828	$0\cdot 2984$	0.3138	$0.3306 \\ 0.3479$	
31° 20′ 1	39° 97'	34° 34′	36° 37′	38° 36'	40° 99'	42° 18′	44° 3′	45° 42′	47° 19′	$48^{\circ} 50'$	50° 17'	51° 41′	53° 0'	54° 17'	55° 31'	56° 40'	57° 46'	58° 51'	59° 50'	60° 49′ 61° 44′	(()
0.95) (- 6-1	i ÷	1.4	۲ <u>۱</u>	9.1	1.7	1.8	6.1	2.0	2.1	5.5 5	2.3	2.4	2.5	2.6	2.7	2.8	3.0 3	•

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TABLE 1-continued

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 $\beta = 10^{\circ}$

ades	lê _T	$0 \cdot 1715$ $0 \cdot 2124$	$0 \cdot 2501$	$() \cdot 285()$	$() \cdot 3129$	0.3431	
4 Bl	$sk_{\rm L}$	$0.0601 \\ 0.0680 \\ 0.0751 \\ 0.0751$	0.0887	0.0957 0.1015	0 · 1116	$0 \cdot 1220$	
ades	Γų	$\begin{array}{c} 0\cdot 1696\\ 0\cdot 2065 \end{array}$	0.2412	0.2717	$0 \cdot 2963$		
3 Bla	T¥S	$\begin{array}{c} 0\cdot 0596 \\ 0\cdot 0661 \\ 0\cdot 0732 \end{array}$	$0.0797 \\ 0.0858$	0 · 0916 0 · 0968	$0 \cdot 1017 \\ 0 \cdot 1057 \\ 0 \cdot 1093$		
ades	μ	$\begin{array}{c} 0 \cdot 1614 \\ 0 \cdot 1753 \\ 0 \cdot 1900 \end{array}$	0.2027 0.2142	$() \cdot 2238$ $() \cdot 2318$	$() \cdot 2397$ $() \cdot 2470$ $() \cdot 2543$	$\begin{array}{c} 0 \cdot 2603 \\ 0 \cdot 2646 \\ 0 \cdot 2682 \\ 0 \cdot 2713 \\ 0 \cdot 2743 \end{array}$	$0.2773 \\ 0.2819 \\ 0.2849 \\ 0.2880$
2 Bla	skL	0.0568 0.0621 0.0674	0.0720 0.0762	0.0798 0.0829	0.0858 0.0884 0.0909	$\begin{array}{c} 0 \cdot 0929 \\ 0 \cdot 0946 \\ 0 \cdot 0957 \\ 0 \cdot 0967 \\ 0 \cdot 0976 \end{array}$	$\begin{array}{c} 0\cdot 0983\\ 0\cdot 0993\\ 0\cdot 1001\\ 0\cdot 1008\end{array}$
	Ω, ^r	$0 \cdot 1234 \\ 0 \cdot 1240 \\ 0 \cdot 1240 \\ 0 \cdot 1247$	0.1255 0.1265	$\begin{array}{c} 0 \cdot 1277 \\ 0 \cdot 1288 \end{array}$	$0 \cdot 1302 \\ 0 \cdot 1317 \\ 0 \cdot 1334$	$\begin{array}{c} 0\cdot 1352\\ 0\cdot 1372\\ 0\cdot 1393\\ 0\cdot 1393\\ 0\cdot 1415\\ 0\cdot 1440\end{array}$	$0 \cdot 1465$ $0 \cdot 1493$ $0 \cdot 1522$ $0 \cdot 1523$
	¢	10° 11° 18′ 11° 36′	13° 54′ 15° 12′	16° 29' 17° 46'	$\begin{array}{ccc} 19^{\circ} & 3' \\ 20^{\circ} & 18' \\ 21^{\circ} & 34' \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{ccc} 28^{\circ} & 49^{\circ} \\ 29^{\circ} & 59^{\circ} \\ 31^{\circ} & 7^{\prime} \\ 32^{\circ} & 15^{\prime} \end{array}$
		0 0 0 0	0.15	$\begin{array}{c} 0\cdot 25\\ 0\cdot 3\\ \end{array}$	$\begin{array}{c} 0.35\\ 0.4\\ 0.45\end{array}$	0.55 0.55 0.66 0.77	$\begin{array}{c} 0.75\\ 0.85\\ 0.9\end{array}$

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2911 2952 3014 3083 3147	3212 3276 3353 3413 3481	.3549 .3612 .3670 .3782 .3782	.3849 .3906 .3959 .4012 .4073	-4124 -4164
$\begin{array}{c} 0 \cdot 1015 \\ 0 \cdot 1024 \\ 0 \cdot 1037 \\ 0 \cdot 1052 \\ 0 \cdot 1063 \\ 0 \cdot 1063 \\ 0 \cdot 0 \end{array}$	$\begin{array}{c} 0 \cdot 1072 \\ 0 \cdot 1081 \\ 0 \cdot 1091 \\ 0 \cdot 1098 \\ 0 \cdot 1105 \end{array} \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array}$	0.1112 0.1116 0.1121 0.1123 0.1123	0.1130 0.1132 0.1132 0.1134 0.1136 0.1136 0.1138	0 · 1139 0 0 0 0 0 0
$\begin{array}{c} 0\cdot 1584 \\ 0\cdot 1619 \\ 0\cdot 1692 \\ 0\cdot 1772 \\ 0\cdot 1859 \end{array}$	$\begin{array}{c} 0\cdot 1953\\ 0\cdot 2055\\ 0\cdot 2166\\ 0\cdot 2282\\ 0\cdot 2410 \end{array}$	$\begin{array}{c} 0 \cdot 2543 \\ 0 \cdot 2685 \\ 0 \cdot 2688 \\ 0 \cdot 2838 \\ 0 \cdot 2996 \\ 0 \cdot 3168 \end{array}$	$\begin{array}{c} 0\cdot 3350\\ 0\cdot 3539\\ 0\cdot 3539\\ 0\cdot 3737\\ 0\cdot 3952\\ 0\cdot 4168\end{array}$	$0.4401 \\ 0.4643$
33°20 34°27 36°34 38°37 40°36	$\begin{array}{cccc} 42^{\circ} & 29' \\ 44^{\circ} & 18' \\ 46^{\circ} & 3' \\ 47^{\circ} & 42' \\ 49^{\circ} & 19' \end{array}$	$50^{\circ} 50'$ $52^{\circ} 17'$ $53^{\circ} 41'$ $55^{\circ} 0'$	57° 31' 58° 40' 59° 46' 60° 51' 61° 50'	$62^{\circ} 49'$ $63^{\circ} 44'$
0.95 1.0 1.3 1.3	$\begin{array}{c} 1 \\ \cdot \\$	55700 55570 55570	22222 4.0.02 8.0.05	2.9 3.0

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ades	μT	$\begin{array}{c} 0\cdot 2426\\ 0\cdot 2651\\ 0\cdot 2871\\ 0\cdot 3084\end{array}$	$0.3282 \\ 0.3464 \\ 0.3639$
4 B	Тųs	$\begin{array}{c} 0\cdot 0870\\ 0\cdot 0955\\ 0\cdot 1039\\ 0\cdot 1119\end{array}$	0 · 1196 0 · 1264 0 · 1330
ades	ĿΨ	$\begin{array}{c} 0\cdot 2430\\ 0\cdot 2576\\ 0\cdot 2769\\ 0\cdot 2959\end{array}$	0.3135
3 Bl	Тųs	$\begin{array}{c} 0 \cdot 0847 \\ 0 \cdot 0928 \\ 0 \cdot 1002 \\ 0 \cdot 1074 \end{array}$	0 · 1142
lades	ΥŢ	$\begin{array}{c} 0\cdot 2186\\ 0\cdot 2332\\ 0\cdot 2472\\ 0\cdot 2593\end{array}$	$\begin{array}{c} 0\cdot 2684\\ 0\cdot 2796\\ 0\cdot 2958\\ 0\cdot 2958\\ 0\cdot 3020\\ 0\cdot 3127\\ 0\cdot 3127\\ 0\cdot 3127\\ 0\cdot 3127\\ 0\cdot 3127\\ 0\cdot 3236\\ 0\cdot 3236\\ 0\cdot 3238\\ 0\cdot 3236\\ 0\cdot 3333\\ 0\cdot 3370\\ \end{array}$
2 Bi	Т <i>ųs</i>	$\begin{array}{c} 0 \cdot 0787 \\ 0 \cdot 0844 \\ 0 \cdot 0897 \\ 0 \cdot 0944 \end{array}$	$\begin{array}{c} 0\cdot 0982\\ 0\cdot 1057\\ 0\cdot 1057\\ 0\cdot 1084\\ 0\cdot 1131\\ 0\cdot 1131\\ 0\cdot 1131\\ 0\cdot 1159\\ 0\cdot 1159\\ 0\cdot 1159\\ 0\cdot 1198\\ 0\cdot 1198\\ 0\cdot 1198\\ 0\cdot 1198\\ 0\cdot 1207\\ 0\cdot 1218\\ 0\cdot 1207\end{array}$
	wc	$\begin{array}{c} 0\cdot 1488\\ 0\cdot 1496\\ 0\cdot 1505\\ 0\cdot 1517\end{array}$	$\begin{array}{c} 0 \cdot 1530 \\ 0 \cdot 1544 \\ 0 \cdot 1561 \\ 0 \cdot 1561 \\ 0 \cdot 1579 \\ 0 \cdot 1599 \\ 0 \cdot 1621 \\ 0 \cdot 1621 \\ 0 \cdot 1621 \\ 0 \cdot 1697 \\ 0 \cdot 1697 \\ 0 \cdot 1757 \\ 0 \cdot 1757 \\ 0 \cdot 1756 \\ 0 \cdot 1756 \\ 0 \cdot 1756 \\ 0 \cdot 1863 \end{array}$
	-6.	12° 13° 18′ 14° 36′ 15° 54′	$\begin{array}{c} 17^{\circ}\\ 18^{\circ}\\ 29^{\circ}\\ 21^{\circ}\\ 21^{\circ}\\ 22^{\circ}\\ 23^{\circ}\\ 33^{\circ}\\ 33^{\circ}\\ 33^{\circ}\\ 29^{\circ}\\ 33^{\circ}\\ 7^{\circ}\\ 33^{\circ}\\ 7^{\circ}\\ 7^{$
	ſ	$\begin{array}{c} 0\\ 0\cdot 05\\ 0\cdot 1\\ 0\cdot 15\end{array}$	$\begin{array}{c} 0.2\\ 0.25\\ 0.3\\ 0.35\\ 0.55\\ 0.55\\ 0.6\\ 0.75\\ 0.75\\ 0.88\\ 0.75\\ 0.75\\ 0.88\\ 0.75\\ 0.75\\ 0.88\\ 0.75\\ 0.75\\ 0.88\\ 0.75\\ 0.88\\ 0.75\\ 0.88\\ 0.75\\ 0.88\\ 0.88\\ 0.75\\ 0.88\\ 0.$

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Table of values of skL and kT for given values of J and for x = 0.7 for 6 blades and infinity blades.

	y Blades	ЪТ	0.0289 0.0474 0.0660 0.0842 0.1021 0.1207 0.1264 0.1738 0.1738 0.1738 0.1738 0.1738 0.2633 0.2636 0.2768 0.2263 0.2768 0.2768 0.2768 0.3123 0.3285 0.3285 0.3276	
0	Infinit	Tys	$\begin{array}{c} 0 \cdot 0096 \\ 0 \cdot 0161 \\ 0 \cdot 0224 \\ 0 \cdot 0285 \\ 0 \cdot 0345 \\ 0 \cdot 0345 \\ 0 \cdot 0345 \\ 0 \cdot 0404 \\ 0 \cdot 0568 \\ 0 \cdot 0568 \\ 0 \cdot 0568 \\ 0 \cdot 0568 \\ 0 \cdot 0711 \\ 0 \cdot 0701 \\ 0 \cdot 0986 \\ 0 \cdot 00986 \\ 0 \cdot 0000 \\ 0 \cdot 00986 \\ 0 \cdot 0000 \\ 0 \cdot 00$	
4	6 Blades	Ides	kт	$\begin{array}{c} 0.0285\\ 0.0470\\ 0.0658\\ 0.0658\\ 0.0839\\ 0.1012\\ 0.1195\\ 0.1195\\ 0.1195\\ 0.1790\\ 0.1790\\ 0.1790\\ 0.1790\\ 0.2154\\ 0.2262\\ 0.2262\\ 0.2262\\ 0.22896\\ 0.22990\\ 0.22990\\ 0.22990\\ 0.22990\\ 0.22990\\ 0.2386\\ 0.2386\\ 0.2386\\ 0.2386\\ 0.23896\\$
		$\Im y_S$	$\begin{array}{c} 0 \cdot 0096 \\ 0 \cdot 0160 \\ 0 \cdot 0223 \\ 0 \cdot 0284 \\ 0 \cdot 0342 \\ 0 \cdot 0449 \\ 0 \cdot 0449 \\ 0 \cdot 0449 \\ 0 \cdot 0449 \\ 0 \cdot 0541 \\ 0 \cdot 0541 \\ 0 \cdot 0541 \\ 0 \cdot 0667 \\ 0 \cdot 0689 \\ 0 \cdot 0731 \\ 0 \cdot 0790 \\ 0 \cdot 0790 \\ 0 \cdot 0810 \\ 0 \cdot 0810 \\ 0 \cdot 0810 \end{array}$	
	r Blades	k_{T}	$\begin{array}{c} 0 & 0.0071 \\ 0 & 0.0168 \\ 0 & 0.0355 \\ 0 & 0.0355 \\ 0 & 0.450 \\ 0 & 0.355 \\ 0 & 0.01980 \\ 0 & 0.01980 \\ 0 & 0.01980 \\ 0 & 0.01980 \\ 0 & 0.01980 \\ 0 & 0.01980 \\ 0 & 0.01980 \\ 0 & 0.01980 \\ 0 & 0.01980 \\ 0 & 0.00105 \\ 0 & 0.00105 \\ 0 & 0.00105 \\ 0 & 0.0005 \\ 0 & 0 & 0.0005 \\ 0 & 0 & 0.0005 \\ 0 & 0 & 0.0005 \\ 0 & 0 & 0.0005 \\ 0 & 0 & 0.0005 \\ 0 & 0 & 0.0005 \\ 0 & 0 & 0 & 0.0005 \\ 0 & 0 & 0 & 0.0005 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 &$	
0	Infinity	Tys	$\begin{array}{c} 0 \cdot 0024 \\ 0 \cdot 0056 \\ 0 \cdot 0088 \\ 0 \cdot 0118 \\ 0 \cdot 0118 \\ 0 \cdot 0178 \\ 0 \cdot 0178 \\ 0 \cdot 0207 \\ 0 \cdot 0235 \\ 0 \cdot 02311 \\ 0 \cdot 02334 \\ 0 \cdot 02311 \\ 0 \cdot 0334 \\ 0 \cdot 03334 \\ 0 \cdot 0334 \\ 0 \cdot 0334$	
5	des	ades	$\mathbf{L} \boldsymbol{\gamma}$	$\begin{array}{c} 0 \cdot 0071\\ 0 \cdot 0168\\ 0 \cdot 0262\\ 0 \cdot 0353\\ 0 \cdot 0353\\ 0 \cdot 03537\\ 0 \cdot 03537\\ 0 \cdot 03537\\ 0 \cdot 0537\\ 0 \cdot 0537\\ 0 \cdot 0537\\ 0 \cdot 0537\\ 0 \cdot 0711\\ 0 \cdot 0537\\ 0 \cdot 0248\\ 0 \cdot 0361\\ 0 \cdot 0301\\ 0 \cdot 1183\\ 0 \cdot 1630\\ 0 \cdot 1580\\ 0 \cdot 1630\\ 0 \cdot 160\\ 0 \cdot 100\\ 0 \cdot 1$
	6 Bl	rt <i>ys</i>	$\begin{array}{c} 0.0024\\ 0.0056\\ 0.0056\\ 0.0117\\ 0.0117\\ 0.0176\\ 0.0176\\ 0.0228\\ 0.0228\\ 0.0228\\ 0.0228\\ 0.0233\\ 0.0233\\ 0.0321\\ 0.0333\\ 0.0333\\ 0.0363\\ 0.0379\\ 0.0386\\ 0.0088\\$	
₿°	No. of blades	F	00000000000000000000000000000000000000	

	Blades	k_{T}	$0 \cdot 1742$ $0 \cdot 2167$ $0 \cdot 2598$ $0 \cdot 3010$
o(Infinity	skL	$\begin{array}{c} 0.0612\\ 0.0769\\ 0.0924\\ 0.1076\\ \cdot\end{array}$
1(ades	$^{\mathrm{T}a}$	$\begin{array}{c} 0 \cdot 1734 \\ 0 \cdot 2577 \\ 0 \cdot 2577 \\ 0 \cdot 2963 \\ 0 \cdot 3314 \\ 0 \cdot 3314 \\ 0 \cdot 3967 \\ 0 \cdot 4518 \\ 0 \cdot 4518 \end{array}$
	6 Bl	sk_{L}	$\begin{array}{c} 0.0610\\ 0.0767\\ 0.0917\\ 0.1058\\ 0.1185\\ 0.1312\\ 0.1513\\ 0.1513\\ 0.1597\\ 0.1597\end{array}$
	r Blades	Γų	$\begin{array}{c} 0 \cdot 1136 \\ 0 \cdot 1485 \\ 0 \cdot 1837 \\ 0 \cdot 25187 \\ 0 \cdot 2518 \\ 0 \cdot 2862 \\ 0 \cdot 2862 \end{array}$
0	Infinity	skL	$\begin{array}{c} 0.0391 \\ 0.0517 \\ 0.0642 \\ 0.0763 \\ 0.0882 \\ 0.0998 \\ 0.0998 \end{array}$
8	ades	k_{T}	$\begin{array}{c} 0.1130\\ 0.1130\\ 0.1480\\ 0.1828\\ 0.2467\\ 0.2467\\ 0.2772\\ 0.3313\\ 0.3313\\ 0.3367\\ 0.3667\\ 0.3667\\ 0.5205\end{array}$
	6 Bl	SkL	$\begin{array}{c} 0.0390\\ 0.0515\\ 0.0639\\ 0.0639\\ 0.0753\\ 0.0967\\ 0.0967\\ 0.1058\\ 0.1144\\ 0.11217\\ 0.11217\\ 0.11217\\ 0.1283\\ 0.1283\\ 0.1283\\ 0.1217\\ 0.1283\\ 0.1510\\ 0.1573\\ 0.1573\\ 0.1598\end{array}$
	Blades	ЪТ	$\begin{array}{c} 0.0642\\ 0.0915\\ 0.1187\\ 0.1722\\ 0.1981\\ 0.2240\\ 0.2754\\ 0.3008\\ 0.3264\\ 0.3264\end{array}$
0	Infinity	I'as	$\begin{array}{c} 0.0219\\ 0.0314\\ 0.0500\\ 0.0500\\ 0.0577\\ 0.0761\\ 0.0842\\ 0.0920\\ 0.0995\\ 0.1065\end{array}$
9	ldes	μT	$\begin{array}{c} 0.0640\\ 0.0912\\ 0.1180\\ 0.1453\\ 0.1453\\ 0.1937\\ 0.1937\\ 0.2587\\ 0.2381\\ 0.2363\\ 0.3407\\ 0.3407\\ 0.3407\\ 0.3407\\ 0.3843\\ 0.3843\\ 0.3843\\ 0.4490\\ 0.4490\\ 0.4490\\ 0.4490\\ 0.4490\end{array}$
-	6 Bl	SkL	$\begin{array}{c} 0.0218\\ 0.0218\\ 0.0406\\ 0.0499\\ 0.0581\\ 0.0662\\ 0.0864\\ 0.0864\\ 0.0864\\ 0.0916\\ 0.0916\\ 0.0916\\ 0.0916\\ 0.1172\\ 0.1172\\ 0.1172\\ 0.1172\\ 0.1172\\ 0.1172\\ 0.1172\\ 0.1172\\ 0.1128\\ 0.1128\\ 0.11236\\ 0.11226\\ 0.11236\\ 0.11226\\ 0$
βο	No. of Blades	, T	2222-1-1-1-1-1-1-0000000000000000000000

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TABLE 2-continued

TABLE 3

Values of $k_{\rm L}$ and $k_{\rm D}$ at x = 0.7 deduced from analysis of performance data of actual model airscrews :---R.A.F. 6 type sections; Ref. 4; standard 2 bladers (Figs. 8 and 9) and 4 bladers; also mean of 2 and 4 blades.

Range of pitch ratio 0.3 to 2.5.

α°	2 Blades		4 Blades		Mean of 2 and 4 Blades	
	$k_{\mathbf{L}}$	k _D	k_{L}	k _D	$k_{\mathbf{L}}$	k _D
$ \begin{array}{c c} -4 \cdot 4 \\ -4 \cdot 25 \\ -4 \cdot 0 \\ -3 \cdot 0 \\ -2 \cdot 0 \\ -1 \cdot 0 \\ 0 \\ +1 \\ 2 \\ 3 \\ 4 \\ 5 \\ \end{array} $	$\begin{array}{c} 0 \\ 0 \cdot 026 \\ 0 \cdot 087 \\ 0 \cdot 141 \\ 0 \cdot 192 \\ 0 \cdot 239 \\ 0 \cdot 239 \\ 0 \cdot 323 \\ 0 \cdot 323 \\ 0 \cdot 360 \\ 0 \cdot 399 \\ 0 \cdot 435 \end{array}$	$\begin{array}{c} 0 \cdot 0212 \\ 0 \cdot 0183 \\ 0 \cdot 0136 \\ 0 \cdot 0107 \\ 0 \cdot 0091 \\ 0 \cdot 0082 \\ \end{array}$ $\begin{array}{c} 0 \cdot 0077 \\ 0 \cdot 0078 \\ 0 \cdot 0078 \\ 0 \cdot 0093 \\ 0 \cdot 0107 \end{array}$	$\begin{array}{c} 0 \\ 0 \cdot 017 \\ 0 \cdot 081 \\ 0 \cdot 137 \\ 0 \cdot 184 \\ 0 \cdot 230 \\ 0 \cdot 273 \\ 0 \cdot 311 \\ 0 \cdot 349 \\ 0 \cdot 385 \\ 0 \cdot 423 \end{array}$	$\begin{array}{c} 0 \cdot 0208 \\ 0 \cdot 0194 \\ 0 \cdot 0137 \\ 0 \cdot 0114 \\ 0 \cdot 0102 \\ 0 \cdot 0096 \\ \end{array}$ $\begin{array}{c} 0 \cdot 0095 \\ 0 \cdot 0099 \\ 0 \cdot 0107 \\ 0 \cdot 0119 \\ 0 \cdot 0134 \end{array}$	$\begin{array}{c} 0\cdot 021\\ 0\cdot 084\\ 0\cdot 139\\ 0\cdot 188\\ 0\cdot 235\\ 0\cdot 235\\ 0\cdot 278\\ 0\cdot 317\\ 0\cdot 355\\ 0\cdot 392\\ 0\cdot 429\\ \end{array}$	0.0187 0.0136 0.0096 0.0089 0.0086 0.0088 0.0088 0.0095 0.0120
$ \begin{array}{c} 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ \end{array} $	$\begin{array}{c} 0 \cdot 470 \\ 0 \cdot 503 \\ 0 \cdot 533 \\ 0 \cdot 563 \\ 0 \cdot 587 \\ 0 \cdot 609 \\ 0 \cdot 629 \\ 0 \cdot 642 \\ 0 \cdot 651 \end{array}$	$\begin{array}{c} 0.0123\\ 0.0143\\ 0.0167\\ 0.0199\\ 0.0243\\ 0.0299\\ 0.0370\\ 0.0450\\ 0.0540\\ \end{array}$	$\begin{array}{c} 0 \cdot 456 \\ 0 \cdot 492 \\ 0 \cdot 526 \\ 0 \cdot 558 \\ 0 \cdot 590 \\ 0 \cdot 620 \\ 0 \cdot 649 \\ 0 \cdot 660 \\ 0 \cdot 665 \end{array}$	$\begin{array}{c} 0 \cdot 0152 \\ 0 \cdot 0173 \\ 0 \cdot 0199 \\ 0 \cdot 0230 \\ 0 \cdot 0265 \end{array}$ $\begin{array}{c} 0 \cdot 0310 \\ 0 \cdot 0370 \\ 0 \cdot 0450 \\ 0 \cdot 0580 \\ 0 \cdot 0580 \end{array}$	$\begin{array}{c} 0 \cdot 463 \\ 0 \cdot 497 \\ 0 \cdot 529 \\ 0 \cdot 560 \\ 0 \cdot 588 \\ 0 \cdot 615 \\ 0 \cdot 639 \\ 0 \cdot 651 \\ 0 \cdot 658 \\ 0 \cdot 658 \\ 0 \cdot 658 \end{array}$	$\begin{array}{c} 0 \cdot 0137 \\ 0 \cdot 0158 \\ 0 \cdot 0184 \\ 0 \cdot 0215 \\ 0 \cdot 0254 \\ \end{array}$ $\begin{array}{c} 0 \cdot 0305 \\ 0 \cdot 0370 \\ 0 \cdot 0450 \\ 0 \cdot 0560 \\ 0 \cdot 0255 \end{array}$
15 20 25 30 35	$ \begin{array}{c} 0.660 \\ 0.688 \\ 0.700 \\ 0.705 \\ 0.698 \end{array} $	$\begin{array}{c} 0 \cdot 0660 \\ 0 \cdot 148 \\ 0 \cdot 252 \\ 0 \cdot 357 \\ 0 \cdot 461 \end{array}$	$ \begin{array}{c} 0.665 \\ 0.652 \\ 0.673 \\ 0.690 \end{array} $	$ \begin{array}{c c} 0.0730 \\ 0.158 \\ 0.249 \\ 0.337 \end{array} $	$ \begin{array}{c} 0.662 \\ 0.670 \\ 0.686 \\ 0.697 \end{array} $	$ \begin{array}{c c} 0.0695 \\ 0.153 \\ 0.2505 \\ 0.347 \end{array} $

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Fig. 4.



Chart 3; k_{T} (or $dk_{T}/d(x^{2})$ against sk_{L} for given J; Two blades; radius 0.7.



<u>Chart 4</u>; w_c against sk_L For given J. Two blades, radius 0.7.





FIG. 8.

R.& M. 1675.

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Analysis of observed performance of model 2 bladed airscrews. Values of k_D plotted against α for radius 0.7 derived from airscrews of pitch values as follows.



Fig. 9.

<u>R. & M. 1675.</u>

Fig. 10.

Lift and Drag <u>Coefficient Curves for a Section at 0.7</u> Radius of an Airscrew Blade.



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