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Some Data Pertaining to the Supersonic Axial-flow Compressor

By

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COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR),
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Summary.—Together with some random considerations concerning possible compressor development, data concerning the flow of air at high speeds is presented in this note in a form suitable for use in the design of supersonic axial-flow compressors. A brief history and description is also given of the work of the German pioneers Weise, Encke and Betz.

CONTENTS

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CONTENTS—*continued*

1.0. *Introduction.*—Now that the problems of high speed and particularly of supersonic flight are receiving serious consideration, a requirement will soon arise for aircraft power plant which is much lighter and much more compact than ever before. Assuming that there is to be no appreciable sacrifice in efficiency these requirements may, in so far as the gas turbine compressor is concerned, be interpreted as involving aerodynamic and mechanical simplicity, compactness and lightness. In orthodox centrifugal or axial-flow units some of the requirements are conflicting, so that, although each type of compressor has its own peculiar advantages, no single unit can satisfy them all. It is nevertheless felt that such a compressor might be evolved and that the key to the problem lies in the realms of supersonic flow.

The primary object of this note is accordingly a presentation of the basic material in such a form that it may easily be used as design data, and this is done in Part I. At the same time an opportunity has been taken in Part II to include some considerations regarding the type of unit which might be evolved. That the deductions in this section are largely hypothetical is obvious, and its conclusions should not therefore be taken as final.

The earliest available document of British origin pertaining to a supersonic axial-flow compressor was a note written by A. A. Griffith and dated the 10th November, 1938. Immediately after this Howell commenced work on the subject at the Royal Aircraft Establishment (R.A.E.) and, in September of the following year, produced the first sketch of a possible two-stage unit, similar schemes being perhaps considered at about the same time in other establishments. However, with the incidence of the latest World war, the work at the R.A.E. could be continued only on very low priority and eventually came to a stop. In January, 1945, the work was recommenced and a preliminary working scheme was completed in June of that year.

On the cessation of hostilities it was discovered, however, that Encke, Betz and Weise had commenced work in the field in or before the year 1935. Encke built one and Weise two experimental units, but without much success, the only recorded results being published by Weise in 1943[1] under severe security restrictions. Part III of this note therefore comprises a brief description of the original German work.

PART I

BASIC DATA

2.0. *Introduction to Part I.*—Compressors of the aerodynamic type generally operate by the vectorial addition of air and blade velocities and by the subsequent conversion into static pressure increments of the relative dynamic heads thus generated. With subsonic flow the conversion process can be performed only by diffusion, but under supersonic conditions an attractive alternative exists in the form of the shock.

As the supersonic compressor is thus merely a machine for harnessing the shock, its design must be based largely on a knowledge of the behaviour of that phenomenon and of the conditions in which it can persist. The following information is intended, however, not as a general statement concerning the properties of the shock but merely as a brief survey of some facts which might prove significant in the design of a transonic compressor cascade. Two dimensions only are considered.

3.0. *The Prandtl-Meyer Expansion.*—Consider the history of a transient disturbance of infinitesimal magnitude which is generated instantaneously at the point O in a compressible fluid moving with velocity v and is transmitted in the fluid at the velocity, v_s, of sound. If the fluid velocity be supersonic the disturbance will take successively the form of the circles a, b and c of Fig. 1, from which it may be observed that no small disturbance generated at O can effect any region of the field other than that between the common tangents OX and OY of these circles.

The lines OX and OY are known alternatively as Mach lines or characteristics (Temple[2], 1944) and may in practice be regarded as shock waves of infinitesimal intensity. The fluid Mach number being M, the Mach angle, μ, is thus given by

$$\sin \mu = \frac{v_s}{v} = \frac{1}{M} \qquad \ldots \quad (1)$$

In the theoretical treatment of two-dimensional potential supersonic flow around a sharp corner (Taylor and Maccoll[3], 1934) due to Prandtl and Meyer, it is shown that all radial lines are Mach lines and so, if the corner be considered as the origin of polar co-ordinates as in Fig. 2,

$$\theta + \mu = \frac{\pi}{2} + \delta, \qquad \ldots \quad (2)$$

where θ is measured from the radial line of unit Mach number. It may then be shewn that

$$\cos 2\left(\frac{\gamma - 1}{\gamma + 1}\right)^{1/2} \theta = \frac{1 - \dfrac{\gamma - 1}{\gamma + 1}(M^2 - 1)}{1 + \dfrac{\gamma - 1}{\gamma + 1}(M^2 - 1)}, \quad \ldots \quad (3)$$

from which equation the tables of Appendix III have been prepared for $\gamma = 1{\cdot}400$.

Should the expansion be around an arbitrary curve the tables may still be used, provided only that there is no point of inflection on the curve and that the total expansion angle, δ, is known. For instance, with an initial Mach number of 1·308 and a deflection of 10·3 deg,

$\delta_1 = 6{\cdot}4$ deg and $\mu_1 = 49{\cdot}8$ deg,

Then $\delta_2 = \delta_1 + \delta = 16{\cdot}7$ deg,

So that $M_2 = 1{\cdot}662$ and $\mu_2 = 37{\cdot}0$ deg.

Boundary conditions excepted, an expansion of this type is both iso-energetic and isentropic, so the consequent changes in the fluid pressure and temperature may be found from the equations of Appendix II or the curves of Fig. 3.

4.0. *The Compression Shock.*—4.1. *Theory.*—Consider a plane shock inclined as in Fig. 4 at an angle ζ to the incident streamlines of a compressible fluid moving with uniform velocity w, conditions before and after the shock being denoted by the suffices 1 and 2 respectively. As the only force acting is due to the differential static pressure, the tangential velocity component, v, cannot be affected and considerations of continuity and momentum need thus be applied only to the normal component, u.

With the aid of the principle of the conservation of energy and the characteristic gas equation it may then be shown that

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma + 1} M_1^2 \sin^2 \zeta - \frac{\gamma - 1}{\gamma + 1}, \quad \ldots \quad (4)$$

$$\frac{\tan \zeta}{\tan (\zeta - \delta)} = \frac{\dfrac{\gamma - 1}{\gamma + 1} + \dfrac{p_2}{p_1}}{1 + \dfrac{\gamma - 1}{\gamma + 1} \cdot \dfrac{p_2}{p_1}}, \quad \ldots \quad (5)$$

and

$$\frac{M_1^2}{M_2^2} = \frac{\dfrac{\gamma - 1}{\gamma + 1} + \dfrac{p_2}{p_1}}{\dfrac{\gamma - 1}{\gamma + 1} + \dfrac{p_1}{p_2}} \cdot \frac{\sin^2 (\zeta - \delta)}{\sin^2 \zeta}, \quad \ldots \quad (6)$$

Since equation (4) is a quadratic in M_1^2, two solutions will be obtained for each value of that variable, and this is evident from charts 1 and 2, of which the former—reproduced in outline in Fig. 5—may be divided into three regions as follows.

(i) The region ABC, between the Mach line condition and the line of unit outlet Mach number, in which simple oblique 'first-solution' shocks occur.

(ii) The region ACD, between the line of unit outlet Mach number and the locus of the turning points of the lines of constant streamline deflection, in which transonic 'first-solution' shocks occur.

(iii) The region ADE, between the turning points locus and the normal shock line, which comprises the invariably transonic 'second-solution' shocks.

In accordance with their positions on the Mach number chart shocks might therefore be described either as 'first-solution,' 'second-solution,' or 'normal' shocks. From equation (6) it may be observed that, if γ be assumed constant, the change produced by any shock in the normal Mach vector of a fluid is a function only of p_2/p_1. The static pressure ratio across a shock is thus a measure of its intensity.

4.2. *Functional Classification.*—An oblique first-solution shock is most readily produced by means of a wedge, as in Fig. 6, and it may be observed that the systems on either side of the wedge are mutually independent. In general, however, a shock will occur in any region on the boundary of a uniform supersonic stream towards which the normal Mach vector is locally directed. At a slightly concave surface in which there is no discontinuity the resultant shock will take the form of an envelope to a 'fan' of converging Mach lines—or with certain boundary-layer conditions small but finite shocks—and may even appear to terminate in the stream a short distance from the surface. Such a system is illustrated in Fig. 7.

There is also to be found in practice a most important type of shock which, as illustrated in Fig. 8, occurs in completely unobstructed flows and is due to the effect of supersonic over-expansion. Shocks of this type are generally of the second-solution variety and may be predicted by means of a one-dimensional theory. As both their properties and their positions may vary considerably with small changes in the boundary conditions, they are, moreover, much less steady in slightly fluctuating flows than shocks which are anchored at obstructions.

Because of this unsteadiness 'overexpansion' shocks may produce heavy 'live' loads on any structural member with which they come in contact and should for this reason be used only with caution in the design of an applied shock system. In systems in which they are undesirable they may nevertheless occur unavoidably during acceleration, and that process should then be accomplished as rapidly as possible.

4.3. *The Detached Shock.*—If in a supersonic stream there be a discontinuity at any boundary sufficiently great to involve a streamline deflection larger than that permitted by the above theory the latter can no longer be applicable, for only by exhibiting the required deflection can the flow satisfy continuity. As an example the wedge of Fig. 6 might be considered when operating at a different incidence, as in Fig. 9. From this it may be observed that the shock moves upstream of the wedge apex and that the requirements for continuity are fulfilled in an intervening subsonic region. The shock is no longer straight but, starting as a normal shock directly upstream of the stagnation point of the wedge, takes successively the second-solution, first-solution transonic and simple oblique forms.

Under such conditions a shock is said to be detached and, at a fixed Mach number, the streamline deflection at which this phenomenon first occurs may be described as the detachment angle of the flow at the point in question. Conversely, if a certain boundary condition involves a streamline deflection of fixed magnitude, the highest Mach number at which a detached shock can exist can be described as the detachment Mach number of the system. The example quoted above serves to demonstrate an important principle which may be enunciated as follows:—

Whenever the requirement for continuity in a supersonic régime necessitates the existence of a shock the latter will, if possible, take the first-solution form, transonic or otherwise. Only if this be impossible will a second-solution shock occur.

It should be noted that there is yet no theoretical support for this principle, which is at present acceptable only inasmuch as it is a statement of experimental fact yet to be contradicted. An illustration of the manner in which the necessity for continuity may indirectly produce second-solution shocks is afforded in the phenomenon of shock wave reflection.

4.4. *Shock Wave Reflection.*—The most general form of a system involving the reflection of a single shock at a flow boundary or line of symmetry is illustrated in Fig. 10. The independent variables to be considered are the initial Mach number, M_1, and the deflection δ_{12}, the mode of formation of the incident shock, 1—2 being clearly irrelevant. Should the magnitude of δ_{12} exceed that of the detachment angle at M_2 a simple reflection will not ensure continuity and is thus impossible. The incident shock is therefore terminated in mid-stream with the formation of the Mach shock, 1—4, and the reflected shock, 2—3.

In order that the requirements imposed by continuity and momentum shall simultaneously be fulfilled the system must then be such that, at the point of intersection of the three shocks, the following equations are satisfied.

$$\delta_{14} = \delta_{12} + \delta_{23}, \qquad (7)$$

$$\frac{p_4}{p_1} = \frac{p_2}{p_1} \times \frac{p_3}{p_2}. \qquad (8)$$

On the application to the problem of charts 1 and 2 it will be found that in general this is possible only when the Mach and reflected shocks are of the second-solution type.

With the exception of the theoretical conditions at the point of intersection of the three shocks, and the fact that in order to satisfy continuity the Mach shock must become nearly normal as it approaches the reflecting boundary or streamline, little quantitative information is as yet available concerning such systems. Qualitatively, however, it is known that the discontinuity in the velocity profile at the boundary 3—4 will generally give rise to a vortex sheet, that the variations in the conditions immediately behind the Mach shock are adjusted in the following subsonic region and that these variations, being functions of M_1 and δ_{12}, determine the length and form of the shock.

It was shown by Bickley[4] in 1943, that, if the magnitude of δ_{12} and hence the incident shock intensity be increased at a constant value of M_1, the reflected shock may become normal at the point of intersection while M_2 is still appreciably greater than unity. If δ_{12} were further increased there could thus be no real solution of the simultaneous equations (7) and (8). That a reflection of the general type is nevertheless possible in these circumstances is most probably due to the existence behind the incident and/or Mach shock of a sudden re-expansion. This was suggested by Lean[5] in 1943 and was strongly supported by the demonstration in 1946 by Ackeret, Feldmann and Rott[6] that, in potential subsonic flow, fluctuations about the mean static pressure occur $(1 - M^2)^{-1/2}$ times as fast in a compressible fluid as they do in an incompressible one.

4.5. *Curvature in a Non-uniform Field.*—Should a shock be situated in a non-uniform field its obliquity will in general vary from point to point, that is, the shock will be curved. From chart 1 it may be demonstrated, moreover, that the assumption of constant shock intensity is not tenable. This is best observed in the case of a Mach shock or of any other shock which extends to and ends at a line of sonic velocity.

In supersonic compressor design the systems which should mainly be of interest are those of Fig. 11 and 12 in which the shock is so crossed by a ' fan ' of expansion waves that its obliquity increases with distance from the ' source.' Even if the actual intensity of the shock did not vary appreciably in the region considered, its value at any point relative to that of a normal shock at the same point would thus decrease substantially with distance from the ' source,' and the potency of the shock as an effective instrument in the production of transonic régimes would therefore be considerably reduced.

The greatest danger arising from this phenomenon in practice is that the shock may be so curved that its reflection, and consequently transonic operation of the system involved, is rendered impossible. Thus, all that is required in design is a rough guide to the approximate shock curvature, and for this purpose the assumption of constant intensity, although incorrect, will be found adequate in most circumstances involving simple oblique shocks in axial compressor cascades.

5.0. *Shock Boundary-Layer Interaction.*—5.1. *General Note.*—The phenomena involved in interactions between shocks and boundary layers have been and are at present the subjects of major investigations in several countries. As the results so far reported have been obtained under vastly different conditions and possibly with different objectives in view they naturally do not agree in all respects.

In the following text the findings of various investigators are briefly summarised and an attempt is made to extract from them rough and tentative design rules. Although these are presumptive in the extreme, it is hoped that they will be sufficiently correct to enable the more serious consequences of interaction to be avoided in the design of any pertinent project.

5.2. *The E.T.H. Investigation.*—This investigation (Ackeret, Feldmann and Rott[6], 1946), the most detailed of the three, was performed in a working section which is illustrated in Fig. 13. The phenomena were studied quantitatively under very closely controlled and reproducable conditions, the most important results obtained being as follows.

(*a*) Whenever the boundary layer upstream of the shock was laminar the system described as a λ-shock (*see* Fig. 14) was established. It was demonstrated that the laminar boundary layer began to thicken at the base of the oblique shock and that its wedge shape was terminated further downstream at the transition point. The latter occurred just before the main shock, there being in all a tenfold increase in the boundary-layer displacement thickness on passage through the system.

The thickening of the laminar boundary layer was found to be a displacement caused by a reversal of the flow, and it was demonstrated quantitatively that this phenomenon might be considered as the cause of the oblique shock. Although the flow was invariably transonic through the main shock the latter was followed by a small supersonic expansion which terminated in the third and final shock, the flow being thereafter subsonic.

(*b*) With a laminar boundary layer at low Mach numbers it was discovered that a series of λ-shocks could be formed as in Fig. 15, the resultant static pressure distribution at the wall (*see* Fig. 16) showing no evidence of the individual shocks. It was demonstrated, moreover, that although each λ-shock was transonic and the laminar boundary layer started to thicken at the first oblique member, transition was delayed until just before the last main shock.

(*c*) With a turbulent boundary layer it was found that a single transonic shock occurred, this being curved as in Fig. 17 and having a very small oblique component at its base. The shock was followed by a slight subsonic expansion, but the system was definitely transonic and, although the boundary-layer displacement thickness was increased by a factor of four or five, no reversed flow or separation was observed.

At a fixed Mach number, the transition from a λ-shock to a single curved shock when the Reynolds number was raised was effected by a gradual 'shrinking' of the oblique component, the main shock remaining substantially unchanged. So long as the boundary layer was turbulent, however, only the single curved shock could be obtained, changes in the Mach number altering merely its intensity.

It was finally demonstrated that 'artificial' turbulence, such as that caused by a wire, produced precisely the same phenomena as 'natural' turbulence.

5.3. *The G.A.L.C.I.T. Investigation.*—The following American results, due to Liepmann[7], were obtained from qualitative tests on a two-dimensional biconvex aerofoil in the system illustrated in Fig. 18. The effects of tunnel constraint might therefore have been significant, especially as the influence of this factor must have been dependent upon the Mach number, the shock systems which were studied approaching the trailing edge as the tunnel speed was raised.

(*a*) With a tunnel speed corresponding to a Mach number at infinity, M_∞, of 0·843 a thick laminar boundary layer was obtained on the model, and the local Mach number just before the shock system was 1·18. The resultant system was not a λ-shock but that of Fig. 19. The boundary layer appeared to act as a free boundary inasmuch as the first shock was followed by an expansion of about the same streamline deflection but in the opposite sense. Although a slight continuous compression was observed upstream of the shock and the boundary layer in that region was thickened considerably, not only separation but also transition was prevented by the thinning effect of the Prandtl-Meyer expansion.

(*b*) At the same Mach number, but with a plate added to the model to produce a turbulent boundary layer, the system obtained was substantially the same as that in the Swiss investigation (*see* Fig. 17).

(*c*) With a laminar boundary layer and $M_\infty = 0·895$ the system obtained (*see* Fig. 20) was similar to a λ-shock but exhibited the following differences :—

(i) The thickening laminar boundary layer was not wedge-shaped but concave, and transition occurred sooner than in the Swiss system.

(ii) As a consequence the well defined oblique member of the λ-shock was replaced by a large compression 'fan.'

(iii) The main shock did not appear to be followed by any subsequent supersonic expansion.

(*d*) When the boundary layer was rendered turbulent at $M_\infty = 0·895$ a single intense shock was obtained as before, but with one major difference, namely, that the boundary layer separated some distance downstream of the shock.

5.4. *The N.P.L. Investigation.*—This investigation (Fage and Sargent[8], 1947) differed from the others inasmuch as the flow was entirely supersonic, and externally-produced shocks of various known intensities were arranged to impinge upon, and in some instances be reflected by, the boundary layer on a plane surface. The system which was used is illustrated in Fig. 21, and the independent variables considered were the Mach number and the incident shock intensity.

The formation of a λ-shock was described as the bifurcation of an existing shock, and it was concluded that with a turbulent boundary layer the phenomenon would usually occur when the reacting shock intensity exceeded, but not when it was less than, a certain limiting value. This limiting intensity, or static pressure ratio, was found to be substantially independent of Mach number and to have a value of about 1·8.

The static pressure distributions measured at the wall in the region of the bifurcated shocks were found to be somewhat different from those recorded for λ-shocks by Ackeret, Feldmann and Rott, the differences being apparent in Fig. 22.

5.5. *Tentative Design Rules.*—From the above data it would be hazardous to draw any general conclusions, except perhaps that the interaction of a transonic shock with a laminar boundary layer, or with a turbulent boundary layer when the static pressure ratio across the shock exceeds 1·8, will tend to produce a λ-shock or some closely related phenomenon. As this will almost certainly cause boundary-layer separation it should, however, be avoided whenever possible.

It thus appears that the effects of the more serious consequences of compression shock boundary layer interaction may perhaps be avoided only when the following design rules are observed.

(i) Should a compression shock be permitted to impinge upon a laminar boundary layer it must be of the simple oblique type and permit of simple reflection.

(ii) If any shock not adhering to the above limitations must inevitably interact with a boundary layer, the latter should be rendered turbulent upstream of the base of the shock.

There is no doubt that the above rules are very rough, but they most probably err on the safe side. It is therefore to be expected that further experience and accumulated data will eventually make possible the adoption of less stringent design limitations.

PART II
FURTHER CONSIDERATIONS

6.0. *Introduction to Part II.*—Should the flow upstream of a compressor cascade be supersonic a substantial pressure rise may be obtained by means of three distinct phenomena—a gradual shockless compression, an anchored shock system, or an 'overexpansion' shock. With only shockless compression the velocity at outlet from the cascade also must be supersonic and that system is considered as a very special case.

In common with the other aerodynamic types of compressor, the supersonic axial-flow unit will undoubtedly produce serious design problems involving matching, stability and radial equilibrium, etc. The ultimate form of the machine will, moreover, most probably be determined by such properties rather than by the precise behaviour of the flow in its transonic cascades, and for this reason they must be considered at an early stage in any investigation.

The text of Part II comprises merely a few miscellaneous and unconnected considerations which are intended neither to be typical nor exhaustive. Whenever the shock system is considered it is assumed to be anchored at the leading edges of the cascade, but this must not be construed as an implication that the 'overexpansion' shock system should be neglected.

7.0. *The Possible Forms of a Compressor.*—7.1. *General Note.*—Axial flow compressors, according as the fluid velocity at inlet to their various components is supersonic or subsonic, may be classified as having either

(i) subsonic rotors and stators,

(ii) subsonic rotors and supersonic stators,

(iii) supersonic rotors and subsonic stators, or

(iv) supersonic rotors and stators.

Group (i) is of course outside the scope of this note and the sketching of a few velocity triangles will demonstrate that group (iv) units would, in practice, involve considerable difficulties. The only supersonic axial-flow compressor which has so far been known definitely to have operated (*see* section 13.2 below) having been of the group (ii) variety, employing substantially the same operating principles as the orthodox centrifugal unit; this type may well be considered first.

7.2. *The Impulse Rotor Unit.*—The operation of a compressor of the group (ii) type is illustrated in Fig. 23, from which it may be observed that considerable deflections must be employed both in the rotor and in the stator elements. To produce a stator inlet Mach number of 1·5, the deflections necessary in a pure impulse rotor are indicated in Fig. 24, and it is evident that these might successfully be achieved with comparatively small losses. In practice the rotor boundary layers could be stabilised and low losses thus ensured by the provision in that element of a little negative reaction, that is, expansion.

It is not, therefore, in the rotor but in the subsequent flow that the inherent disadvantages of the group (ii) compressor are to be found, these being substantially as follows.

(*a*) *Wall Friction.*—As the flow is supersonic relative to the compressor casing some distance upstream of the transonic stator cascade, considerable supersonic boundary layers may be formed on both the inner and the outer walls of the annulus and so effect the production of disproportionately large annulus frictional and interaction losses.

(*b*) *Radial Instability.*—Owing to the relative magnitude of the mean fluid peripheral Mach vector it will in general not be practicable to twist the rotor blades sufficiently to ensure radial equilibrium in the rotor outlet plane. As a consequence considerable radial flow components are to be expected immediately downstream of the rotor, and these will most probably effect a separation of the boundary layer on the inner annulus wall, with perhaps even a substantial flow reversal.

(*c*) *Velocity Profiles.*—Assuming for the moment that the difficulty envisaged above does not arise and that radial equilibrium is satisfactorily achieved in the stator inlet plane, it is evident that the desirable velocity and pressure gradients thus obtained will be destroyed and most probably reversed on passage through the stator shock-wave system. With the radial pressure gradient in the high-speed subsonic diffuser not merely zero but negative, the resulting chaos might thus be somewhat difficult to visualize. Moreover, even if separation did occur on the inner annulus wall before the stator, the conditions in the latter could scarcely be much better.

(*d*) *Subsonic Diffusion.*—Even after the transonic stator cascade, there arises a considerable problem inasmuch as the high-speed stream must be deflected by a compressor cascade or cascades through an angle of the order of 30 deg in the initial stages of a multi-stage unit and of 60 deg in the final stage. If high losses are to be avoided, this operation will, with the best modern practice, involve respectively the use of one or two subsequent stator cascades. Including the inlet guides, single, two and three stage group (ii) compressors must therefore have respectively four, six and eight stator cascades.

Because of these several inherent disadvantages no further reference will accordingly be made to compressors of the group (ii) variety.

7.3. *The Supersonic Rotor Unit.*—Should the flow through the rotor of a compressor be entirely supersonic, or—more precisely—should it be supersonic in both the inlet and the outlet planes, compression may be effected in several ways. Nevertheless, the various arrangements are merely alternative means of so contracting the passage area that the gas must be compressed in order to satisfy the requirement of continuity. An examination of Fig. 3 will therefore reveal the fact that, should a rotor with a fixed contraction ratio be accelerated under the conditions envisaged, the pressure ratio developed will drop with increasing speed.

A more detailed analysis will show, moreover, that this decrease is of such an order that, as the compressor is accelerated, it may at first require not only a decreasing torque but actually a steadily decreasing amount of power to drive it. Such a unit would thus be mechanically unstable, and so it appears that the only form of supersonic axial-flow compressor which might be of much practical value is that having subsonic stator elements and a transonic rotor. Any further reference to a supersonic compressor will accordingly be understood to refer only to a unit of this type.

8.0. *Supersonic Cascade Entry Conditions.*—8.1. *With a Cascade of Laminæ.*—Consider a cascade of laminæ identically disposed in a unform field at a stagger β, as in Fig. 25. Should one blade be considered at a negative incidence, i, it may be observed to give rise to a shock of inclination ζ and, provided that $\beta + \zeta + i \nless \pi/2$, the fluid will be so deflected that adjacent blades of the cascade no longer operate under similar conditions. Should a positive incidence be postulated a Prandtl-Meyer expansion will result and an equally impossible system be obtained. It must therefore be concluded that, in theory, an isolated cascade of laminæ can operate supersonically only at zero incidence.

The condition $\beta + \zeta + i \nless \pi/2$ being unlikely to arise in practice, it thus appears that an isolated rotor bladed with laminæ will pass theoretically only one mass flow at each rotational speed, namely that giving zero incidence, and so can do no useful work. Should the mass flow through such a system tend to increase the algebraic blade incidence will tend to decrease, and it is suggested that a tendency towards increased fluid static pressure might then exist before the rotor, thus dropping the mass flow to its theoretically correct value. If this hypothesis were correct a dropping mass flow would similarly be restored, the system being therefore in a state of stable equilibrium.

8.2. *With a Cascade of Thin Aerofoils.*—Should a cascade of thin arbitrary aerofoils be considered at zero incidence, as in Fig. 26(a), the shock due to the curvature of one blade will be observed to give rise to the operation at a positive incidence of one or more of the other blades, so that such a régime is unstable and therefore cannot persist. At positive incidence, however, the shock will be preceded by a Prandtl-Meyer expansion, as in Fig. 26(b), and there must be some incidence at which the total deflection produced by each expansion-shock pair is zero, the cascade being then in a state of apparent equilibrium.

As a shock involves an irreversible process such an isolated régime might, however, be indeterminate. Thin aerofoils in an isolated supersonic cascade cannot therefore be of entirely arbitrary shape, but should be straight for a distance from the leading edge sufficient to ensure that, at zero incidence, no fluid discontinuities can be propogated upstream. Provided only that there is no sudden discontinuity in the curvature of the aerofoil, it may be observed from Fig. 26(c) that the minimum straight entry length is given by

$$\frac{b}{c} = \frac{s}{c}\left\{ \sin \beta - \frac{\cos \beta}{\tan \mu} \right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (9)$$

8.3. *With Aerofoils of Finite Thickness.*—In this note the classical conception of a compressor blade or aerofoil (Howell, 1942) will be replaced by that of two independent surfaces, designated the upper and lower surfaces, which intersect at angles θ_1 and θ_2 at the leading and trailing edges respectively. Of these two the lower or leading surface is that which, at design incidence, has the greater normal incident Mach vector.

Unless $\theta_1 > \delta_{d1}$ the entry conditions to a cascade at design incidence can be affected only by the upper surfaces of the blades, and in this respect these surfaces will behave as would thin aerofoils of the same shape. The entry design requirements to be satisfied by blades of finite thickness in an isolated supersonic cascade at a Mach number M_1 are therefore as follows.

(i) The leading edge of each blade must be sharp and its included wedge angle, θ_1, less than the detachment angle δ_{d1}.

(ii) The upper surface of each blade must be straight for a minimum distance, b, from the leading edge defined by equation (9).

(iii) The pitch must be sufficiently small to ensure that no shock arising from the upper surface of one blade can pass upstream of the leading edge of an adjacent blade.

The operation of a compressor at other than design incidence will, in practice, give rise to the propagation upstream of the rotor of compression and/or expansion 'waves,' but as these disturbances will be interrupted by the upstream stator element the argument concerning the stability of the flow through an isolated rotor need not necessarily be applicable.

9.0. *Single Stage Compressor Stability.*—9.1. *General Note.*—As no pertinent experimental characteristics are available it is not yet possible to predict with any degree of certainty the stability limits of a supersonic compressor or to estimate its efficiency except under design conditions. There is, however, sufficient evidence to provide a reasonably good qualitative indication of the form that the characteristics and limitations are likely to take in the pressure-ratio mass-flow field.

Should the stability of a single-stage unit be considered in terms of the limiting angles of incidence of the air on its rotor blades, the line of operation at design incidence will be found a convenient datum around which to construct the characteristics. This is done in Fig. 27, where the useful operating regions are shaded.

9.2. *The Choking Line.*—If the subsonic performance of a compressor accelerated at its design incidence be considered, it may be observed that a point C will be reached at which the rotor cascade becomes choked. Should the speed now be kept constant and the aperture of the controlling throttle varied, unchoked operation may be obtained at any point on the characteristic between A and C, the cascade area ratio (inlet to 'throat' changing with incidence. Between C and B, however, the drop in pressure ratio is due not to a reduction in incidence, for with a constant mass flow the latter must remain constant and equal to the design incidence, but to a considerable increase in the losses associated with the transonic flow behind the rotor cascade 'throat.'

On other characteristics there will be found points such as C′ and C″ which are similar to C, and the locus of these points is a line which extends into the supersonic performance region and which will be described henceforth as the compressor rotor choking line.

The practical significance of the rotor choking line of a compressor is that operation below it is possible but will be accompanied by the formation of undesirable overexpansion shocks. These shocks may effect boundary-layer separation and so provide losses of considerable magnitude. Being unstable, they may also excite severe blade vibration and thus, under high-speed conditions, possibly destroy a well designed supersonic rotor.

9.3. *The Surge Line.*—As the surging of an orthodox axial-flow compressor is connected with the stalling of its blades this phenomenon may also be expected to arise in high-speed rotors and, in view of the sharp leading edges employed, to occur at a rather small incidence.

However, as the rotor inlet Mach number approaches unity, the existing flows around the leading edges at positive incidence will be replaced by Prandtl-Meyer expansions and the possibility of separation thus avoided. It is possible of course that separations caused by compression-shock boundary-layer interaction will occur further downstream, but this phenomenon should not produce a compressor surge near the design incidence.

The surge line of a typical supersonic axial-flow compressor might therefore be expected to exhibit a considerable change in curvature in the transonic region as shown.

9.4. *Supersonic Operation at Positive Incidence.*—Should the mass flow through a supersonic compressor be reduced at constant rotor speed two conflicting changes will occur in the operating conditions. Firstly the rotor inlet Mach number will be reduced, so tending to lower the pressure ratio, and secondly the rotor blade incidence and consequently the rotor deflection will be increased.

The precise effect of this phenomenon will vary to a considerable extent with the details of the rotor cascade design, but in general it should at first cause a slight increase in the pressure ratio and later a considerable drop. This is due to the production by the leading-edge expansions of undesirably high fluid Mach numbers, with their associated shock and interaction losses. In extreme cases the effect may become so severe that, if a surge does not occur, the designed shock system is replaced by one or more overexpansion shocks of considerable intensity. The position of the point of maximum pressure ratio in relation to the design incidence line will of course depend upon the detail design of the rotor.

9.5. *The Hunting Line.*—Should the mass flow through a compressor be increased the rotor blade incidence will be reduced, and in general the shape of the characteristics should not be very different on either side of the design incidence line. At high speeds, however, a point must be reached at which the obliquity of the rotor blade leading-edge shocks is such that the latter pass directly out of the rotor without reflection.

When this occurs the unit will in theory change instantaneously to a different mode of operation and the characteristics will become vertical at the point in question. Depending upon the detail design of the rotor, the secondary mode of operation will be either that of a high-speed reaction turbine or of a completely supersonic compressor rotor, but, as the operating conditions will in any case oscillate catastrophically between those of the primary and secondary modes, this difference is of little importance.

In practice, however, the change may not occur so suddenly, for the effects of the inlet guide wakes will be to produce a rotor shock system which in normal operation oscillates with a small amplitude at a relatively high frequency—of the order of ten kilocycles per second. The shocks involved will thus be effectively broadened and so will require a measurable incidence range to pass completely out of the rotor.

Although in practice the characteristics of a compressor at negative incidence will probably be curved as shown and no catastrophic effects may result from operation on the sharply curved portion of the characteristic, running in this region will nevertheless be very ' rough ' and in all cases inadvisable. The limiting line defined by the phenomenon on the pressure-ratio mass-flow field will be described hereinafter as the hunting line.

10.0. *General Rotor Design Considerations*.—10.1. *Stagger*.—Being the only one which can appreciably affect the Mach vector triangles of a unit, stagger is by far the most important of the supersonic rotor design variables and its major effects may perhaps more easily be understood after reference to Fig. 28. These effects may be divided into two groups according as they arise from variations of the Mach vector triangles or from variations of the supersonic cascade performance.

In the first group the results of a change in stagger affect only the unit design point performance, an increase in stagger raising both the pressure ratio and the adiabatic efficiency at the expense of a considerable reduction in mass flow.

The main effects on the supersonic cascade performance are that an increase in stagger will raise the hunting line and drop the rotor choking line, so by this means the stability of the unit, both during acceleration and at high speeds, can be considerably enhanced. Some of these effects are demonstrated in the following example.

In the theoretical supersonic performance estimate for a two-stage aircraft unit designed for 990 m.p.h. at 50,000 ft and operating at design incidence, it was found that at the design point the mean diameter stage conditions would be as in Table 1.

TABLE 1

Theoretical Performance Estimate

Quantity	*1st Stage*	*2nd Stage*
Stagger, β	60 deg	73·5 deg
Upstream stator outlet angle, α_{s1}	25 deg	54 deg
Upstream stator outlet Mach number, M_{s1}	0·90	0·78
Rotor inlet Mach number, M_{r1}	1·60	1·60
Total head inlet temperature	314 deg K	380 deg K
Total head inlet pressure	5·53 lb/in² abs.	9·32 lb/in² abs.
$\dfrac{\text{Supersonic frictional loss}}{\text{Useful temperature rise}}$	0·0256	0·0408
Relative corrected mass flow (at N.T.P.)	1·000	0·539
Stage pressure ratio	1·685	1·732
Stage adiabatic efficiency	76·3 per cent	78·2 per cent

The reason for the low pressure ratios and efficiencies is that, whilst the upstream stator outlet velocities are relatively high, the rotor inlet Mach numbers must be limited to 1·60 because of the high total head inlet temperatures. For the same reason the large difference between the total head inlet temperatures of the two stages markedly reduces the differences which would otherwise occur in their pressure ratios and efficiencies. With this fact in view it may thus be realised that the increase of 13·5 deg in stagger has resulted in a considerable improvement in performance, especially as the higher gas velocity and density in the second stage produce increased supersonic blade and annulus frictional losses. The estimated supersonic design line performance of the compressor and its stages is shown in Fig. 29.

10.2. *Pitch.*—The main effect of an increase in rotor blade pitch is to drop both the rotor choking line and the hunting line so that, whilst better acceleration may be obtained, the maximum attainable pressure ratio may be determinated not by mechanical considerations but by the necessity for avoiding instability at high speeds.

A change of pitch should, however, produce but little secondary effect, for at a given speed neither the pressure ratio and the adiabatic efficiency of, nor the mass flow through, a unit should be affected greatly by the number of rotor blades used.

10.3. *Blade Form.*—The effect of blade form in a supersonic compressor is a problem with many aspects, but in general the most important effects are those which detract from the performance or the stability of the unit and so may be considered qualitatively with a view to their elimination in the early design stages.

Because of the attenuation of the designed shock system by expansion zones, difficulty may be experienced in ensuring transonic operation at high speeds. That is, the position of the rotor hunting line on the pressure-ratio mass-flow field may be influenced to a considerable extent by the detail design of the supersonic blade profiles. There exists in addition the possibility that, despite the fact that a transonic compression shock system is confined within the rotor, the flow may become supersonic in the outlet plane as a result of the disturbance caused by a separated boundary layer.

The properties of a good profile are therefore as follows.

(i) At the selected stagger and pitch-chord ratio it should have a leading-edge angle, θ_1, and a form such that, at design incidence, the flow through the cascade is barely transonic at the maximum compressor speed. That is, such that the hunting and mechanical speed limitations of the unit should coincide on its design incidence line.

(ii) It should have the lowest possible 'boundary-layer drag,' the latter term including the total boundary-layer frictional and separation losses.

(iii) It should provide a reasonably good subsonic performance.

As the total supersonic expansion around a profile is a measure of the shock wave attenuation and the extraneous shock loss which it produces, a useful criterion of the desirability of a blade might be the smallness of its total convexity at design incidence. This expression is defined quantitatively in Fig. 30 and a useful design value might be $2\theta_1$, which is twice the theoretical minimum with an infinite chord.

If serious attenuation and/or subsonic boundary-layer separation is to be avoided, this convexity should, moreover, be distributed as evenly as possible over the profile and in particular it should be distributed on both sides of the blade. In accordance with the overriding entry design requirements the leading part of the upper blade surface should nevertheless be straight, thus providing a profile of the type illustrated in Fig. 31. Such profiles should automatically satisfy the requirement of a reasonable subsonic performance, for a deflection of approximately $\theta/2$ will be obtained in the desired sense at design incidence.

10.4. *Boundary-Layer Conditions.*—In fulfilment of the requirement for low boundary-layer drag it is essential to maintain a laminar boundary-layer over as large a portion of the profile as possible, whilst at the same time avoiding when possible any separation due to shock interaction. Assuming that the boundary-layer on either side of the blade is initially laminar and will remain so unless it is deliberately disturbed, there appear to be three main practicable arrangements, these being illustrated in Fig. 32. Here the points marked X are the sites on the profile of small steps designed to promote boundary-layer transition, the turbulent region being shown broken.

The first method is of use in units which must have a large working range and in which separation must be avoided at all speeds. For units in which it is desirable to have the highest possible cruising efficiency and it is expedient to obtain this at the expense of a slightly inferior low speed performance, either of the remaining schemes may be used. In the last, which will involve considerable precision in design, the expansion around the upper blade surface is used so to attenuate the incident shock that a simple reflection is obtained in the laminar boundary layer at cruising speed.

Although the boundary layer will most probably remain laminar on the curved lower surface, it may exhibit automatic transition on the plane upper surface at large Reynolds numbers. The precise design of the boundary layers on a given rotor blade must therefore depend largely on its most frequent operating conditions.

The above discussion does not, of course, consider the fact that the wakes from a preceding subsonic cascade might render the boundary layers completely turbulent. Should this be the case the arguments will have been in vain, but whether or no periodically encountered wakes can in general produce such an effect, and if so in what magnitude, is yet to be established.

PART III.

Development in Germany

11.0. *Introduction to Part III.*—As is so often the case it is not possible definitely to ascribe the invention of the supersonic axial-flow compressor to any one man or even to one group, for the idea was apparently first conceived at about the same time by Weise of the D.V.L., Berlin, and by Encke and Betz of the A.V.A., Göttingen.

The first compressor to be built and tested was undoubtedly Encke's, its design being commenced in 1935 and the tests performed in 1936. However, as the first patents were taken out by Weise in 1936 the latter must have been working on the subject at about the same time as Encke and Betz.

After the failure of the Göttingen unit a small amount of pertinent supersonic wind tunnel work was continued at the A.V.A., but the responsibility for the supersonic compressor was transferred at the direction of the Government to the D.V.L., officially for economic reasons. All further development was thus continued under Weise, his first experimental unit, type I, being tested in 1942, and the type II unit in 1944.

12.0. *The Göttingen Unit.*—Unfortunately Encke never reported his results, and as no drawings or other documents are now available it will not be possible to describe the investigation exactly, the circumstances being apparently as follows. The compressor was a single-stage unit having a supersonic rotor of just under 30 cm outside diameter and with a maximum speed of about 30,000 r.p.m. After the fashion of the 'Busemann biplane' the blades were triangular in section and were so disposed that the head shock from each blade impinged on the obtuse vertex of the next. It was thus supposed that at the throat so formed the flow would change from supersonic to subsonic and that the remainder of the cascade channel would act as a high-speed subsonic diffuser.

As the German ball-bearing industry of the period could not supply a high duty thrust bearing suitable for the project it was necessary to overload a commercial unit of the type then available, the design of the rig being to this extent mechanically unsound. In order to reach its design point the compressor was accelerated with the controlling throttle wide open, the mass flow being thus high, the pressure rise of the order of 1 cm of mercury and the rotor blades operating supersonically at a very small lift coefficient.

After a few measurements had been made under these conditions the throttle was gradually closed and the pressure rise increased to about 10 cm of mercury, at which point a violent aerodynamic oscillation commenced and the thrust bearing failed with the consequent destruction of the rotor. An inspection of the remains indicated that considerable axial forces had been acting on the rotor, principally in a direction opposed to that of the airstream through the unit.

13.0. *Development under Weise.*—13.1. *The Test Rig.*—Although not widely reported, Weise's results did appear in 1943 in a paper in the *Lilienthal-Gesellschaft Bericht* 171[1] and the rig on which the testing was performed is illustrated in Figs. 33, 34 and 35. It consisted substantially of a rotor with two discs mounted between journal bearings and driven by a swinging field dynamometer at a maximum speed of 25,000 r.p.m. Either single or two-stage units could thus be tested, the i/d : o/d ratio being 0·8 and the maximum tip speed 1,370 ft/sec.

For the test systems involving supersonic stators, boundary-layer suction could be applied both to the inner and to the outer walls of the stator elements, the annular passages provided for this purpose being visible in Fig. 35 and the suction pipe flanges obvious in Fig. 33. Aerodynamic data was provided by numerous static holes and thermocouples in the casing and guide vanes, the compressor having an annular entry and a conventional type of exhaust volute.

13.2. *Compressor Type I.*—This unit had a subsonic impulse rotor and a supersonic stator, its velocity triangles and cascades being illustrated in Fig. 36. The rotor was designed for positive reaction at the tip, impulse at mean diameter and negative reaction at the root and, in order to minimise the losses, the profiles were carefully designed with a lift coefficient of about one.

The stator consisted of thin uncambered blades in a radially diverging annulus—the high-speed flow being thus deflected as little as possible—and the compression was effected mainly by substantially normal shocks situated at the leading edges of these blades. In order to reduce the annulus losses to a minimum the axial clearance between the rotor and stator was made as small as possible and shock-boundary layer interaction was avoided by the introduction of suction slots in the stator element as indicated.

In view of the amount of negative reaction at its blade roots Weise himself was rather worried about the starting of the rotor, and as a consequence the leading edges were made more blunt and the trailing edges more sharp than is the usual practice. No trouble was, however, experienced, the circulation starting invariably in the desired sense. An additional effect of the rotor twist was that it should have helped to reduce the radial flow components upstream of and within the stator element.

The unit was apparently designed for a pressure ratio of about 2·0 at 24,000 r.p.m. with an inlet temperature of 273 deg K and the test results are shown in Fig. 37. As the low efficiencies and pressure ratios were ascribed to secondary flows the stator was removed and traverses performed at low speed in a cylindrical annulus behind the rotor. The results of these traverses are shown in Fig. 38, from which the severity of the effect may be appreciated.

13.3. *Compressor Type Ia.*—This unit was described by Weise as a ' pure shock compressor,' the principles involved being demonstrated in Fig. 39. The intention was evidently to use a pure impulse untwisted rotor, and so to design the system that the rotor inlet and stator outlet velocities would be the same. Most of the compression was thus to be effected by the supersonic stator inlet shock, and the change of annulus shape was intended to reduce to some extent the losses caused by boundary-layer separation in the stator.

However, it was apparently realised that a mere change of annulus shape would not suffice to overcome the undesirable secondary flows and their associated losses, for, instead of building a unit of the type Ia, Weise then resorted to more fundamental and important changes.

13.4. *Compressor Type II.*—The unit evolved—designated type II—had a supersonic rotor and is illustrated in Fig. 40, the shock wave pattern shown being what was then assumed would occur. Despite the progressive change to the supersonic rotor the potential flow in the cascade was apparently not considered, and it was hoped that a substantially normal shock would occur as in type I unit.

Before the test the tip clearance of the light alloy rotor was about 0·5 mm and, in order to reach the design point, the method adopted was that used by Encke, namely to accelerate to a high speed at a large mass flow and then gradually to close the throttle until the required conditions were attained. However, after some fifteen or twenty minutes steady operation at 90 per cent full speed with the throttle fully opened, slight vibration occured and the rotor blading failed before the unit could be decelerated. During the process of failure no distinctive noise was emitted, and the vibration was observed to be of too low a frequency to be associated with any known natural frequency of the compressor.

The casing was scored, the rotor blades were crushed slightly from root to tip and their leading edges were cracked (parallel to the chord) at regular intervals and bent in alternate senses between adjacent pairs of cracks, but no other damage occurred. The blade crushing and cracking may not have been associated phenomena, but it is most probable that the regular cracks occurred at the sites of the nodes of a peculiar form of blade vibration of an exceedingly high frequency excited by tip rubbing. If this were not so the nature of the failure would be difficult to understand, for there is no known way in which any aerodynamic phenomenon could produce such a result.

13.5. *Compressor Type III.*—The last system to be considered was that shown in Fig. 41, it being intended that the inlet guide vanes should be abandoned in favour of an axial inlet and that a purely supersonic rotor should be used.

It was fortunate, however, that a unit of this type was never built, for as mentioned in section 7.3 above, such a compressor would be inherently unstable under supersonic conditions.

14.0. *General Note.*—In the field concerned Encke, Betz and Weise were undoubtedly many years ahead of any other workers, and the test results of Weise's type I unit, disappointing as they may well have been, proved that a supersonic axial-flow compressor could be made to operate successfully and stably with acceptable pressure ratio characteristics.

It was unfortunate that both the attempts made at a compressor with a supersonic rotor should have succumbed to mechanical difficulties, for it seems fairly certain that Encke's failure was due to the lack of a suitable thrust bearing and Weise's to the necessity for manufacturing the rotor in light alloy. Further investigations should not therefore be influenced in any way by the premature failure of these two German units, for the means are now available to ensure that future compressors need not be fraught with such mechanical dangers as were experienced in those days.

The progress in Germany of the work on the supersonic axial-flow compressor was rather unfortunate in so far as the ad hoc nature of some of the tests and proposals are concerned, for there certainly was not any great dearth of basic data. Indeed, Weise himself in his preliminary investigations made and reported in 1943 an independent discovery of the general form of a shock wave reflection[10] and of the effects of boundary-layer interaction in a high-speed diffuser[1]. He also developed the ' heart curve ' method for the theoretical treatment of shock-wave intersection and used it to demonstrate the connection between boundary-layer separation and the bifurcated or λ-shock in cases of interaction.

However, the aerodynamic design of the type I unit and the proposals for type III indicate that, despite these achievements, the importance of the radial equilibrium of the flow was not fully appreciated in the design stage and that a survey of the problem of compressor stability could not properly have been undertaken.

15.0. *Conclusion.*—Some of the basic data which is at present available and might be of use in the design of a supersonic axial-flow compressor is summarised in the first part of this note. Of the information presented, that concerning shock boundary-layer interaction is, however, rather scant, and so should be considered more as evidence than as design data.

From a study of the available facts it is felt that although several varieties of supersonic compressor might be possible only one major type appears likely to be of much practical significance. This type has accordingly been considered in detail and some suggestions have been made regarding its possible development.

Early German work has demonstrated that a supersonic axial-flow compressor can be made to operate successfully, and the failures encountered in those days should not be considered as a deterrent, for they were due largely to mechanical hazards which could, with good modern design, be eliminated.

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APPENDIX I

Nomenclature

General.—Symbols.—

- p Absolute pressure
- t Absolute temperature.
- ρ Density.
- γ The isentropic exponent.

Nomenclature

General.—Symbols.—

J	Joule's equivalent.
c_p	Specific heat at constant pressure.
c_v	Specific heat at constant volume.
u	A velocity.
v	A velocity.
w	A velocity.
M	Mach number.
A	Area.
A_t	Throat area.
μ	Mach angle.
ζ	Shock inclination.
δ	Streamline deflection.

Subscripts.—

d	Detachment.
tot	Total.
∞	At infinity.

Compressor—Symbols.—

s	Blade pitch.
c	Blade chord.
t	Blade thickness.
b	Rotor blade straight entry length.
l	Rotor stage axial length.
θ	Rotor blade subtended wedge angle.
β	Stagger.
α	Air angle.
i	Incidence.
N	Speed in r.p.m.
Q	Mass flow.

Subscripts.—

0	Stator inlet.
1	Stator outlet and rotor inlet.
2	Rotor outlet.
r	Relative to rotor.
s	Relative to stator.
p	Peripheral.

Abbreviations.—

D.V.L.	Deutsche Versuchsanstalt für Luftfahrt.
E.T.H.	Institute für Aerodynamic Eidgenössiche Technische Hochschule Zürich.
G.A.L.C.I.T.	Guggenheim Aeronautics Laboratory, California Institute of Technology.
N.P.L.	National Physical Laboratory.
R.A.E.	Royal Aircraft Establishment.

APPENDIX II

The Expansion of a Compressible Fluid

In accordance with the procedure adopted in the main part of this note the following is not a proof but merely a reduction of some well known equations to forms more suited to constant usage in the field considered.

Should the isentropic expansion of a compressible fluid be considered by means of the orthodox one-dimensional theory the following equations will be found to apply.

$$\rho_1 . A_1 . v_1 = m = \rho_2 . A_2 . v_2, \qquad (1)$$

$$v_2{}^2 - v_1{}^2 = 2.J.c_p\,(T_1 - T_2), \qquad (2)$$

$$M = v(\gamma RT)^{-1/2}, \qquad (3)$$

$$p_2/p_1 = (T_2/T_1)^{\frac{\gamma}{\gamma-1}}, \qquad (4)$$

$$p/\rho = R.T, \qquad (5)$$

$$R/J = c_p - c_v. \qquad (6)$$

Equations (1) and (2) are statements respectively of the continuity of mass flow and the principle of the conservation of energy, equation (3) is the definition of Mach number and remaining equations deal with the various physical properties of the fluid.

Then, from equations (2) and (3),

$$\gamma R\,(T_2\,M_2{}^2 - T_1\,M_1{}^2) = 2.J.c_p\,(T_1 - T_2).$$

Therefore $$T_1\left\{\frac{2.J.c_p}{\gamma . R} + M_1{}^2\right\} = T_2\left\{\frac{2.J.c_p}{\gamma . R} + M_2{}^2\right\}$$

But, from equation (6),

$$\frac{J.c_p}{\gamma . R} = \frac{1}{\gamma - 1}$$

Therefore $$\frac{T_1}{T_2} = \frac{1 + \dfrac{\gamma - 1}{2} M_2{}^2}{1 + \dfrac{\gamma - 1}{2} M_1{}^2}$$

However, if the expansion commences with the gas at rest, the initial temperature will be T_{tot} and the Mach number zero.

Therefore $$\frac{T_{\text{tot}}}{T} = 1 + \frac{\gamma - 1}{2} M^2, \qquad (7)$$

and, from equation (4),

$$\frac{p_{\text{tot}}}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}. \qquad (8)$$

Then, from equations (1) and (5),

$$\frac{A_2}{A_1} = \frac{\rho_1 v_1}{\rho_2 v_2} = \frac{M_1}{M_2}\left(\frac{T_1}{T_2}\right)^{\frac{1}{2}\frac{\gamma+1}{\gamma-1}}$$

Or, from equation (7),

$$\frac{A_2}{A_1} = \frac{M_1}{M_2}\left\{\frac{1 + \dfrac{\gamma - 1}{2} M_2^2}{1 + \dfrac{\gamma - 1}{2} M_1^2}\right\}^{\frac{1}{2}\frac{\gamma + 1}{\gamma - 1}}$$

But if the fluid is expanding isentropically in a nozzle the Mach number at the throat will be unity.

Therefore $$\frac{A}{A_t} = \frac{1}{M}\left(\frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1}M^2\right)^{\frac{1}{2}\frac{\gamma + 1}{\gamma - 1}} \qquad (9)$$

Thus, taking $\gamma = 1{,}400$, we have for air

$$\frac{T_{tot}}{T} = 1 + \frac{M^2}{5},$$

$$\frac{p_{tot}}{p} = \left(1 + \frac{M^2}{5}\right)^{3\cdot 5},$$

and $$\frac{A}{A_t} = \frac{1}{M}\left(\frac{5}{6} + \frac{1}{6}M^2\right)^3,$$

from which expressions the curves of Fig. 3 have been prepared.

APPENDIX III

Expansion Tables

I. *Mach Number Table*

Deflection deg	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
0	1·000	1·017	1·027	1·036	1·044	1·051	1·058	1·064	1·070	1·076
1	1·082	1·087	1·093	1·098	1·103	1·108	1·113	1·118	1·123	1·128
2	1·133	1·137	1·142	1·146	1·151	1·155	1·160	1·164	1·168	1·173
3	1·177	1·181	1·185	1·189	1·194	1·198	1·202	1·206	1·210	1·214
4	1·218	1·222	1·226	1·230	1·233	1·237	1·241	1·245	1·249	1·253
5	1·256	1·260	1·264	1·268	1·272	1·275	1·279	1·283	1·286	1·290
6	1·294	1·297	1·301	1·305	1·308	1·312	1·316	1·319	1·323	1·326
7	1·330	1·334	1·337	1·341	1·344	1·348	1·351	1·355	1·358	1·362
8	1·366	1·369	1·373	1·376	1·380	1·383	1·387	1·390	1·394	1·397
9	1·400	1·404	1·407	1·411	1·414	1·418	1·421	1·425	1·428	1·432
10	1·435	1·438	1·442	1·445	1·449	1·452	1·456	1·459	1·462	1·466
11	1·469	1·473	1·476	1·479	1·483	1·486	1·490	1·493	1·496	1·500
12	1·503	1·507	1·510	1·513	1·517	1·520	1·524	1·527	1·530	1·534
13	1·537	1·540	1·544	1·547	1·551	1·554	1·557	1·560	1·564	1·568
14	1·571	1·574	1·578	1·581	1·584	1·588	1·591	1·595	1·598	1·601
15	1·605	1·608	1·611	1·615	1·618	1·622	1·625	1·628	1·632	1·635
16	1·639	1·642	1·645	1·649	1·652	1·655	1·659	1·662	1·666	1·669
17	1·672	1·676	1·679	1·683	1·686	1·689	1·693	1·696	1·700	1·703
18	1·706	1·710	1·713	1·717	1·720	1·724	1·727	1·730	1·734	1·737
19	1·741	1·744	1·747	1·751	1·754	1·758	1·761	1·765	1·768	1·772
20	1·775	1·778	1·782	1·785	1·789	1·792	1·796	1·799	1·803	1·806
21	1·810	1·813	1·816	1·820	1·823	1·827	1·830	1·834	1·837	1·841
22	1·844	1·848	1·851	1·855	1·858	1·862	1·865	1·869	1·872	1·876
23	1·879	1·883	1·886	1·890	1·893	1·897	1·900	1·904	1·908	1·911
24	1·915	1·918	1·922	1·925	1·929	1·932	1·936	1·940	1·943	1·947
25	1·950	1·954	1·957	1·961	1·964	1·968	1·972	1·975	1·979	1·983
26	1·986	1·990	1·993	1·997	2·001	2·004	2·008	2·012	2·015	2·019
27	2·023	2·026	2·030	2·034	2·037	2·041	2·045	2·048	2·052	2·056
28	2·059	2·063	2·067	2·070	2·074	2·078	2·081	2·085	2·089	2·093
29	2·096	2·100	2·104	2·108	2·111	2·115	2·119	2·123	2·126	2·130

II. *Mach Angle Table*

Deflection deg	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
0	90·0	79·5	76·8	74·9	73·4	72·1	71·0	70·0	69·1	68·3
1	67·6	66·9	66·2	65·6	65·0	64·5	63·9	63·4	62·9	62·4
2	62·0	61·6	61·1	60·7	60·3	59·9	59·6	59·2	58·9	58·5
3	58·2	57·9	57·5	57·2	56·9	56·6	56·3	56·0	55·8	55·5
4	55·2	54·9	54·7	54·4	54·2	53·9	53·7	53·4	53·2	53·0
5	52·7	52·5	52·3	52·1	51·9	51·6	51·4	51·2	51·0	50·8
6	50·6	50·4	50·2	50·0	49·8	49·7	49·5	49·3	49·1	48·9
7	48·8	48·6	48·4	48·2	48·1	47·9	47·7	47·6	47·4	47·2
8	47·1	46·9	46·8	46·6	46·5	46·3	46·2	46·0	45·9	45·7
9	45·6	45·4	45·3	45·1	45·0	44·9	44·7	44·6	44·4	44·3
10	44·2	44·0	43·9	43·8	43·7	43·5	43·4	43·3	43·1	43·0
11	42·9	42·8	42·6	42·5	42·4	42·3	42·2	42·1	41·9	41·8
12	41·7	41·6	41·5	41·4	41·2	41·1	41·0	40·9	40·8	40·7
13	40·6	40·5	40·4	40·3	40·2	40·1	39·9	39·8	39·7	39·6
14	39·5	39·4	39·3	39·2	39·1	39·0	38·9	38·8	38·7	38·6
15	38·5	38·5	38·4	38·3	38·2	38·1	38·0	37·9	37·8	37·7
16	37·6	37·5	37·4	37·3	37·2	37·2	37·1	37·0	36·9	36·8
17	36·7	36·6	36·5	36·5	36·4	36·3	36·2	36·1	36·0	36·0
18	35·9	35·8	35·7	35·6	35·5	35·5	35·4	35·3	35·2	35·1
19	35·1	35·0	34·9	34·8	34·8	34·7	34·6	34·5	34·4	34·4
20	34·3	34·2	34·1	34·1	34·0	33·9	35·8	33·8	33·7	33·6
21	33·5	33·5	33·4	33·3	33·3	33·2	33·1	33·0	33·0	32·9
22	32·8	32·8	32·7	32·6	32·6	32·5	32·4	32·4	32·3	32·2
23	32·1	32·1	32·0	31·9	31·9	31·8	31·7	31·7	31·6	31·6
24	31·5	31·4	31·4	31·3	31·2	31·2	31·1	31·0	31·0	30·9
25	30·8	30·8	30·7	30·7	30·6	30·5	30·5	30·4	30·4	30·3
26	30·2	30·2	30·1	30·0	30·0	29·9	29·9	29·8	29·7	29·7
27	29·6	29·6	29·5	29·5	29·4	29·3	29·3	29·2	29·2	29·1
28	29·1	29·0	28·9	28·9	28·8	28·8	28·7	28·7	28·6	28·5
29	28·5	28·4	28·4	28·3	28·3	28·2	28·2	28·1	28·1	28·0

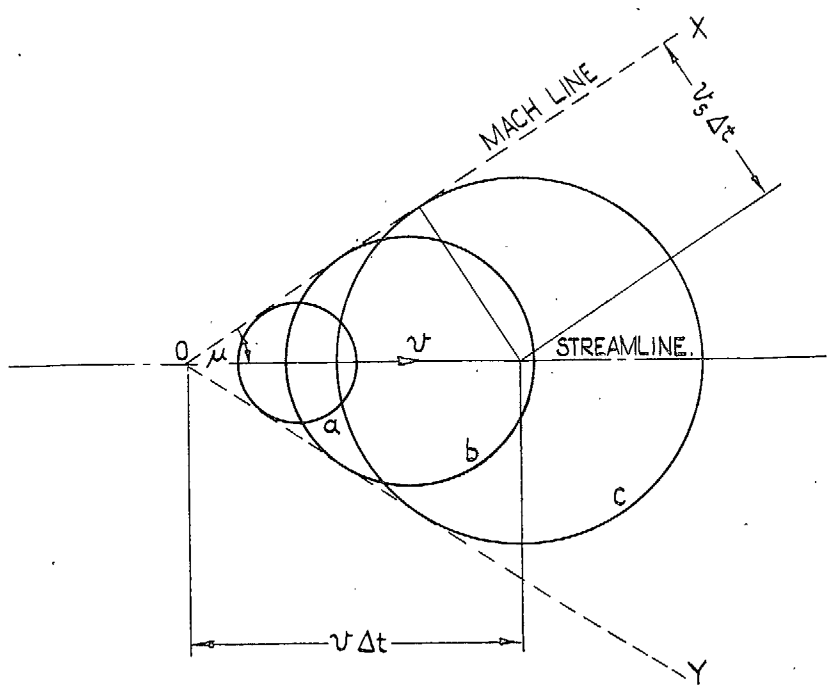


FIG. 1. Mach line definition.

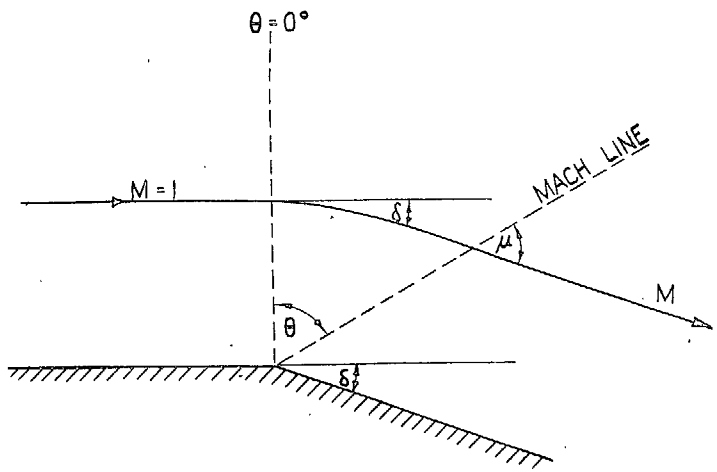


FIG. 2. The Prandtl-Meyer expansion.

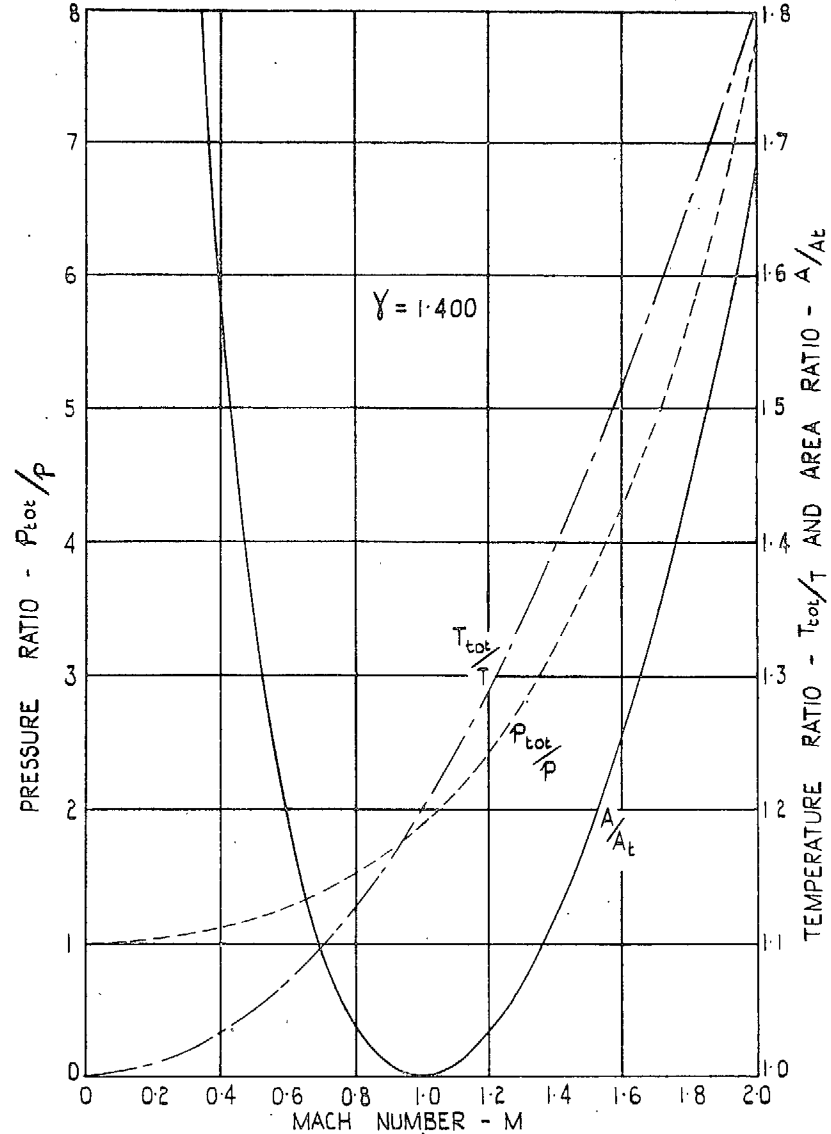


FIG. 3. The isentropic expansion of air.

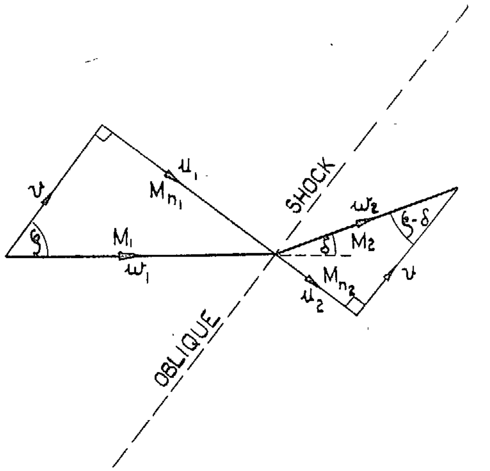


FIG. 4. Oblique shock theory.

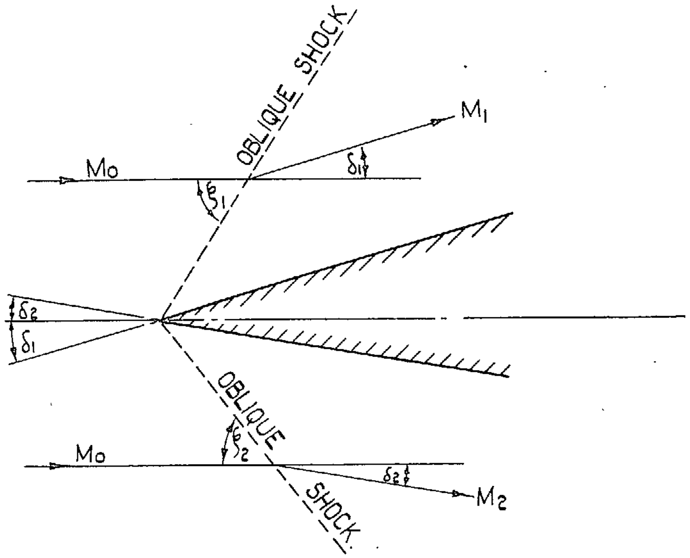


FIG. 6. The simple oblique shock.

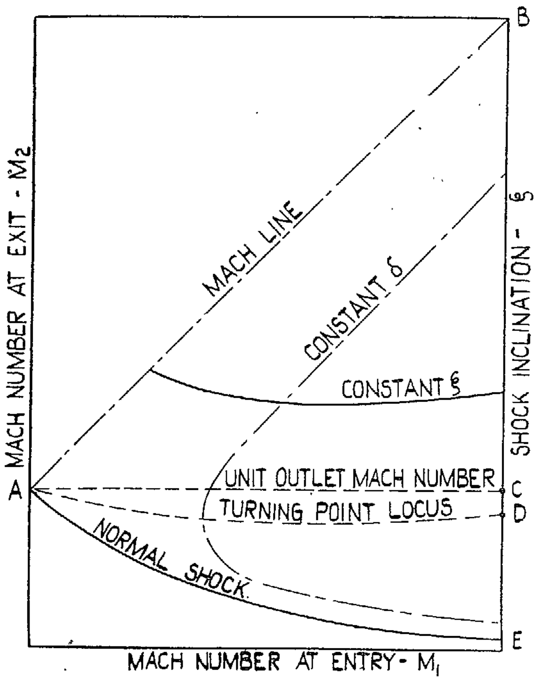


FIG. 5. Skeleton mach number chart.

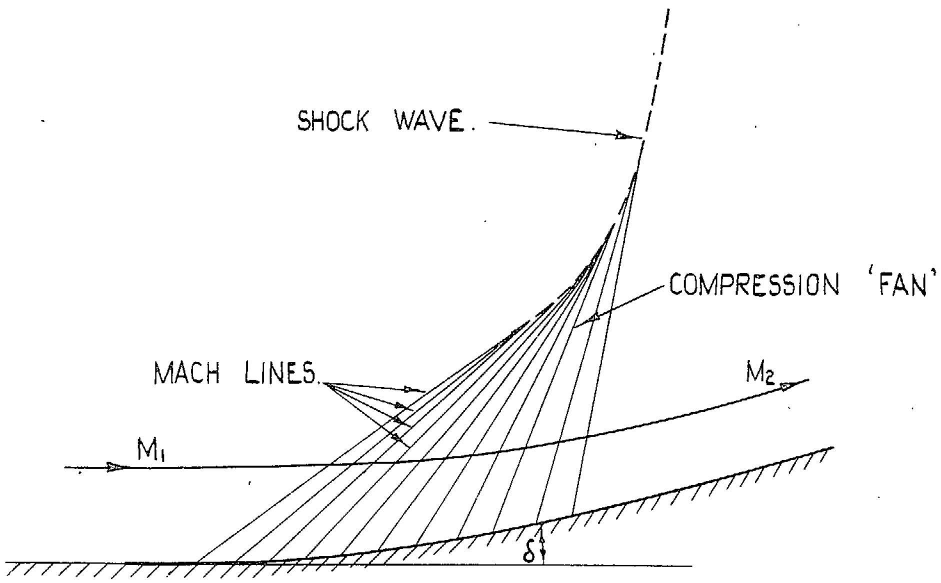


FIG. 7. The shock at a concave surface (exaggerated).

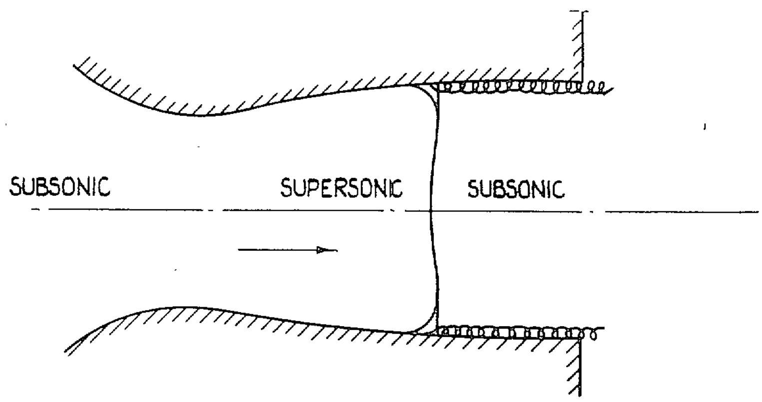


FIG. 8. A simple ' over-expansion ' shock.

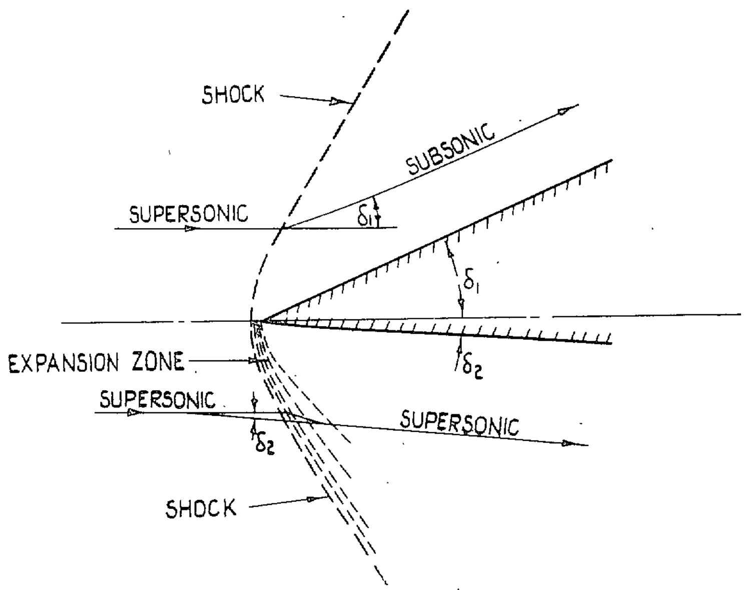


FIG. 9. The detached shock.

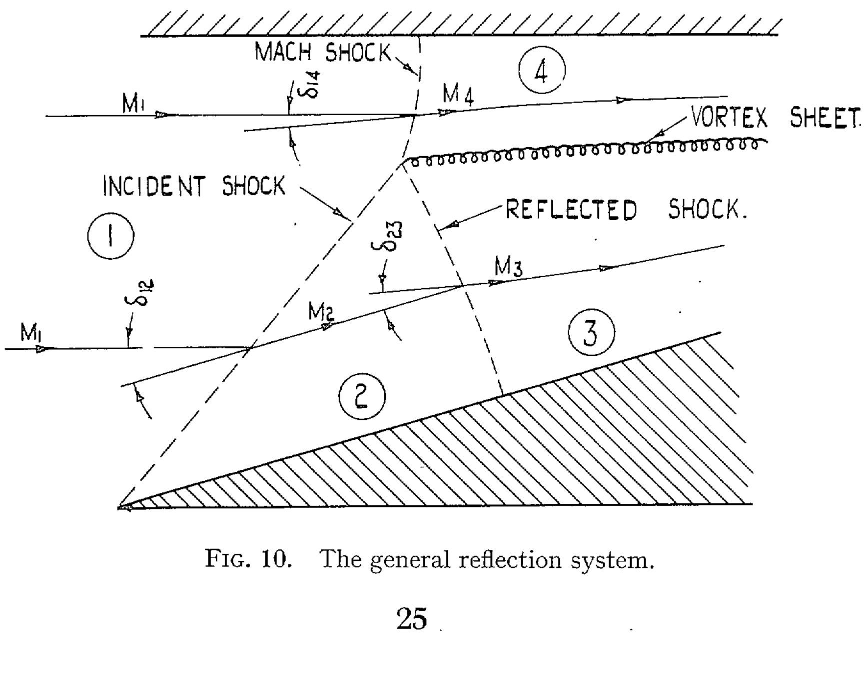


FIG. 10. The general reflection system.

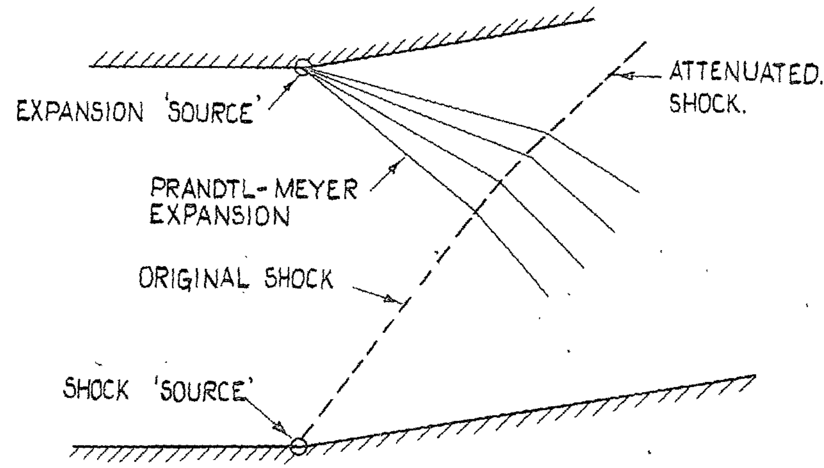


FIG. 11. Shock wave attenuated in a duct.

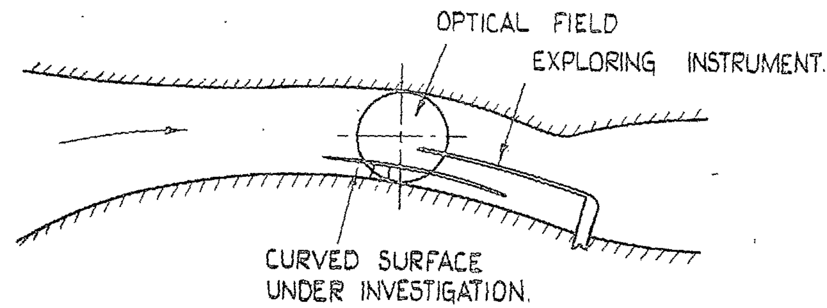


FIG. 13. The E.T.H. working section.

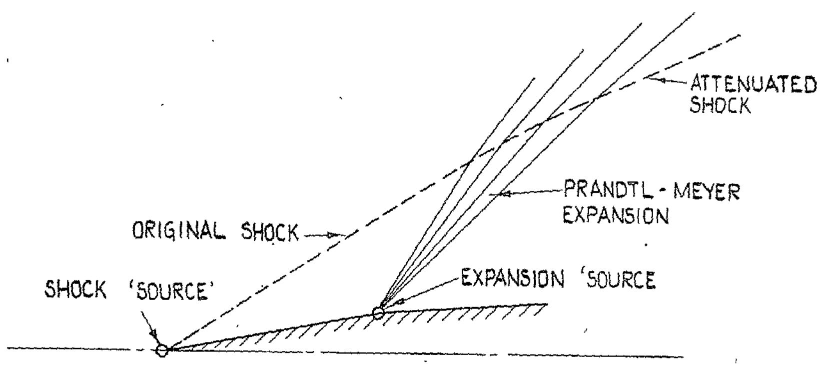


FIG. 12. Attenuation on an isolated aerofoil.

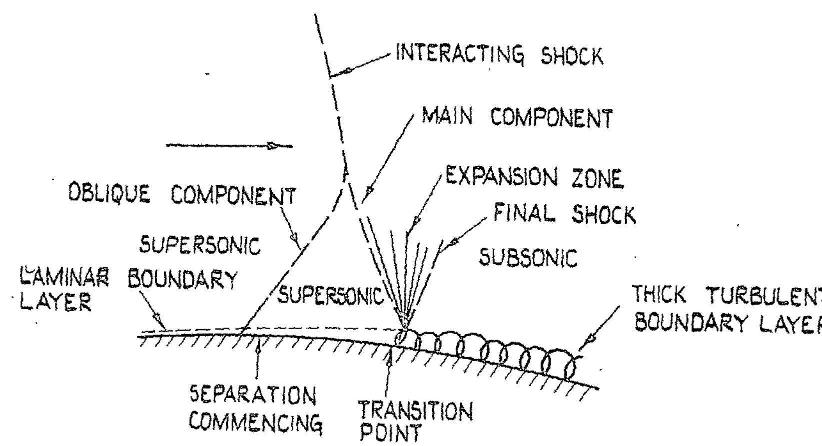


FIG. 14. The λ-shock.

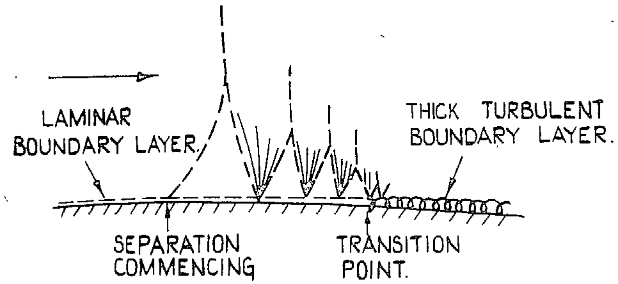


FIG. 15. A λ-shock series.

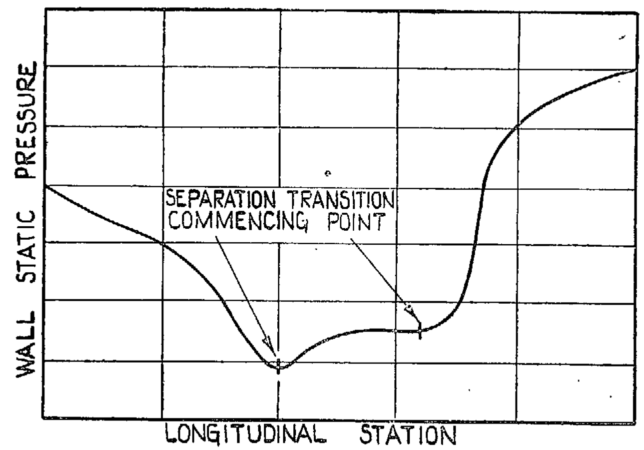


FIG. 16. Pressure distribution under λ-shock series.

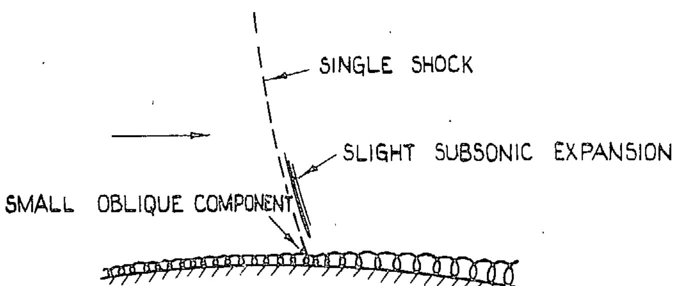


FIG. 17. Interaction with turbulent boundary layer.

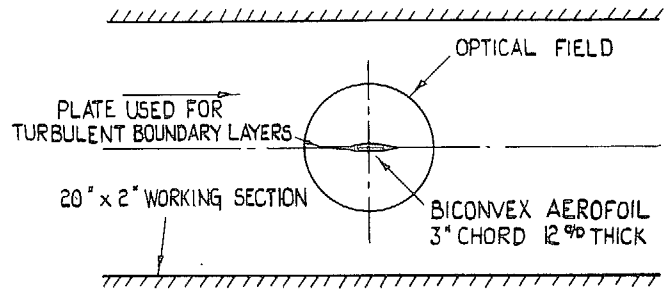


FIG. 18. The G.A.L.C.I.T. working section.

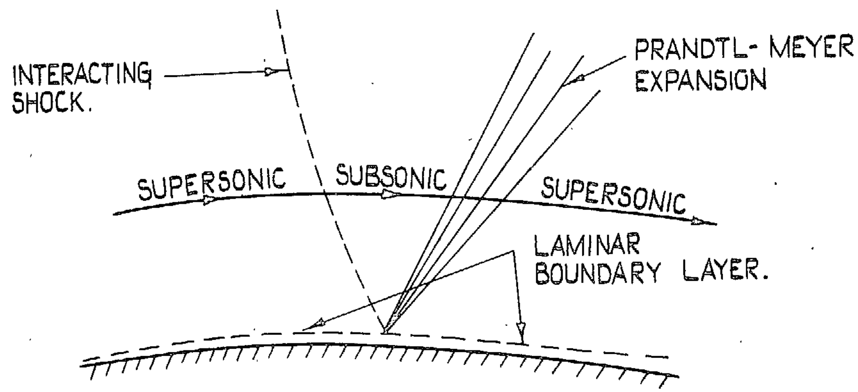


FIG. 19. A "freely reflecting" boundary layer.

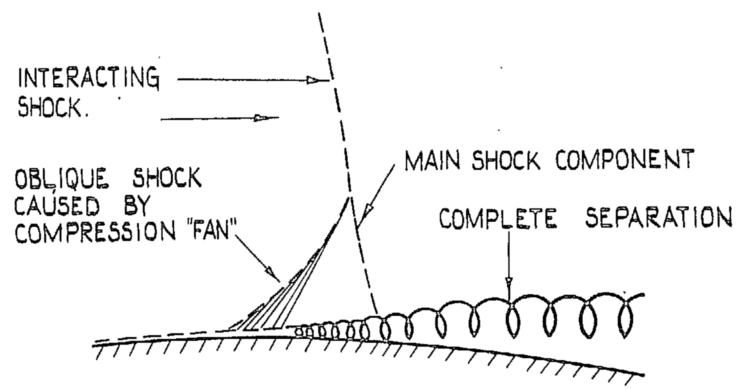


FIG. 20. The G.A.L.C.I.T. λ-system.

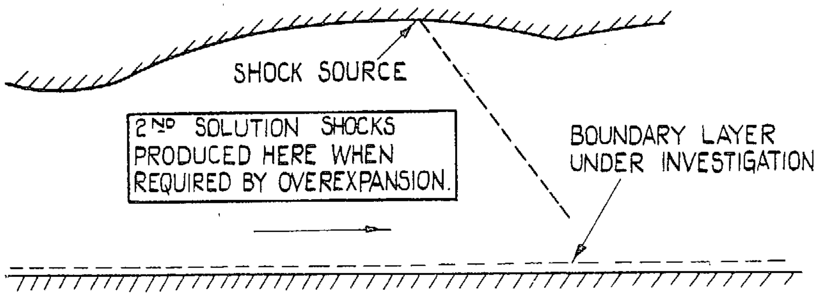


FIG. 21. The N.P.L. working section.

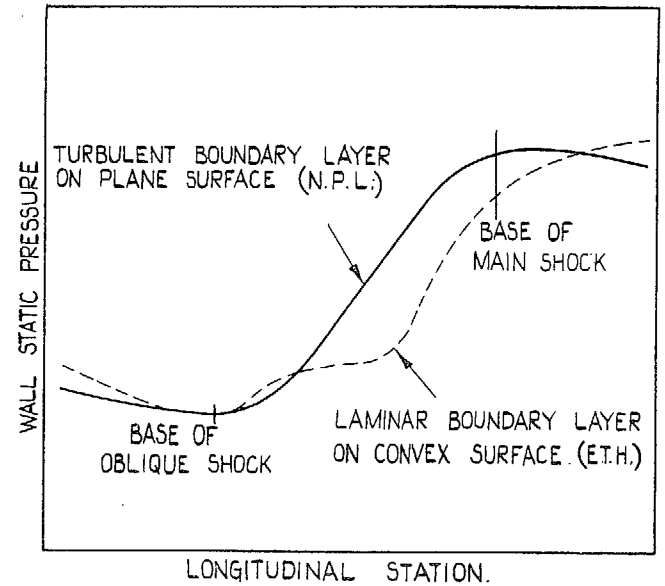


FIG. 22. The pressure distribution under a single λ-shock.

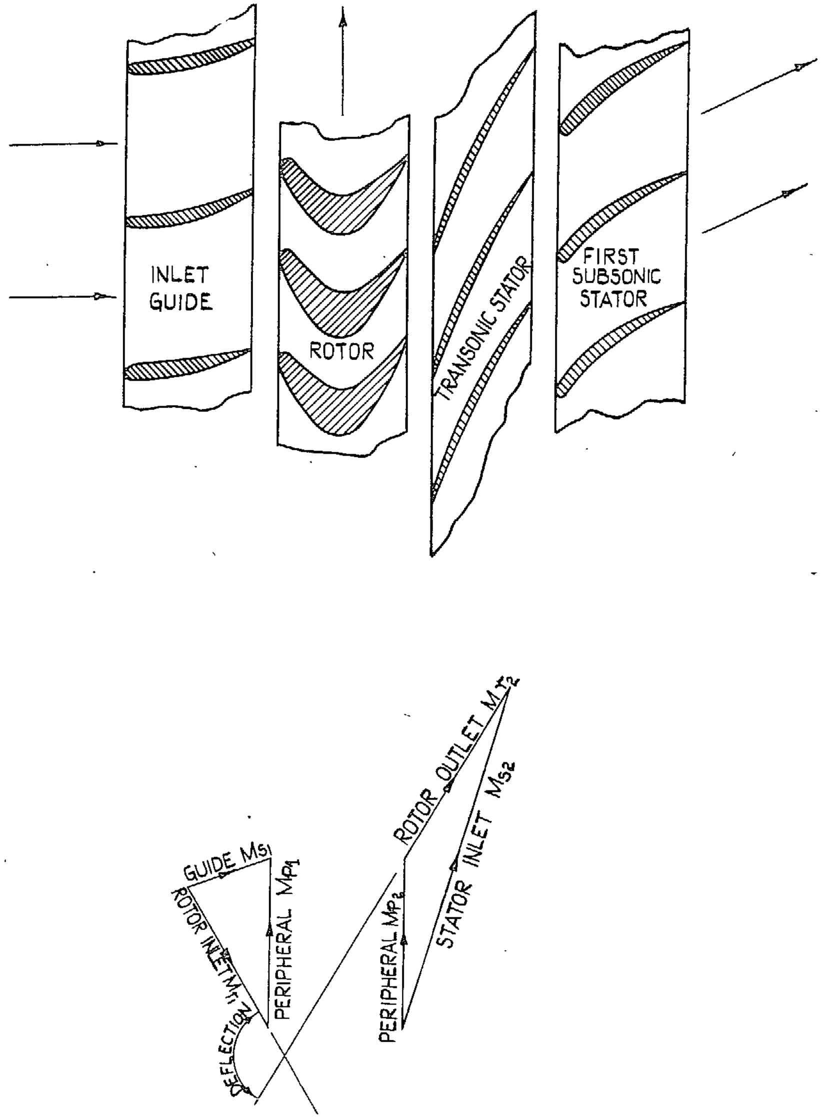


FIG. 23. The impulse rotor unit.

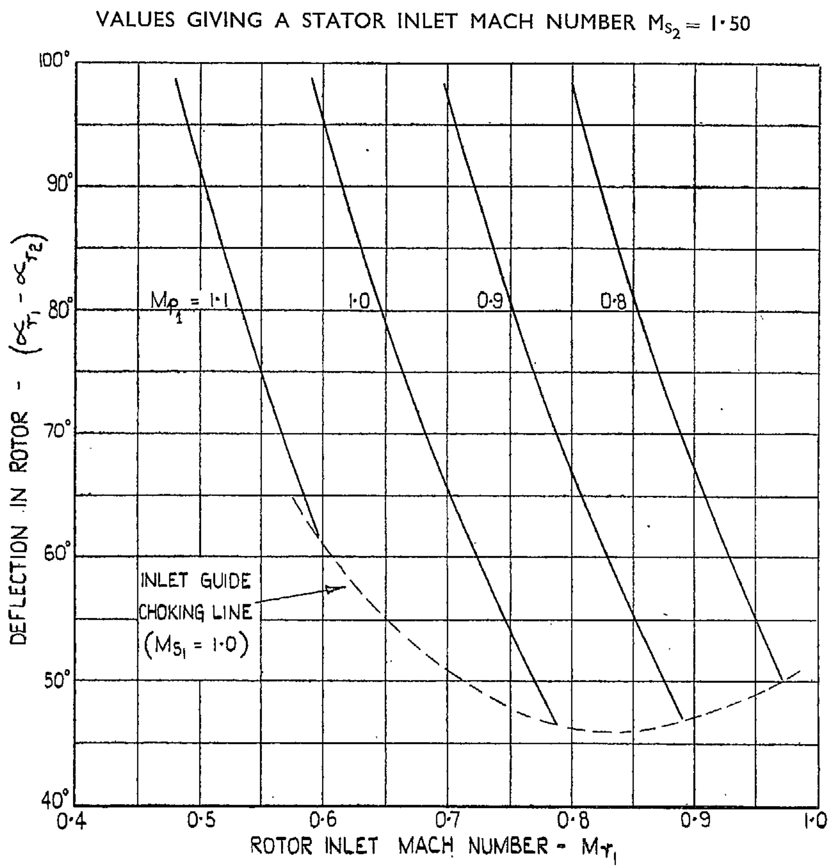


FIG. 24. Impulse rotor deflections for $Ms_2 = 1{\cdot}50$.

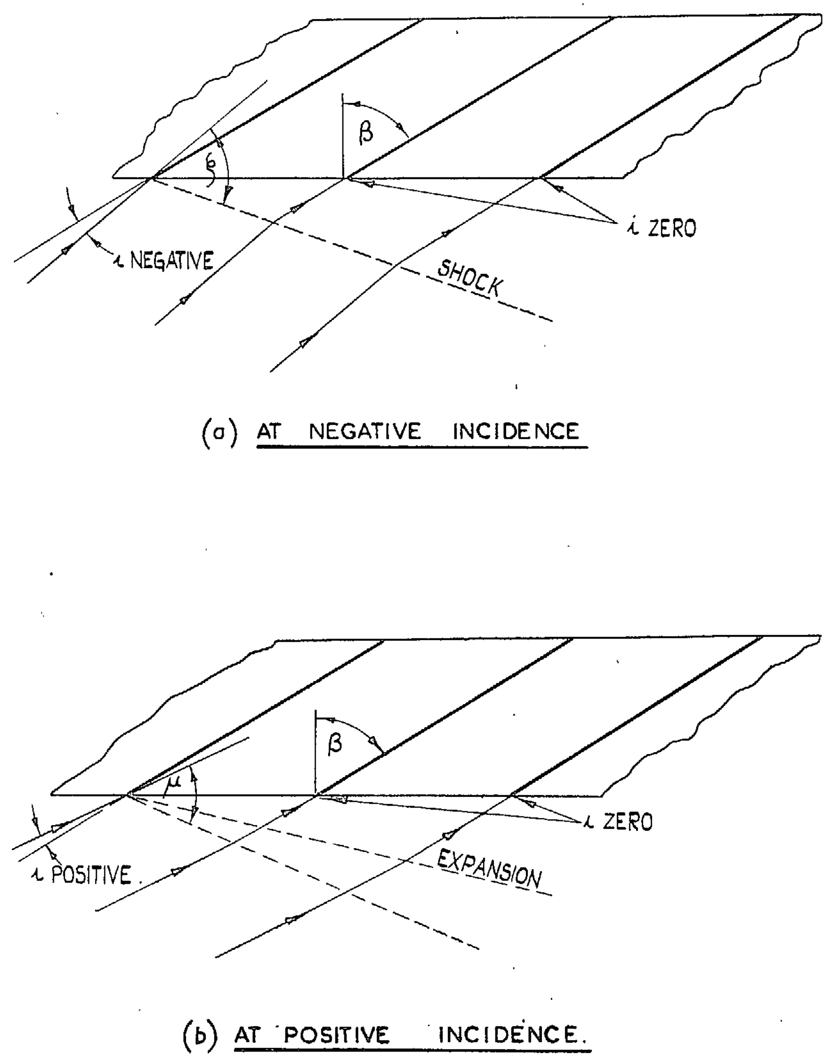


FIG. 25. A cascade of laminæ.

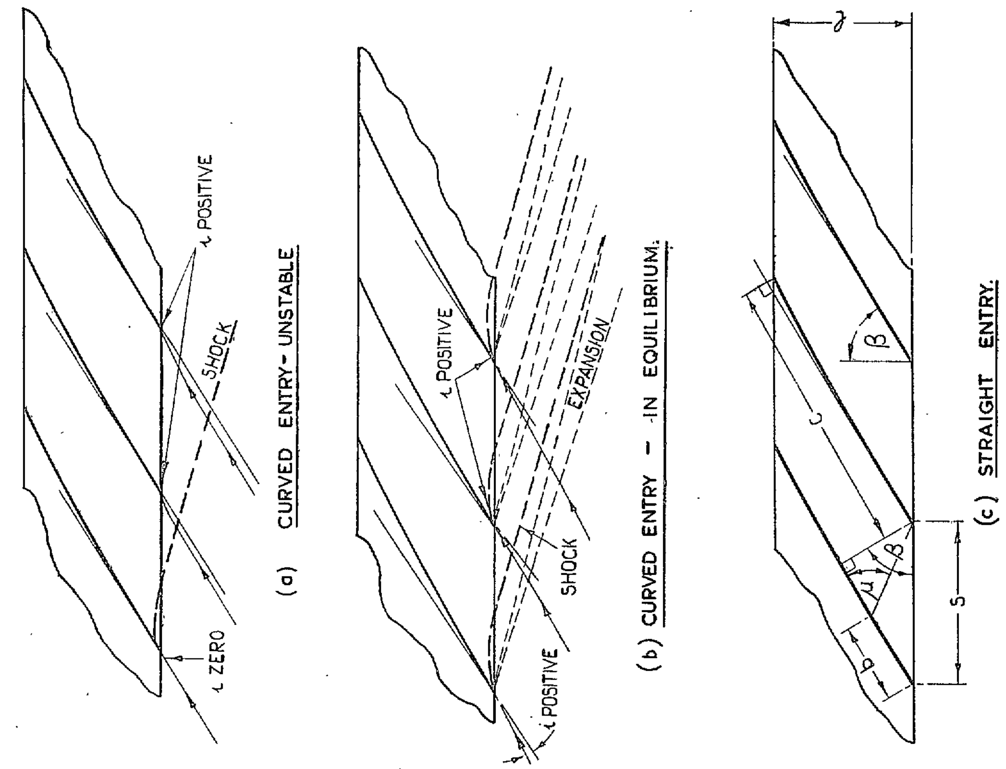


FIG. 26. A cascade of thin aerofoils.

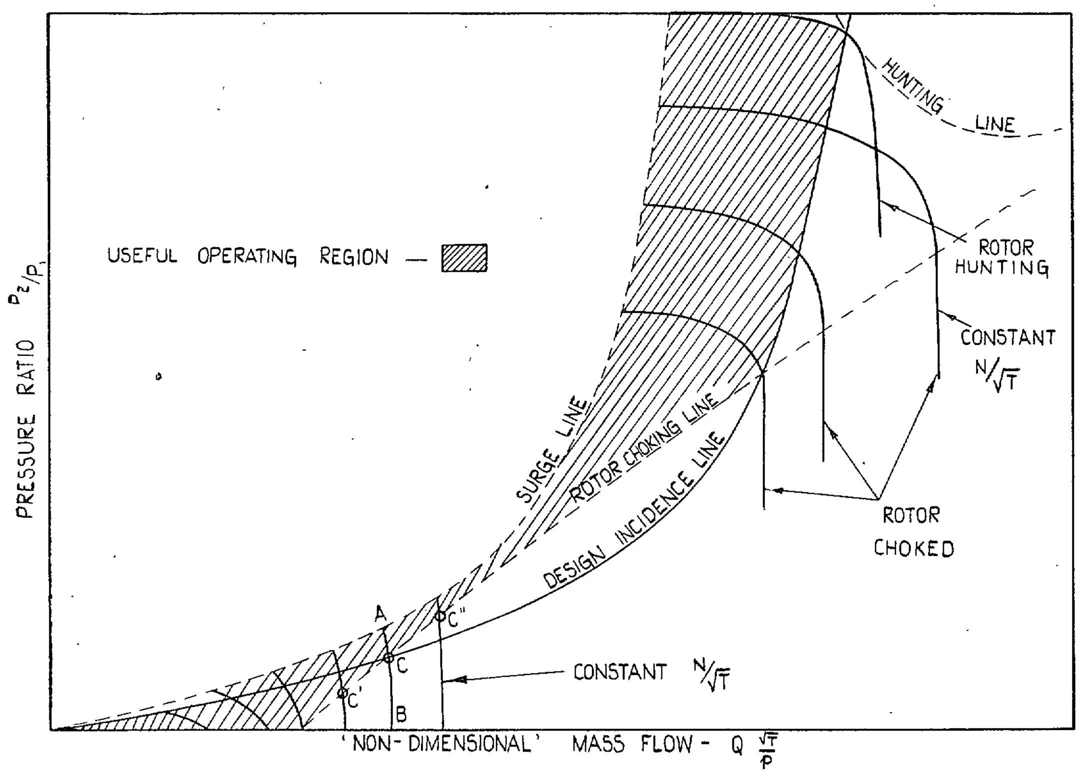
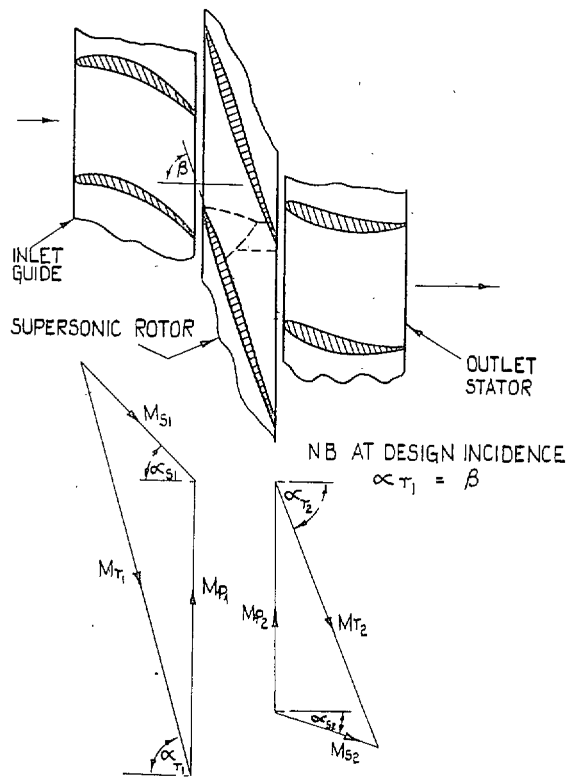


FIG. 27. Generalised compressor stability.



MASS FLOW, PRESSURE RATIO AND ADIABATIC EFFICIENCY ARE FUNCTIONS OF M_{S_1} M_{T_1} AND α_{S_1} SUCH THAT WHEN THESE INDEPENDENT VARIABLES ARE GIVEN SEPARATE POSITIVE INCREMENTS THE FOLLOWING FIRST ORDER DESIGN POINT CHANGES OCCUR

	β	Q	P_2/P_1	η
M_{S_1}	-	+	0	-
M_{T_1}	+	0	+	+
α_{S_1}	+	-	0	0

FIG. 28. The effects of a change in stagger.

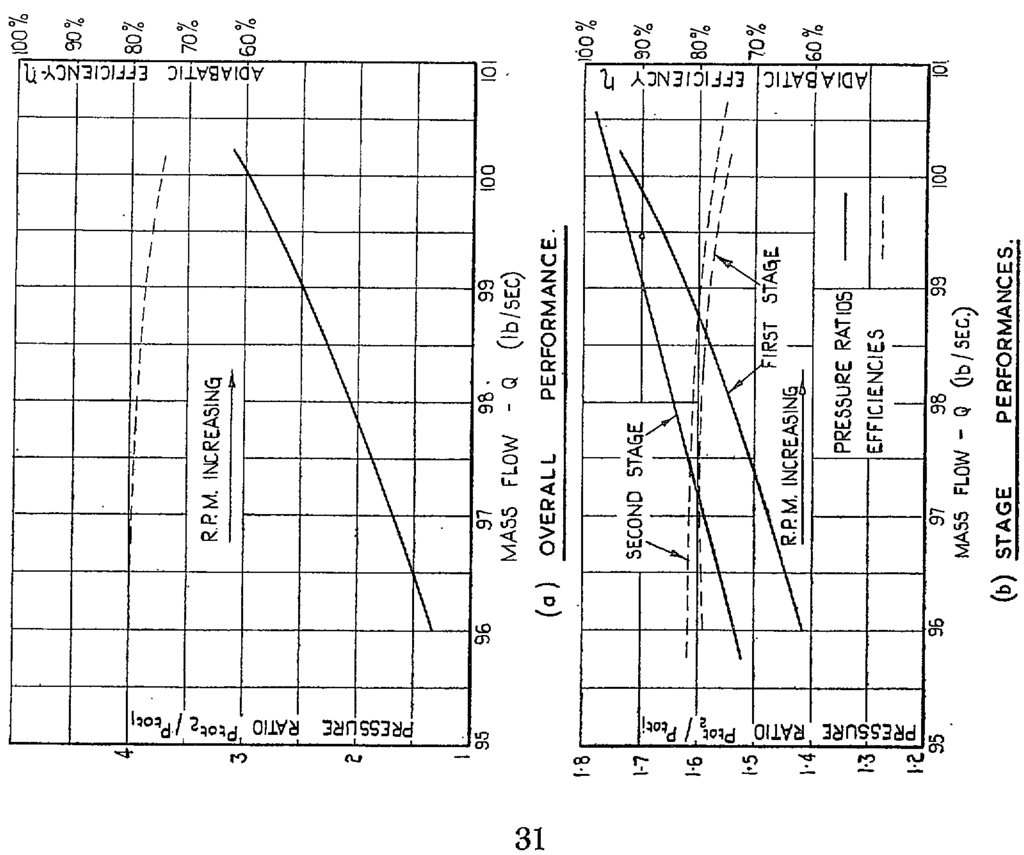


FIG. 29. Hypothetical two-stage unit performance.

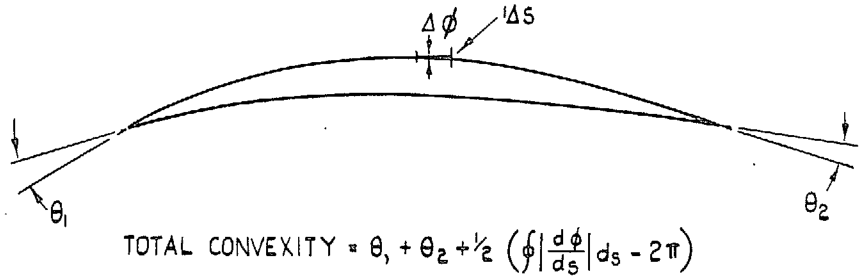


FIG. 30. Rotor blade convexity.

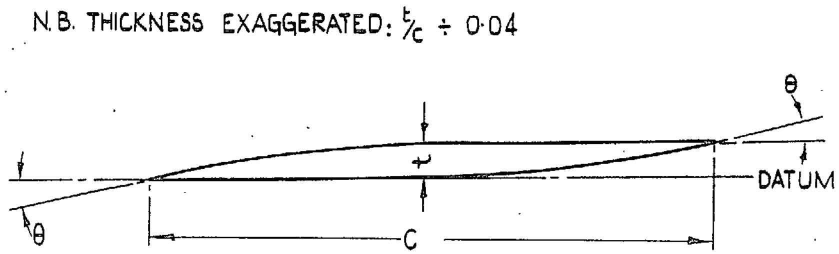


FIG. 31. Typical blade with point symmetry.

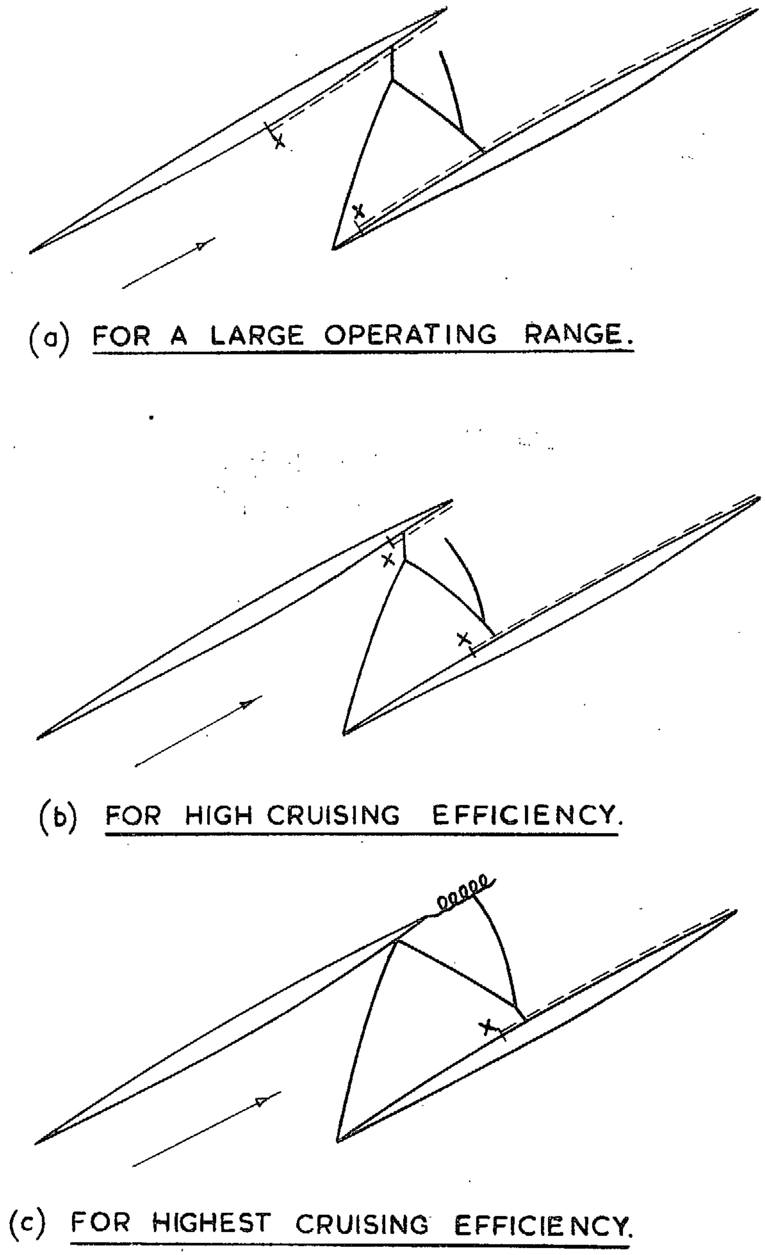


FIG. 32. Rotor boundary-layer conditions.

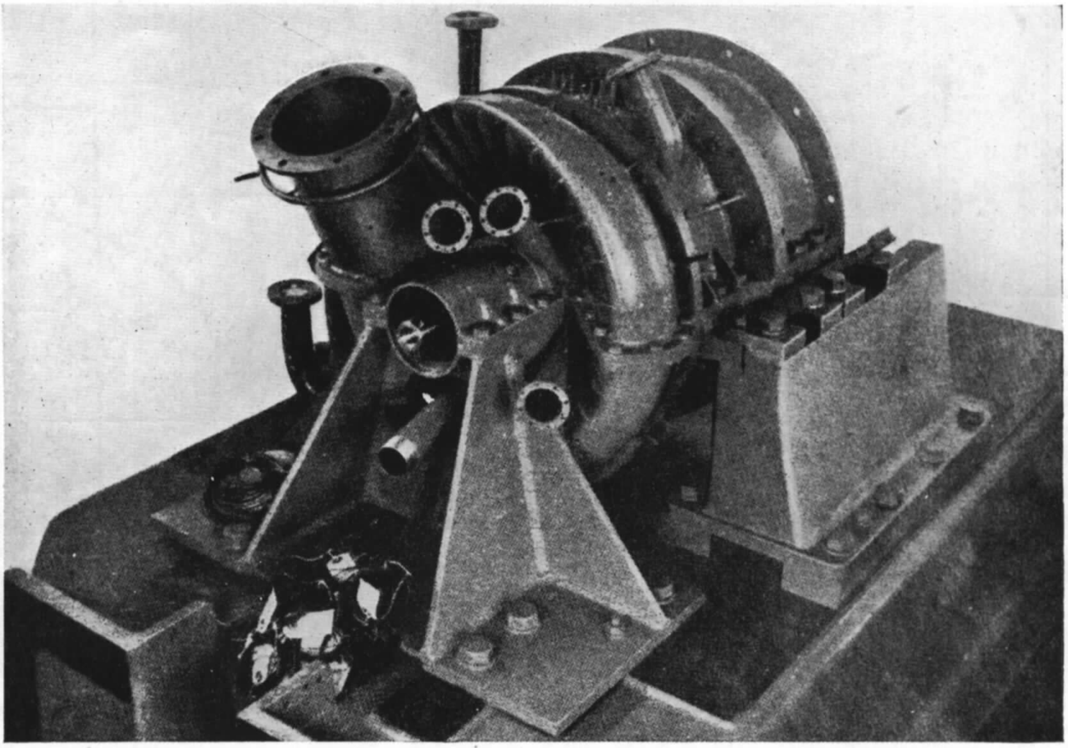


FIG. 33. View of D.V.L. rig showing suction pipe flanges.

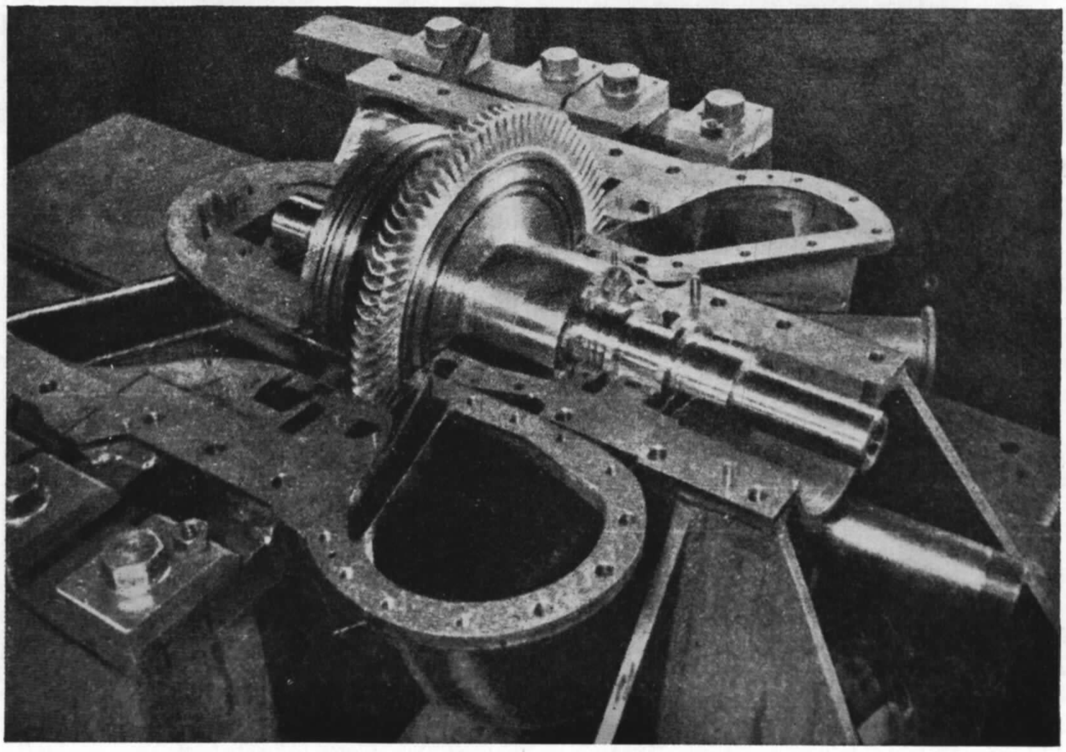


FIG. 34. Interior of ducting and type I rotor.

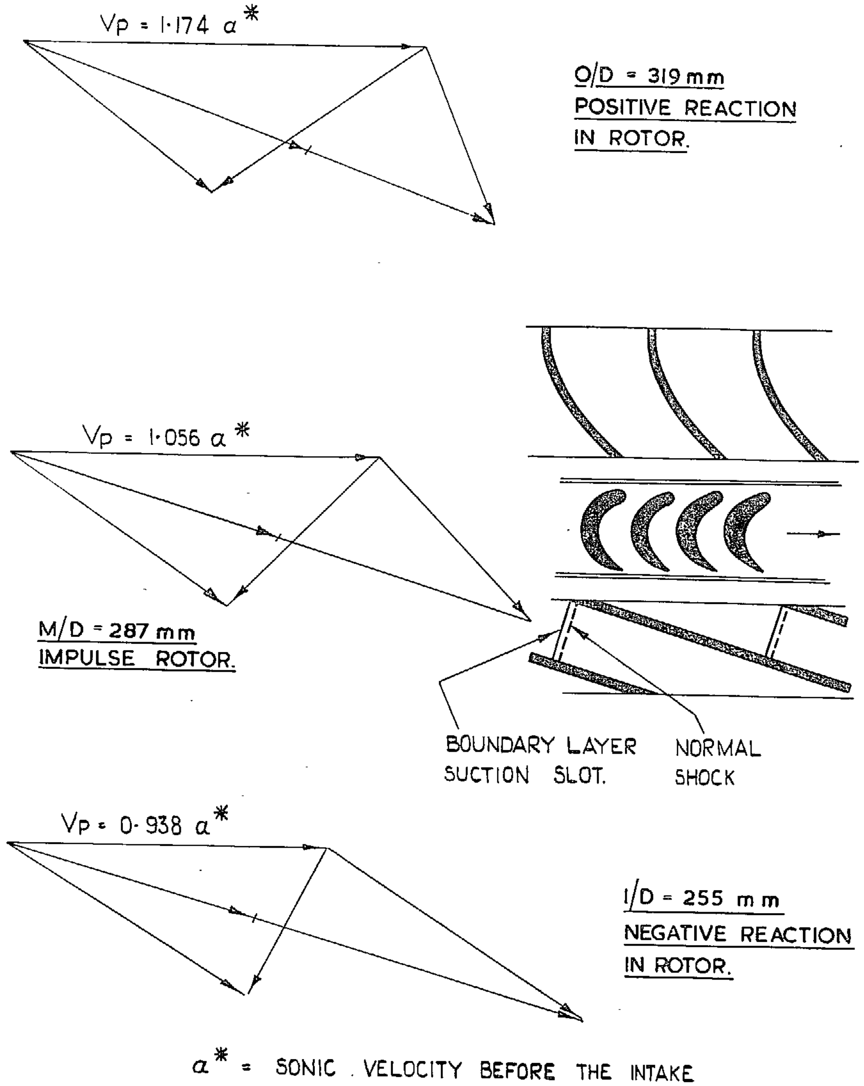


FIG. 36. Type I compressor details.

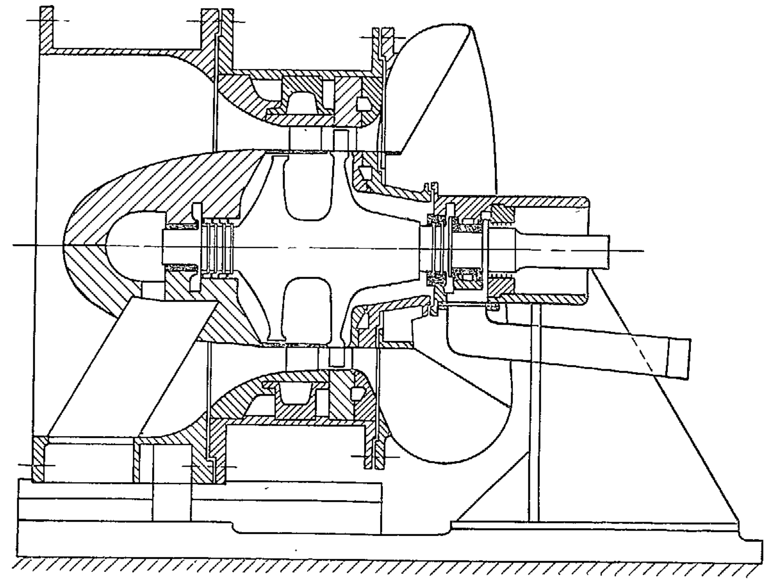


FIG. 35. Section through the D.V.L. test rig.

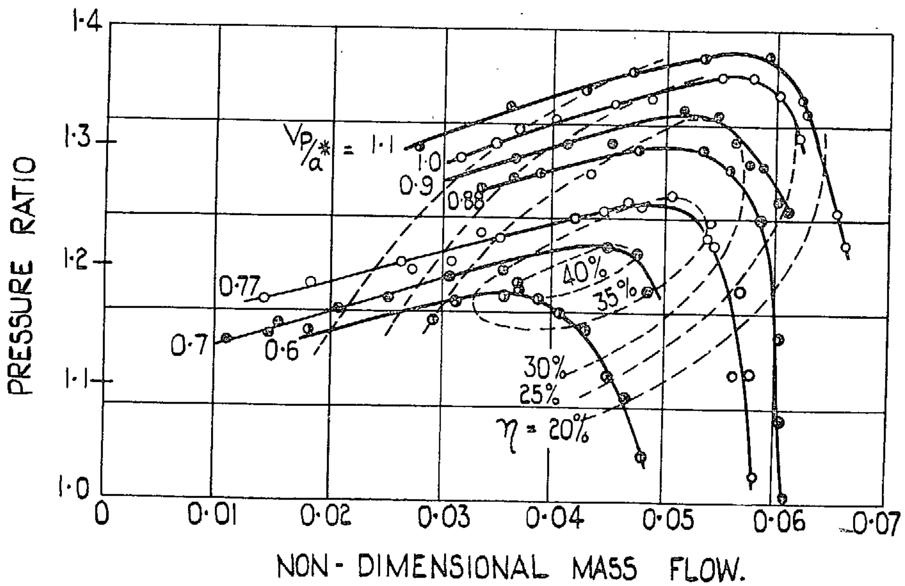


FIG. 37. Type I compressor performance.

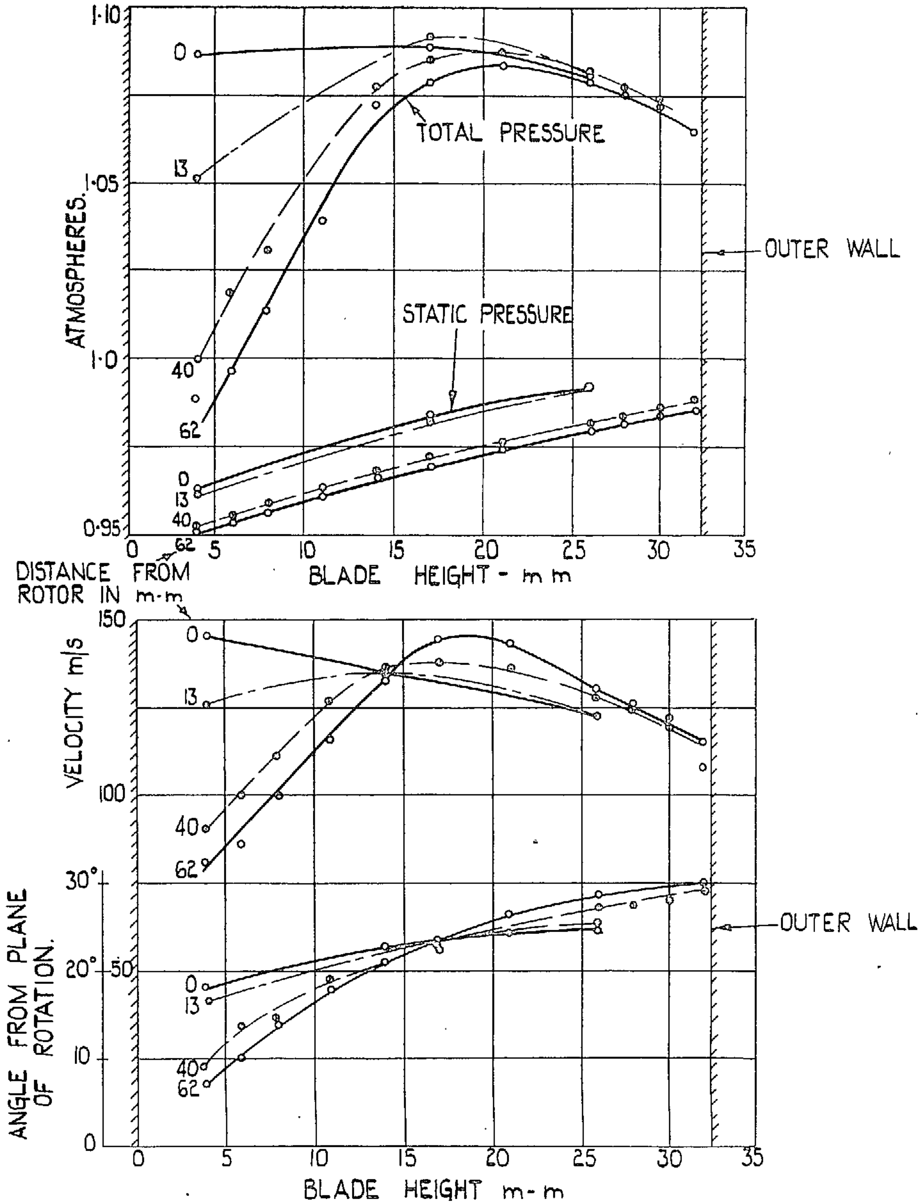


Fig. 38. Radial flow behind type I rotor.

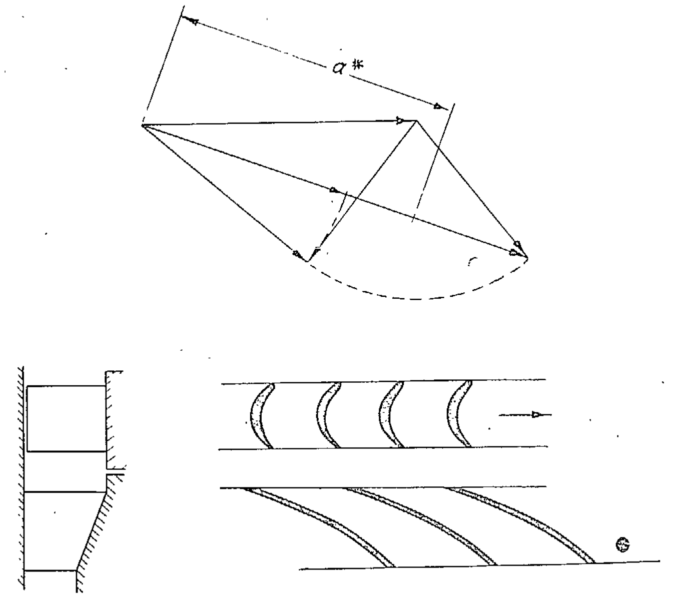


Fig. 39. Type Ia compressor details.

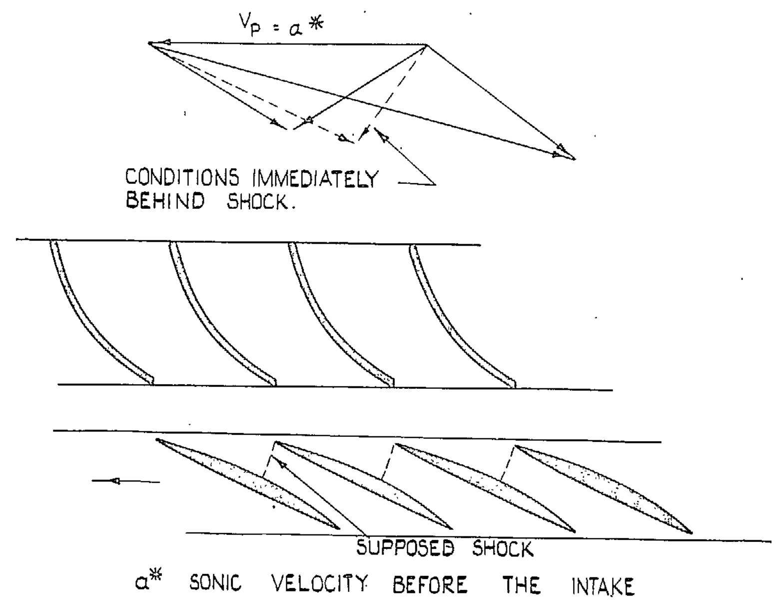


Fig. 40. Type II compressor details.

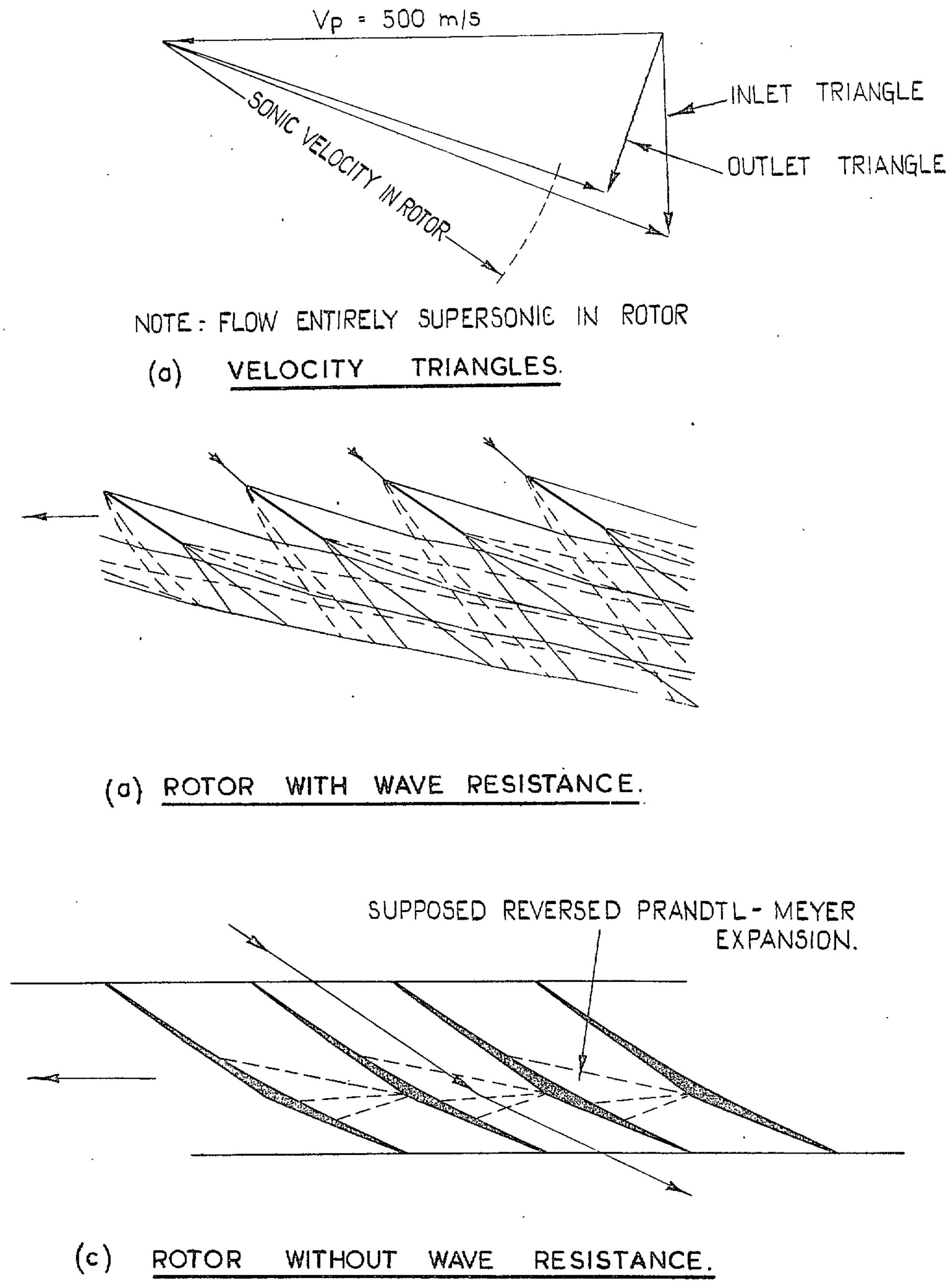


FIG. 41. Type III compressor details.

J4249 Wt.13/806 K.9 5/51 D&Co. 34/263

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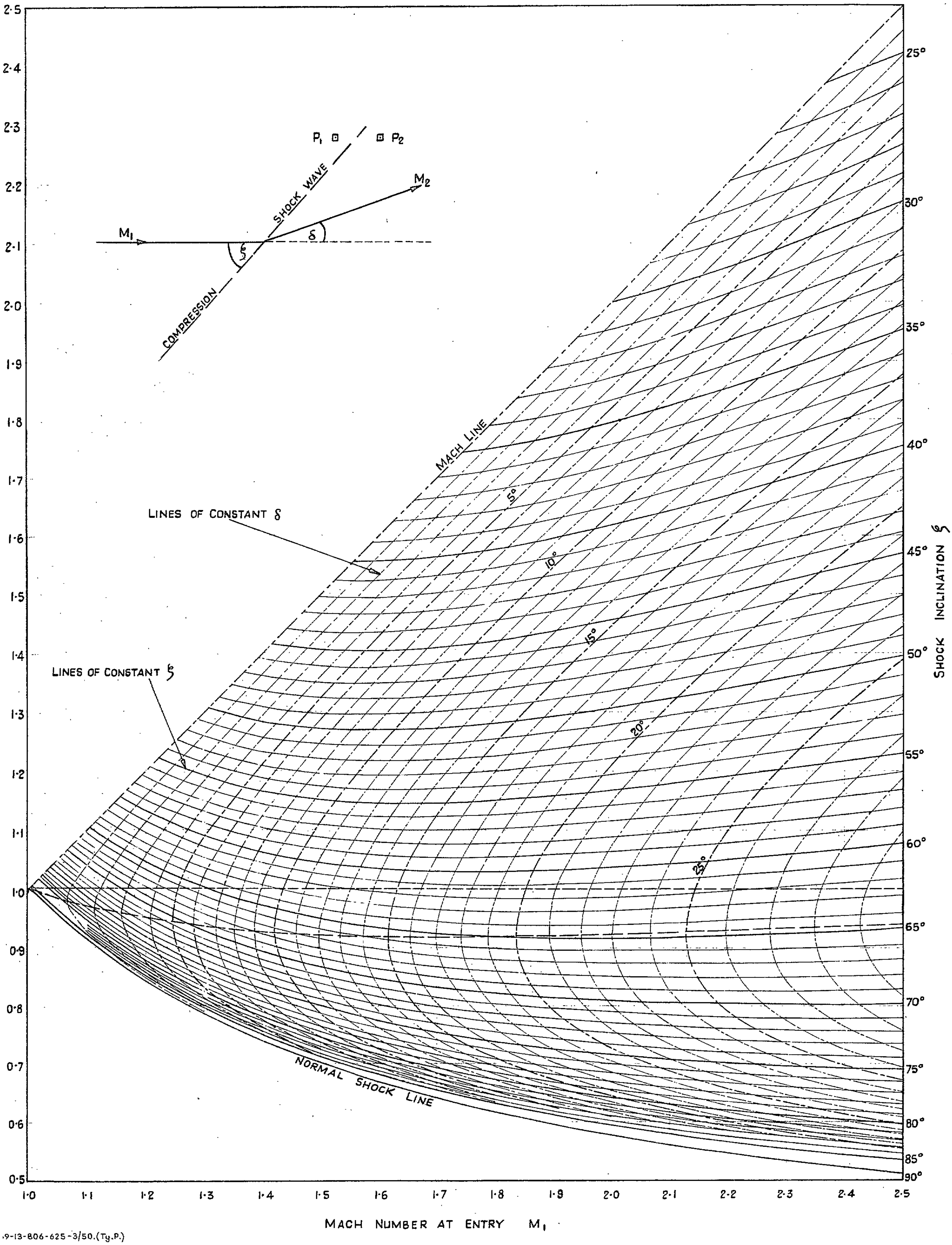
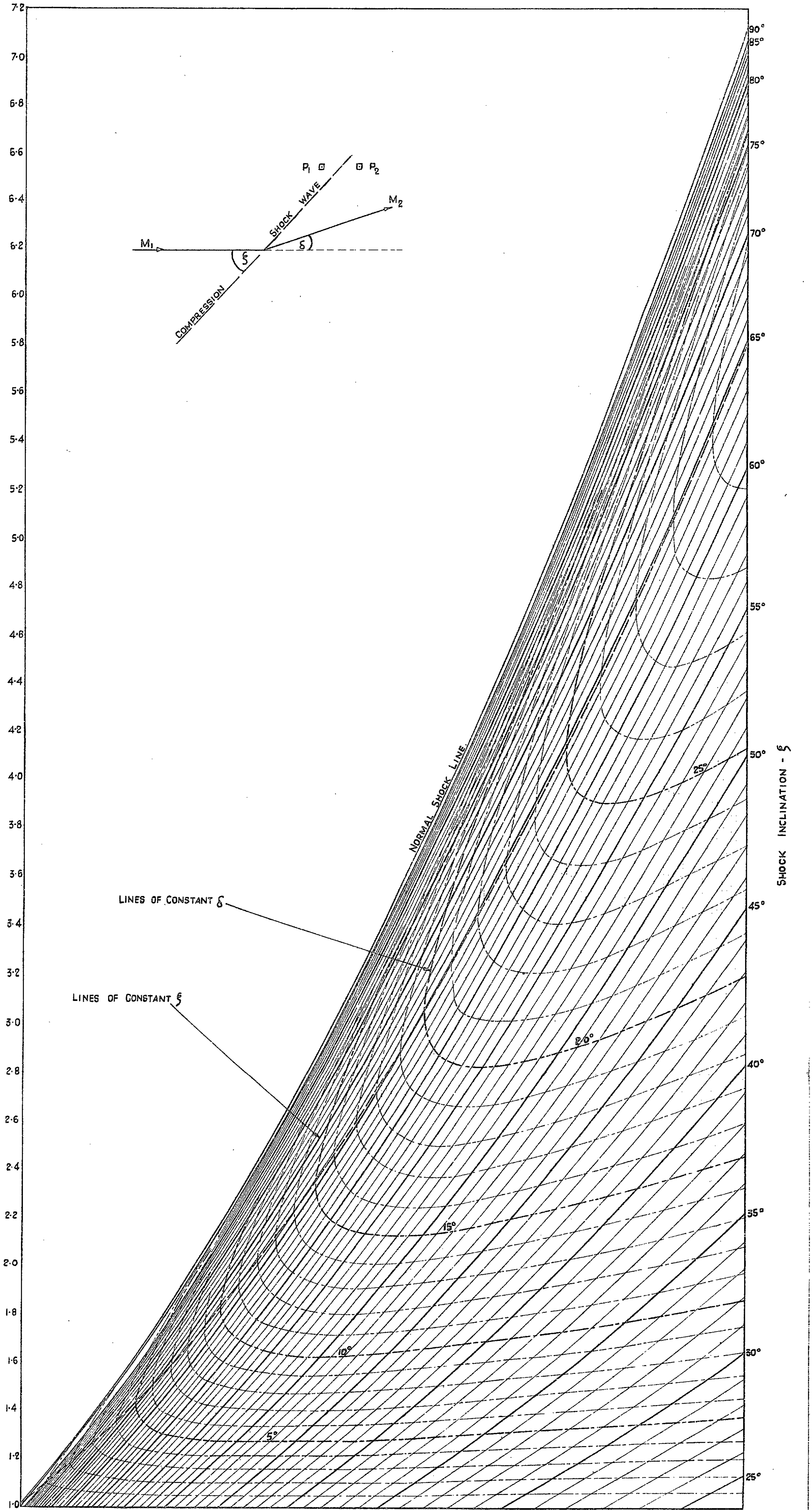
MACH NUMBER CHART.

FIG. 3.



9-13-806-625-3/50.(Ty.P.)

PRESSURE RATIO CHART. FIG. 4.



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