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The Shear Stiffness of a Corrugated Web

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Summary.

An analysis is made of the shear deformation of a corrugated web attached to the flanges at discrete points. The shear stiffness is calculated for a wide variation of the dimensions of the web, and two positions of the points of attachment to the flanges. Results are presented showing this stiffness relative to that of the continuously attached corrugation.

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1. *Introduction.*

A corrugated web has a number of advantages and disadvantages in comparison with a plane web of equal weight. There is the obvious disadvantage of increased cost of manufacture and attachment. The shear stiffness will also be slightly less at low stress levels, but this is more than

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offset by the greater stiffness at higher stress levels where the plane web would be in a buckled state. The method of attachment can also be a critical factor; if the attachment is at discrete points, rather than continuous, bending deformations occur which may cause a marked reduction in stiffness. It is this drop in stiffness due to bending deformation which is considered in this report. The analysis is for a corrugated web whose cross-section consists of a series of equal circular arcs attached either on the crests of the waves or along the centre-line. Results are presented showing the shear stiffness relative to that of a continuously attached corrugation for a wide variation of web parameters.

Corrugated webs may also be used to avoid thermal stresses[1]. However, in this connection the corrugations may be very shallow with the result that, although attachment at discrete points is required, the ensuing loss of shear stiffness is small.

2. *Outline of Problem, Assumptions and Boundary Conditions.*

The shear stiffness of a corrugated web whose cross-section consists of a series of circular arcs is derived first when the points of attachment of the web to the flanges lie on the crests of the waves in the web and then when they lie along the centre-line of the corrugation. (*See* Figs. 1 and 2.)

The analysis is based on the assumptions that the deformation is small and consists of a simple shearing displacement of the middle surface together with an arbitrary inextensional deformation.

Since these two types of deformation occur independently of each other, the flexibilities arising from them can be added, and thus the following expression is obtained for the stiffness of a panel of corrugation relative to that of a similar panel continuously attached to the flanges,

$$\Theta = \frac{1}{1 + \dfrac{1}{\Gamma/\Gamma_0}} \tag{1}$$

where Γ_0 and Γ are the stiffnesses obtained assuming a simple shear of the middle surface and an inextensional deformation respectively. It should be noted that because the type of deformation assumed is an approximation to that occurring in practice, the stiffness obtained in this way is an over-estimate.

Boundary Conditions.

From the fact that the generators of the web remain straight and unstretched in inextensional deformation, three of the boundary conditions applying along the generators joining the points of attachment may be deduced. These are that (referring to Fig. 1 for the notation)

$$\left.\begin{aligned} u &= \text{constant} \\ v &= 0 \\ w &= 0 . \end{aligned}\right\} \tag{2}$$

Now the deflected form in shear is antisymmetrical about the generator midway between the points of attachment. Examination of Fig. 2 in this light reveals the fourth boundary condition to be

$$\frac{\partial^2 w}{\partial y^2} = 0 \tag{3}$$

when the points of attachment are at the crests of the waves in the corrugation and

$$\frac{\partial w}{\partial y} = 0 \tag{4}$$

when the points of attachment are along the centre-line of the corrugation.

3. *Analysis.*

Using the system of co-ordinates shown in Fig. 1, the vanishing of the three strain components in the middle surface of the shell is equivalent to the equations

$$\left.\begin{aligned} \frac{\partial u}{\partial x} &= 0 \\ \frac{\partial v}{\partial y} + \frac{w}{R} &= 0 \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} &= 0 . \end{aligned}\right\} \tag{5}$$

Thus, writing

$$\left.\begin{aligned} \xi &= x/a \\ \eta &= y/a , \end{aligned}\right\} \tag{6}$$

the displacements may be written in terms of two independent functions ϕ and ψ as follows,

$$\left.\begin{aligned} u &= h\phi(\eta) \\ v &= -h\xi\phi^{I}(\eta) + h\psi(\eta) \\ w &= \frac{h\xi}{\kappa}\phi^{II}(\eta) - \frac{h}{\kappa}\psi^{I}(\eta) . \end{aligned}\right\} \tag{7}$$

In the present case however, the deflection w is zero from symmetry on $x = 0$, so that $\psi(\eta) = 0$ for all η. When the middle surface is inextensional, the usual expression for the strain energy of a shell[2] becomes

$$V = \frac{D}{2} \iint\limits_{\substack{\text{middle}\\\text{surface}}} \left[\left(-\frac{\partial^2 w}{\partial y^2} + \frac{1}{R}\frac{\partial v}{\partial y}\right)^2 + 2(1-\nu)\left(-\frac{\partial^2 w}{\partial x \partial y} + \frac{1}{R}\frac{\partial v}{\partial x}\right)^2\right] dx\, dy . \tag{8}$$

Thus, substituting from equations (7) and performing the integration with respect to ξ, the strain energy of a curved panel of length $2b$ and developed width $2a$ is given by

$$V = \frac{Dh^2\beta^3}{3a^2\kappa^2} \int_{-1}^{1} [\{\phi^{IV}(\eta) + \kappa^2\phi^{II}(\eta)\}^2 + \mu^2\{\phi^{III}(\eta) + \kappa^2\phi^{I}(\eta)\}^2]\, d\eta \tag{9}$$

where $\beta = b/a$, $\mu^2 = 6(1-\nu)/\beta^2$. In the problem under consideration there is no work done by external forces since the edges $y = \pm a$ are assumed to undergo a constant shearing displacement $\pm u_0$. Thus the solution of the problem is given by that value of the function $\phi(\eta)$ for which the variation of the integral (9) is zero and which satisfies the boundary conditions on the edges $\eta = \pm 1$. The statement

$$\delta V = 0$$

is equivalent, after applying the usual processes of the calculus of variations, to the equation

$$\begin{aligned} &\int_{-1}^{1} [\phi^{VIII}(\eta) + 2\kappa^2\phi^{VI}(\eta) + \kappa^4\phi^{IV}(\eta) - \mu^2\{\phi^{VI}(\eta) + 2\kappa^2\phi^{IV}(\eta) + \kappa^4\phi^{II}(\eta)\}]\delta\phi\, dy + \\ &\quad + [\delta\phi^{III}(\phi^{IV} + \kappa^2\phi^{II}) - \delta\phi^{II}\{\phi^{V} + \kappa^2\phi^{III} - \mu^2(\phi^{III} + \kappa^2\phi^{I})\} + \\ &\quad + \delta\phi^{I}\{\phi^{VI} + 2\kappa^2\phi^{IV} + \kappa^4\phi^{II} - \mu^2(\phi^{IV} + \kappa^2\phi^{II})\} - \\ &\quad - \delta\phi\{\phi^{VII} + 2\kappa^2\phi^{V} + \kappa^4\phi^{III} - \mu^2(\phi^{V} + 2\kappa^2\phi^{III} + \kappa^4\phi^{I})\}]_{-1}^{1} = 0 . \end{aligned} \tag{10}$$

At this point it is convenient to treat the two methods of attachment separately.

3.1. *The Points of Attachment at the Crests of the Waves.*

In this case the panel is regarded as consisting of three parts as shown in Fig. 1, each part being a segment of a cylinder of radius R. Furthermore, since the deflection is antisymmetrical about $y = 0$ it is necessary only to consider the behaviour of one half of the panel ($0 \leqslant y \leqslant 2a$). Using suffix 1 for quantities in the region $0 \leqslant y \leqslant a$, suffix 2 for quantities in the region $a \leqslant y \leqslant 2a$, and the co-ordinate systems shown in Fig. 1, the boundary conditions on $y = a$ are

$$\left.\begin{array}{ll} u_1 = u_2 & w_1 = -w_2 \\ v_1 = -v_2 & \dfrac{\partial w_1}{\partial y_1} = \dfrac{\partial w_2}{\partial y_2}. \end{array}\right\} \qquad (11)$$

Expressed in terms of ϕ, these become

$$\left.\begin{aligned} \phi_1(1) &= \phi_2(1) \\ \phi_1{}^I(1) &= -\phi_2{}^I(1) \\ \phi_1{}^{II}(1) &= -\phi_2{}^{II}(1) \\ \phi_1{}^{III}(1) &= \phi_2{}^{III}(1). \end{aligned}\right\} \qquad (12)$$

Now equations similar to equation (10) can be set up for both regions of the panel, the only difference in this case being that the integrations are carried out over the range 0 to 1. Consideration of these equations, and equations (12) gives four more boundary conditions on $\eta_1 = 1 = \eta_2$, namely

$$\left.\begin{aligned} &\phi_1{}^{IV} + \kappa^2\phi_1{}^{II} = -(\phi_2{}^{IV} + \kappa^2\phi_2{}^{II}) \\ &\phi_1{}^{V} + \kappa^2\phi_1{}^{III} - \mu^2(\phi_1{}^{III} + \kappa^2\phi_1{}^{I}) \\ &\qquad = \phi_2{}^{V} + \kappa^2\phi_2{}^{III} - \mu^2(\phi_2{}^{III} + \kappa^2\phi_2{}^{I}) \\ &\phi_1{}^{VI} + 2\kappa^2\phi_1{}^{IV} + \kappa^4\phi_1{}^{II} - \mu^2(\phi_1{}^{IV} + \kappa^2\phi_1{}^{II}) \\ &\qquad = \phi_2{}^{VI} + 2\kappa^2\phi_2{}^{IV} + \kappa^4\phi_2{}^{II} - \mu^2(\phi_2{}^{IV} + \kappa^2\phi_2{}^{II}) \\ &\phi_1{}^{VII} + 2\kappa^2\phi_1{}^{V} + \kappa^4\phi_1{}^{III} - \mu^2(\phi_1{}^{V} + 2\kappa^2\phi_1{}^{III} + \kappa^2\phi_1{}^{I}) \\ &\qquad = -\{\phi_2{}^{VII} + 2\kappa^2\phi_2{}^{V} + \kappa^4\phi_2{}^{III} - \mu^2(\phi_2{}^{V} + 2\kappa^2\phi_2{}^{III} + \kappa^4\phi_2{}^{I})\}. \end{aligned}\right\} \qquad (13)$$

The conditions on $\eta_1 = 0$ are automatically satisfied by the fact that $\phi_1(\eta_1)$ is an odd function. The conditions on $\eta_2 = 0$ are

$$\left.\begin{aligned} \phi_2(0) &= u_0/h \\ \phi_2{}^I(0) &= 0 = \phi_2{}^{II}(0) = \phi_2{}^{IV}(0). \end{aligned}\right\} \qquad (14)$$

The differential equation to be satisfied by both ϕ_1 and ϕ_2 is that under the integral sign in equation (10). The solutions of this equation are

$$\phi_1{}^*(\eta_1) = \frac{h\phi_1(\eta_1)}{u_0} = A_1 \sinh \mu\eta_1 + A_2 \sin \kappa\eta_1 + A_3\eta_1 \cos \kappa\eta_1 + A_4\eta_1 \qquad (15)$$

$$\begin{aligned} \phi_2{}^*(\eta_2) = \frac{h\phi_2(\phi_2)}{u_0} &= B_1 \sinh \mu\eta_2 + B_2 \sin \kappa\eta_2 + B_3\eta_2 \cos \kappa\eta_2 + B_4\eta_2 + \\ &\quad + B_5 \cosh \mu\eta_2 + B_6 \cos \kappa\eta_2 + B_7\eta_2 \sin \kappa\eta_2 + B_8. \end{aligned} \qquad (16)$$

The boundary conditions (12), (13) and (14) give rise to a system of twelve simultaneous equations which may be expressed in matrix form as

$$Pq = r \tag{17}$$

where

$$P = \left[\begin{array}{cccccccccccc}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 1, & 0, & 1 \\
0, & 0, & 0, & 0, & \mu, & \kappa, & 1, & 1, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \mu^2, & -\kappa^2, & 2\kappa, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \mu^4, & \kappa^4, & -4\kappa^3, & 0 \\
\sinh\mu, & \sin\kappa, & \cos\kappa, & 1, & -\sinh\mu, & -\sinh\kappa, & -\cos\kappa, & -1, & -\cosh\mu, & -\cos\kappa, & -\sin\kappa, & -1 \\
\mu\cosh\mu, & \kappa\cos\kappa, & \cos\kappa - \kappa\sin\kappa, & 1, & \mu\cosh\mu, & \kappa\cos\kappa, & \cos\kappa - \kappa\sin\kappa, & 1, & \mu\sinh\mu, & -\kappa\sin\kappa, & \sin\kappa + \kappa\cos\kappa, & 0 \\
\mu^2\sinh\mu, & -\kappa^2\sin\kappa, & -\kappa^2\cos\kappa - 2\kappa\sin\kappa, & 0, & \mu^2\sinh\mu, & -\kappa^2\sin\kappa, & -\kappa^2\cos\kappa - 2\kappa\sin\kappa, & 0, & \mu^2\cosh\mu, & -\kappa^2\cos\kappa, & -\kappa^2\sin\kappa + 2\kappa\cos\kappa, & 0 \\
\mu^3\cosh\mu, & -\kappa^3\cos\kappa, & \kappa^3\sin\kappa - 3\kappa^2\cos\kappa, & 0, & -\mu^3\cosh\mu, & \kappa^3\cos\kappa, & -\kappa^3\sin\kappa + 3\kappa^2\cos\kappa, & 0, & -\mu^3\sinh\mu, & -\kappa^3\sin\kappa, & \kappa^3\cos\kappa + 3\kappa^2\sin\kappa, & 0 \\
\mu^2(\mu^2+\kappa^2)\times\sinh\mu, & 0, & 2\kappa^3\sin\kappa, & 0, & \mu^2(\mu^2+\kappa^2)\times\sinh\mu, & 0, & 2\kappa^3\sin\kappa, & 0, & \mu^2(\mu^2+\kappa^2)\times\cosh\mu, & 0, & -2\kappa^3\cos\kappa, & 0 \\
0, & 0, & 2(\kappa^2+\mu^2)\times\cos\kappa, & -\mu^2, & 0, & 0, & -2(\kappa^2+\mu^2)\times\cos\kappa, & \mu^2, & 0, & 0, & -2(\kappa^2+\mu^2)\times\sin\kappa, & 0 \\
(\mu^2+\kappa^2)\times\sinh\mu, & 0, & -2\kappa\sin\kappa, & 0, & -(\mu^2+\kappa^2)\times\sinh\mu, & 0, & 2\kappa\sin\kappa, & 0, & -(\mu^2+\kappa^2)\times\cosh\mu, & 0, & -2\kappa\cos\kappa, & 0 \\
0, & 0, & 0, & 1, & 0, & 0, & 0, & 1, & 0, & 0, & 0, & 0
\end{array}\right] \tag{18}$$

$q = \{A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8\}$ and $r = \{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$.

Now the shear stiffness Γ is given by

$$\Gamma = \frac{1}{4}\frac{\partial V}{\partial u_0}\Big/u_0 \tag{19}$$

and

$$V = \frac{2D\beta^3}{3a^2\kappa^2}u_0^2\left[\int_0^1 \{(\phi_1^{*IV} + \kappa^2\phi_1^{*II})^2 + \mu^2(\phi_1^{*III} + \kappa^2\phi_1^{*I})^2\}\,d\eta_1 + \right.$$
$$\left. + \int_0^1 (\phi_2^{*IV} + \kappa^2\phi_2^{*II})^2 + \mu^2(\phi_2^{*III} + \kappa^2\phi_2^{*I})^2\}\,d\eta_2\right]. \tag{20}$$

Therefore performing the integrations and noting that the stiffness of the developed flat sheet is given by

$$\Gamma_0 = \frac{Eh}{2(1+\nu)}\left(\frac{b}{2a}\right), \tag{21}$$

we obtain

$$\frac{\Gamma}{\Gamma_0} = \frac{2}{9(1-\nu)}\left(\frac{h}{2a}\right)^2\frac{\beta^2}{\kappa^2}\,[\mu^3(\mu^2+\kappa^2)^2\{(A_1^2+B_1^2+B_5^2)\sinh 2\mu + 2B_1B_5(\cosh 2\mu - 1)\} +$$
$$+ 2\kappa^3\{2(A_3^2+B_3^2+B_7^2)\,\kappa(\kappa^2+\mu^2) + (\mu^2-\kappa^2)(A_3^2+B_3^2-B_7^2)\sin 2\kappa +$$
$$+ 2(\mu^2-\kappa^2)\,B_3B_7(1-\cos 2\kappa)\} +$$
$$+ 2\mu^2\kappa^4(A_4^2+B_4^2) -$$
$$- 8\mu^2\kappa^2(\mu^2+\kappa^2)\{(A_1A_3+B_1B_3)\cos\kappa\sinh\mu + B_3B_5(\cos\kappa\cosh\mu - 1) +$$
$$+ B_5B_7\sin\kappa\cosh\mu + B_7B_1\sin\kappa\sinh\mu\} +$$
$$+ 4\mu^2\kappa^2(\mu^2+\kappa^2)\{(A_1A_4+B_1B_4)\sinh\mu + B_4B_5(\cosh\mu - 1)\} -$$
$$- 8\mu^2\kappa^3\{(A_4A_3+B_4B_3)\sin\kappa + B_4B_7(1-\cos\kappa)\}] \tag{22}$$

where the A's and B's are given by the solution of equation (17).

Finally the relative stiffness is given by equation (1).

3.2. *Points of Attachment along the Centre-Line of the Corrugation.*

In this case the panel consists of one segment of a cylinder of radius R. The boundary conditions on $y = a$ are

$$\left.\begin{aligned} u &= u_0 \\ v &= 0 \\ w &= 0 \\ \frac{\partial w}{\partial y} &= 0, \end{aligned}\right\} \tag{23}$$

or in terms of ϕ

$$\left.\begin{aligned} \phi(1) &= u_0/h \\ \phi^I(1) &= 0 = \phi^{II}(1) = \phi^{III}(1); \end{aligned}\right\} \tag{24}$$

and since the deformation is again antisymmetrical about $y = 0$ the solution of the differential equation is given by

$$\phi^* = \frac{\phi h}{u_0} = A_1\sinh\mu y + A_2\sin\kappa y + A_3 y\cos\kappa y + A_4 y. \tag{25}$$

On substitution of this expression into equations (24) the four boundary conditions become:

$$\left.\begin{aligned} A_1 \sinh\mu + A_2 \sin\kappa + A_3 \cos\kappa + A_4 &= 1 \\ A_1\mu \cosh\mu + A_2\kappa \cos\kappa + A_3(\cos\kappa - \kappa \sin\kappa) + A_4 &= 0 \\ A_1\mu^2 \sinh\mu - A_2\kappa^2 \sin\kappa - \kappa A_3(\kappa\cos\kappa + 2\sin\kappa) &= 0 \\ A_1\mu^3 \cosh\mu - A_2\kappa^3 \cos\kappa + \kappa^2 A_3(\kappa\sin\kappa - 3\cos\kappa) &= 0\,. \end{aligned}\right\} \qquad (26)$$

These four equations may be explicitly solved for A_1, A_2, A_3 and A_4, giving

$$\left.\begin{aligned} A_1 &= \kappa^3(\kappa - \cos\kappa\sin\kappa)/Q \\ A_2 &= \mu^2[\kappa\sinh\mu(\kappa\sin\kappa - 3\cos\kappa) + \mu\cosh\mu(\kappa\cos\kappa + 2\sin\kappa)]/Q \\ A_3 &= \mu^2\kappa(\kappa\cos\kappa\sinh\mu - \mu\cosh\mu\sin\kappa)/Q \\ A_4 &= \frac{\kappa\cos\kappa}{\kappa\cos\kappa - \sin\kappa} - \frac{\kappa(\mu^2+\kappa^2)(\kappa\cos\kappa\sinh\mu - \mu\cosh\mu\sin\kappa)(\kappa - \cos\kappa\sin\kappa)}{Q(\kappa\cos\kappa - \sin\kappa)} \end{aligned}\right\} \qquad (27)$$

where

$$\begin{aligned} Q = {} & (\mu^2+\kappa^2)(\sinh\mu - \mu\cosh\mu)(\kappa^2 - \kappa\cos\kappa\sin\kappa) - \\ & - 2\mu^2(\kappa\cos\kappa\sinh\mu - \mu\cosh\mu\sin\kappa)(\sin\kappa - \kappa\cos\kappa)\,. \end{aligned} \qquad (28)$$

Again the stiffness is given by

$$\Gamma = \frac{1}{4}\frac{\partial V}{\partial u_0}\Big/ u_0$$

but now

$$\Gamma_0 = \frac{Eh}{2(1+\nu)}\frac{b}{a} \qquad (29)$$

so that

$$\begin{aligned} \frac{\Gamma}{\Gamma_0} = {} & \frac{1}{9(1-\nu)}\left(\frac{h}{2a}\right)^2 \frac{\beta^2}{\kappa^2}\,[\mu^3(\mu^2+\kappa^2)A_1{}^2 \sinh 2\mu + 2\kappa^3 A_3{}^2\{2\kappa(\kappa^2+\mu^2) + (\mu^2-\kappa^2)\sin 2\kappa\} + \\ & + 2\mu^2\kappa^4 A_4{}^2 - 8\kappa^3\mu^2 A_3 A_4 \sin\kappa + \\ & + 4\kappa^2\mu^2(\kappa^2+\mu^2)A_1 \sinh\mu\{A_4 - 2A_3\cos\kappa\}] \end{aligned} \qquad (30)$$

and as before the relative stiffness is obtained from equation (1).

4. *Results.*

In presenting the results of the analysis in graphical form it is convenient to introduce the parameters Ω, $2b/d$ and d/h. These are related to the parameters κ, b/a, $2a/h$ by the simple formulae

$$\left.\begin{aligned} \Omega &= 180\kappa/\pi \\ 2b/d &= \frac{\kappa}{\sin\kappa}\frac{b}{a} \\ d/h &= \frac{\sin\kappa}{\kappa}\left(\frac{2a}{h}\right). \end{aligned}\right\} \qquad (31)$$

The shear stiffness Γ_f of a straight web of the same weight as the corrugation web and attached to the flanges at the same points is given by

$$\Gamma_f = \left(\frac{\kappa}{\sin \kappa}\right)^2 \Gamma_0. \tag{32}$$

The relative shear stiffness Θ is plotted against Ω in Figs. 3 to 6 inclusive for a number of values of $2b/d$ and d/h. The expected trends are displayed, namely that Θ decreases as Ω and d/h increase and as $2b/d$ decreases, Θ also decreases as the distance between the points of support increases. For comparison Γ_f/Γ_0 is also shown on each of these figures. Fig. 7 shows how typical displacements vary across the width of the panel and it is of interest to note that the bending of the panel takes place in opposite directions for the two positions of the points of support. This is due to the different positions of the shear centre in the two cases. The discontinuity in the slope of v when the web is supported at the crests of its waves is due to the abrupt change of curvature at that point and to the assumption of inextensibility.

NOTATION

x, y		Co-ordinate axes shown in Fig. 1
Ω, d, a, b, h, R		Dimensions of web shown in Fig. 1
ξ	=	x/a
η	=	y/a
u, v, w		Displacements of middle surface of web
κ	=	a/R
β	=	b/a
μ^2	=	$6(1-\nu)/\beta^2$
ϕ, ψ		Displacement functions defined by equations (7)
D, E, ν		Flexural rigidity, Young's modulus and Poisson's ratio
V		Strain energy of bending
Q		Defined by equation (28)
u_0		Applied shearing displacement
Γ		Shear stiffness of inextensional web
Γ_0		Shear stiffness of web continuously attached to the flanges
Θ		Relative shear stiffness
Γ_f		Shear stiffness of the equal-weight straight web

Roman superscripts refer to differentiation with respect to η.

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2	V. V. Novozhilov	*The theory of thin shells.* Noordhoff. 1959.

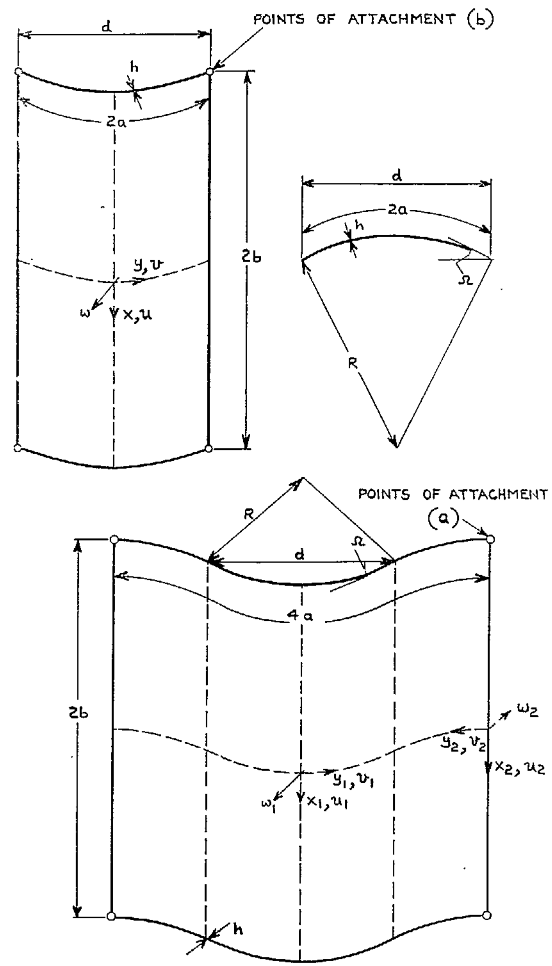


FIG. 1. Co-ordinate systems and notation.

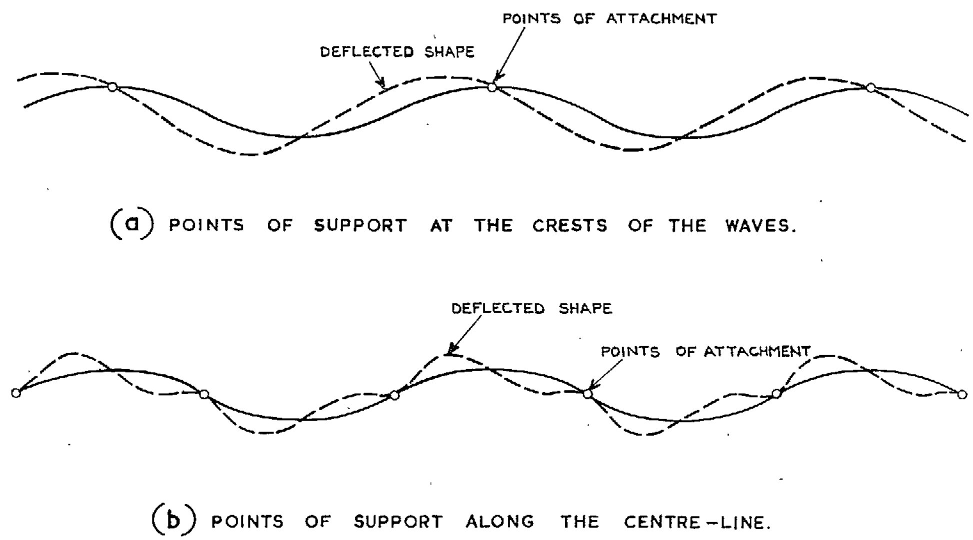


FIG. 2a and b. The two arrangements of the points of attachment of the web to the flanges and the form of the bending deflection.

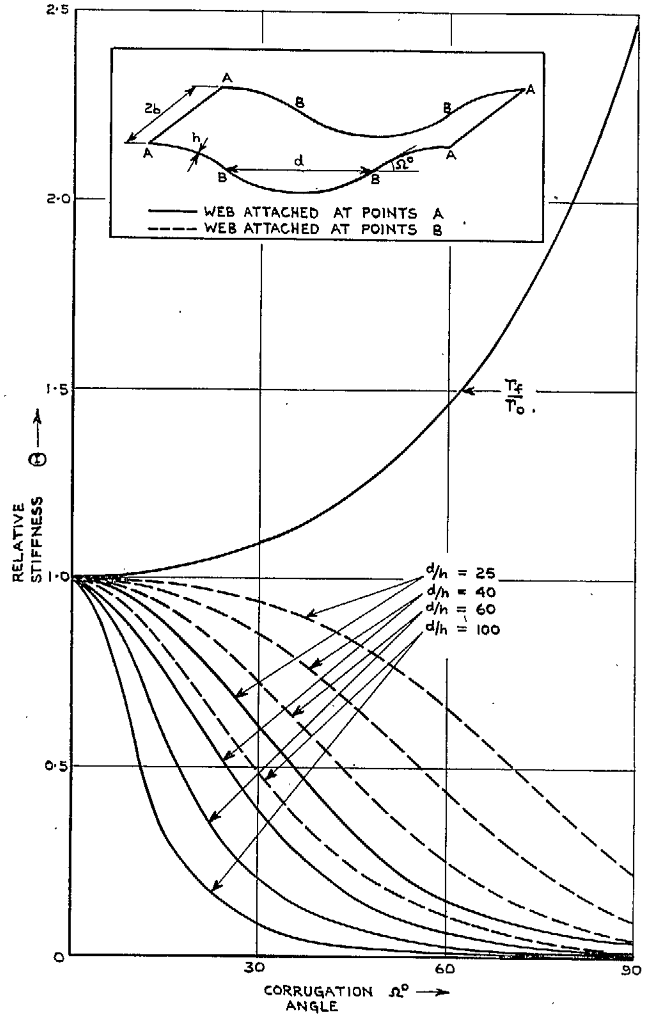


FIG. 3. The shear-stiffness variation when $2b/d = 2 \cdot 5$.

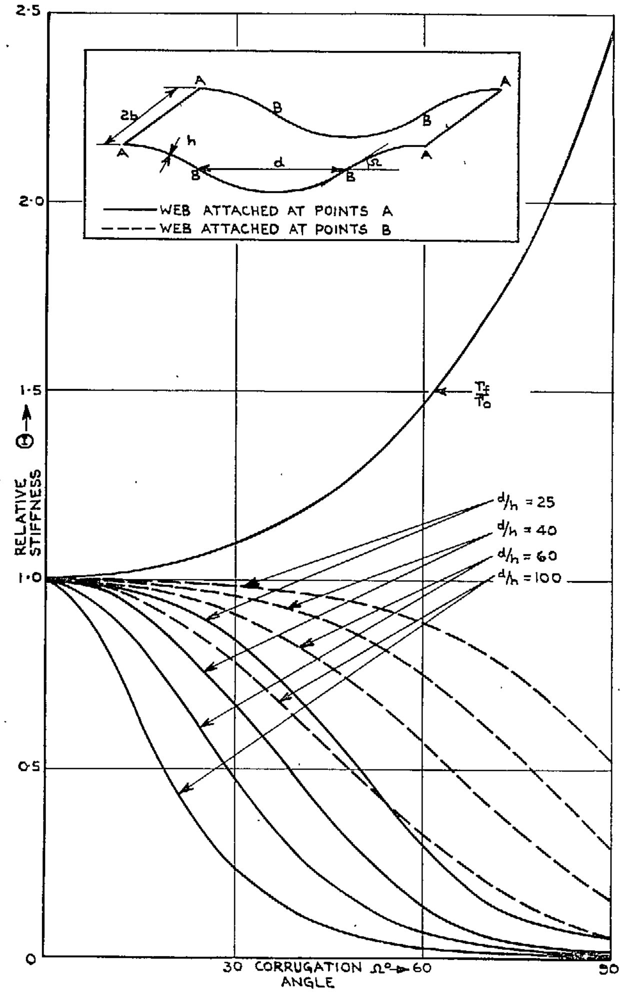


FIG. 4. The shear-stiffness variation when $2b/d = 5$.

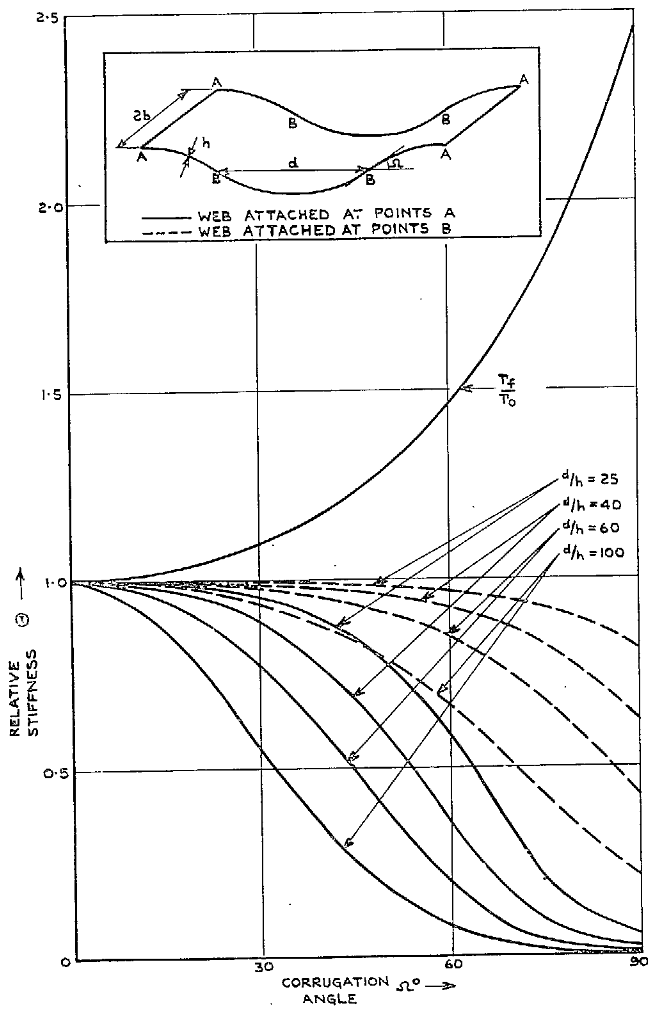


FIG. 5. The shear-stiffness variation when $2b/d = 10$.

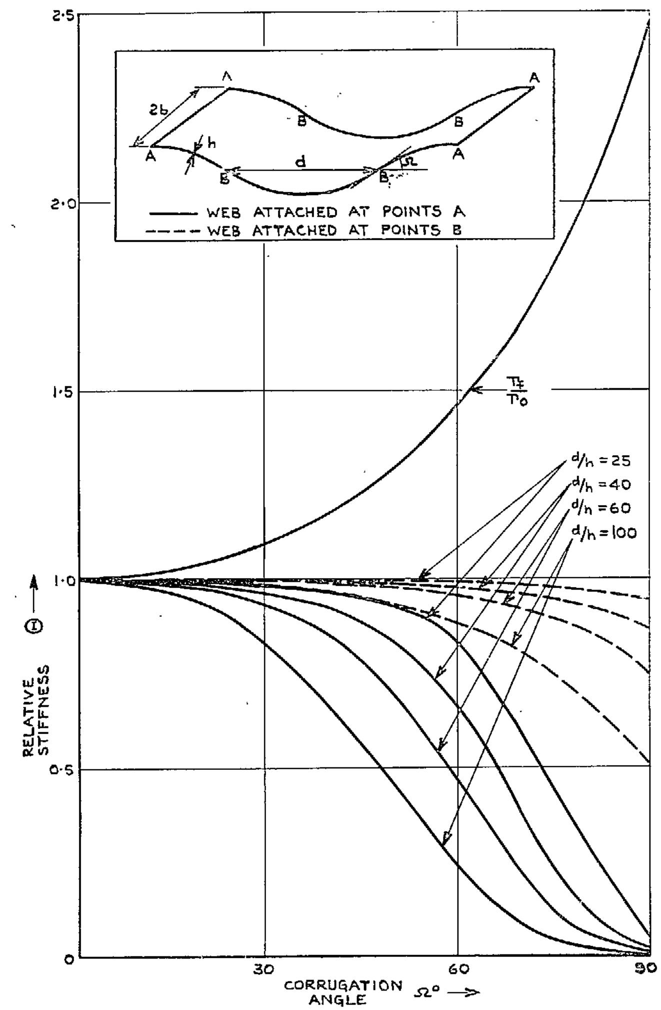
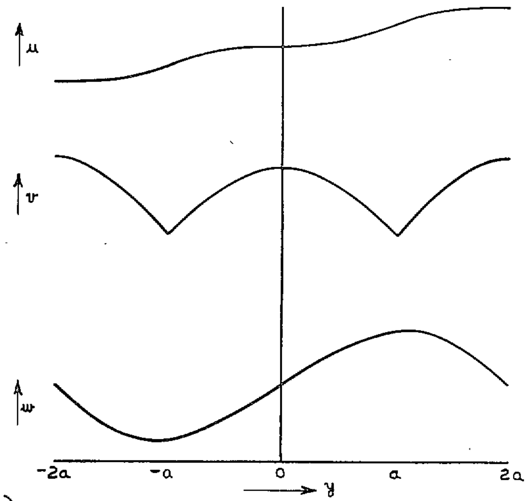
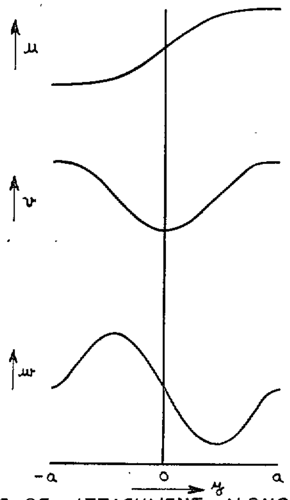


FIG. 6. The shear-stiffness variation when $2b/d = 20$.



(a) POINTS OF ATTACHMENT AT CRESTS OF WAVES.



(b) POINTS OF ATTACHMENT ALONG CENTRE-LINE.

FIGS. 7a and b. Some typical displacement variations.

(87840) Wt. 65/1418 K.5 11/63 Hw.

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