R. & M. No. 3360

MINISTRY OF AVIATION

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

On the Design of a Row of Windows in a Pressurized Cylindrical Fuselage

By E. H. Mansfield, Sc.D., F.R.Ae.S.

LONDON: HER MAJESTY'S STATIONERY OFFICE
1964

FIVE SHILLINGS NET

On the Design of a Row of Windows in a Pressurized Cylindrical Fuselage

By E. H. Mansfield, Sc.D., F.R.Ae.S.

Communicated by the Deputy Controller Aircraft (Research and Development), Ministry of Aviation

*Reports and Memoranda No. 3360**
May, 1963

Summary.

Some factors affecting the design of a row of windows in a pressurized cylindrical fuselage are discussed. The concept of the neutral hole is adopted, but account is taken of the change in the basic 2:1 stress field occasioned by the presence of stringers, frames, longerons and, in particular, reinforced strips surrounding the windows and running the length of the fuselage. The influence of such reinforcement is to make the shape of the neutral hole slightly more circular than the standard $\sqrt{2}:1$ ellipse.

Results are presented in graphical form.

LIST OF CONTENTS

Section

1. Introduction
    1.1 Assumptions
2. Analysis
    2.1 Stresses in the reinforced fuselage
    2.2 The neutral hole
    2.3 Effect of fuselage frames
3. Numerical Example
4. Conclusions

List of Symbols

List of References

Illustrations—Figs. 1 to 4

Detachable Abstract Cards

* Replaces R.A.E. Report No. Structures 287—A.R.C. 24 824.

LIST OF ILLUSTRATIONS

Figure

1. Figure showing notation
2. Variation of height/width of neutral ellipse
3. Typical variation of neutral reinforcement
4. Variation of reinforcement area A_0

1. *Introduction.*

The design of windows in a pressurized fuselage depends upon many factors. These include the choice of window shape, edge reinforcement and the use of adjacent reinforced skin. But the design is not necessarily governed purely by considerations of weight, structural efficiency or cost of manufacture, for some window shapes may be preferred because they command a better field of vision or have greater aesthetic appeal.

If the only loading on the fuselage is that due to pressurization it is well known that a window whose shape is that of a 'neutral hole' may be designed, in theory[1], to give no stress concentration. If the frames carry a negligible proportion of the total hoop loads it can be shown that this neutral shape is an ellipse whose diameters in the hoop and axial directions are in the ratio $\sqrt{2}:1$. Further, because such a window does not alter the stress state in the surrounding structure it is possible to have a 'neutral row' of such windows. In practice, however, some slight stress concentration is unavoidable and, what may be more important, the fuselage must carry other forms of loading, including shear, torsion and (lateral) bending. These additional loads necessarily cause stress concentrations which may, of course, be reduced by the use of thicker sheet material around the windows. For each row of windows, such as shown in Fig. 1, the thicker sheet might extend over a strip running the length of the fuselage.

At this conceptual stage let us suppose that such reinforced strips form part of the 'uncut' shell. Further, if the primary fuselage loading is that due to pressurization it is natural to stipulate that the window holes *in the reinforced fuselage* should be neutral for this primary loading. Now the shape of such a neutral hole is governed by the stress field in the reinforced strip, and this stress field differs from that in the unreinforced regions. This is because the hoop stress is governed simply by radial equilibrium with the internal pressure, whereas the longitudinal stress is governed by compatibility of longitudinal strains together with equilibrium of the complete cross-section of the fuselage. The resultant ratio of hoop to longitudinal stress is thus less than 2:1 and the appropriate neutral shape is more nearly circular, and this is itself an advantage over the $\sqrt{2}:1$ ellipse in that the corresponding stress-concentration factors due to shear, torsion and bending are reduced. Similarly there is a reduction in the stress ratio if the frames are effective in resisting the pressurization. Finally, it is to be noted that if longerons are present an identical argument shows that they tend to *increase* the ratio of hoop to longitudinal stress in the region of the windows.

1.1. *Assumptions.*

The following assumptions are made.

(*a*) The material is everywhere elastic.

(*b*) The stress distribution due to pressurization is unaffected by any change in the stress pattern at the ends of the fuselage.

(*c*) The fuselage frames play a negligible part in resisting the pressurization.

(*d*) The c.g. of the cross-sectional area of all longitudinal-stress-bearing material lies at the centre of the (circular) fuselage.

(*e*) Any stringers between windows may be adequately represented by a 'stringer-sheet' with equivalent directional properties.

(*f*) There is no offset between the 'stringer-sheet plus reinforced strip' and the shear centre of the edge reinforcement around a window.

With reference to assumption (*b*) it will be appreciated that any change in the stress pattern at the ends of the fuselage is quite localised and, furthermore, is of small magnitude in any realistic design which necessarily allows for continuity of longitudinal load. Assumption (*c*) is not strictly necessary because an approximate treatment, given in Section 2.3, is available if the frames do carry a significant proportion of the hoop loads resulting from pressurization. Assumptions (*d*) and (*e*) are not essential to the analysis but it is felt that the resulting increase in complexity is not justified in view of the generally small nature of the effects. As regards assumption (*f*) the influence of an offset is to reduce the effective tensile stiffness of the reinforcement, a problem which was treated in greater detail in Ref. 2.

2. *Analysis.*

In this section we consider first the stresses in the reinforced fuselage, and then the form of the associated neutral hole. Finally, the effect of fuselage frames is considered.

It will be recalled that, by definition and design, the neutral hole is such that the stresses in the surrounding structure are unaffected by the presence of the hole.

2.1. *Stresses in the Reinforced Fuselage.*

The hoop stresses are given immediately by radial equilibrium, whence

$$\sigma_y = \frac{pd}{2t}, \tag{1}$$

$$\sigma_y^{\,r} = \frac{pd}{2t_r}. \tag{2}$$

Longitudinal equilibrium gives

$$(\pi d - 2w)t\sigma_x + 2wt_r\sigma_x^{\,r} + F_s\sigma_x^{\,s} = \tfrac{1}{4}\pi p d^2, \tag{3}$$

and equality of longitudinal strains gives

$$\left.\begin{aligned}\sigma_x^{\,s} &= \sigma_x - \nu\sigma_y\\ &= \sigma_x^{\,r} - \nu\sigma_y^{\,r}.\end{aligned}\right\} \tag{4}$$

These equations suffice to determine the longitudinal stresses, which may be expressed in the form:

$$\left.\begin{aligned}\sigma_x &= \frac{pd}{4}\left(\frac{1-2\nu}{t_A} + \frac{2\nu}{t}\right),\\ \sigma_x^{\,r} &= \frac{pd}{4}\left(\frac{1-2\nu}{t_A} + \frac{2\nu}{t_r}\right),\\ \sigma_x^{\,s} &= \frac{pd}{4}\left(\frac{1-2\nu}{t_A}\right).\end{aligned}\right\} \tag{5}$$

The simplicity of these equations suggests that they have a simple physical interpretation, and this is indeed the case. If we first restrain all longitudinal extension of the fuselage the stringer stresses are zero and the longitudinal stresses in the sheet are equal to ν times the hoop stresses, i.e. the second terms in equation (5). The associated longitudinal tensile force in the sheet is thus independent of the sheet thickness and equal to $\frac{1}{2}\nu pd$ per unit circumferential length, and the total longitudinal force is therefore $\frac{1}{2}\nu\pi pd^2$. Now in the actual, unrestrained fuselage the total longitudinal force required for equilibrium is $\frac{1}{4}\pi pd^2$, and the difference between these forces, namely $(\frac{1}{4}-\frac{1}{2}\nu)\pi pd^2$, is responsible for the constant stress component in equation (5). Note that if assumption (*d*) is invalid there would be some bending of the fuselage due to pressurization, but the amount of bending may be readily determined from the above physical considerations. So, too, may the stresses in a 'double-bubble' fuselage.

2.2. *The Neutral Hole.*

The shape of the neutral hole is governed by the forces per unit length in the (reinforced) sheet and stringers or, more conveniently, by the associated force function Φ. Thus in the hoop direction we have

$$t_r\sigma_y^r = \frac{\partial^2\Phi}{\partial x^2} = \tfrac{1}{2}pd, \tag{6}$$

and in the longitudinal direction

$$t_r\sigma_x^r + t_s\sigma_x^s = \frac{\partial^2\Phi}{\partial y^2} = \frac{pd}{4}\left\{(1-2\nu)\left(\frac{t_r+t_s}{t_A}\right)+2\nu\right\}. \tag{7}$$

The associated force function Φ may therefore be written as

$$\Phi = \frac{pd}{4}\left(x^2+\frac{y^2}{k^2}-b^2\right) \tag{8}$$

where

$$k^2 = \frac{2}{(1-2\nu)\left(\dfrac{t_r+t_s}{t_A}\right)+2\nu}, \tag{9}$$

and the constant term has been chosen so that the shape of the neutral hole is the ellipse given by

$$\Phi = 0. \tag{10}$$

The variation of k with ν and $(t_r+t_s)/t_A$ is shown in Fig. 2.

The section area of the compact reinforcement is given by equation (33) of Ref. 1:

$$A = (t_r+t_s)\left\{\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial y}\right)^2\right\}^{3/2}\left[\left\{1+(1-\nu^2)t_s/t_r\right\}\frac{\partial^2\Phi}{\partial x^2}\left(\frac{\partial\Phi}{\partial x}\right)^2+\frac{\partial^2\Phi}{\partial y^2}\left(\frac{\partial\Phi}{\partial y}\right)^2-\right.$$
$$\left.-\nu\left\{\frac{\partial^2\Phi}{\partial x^2}\left(\frac{\partial\Phi}{\partial y}\right)^2+\frac{\partial^2\Phi}{\partial y^2}\left(\frac{\partial\Phi}{\partial x}\right)^2\right\}\right]^{-1}$$
$$= \frac{(t_r+t_s)(k^4x^2+y^2)^{3/2}}{k^4x^2\left[k^2\{1+(1-\nu^2)t_s/t_r\}-\nu\right]+y^2(1-\nu k^2)} \tag{11}$$

in virtue of equation (8).

A typical variation of the section area A around the ellipse is shown in Fig. 3. It is seen that the variation is slight except in the neighbourhood of the ends of the major axis, where Poisson's ratio effects assume an exaggerated importance. Now, practically there are obvious advantages in having a constant value for A and it is suggested[2, 3] that a nearly neutral hole will be achieved by making A constant at its correct value A_0 at the ends of the minor axis, where the load in the reinforcement is a maximum. Substituting $x = \pm\frac{1}{2}b$, $y = 0$ into equation (11) and simplifying then gives

$$A_0 = \frac{\frac{1}{2}bt_r}{1 - \nu^2 - \nu(\frac{1}{2} - \nu)t_r/t_A}, \tag{12}$$

a result which could also have been derived directly by physical argument. As shown in Fig. 4 the ratio $A_0/\frac{1}{2}bt_r$ is almost constant at the value 1·2, varying only slightly with ν and t_r/t_A.

2.3. *Effect of Fuselage Frames.*

The contribution of fuselage frames in resisting the pressurization depends upon their cross-sectional area and *shape*, upon their method of attachment to the stringers, upon the flexural rigidity of the stringers, and upon the frame spacing. These points have been discussed in greater detail in Ref. 2 and elsewhere, and it can be shown that the contribution is generally small. With this in mind the following approximate treatment is given, in which perturbations in the stress pattern between frames are ignored.

If the proportion of the total hoop load carried by the frames is δ, that carried by the skin is $(1-\delta)$, and the hoop stresses are accordingly given by

$$\left.\begin{aligned} \sigma_y &= (1-\delta)\frac{pd}{2t}, \\ \sigma_y{}^r &= (1-\delta)\frac{pd}{2t_r}. \end{aligned}\right\} \tag{13}$$

If now we again restrain all longitudinal extension of the fuselage, the longitudinal stresses in the sheet are ν times the hoop stresses while the stringer and longeron stresses are zero. The total longitudinal force in the fuselage is now $\frac{1}{2}(1-\delta)\nu\pi pd^2$ compared with $\frac{1}{4}\pi pd^2$ required for equilibrium, and the difference between these forces, namely $\{\frac{1}{4} - \frac{1}{2}(1-\delta)\nu\}\pi pd^2$, must be applied to give the stresses in the actual unrestrained fuselage. The longitudinal stresses are finally given by

$$\left.\begin{aligned} \sigma_x &= \frac{pd}{4}\left(\frac{1-2\nu(1-\delta)}{t_A} + \frac{2\nu(1-\delta)}{t}\right) \\ \sigma_x{}^r &= \frac{pd}{4}\left(\frac{1-2\nu(1-\delta)}{t_A} + \frac{2\nu(1-\delta)}{t_r}\right) \\ \sigma_x{}^s &= \frac{pd}{4}\left(\frac{1-2\nu(1-\delta)}{t_A}\right) \end{aligned}\right\} \tag{14}$$

and the corresponding values of k^2 and A_0 are given by

$$k^2 = \frac{2}{\left(\dfrac{1}{1-\delta} - 2\nu\right)\left(\dfrac{t_r + t_s}{t_A}\right) + 2\nu} \tag{15}$$

and

$$A_0 = \frac{\frac{1}{2}bt_r}{1 - \nu^2 - \frac{1}{2}\nu\left(\dfrac{1}{1-\delta} - 2\nu\right)t_r/t_A}. \tag{16}$$

These equations show that although the variation of k with δ is significant, the corresponding variation of A_0 is less so. For comparison, the case in which $\delta = 0 \cdot 2$, $\nu = 0 \cdot 3$ is shown in Figs. 2 and 4.

3. *Numerical Example.*

A circular cylindrical fuselage has the following dimensions:

$$d = 12 \text{ ft}, \ (\pi d = 452 \text{ in.})$$

$$t = 0 \cdot 048 \text{ in.}$$

$$t_r = 0 \cdot 072 \text{ in.}$$

$$w = 4 \text{ ft}$$

$$b = 15 \text{ in.}$$

$$\nu = 0 \cdot 3 .$$

There are 90 equally spaced stringers each of developed section 3 in. × 0·036 in. The skin thicknesses are such as to produce the following hoop stresses due to a pressurization of 8 lb/in.²:

$$\sigma_y = 12{,}000 \text{ lb/in.}^2$$

$$\sigma_y{}^r = 8{,}000 \text{ lb/in.}^2 .$$

What are the corresponding longitudinal stresses, and what are the values of kb and A_0 for a neutral hole in the reinforced strip?

From the definition of t_A we find

$$t_A = \{90 \times 3 \times 0 \cdot 036 + (452 - 96) \times 0 \cdot 048 + 96 \times 0 \cdot 072\}/452$$

$$= 0 \cdot 0745 \text{ in.},$$

and similarly

$$t_s = 0 \cdot 0215 \text{ in.}$$

The longitudinal stresses are now given by equation (5):

$$\sigma_x = 5{,}150 \text{ lb/in.}^2$$

$$\sigma_x{}^r = 3{,}850 \text{ lb/in.}^2$$

$$\sigma_x{}^s = 1{,}550 \text{ lb/in.}^2$$

and it will be noted that these differ significantly.

Also,

$$(t_r + t_s)/t_A = 1 \cdot 25 ,$$

and

$$t_r/t_A = 0 \cdot 967 ,$$

so that equation (9) gives

$$k^2 = 1 \cdot 82$$

and hence

$$kb = 20 \cdot 2 \text{ in.},$$

which may be compared with 21·2 in. for a $\sqrt{2}$:1 ellipse.

Similarly from equation (12)

$$A_0 = 0 \cdot 633 \text{ in.}^2.$$

Finally, if the frames carry 15% of the hoop loads, equations (15) and (16) yield

$$kb = 18 \cdot 45 \text{ in.},$$

$$A_0 = 0 \cdot 648 \text{ in.}^2.$$

4. *Conclusions.*

Simple expressions have been presented for the longitudinal stresses in a pressurized fuselage reinforced by stringers, frames, longerons and strips of thicker skin running the length of the fuselage. Because of Poisson's ratio effects the longitudinal stresses in the basic skin, in the reinforced strips, and in the stringers and longerons differ markedly from each other. The ratio of hoop to longitudinal forces per unit length in the reinforced strips is shown to be slightly less than 2:1 —indeed, were it not for Poisson's ratio effects it would be significantly less than 2:1—and the associated neutral hole is accordingly more circular than the usual $\sqrt{2}$:1 ellipse. This is an advantage in that the stress concentrations due to other forms of loading, e.g. shear, torsion and bending, are reduced.

LIST OF SYMBOLS (*See* Fig. 1)

x, y	Cartesian co-ordinates on developed surface of fuselage, Ox measured longitudinally from centre of typical window
d	Diameter of fuselage
t	Thickness of sheet (away from reinforced strip)
t_r	Thickness of reinforced strip ($t_r > t$)
w	Width of each reinforced strip
F_s	Total section area of all stringers and longerons
t_A	(Total section area of all longitudinal-stress-bearing material)$/\pi d$
$=$	$\{F_s + (\pi d - 2w)t + 2wt_r\}/\pi d$
t_s	Effective stringer-sheet thickness in reinforced strip
	(stringer section area)/(stringer pitch) in reinforced strip
b	Width of elliptical window
kb	Height of elliptical window
A	Section area of compact edge reinforcement for window
A_0	Value of A at ends of minor axis ($x = \pm \frac{1}{2}b$)
ν	Poisson's ratio
p	Fuselage pressurization
σ_x	Longitudinal stress
σ_y	Hoop stress
Φ	Force function introduced in Section 2.2
δ	Proportion of total hoop load carried by the frames

Indices r, s (as in $\sigma_x{}^r$) refer to stress in reinforced strip (r) or stringers and longerons (s).

REFERENCES

No.	*Author*	*Title, etc.*
1	E. H. Mansfield	Neutral holes in plane sheet: reinforced holes which are elastically equivalent to the uncut sheet. A.R.C. R. & M. 2815. September, 1950.
2	E. H. Mansfield	Stress considerations in the design of pressurized shells. A.R.C. C.P. 217. April, 1955.
3	W. H. Wittrick	Stresses around reinforced elliptical holes with applications to pressure cabin windows. *Aero. Quart.*, Vol. 10, p. 373. November, 1959.

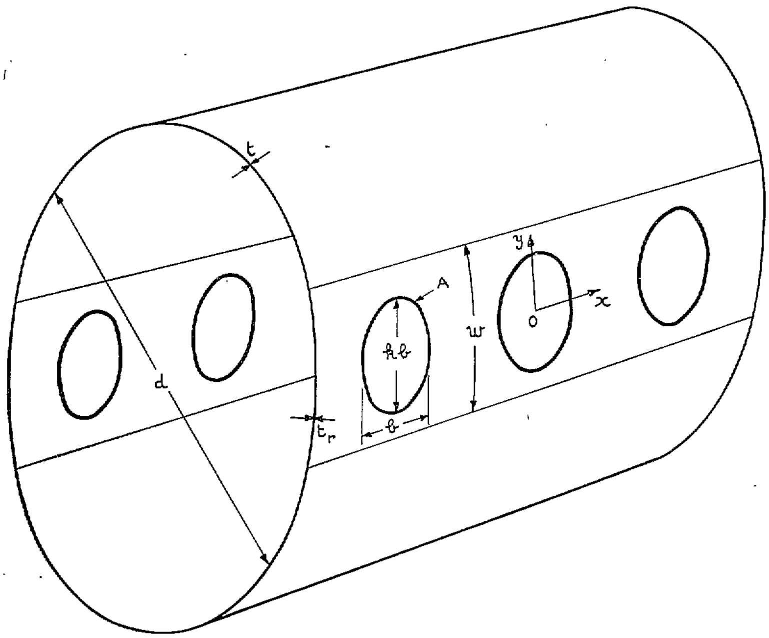

FIG. 1. Figure showing notation.

(88760) Wt. 65/1418 K.5 2/64 Hw.

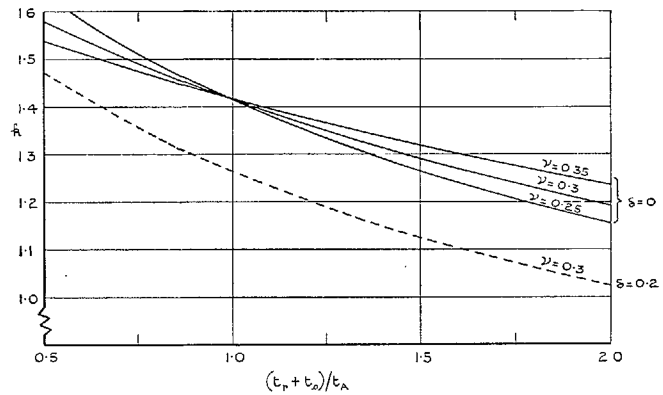

FIG. 2. Variation of height/width of neutral ellipse.

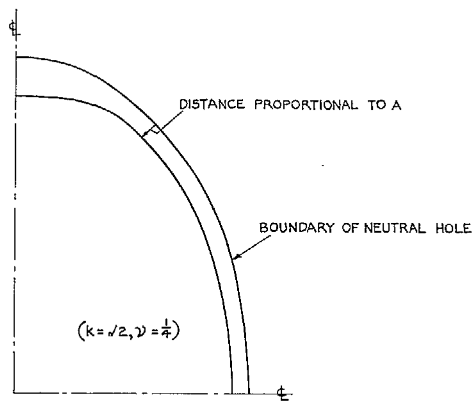

FIG. 3. Typical variation of neutral reinforcement.

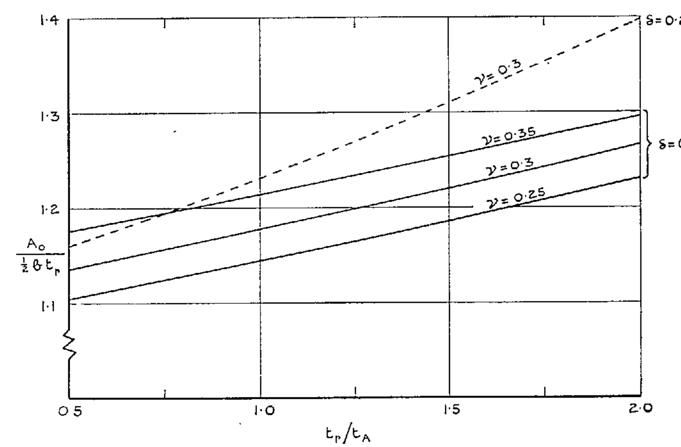

FIG. 4. Variation of reinforcement area A_0.

Publications of the Aeronautical Research Council

ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)

1942 Vol. I. Aero and Hydrodynamics, Aerofoils, Airscrews, Engines. 75*s*. (post 2*s*. 9*d*.)
Vol. II. Noise, Parachutes, Stability and Control, Structures, Vibration, Wind Tunnels. 47*s*. 6*d*. (post 2*s*. 3*d*.)

1943 Vol. I. Aerodynamics, Aerofoils, Airscrews. 80*s*. (post 2*s*. 6*d*.)
Vol. II. Engines, Flutter, Materials, Parachutes, Performance, Stability and Control, Structures. 90*s*. (post 2*s*. 9*d*.)

1944 Vol. I. Aero and Hydrodynamics, Aerofoils, Aircraft, Airscrews, Controls. 84*s*. (post 3*s*.)
Vol. II. Flutter and Vibration, Materials, Miscellaneous, Navigation, Parachutes, Performance, Plates and Panels, Stability, Structures, Test Equipment, Wind Tunnels. 84*s*. (post 3*s*.)

1945 Vol. I. Aero and Hydrodynamics, Aerofoils. 130*s*. (post 3*s*. 6*d*.)
Vol. II. Aircraft, Airscrews, Controls. 130*s*. (post 3*s*. 6*d*.)
Vol. III. Flutter and Vibration, Instruments, Miscellaneous, Parachutes, Plates and Panels, Propulsion. 130*s*. (post 3*s*. 3*d*.)
Vol. IV. Stability, Structures, Wind Tunnels, Wind Tunnel Technique. 130*s*. (post 3*s*. 3*d*.)

1946 Vol. I. Accidents, Aerodynamics, Aerofoils and Hydrofoils. 168*s*. (post 3*s*. 9*d*.)
Vol. II. Airscrews, Cabin Cooling, Chemical Hazards, Controls, Flames, Flutter, Helicopters, Instruments and Instrumentation, Interference, Jets, Miscellaneous, Parachutes. 168*s*. (post 3*s*. 3*d*.)
Vol. III. Performance, Propulsion, Seaplanes, Stability, Structures, Wind Tunnels. 168*s*. (post 3*s*. 6*d*.)

1947 Vol. I. Aerodynamics, Aerofoils, Aircraft. 168*s*. (post 3*s*. 9*d*.)
Vol. II. Airscrews and Rotors, Controls, Flutter, Materials, Miscellaneous, Parachutes, Propulsion, Seaplanes, Stability, Structures, Take-off and Landing. 168*s*. (post 3*s*. 9*d*.)

1948 Vol. I. Aerodynamics, Aerofoils, Aircraft, Airscrews, Controls, Flutter and Vibration, Helicopters, Instruments, Propulsion, Seaplane, Stability, Structures, Wind Tunnels. 130*s*. (post 3*s*. 3*d*.)
Vol. II. Aerodynamics, Aerofoils, Aircraft, Airscrews, Controls, Flutter and Vibration, Helicopters, Instruments, Propulsion, Seaplane, Stability, Structures, Wind Tunnels. 110*s*. (post 3*s*. 3*d*.)

Special Volumes

Vol. I. Aero and Hydrodynamics, Aerofoils, Controls, Flutter, Kites, Parachutes, Performance, Propulsion, Stability. 126*s*. (post 3*s*.)
Vol. II. Aero and Hydrodynamics, Aerofoils, Airscrews, Controls, Flutter, Materials, Miscellaneous, Parachutes, Propulsion, Stability, Structures. 147*s*. (post 3*s*.)
Vol. III. Aero and Hydrodynamics, Aerofoils, Airscrews, Controls, Flutter, Kites, Miscellaneous, Parachutes, Propulsion, Seaplanes, Stability, Structures, Test Equipment. 189*s*. (post 3*s*. 9*d*.)

Reviews of the Aeronautical Research Council

1939–48 3*s*. (post 6*d*.) 1949–54 5*s*. (post 5*d*.)

Index to all Reports and Memoranda published in the Annual Technical Reports

1909–1947 R. & M. 2600 (out of print)

Indexes to the Reports and Memoranda of the Aeronautical Research Council

Between Nos. 2351–2449	R. & M. No. 2450	2*s*. (post 3*d*.)
Between Nos. 2451–2549	R. & M. No. 2550	2*s*. 6*d*. (post 3*d*.)
Between Nos. 2551–2649	R. & M. No. 2650	2*s*. 6*d*. (post 3*d*.)
Between Nos. 2651–2749	R. & M. No. 2750	2*s*. 6*d*. (post 3*d*.)
Between Nos. 2751–2849	R. & M. No. 2850	2*s*. 6*d*. (post 3*d*.)
Between Nos. 2851–2949	R. & M. No. 2950	3*s*. (post 3*d*.)
Between Nos. 2951–3049	R. & M. No. 3050	3*s*. 6*d*. (post 3*d*.)
Between Nos. 3051–3149	R. & M. No. 3150	3*s*. 6*d*. (post 3*d*.)

HER MAJESTY'S STATIONERY OFFICE

from the addresses overleaf

Printed and published by
HER MAJESTY'S STATIONERY OFFICE

To be purchased from
York House, Kingsway, London W.C.2
423 Oxford Street, London W.1
13A Castle Street, Edinburgh 2
109 St. Mary Street, Cardiff
39 King Street, Manchester 2
50 Fairfax Street, Bistol 1
35 Smallbrook, Ringway, Birmingham 5
80 Chichester Street, Belfast 1
or through any bookseller

Printed in England

R. & M. No. 3360

SO. Code No. 23-3360