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The Stability of an Aircraft under Automatic Throttle Control and the Cross-Coupling Effects with Elevator Control

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Summary.

The stability of an aircraft with automatic throttle and elevator controls has been investigated theoretically using an analogue computer. Throttle application proportional to change in airspeed, incidence, or rate of pitch may provide damping of the long-period motion, but speed is shown to be the most suitable variable.

The control of an aircraft having negative static margin is considered and shown to require either the addition of an integral control on the throttle or a combination of throttle and elevator controls.

Where it is required to control height by means of the elevator, some aircraft flying under certain conditions can only be adequately stabilised by means of an automatic throttle control.

1. *Introduction.*

The longitudinal stability of various aircraft with automatic throttle control has been investigated theoretically. Several types of throttle control have been studied.

The main effects of elevator controls are known[1], but are summarised in this report. When a height lock is added to an elevator control an unstable mode is often introduced. This instability may be removed by the addition of a speed control on the throttle. Each type of throttle control has been shown to have a different effect on the longitudinal stability, when operated in conjunction with a typical elevator control. The cross-coupling effects of the two types of control are not always beneficial.

Use has been made of a Shorts' analogue computer to substantiate the theoretical analysis, results being given as actual responses in pitch, height, etc. to a horizontal step gust.

* Replaces R.A.E. Tech. Note No. I.A.P. 1079—A.R.C. 21,040.

2. *Aircraft and Control Equations.*

The aircraft equations of motion in non-dimensional form are:

$$\hat{D}\hat{u} = x_u\hat{u} + x_w\hat{w} - k\theta + T \tag{1}$$

$$\hat{D}(\hat{w} - \theta) = z_u\hat{u} + z_w\hat{w} \tag{2}$$

$$\hat{D}q = -\kappa\hat{u} - \chi\hat{D}\hat{w} - \omega\hat{w} - \nu q - \delta\eta \tag{3}$$

where

$$\hat{D} = \frac{d}{d\tau}; \qquad \tau = \frac{t}{\hat{t}}$$

$$\hat{t} = \frac{m}{\rho SV}; \qquad q = \frac{d\theta}{d\tau}$$

$$\kappa = -\mu_1\frac{m_u}{i_B}; \qquad \chi = -\mu_1\frac{m_{\dot{w}}}{i_B}$$

$$\omega = -\mu_1\frac{m_w}{i_B}; \qquad \nu = -\frac{m_q}{i_B}$$

$$\delta = -\mu_1\frac{m_\eta}{i_B}; \qquad k = \tfrac{1}{2}C_L$$

and T is a small change of thrust produced by the automatic control.

The elevator-control equation used was:

$$\eta = G_\theta\theta + G_q q + G_h\hat{h} + G_{\bar{h}}\int\hat{h}\,d\tau \tag{4}$$

where G_θ, G_q, G_h, $G_{\bar{h}}$ are control gearings, and the height deviation $\hat{h}$ is related to θ and $\hat{w}$ by the equation

$$\hat{D}\hat{h} = \theta - \hat{w}. \tag{5}$$

The throttle equation was of the form:

$$T = A_u\hat{u} + A_w\hat{w} + A_\theta\theta\ \ldots \text{ etc.} \tag{6}$$

A_u, A_w, A_θ etc. are control gearings.

The meaning of all symbols other than those defined can be found in R. & M. 1801[2].

Since all equations in this report are in non-dimensional form, $\hat{u}$, $\hat{w}$, etc. will be written as u, w, etc.

The stability polynomial for the aircraft with controls fixed is a quartic. With the addition of control laws, the order of the equation may be raised.

Table 2 gives the contributions from the control terms to the coefficients K_i of the stability equation

$$\sum K_i\lambda^i = 0.$$

Thus for the control law $\eta = G_\theta\theta + G_h h$, the coefficients would be

$$\begin{aligned}
K_4 &= K_{04}\\
K_3 &= K_{03}\\
K_2 &= K_{02} + \delta G_\theta\\
K_1 &= K_{01} + N_1\delta G_\theta\\
K_0 &= K_{00} + P_1\delta G_\theta - z_w\delta G_h,\\
K_{-1} &= (P_1 - R_1)\delta G_h,
\end{aligned}$$

where K_{0i} are the stability coefficients for the basic aircraft.

The contributions from any derivative or integral of the control variable are obtained by moving the value in the table up or down one square. For example, the control law $\eta = G_q q$ would add δG_q, $N_1 \delta G_q$ and $P_1 \delta G_q$ to the coefficients K_3, K_2, K_1 respectively.

It should be noted that the coefficient K_3 is equal to minus the sum of the roots of $\sum K_i \lambda^i = 0$, and since negative real parts of roots correspond to positive damping it is also equal to the sum of the damping terms. Thus from Table 2 it can be seen that examples of controls that actually increase the total damping of the system are: elevator/rate of pitch, (δG_q); throttle/speed, (A_u); throttle/vertical acceleration, ($A_{\dot{w}}$). A control which does not add to K_3 cannot affect the total system damping, but will re-distribute the damping between the several modes of motion.

3. *Basic Principles of Elevator Control.*

The main features of automatic elevator control are outlined below, but a more complete description has been given by Hopkin and Dunn[1].

3.1. $\eta = G_\theta \theta$.

This control does not add any damping to the system. The effect of a G_θ gearing is to increase the frequency of the short-period motion, and decrease the frequency of the long-period motion, damping being transferred from the short to the long-period oscillation (Fig. 1).

3.2. $\eta = G_q q$.

This control adds damping to the system mainly to the short-period motion.

3.3. $\eta = G_\theta \theta + G_h h = G_\theta \theta + G_h \int (\theta - w) d\tau$.

This height control has the effect of increasing the order of the stability equation and thus introducing a new mode of motion. The damping of the long-period oscillation is reduced and the frequency increased (Fig. 2). There is hardly any effect on the short-period motion. The new mode may be stable or unstable depending on the sign of the combination of aircraft derivatives, $(x_u z_w - x_w z_u) + z_u C_L/2$, i.e., $P_1 - R_1$. In Fig. 2, $P_1 - R_1$ is negative and therefore there is a divergence. The stability criterion just quoted is only strictly true if $z_\eta = 0$.

4. *Throttle Control.*

The three basic throttle controls are throttle proportional to speed, incidence and attitude or any derivative of the above three variables. Throttle proportional to errors in speed, incidence, or rate of pitch[3] increases the damping of the long-period motion (Figs. 3, 4), but only speed control actually adds damping to the system; throttle control has little effect on the short-period motion.

Lag on the throttle control of up to about five seconds time constant also has very little effect on the stability of the aircraft. This is shown in Fig. 5 which gives the effect of different lags on a speed-controlled aircraft.

5. *Comparison of Various Throttle Controls.*

The damping power of the three long-period throttle controls A_u, A_w, A_q depends approximately on certain combinations of aircraft derivatives. The analysis is simplified if we assume that the long-period damping is determined mainly by the ratio of the two coefficients K_1, K_2 of the stability equation. That this is reasonable can be seen from the following examples.

(86574) A*

The stability polynomial

$$\lambda^4 + K_3\lambda^3 + K_2\lambda^2 + K_1\lambda + K_0 = 0$$

usually factorises into two quadratics $(\lambda^2+A\lambda+B)(\lambda^2+a\lambda+b)$ where B is large, A is often large, and a, b are small. The first factor corresponds to a well-damped short-period motion and the second to a long-period motion. Comparing coefficients of the stability polynomial with those obtained from the product of the quadratic factors, we have

$$\begin{aligned} K_3 &= A + a \\ K_2 &= B + b + Aa \\ K_1 &= Ba + Ab \\ K_0 &= Bb\,. \end{aligned}$$

It is seen that K_0 and K_1 will be small compared with K_2 and K_3, and a first approximation to the factors will be given by $K_3 = A_1$, $K_2 = B_1$, $K_1 = B_1a_1$, $K_0 = B_1b_1$. The coefficient K_2 is said to be pivotal[4] because the approximate factors are

$$(\lambda^2+K_3\lambda+K_2)\left(\lambda^2+\frac{K_1}{K_2}\lambda+\frac{K_0}{K_2}\right).$$

A better approximation for a is obtained from the relation

$$K_1 = B_1a_2 + A_1b_1 \approx K_2a_2 + K_3K_0/K_2\,,$$

i.e.,

$$a_2 = \frac{1}{K_2}\left[K_1 - K_3K_0/K_2\right]$$

and still better values obtained by writing

$$\begin{aligned} A_2 &= K_3 - a_2\,, \\ B_2 &= K_2 - b_1 - A_2a_2\,, \text{ etc.} \end{aligned}$$

However, it is not necessary to proceed any further in order to see that the major contribution to the long-period damping coefficient a is equal to K_1/K_2 provided K_2 is sufficiently large. It is not essential that K_3 be large as well. Two examples are given to illustrate this:

	Example 1	Example 2
K_3	3·2564	2·6815
K_2	113·696 (highly pivotal)	9·5469 (pivotal)
K_1	2·5133	1·2253
K_0	−0·1467	1·0742

The approximate and accurate values of a and b are compared in the following table:

	a_1	a_2	a	b_1	b
Example 1	0·0221	0·0221	0·0222	−0·00129	−0·00130
Example 2	0·1283	0·0967	0·1007	0·1125	0·1171

Throttle-control terms are important in K_1 (Table 2) but are relatively unimportant in K_2, so that the effects on damping of the three types of control are almost directly proportional to the K_1 contributions, which are $-A_u M_1$, $A_w M_2$ and $-A_q S$. M_1, M_2 and S are functions of the aircraft derivatives: M_1 is proportional to the standard manoeuvre margin, and S to the static margin, while M_2 is proportional to another kind of manoeuvre margin suggested by Hopkin. These three quantities are

$$M_1 = (\omega - \nu z_w),$$

$$M_2 = (\kappa - \nu z_u),$$

$$S = (\kappa z_w - \omega z_u).$$

Thus, by using the known values of the aircraft derivatives, the relative values of A_u: A_w: A_q can be determined to give the same amount of long-period damping. Examples are given in Figs. 3 and 4 for different aircraft and flight conditions.

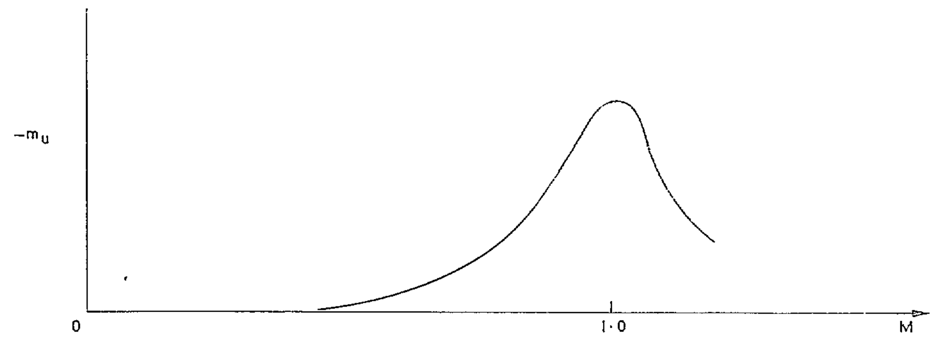
With 'normal' values of derivatives the two manoeuvre margins and the static margin are all positive, and therefore the coefficient K_1 and the damping will be increased by negative values for A_u and A_q, and positive A_w.

M_1, the manoeuvre margin, is invariably positive for subsonic aircraft, but it is possible for it to be negative for supersonic aircraft in the subsonic condition.

M_2 is usually positive, but could possibly be negative for large negative $\kappa(= -\mu_1 m_u/i_B)$.

The static margin S is also usually positive, but like M_1 may be negative for supersonic aircraft in the subsonic condition. This is because m_w may have to be made positive ($\omega < 0$) in order to avoid large negative values at supersonic speed. Quite apart from this the static margin may be negative because m_u is large (negative); so that S will change sign if $z_w \kappa$ becomes larger than $z_u \omega$ (Aircraft 2 and 3).

This may happen at transonic speeds (*see* diagram).



Typical relation between m_u and Mach number.

Thus to keep the increment to K_1 of the same sign over the flight range, the sign of the throttle-control gearing A_q may have to change. Also, since the amount of long-period damping depends approximately on $-A_q(\kappa z_w - \omega z_u)$ and the value in the bracket goes from positive to zero to negative, and probably to zero and positive again, the value of the A_q gearing would have to vary considerably and also change in sign, over a comparatively short flight range. Thus it would appear that a rate of pitch control on the throttle would not be suitable for continuous long-period damping.

Examples of the relative strengths of the three control gearings for different flight conditions of the same aircraft are:

Aircraft	A_u	:	A_w	:	A_q
(1)	1	:	$13 \cdot 13$	:	$2 \cdot 26$
(2)	1	:	$4 \cdot 85$	:	$-198 \cdot 6$
(3)	1	:	$13 \cdot 92$	:	$-61 \cdot 61$

In order to get the approximate equivalent damping contributions to the long-period motion it can be seen from the above Table that it is necessary to have an extremely large gearing A_q {Aircraft (2), (3)}. This means that we not only affect coefficient K_1 but also make a large contribution to K_2, since $-RA_q$ is now comparable with M_1 (Table 2). This will increase the frequency of the short-period motion and thus the damping per cycle is reduced. The resulting high-frequency oscillation can be seen in Fig. 6.

The derivatives for the aircraft 1, 2, 3, etc. are given in Table 1.

For aircraft 2, 3 with negative static margin, i.e., coefficient K_0 negative, the uncontrolled long-period motion is either an unstable oscillation or consists of two exponential modes one of which must be a divergence. None of the three throttle damping controls will make the system stable, since no increments of A_u, A_w or A_q are added to coefficient K_0, which must always remain negative. One remedy would be to use the integral of the control variable in combination with the basic control itself, e.g.,

$$T = A_u u + A_{\bar{u}} \int u \, d\tau .$$

The strength of the integral term necessary will depend on how negative the static margin is. For the cases illustrated in Figs. 6 and 7 the static margins are $-0 \cdot 0398$ and $-0 \cdot 1521$ respectively, and the curves show that a compromise must be made between effective control over the divergent tendency caused by the negative static margin and a conflicting tendency for the integral term to introduce a long-period oscillation.

If A_u is kept constant and $A_{\bar{u}}$ increased to a very large value, the stability quartic has two quadratic factors:

$$(\lambda^2 + L_1\lambda + M_1)\,[\lambda^2 - (A_u + x_u)\lambda - A_{\bar{u}}] = 0 ,$$

i.e., the basic highly damped short-period oscillation $(\lambda^2 + L_1\lambda + M_1)$, and a barely stable very-high-frequency oscillation.

If, on the other hand, the throttle gearings A_u and $A_{\bar{u}}$ are both made large together, but having a fixed ratio (time constant), $\tau_1 = A_u/A_{\bar{u}}$, the stability quartic splits up approximately as follows:

$$(\lambda^2 + L_1\lambda + M_1)\left(\lambda + \frac{1}{\tau_1}\right)(\lambda - A_{\bar{u}}\tau_1) = 0 ,$$

i.e., the basic short period again, and two subsidences, one heavily damped and the other lightly damped. The above two approximations indicate that too large an increase in $A_{\bar{u}}$ without a corresponding increase in A_u will lead to a poorly damped oscillation.

An alternative suggestion for counteracting the negative components of coefficient K_0 is given in the next paragraph.

6. *Cross-coupling Effects of Various Throttle Controls with Basic Elevator Control.*

An elevator control proportional to change in angle of pitch ($\eta = G_\theta \theta$) could overcome the negative static-margin effect, but with the addition of a height lock ($\eta = G_h h$), and with z_η assumed zero, $(P_1 - R_1)$ still determines the sign of the last coefficient of the stability equation (*see* Table 2).

$$P_1 - R_1 = x_u z_w - x_w z_u + z_u \frac{C_L}{2}.$$

This combination of derivatives is often positive but is usually very small, and with relatively little change in derivatives it can become negative, e.g., aircraft (1) and (2). Since one of the conditions for positive stability is that the last coefficient be positive, an elevator control with height lock will always produce an unstable mode if $P_1 - R_1$ is negative (Fig. 2). The condition $P_1 = R_1$ is almost identical with the condition for minimum drag in steady straight and level flight ~~suggested by Hopkin~~.

To check the cross-coupling effects of a basic elevator-pitch attitude control and a throttle-speed control, the computer was used to give responses in speed and pitch for various values of G_θ and A_u (Fig. 8).

$$\eta = G_\theta \theta$$
$$T = A_u u.$$

Increasing A_u damped the long-period motion without materially affecting the short, while increasing G_θ improved the long-period mode by subtracting damping from the quick oscillation. The cross-coupling terms due to the product $A_u G_\theta$ were beneficial, adding increments to coefficients K_1 and K_0.

The cross-coupling of the throttle-speed control with any of the elevator controls appears to be beneficial. Fig. 9b gives the aircraft response in pitch with an elevator control only. Fig. 9c shows the effect of a height lock, giving a divergence ($P_1 - R_1$ being negative), i.e., the final coefficient K_{-2} of the stability equation being negative. Immediately a throttle-speed control is introduced, the cross-coupling contribution $A_u G_{\bar{h}}$ swamps the small negative component and makes the coefficient K_{-2} positive. This has the effect of making the extra mode introduced by the integral height control a subsidence instead of a divergence. By inspecting Table 2 it can be seen that all the cross-products of elevator-control terms with A_u (speed control) are positive and associated with the derivative z_w.

To show the variations in the cross-coupling effects of the three long-period damping throttle controls, A_u, A_w, A_q, with elevator control and height lock, 'equivalent' values of the throttle gearings were used, i.e.,

$$\eta = G_\theta \theta + G_h h + G_{\bar{h}} \int h \, d\tau$$

with

$$T = A_u u$$

or

$$T = A_w w$$

or

$$T = A_q q.$$

The cross-coupling terms are shown in Table 2 and examples are given in Figs. 10, 11 for which aircraft the value of $(P_1 - R_1)$ is negative. Thus the speed control gives a damped response in height and pitch, the incidence control (A_w) a damped response, but less damped, while the rate of pitch control gives a damped oscillation and a divergence. The explanation for these variations in stability comes from the cross-couplings terms $A_u G_{\bar{h}}$ etc. $A_u G_{\bar{h}}$ is positive and cancels out the negative $P_1 - R_1$ of K_{-2}, associated with the integral height gearing. The $A_w G_{\bar{h}}$ cross-coupling term is beneficial (positive), but is associated with the relatively small derivative z_u, as opposed to the $A_u G_{\bar{h}}$ term associated with z_w. Finally, the $A_q G_{\bar{h}}$ term is negative, reducing the coefficient K_{-1}, but still leaving the final coefficient K_{-2} negative.

The above controls do not add damping to the short-period motion, the remedy for any lack of short-period damping being an elevator control proportional to rate of pitch. Fig. 11 gives the aircraft and pitch response with the addition of a G_q control as compared with Fig. 10.

Although elevator proportional to rate of pitch is necessary for short-period damping, it also has an effect on the frequency and damping of the long period. This is apparent for the large q control gearing ($G_q = 2 \cdot 0$) of Fig. 12. In the practical case of an autostabiliser which incorporates a 'high-pass filter' the elevator signal will be proportional to $\{\tau_1 D/(1+\tau_1 D)\} q$. Comparison of Figs. 12 and 13 shows that an unsuitable choice of time constant τ_1 may introduce instability. There is scope for further investigation into this particular problem, but it is not intended to deal with it in this report.

Two other flight conditions with both elevator and throttle controls are shown in Fig. 14:

(*a*) with negative static margin and negative $P_1 - R_1$

(*b*) with negative static margin and positive $P_1 - R_1$.

Both are completely stable when a combination of elevator and speed-throttle controls are used. Condition (*a*) is unstable if either set of controls is used separately, while condition (*b*) is unstable with throttle control alone.

7. *Conclusions.*

Automatic throttle controls mainly affect the long-period motion of the aircraft. Moving the throttle proportional to the speed of the aircraft seems to be the most suitable of the various throttle controls, the other types sometimes needing large changes in gearing to keep the same amount of damping over the flight range. The unstable mode introduced by negative static margin cannot be stabilised by basic throttle controls themselves, but addition of an integral term will remove the instability. However, elevator-pitch attitude control seems a better method of counteracting negative static margin.

Analogous results are found when automatic height control is obtained by elevator action alone. A new mode of motion is introduced which may or may not be stable depending only on the sign of a certain combination of aircraft derivatives ($P_1 - R_1$). When this mode is unstable it cannot be made stable by changing the normal elevator-control gearings, but adding an integral term can be effective. Alternatively the instability can be overcome by throttle-speed control.

The combination of a throttle-speed control and an elevator control (elevator proportional to pitch attitude, rate of pitch and height deviation) has an appreciable stabilising effect on the aircraft motion even for aircraft with both negative static margin and negative ($P_1 - R_1$). The other throttle controls (proportional to rate of pitch or incidence) with elevator control are not so powerful in their stabilising effect and may even be de-stabilising in some flight conditions.

LIST OF SYMBOLS

A_u, A_w, A_θ etc.		Non-dimensional throttle-control gearings
$D(\hat{D})$	$=$	$\dfrac{d}{d\tau}$
G_θ, G_h etc.		Non-dimensional elevator-control gearings
$h(\hat{h})$		Non-dimensional height deviation
k	$=$	$\frac{1}{2}C_L$
K_4, K_3 etc.		Coefficients of stability equation
T		Incremental change in thrust
t		Time in true seconds
$\hat{t}$	$=$	$m/\rho VS$, unit of time in non-dimensional form
τ	$=$	$\dfrac{t}{\hat{t}}$, time in air-seconds
τ_1		Non-dimensional lag time constant
$u(\hat{u})$		Non-dimensional speed error along x axis
$w(\hat{w})$		Non-dimensional component of speed along z axis
θ		Pitch deviation of aircraft
η		Elevator angle from equilibrium position

$$\left.\begin{aligned} \omega &= -\mu_1 m_w/i_B \\ \nu &= -m_q/i_B \\ \chi &= -\mu_1 m_{\dot{w}}/i_B \\ \kappa &= -\mu_1 m_u/i_B \\ \delta &= -\mu_1 m_\eta/i_B \end{aligned}\right\} \text{Portmanteau functions of pitching-moment derivatives}$$

$$\mu_1 = \frac{m}{\rho Sl}$$

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TABLE 1

Aerodynamic Data and Derivatives

Aircraft	(1)	(2)	(3)	(4)
Altitude	0	40,000	40,000	40,000
Vi (kts)	172	256	313	214
M	0·26	0·9	1·1	0·75
C_L	0·55	0·242	0·165	0·274
μ_1	40·5	164	164	69
$\hat{t}$	2·44	3·31	2·71	3·09
x_u	−0·0585	−0·0115	−0·0284	−0·020
x_w	+0·0578	+0·0085	−0·0065	+0·011
z_u	−0·55	−0·327	−0·1	−0·365
z_w	−1·403	−1·61	−1·618	−2·56
m_u	0	−0·0276	−0·017	+0·00123
m_w	−0·061	−0·134	−0·237	−0·0282
$m_{\dot{w}}$	0	0	0	−0·00457
m_q	−0·428	−0·508	−0·565	−0·45
m_η	−0·204	−0·233	−0·244	−0·24
i_B	0·35	0·35	0·35	0·1
κ	0	13	8	0·849
ω	7·1	63	111	19·5
χ	0	0	0	3·15
ν	1·22	1·45	1·61	4·5
δ	24	109	114	165·6
$P_1 - R_1$	−0·0377	−0·0182	0·0373	0·1052

TABLE 2

Coefficients of Stability Equation $\sum K_i \lambda^i = 0$

	Basic Aircraft	Automatic-Control Terms									
	1	δG_θ	δG_h	$-A_u$	A_w	$-A_q$	$-\delta G_\theta A_u$	$\delta G_\theta A_w$	$-\delta G_h A_u$	$\delta G_h A_w$	$-\delta G_h A_q$
K_4	K_{04}										
K_3	K_{03}			1							
K_2	K_{02}	1		L_1	$-z_u$	R					
K_1	K_{01}	N_1		M_1	M_2	S	1				
K_0	K_{00}	P_1	$-z_w$				$-z_w$	$-z_u$			z_u
K_{-1}			$P_1 - R_1$						$-z_w$	$-z_u$	

Control equations

$$\eta = G_\theta \theta + G_q q + G_h h + G_{\bar{h}} \int h \, d\tau$$

$$T = A_u u + A_w w + A_q q$$

Aircraft equations

$$Du = x_u u + x_w w - k\theta + T$$

$$D(w - \theta) = z_u u + z_w w$$

$$D^2\theta = -\kappa u - \chi Dw - \omega w - \nu q - \delta\eta$$

$$L_1 = \nu + \chi - z_w$$

$$M_1 = \omega - \nu z_w$$

$$M_2 = \kappa - \nu z_u$$

$$N_1 = -x_u - z_w$$

$$P_1 = x_u z_w - x_w z_u$$

$$R_1 = -k z_u$$

$$R = -(\kappa + z_u \chi)$$

$$S = \kappa z_w - \omega z_u$$

$$K_{04} = 1$$

$$K_{03} = L_1 - x_u$$

$$K_{02} = M_1 - x_u L_1 - x_w z_u$$

$$K_{01} = -x_u M_1 + x_w M_2 + kR$$

$$K_{00} = kS$$

To illustrate the use of the Table, we write

$$K_2 = (K_{02}) + \delta G_\theta - L_1 A_u - z_u A_w - R A_q$$

$$K_{-1} = (P_1 - R_1)\delta G_h + z_w \delta G_h A_u - z_u \delta G_h A_w \text{ etc.}$$

NOTE

The following figures have all been reduced to half linear size. The various scales should be adjusted accordingly.

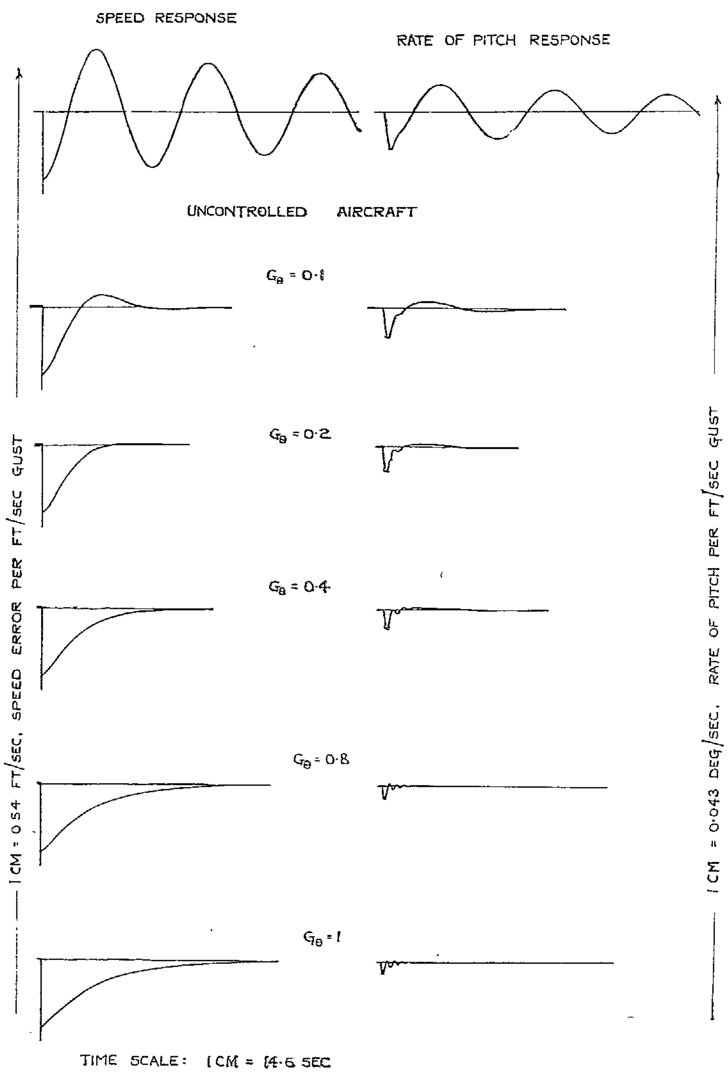

FIG. 1. Response of Aircraft (1) to a horizontal gust; pitch control on elevator ($\eta = G_\theta \theta$).

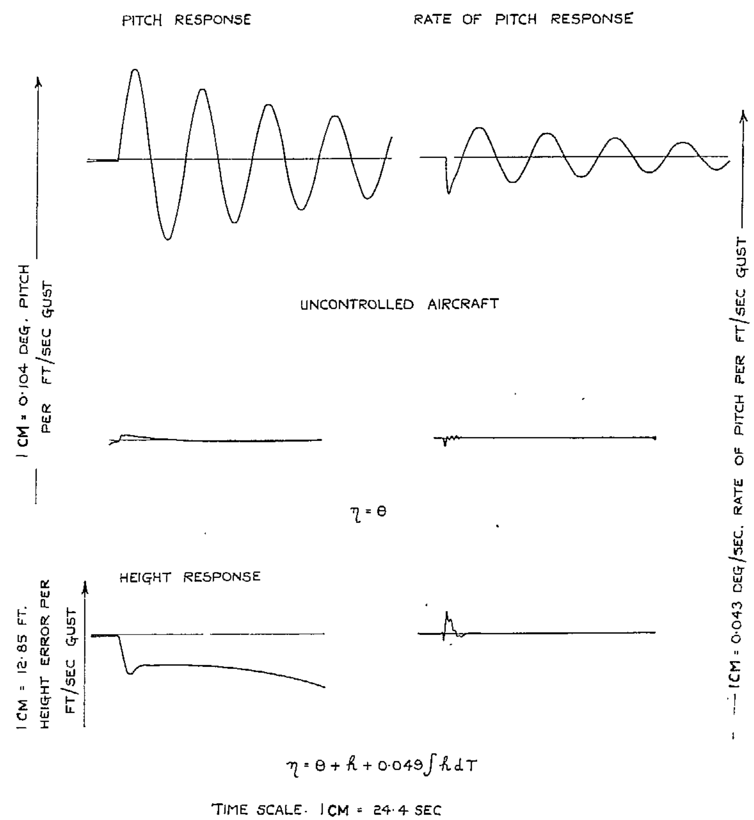

FIG. 2. Aircraft (1), effect of height lock on elevator-pitch control.

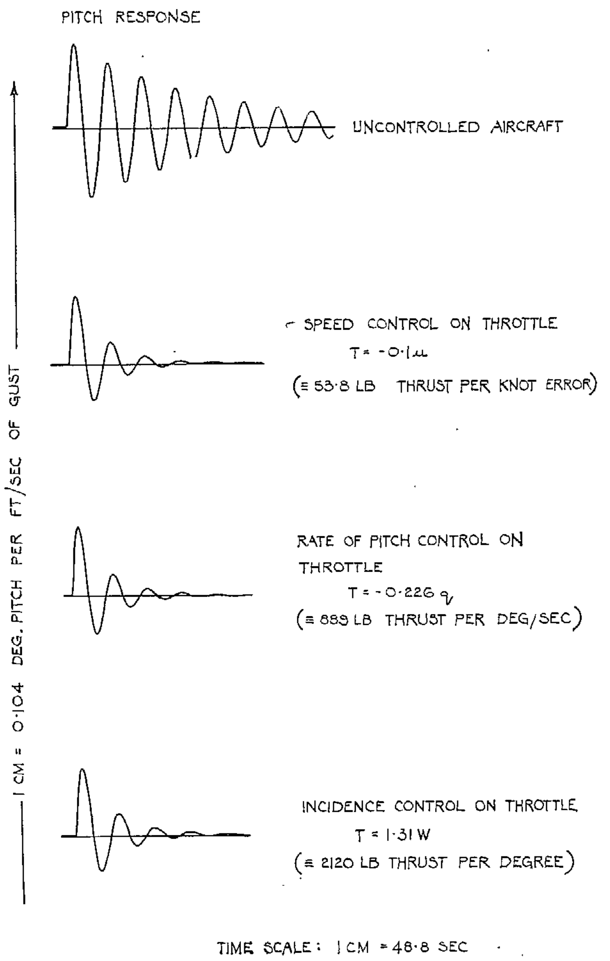

Fig. 3. Pitch response of Aircraft (1) for three different types of throttle control.

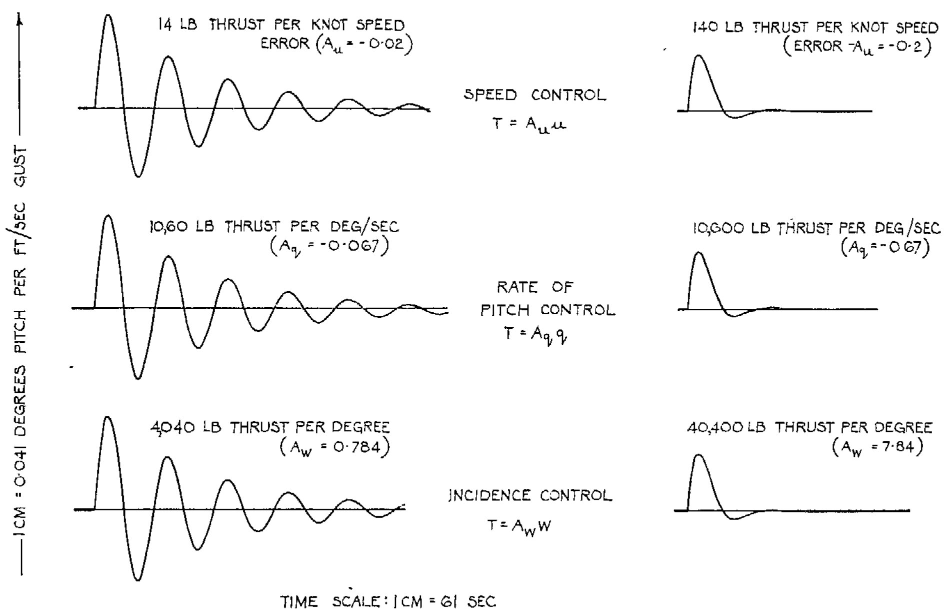

Fig. 4. Pitch responses for three different types of throttle control [Aircraft (4)].

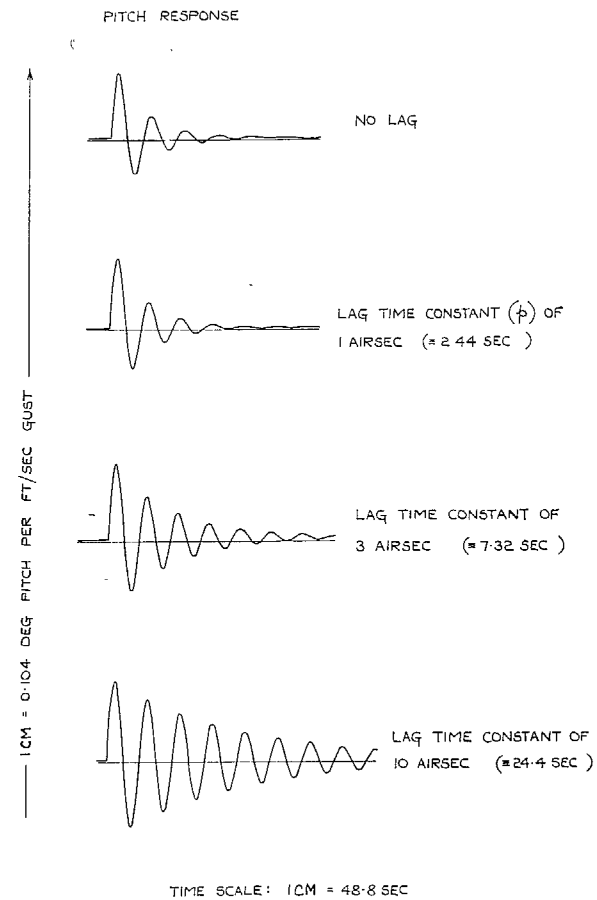

FIG. 5. Effect of lag on throttle-speed and integral speed control.

$$\left([1+pD]T = A_u\left[u + \frac{1}{T_i}\int u\,d\tau\right];\ \begin{array}{l} A_u \equiv 53\cdot 8\,\text{lb/knot} \\ T_i \equiv 40\,\text{sec.} \end{array}\right)$$

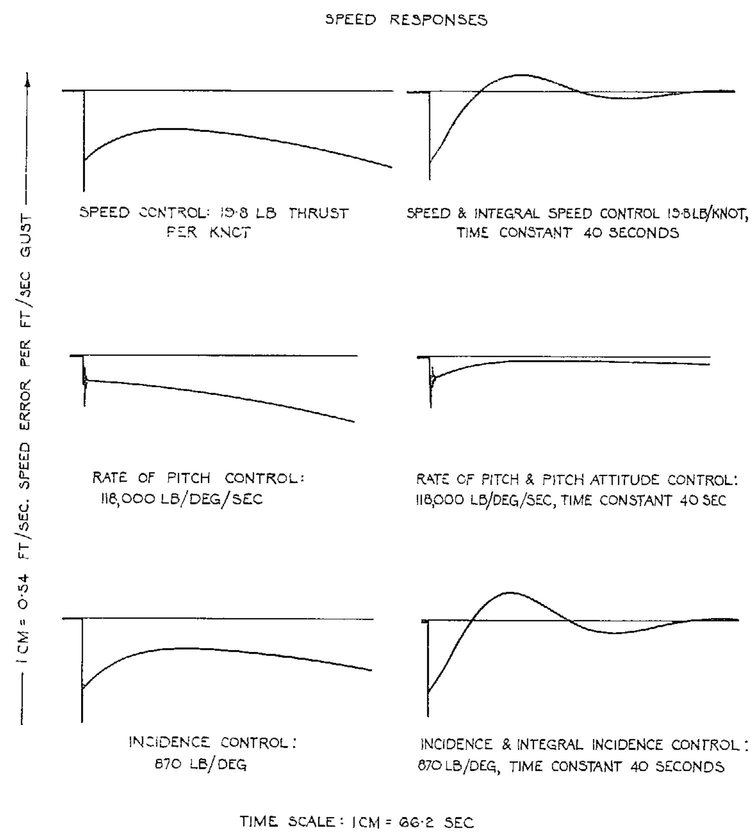

FIG. 6. Effect of addition of integral controls to the three types of throttle control [Aircraft (2)].

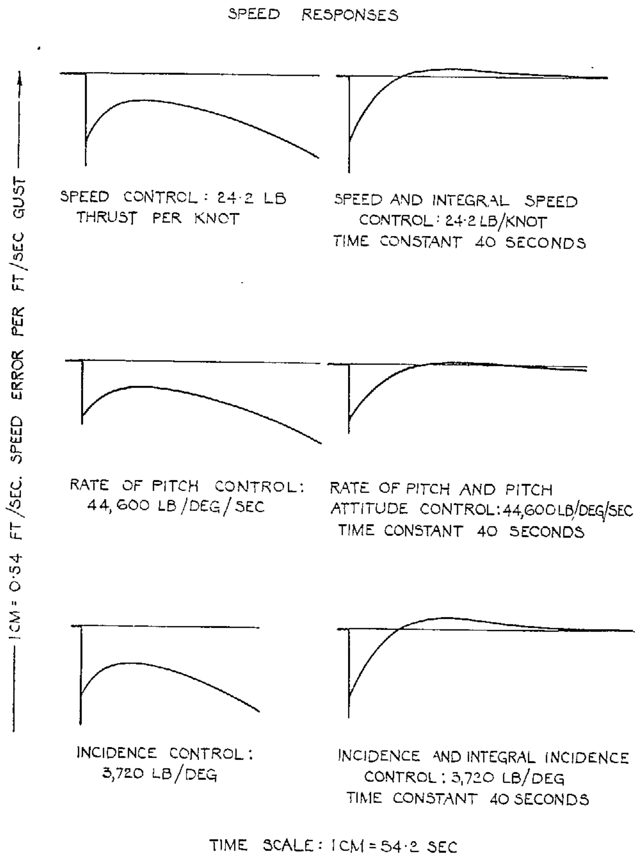

FIG. 7. Effect of addition of integral controls to the three types of throttle control [Aircraft (3)].

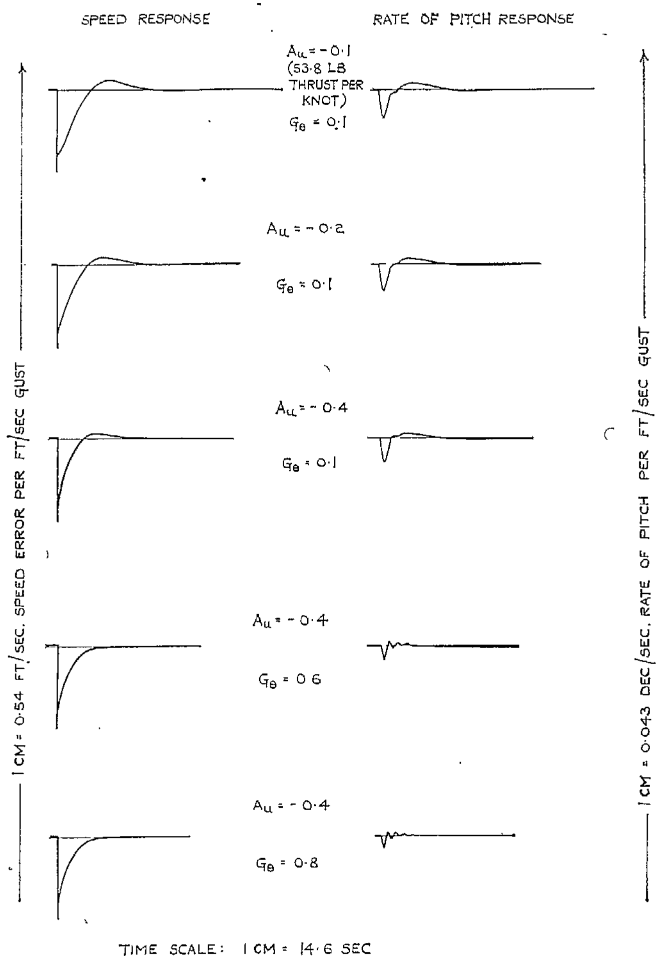

FIG. 8. Response of Aircraft (1), pitch control on elevator, speed control on throttle ($\eta = G_\theta \theta$; $T = A_u U$).

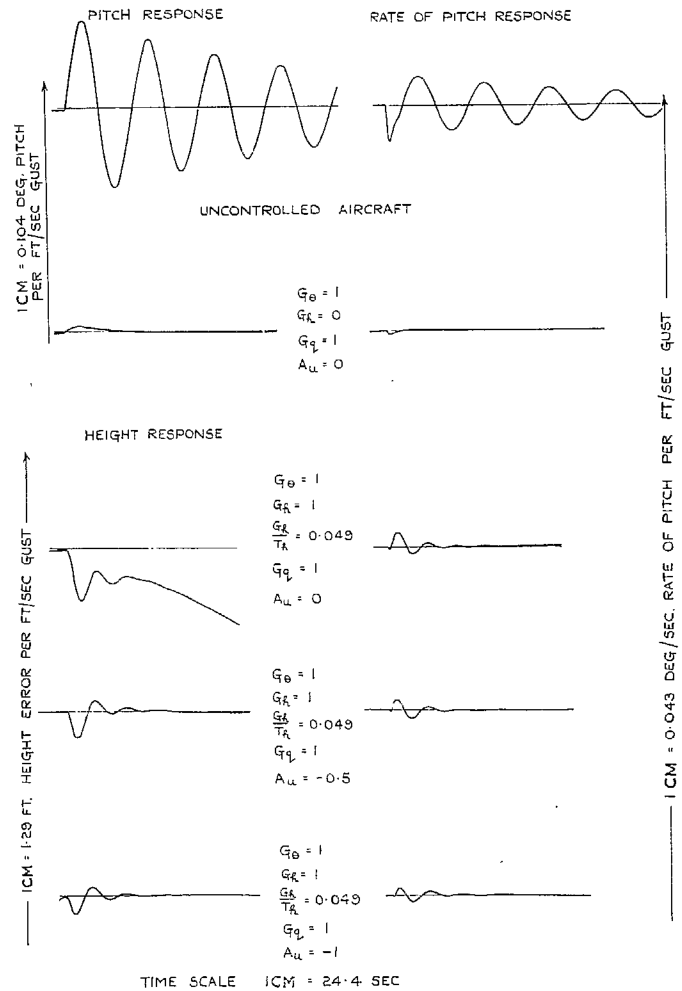

FIG. 9. Response of Aircraft (1) showing effect of throttle-speed control on elevator height lock.

$$\left\{\eta = G_\theta \theta + G_q q + G_h \left(h + \frac{1}{T_h} \int h \, d\tau\right) ; \; T = A_u u\right\}$$

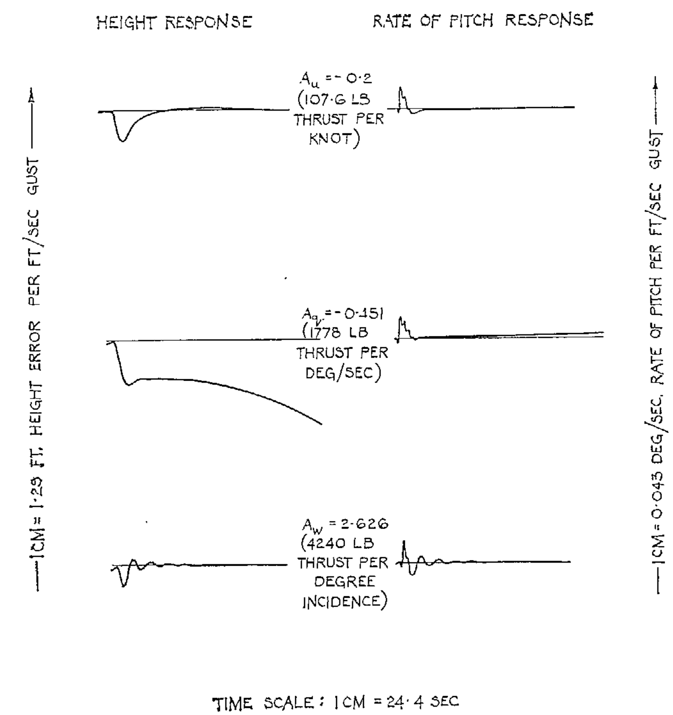

FIG. 10. Aircraft (1), comparison of three types of throttle control combined with elevator-pitch attitude control and height lock.

$$(\eta = \theta + h + 0{\cdot}049 \int h \, d\tau \; ; \; T = A_u u \text{ or } A_q q \text{ or } A_w w)$$

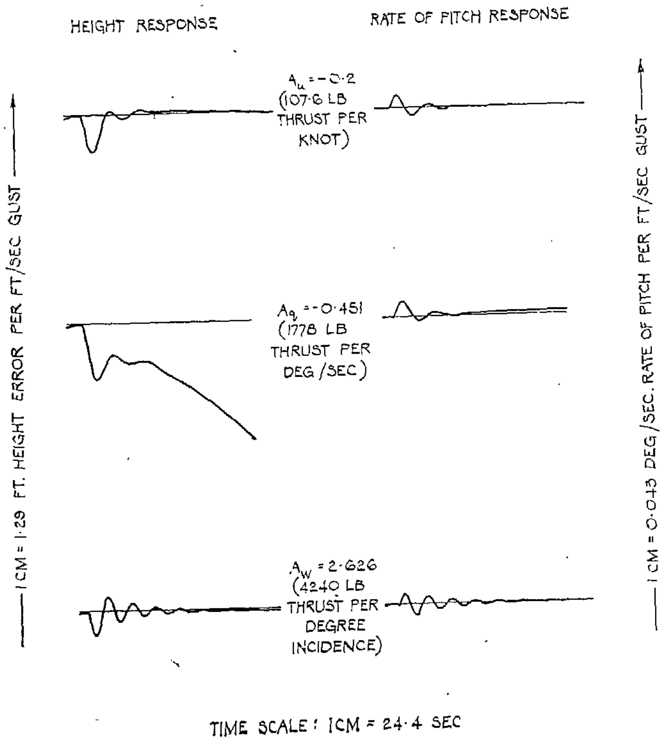

FIG. 11. Aircraft (1), addition of elevator-rate of pitch control to Fig. 10 examples.

$$\left(\eta = \theta + q + h + 0{\cdot}049 \int h\, d\tau\ ;\ T = A_u u \text{ or } A_q q \text{ or } A_w w\right)$$

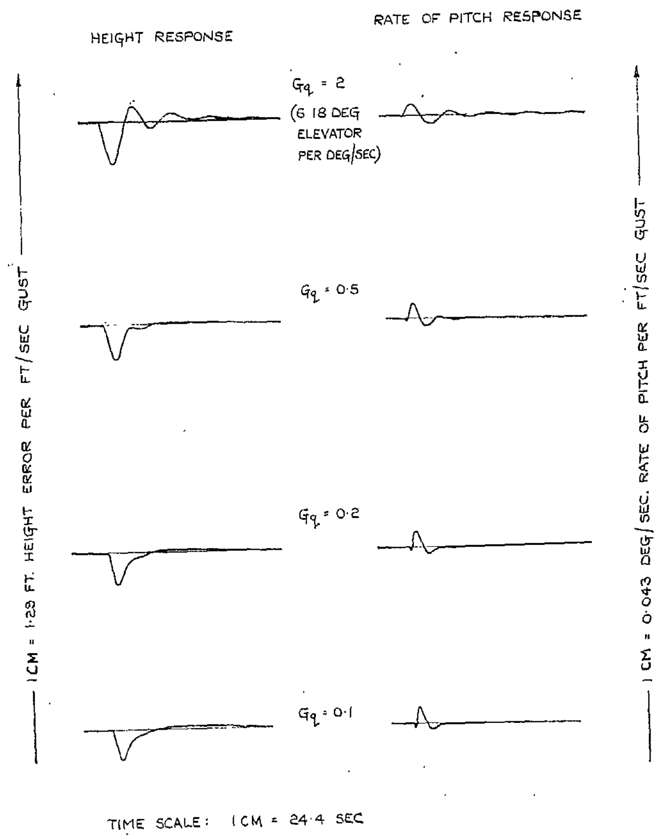

FIG. 12. Aircraft (1), effect of elevator-rate of pitch control with throttle control.

$$\left(\eta = \theta + G_q q + h + 0{\cdot}049 \int h\, d\tau\ ;\ T = -\ 0{\cdot}2u\right)$$

(86574) Wt. 64/1857 K.5 3/63 Hw.

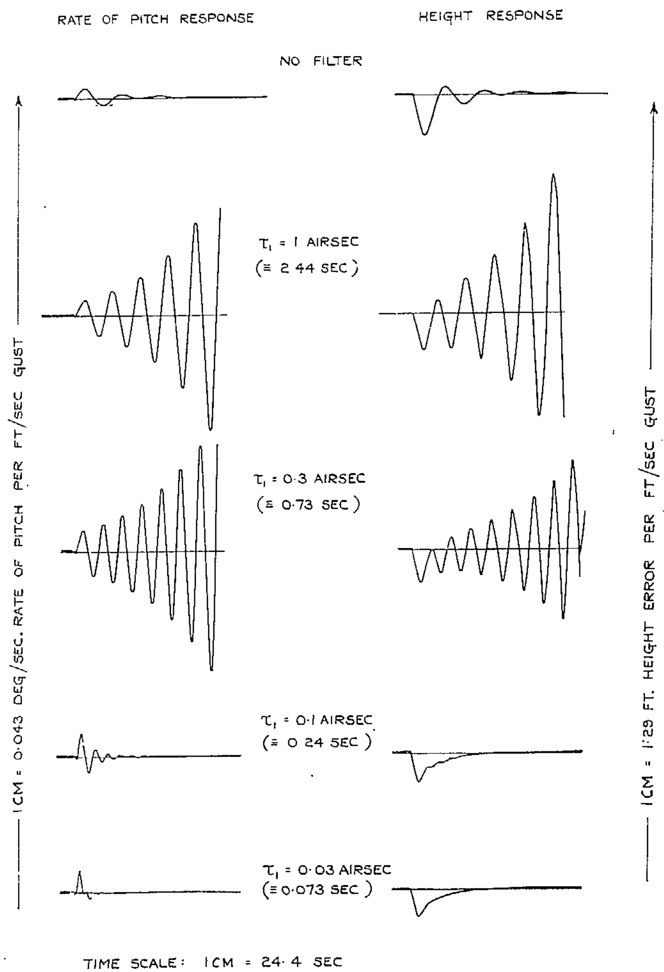

FIG. 13. Effect of 'high-pass filter' on q signal.

$$\left(\eta = \theta + h + 0{\cdot}049 \int h \, d\tau + \frac{2\tau_1 D}{1 + \tau_1 D} q \, ; \; T = -\, 0{\cdot}2u\right)$$

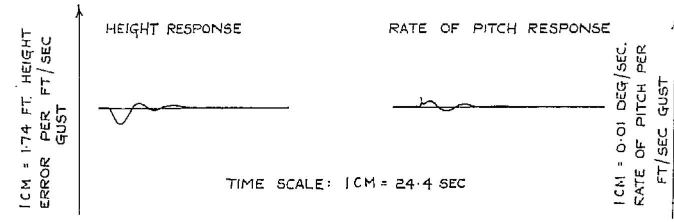

FIG. 14a. Response of elevator and throttle controlled aircraft, with height lock [Aircraft (2)].

$$\left(\eta = \theta + q + h + 0{\cdot}0663 \int h \, d\tau \, ; \; T = -\, 0{\cdot}2u\right)$$

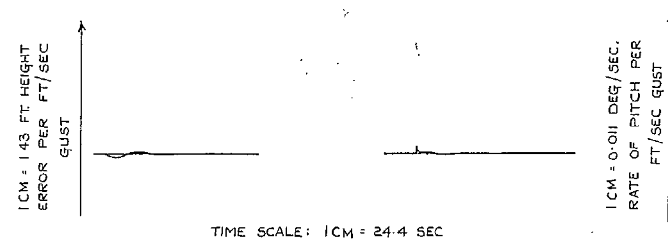

FIG. 14b. Response of elevator and throttle controlled aircraft, with height lock [Aircraft (3)].

$$\left(\eta = \theta + q + h + 0{\cdot}0542 \int h \, d\tau \, ; \; T = -\, 0{\cdot}2u\right)$$

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