R. & M. No. 3343

MINISTRY OF AVIATION

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

The Forces on a Cylinder in Shear Flow

By A. THOM, LL.D.

LONDON: HER MAJESTY'S STATIONERY OFFICE
1963
FOUR SHILLINGS NET

The Forces on a Cylinder in Shear Flow

By A. THOM, LL.D.

*Reports and Memoranda No. 3343**
July, 1962

Summary.

An arithmetical solution is obtained for the viscous creeping flow past a circular cylinder on the bottom of a channel roughly two diameters deep. The top of the channel is assumed to be in uniform motion so a shear layer exists in the undisturbed channel. The drag of the cylinder is found to be equivalent to 29 times the floor drag on an area equal to the projected area of the cylinder. The resultant vertical force on the cylinder is zero.

Introduction.

A solution for the flow past a cylinder at the bottom of a shear layer would enable an estimate to be made of the force produced by tidal currents on a cable lying on the bed of the sea. It was accordingly decided to work through an approximate solution by hand to see what kind of difficulties arise. To reduce the problem to a manageable size it was decided to take the case of a cylinder of diameter D in a passage of depth roughly $2D$, the roof of the passage being assumed to move at a uniform speed across the cylinder. Only the case of zero Reynolds number has been attempted but, even so, the results may be of interest.

The Non-Viscous Case.

In order to obtain a grid on which to work the viscous case a Laplacian solution had first to be obtained. This is shown in Fig. 1. It will be seen that matters have been arranged so that an integral equipotential line forms the line of symmetry (at the right) and another runs into the cusp or stagnation point. Elsewhere (Thom and Apelt[2]) it is suggested that figures with properties of this kind be called symmetromorphic. The method of obtaining the solution to $\nabla^2\psi = 0$ for cases like this has been described several times and only the difficulties need be mentioned here. These centre round the cusp. In problems previously worked the corners had definite angles and the function $w = Z^n$ was available. With zero angle a suitably simple function was not known and the region had to be subdivided and worked over until it was certain that no error was being spread through the field. In Fig. 2 suppose O to be the apex of the cusp (in the $w = \xi + i\eta$ field). As practically all the work was done with the four-square molecule we need to know the values of θ to put at O for use in each of the three molecules involved, namely, OBCE, JADF and HOEG. As the size of the molecule shrinks towards zero presumably the values tend to $\pi/4$, $\pi/2$ and $3\pi/4$. The values found empirically are:

For $a = 1$, 0·50 0·89 1·38

For $a = \frac{1}{2}$, 0·55 1·00 1·53

For $a = \frac{1}{4}$, 0·60 1·09 1·61

Having, from the run of the figures and the limiting values just stated, guessed the next value the local subdivision can be made and the previous boundary values adjusted to suit.

* Replaces A.R.C. 23,930.

As every round of operations involves finding the conjugate function (log $1/Q$) the process is long but it is believed that a reasonably good solution has been obtained. Unfortunately the size of the cylinder (which is only known when the solution is complete) is rather large. The radius is $1 \cdot 04$ for a channel width of 4. As so much work had gone into the production of the grid it was decided to go ahead with the viscous solution. Skeleton details of the Laplacian solution will be found in Table 1. The usual squaring formulae enable any necessary subdivisions to be made.

Viscous Solution, $R = 0$.

We write

$$\nabla^2\psi = \zeta, \quad \nabla^2\zeta = 0$$

and operate alternately on ψ and ζ. The four-square-molecule formulae were used throughout except when changing to a smaller grid size. This change of size was made for the region of the cusp shown by a rectangle in Table 2. The formulae used are

$$20\zeta_0 = 4s_1 + s_2$$

$$20\psi_0 = 4S_1 + S_2 - 6(a^2/Q^2)\zeta$$

where s_1 is the sum of the ζ values in the centre of the side

s_2 is the sum of the corner values

S_1 and S_2 refer similarly to ψ values and Q is the modulus of the transformation.

The boundary formula (Woods[3]) used was

$$\zeta_B = 3Q_B^2(\psi_A - \psi_B)/a^2 - \tfrac{1}{2}\zeta_A(Q_B/Q_A)^2$$

for the lower boundary. For the upper boundary $\psi_A - \psi_B$ is replaced by $\psi_A - \psi_B - u/Q_B$, u being the velocity of the moving boundary.

The depth of the passage is 4 and the undisturbed ψ is taken as

$$\psi_u = 5y^2$$

which entails $\zeta = 10$ and the speed of the moving top boundary $u = -40$.

The usual boundary troubles presented themselves (Thom[1]) but by using roughly $\frac{1}{2}$ movement a reasonably good solution was obtained. It could be improved by subdividing a larger area of the field. The results are shown in Table 2.

The Pressures.

Since $uD\rho/\mu$ is zero and u is finite then ρ is zero and the pressures are found by integrating, e.g. along $\eta = \text{const.}$

$$p_A - p_B = \mu \int_A^B \frac{\partial\zeta}{\partial\xi}\, d\eta \,.$$

Thus $(p_A - p_B)/\mu$ is the function conjugate to ζ and the formulae developed elsewhere (Thom and Apelt[2], Section 5.4) are available.

The pressure drop from $\xi = 0$, the top of the cylinder to the extreme left was found to be 89 by integrating along $\eta = 2$, so the total drop caused by the cylinder is 178. An identical value was found by integrating along $\eta = 1$. The pressures throughout the field are given along with ψ and ζ in Table 2 and the conformal net (ζ, p/μ) is shown in Fig. 4. The pressure and vorticity along the top and bottom passage walls are shown in Fig. 5.

Forces on the Cylinder.

The pressures round the cylinder are shown plotted on the vertical projection in Fig. 6a. The area gives the resultant horizontal force. Fig. 6b shows the same thing for the vertical forces. From the type of symmetry which obtains it appears that there is no resultant vertical force on the cylinder from the pressures. Evidently there is also no vertical force from the skin drag. The horizontal resultant skin drag is obtained from Fig. 3.

As an overall check consider the total forces acting on the water from the extreme left to the extreme right.

We have:

From pressure difference on ends	$-4 \times 178\mu$	$= -712\mu$
From skin drag on passage walls (Fig. 5)	$+104\mu$	
From pressure on cylinder (Fig. 6)	$+440\mu$	$= +711\mu$.
From skin drag on cylinder (Fig. 3)	$+167\mu$	

This is certainly a better balance than was expected in view of the somewhat rough solution. The total horizontal force F_C on the cylinder per foot length is $440\mu + 167\mu$ or 607μ. The vorticity in the undisturbed flow is 10 so the skin drag on the bottom is 10μ.

Put F_B = skin friction on an area equal to 1 foot length of cylinder. The cylinder diameter is 2·07 so

$$F_B = 2{\cdot}07 \times 1 \times 10\mu = 20{\cdot}7\mu .$$

Thus our final conclusion is that

$$F_C = 29F_B$$

or the force on the cylinder is equal to the undisturbed friction on an area 29 diameters wide.

Conclusion.

The force on the cylinder has been found for a somewhat artificial case but with the knowledge gained it ought to be possible to programme the problem with a wider channel and a finite if small Reynolds number. The lack of symmetry introduced with the Reynolds number would produce the lift which is actually found experimentally.

REFERENCES

No.	*Author(s)*	*Title, etc.*
1	A. Thom	Boundary troubles in arithmetical solutions of the Navier-Stokes equations. A.R.C. 20,289. July, 1958.
2	A. Thom and C. J. Apelt ..	*Field computations in engineering and physics.* D. van Nostrand Co., London, Toronto, New York and Princeton New Jersey. 1961.
3	L. C. Woods	A note on the numerical solution of fourth order differential equations. *Aero. Quart.*, Vol. 5, Part 3, p. 176. 1954.

TABLE 1

η ξ	−8	−7	−6	−5	−4	−3	−2	−1	0
4	0 15 629 400	0 32 531 400	0 66 436 400	0 127 345 400	0 222 261 400	0 340 185 400	0 455 118 400	0 537 57 400	0 567 0 400
3	10 11 628 301	22 24 531 303	44 52 434 306	77 109 341 312	114 208 256 320	128 341 181 329	107 473 115 337	60 564 56 342	0 597 0 344
2	15 1 627 202	33 2 529 204	73 9 428 209	149 44 331 220	251 155 242 240	296 345 166 260	243 533 104 277	131 649 51 287	0 694 0 291
1	11 − 10 626 101	26 − 24 528 103	64 − 53 421 107	174 − 97 314 120	457 +145 210 154	582 368 137 196	431 671 87 225	219 820 42 240	0 870 0 245
0	0 − 15 625 0	0 − 35 527 0	0 − 85 418 0	0 −240 300 0		1095 + 620 92 152	651 955 63 186	312 1089 32 202	0 1124 0 207

Each square contains
1000 θ
1000 Ln Q
100 x
100 y

TABLE 2

η \ ξ	−11	−10	−9	−8	−7	−6	−5	−4	−3	−2	−1	0
4	80 10	80 9·98	80 10·02	80 10·15 − 1·9	80 9·98 − 0·4	80 9·83 − 1·0	80 7·8 − 2·0	80 + 2·8 − 1·3	80 − 6·8 + 4·6	80 −19·6 20	80 −34·3 49	80 −40·1 89
3	45·04 10	45·10 9·91	45·24 9·98	45·50 9·93 − 1·2	45·96 9·73 − 0·2	46·74 9·27 − 0·2	47·91 8·32 + 0·3	49·32 6·8 3·3	50·73 4·6 11·6	51·89 + 1·5 29	52·63 − 1·3 56	52·89 − 2·6 89
2	20·04 10 0·002	20·10 9·99 0·005	20·24 9·95 0·01	20·50 9·87 0·04	20·93 9·67 0·06	21·58 9·37 0·3	22·36 9·51 0·5	23·20 11·5 2·4	23·97 16·8 20	24·70 24 27	25·22 30 55	25·48 32 89
1	5·02 10	5·04 10·00	5·09 10·00	5·19 9·99 + 0·2	5·36 9·76 + 0·4	5·56 9·31 + 0·62	5·53 8·9 − 0·5	5·36 12·4 − 3·6	5·73 26·5 − 4·5	5·99 48 +14	6·23 65 47	6·36 71 89
0	0 10	0 10·00	0 10·03	0 10·05 + 0·2	0 10·20 + 0·6	0 9·6 + 1·8	0 6·5 + 2·1	 −12·2	0 31 −33	0 72 −16	0 108 + 28	0 121 89

Grid subdivided in this Region

Each rectangle contains ψ, ζ, p/μ

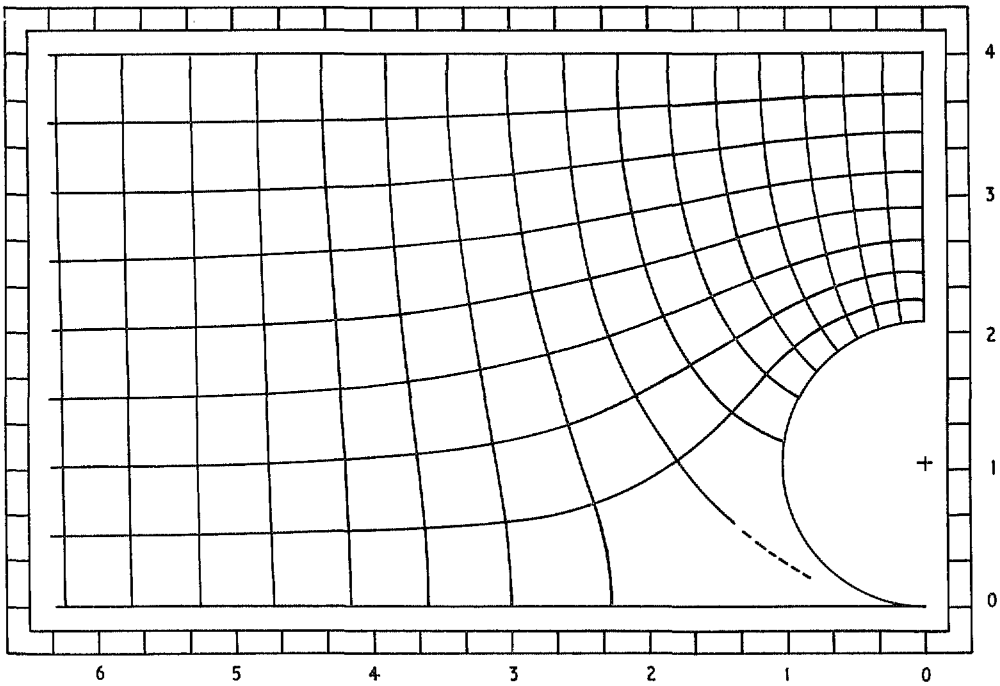

FIG. 1.

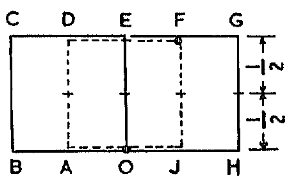

FIG. 2.

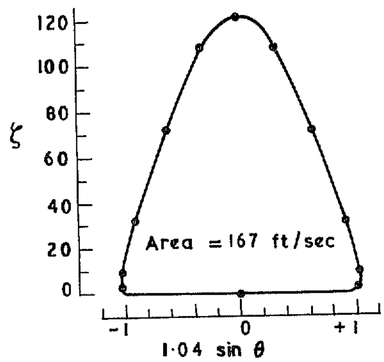

FIG. 3. Skin drag on cylinder.

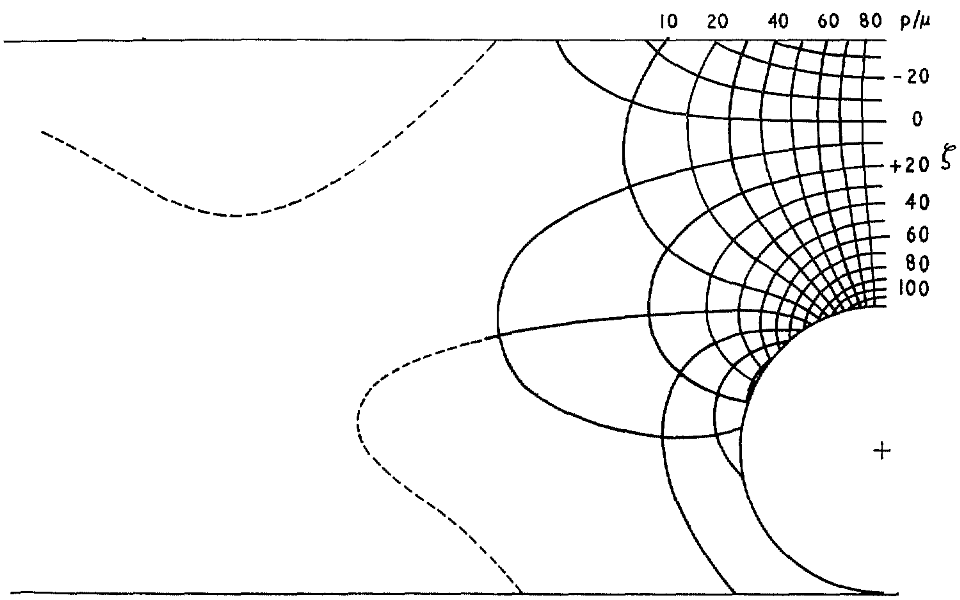

FIG. 4. Vorticity and pressure.

(87995) Wt. 65/1418 K.5 11/63 Hw.

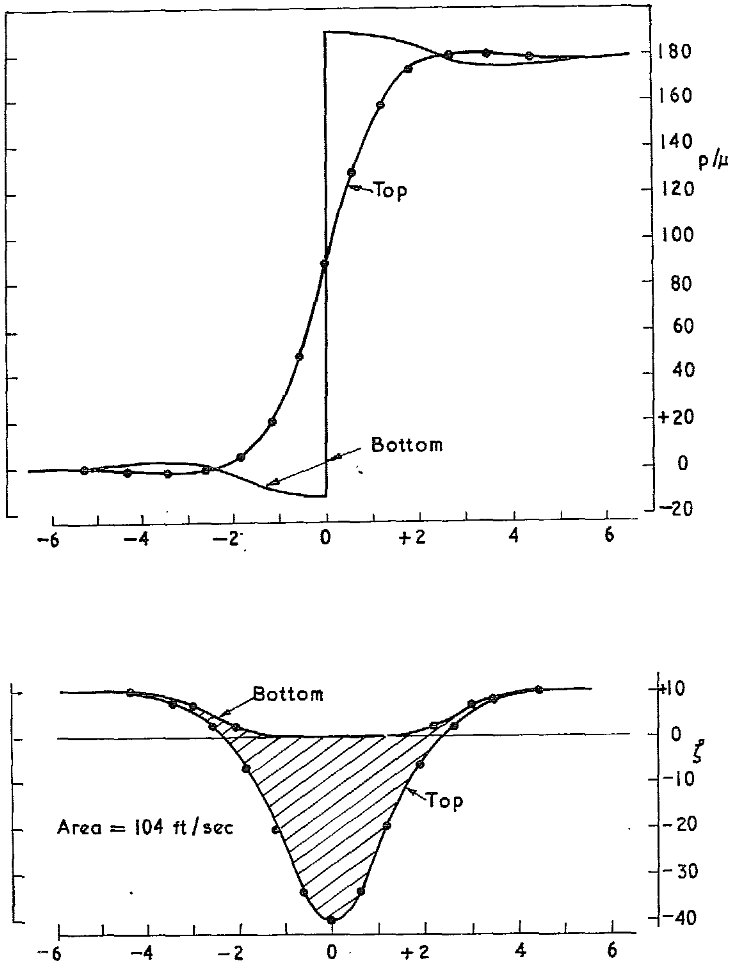

FIG. 5. Pressure and vorticity on channel walls.

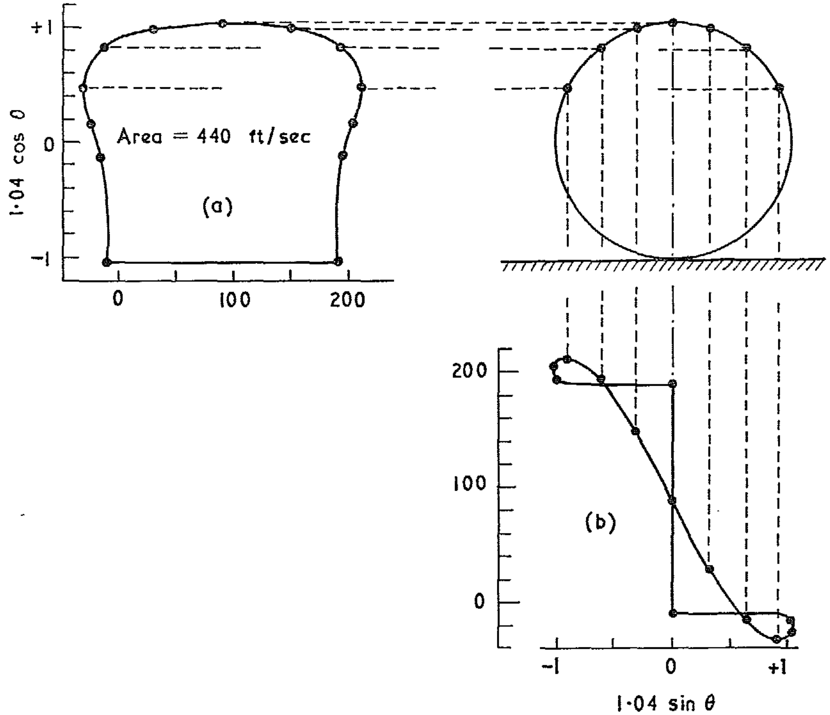

FIG. 6. Pressure round cylinder plotted on its horizontal and vertical projections.

Publications of the Aeronautical Research Council

ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)

1942 Vol. I. Aero and Hydrodynamics, Aerofoils, Airscrews, Engines. 75*s.* (post 2*s.* 9*d.*)
Vol. II. Noise, Parachutes, Stability and Control, Structures, Vibration, Wind Tunnels. 47*s.* 6*d.* (post 2*s.* 3*d.*)

1943 Vol. I. Aerodynamics, Aerofoils, Airscrews. 80*s.* (post 2*s.* 6*d.*)
Vol. II. Engines, Flutter, Materials, Parachutes, Performance, Stability and Control, Structures. 90*s.* (post 2*s.* 9*d.*)

1944 Vol. I. Aero and Hydrodynamics, Aerofoils, Aircraft, Airscrews, Controls. 84*s.* (post 3*s.*)
Vol. II. Flutter and Vibration, Materials, Miscellaneous, Navigation, Parachutes, Performance, Plates and Panels, Stability, Structures, Test Equipment, Wind Tunnels. 84*s.* (post 3*s.*)

1945 Vol. I. Aero and Hydrodynamics, Aerofoils. 130*s.* (post 3*s.* 6*d.*)
Vol. II. Aircraft, Airscrews, Controls. 130*s.* (post 3*s.* 6*d.*)
Vol. III. Flutter and Vibration, Instruments, Miscellaneous, Parachutes, Plates and Panels, Propulsion. 130*s.* (post 3*s.* 3*d.*)
Vol. IV. Stability, Structures, Wind Tunnels, Wind Tunnel Technique. 130*s.* (post 3*s.* 3*d.*)

1946 Vol. I. Accidents, Aerodynamics, Aerofoils and Hydrofoils. 168*s.* (post 3*s.* 9*d.*)
Vol. II. Airscrews, Cabin Cooling, Chemical Hazards, Controls, Flames, Flutter, Helicopters, Instruments and Instrumentation, Interference, Jets, Miscellaneous, Parachutes. 168*s.* (post 3*s.* 3*d.*)
Vol. III. Performance, Propulsion, Seaplanes, Stability, Structures, Wind Tunnels. 168*s.* (post 3*s.* 6*d.*)

1947 Vol. I. Aerodynamics, Aerofoils, Aircraft. 168*s.* (post 3*s.* 9*d.*)
Vol. II. Airscrews and Rotors, Controls, Flutter, Materials, Miscellaneous, Parachutes, Propulsion, Seaplanes, Stability, Structures, Take-off and Landing. 168*s.* (post 3*s.* 9*d.*)

1948 Vol. I. Aerodynamics, Aerofoils, Aircraft, Airscrews, Controls, Flutter and Vibration, Helicopters, Instruments, Propulsion, Seaplane, Stability, Structures, Wind Tunnels. 130*s.* (post 3*s.* 3*d.*)
Vol. II. Aerodynamics, Aerofoils, Aircraft, Airscrews, Controls, Flutter and Vibration, Helicopters, Instruments, Propulsion, Seaplane, Stability, Structures, Wind Tunnels. 110*s.* (post 3*s.* 3*d.*)

Special Volumes

Vol. I. Aero and Hydrodynamics, Aerofoils, Controls, Flutter, Kites, Parachutes, Performance, Propulsion, Stability. 126*s.* (post 3*s.*)
Vol. II. Aero and Hydrodynamics, Aerofoils, Airscrews, Controls, Flutter, Materials, Miscellaneous, Parachutes, Propulsion, Stability, Structures. 147*s.* (post 3*s.*)
Vol. III. Aero and Hydrodynamics, Aerofoils, Airscrews, Controls, Flutter, Kites, Miscellaneous, Parachutes, Propulsion, Seaplanes, Stability, Structures, Test Equipment. 189*s.* (post 3*s.* 9*d.*)

Reviews of the Aeronautical Research Council

1939–48 3*s.* (post 6*d.*) 1949–54 5*s.* (post 5*d.*)

Index to all Reports and Memoranda published in the Annual Technical Reports

1909–1947 R. & M. 2600 (out of print)

Indexes to the Reports and Memoranda of the Aeronautical Research Council

Between Nos. 2351–2449	R. & M. No. 2450	2*s.* (post 3*d.*)
Between Nos. 2451–2549	R. & M. No. 2550	2*s.* 6*d.* (post 3*d.*)
Between Nos. 2551–2649	R. & M. No. 2650	2*s.* 6*d.* (post 3*d.*)
Between Nos. 2651–2749	R. & M. No. 2750	2*s.* 6*d.* (post 3*d.*)
Between Nos. 2751–2849	R. & M. No. 2850	2*s.* 6*d.* (post 3*d.*)
Between Nos. 2851–2949	R. & M. No. 2950	3*s.* (post 3*d.*)
Between Nos. 2951–3049	R. & M. No. 3050	3*s.* 6*d.* (post 3*d.*)
Between Nos. 3051–3149	R. & M. No. 3150	3*s.* 6*d.* (post 3*d.*)

HER MAJESTY'S STATIONERY OFFICE

from the addresses overleaf

Printed and published by
HER MAJESTY'S STATIONERY OFFICE

To be purchased from
York House, Kingsway, London W.C.2
423 Oxford Street, London W.1
13A Castle Street, Edinburgh 2
109 St. Mary Street, Cardiff
39 King Street, Manchester 2
50 Fairfax Street, Bristol 1
35 Smallbrook, Ringway, Birmingham 5
80 Chichester Street, Belfast 1
or through any bookseller

Printed in England

R. & M. No. 3343

S.O. Code No. 23-3343