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Summary.
The rotation and flare-up phases of the take-off of a slender-wing supersonic transport aircraft are studied by calculating the time histories of speed, height, etc. when the pitch-attitude time history is specified. The shape of the attitude function is fixed but its maximum attitude and duration, and the rotation speed at which it starts, may be varied to show the effect on the performance of variations in the take-off procedure. Both three and four engine take-offs are examined and the influence of the ground on lift, drag and pitching moment is included. Only small variations in achieved rotation and unstick speeds can be permitted if low climb gradients or large take-off distance are to be avoided. Low rotation rates of about 1–2 deg/sec are preferable.

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1. *Introduction.*

To permit a satisfactory take-off, an aircraft must have good performance and manoeuvrability. Good performance requires an adequate margin of thrust over drag to provide both sufficient acceleration to reach a safe flying speed and an adequate rate of climb in the event of failure of one of the engines. Good manoeuvrability requires that the changes in attitude and speed during the rotation, flare-up and climb-away phases may easily and accurately be achieved without exceptional skill on the part of the pilot. Performance and manoeuvrability are linked because errors in technique, due, perhaps, to poor aircraft handling characteristics, could lead to a reduction in the achieved performance.

A typical take-off manoeuvre of a jet transport aircraft consists of four phases. First, the aeroplane is accelerated from rest while being held in a low lift, low drag attitude. Next, at a certain speed, called the rotation speed, V_R, which is close to the lift-off speed, V_{LOF}, the aircraft is rotated rapidly to an attitude at which the lift is sufficient to balance the weight. After the aircraft has become airborne, the third phase is a 'flare-up' or transition from horizontal to climbing flight, and the final phase is the climb-out up to a height of about 1500 ft.

During the rotation phase, considerable errors may occur in both the speed at which rotation is initiated and the pitch attitude achieved on the ground. In practice, errors of as much as 20 knots in the rotation speed have been recorded[1] but it is not known whether such errors are deliberate or whether they arise from the difficulty of the task. The dangers of under-rotation, which can lead to a shallow flight path and an increased take-off distance, have been emphasized in a theoretical study[2] of the take-off performance of an American supersonic transport configuration.

For slender wings of the type proposed in Europe[3,4] for supersonic transport aircraft, the nominal range of take-off and landing speeds lies well below the minimum drag speed, so that an efficient take-off procedure will require that a relatively high speed be attained before rotation and lift-off. An upper limit is imposed, however, by considerations of tyre safety or runway length. The margin between safe flying speed and tyre limit speed may be small, thus calling for both a rapid and a careful manouvre.

Handling problems may arise for the following reasons. The low-aspect-ratio wing shapes proposed will require that a high take-off incidence be used to generate sufficient lift to unstick. The cockpit position so far ahead of the centre of gravity of the aircraft, combined with this large rotation angle, may cause considerable pilot disorientation during the rotation manoeuvre. Any resulting errors in rotation angle may then produce a large increase in take-off distance because of the high induced drag at incidence. Ground effect on lift, drag and pitching moment is important. As the aircraft climbs away from the ground, the lift due to the ground will disappear, implying a need for an increase of incidence with height. At the same time, the reduction of the nose-down pitching moment due to the ground will tend to raise the nose. Since these two effects are unlikely to be perfectly in accord, the pilot action may need to be quite complicated. A further important characteristic of slender supersonic transport designs is that the response to controls (or disturbances) is sluggish in pitch and yaw but very lively in roll, a situation which might present the pilot with control co-ordination difficulties. The combination of these performance and handling factors could mean that the execution of a satisfactory take-off manoeuvre will require very precise control, with consequent implications for the pilot's information display.

It has been stated[5] that existing instruments for use during take-off provide very little information during the flare-up, and even this is not in an easily assimilable form. Because operational experience with subsonic jet transport aircraft has strengthened the case for the development of new instrumentation to reduce the variability of take-off performance, work is proceeding[6,7,8] on possible schemes for take-off directors which present information to the pilot in such a way as to indicate the required control movements. It is thought that if present day jet transport aircraft require improved instrumentation, the supersonic transport aircraft might have an even greater need. This point is examined in the present Report.

The work recorded here was undertaken to examine the dependence of take-off performance on the take-off procedure and piloting technique, and in particular to see how variations in the achieved manoeuvre affect the take-off distance and screen speed. Aircraft characteristics for a typical slender-wing supersonic transport design are used. The calculations are performed by constraining the aircraft to follow a prescribed pitch attitude time-history; the necessary control movements are then derived from the equations of motion. The general form of the manoeuvre is fixed, but a particular take-off manoeuvre is determined by three parameters: the rotation speed, at which the manoeuvre starts; the amplitude of the change in pitch attitude; and the time taken to achieve the final attitude. By manipulating these three parameters, the effects of such errors as early or late rotation, too large or too small an attitude change, and too rapid or too slow a pitching manoeuvre can be studied. Conclusions can then be drawn regarding the accuracy with which take-off manoeuvres must be performed. Both three and four engine take-offs are considered but transient effects of engine failure are not included.

Most of the take-off calculations described in this paper used only the one generalised shape of pitch-attitude time-history. A few calculations were made with a modified shape. This modification changed the sensitive variables such as peak normal acceleration and maximum elevator deflection but the overall take-off performance was not affected. Thus although no exhaustive attempt was made to find the effect of variations in the shape of the pitch-attitude time-history, it is felt that the results and conclusions obtained are representative for the type of aircraft considered.

The method of calculation works in an inverse sense. This implies that the pilot maintains very tight control over the manoeuvre. In the real situation, his ability to do so depends on whether his attention is diverted by such disturbances as gusts or engine failure. Even if the calculation proceeded in the usual, direct way, it is doubtful whether the effects of such disturbances on the control task could be included except in a piloted simulator. Some assessment of the piloting task is made in this paper based on the derived elevator time-histories. This is possible because the assumed manoeuvre does not produce any step changes in the primary variables, although kinks (discontinuities in slope) do occur occasionally.

Thus the main emphasis of the work is on the sensitivity of the take-off performance to variations in the manoeuvre. The results indicate practical limits to the rotation and unstick speeds and to the pitch rates during rotation and flare-up, and provide a background for possible handling and instrumentation studies.

2. *Method of Attack.*

2.1. *Specification of the Manoeuvre.*

The difficulty as always with this type of manoeuvrability study lies in the choice of a simply definable but sufficiently realistic representation of the manoeuvre. Several possibilities, in increasing order of complication, are

(i) to define the variation of one of the parameters[2], for example speed, attitude or height, and derive the time histories of all the other variables which exactly satisfy the specified manoeuvre;

(ii) to devise an elevator input which will give suitable behaviour;

(iii) to set up a control law which is followed by an autopilot[8];

(iv) to represent the pilot mathematically, as a transfer function, operating on errors in particular variables;

(v) to set up a simulator[9] with the pilot in the loop.

In order to illustrate trends, such as the effect on take-off distance and screen speed*, V_{35}, of errors in the manoeuvre, and to discover sensitive areas, it is necessary to exercise close control over the form of the take-off. This can best be done analytically by forcing an output variable. The question is, which one?

A climb-away at a steady speed, far from the ground and without configuration changes, implies constant incidence and attitude. Incidence is an indication of the lift being generated and of the available margin before the stall or some other undesirable phenomenon occurs. An incidence display has in fact been used, in both flight test[6] and simulator experiments[9]. But incidence is difficult to measure and as a control parameter does not provide sufficient damping for the long period mode. The combination of a constant attitude and more thrust than is needed for a steady climb at that attitude will also lead to a nearly constant forward acceleration. This may be desirable for noise abatement reasons[10]. All these considerations, combined with the analytical convenience of the simpler functional form of the attitude time history, led to the choice of attitude as the defined variable.

The variation of attitude up to its final steady value could take many forms but since it was the intention to avoid serious discontinuities, a function with smooth changes in pitch acceleration was selected. Some slight kinks still appear in the elevator-angle time-history: these are due to the non-zero value of the rate of change of pitching acceleration at the beginning and end of the manoeuvre, and to the sudden elimination, at the instant of lift-off, of the ground reaction.

In this analytical study it is convenient to use a simple mathematical function which may be easily generated. It is symmetrical, with a sinusoidal variation of pitching acceleration.

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*During the take-off, the obstacle clearance height or screen height is usually taken as 35 ft for civil aircraft. The take-off distance is then the horizontal distance travelled to accelerate from rest until a wheel height of 35 ft is attained shortly after unstick, at which height the speed is V_{35}, the screen speed.

For $0 \leqslant t \leqslant t_1$

$$\left.\begin{aligned}
\theta &= \theta_0+\theta_1\left(\frac{t}{t_1}-\frac{1}{2\pi}\sin\frac{2\pi t}{t_1}\right), \\
\dot{\theta} &= \frac{\theta_1}{t_1}\left(1-\cos\frac{2\pi t}{t_1}\right), \\
\text{and} \quad \ddot{\theta} &= \frac{\theta_1}{t_1^2}2\pi\sin\frac{2\pi t}{t_1}. \\
\text{For } t>t_1 \quad \theta &= \theta_F = \theta_0+\theta_1 .
\end{aligned}\right\} \quad (1)$$

The pitch attitude angle θ is the angle of elevation of the aircraft datum line to the horizontal (*see* Fig. 1). θ_1 is the change in attitude from the initial value θ_0, with all wheels on the ground, to the final value θ_F; and t_1 is the duration of the manoeuvre. (It is important to note that the manoeuvre duration t_1 includes the whole of the rotation phase *and* much of the flare-up phase.) Both the final attitude and the time taken to reach this attitude are varied, so that the effect of different techniques, or of piloting errors, can be studied.

Fig. 2 shows the normalised values $(\theta-\theta_0)/\theta_1$, $\dot{\theta}t_1/\theta_1$ and $\ddot{\theta}t_1^2/\theta_1$ plotted *versus* t/t_1. The peak demanded pitch rate is exactly double the mean value θ_1/t_1. An advantage of the function of equation (1), that it can be described by only two parameters θ_1(or θ_F) and t_1, makes it possible to plot the variations of various quantities in terms of the take-off manoeuvre.

2.2. *Equations of Motion during Take-off after Nose-lift.*

The take-off of an aircraft with tricycle undercarriage can be considered in four stages – acceleration to the nose-lift or rotation speed, rotation about the main wheels until lift-off, flare-up or transition to climbing flight, and climb-out. During the first stage, which is essentially a simple problem of performance, aircraft stability and controllability are not important except for directional control and prevention of early rotation. Necessary control movements will have little effect on nose-lift speed and on the time and distance to nose-lift. During the other stages, the attitude and height changes will be sufficiently great for longitudinal stability and control to become important. Among the parameters whose effect on take-off patterns is considered here are rotation (or nose-lift) speed, final attitude and time to reach final attitude. The equations of motion are developed to cover the stages from rotation onwards, although the ground run from rest to any speed is also calculated so that total take-off distance can be found.

A number of considerations suggest that the equations of motion should be used in a non-linear form. During the rotation and flare-up phases, changes in incidence and attitude are large; the aircraft is continuously accelerating; and the influence of the ground (loosely termed the 'ground effect') on the lift, drag and pitching moment is a non-linear function of height. It is then far simpler to express the forces and moments in the form of coefficients which are functions of incidence and height.

Axes must be chosen. For a take-off manoeuvre in which only longitudinal motion is considered, it is convenient to use flight path axes, since much of the aerodynamic data is given relative to this system of axes. No difficulties arise in the formulation of the pitching moment equation because, for symmetric flight, the y-axis in the flight path becomes the same as the body axis, and so the moment of inertia I_y is constant. The equations of motion are then

$$m\dot{V} = X-mg\sin\gamma\,, \quad (2)$$

$$-mV\dot{\gamma} = Z+mg\cos\gamma\,, \quad (3)$$

and

$$I_y \dot{q} = M \,, \tag{4}$$

where

$$q = \dot{\theta} = \dot{\gamma} + \dot{\alpha} \,. \tag{5}$$

The symbols and notation used are defined in Fig. 1. Dots over a variable denote differentiation with respect to time. To include the effect of ground presence on the aerodynamic forces and moment, the following equation is required for the relation between the rate of change of centre of gravity height, $\dot{h}$, and the flight path angle:

$$\dot{h} = V \sin \gamma \,. \tag{6}$$

Equations (2) to (6) apply during both rotation and flare-up. During the rotation phase, however, an extra equation is required to express the condition that the main wheels remain in contact with the ground. Considering motion about the main wheels, this results in the following expression:

$$\dot{h} - l_1 \dot{\theta} = 0 \,, \tag{7}$$

where l_1, the moment arm of the weight about the main wheel contact point, is given by

$$l_1 = d_1 \cos \theta - d_2 \sin \theta \,.$$

The undercarriage is assumed to be rigid. Although this assumption will affect the ground clearance at unstick and the observed value of unstick speed, the errors are unlikely to be large enough to require the conclusion of complicated undercarriage dynamics.

Before lift-off, $\dot{h}$ is determined from (7) and γ could then be found from (6) but $\dot{h}$ is so small at this stage of the manoeuvre that γ and $\dot{\gamma}$ can quite justifiably be neglected. After lift-off (7) is discarded.

In addition to equations (2) to (7), which completely determine the aircraft motion, further equations are necessary to provide other useful variables:

c.g. height	$h = h_0 + \int \dot{h} \, \mathrm{d}t \,,$
wheel height	$h_w = h - l_2 \,,$
rear extremity height	$h_t = h - l_3 \,,$
horizontal distance travelled	$s = \int_{V_r}^{V} V \cos \gamma \, dt \,,$

where l_2, the height of the c.g. above the main wheel ground contact point, and l_3, the vertical distance between the c.g. and the rear extremity, are given by

$$l_2 = d_1 \sin \theta + d_2 \cos \theta \,,$$

$$l_3 = d_3 \sin \theta + d_4 \cos \theta \,.$$

h_0 is obtained by putting $h_w = 0$ and $\theta = \theta_0$ (the ground attitude). The extremity height gives the ground clearance of the part of the aircraft most likely to scrape the ground during take-off. The distances d_1, d_2, d_3 and d_4 (shown in Fig. 1) depend only on the geometry and c.g. position of the aircraft (for a rigid undercarriage).

The forces X, Z and pitching moment M in equations (2) to (4) contain aerdynamic and thrust terms, together with contributions from the ground reactions. During flare-up and climb-out the ground reactions are zero and the condition in equation (7) need not be satisfied. X, Z and M are given by

$$X = -\tfrac{1}{2}\rho V^2 S C_D + T\cos\alpha + R(\sin\gamma - \mu\cos\gamma), \tag{8}$$

$$Z = -\tfrac{1}{2}\rho V^2 S C_L - T\sin\alpha - R(\cos\gamma + \mu\sin\gamma), \tag{9}$$

$$M = \tfrac{1}{2}\rho V^2 S c_0 C_m + Td - R(l_1 + \mu l_2). \tag{10}$$

In these equations T, the total thrust, is assumed independent of forward speed; R is the reaction force, normal to the ground, at the main wheels (the reaction at the nosewheel is assumed to be just zero); and μ is the coefficient of rolling friction. The γ terms are included to take account of the path of the centre of gravity during rotation. The thrust line is parallel to the fuselage datum but offset by a distance d, as shown in Fig. 1. The friction terms, included for completeness, did not introduce any difficulties into the computation because a digital computer was used for the calculations. Since making μ zero produced only very slight differences in the results (after rotation) it would seem justified to neglect μ when its inclusion is inconvenient, as in analogue computer calculations.

It is assumed that the lift, drag and pitching-moment coefficients can be represented in the following way.

$$C_L = C_{L1} + C_{L\eta}\eta, \tag{11}$$

and

$$C_D = C_{D0} + K(h)C_{L1}^2 + \Delta C_D(\alpha,\eta), \tag{12}$$

where

$$C_{L1} = C_{L\alpha}(h)(\alpha - \alpha_e),$$

$$C_m = C_{md} + C_{m\alpha}(h)(\alpha - \alpha_d) + C_{m\eta}\eta + C_{m\dot{\alpha}}\dot{\alpha}_\gamma + C_{mq}q_\gamma. \tag{13}$$

In equations (11) to (13), h is the centre of gravity height but in all the results given later any mention of height usually refers to the *wheel* height.

One of the novel aerodynamic features of slender wings is the non-linear lift curve. But in deriving empirical relationships a mean slope has been used which is closer to the actual slope at high rather than low incidences. During the early part of the rotation phase when the incidence is low, the lift force is a small term in the normal force equation and errors in its representation are not serious. For the remainder of the take-off, the incidence is high and the lift force representation accurate. It is not expected that large errors will result from approximating the non-linear lift-curve slope by a mean slope. In expression (11) for C_L, only the lift-curve slope is assumed to be a function of the centre-of-gravity height h. The meaning of the constant α_e is explained in Appendix A.

The drag coefficient, based on a parabolic drag polar approximation, has three components: a constant, C_{D0}, comprising the basic zero-lift drag plus the undercarriage drag; a lift-dependent term which is assumed to be proportional to the square of the lift coefficient C_{L1}; and an elevator drag term $\Delta C_D(\alpha,\eta)$.

While this is not subject to ground effect, it does vary with both the aircraft incidence, α, and the elevator deflection, η. An empirical expression derived from wind-tunnel results, as outlined in Appendix A, is used:

$$\Delta C_D(\alpha,\eta) = 0{\cdot}131\,\eta^2 + 0{\cdot}460\,\eta\alpha + 0{\cdot}015\eta\,, \tag{14}$$

where α and η are in radians. For negative elevator angles, the drag is reduced. Results from the solution of the equations of motion show that, in fact, the elevator drag has only a small effect and little error would be introduced by neglecting it completely.

As well as C_{D0} and $C_{L\eta}$, the aerodynamic terms C_{md}, $C_{m\eta}$, $C_{m\dot{\alpha}}$ and C_{mq} are all assumed to be constant (as the undercarriage is down all the time) and independent of the variables in the equations. Typical values are taken from wind-tunnel tests. It is assumed that any effect of the ground presence on the control powers is sufficiently small to be neglected. The two pitching velocity derivatives also will probably be modified near the ground; insufficient data are available, however, and values without ground effect are used in the computations.

The ground effect is confined to the terms $C_{L\alpha}(h)$, $K(h)$ and $C_{m\alpha}(h)$. Again wind-tunnel results were used to form empirical relationships.

$$C_{L\alpha}(h) = \quad 3{\cdot}15 \left(\frac{h-4{\cdot}9}{h-8}\right) \text{ per radian}\,. \tag{15}$$

$$K(h) = \quad 0{\cdot}325 \left(\frac{h-5{\cdot}3}{h-0{\cdot}4}\right). \tag{16}$$

$$C_{m\alpha}(h) = -0{\cdot}0802\left(\frac{h+24{\cdot}1}{h-3{\cdot}5}\right) \text{ per radian}\,. \tag{17}$$

The derivation and form of these expressions are discussed in Appendix A. They are based, for convenience, on the centre-of-gravity height, h (in feet), although this height is not, of course, fundamental to an essentially aerodynamic effect. A reference centre-of-gravity position is used but there is facility in the computation for changing the c.g. position. For the reference position chosen, the ground presence has its largest effect on $C_{m\alpha}$.

The differential equations (2) to (6) must be solved using the substitutions from (8) to (17) and in conjunction with the attitude time history of equation (1). The condition of equation (7) is used when the ground reaction R is positive. After lift-off the terms in R are dropped from the equations. The process of solution of the equations of motion is started by calculating the elevator deflection which is just sufficient to lift the nose wheel off the ground (i.e. make the nosewheel reaction zero). This is why all elevator time histories start with a step function. Once the nosewheel is about to rise, the attitude time-history is brought into action and the equations of motion are solved by a step-by-step integration process on a Mercury digital computer. (The standard Mercury Autocode subroutine INTSTEP was used.)

The aircraft data used in this paper are listed in Table 1. They have been used to calculate certain basic performance characteristics of the example aircraft. Fig. 3 shows how the elevator angle to lift the nosewheel varies as a function of speed, centre of gravity position and number of engines. The derivation is given in Appendix B. At rotation speeds above 300 ft/sec the deflections are small enough to be applied in practice in less than half a second, assuming a maximum elevator rate of 20 deg/sec. The steady state three-engine climb-gradient as a function of speed is illustrated in Fig. 4 and the ground run to any speed V up to and including V_R in Fig. 5.

3. *Results.*

The results are presented in the form of time histories of important variables for a particular manoeuvre (e.g. Figs. 6, 8, 9), and as summary plots to show how various parameters vary with the nature of the rotation manoeuvre (e.g. Figs. 13 to 15).

3.1. *Choice of Cases.*

A number of cases are studies in which rotation speed, final attitude and manoeuvre duration are taken as the independent variables. Both four engine and three engine take-offs are considered. It is always assumed that engine failure occurs at or before rotation. The dynamic response to engine failure is thus ignored, not because it is trivial, but because a study of the motion requires a different treatment from that considered here. It is a problem which must be studied because it has been shown[9] that the lateral disturbances due to engine failure can lead to longitudinal difficulties.

Rotation speed is varied from 220 ft/sec (130 kt) to 348 ft/sec (206 kt). A large number of cases is examined at rotation speeds of 324 ft/sec (192 kt) and 300 ft/sec (178 kt). Note that for these speeds the Mach number for ISA conditions is about 0·3 and may be sufficient for compressibility effects to appear. Fig. 3 shows the variation with speed of the elevator deflection necessary merely to lift the nosewheel off the ground. To rotate the aircraft requires a bigger deflection. If the maximum up elevator deflection is 25 deg, the minimum rotation speed is 200 ft/sec (130 kt) for a forward c.g. position and full thrust and about 150 ft/sec (89 kt) for a rear c.g. position. Because of the large effect of c.g. position on the elevator angle to lift the nosewheel it would not be possible to apply the appropriate elevator deflection at the start of the take-off run and allow rotation to occur naturally. Small errors in c.g. position or in elevator setting could lead to large errors in rotation speed. Thrust variation has little effect on these elevator requirements. An approximation to the value of the minimum unstick speed V_{mu} may be found by trimming the aircraft on the ground at its maximum ground attitude of 14 deg. The resulting V_{mu} is 273 ft/sec (162 kt). The elevator angle to trim is -10 deg and has a large effect on this speed. For example, if the elevator deflection is zero the minimum unstick speed is reduced to 258 ft/sec (153 kt). The change is roughly 1 knot per degree of elevator.

The duration of the rotation manoeuvre varies from 3 to 7 seconds at $V_R = 324$ ft/sec and up to 10 seconds or more at $V_R = 300$ ft/sec. It must be emphasised that this 'duration' t_1, defined earlier, is the time from the initiation of rotation on the ground to the achievement of a constant final attitude. It is *not* simply the time from rotation to unstick. A simulator study[9] has suggested that 4 to 6 seconds is a suitable duration, a conclusion which is open to question as a result of the present study. In general, the limiting factor is not the maximum possible pitch rate but the pilot's ability to execute a well controlled manoeuvre.

The maximum demanded attitude ranges from 12 to 20 deg. In the NASA work previously mentioned[2] the standard manoeuvre involved rotation at various rates to 13·9 deg *incidence*, which was then held constant. Unfortunately, no attitude time histories were given for comparison with the present work, but incidence-time histories are given here so that comparison can be made.

Throughout this paper a maximum take-off weight of 290 000 lb and a forward c.g. position are used unless otherwise stated. Standard atmosphere conditions at sea level are assumed, with no wind. Aerodynamic and other data are listed in Table 1.

In a tightly controlled manoeuvre, questions of stability or instability tend to become insignificant but will reassert themselves if, and to the extent that, pilot attention is directed to some other task. For these reasons it is useful to look briefly at the longitudinal stability characteristics of the basic aircraft.

For the aircraft data of Table 1 and Appendix A, and a forward c.g. position, the solution of the stability equation yields the conventional short period and long period oscillatory modes. Free-air values of $C_{L\alpha}$, K and $C_{m\alpha}$ are used. The 'short-period' oscillation is well damped ($\zeta = 0{\cdot}72$) and has a period of 12 seconds. This relatively long time is caused by the low value of the aerodynamic restoring-moment derivative $C_{m\alpha}$ and will make the response to elevator movement rather sluggish. The other mode has a period of 90 seconds and, mainly as a result of the low stiffness and the offset thrust line, is unstable, with a time to double amplitude of 360 seconds. Such a feeble divergence should not produce any difficulty but could, if necessary, be eliminated by appropriate autostabilization.

3.2. *Time Histories.*

3.2.1. *Four engines and three engines.* A typical four-engine take-off is shown in Fig. 6 (full line). Rotation is initiated at 324 ft/sec and takes 5 seconds to achieve a final attitude of 16 deg. Until lift-off occurs at $t = 2{\cdot}6$ sec, the velocity increases steadily at 9 ft/sec^2. Soon after unstick the acceleration reduces to about 1 ft/sec^2 but it then increases during the climb-out to more than 2 ft/sec^2. At lift-off the incidence is 9·7 deg and is increasing. It reaches a maximum of 13·3 deg about $1\frac{1}{2}$ seconds later and then slowly declines to a steady value of just over 11 deg, thus giving a climb angle slightly less than 5 deg (8·7 per cent). The manoeuvre must provide for the incidence to be increased above the unstick value because of the ground effect. If the incidence is not increased above this value, the aircraft will lift-off but will only gain height slowly. The elevator angle time history is somewhat kinky. It starts with a step change of $-6{\cdot}8$ deg, this being the deflection needed to lift the nosewheel off the ground at 324 ft/sec (see Fig. 3) and reaches $-13{\cdot}5$ deg. A kink in the curve occurs at lift-off because no undercarriage dynamics are represented and therefore when the ground reaction R becomes zero, its rate of change is still finite. A second kink in the curve occurs at the end of the rotation ($t_1 = 5$ sec) because the demanded pitching acceleration does not terminate with zero slope. Once the manoeuvre is ended, the elevator angle moves slowly to its trimmed position. The elevator angle components after lift-off are shown in Fig. 7 to clarify the origin of the kinks and bumps in the curve. The bump just after lift-off arises because the incidence α increases more rapidly than the ground effect on the pitching moment decreases, as shown by curve H. These kinks would not occur in practice because they are produced by the form of the input function. And since smoothing them out would make little or no difference to the manoeuvre the overall results will not be invalidated. The finite time which would be necessary to apply the initial elevator angle could affect the results slightly, but this time is unlikely to exceed half a second except at minimum rotation speed. Returning to Fig. 6, the normal acceleration rises rapidly to a peak value of $1{\cdot}35g$ before declining slowly to about $1{\cdot}02g$. This peak value is considerably higher than the mean value recorded during operational take-offs of contemporary subsonic jets, although $1{\cdot}6g$ has been noted on occasions. Values much higher than $1{\cdot}35g$ are reached in some of the examples considered here. This aspect of the chosen attitude time history is discussed later. Because of the high peak normal acceleration, a wheel height of 35 ft is reached quite quickly, 5·5 seconds after rotation and only 2·9 seconds after lift-off. The horizontal distance travelled between initiation of rotation and the 35 ft point is 1896 ft. The calculated ground run for any speed up to and including rotation speed is shown in Fig. 5. For $V_R = 324$ ft/sec, $S_R = 5470$ ft, giving a total take-off distance to 35 ft of 7370 ft.

Fig. 6 also shows the effect of an engine failure occurring before rotation at 324 ft/sec, if the same attitude time history is followed. The initial acceleration is now diminished to about 6 ft/sec^2 and after unstick, which occurs slightly later, the aircraft decelerates slowly because it is being constrained to climb out at too steep an angle. An attitude of 16 deg corresponds to a steady speed of 310 ft/sec. The curve of attitude for a given speed in Fig. 4 is so flat that only a slight reduction of attitude, to about 15·5 deg, will give a speed of 340 ft/sec with no deceleration. In Fig. 6, the incidence and elevator angle time histories for three engines are very similar to the four-engine case, although the final incidence is increased to nearly 13 deg. As a result of the lower speed after lift-off, the peak normal acceleration is reduced to $1{\cdot}31g$. A wheel height of 35 ft is reached 5·9 seconds after rotation (3·2 sec after lift-off) and 1990 ft from the point of rotation. The table below compares the two cases.

$V_R = 324$ ft/sec, $\theta_F = 16$ deg, $t_1 = 5$ sec					
Engines	V_{LOF}	V_{35}	Total S	S_R	γ_2
4	346	351	7370	5470	4·7 deg
3	339	337	8210	6220	3·2 deg

γ_2 is the climb gradient measured at $t = (t_1 + 5)$ sec and is intended to be an approximate indication of the steady value. In calculating the total take-off distance for the three-engine case, engine failure is assumed to occur at 275 ft/sec. Most of the 840 ft increase in the distance to 35 ft occurs on the ground (*see* Fig. 5). The distance from rotation to 35 ft is only increased by 94 ft. It is of particular interest to compare the achieved climb gradient, for one engine inoperative, with the value predicted from steady flight conditions (Fig. 4). At a speed of 335 ft/sec the predicted gradient is 4·2 per cent, whereas the actual value is 5·6 per cent (3·2 deg), albeit with a slight deceleration of $-0{\cdot}4$ ft/sec^2.

The results of variations in the three basic parameters will now be illustrated.

3.2.2. *Final attitude.* Fig. 8 shows the effect of changing the final attitude by ± 4 deg relative to the datum four-engine case of Fig. 6. Decrease of final attitude increases the forward acceleration in exchange for a reduction in normal acceleration and climb gradient. The obstacle clearance height is therefore achieved later. Increase of final attitude produces a slight deceleration after lift-off, and then the speed stays constant at 340 ft/sec. It will ultimately increase again because the steady attitude appropriate to 340 ft/sec is higher than 20 deg. The applied normal acceleration is very high. Despite the sharpness of the manoeuvre, however (peak pitch rate is 7·2 deg/sec), the most extreme elevator deflection is only -15 deg and ample ground clearance is maintained. Although the variations between the elevator time histories are fairly small, the achieved manoeuvres differ considerably.

The results of Fig. 8 are the first indication of the sensitivity of the manoeuvre to different inputs. The difference between the manoeuvres with final attitudes of 16 deg and 12 deg is much more serious than the difference between the 20 and 16 deg manoeuvres. While unstick speed is almost unchanged, and would give no warning of these variations, there are marked changes in the normal acceleration. Questions of variability are discussed later in Section 4.

3.2.3. *Manoeuvre duration.* A short duration manoeuvre requires a very sharp application of elevator with a rapid reversal to check the motion (*see* Fig. 9). Such a severe control movement is unreasonable because it could lead to pilot-induced oscillations. Clearly manoeuvre durations of 5 seconds or longer would place no great burden on the pilot. As the manoeuvre becomes longer, the speed achieved at 35 ft increases, although the subsequent acceleration, 2 ft/sec^2, is about the same for the three cases illustrated. Increasing the duration decreases the peak incidence slightly and the peak normal acceleration quite considerably. Because it has the highest normal acceleration the most rapid manoeuvre has the shortest take-off distance to 35 ft. The longer manoeuvre could be taken to a higher final attitude for a given steady climb speed.

These results from Sections 3.2.2. and 3.2.3. are collected in the Table below.

V_R = 324 ft/sec. Four engines

θ_F	t_1	V_{LOF}	V_{35}	S	γ_2	n_{max}
16	5	346	351	7370	4·7 deg	1·35
20	5	344	342	7105	7·6 deg	1·50
12	5	350	371	8250	1·7 deg	1·20
16	7	355	362	7800	5·3 deg	1·29
16	3	338	342	6910	4·1 deg	1·50

3.2.4. *Rotation speed.* The results so far show that a longer manoeuvre gives a higher speed at 35 ft, lower maximum normal acceleration and higher climb gradient. For the variable rotation speed cases, therefore, the datum manoeuvre is assumed to have a duration of 7 seconds (rather than 5 seconds) with a final attitude of 16 deg for four-engine take-offs, and 14 deg for three-engine take-offs. This reduction in attitude is made to prevent deceleration after unstick.

Four rotation speeds are considered, ranging from 280 ft/sec (166 kt) to 348 ft/sec (206 kt). The flight path and speed variations are shown in Figs. 10 and 11. Reduction of rotation speed reduces the take-off distance but also the speed at a given height. Fig. 12 shows time histories of attitude, incidence, elevator angle and normal acceleration for the four-engine take-offs. A rotation speed of 280 ft/sec is still well above that for minimum four-engine take-off distance.

To decide whether a reduced rotation speed is feasible, it is important to examine what happens during a take-off with one engine failed (Fig. 11). As V_R is decreased, V_{35} is reduced, the unstick incidence increases but the take-off distance decreases. The 'steady' climb gradient γ_2 falls with V_R but only when V_R is as low as 300 ft/sec does the initial flight path begin to droop noticeably (Fig. 11). The total take-off distance to 35 ft is a minimum near $V_R = 300$ ft/sec. For a still lower rotation speed (280 ft/sec) the aircraft unsticks easily at 306 ft/sec but, as it climbs away and loses the lift increment due to the ground, the high climb gradient cannot be maintained and the aircraft rises only very slowly. The take-off distance is increased considerably. An additional case for $V_R = 310$ ft/sec has been computed. It shows that this drooping tendency only sets in for lower rotation speeds, when the unstick speed is close to the speed for zero rate of climb (in free air) of 295 ft/sec. With the possible difficulties of the dynamic situation added it is unlikely that much reduction in V_R could be contemplated.

3.3. *Summary Plots.*

A number of cases have been computed, with different values of θ_F and t_1, for both four-engine and three-engine take-offs. An attempt has been made to extract most of the useful information in the form of summary plots, such as Figs. 13–15, where various quantities are plotted in the form of carpets with θ_F and t_1 as independent variables. V_R is constant at 324 ft/sec.

As mentioned earlier, the normal acceleration reaches very large values. For many of the cases considered here, n_{max} is of the order $1{\cdot}4g$ (Fig. 13). It varies linearly with final attitude and increases rapidly for the shorter duration manoeuvres. Similarly, the maximum (negative) elevator angle increases sharply with reduced manoeuvre duration. For a case calculated with a simple pitch rate autostabilizer*, with $G_q = 1{\cdot}0$, an additional up elevator movement of nearly 4 deg was required (*see* Fig. 7). For the pitch rate autostabilizer to be effective during take-off it would require an authority of about 6 deg to cope with these fairly high rotational rates used. The maximum incidence varies between about 11 and 16 deg. It increases with final attitude and decreases with manoeuvre duration, in an approximately linear fashion in each case. The exchange rate is about 0·6 deg incidence for 1 deg attitude.

The conditions at lift-off are shown in Fig. 14. The speed at unstick varies from 337 ft/sec to 355 ft/sec, the variation depending mainly on the manoeuvre duration. If the same unstick speed is required with three engines as with four, the manoeuvre duration needs to be extended by about 2 seconds. The time taken from rotation to unstick is plotted in the middle picture. Not surprisingly, this time interval is a linear function of the manoeuvre duration, with $\Delta t_{LOF} \simeq \frac{1}{2}\Delta t_1$, while it varies only slightly with final attitude. Take-offs with three engines take only slightly longer to unstick than with four. At unstick the elevator angle is usually between -5 and -10 deg, thus decreasing the available C_L by as much as 0·1. For a given take-off speed the take-off incidence can vary by half a degree, depending on the nature of the manoeuvre. The lower incidence occurs for lower final attitude.

Although the airworthiness requirements normally specify the performance of an aircraft in the take-off configuration, only one of the criteria must be satisfied during an actual take-off. This is that the speed at a height of 35 ft must not be less than some specified value V_2. The speed at 35 ft and the distance from the start of rotation until a height of 35 ft is reached are shown in Fig. 15 for $V_R = 324$ ft/sec. With one engine inoperative, V_2 is taken to be 340 ft/sec and will be achieved if the final attitude is less than 15 deg for $t_1 = 5$ sec or if t_1 is greater than 6 sec for a final attitude of 16 deg. It may then be deduced that the distance from rotation (V_R) to 35 ft will not be less than 2200 ft. The bottom plot in Fig. 15 shows that a final attitude of 16 deg achieved in 6 sec or more gives a higher climb gradient than does an attitude of 15 deg or less in 5 sec.

One of the aims of this study is to see what parameters can be changed to reduce the total take-off distance. The previous results show that this can be achieved either by reducing the rotation speeds or by using a short sharp manoeuvre (e.g. 20 deg final attitude in 5 seconds). At rotation speeds in the region of 300 ft/sec the ground run varies by nearly 35 ft for every ft/sec change in V_R. The effect of V_R on take-off distance and speed at 35 ft is shown in Fig. 16.

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*The assumed control law is $\eta = G_q\, q$.

For a given manoeuvre, both V_{35} and S decrease as rotation speed is reduced from 324 ft/sec. The take-off distance does not, however, decrease indefinitely with decreasing rotation speed because at a certain stage the airborne performance is so poor that the airborne distance begins to increase faster than the ground run decreases. For four engines the minimum take-off distance occurs at a V_R of 260 ft/sec and for three engines at 300 ft/sec, for the appropriate manoeuvres of Fig. 16. The rotation speed for minimum distance decreases slightly as the manoeuvre duration increases (for a given final altitude) but the actual minimum distance increases, an effect also found by Hall[2]. The three-engine rotation speed for minimum distance appears to be independent of the choice of engine failure speed, assuming always that it is less than V_R. The three engine minimum distance is 8 per cent less than the value at $V_R = 324$ ft/sec and occurs when V_{35} is 330 ft/sec. This is 1·12 times the speed for zero rate of climb with the undercarriage down (*see* Fig. 4). A point of interest is that the choice of manoeuvres in Fig. 16 has resulted in the three engine take-off distance being equal to 1·15 times the four engine distance, for a given V_R above that for minimum distance. This factor of 1·15 just happens to be the same as that specified in the airworthiness requirements.

When S is plotted against V_{35}, as in Fig. 17, it is apparent that final attitude is much more important than manoeuvre duration in determining total take-off distance as a function of speed at 35 ft. This is true provided t_1 is neither too short or too long. Furthermore, the curves of distance against V_{35} merge for increasing values of V_{35} beyond the speed for minimum distance appropriate to a particular attitude. This appears to be true for both three and four engine results*. For a given manoeuvre duration the rotation speed (and hence the unstick speed) for minimum distance decreases as the final attitude increases and the actual minimum distance decreases. For speeds below the minimum distance speed the distance rises sharply and a particular combination of attitude and duration seems to produce a nearly constant value of V_{35}.

To supplement these results, Fig. 18 shows, for $V_R = 324$ ft/sec, the effect of normal acceleration on the speed at 35 ft and on the airborne part of the take-off distance. If V_{35} and airborne distance are plotted against n_{max} it is possible to draw a single trend line irrespective of the particular manoeuvre of the type used here. This is till true if the airborne path is plotted against the mean effective normal acceleration, $\bar{n}$, but is not true for V_{35}, which is affected quite noticeably by the manoeuvre. It can be seen in Fig. 18 that reducing $\bar{n}$ from $1{\cdot}3g$ to $1{\cdot}1g$ increases the airborne distance by 700 ft, which is nearly 10 per cent of the normal four-engine total take-off distance.

Some other aspects of the effect of rotation speed are summarised in Fig. 19 for the four engine manoeuvre case. The unstick speed increases linearly with rotation speed with a slope of 0·92. Variations in V_R will therefore change the unstick speed by nearly the same amount. Within a certain range of rotation speeds, V_{35} also varies linearly with V_R and the speed increment between unstick and 35 ft is nearly constant at 7 ft/sec (4 knots). At $V_R = 300$ ft/sec the unstick incidence is about 10 deg and varies by nearly $\frac{1}{2}$ deg for every 10 ft/sec change in V_R.

3.4. *Other Effects.*

In addition to all these variations in the nature of the manoeuvre, a few variations have also been made in the aerodynamic and other assumptions using, as the datum manoeuvre, a rotation at 324 ft/sec to a final attitude of 16 deg in 7 seconds.

First, the centre of gravity position was changed. So far, all the calculations have been with the c.g. at its forward limit of $0{\cdot}50c_0$. If the c.g. is moved aft, to $0{\cdot}53c_0$, only slight changes in the take-off behaviour occur (Fig. 20) for the same attitude time history. Unstick occurs slightly earlier and a higher normal acceleration is pulled, with the result that the 35 ft point is reached after a take-off run 140 ft shorter than for the forward c.g. case. The only big change is in the general level of the elevator movement, which is made more positive by the rearward c.g. shift. A maximum positive deflection of 7 deg occurs, which

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*Not all the data points in Fig. 17 are for the attitudes and durations shown – since only a few manoeuvres were examined for a wide range of rotation speeds – but nevertheless most points lie on or close to the curves.

is very close to the suggested maximum available of 10 deg. Thus the maximum up elevator movement, which usually occurs soon after the initiation of rotation, is likely to be needed when the c.g. is at its forward limit. Conversely, the maximum down elevator will be needed at the aft c.g. limit. This is particularly so at the lower weights, when the trim deflection is more positive and more down (positive) elevator must be used to check the pitching motion and prevent high normal accelerations. The changes relative to the final trim setting are important: with the c.g. at 0·50 c_0 the initial elevator deflection is then 9 deg while at 0·53c_0 the value is 5 deg.

Second, to clarify what influence the ground effect has on the performance and necessary pilot action, an example has been calculated with the ground effect assumed to be negligible. The empirical relationships in equations (15) to (17) are based on measurements at only three heights and there is no theoretical basis for their functional forms. If the ground presence has a large effect, accurate measurement and representation are important. When the free-air values of $C_{L\alpha}$, K and $C_{m\alpha}$ are used throughout the take-off, lift-off occurs slightly later, at 2 deg higher incidence, and the distance from the rotation point to 35 ft is increased by 270 ft (3·5 per cent). Maximum incidence reached is also increased by 2 deg, to nearly 14 deg. As might be expected, the most significant changes occur in the elevator movement required to take-off (Fig. 21). With no ground effect, the initial elevator angle required to raise the nose wheel is increased. This is because, for the assumptions made in this Report, the presence of the ground produces a positive (nose-up) pitching-moment increment at low incidences (compared with a negative increment at high incidences). Following this increased pull to start the rotation more push is required to check the motion, as the retarding influence of the ground effect is absent. These results show that the ground presence has the beneficial effect of smoothing and reducing the elevator demands, as well as improving the performance, and the changes are sufficiently marked for accurate representation of the ground effect to be important.

Fig. 22 is included to show a typical variation of forward acceleration and lift coefficient during the take-off, particularly the contribution of the ground effect to the lift coefficient. The forward acceleration, initially about 9 ft/sec^2 (0·28g), remains steady for 2 sec until, with the attitude starting to increase noticeably, it falls sharply to just over 1 ft/sec^2. Thereafter, for the particular manoeuvre shown, the acceleration increases to a steady value of about 2 ft/sec^2. This acceleration could be traded for climb gradient by increasing the attitude, since for a steady speed of 370 ft/sec the equilibrium attitude is nearly 20 deg.

4. *Discussion.*

4.1. *Variability and Error Cases.*

4.1.1. *Variability.* A point of considerable concern with aircraft of the type considered here, requiring large incidence changes to take-off and having a high induced drag, is the effect on the achieved performance of deviations from the normal procedure.

An American paper[2], for example, showed that serious increases in take-off distance were produced by under-rotation. The paper assumed that, starting at a certain speed V_R, the aircraft rotated at a steady rate up to an *incidence* which was then held constant. It was found that reducing the final incidence by 1·4 deg increased the take-off distance to 35 ft by 900 ft, nearly 15 per cent of the normal distance.

This is a dangerous extension of the take-off run for such a small incidence error, but assessment of the result must be qualified by the comment that, since the rotation speed used is such that 95 per cent of the maximum permissible incidence must be attained before lift-off occurs, the manoeuvre will inevitably be extremely sensitive to a reduction in the maximum achieved incidence. Comparison with the present work is difficult because here it is attitude that is varied and not incidence. But at $V_R = 260$ ft/sec (the rotation speed for minimum four engine distance when $\theta_F = 16$ deg, $t_1 = 7$ sec) a reduction of θ_F by 2 deg decreases α_{max} by 1·4 deg and increases S by 1059 ft (17 per cent). For $V_R = 300$ ft/sec the corresponding figures are 1·2 deg and 346 ft (5·0 per cent), and for $V_R = 324$, 1·1 deg and 250 ft (3·1 per cent). At practical operating speeds, therefore, the sensitivity to under-rotation is much reduced but it is still necessary to consider the effect of errors.

4.1.2. *Error cases.* It has been proposed[11] that the certificated take-off performance of supersonic transport aircraft should in part be determined by certain error cases. These cases, usually considered with all engines operating, are deemed necessary for two reasons. Operational statistics on current jets show that appreciable errors may occur in the speed at which rotation is initiated. And once rotation is started the pilot may misjudge his attitude, since there is at present no satisfactory attitude indicator in widespread use. To ensure that misjudgement of speed or attitude will not lead to a hazardous situation in terms of take-off distance, four exceptional procedures have been suggested. Briefly these are:

(a) rotation 3 seconds early;

(b) rotation 3 seconds lare, both relative to the normal rotation speed;

(c) checked rotation at the normal V_R;

(d) rotation at such a speed that the aircraft lifts off at an angle 2 deg greater than that at which it would lift-off in the 'normal' take-off procedure.

The late rotation and checked rotation cases are likely to be the most important. Regarding the checked rotation case, Ref. 11 describes as a 'common occurrence' the situation where rotation is stopped early and the aircraft runs along the ground, at something less than the normal unstick attitude, until lift-off occurs or further rotation is initiated. It has therefore been suggested that error case (c) should comprise rotation at the normal V_R to an attitude 2 deg less than the normal unstick attitude, the attitude to be held until 'it is readily apparent to the pilot that an error exists', when rotation is recommenced. 'Readily apparent' is defined as reaching a speed 10 knots above the normal unstick speed.

An important factor in the consideration of these error cases is that of tyre limit speeds. A suggested value for the 'never-to-be-exceeded' tyre limit speed is about 220 knots or 370 ft/sec. To cope with emergencies and errors, such as late rotation, a factor of saftey must be applied to this figure. A likely value for the normal maximum unstick speed is then about 205 knots (346 ft/sec). These figures are true ground speeds and not equivalent or indicated air speeds. For the 'standard' four-engine manoeuvre used here (*see* Fig. 19) the absolute tyre limit speed occurs when V_R is 337 ft/sec (200 knots). The normal maximum V_R would then need to be 3 seconds earlier than this (i.e. 310 ft/sec) to allow for the late rotation case (b). It can also be shown that the checked rotation case, described above, satisfies the absolute limitation when V_R = 310 ft/sec. It would seem, therefore, that for the particular aircraft characteristics considered, the target V_R should not exceed 310 ft/sec (184 kt). This value of V_R is close to that for minimum take-off distance after an engine failure (*see* Fig. 16) and is such as to enable the safe flying speed just to be achieved.

If, as a result of the above discussion, the datum 4-engine take-off is taken as rotation at 310 ft/sec to a final attitude of 16 deg in 7 seconds, the effect of each error case may be determined. The results for these cases have been obtained from Figs. 16, 17 and 19, and are given in the Table.

Case	N	V_R	V_{LOF}	V_{35}	S	F	FS
Basic	4	310	342	350	7320	—	—
Basic	3	310	334	339	8470	1·0	8470
(a) Early rotation	4	283	318	328	6550	1·05	6870
(b) Late rotation	4	337	367	374	8290	1·05	8700
(c) Checked rotation	4	310	366	370	8320	1·05	8740
(d) Over rotation	4	268	305	316	6280	1·05	6600
(e) $V_{35} = V_2(3E)$	4	310		340	7000	1·15	8050
(f) $V_R - 5$ kt	3	302		332	8270	1·0	8270

N is the number of engines and F is a safety factor applied by the airworthiness authorities. Cases (a) to (d) are defined above, while (e) and (f) are two other suggested error cases. In (e), the speed at 35 ft is allowed to drop to the 3-engine take-off safety speed V_2, while (f) is a three engine take-off in which rotation is commenced 5 knots early.

Of all these cases, the checked rotation and late rotation are almost equally serious, followed by the one engine failed cases. A comparison between the checked rotation and the basic four-engine manoeuvres is shown in Fig. 23. The second phase of the checked take-off, which begins at $t = 6$, involves rapid movement of the elevator, with $\dot{\eta} \simeq 15$ deg/sec, and a very high normal acceleration. Both the early and the over rotation cases require less take-off distance than the standard 4-engine case.

4.1.3. *Two stage and continuous manoeuvres.* A factor which may contribute to this sort of variability is the piloting technique. It has sometimes been thought that it would be a good technique if the pilot attempted to achieve a steady attitude at unstick speed before initiating further increase of attitude to flare-up. This is called here a 'two-stage' manoeuvre and is compared with the 'continuous' manoeuvre, used throughout this Report, in which no break or pause of any sort is made at unstick. Results for the two types of manoeuvre are shown in Fig. 24. Because the attitude changes in the two-stage manoeuvre must take place more quickly than in the continuous one, higher pitch accelerations and therefore larger and more rapid elevator movements are necessary. The speed time-histories are similar but the overall distance to 35 ft is increased for the two-stage manoeuvre by 600 ft or nearly 8 per cent. It is not surprising that the two-stage manoeuvre is inefficient because, after unstick, effort must be exerted to increase the incidence and flare-up, while the continuous manoeuvre is increasing its attitude at almost its maximum rate. This difference is vividly illustrated by the curves of normal acceleration. A further contribution to the increased take-off distance in the two-stage example is the one second time lag between the start of the second stage and lift-off. At point A (*see* Fig. 24) the aircraft is about to unstick, but application of elevator to increase the attitude decreases the lift so that more speed and more incidence are necessary to lift-off. The Table below summarises the results for the two cases.

	Distance (ft) to			Achieved	
	V_R	V_{LOF}	V_{35}	V_{LOF}	V_{35} ft/sec
Continuous	4660	6340	7630	346	356
Two-stage	4660	6850	8220	351(346)	363
Change		+510	+590		

Since the two-stage manoeuvre enlarges the scope for piloting errors and requires an increased take-off distance, it is not considered suitable as a rechnique.

4.2. *Performance Summary.*

To throw some light on the question of what ranges of speeds and manoeuvres are satisfactory, Fig. 25 is plotted to summarise the performance results. This figure shows the variation of total take-off distance as a function of lift-off speed and manoeuvre, for both full thrust and one-engine-failed situations. Three important speeds are marked on the base-line. V_{mu} is the minimum unstick speed obtained from trim considerations (*see* Section 3.1), V_{ZRC} is the zero rate of climb speed with three engines, obtained from Fig. 4, and the tyre limit speed is the normal limit discussed in Section 4.1.2.

Three engine take-off distances can vary widely for a given lift-off speed because the minima occur within the normal operating range of V_{LOF}. Thus if the final attitude is slightly lower than intended, the manoeuvre will lie on the wrong side of the minimum. Take-off at a speed 10 kt below the speed for minimum distance can increase the distance by anything from 1250 to 2300 ft (13 to 28 per cent), depending on the manoeuvre. Lift-off at a speed 10 kt higher only increases the distance by 5 per cent. There is less variation for four engine cases because the region of minima is at much lower speeds. This is the effect of the higher thrust. Hence there is a need for sufficient thrust to ensure that in three engine cases the minima are well away from the tyre limit speed. Early lift-off, in both three and four engine cases, requires a reduced rate of rotation if the specified V_{35} is to be attained, and in three engine cases inevitably results in extended take-off distance. Late lift-off can lead to increased distance and/or tyre damage. The pitch rate used must be a function of the *achieved* lift-off speed.

If the three engine results had the thrust appropriate to three engines during the entire take-off instead of only from 275 ft/sec, the shape of the curves in Fig. 25 would not be altered but the distance scale would be increased by 1500 ft. The curves of Fig. 25 show how very sensitive the results are to changes in the thrust-weight ratio. Short take-off distance for three engine manoeuvres with a final attitude of 16 deg are somewhat deceptive because they are obtained at the expense of slight deceleration. The results do show, however, that if the lowest possible three-engine distance is required, it is necessary to flare-up fairly rapidly in order to reach the maximum climb attitude while still in the ground effect. If speed is gained close to the ground, the distance to 35 ft will be unduly prolonged. This may be necessary, however, if climb gradient requirements in later stages of the climb-out demand a higher speed.

4.3. *Satisfactory Take-off Manoeuvres.*

This section attempts to derive some general conclusions about what constitutes a satisfactory take-off manoeuvre. It is assumed that certain conditions must be satisfied. These are listed in the Table below.

	Four engines	Three engines
1. V_{LOF}	$\leqslant$ 346	$\leqslant$ 346 ft/sec
2. V_{35}	$\geqslant$ 348	$\geqslant$ 340 ft/sec
3. S	$\leqslant$ 10000/1·15	$\leqslant$ 10000 ft

Maximum V_{LOF} is set by the tyre limit speed and minimum V_{35} by climb gradient considerations. The maximum three-engine (and factored four-engine) take-off distance is arbitrarily limited to 10000 ft. In the case of three engine take-offs a further limitation is provided by the maximum attitude which can be used without deceleration occurring. This can be found from Fig. 4.

From the previous results (e.g. Figs. 13 to 15) it is possible to derive boundaries in terms of θ_F and t_1, along which the equalities of the table are satisfied. The area enclosed by these boundaries then shows what range of manoeuvres is 'satisfactory'. Such satisfactory areas are illustrated in Figs. 26 and 27 for two rotation speeds.

Several important points are immediately apparent. At a given rotation speed the four engine and three engine zones barely overlap. (This is particularly noticeable at the lower rotation speed.) At different rotation speeds corresponding zones are completely distinct. When $V_R = 324$ ft/sec (Fig. 27), the tyre limit speed severely restricts the four engine cases and vary rapid manoeuvres, using high pitch rates, are necessary. Reducing the achieved speed at 35 ft to the three-engine value only increases the permissible maximum attitude but does not extend the allowable duration. Previous results (*see* Fig. 9) have shown that at this rotation speed the minimum practicable duration is about 5 seconds if very rapid control movements are to be avoided. Thus $V_R = 324$ ft/sec is an upper limit to the rotation speed with four engines, unless the tyre limit speed can be raised.

Lines showing the necessary mean pitch rates* are also included in the figures. For $V_R = 300$ ft/sec the satisfactory four-engine region implies a mean pitch rate of about 1·5 deg/sec and the three engine region a mean rate of about 1·0 deg/sec. For the higher rotation speed in Fig. 27 the necessary mean rates are increased to about 2 deg/sec for three engines and between 3 and 4 deg/sec for four engines. On the basis of these results, the lower rotation speed is better because it enlarges the satisfactory region and reduces the necessary pitch rates. An example of a suitable slow manoeuvre is illustrated in Fig. 28.

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*The mean pitch rate is defined as $(\theta_F - \theta_0)/t_1$, where $\theta_0 = 2$ deg.

From Fig. 26 the precision required can be expressed in terms of mean rate of pitch.

Four engines: 1·5 ±0·3 deg/sec

Three engines: 1·0 ±0·2 deg/sec

The useable range of attitude at a given t_1 is always less than 4 deg, and that of manoeuvre duration at a given attitude is generally less than 3 seconds. Achievement of a satisfactory manoeuvre would therefore require very precise control.

If the conditions defined at the start of this Section are applied to Fig. 25, it is possible to derive further regions within the take-off manoeuvres must lie. These are shown in Fig. 29. Normal acceleration limits are also included, but they hardly affect the conclusions regarding speed variations, which are:

$$V_R = 305 \pm 10 \text{ ft/sec } (180 \pm 5\text{kt})$$

and

$$V_{LOF} = 335 \pm 11 \text{ ft/sec } (198 \pm 6\kappa t).$$

These variations apply to normal operations and not to those error cases discussed in Section 4.1.2. The speed margins, which are set mainly by the three engine performance, may be converted into time margins by noting that the full thrust accelaration is about 9 ft/sec^2 at V_R and 5 ft/sec^2 at V_{LOF}; the figures for three engines are about 6 ft/sec^2 and 3 ft/sec^2. These tolerances are so small that assessment of the performance must include the dynamics of the system, in particular the smoothness of the required control application, the finite time to apply elevator at V_R and the ability of the pilot to achieve a controlled pitching manoeuvre. The results of Fig. 29 imply that longer runways would not help very much, since the tyre limit speed is more restricting than the runway length.

4.4. *Ground Clearance.*

While still on the ground, the aircraft considered here has a maximum permissible pitch attitude of 14 deg. This value is calculated assuming zero bank angle and the undercarriage extension at its static value. At speeds in the region of 340 ft/sec (200 kt), the attitude at unstick is about 10 deg, thus leaving a margin of 4 deg before the rear extremity strikes the ground. If, at the instant of take-off, the pitch rate is high (Fig. 30) all ground clearance could rapidly disappear before the aircraft gains sufficient height to counteract the rotation.

In fact, of all the cases considered at $V_R = 300$ and $V_R = 324$ ft/sec not one resulted in zero ground clearance. In the majority of cases, the minimum ground clearance exceeded 1 ft and was generally about 2 ft. During the most extreme manoeuvre considered, rotation at 324 ft/sec to 18 deg in 3 seconds, the clearance was reduced to 0·14 ft but this required 28 deg of up elevator and the pilot would have experienced more than 0·5g incremental normal acceleration *before* unstick (Fig. 31). It is important to remember that all these figures for minimum ground clearance would be increased, in practice, by 1 ft or more of undercarriage extension as the lift balances the weight.

At all speeds above 220 ft/sec (130kt), the elevator effectiveness is such that if the maximum up deflection is applied rapidly and held, the rear extremity will hit the ground. But the pilot's sensation of vertical acceleration (*see* Section 4.7.) could well act either as a deterrent to excessively vigorous control movements or as an effective indicator of the need for rapid corrective action.

For the 'standard' four engine manoeuvre ($\theta_F = 16$ deg, $t_1 = 7$) V_R could be reduced to 250 ft/sec before ground clearance becomes zero (*see* Fig. 19), at which stage the pilot has used 20 deg of up elevator.

4.5. *Comments on the Prescribed Attitude Time History.*

It has been pointed out in Section 3.2. that the maximum normal accelerations during take-off manoeuvres with the assumed symmetrical forcing function are large, often reaching 1·3g or more for $V_R = 324$ ft/sec. Measurements on subsonic jet transports[1] indicated that although the normal acceleration immediately after unstick might occasionally exceed 1·3g, in about 65 per cent of the cases it was less than 1·2g. The trend towards low maximum g was particularly apparent for take-offs at high weight, but this was probably because the available maximum normal acceleration, limited by the stall, was also low.

The reason for the high normal acceleration produced by the symmetrical manoeuvre used here is that high rates of pitch still exist at the instant of unstick (Fig. 30). They exist because, in most cases, unstick occurs half-way through the manoeuvre (or soon after) when the pitch rate is a maximum (see Fig. 2), and because short manoeuvres are necessary if the rotation speed is high. These high pitch rates at and after unstick could be avoided by using either a modified manoeuvre which is no longer symmetrical or a manoeuvre of the present form with a long duration.

A possible non-symmetrical form, sketched in Fig. 32, has an initially rapid build-up of attitude followed by a period, after unstick, in which the attitude is increased steadily at a low rate. Such a pitch rate time history, obtained from flight tests, is illustrated in Fearnside's paper[7]. A few calculations with an attitude time history of this modified form show that, at unstick speeds in the region of 350 ft/sec, a steady pitch rate of 1·0 deg/sec after lift-off produces a nearly constant normal acceleration of $1{\cdot}2g$.

From a piloting point of view, however, it would be valuable if the techniques before and after lift-off were the same. This would be so if, from rotation, the pilot were to set up a steady rate of pitch and hold it until well after lift-off. Since even 1·0 deg/sec produces a normal acceleration increment of about $0{\cdot}2g$ such a low rate would be necessary (instead of the oft-quoted 3 deg/sec) and would imply a rotation speed of 300 ft/sec or less in order to keep the unstick speed below the normal tyre limit speed of 346 ft/sec (205 kt).

As an illustration of the possibilities, a long duration manoeuvre, with $\theta_F = 16$ deg, $t_1 = 10$ seconds and $V_R = 300$ ft/sec is shown in Fig. 28. The mean pitch rate is 1·4 deg/sec. The main disadvantage of the present function, however, is that the peak pitch rate is twice the mean value. What is wanted are quicker initial and final changes of θ with a steady value in between, so that the mean value is much closer to the maximum value than in the present function. This would, of course, involve more rapid action by the pilot than is shown here, or the assistance of a pitch rate demand system.

4.6. *On the Information Available to the Pilot from his Instruments.*

Both of the main instruments at present used during take-off, the ASI and the artificial horizon, suffer from disadvantages. From the initiation of rotation to the end of the transition the ASI reading can be very unreliable due to errors and lags. The artificial horizon may also be in error as a result of the long period of steady acceleration during the take-off roll.

The accuracy of the ASI indication is of great importance to the pilot because up to and including the point of lift-off the distance travelled is a direct and simple function of the speed. For example, the results obtained during the present work show that the actual distance to lift-off is only very slightly greater (about 50 ft) than the distance obtained assuming no pitch attitude changes. This is because the deceleration resulting from increased incidence is only apparent in the final second before unstick (*see* for example Fig. 22).

In the climb-out phase, over-concentration on attitude might lead to errors because the attitude appropriate to a given steady speed changes very little with speed (*see* Fig. 4). Incidence is much better in this respect.

Although accurate indications from the ASI and artificial horizon would convey valuable information about the current situation, they do not indicate the future behaviour of the aircraft during the take-off. To do this such anticipatory signals as pitch rate or forward acceleration are necessary.

Forward acceleration is a primary variable because it shows whether the attitude is too high or too low and immediately indicates an engine failure. In both three-engined and four-engined flight, a forward acceleration indicator could be used as an energy management device, to assist the pilot in the allocation of the available energy increase to forward acceleration and climb gradient. The achieved gradient for a given speed can differ appreciably from the theoretical steady value at that speed. Maximum gradient at a given speed is obtained for zero forward acceleration (assuming deceleration is not allowed) but because the aircraft is flying well below minimum drag speed a higher ultimate gradient can be obtained by accelerating to a higher speed. This technique has in fact been advocated[10] as being beneficial in reducing fly-over noise at the 4-mile check point. A measure of forward acceleration is the essential ingredient of a take-off director and is already in service in the SCAT system.

To illustrate the limitations of even an accurate ASI and artificial horizon and to show the need for additional instrumentation, the following points are noted. For the four engine manoeuvre of Fig. 16, a 10 knot error in V_R produces less than 8 per cent in take-off distance, provided V_R is greater than the value for minimum distance. An error in V_R would produce a similar change in V_{LOF} (*see* Fig. 19), and hence the speed error would be an indication of the distance error. For a given V_R, different manoeuvres will produce a different take-off distance. For example, at $V_R = 324$, S can vary from 6760 to 8250 ft but V_{LOF} only differs by 13 ft/sec, equivalent to an increase of 29 per cent per 10 knot increase in V_{LOF}. Very small differences in V_{LOF} can be followed by large errors in S. Increase in S due to errors in manoeuvre duration for constant final attitude would be indicated by V_{LOF} but errors in attitude for constant t_1 are not shown by V_{LOF} (but would be by V). Extra long take-off distances result from low final attitudes or very slow manoeuvres; both sorts of manoeuvre would show too high a forward acceleration. At a given V_{LOF}, say 340 ft/sec, a possible variation in S could be 7000 to 8000 ft, a change of 14 per cent. Thus V_{LOF} is not a sufficient indication of the ultimate take-off distance as this also depends on the manoeuvre which is in progress.

An accurate inertial platform could give all the information required for instrumenting the take-off in a variety of possible ways – accelerations, speeds, incidence, attitude and pitch rate would be available.

4.7. *The Pilot's Sensations.*

Because the pilot is situated about 100 ft ahead of the main wheels, he will experience considerable acceleration forces as a result of rotational accelerations. Before unstick, while the aircraft is still on the ground, he will experience a positive incremental normal acceleration which can exceed $0{\cdot}5g$ (Fig. 31) in a very sharp manoeuvre. After unstick he will be subjected to a negative incremental acceleration due to checking the rotation and a positive increment due to the upward acceleration of the aircraft as a whole. Depending on the rapidity of the manoeuvre, the normal acceleration at the cockpit during the flare-up will not only be less than the normal acceleration at the c.g. but the peak value can also be less than the peak value experienced before unstick (Fig. 33). The pilot may well become confused. Analysis of a few cases show that the pilot's sensation of being pressed back in his seat remains fairly steady, because the reduction of forward acceleration due to increasing incidence and drag is balanced by the gravity component due to the changing attitude. Structural bending may also affect the pilot's sensations but has not been included in this Report.

Fortunately, height judgment during take-off is not as critical as during landing[12], so that the fact that the pilot is 35 ft above the main wheels at unstick, as well as nearly 100 ft in front of them, will not be discussed here. But as a consequence of being so remote from the main wheels, the pilot may need some definite indication of the instant of unstick. Even on a Comet, apparently, the pilot does not sense the unstick point at all accurately.

5. *Conclusions.*

Take-off calculations for a slender-wing supersonic transport design have been performed assuming that, from rotation speed on, a particular pitch-attitude time-history is followed exactly. Time-histories of all other relevant variables are derived from the equations of motion. Although most of the results were obtained using only one generalised shape of pitch-attitude time-history, the shape chosen gave reasonably realistic results and could in fact form a basis for performing satisfactory take-offs.

Rotation speeds from 220 ft/sec (130 kt) to 348 ft/sec (206 kt) were used but two speeds received much attention: 300 ft/sec (178 kt) and 324 ft/sec (192 kt). Unstick speeds ranging from 262 ft/sec (155 kt) to 377 ft/sec (224 kt) were obtained but an upper limit of 346 ft/sec (205 kt), set by the tyres, was applied in much of the subsequent analysis. Unstick occurred anything from 13 ft/sec (8 kt) to 48 ft/sec (28 kt) after the start of rotation but, for piloting reasons, *h* lift-off speed about 42 ft/sec (25 kt) above V_R is best. If this speed difference is much less than 25 ft/sec (15 kt) the pilot has to perform very rapid elevator movements to fit the manoeuvre in, because the aircraft accelerates rapidly up to unstick. A speed difference of 42 ft/sec (25 kt) allows about 5 seconds for the rotation phase of the manoeuvre: this implies a pitch rate of about 2 deg/sec and a rotation speed near 304 ft/sec (180 kt). High pitch rates should not be used because of difficulty in checking the rotation sufficiently rapidly to avoid bumping the tail or producing high normal accelerations after unstick. Rapid manoeuvres will also subject the pilot and some of the passengers to high normal accelerations before unstick. If rotation speeds much lower than 300 ft/sec were used, the pilot would have to pull the stick back smartly to lift the nose (because the necessary elevator angle increases rapidly as rotation speed decreases) and then push nearly as smartly to avoid striking the tail on the ground. At all speeds above 220 ft/sec (130 kt), application of the available up elevator deflection of 25 deg is sufficient to provoke a tail-strike but the pilot's sensations due to sitting so far ahead of the centre of rotation should help to prevent this. The assumed maximum down elevator deflection of 10 deg is considered inadequate at aft c.g. to check high rates of rotation soon after unstick and to cope with gusts.

Ground clearance considerations prevent the aircraft from attaining its maximum or even a very high lift coefficient while on or close to the ground but this mainly affects only the minimum unstick speed. The ground effect on the lift and pitching moment is beneficial: it improves the take-off distance and, what is perhaps more important, it smooths and reduces the elevator demands.

The primary aim of this paper was to study how errors in the take-off manoeuvre affected the performance, and to see how accurately the take-off needs to be controlled. The errors considered were early and late rotation, over rotation and checked rotation. In terms of take-off distance, the late and checked rotation cases were equally serious, requiring about 3 per cent (270 ft) more distance than the basic three engine case, and 19 per cent (1400 ft) more than the standard four engine take-off. A very important limitation is imposed by the tyres, which must not normally be run at ground speeds exceeding about 346 ft/sec (205 kt). It has been shown that, if this unstick speed is not to be exceeded while at the same time a specified minimum airspeed is to be attained before a height of 35 ft is reached, the range of satisfactory manoeuvres is small. For a rotation speed of 300 ft/sec (178 kt), the manoeuvres are such that the mean pitch rate is close to $1\frac{1}{2}$ deg/sec (four engines) and 1 deg/sec (three engines). The spread of satisfactory rotation speeds lies between 295 ft/sec (175 kt) and 315 ft/sec (187 kt), and of unstick speeds between 323 ft/sec (191 kt) and 346 ft/sec (205 kt). The lift off speed needs to be as high as possible within this range to give the best climb performance after unstick. More thrust, less drag or a higher tyre limit speed are necessary to improve these speed margins. Because they are small, it may be necessary to include the dynamic characteristics of the aircraft in the assessment of the performance.

For a given manoeuvre of the form considered here, the take-off distance is a minimum for some particular value of rotation (or lift-off) speed, and increases very rapidly with reduction of speed. With four engine take-offs, the region of minima is outside the operating zone, but with three engines the minima occur within the primary zone. There is thus a need for sufficient thrust to ensure that, in the event of an engine failure, the region of minima lies outside the main area of interest.

Since close control of the take-off is necessary if serious errors are to be avoided, the information available to the pilot must be of the right form and easily assimilable. And because the aircraft only responds slowly to elevator inputs, it is important that lags between stick and control surface, and limits on the rate of elevator movement be eliminated as much as possible. It would seem that not only is improved instrumentation needed to tell the pilot *what* to do, but some improved means of control, such as a pitch rate demand system, may also be necessary to help him to do it.

LIST OF SYMBOLS

C_D	Drag coefficient: $D/(\frac{1}{2}\rho V^2 S)$
C_{D0}	Zero-lift drag – includes drag of undercarriage
C_{D1}	$C_{D0}+K\,C_{L1}^2$
C_L	Lift coefficient: $L/(\frac{1}{2}\rho V^2 S)$
C_{L1}	$C_{L\alpha}(\alpha-\alpha_e)$
$C_{L\alpha}$	$\partial C_L/\partial\alpha$
$C_{L\eta}$	$\partial C_L/\partial\eta$
C_m	Pitching-moment coefficient: $M/(\frac{1}{2}\rho V^2 S c_0)$
C_{m1}	$C_{md}+C_{m\alpha}(\alpha-\alpha_d)$
C_{md}	with α_d: co-ordinates of datum point. (*See* Appendix A)
C_{mq}	$\partial C_m/\partial\left(\frac{qc_0}{V}\right)$
$C_{m\alpha}$	$\partial C_m/\partial\alpha$
$C_{m\dot{\alpha}}$	$\partial C_m/\partial\left(\frac{\dot{\alpha}c_0}{V}\right)$
$C_{m\eta}$	$\partial C_m/\partial\eta$
c_0	Reference length
D	Drag
d	Distance between fuselage datum line and line of action of thrust
d_1, d_2, d_3, d_4	Various distances. (*See* Fig. 1)
F	Airworthiness weighting factor
g	Acceleration due to gravity
h	Height of aircraft c.g. above ground
h_0	Height of aircraft c.g. above ground when $\theta=\theta_0$
h_w	Height of main wheels above ground
h_t	Height of rear extremity (tail, nacelles, etc.) above ground
I_y	Moment of inertia about $0y$: $m\,k_y^2$
K	Induced-drag factor : $(C_{D1}-C_{D0})C_{L1}^2$
k_y	Radius of gyration about axis $0y$
L	Lift
l_1, l_2, l_3	Distances (*See* Section 2.2.)
M	Pitching moment about axis $0y$
m	Mass of aircraft
n	Normal acceleration

$\bar{n}$	Mean effective normal acceleration : $\frac{1}{2}\bar{n}g(t_{35}-t_{LOF})^2 = 35$
Q	$\frac{1}{2}\rho V^2$
q	Rate of pitch about axis $0y$
q_γ	qc_0/V
R	Ground reaction at main wheels after nose lift
S	Reference wing area
S	Total take off distance from rest to 35 ft
T	Thrust
t	Time
t_1	Duration of pitching manoeuvre
V	Velocity along the flight path
V_{mu}	Minimum unstick speed
V_2	Take-off safety speed, to be reached at or before 35 ft
V_R	Rotation speed. Speed at which the manoeuvre is initiated
V_{ZRC}	Speed for zero rate of climb with three engines in free air and with undercarriage down
W	Aircraft weight : $W = mg$
X	Force along x-axis
Z	Force along z-axis
$\Delta C_D(\alpha,\eta)$	Increment of drag coefficient due to elevator deflection (*see* Appendix A)
α	Incidence (*see* Fig. 1)
α_d	With C_{md} : co-ordinates of datum point. (*see* Appendix A)
α_e	Incidence for zero lift when $\eta = 0$. (*see* Appendix A)
$\dot{\alpha}_\gamma$	$\dot{\alpha}c_0/V$
γ	Flight path angle or climb gradient
γ_2	Climb gradient measured at $t = t_1+5$ seconds
ζ	Relative damping ratio
η	Elevator angle
θ	Aircraft attitude : angle of elevation of fuselage datum line to horizontal
θ_0	Attitude of aircraft at rest with all wheels on ground
θ_1	Attitude change during pitching manoeuvre
θ_F	Final attitude – reached and held at end of manoeuvre
μ	Coefficient of rolling friction
ρ	Air density

Overscript

	Denotes differentiation with respect to time

Suffixes

R	Refers to instant of start of rotation
LOF	Refers to instant of lift off
35	Refers to conditions at wheel height of 35 ft
max	Maximum value of a variable

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APPENDIX A

Empirical Expressions for the Lift, Drag and Pitching Moment

A.1. *Drag Increment due to Elevator Deflection.*

For the supersonic transport configuration considered, wind-tunnel results show that the increment in drag coefficient due to elevator is a function of both incidence α and elevator angle η. The increment, symbolised by $\Delta C_D(\alpha,\eta)$ in equation (12), has a roughly parabolic variation with η at constant incidence. An empirical relationship fitting the tunnel results is

$$\Delta C_D(\alpha,\eta) = 0{\cdot}131\,\eta^2 + 0{\cdot}460\,\eta\alpha + 0{\cdot}015\,\eta\,,$$

where α and η are measured in radians. Computed results suggest that the elevator drag is not a very large term compared with the other drag terms and little error would be introduced if it is neglected altogether. For the present purposes the small effect of ground presence on ΔC_D has been ignored.

A.2. *The Ground Effect Functions.*

The effect of ground presence on the aerodynamic forces in equations (11), (12) and (13) is incorporated in the functions $C_{L\alpha}(h)$, $K(h)$ and $C_{m\alpha}(h)$. Here h is the height of the centre of gravity above the ground, although there is, of course, no reason why this particular height should be fundamental to the ground effect. However, since the equations of motion are referred to the centre of gravity, it is convenient to look for empirical relationships in terms of the c.g. height. The full expressions for the incidence-dependent lift, drag and pitching moment coefficients are, for zero elevator angle,

$$C_{L1} = C_{L\alpha}(h)(\alpha-\alpha_e)\,, \qquad \text{(A.1)}$$

$$C_{D1} = C_{D0} + K(h)\,C_{L1}^2\,, \qquad \text{(A.2)}$$

and

$$C_{m1} = C_{md} + C_{m\alpha}(h)(\alpha-\alpha_d)\,. \qquad \text{(A.3)}$$

A characteristic feature of slender wings is the non-linear variation of C_L with incidence, which enables low-speed operation to take place at moderate incidences. This non-linear effect is, in general, important; but for the present work a linear variation is assumed to simplify the derivation and form of the empirical expressions. The values of $C_{L\alpha}(h)$ were chosen to give the best fit near α = 12 deg, which is the region of operational interest, since a typical value of unstick incidence is 10 deg. The incidence for zero lift, α_e, is taken to be constant at 2 deg and independent of height. The fitted $C_{L\alpha}$ overestimates the slope at low incidence and underestimates it above α = 12 deg. It is least accurate when the incidence and lift are small but is a good representation after unstick. Only small errors are to be expected from the simplification.

A parabolic approximation (equation A.2) is assumed for the drag coefficient. The wind-tunnel results show that the profile-drag coefficient is unaffected by ground presence but that the induced-drag factor K is a function of h.

The results for pitching-moment coefficient as a function of incidence at various heights can be well approximated by a set of straight lines, all of which happen to pass through a single point, the co-ordinates of which are (C_{md}, α_d). The pitching-moment coefficient can then be very simply written (equation A.3), with only $C_{m\alpha}$ a function of height.

These various considerations enabled the following functional forms to be derived from the wind tunnel results:

$$C_{L\alpha}(h) = 3{\cdot}15 \frac{(h-4{\cdot}9)}{(h-8)} \text{ per radian} \tag{A.4}$$

$$K(h) = 0{\cdot}325 \frac{(h-5{\cdot}3)}{(h-0{\cdot}4)} \tag{A.5}$$

$$C_{m\alpha}(h) = -0{\cdot}0802 \frac{(h+24{\cdot}1)}{(h-3{\cdot}5)} \text{ per radian}. \tag{A.6}$$

It should be emphasised that these are empirical functions, for which there is no theoretical basis. They are valid only for h greater than about 12 ft. In this paper the minimum value of h occurs at full load with the aircraft stationary and its oleos compressed. This minimum value is 13·2 ft and the expressions above are only applicable for values of h greater than this.

One result of the linearised form for C_L is that if the effect of the ground is expressed as the ratio of the C_L increment due to ground over the free air C_L (as suggested by Kirby) this ratio is solely a function of height and independent of incidence. Kirby's results showed a definite dependence on incidence as well as height but they were mainly for slender wings without bodies. The aircraft considered here has a discrete body.

In the absence of ground effect, the limiting values are:

$$C_{L\alpha}(\infty) = 0{\cdot}055 \text{ per degree} = 3{\cdot}15 \text{ per radian},$$

$$K(\infty) = 0{\cdot}325,$$

and

$$C_{m\alpha}(\infty) = -0{\cdot}0014 \text{ per degree} = -0{\cdot}0802 \text{ per radian}.$$

In order to show the magnitude of the ground effect, the ratios of the functions with the ground at its closest to the functions in free air are calculated to be:

$$C_{L\alpha}/C_{L\alpha}(\infty) = 1{\cdot}60,$$

$$K/K(\infty) = 0{\cdot}62,$$

and

$$C_{m\alpha}/C_{m\alpha}(\infty) = 3{\cdot}85.$$

It can be seen that the ground effect is greatest for the slope of the pitching moment-incidence relation.

APPENDIX B

Elevator Deflection to Raise Nosewheel.

At the instant of elevator deflection the aircraft is accelerating along the runway but both incidence and attitude are constant. Thus $\dot{\alpha} = \dot{\theta} = \dot{q} = \gamma = 0$. (Note that a rigid undercarriage is assumed throughout this analysis). Therefore equations (3), (9) and (11) yield

$$W = \tfrac{1}{2}\rho V^2 S[C_{L\alpha}(h)(\alpha-\alpha_e)+C_{L\eta}\eta]+T\sin\alpha+R\,. \tag{B.1}$$

Equations (4), (10) and (13) yield

$$\tfrac{1}{2}\rho V^2 Sc_0[C_{md}+C_{m\alpha}(h)(\alpha-\alpha_d)+C_{m\eta}\eta]+Td-R(l_1+\mu l_2) = 0\,. \tag{B.2}$$

$$\text{Put } Q = \tfrac{1}{2}\rho V^2\,, \qquad C_{L1} = C_{L\alpha}(\alpha-\alpha_e)\,, \qquad C_{m1} = C_{md}+C_{m\alpha}(\alpha-\alpha_d)\,.$$

C_{L1} and C_{m1} are the incidence-dependent values of the lift and pitching moment coefficients. Elimination of the ground reaction R from equations (B.1) and (B.2) gives

$$QSc_0(C_{m1}-C_{m\eta}\eta)+Td-(l_1+\mu l_2)[W-T\sin\alpha-QS(C_{L1}+C_{L\eta}\eta)] = 0\,, \tag{B.3}$$

which may be arranged as

$$\eta = -\frac{[c_0C_{m1}+C_{L1}(l_1+\mu l_2)]+[(W-T\sin\alpha)(l_1+\mu l_2)-Td]/QS}{c_0C_{m\eta}+C_{L\eta}(l_1+\mu l_2)}\,. \tag{B.4}$$

C_{m1} and C_{L1} are found by putting $\alpha = \theta_0$, the attitude with all wheels on the ground. The results of using expression (B.3) are given in Fig. 3. Similarly, the rotation speed for a given elevator deflection may be found by rearranging equation (B.3) in a different way.

$$V_R^2 = \frac{2}{\rho S}\left[\frac{(l_1+\mu l_2)(W-T\sin\alpha)-Td}{c_0(C_{m1}+C_{m\eta}\eta)+(l_1+\mu l_2)(C_{L1}+C_{L\eta}\eta)}\right].$$

For $\eta = -25$ deg and with the data of Table 1, $V_R = 220$ ft/sec (130 kt). If the effect of the ground on C_L and C_m is ignored, this figure rises to 232 ft/sec (137 kt), an increase of 5 per cent. Other variations, such as making $\alpha_d = 2$ deg instead of 4 deg, or $C_{m\eta} = -0{\cdot}15$ instead of $-0{\cdot}175$, also produced increases in V_R of a similar size.

TABLE 1

Aircraft Characteristics

W	290 000 lb
T	4×25000 lb
S	3337 ft^2
W/S	87 lb/ft^2
T/W	0·345 (4 engines)
	0·259 (3 engines)
$c.g.$	0·50c_0 for most cases
k_y	31 ft
μ	0·03
θ_0	2 deg
α_e	2 deg
α_d	4 deg
$C_{L\eta}$	0·587
$C_{m\eta}$	−0·175
C_{md}	0·01
$C_{m\dot{\alpha}}$	−0·17
C_{mq}	−0·32
C_{D0}	0·02
d_1	5·5 for c.g. at 0·50c_0
d_2	13·0
d_3	41·3 for c.g. at 0·50 c_0
d_4	4·06
d	2·5
$c_0 =$	84·4 ft

See Fig. 1 for definition of these distances.

See Appendix A for C_L, C_D and C_m as functions of c.g. height h, incidence α and elevator angle η.

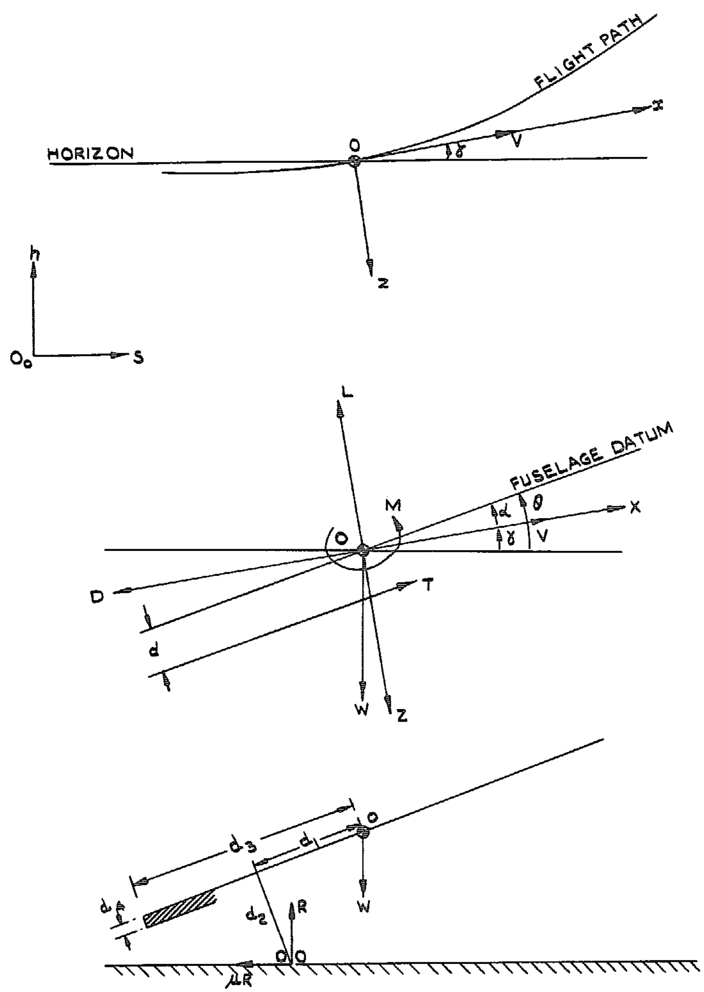


FIG. 1. Axes, forces, angles and geometry.

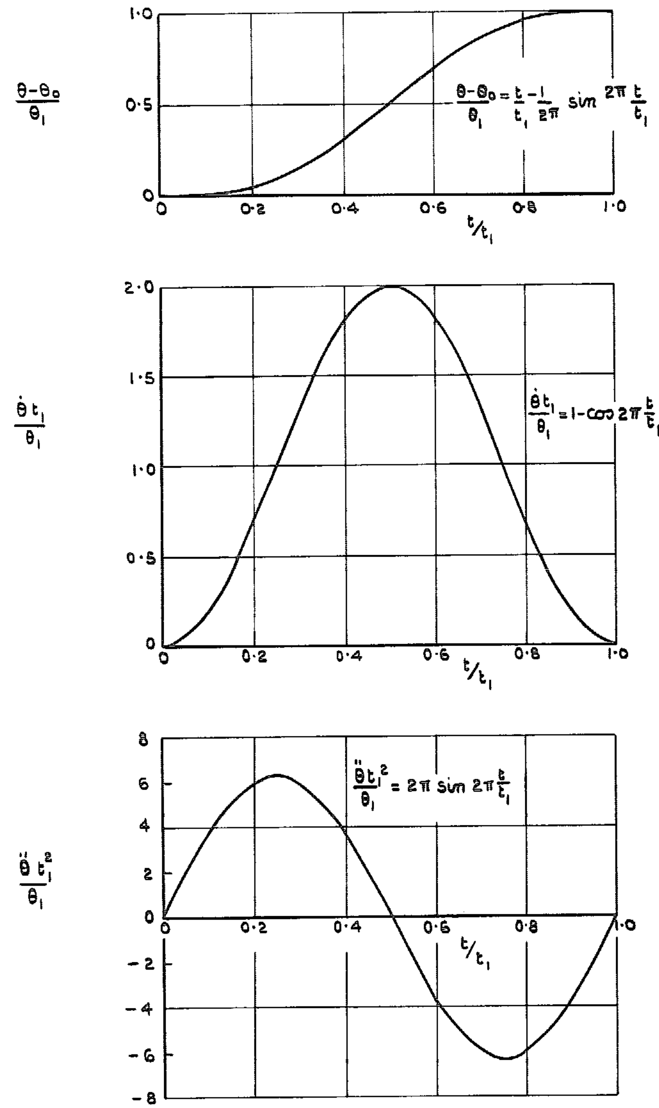


FIG. 2. The specified attitude time history.

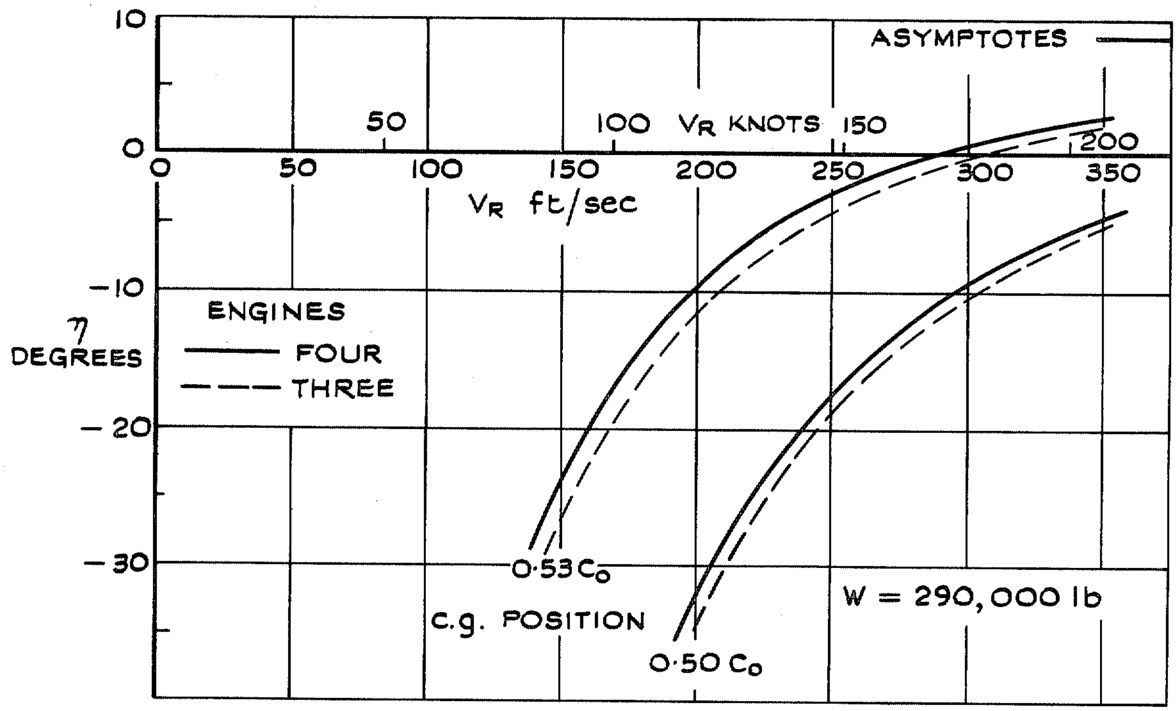


FIG. 3. Elevator angle to lift nose as a function of speed.

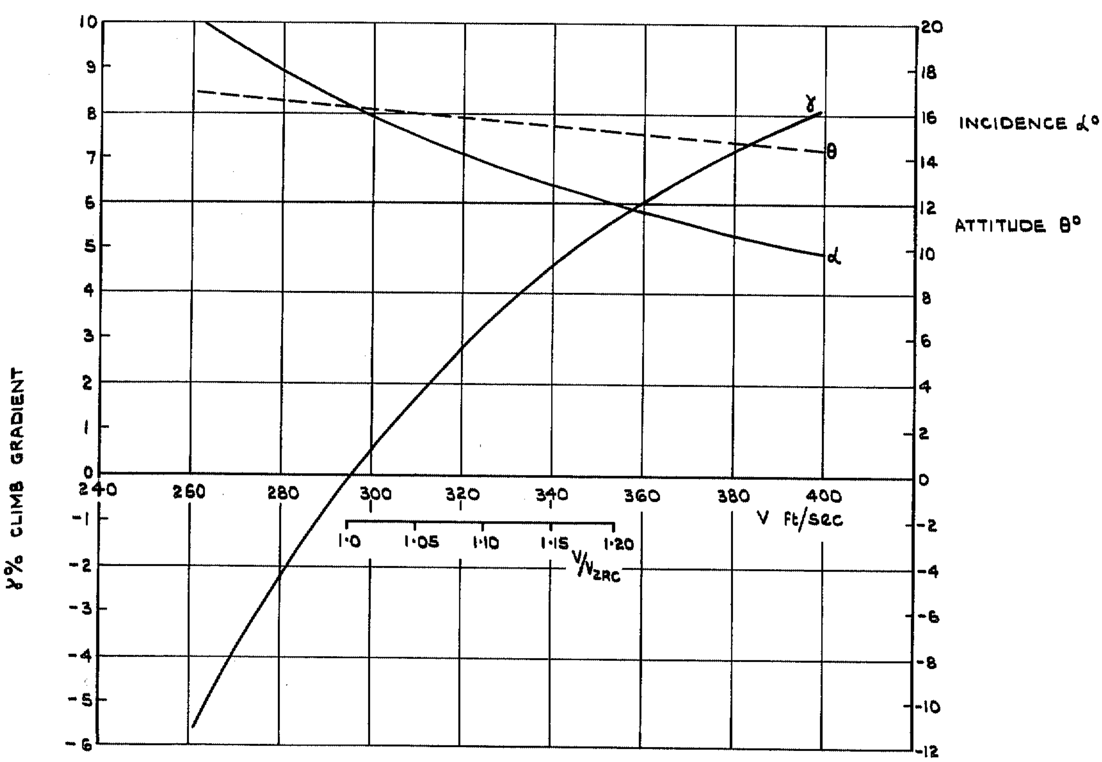


FIG. 4. Three engine climb performance, no ground effect, undercarriage down.

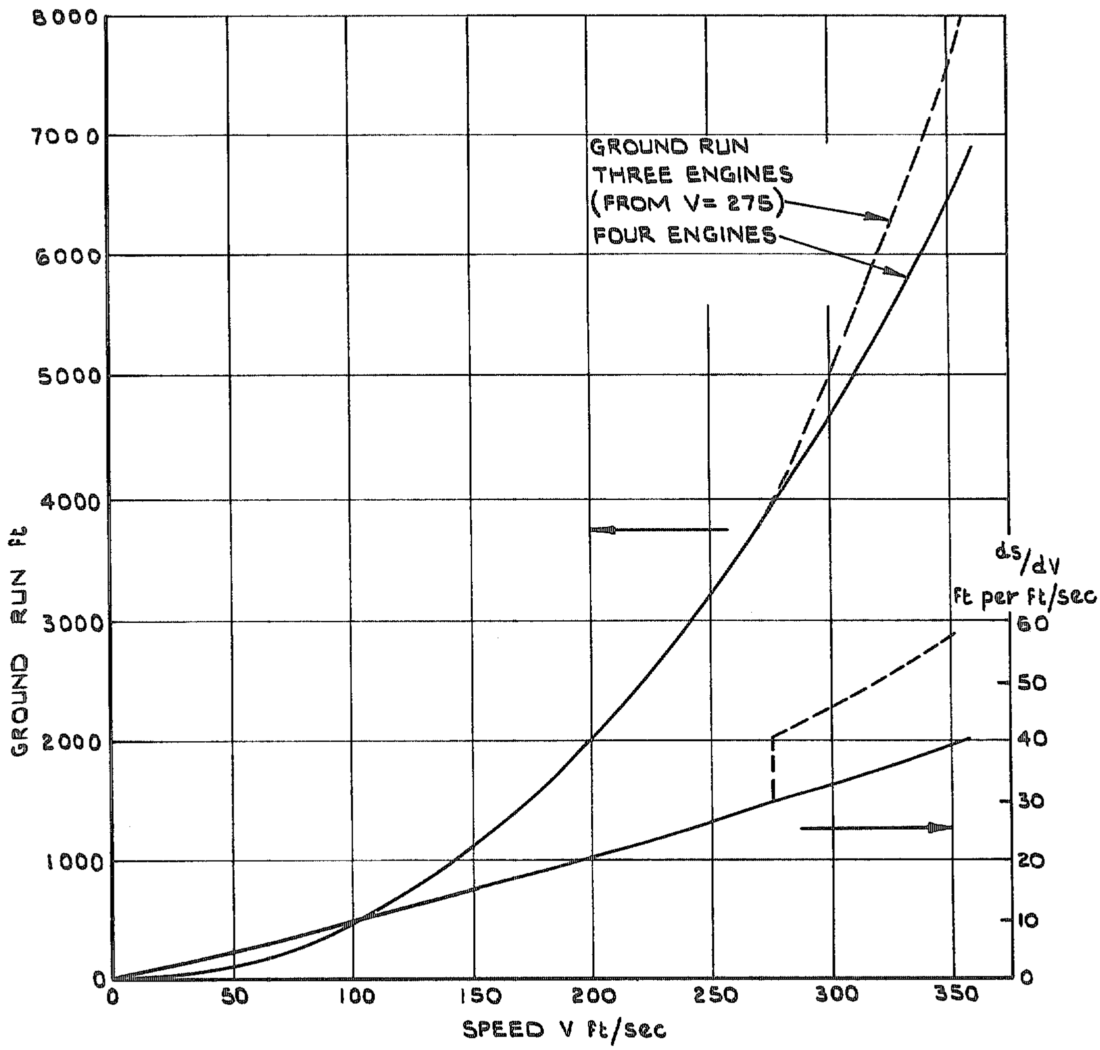


FIG. 5. Ground run from rest to speed $V \leqslant V_R$.

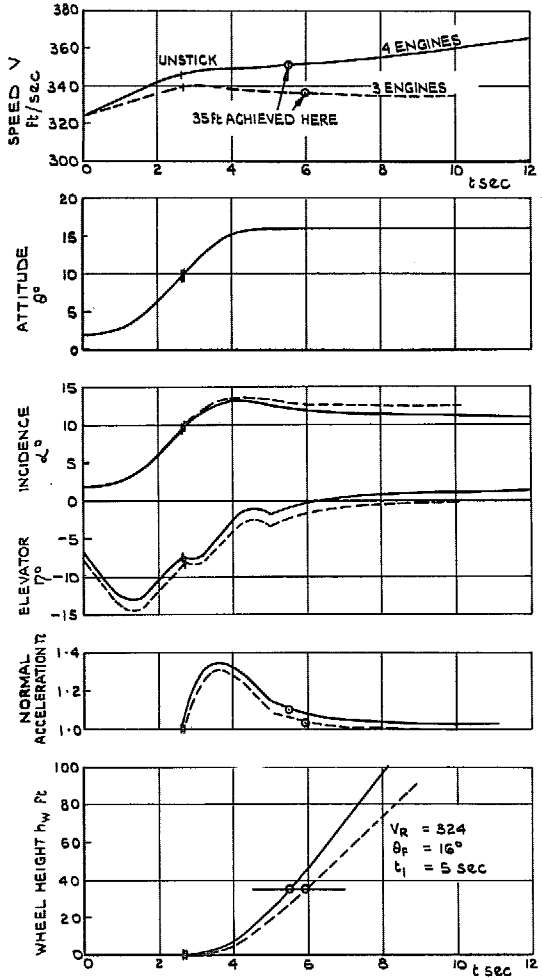


FIG. 6. Typical take-off behaviour including the effect of engine failure.

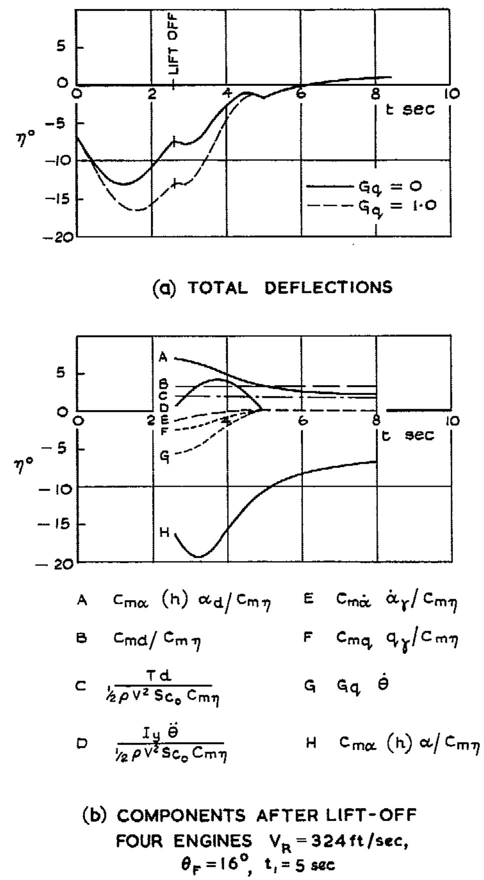


FIG. 7. Elevator time histories with and without simple pitch autostabilisation, showing components.

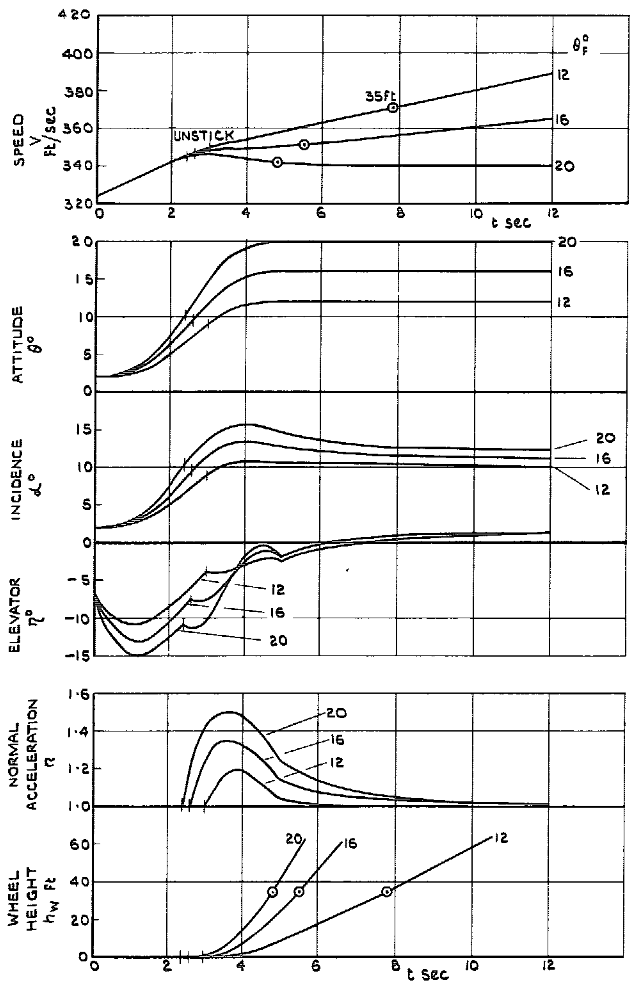


FIG. 8. The effect of final attitude, 4 engines, $V_R = 324$ ft/sec, $t_1 = 5$ sec.

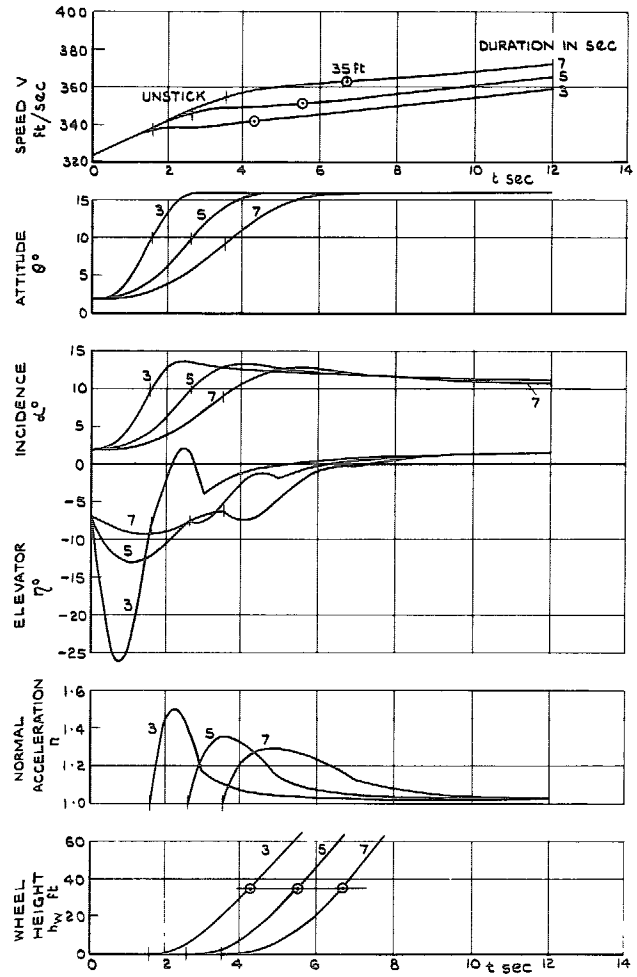


FIG. 9. The effect of duration, 4 engines, $V_R = 324$ ft/sec, $\theta_F = 16$ deg.

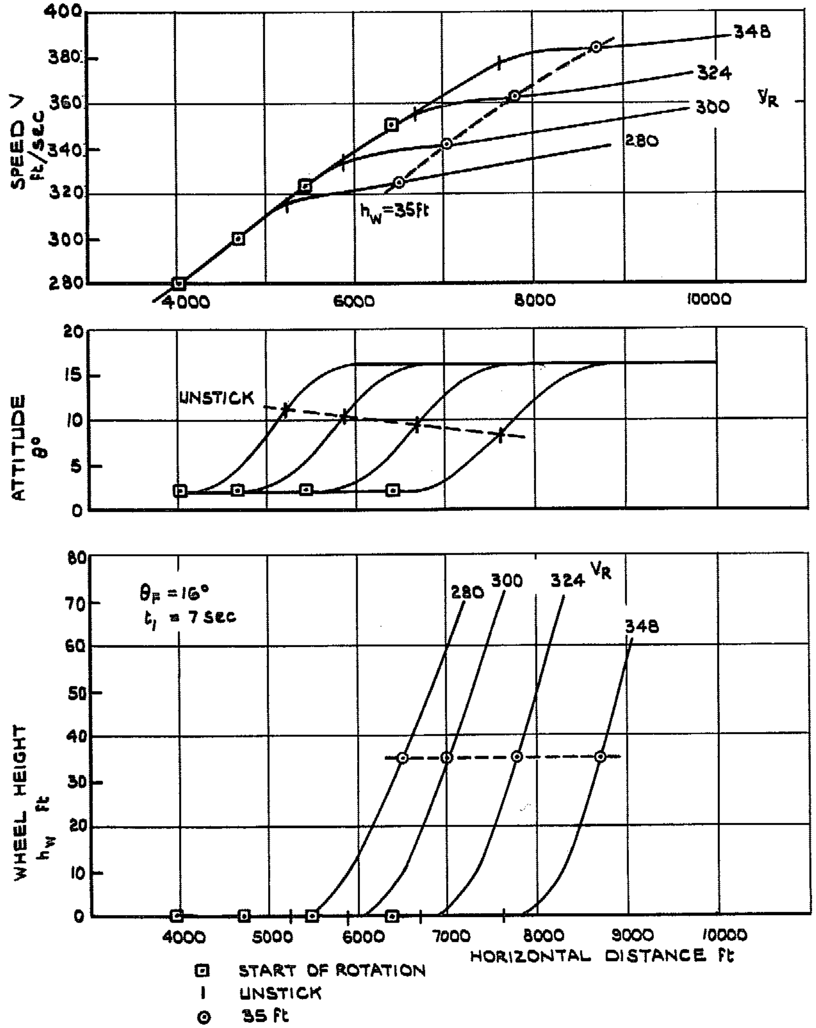


FIG. 10. The effect of rotation speed, V_R, on the take-off performance for a given manoeuvre–four engines.

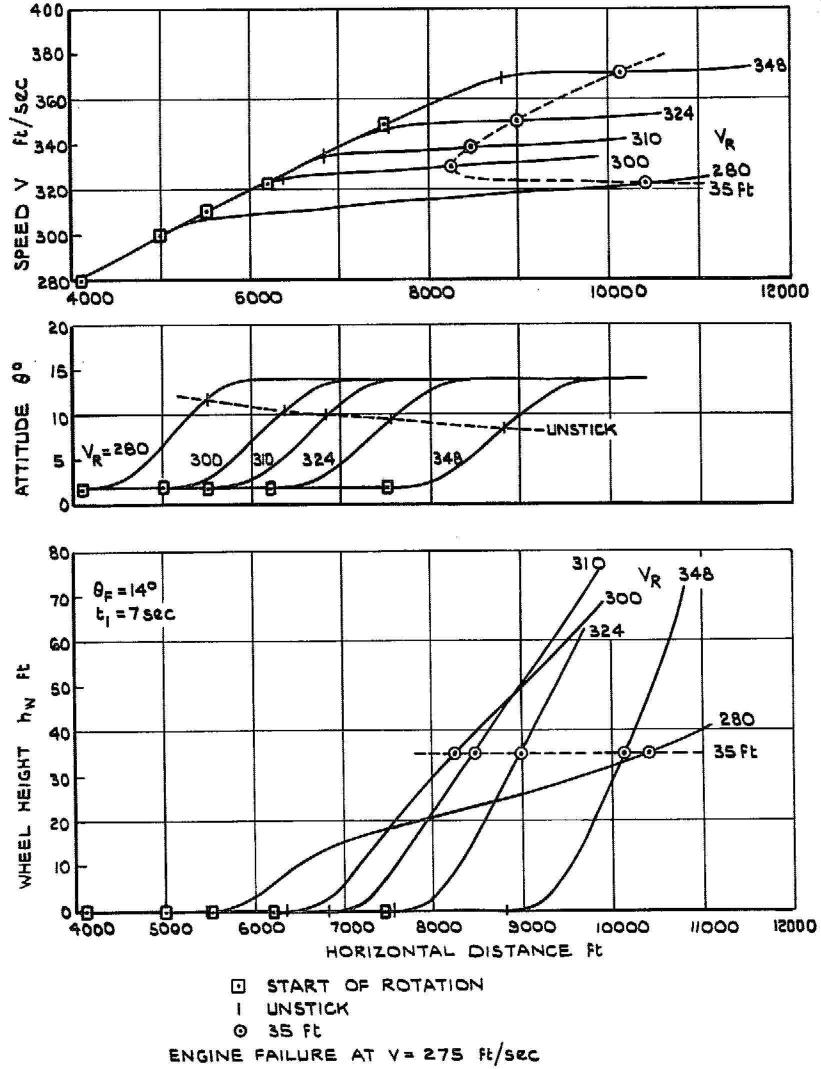


FIG. 11. The effect of rotation speed, V_R, on the take-off performance for a given manoeuvre–three engines.

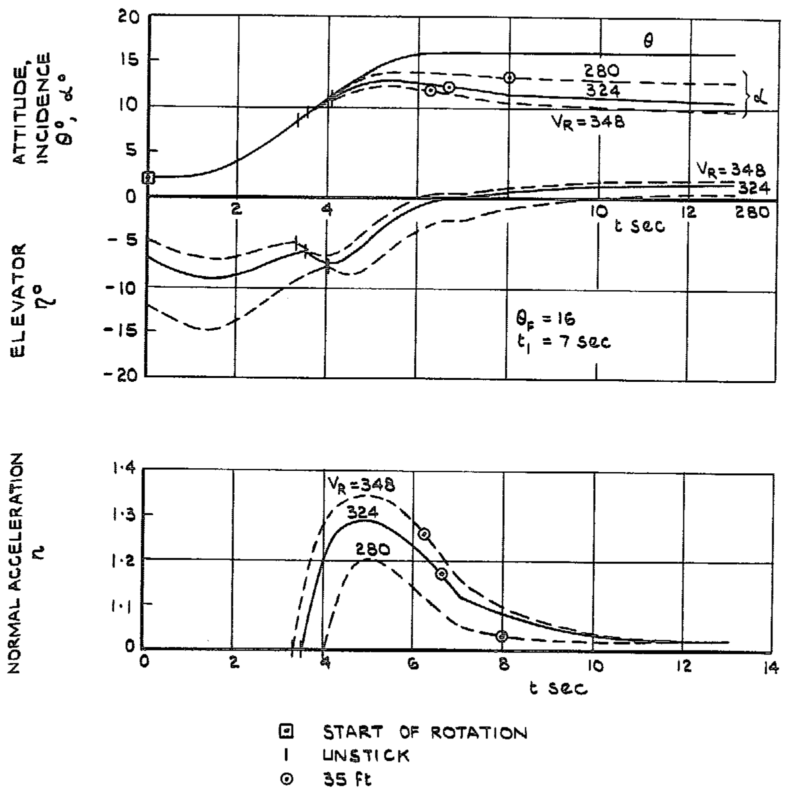


FIG. 12. Some time histories corresponding to the manoeuvres of Fig. 10.

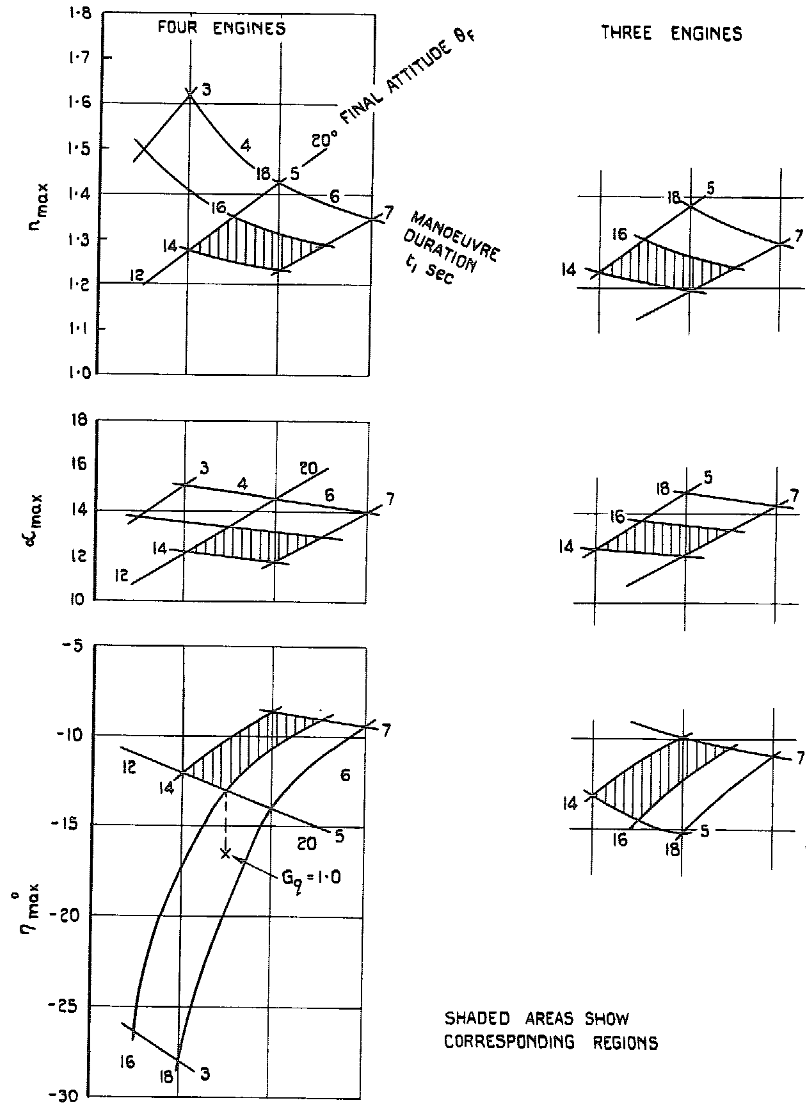


FIG. 13. Summary of the variation with take-off procedure of n_{max}, α_{max} and η_{max} for $V_R = 324$ ft/sec.

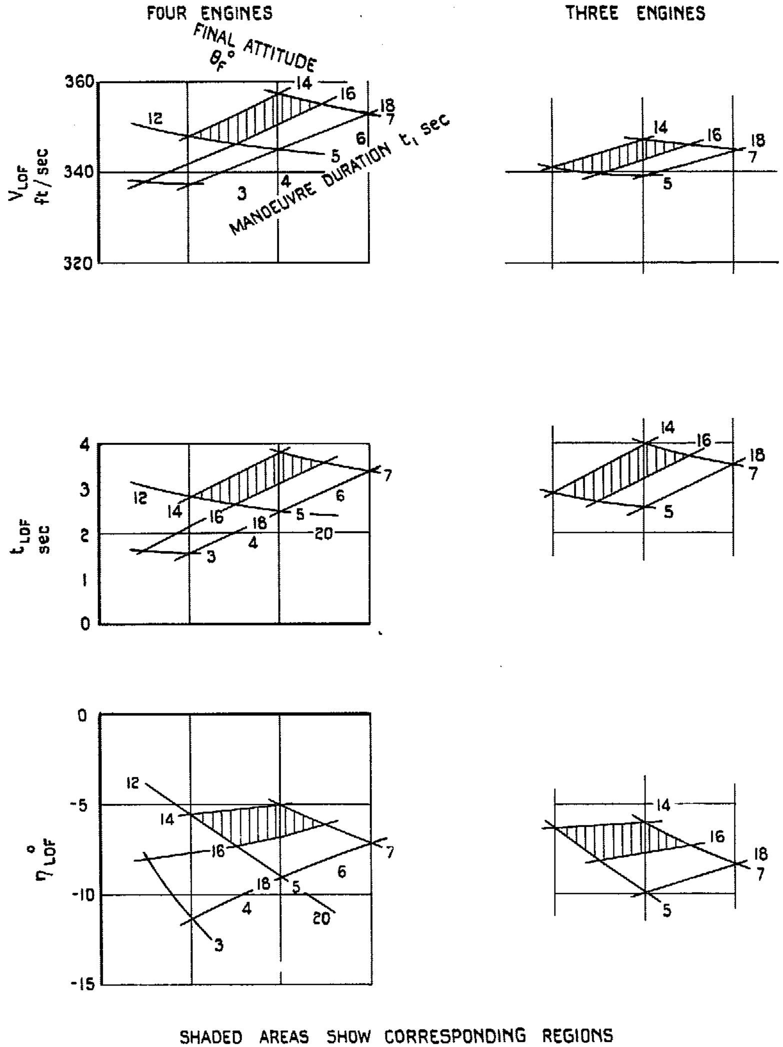


Fig. 14. Conditions at lift off, $V_R = 324$ ft/sec.

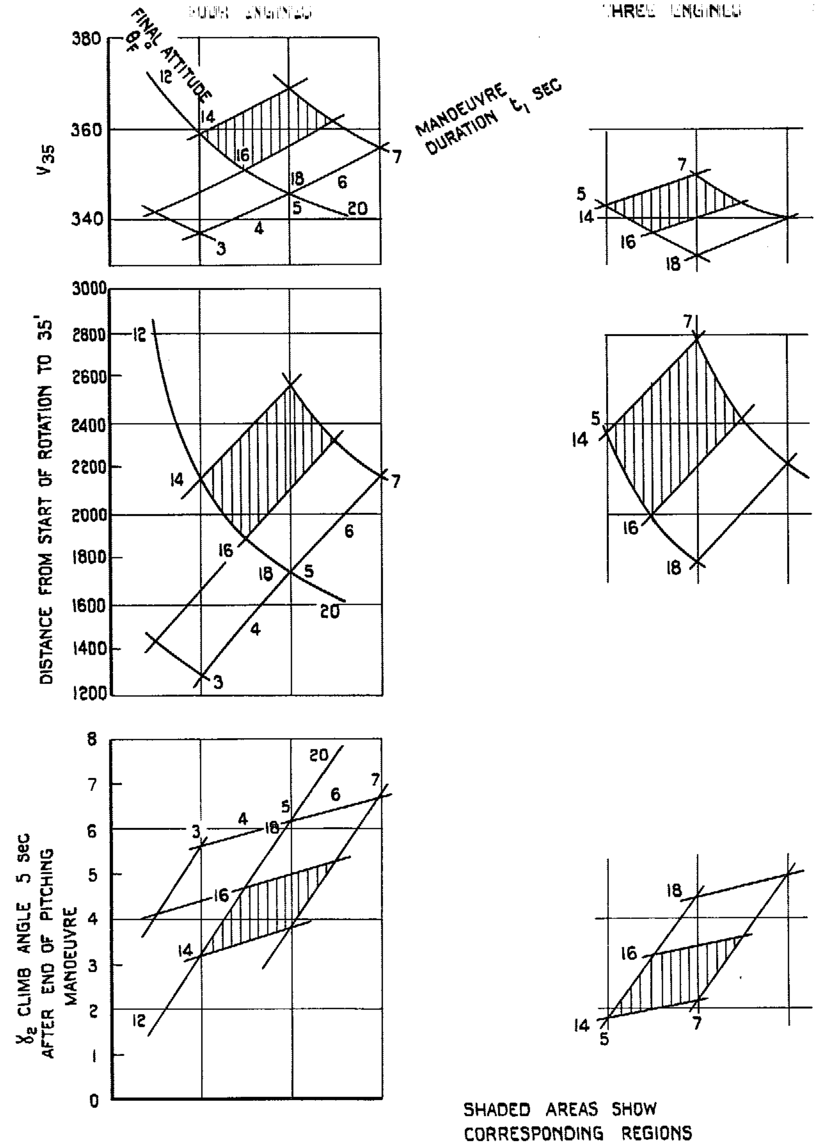


Fig. 15. Conditions at 35 ft and climb gradient, $V_R = 324$ ft/sec.

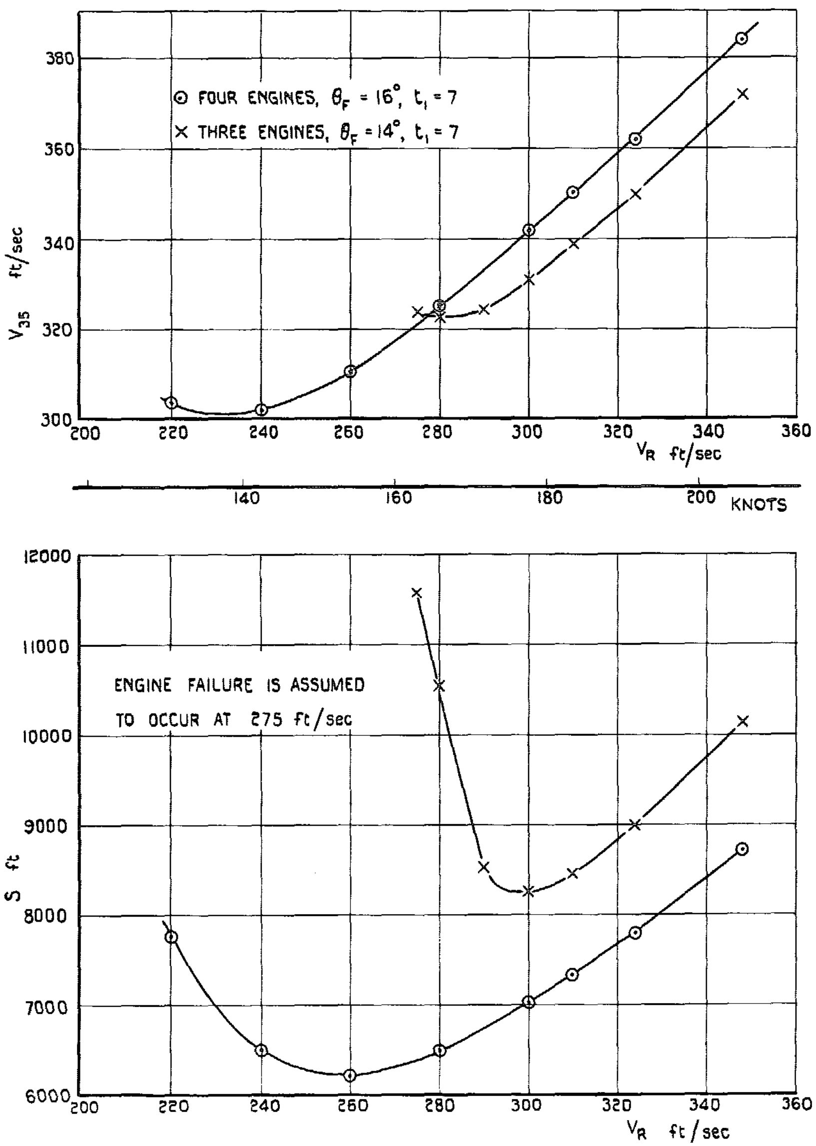


FIG. 16. The variation with rotation speed of speed at and total take-off distance to 35 ft.

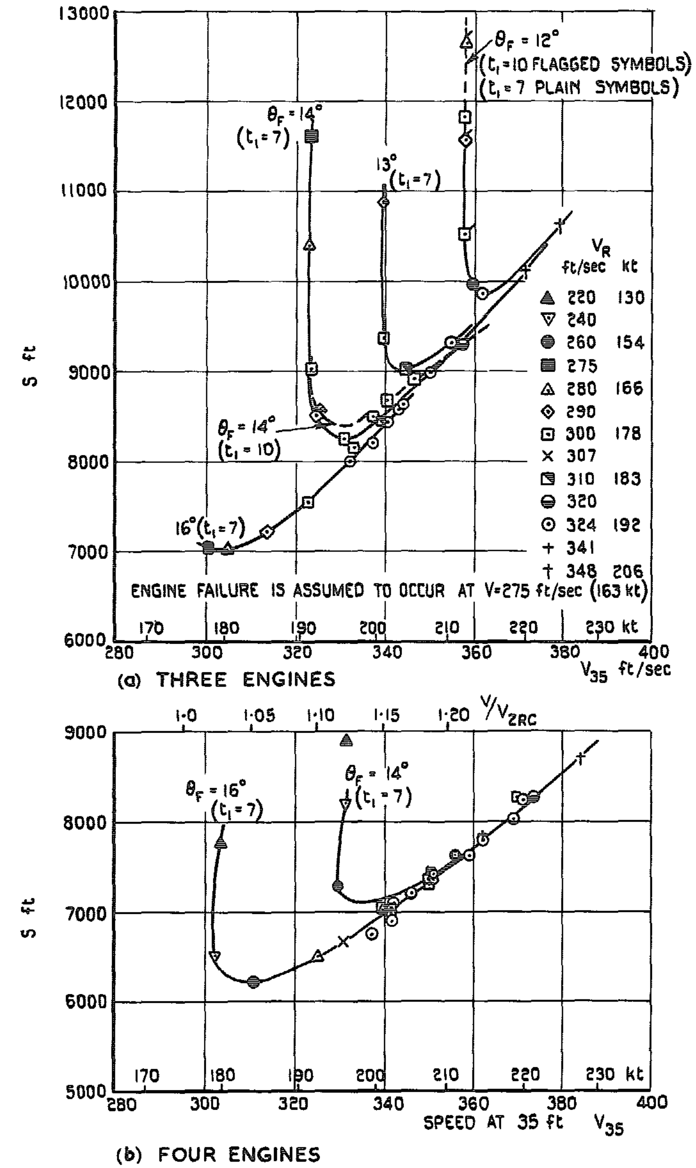
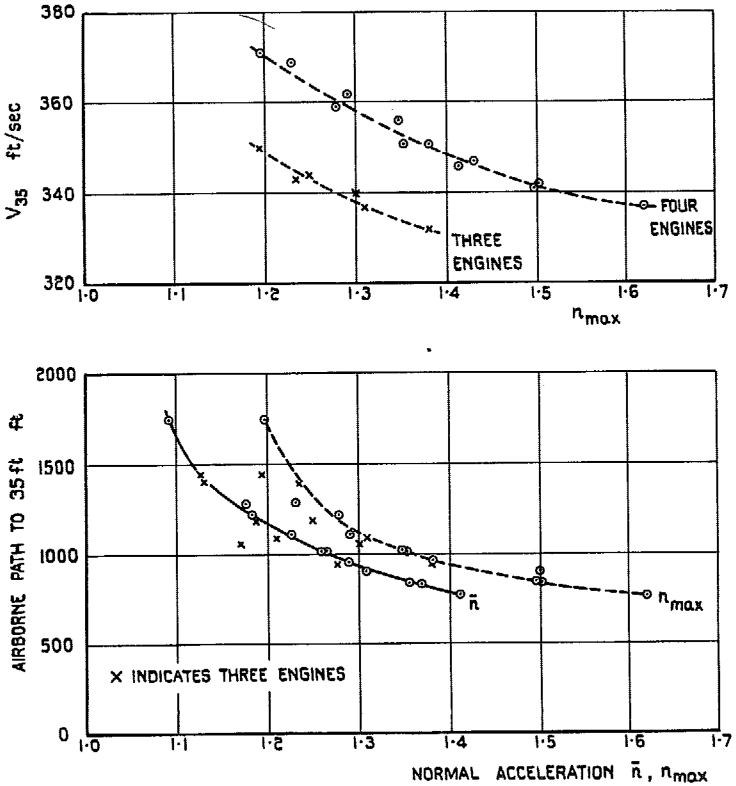


FIG. 17. Total distance to 35 ft as a function of V_{35}.



AIRBORNE PATH EQUALS HORIZONTAL DISTANCE FROM UNSTICK TO 35 ft
MEAN NORMAL ACCELERATION, $\bar{n}$, IS CALCULATED ASSUMING A CONSTANT INCREMENTAL 'g' PULL-UP TO 35 ft IN THE SAME TIME AS IS ACTUALLY TAKEN

FIG. 18. Airborne path as a function of normal acceleration, $V_R = 324$ ft/sec.

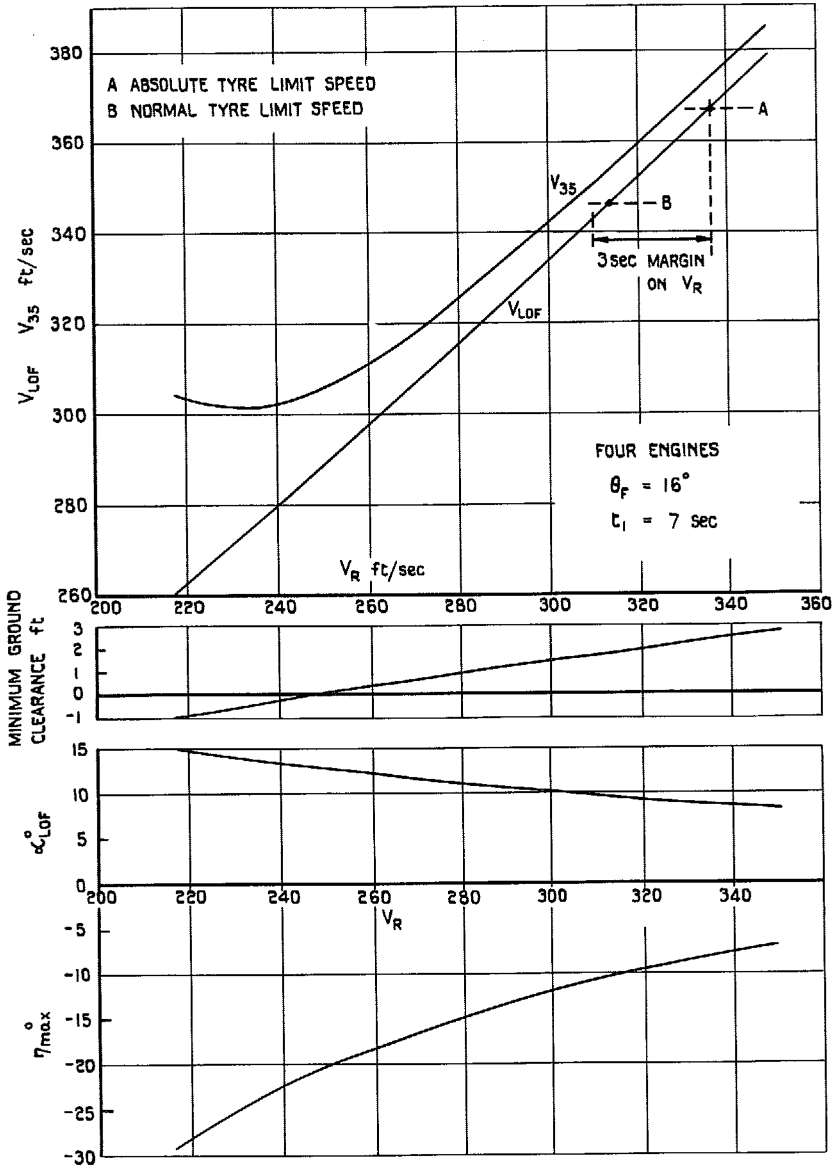


FIG. 19. The effect of rotation speed on lift-off conditions, ground clearance and maximum elevator angle.

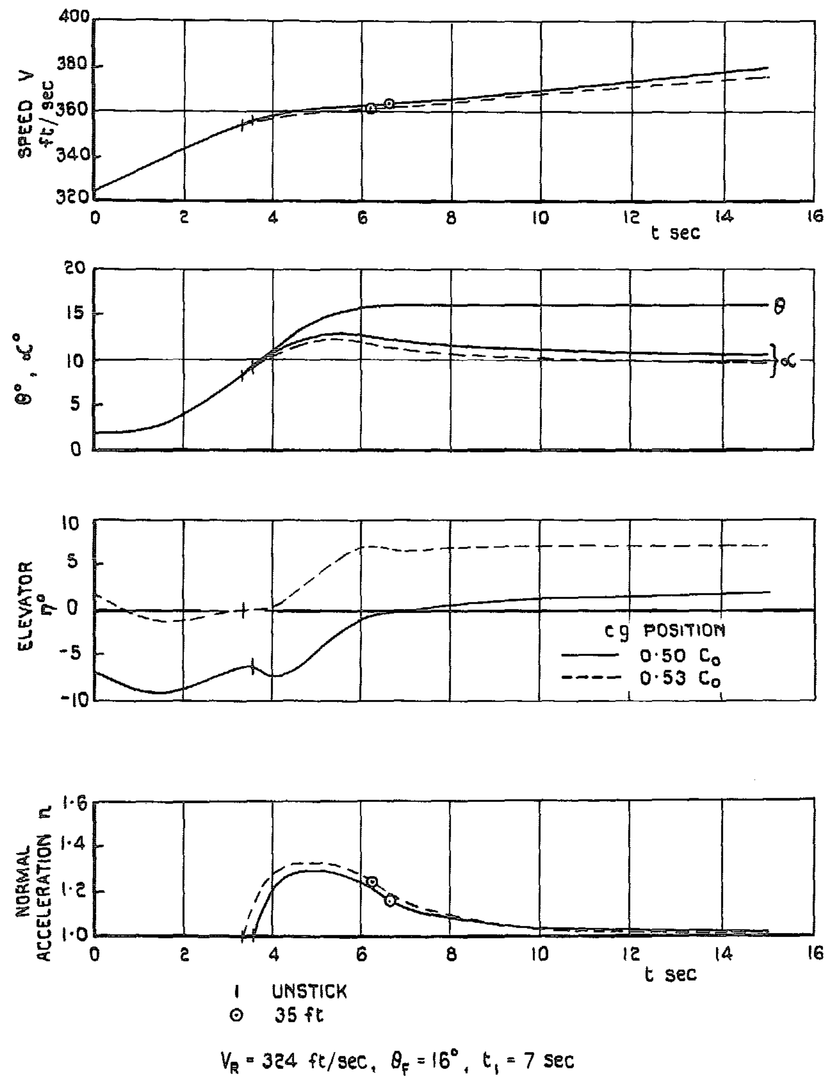


FIG. 20. The effect of c.g. position for a given manoeuvre.

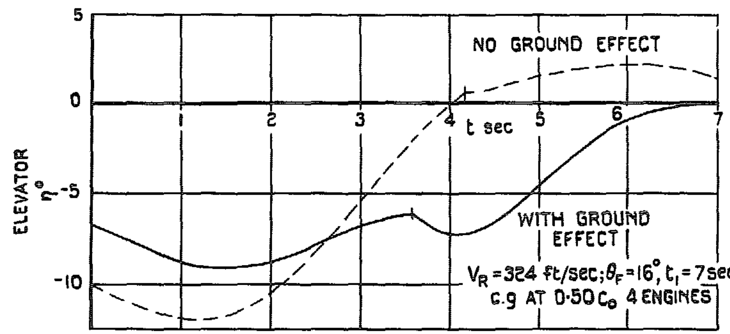


FIG. 21. Effect of ground on elevator time history.

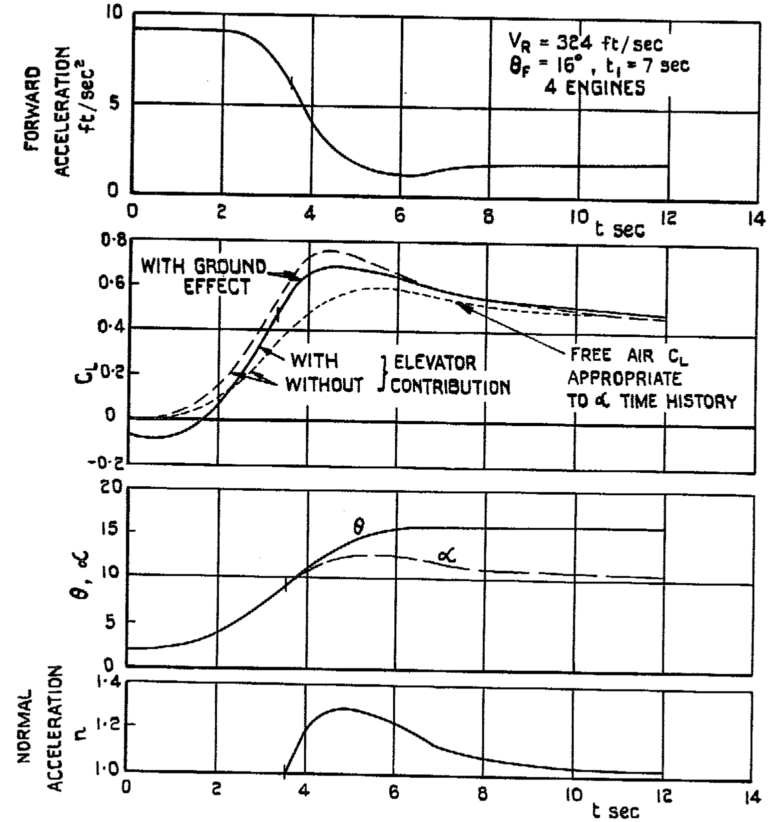


FIG. 22. Typical variation of forward acceleration and C_L.

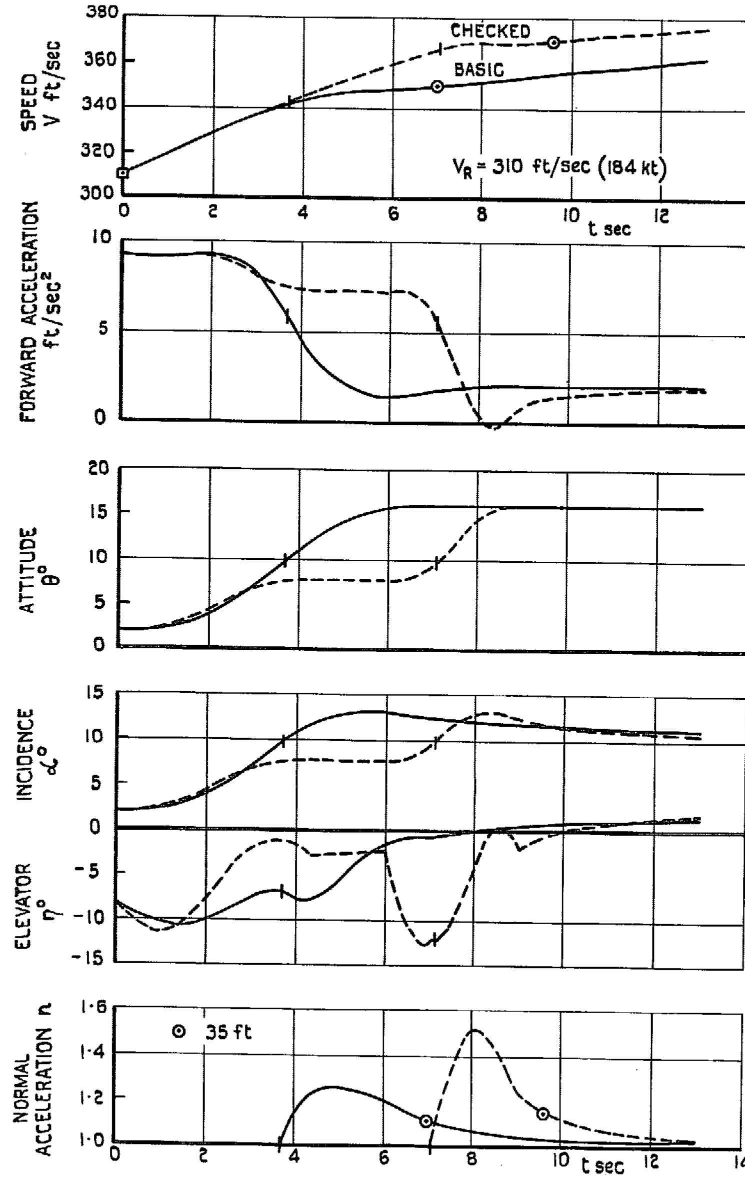


FIG. 23a. Checked rotation case compared with basic 4-engine manoeuvre (θ_F = 16 deg, t_1 = 7).

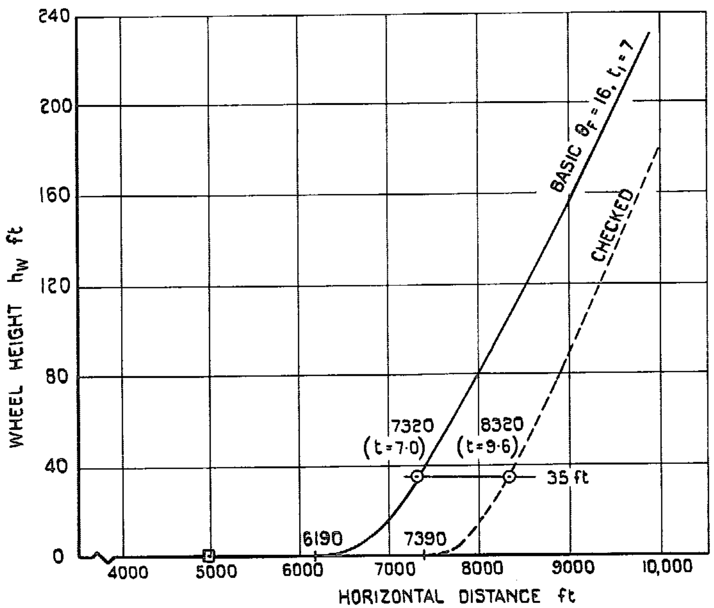


FIG. 23b. Flight paths for checked rotation and basic manoeuvres.

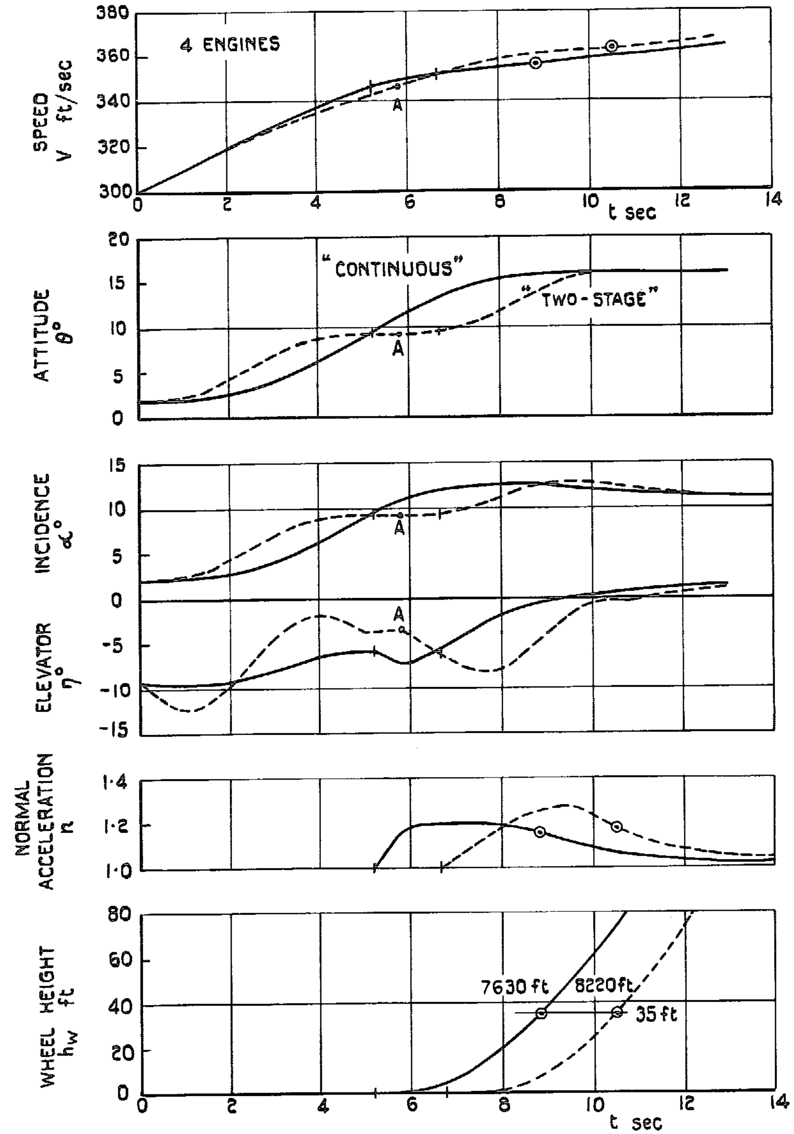


FIG. 24. Comparison between 'continuous' and 'two-stage' manoeuvres, V_R = 300 ft/sec.

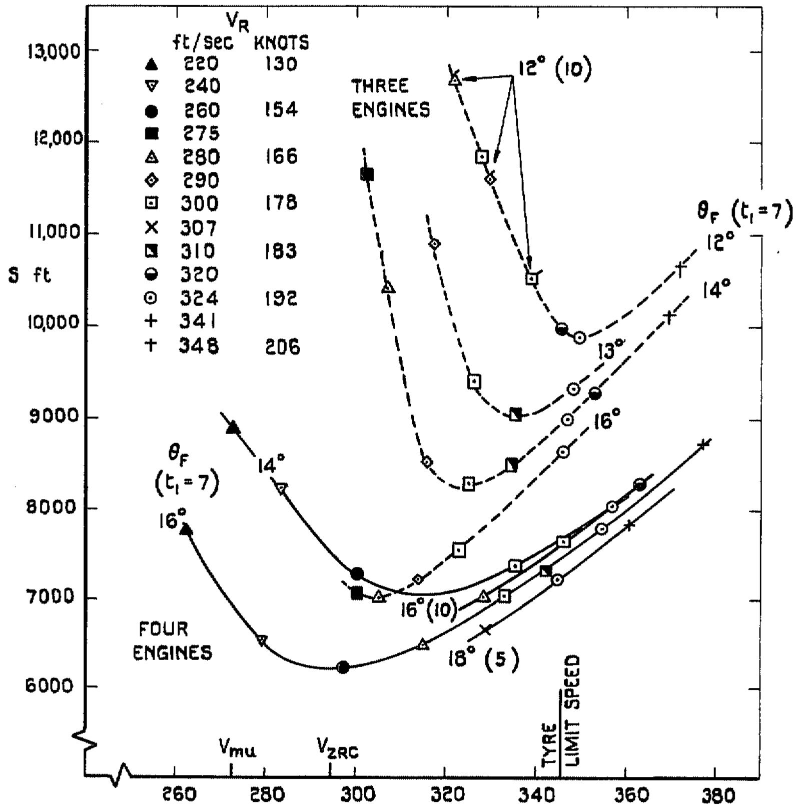


FIG. 25. Take-off distance as a function of lift-off speed for various manoeuvres.

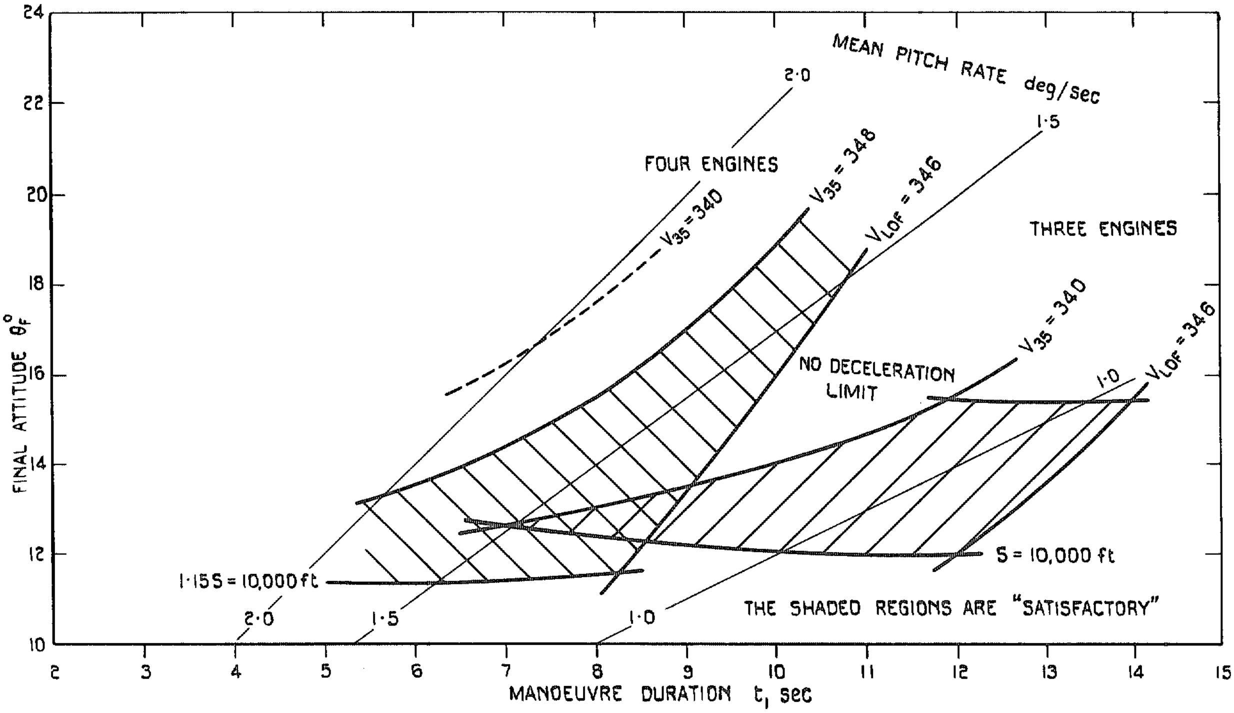


FIG. 26. Regions of satisfactory take-off manoeuvres, V_R = 300 ft/sec (178 kt).

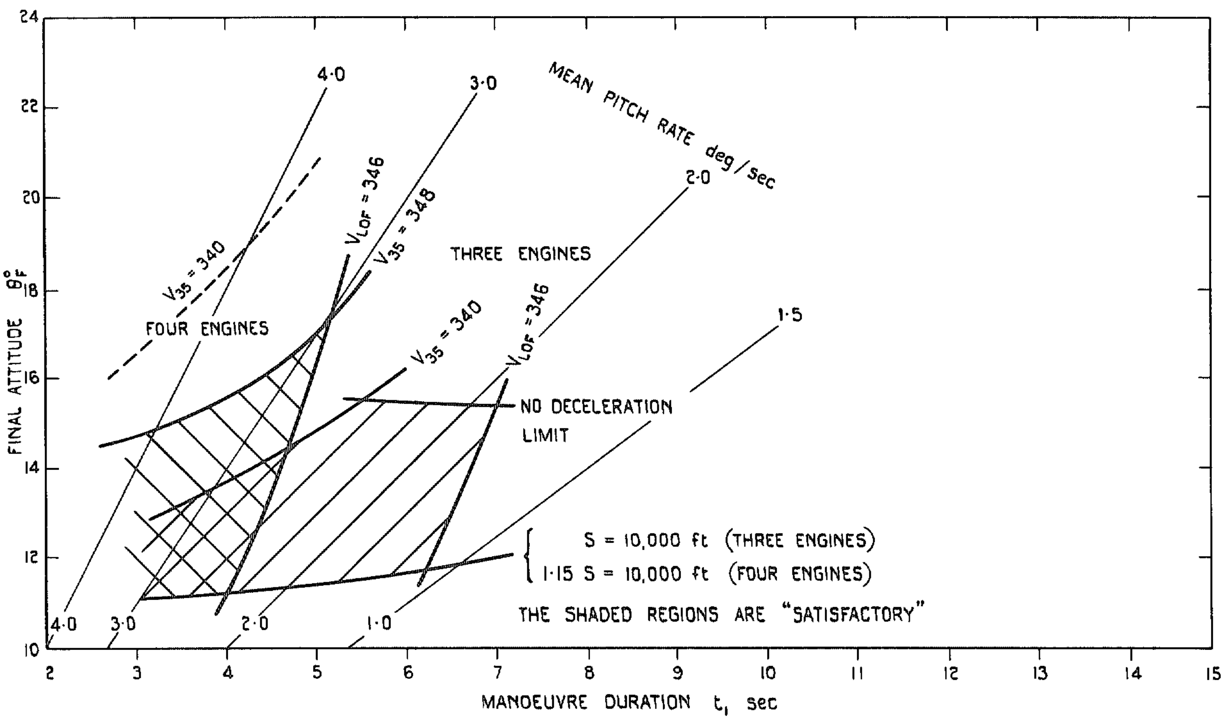


FIG. 27. Regions of satisfactory take-off manoeuvres, V_R = 324 ft/sec (192 kt).

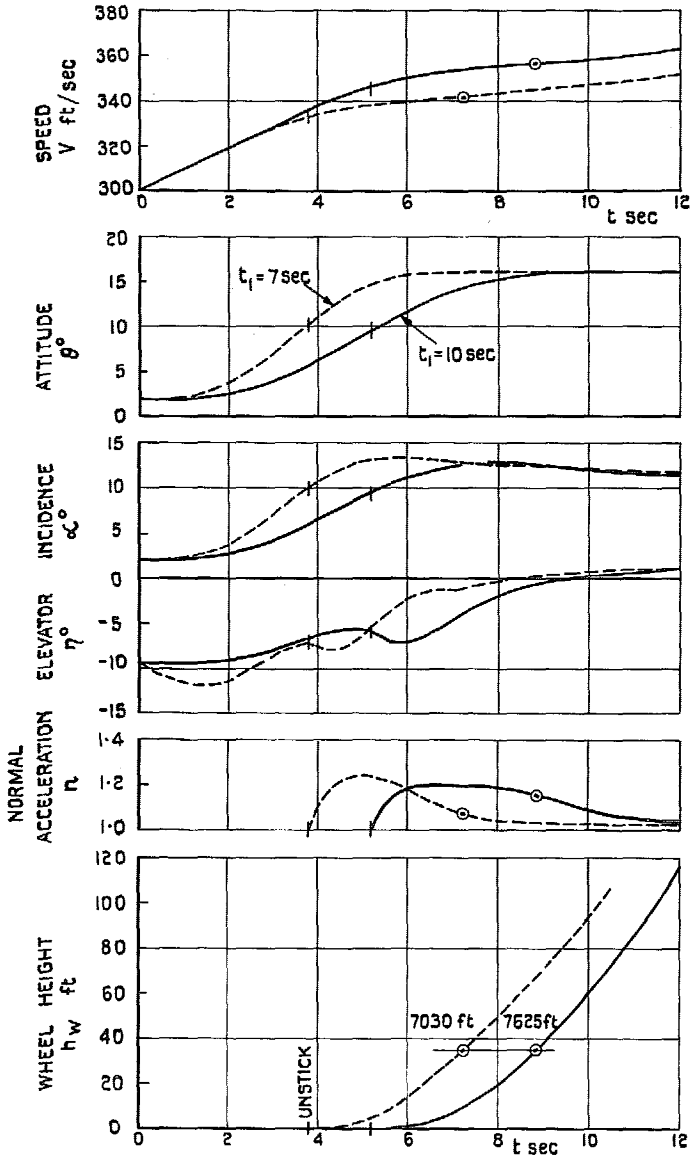


FIG. 28. Results for a slow take-off manoeuvre (t_1 = 10 sec) compared with a basic, (t_1 = 7 sec) case for V_R = 300 ft/sec.

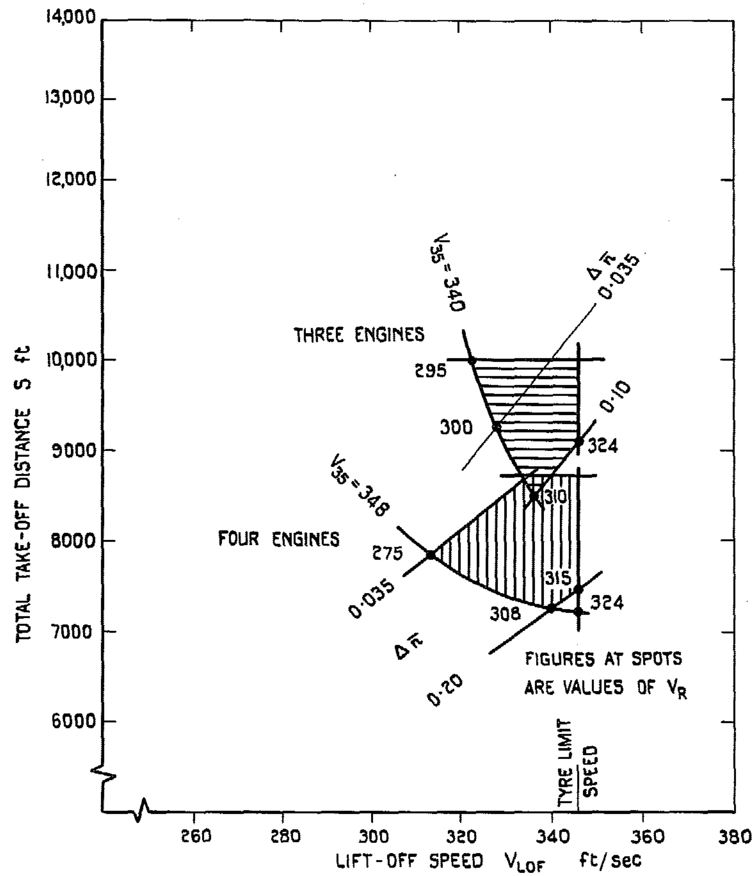


FIG. 29. Take-off limits.

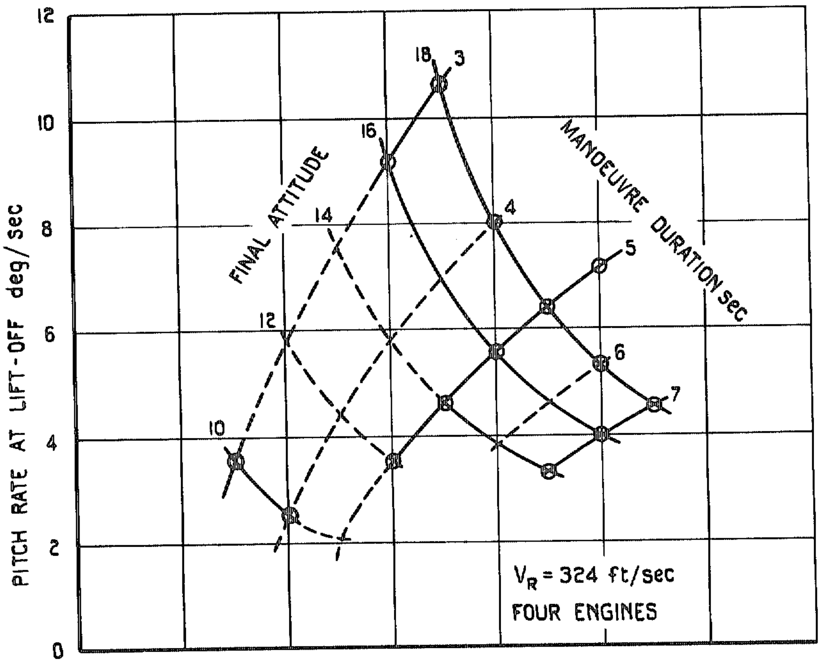


FIG. 30. Pitch rate at lift-off.

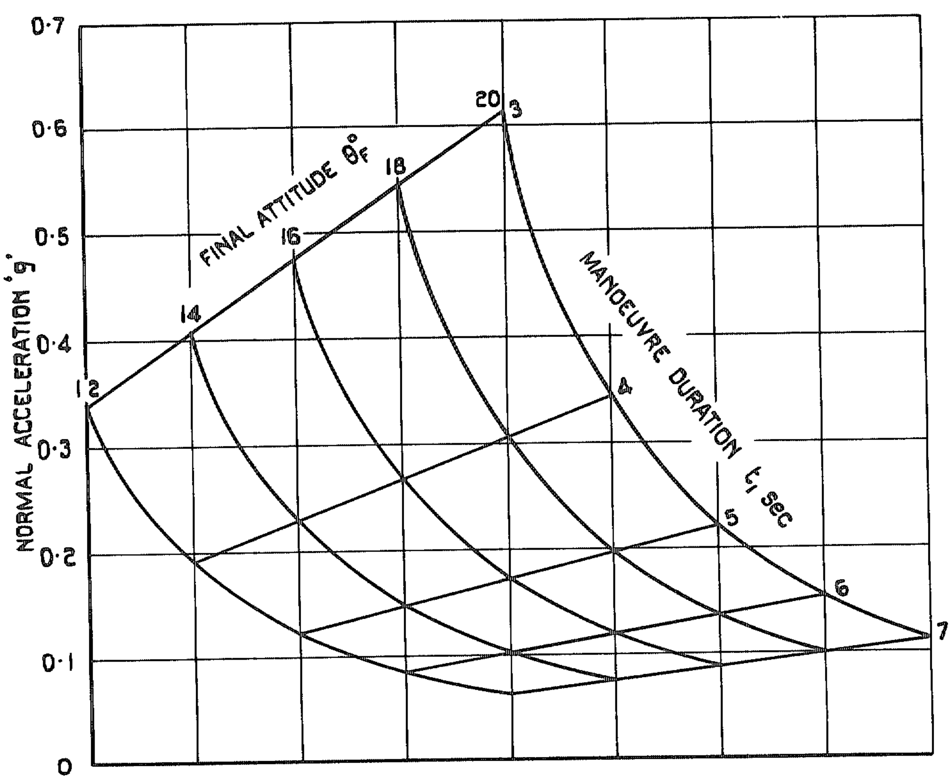


FIG. 31. Incremental normal acceleration experienced by pilot 90 ft ahead of c.g. as a result of the rotation manoeuvre.

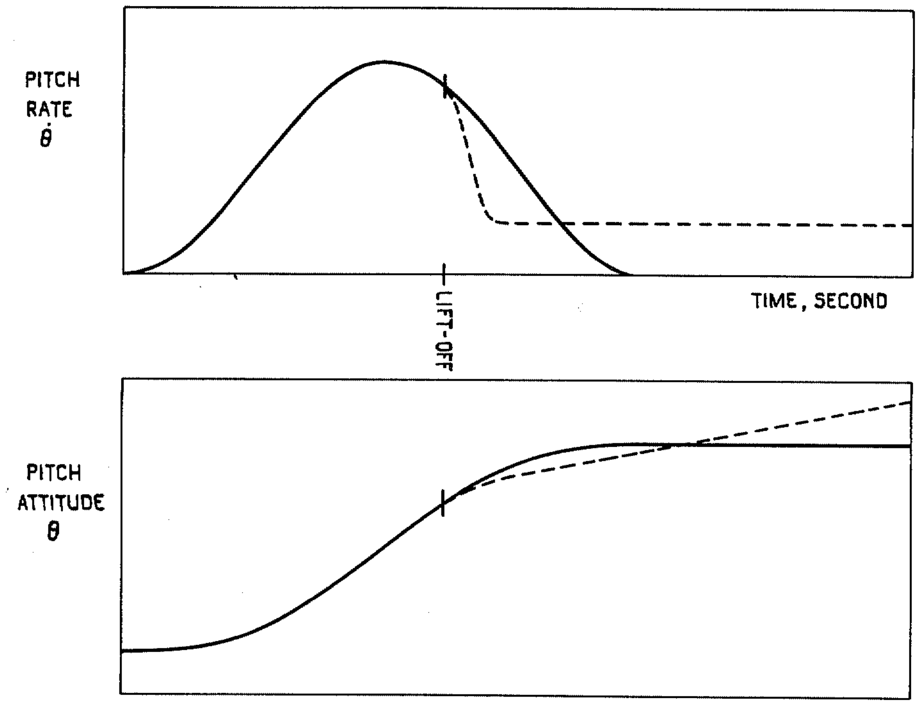


FIG. 32. Modified attitude time history.

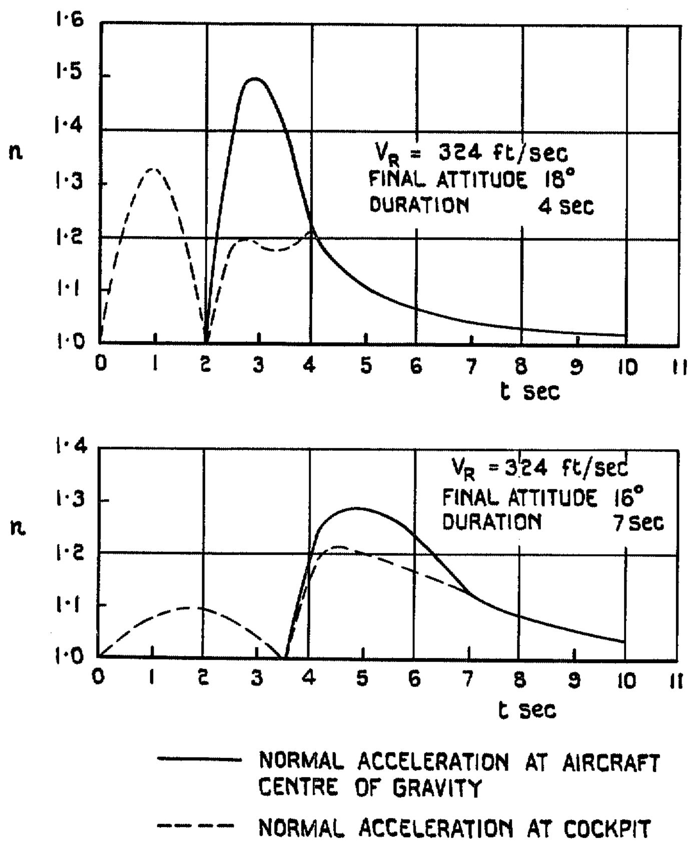


FIG. 33. Comparison between the normal acceleration at cockpit and centre of gravity.

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