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A Preliminary Data Report on Ground Pressure Disturbances Produced by the *Fairey Delta* 2 in Level Supersonic Flight

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Summary. Measurements have been made of pressure disturbances on the ground resulting from approximately straight and level supersonic flight of the *Fairey Delta* 2 in the altitude range 30,000 ft to 3,500 ft above sea level, and with flight Mach numbers up to about 1·5.

Pressure against time, flight histories, radar plots, and other data are given. The results, with allowance for refraction, show satisfactory agreement with the estimates of peak bang pressures and impulses given by Warren.

At low flight Mach numbers, the bang intensities were below theory, and the spread of the bangs each side of the flight track was limited; these effects were attributed to atmospheric refraction.

On a few flights, the normal 'N' pressure/time pattern was followed closely by additional pressure disturbances which, on one occasion, reached a peak positive value of nearly 2 lb/ft^2. These effects seem attributable to flight of an alternately accelerated and decelerated nature.

The maximum positive and negative peak pressures recorded were respectively, $+4 \cdot 05$ ($\pm 0 \cdot 28$) lb/ft^2 and $-5 \cdot 25$ ($\pm 0 \cdot 37$) lb/ft^2, and the maximum impulses were respectively $+0 \cdot 154$ ($\pm 0 \cdot 011$) lb sec/ft^2 and $-0 \cdot 148$ ($\pm 0 \cdot 010$) lb sec/ft^2.

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1. *Introductory Notes.* This report describes the results of a series of measurements of ground pressure disturbances, or 'sonic bangs', produced by approximately straight and level supersonic flight of the *Fairey Delta* 2 (ER 103) at various altitudes between 30,000 ft and 3,500 ft above sea level.

Owing to the need to extend the flight-test envelope of the aircraft, and because of the then prevailing restrictions on low-level supersonic flying in England, together with uncertain climatic conditions there, arrangements were made for the aircraft to be based at Cazaux, France, during October and November, 1956.

The data in this report were obtained from 43 flights and should permit a more comprehensive check of present theoretical predictions of sonic-bang intensities than hitherto possible. As a preliminary assessment of the data, a rough comparison of the results is made here with the theory of Warren, but the full flight data and radar plots etc. are included to permit subsequent more detailed analysis, should the need arise.

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[*] Previously issued as R.A.E. Tech. Note No. Aero. 2520—A.R.C. 20,782.

Data are also included on the lateral spread of sonic bangs from level flight at various altitudes and the physiological sensations accompanying the bangs. Details of damage to property during the flights are also given.

The supersonic 'bang' is a popular description of the sensation which occurs when the shock wave (or waves), produced by an aircraft in supersonic flight, reach the ears of an observer.

In general, the number of bangs heard will be equal to the number of shock waves passing the observer, although separate bangs may not be resolved by the ear if the time interval between the arrival of separate shocks is less than about 0·01 second, or if the rate of rise of pressure is too small.

For the special case of straight and level flight investigated here, the bang intensity is predominantly governed by altitude[1, 2, 3, 4, 5] and longitudinal acceleration of the aircraft. The latter motion may result in considerably enhanced pressure jumps by reason of the formation of cusps in the shock wave pattern and the introduction of concavity into the shock wave front which may then happen to have a point of focus at ground level, depending on the particular flight conditions. In theory the bang intensity is not very dependent on the aircraft shape or Mach number although the former governs the number of shock waves produced, while in practice the flight Mach number determines the extent to which refraction in the atmosphere affects the bang intensity and limits the spread of the bangs each side of the flight track.

The criterion for the occurrence of a bang at a particular point on the ground in the absence of refraction is now well established (*see*, for example, Refs. 1, 3, 4 and 6); it is that a bang will 'originate' from each point (called the 'bang origin') on the flight path for which the component of the aircraft's velocity in the direction of the observer is equal to the speed of sound. Strictly speaking, however, this criterion alone will not determine the number of bangs heard in a short (about 0·1 sec) time interval, as Warren[3] has pointed out, since in the simplest case of steady speed in a straight line at least two shock waves are created by the aircraft's motion[4] so that at least two bangs may be heard each time the criterion is satisfied.

The lines joining the bang origins to the observer are referred to as 'rays'* and these mark the direction of propagation of parts of the shock wave fronts. Because the atmospheric temperature gradient leads to refraction the rays in practice normally curve away from the ground and this results in a lowering of the bang intensity, or, if the curvature of the rays is sufficiently great, in the elimination of the bang on the ground. Randall[7] has shown, theoretically, however, that just before the bang disappears it is intensified; intensification by a factor of up to five is predicted for bangs received on the track. This value is an upper limit and depends on altitude and Mach number.

Elementary theoretical considerations (*see* Fig. 1) show that for the case of supersonic level flight in a standard atmosphere with no wind there is, due to refraction, a critical flight Mach number, ranging from 1·00 at sea level to 1·10 at 25,000 ft, below which no bangs should be heard on the ground. For the case of an aircraft diving at 20° or more, however, this effect disappears.

If the flight conditions are such as to render refraction effects negligible then, for level flight at constant velocity, the bang pressure measured near the ground† is theoretically a function entirely of the flight conditions which prevailed at the 'bang origin', i.e., altitude and Mach number. In the case of accelerated flight, however, the flight history behind and in front of the 'bang origin' must be taken into consideration[5] because of the focusing effects previously mentioned.

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* Warren, in Ref. 1, called them 'bang lines', but the term 'rays' would now seem more acceptable.

† i.e., at such a height above ground where the shock strength is not affected by reflection and surface topography, trees, etc.

2. *Description of Instrumentation and Recording Procedure.* A schematic diagram of the apparatus used at each recording site for obtaining the pressure/time records is shown in Fig. 2. Three complete sets of equipment were used, each having its own power supply in the form of a 240 volt, 2 kilowatt petrol-driven alternator.

Each set consisted of a condenser-type microphone, the Brüel and Kjaer type 4111, used in conjunction with the Southern Instruments type MR 220 F frequency-modulated pre-amplifier and type M 700 L gauge oscillator, with a Cossor type 1049 double-beam oscilloscope and recording camera.

The capacitance changes in the microphone, due to the incident pressure waves, cause a change in the frequency of the gauge oscillator which normally runs at about 2 Mc/s. This change of frequency is detected in a discriminator circuit which yields a direct-voltage output which is practically a linear function of the impressed capacitance change of the microphone. The d.c. signal is amplified in the oscilloscope A_1 amplifier (ranges 10 or 30), and then passes to the Y_1 plates. To provide a series of timing marks, a 100 c/s phantastron oscillator (with its output differentiated and clipped) was coupled to Y_2. No time-base was used, the Y_1 spot being positioned centrally on the tube face for optimum focus. HP3 or TRI-X 35 m.m. film was run at 5 inches per second to give a recording time of approximately 60 seconds. Thus prior knowledge of the arrival of the bangs was essential. This was obtained from a suitably-marked radar plot and warnings were passed to the other recording sites from the master site *via* V.H.F. radio links (Pye 'Reporter' sets operating on 131·3 Mc/s). Except when the flight altitude exceeded about 20,000 ft, the range of the radar was sufficient to give at least 60 seconds warning of the arrival of the bangs.

As shown in Plate 1 of Fig. 3, the microphones were positioned with their diaphragms horizontal and about 10 inches above ground level. A short length of co-axial cable connected the microphone to the oscillator which was wrapped in sponge rubber and packed in the large steel box. The microphone housing itself was also insulated from the steel cylindrical case by means of sponge rubber.

In all the measurements reported here the microphone diaphragms were horizontal; the effect of microphone attitude on the recorded pressure could not be investigated conveniently during the trials since it was most unlikely that successive runs of the aircraft would yield identical experimental conditions*.

It should be mentioned here that the microphone was constructed by the manufacturers to have a level free-field response for normally-incident sound waves. This means that with grazing incidence there will be some loss of high-frequency response. However, the sharpness of the pressure/time records indicates that this did not introduce a significant error in the determination of the peak amplitudes of the disturbances.

The shock transit time over the diaphragm was sufficiently small (about 60 microseconds) compared with the decay time of the pressure transient (about 0·01 second), for its effect to be neglected. In addition, because of the wide frequency response of the system (0 to 10,000 c/s) and good diaphragm damping, the pressure/time records are free of any objectionable 'ringing' or differentiation due to inadequate low-frequency response.

A useful feature of the apparatus used was the facility for static calibration. Calibration is discussed in Section 3.

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* It is hoped to check the effect of microphone attitude later using a spherical shock front.

The microphone had a number of other attractive features: e.g.,

(*a*) A negligible temperature coefficient of sensitivity of only − 0·02 dB per degree centigrade in the range − 20° to + 60°C which was important for use in a location where the microphone was exposed to the direct sun all day. As pressure levels in excess of 130 dB (referred to 2×10^{-4} microbar) were being measured, the change in sensitivity due to temperature effects was unimportant.

(*b*) Negligible distortion; less than 1% up to 120 dB, and less than 4% up to 140 to 160 dB. (160 dB is equivalent to a pressure of 40 lb/ft^2.)

The oscillator design permitted the use of cables about 30 ft long between the microphone and recording equipment. This enabled the microphones to be sited well away from ambient noise such as that from petrol generators etc. No cable 'noise' due to the bangs was observed.

Inevitably, in a recording apparatus of this type, a slow zero drift was present. Its effects were rendered negligible by allowing for at least a half-hour 'warm-up' period and then retuning the F.M. equipment just prior to obtaining a recording. When the F.M. system was correctly tuned, the d.c. output was zero. During the recording time of about 1 minute, the drift was effectively non-existent.

As will be seen from the records obtained (Figs. 45 to 76), the pressure resolution was ± 0·15 lb/ft^2 on the low-sensitivity range and ± 0·05 lb/ft^2 on the high-sensitivity range. Owing to exceptionally fine weather during most of the trials period, and apart from one isolated case (*see* Fig. 50), no interference due to wind noise was experienced even with the equipment working at maximum sensitivity.

3. *Calibration of the Recording Equipment.* Each set of recording equipment was calibrated by applying steady pressures, above and below atmospheric, to the microphone diaphragm, and then measuring these with a Chattock gauge*. The corresponding position of the C.R. tube trace was then photographed and complete calibration obtained, for both gain settings on the d.c. amplifiers (i.e., high- and low-sensitivity ranges), from × 8 enlargements. A typical calibration result, at low sensitivity, is shown in Fig. 4. Before commencing calibration the apparatus was left on for 45 minutes so that drift would be small during the calibration period of about 30 minutes. In order to obtain reproducible results it was found necessary to have the glass tubing of the Chattock manometer scrupulously clean: the cautious use of a concentrated nitric acid/ethyl alcohol mixture was found effective in achieving this.

No hysteresis effects were observed during the calibrations.

4. *Discussion of Errors.* There are three sources of error to be considered in these measurements; first, experimental errors inherent in the calibration technique itself and the accuracy with which the pressures may be derived from the films. Secondly, errors arising from the attitude of the microphone in relation to the shock wave front, and thirdly, errors which arise due to the deflection of the shock-induced flow around the microphone—the so-called Bernoulli effect. These will be considered briefly in turn.

The position of the meniscus in the Chattock gauge could be read to ± 0·001 inch of water (± 0·005 lb/ft^2) so this introduces negligible error. The experimental errors associated with the calibration technique indicate that the quoted bang pressures are accurate to within ± 7%.

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* An accurate form of water manometer.

Some remarks are also necessary regarding the possible effects of an inadequate high-frequency response in the microphone and associated amplifiers, and errors which may arise through the use of a static calibration of the microphone when, in fact, it was being used for the measurement of a transient pressure.

In the former case, the peak of the N-wave will be rounded off, i.e., undershoot will occur, giving uncertainty as to the true peak-pressure value. However, taking into account the high rates of rise of pressure recorded, about 1 lb/ft^2 in 1 millisec, on several occasions, it is considered the frequency response of the recording system was adequate and that the pressure/time histories are free of serious instrumental defects.

When subject to sound disturbances of a given pressure, the diaphragm motion of a condenser microphone is partly damped due to vibrations in the air trapped behind the diaphragm. Under static conditions, however, this air has time to escape from the back, so that the same applied pressure may produce a greater displacement of the diaphragm. The effect of this will be to cause an underestimate of the bang pressures, if a static calibration technique has been used.

However, dynamic calibration of similar microphones has been carried out here with high-intensity sound sources, covering the useful frequency range of the microphone, and has shown no difference from a static calibration, so that errors arising from choice of calibration technique in the present results may be small. Nevertheless a reliable calibration technique employing a pressure pulse, of waveform similar to a sonic bang, has yet to be developed.

Theoretical estimates[3, 4, 5] of the bang pressure jump (or shock strength) at ground level are 'free-field' values. If one wishes to measure these, the microphone position must be such as to result in grazing incidence of the shock wave, for then the diaphragm is subjected to the hydrostatic pressure behind the shock. Any other configurations will result in an enhanced pressure and consequent error.

In practice, owing to the combined effects of refraction, ground surface irregularities, trees, houses, etc., and reflection of the shock (with formation of a Mach stem), the shock strength measured on the ground may be enhanced or weakened compared with the 'free-field' value.

The degree to which these effects have influenced the present results is not known but it is clear they must *not* be regarded as 'free-field' values.

The Bernoulli effect introduces errors when the physical shape of the microphone and its housing is such as to cause a distortion of the field of mass flow behind the shock wave. This results in the diaphragm of the microphone being subjected to a pressure which is lower than the true hydrostatic pressure. As the shock strength increases, and with it the mass velocity, the pressure indicated by the microphone becomes progressively lower than the true value, that is, although the microphone may have a linear static response, its dynamic response will be non-linear.

Calculations have shown however that the shock strengths in the present measurements were insufficient to make the Bernoulli effect a significant source of error.

In conclusion, it is considered that the technique of static calibration of the condenser-type pressure transducer used is satisfactory since a high order of experimental accuracy was not called for and it is very doubtful if the trouble of dynamic calibration of microphones for sonic-bang measurements is worth while: the experimental procedure is difficult since it is necessary when using a shock tube, for example, to measure the Mach number of weak shocks to high accuracy, and there is the problem of obtaining a suitable range of shock strengths for satisfactory calibration. Another method, applicable in the case of condenser microphones, is to use an electrostatic actuator to simulate a step function of pressure of known amplitude.

5. *Description of Recording Sites.* As shown in the sketch map of Fig. 5, the three recording sites were positioned on a line roughly at right angles to the flight track of the aircraft. The two westerly sites 'Radar' and 'Tower' were only 2·64 statute miles apart and were intended to monitor bangs produced on or near the flight track. The easternmost site, 'Beehive', was 8·15 statute miles from 'Tower' and, in conjunction with the measurements at 'Radar' and 'Tower' site, yielded valuable information on the lateral spread of the bangs. This aspect of the results of the trials is described in Section 9.

The 'Radar' or master recording site was situated adjacent to the French radar equipment on the outskirts of Lacanau-Océan at a position roughly 500 yards north-east of the town centre. Two aerial photographs of the site are given in Fig. 3, Plate 3. The recording equipment was set up in the back of a Citroën van (Plate 2, Fig. 3) with the microphone well away from all neighbouring vehicles. The nearest obstructions were a few trees 20 yards away.

'Tower' site was situated at the base of a 150 ft high fire watch tower in a pine forest, and near to the village of Le Moutchic. The microphone was initially placed at the top of the tower but during one run (No. 16) excessive wind disturbances occurred (*see* Fig. 50) and so, as a precaution against further trouble, the microphone was placed on the ground for run 29 and onwards. While it is difficult to assess the possible effects of the trees on the peak pressures and the pressure/time records, the latter do not appear to have been significantly affected by the change in microphone situation.

'Beehive' site was situated near Méogas, on the left-hand side of the road to Brach at a position about $5\frac{1}{2}$ statute miles from Lacanau-Médoc. The surrounding country was flat and open, there being no trees within a radius of 1,000 yards.

6. *Flight Conditions, Communications, Determination of Position of 'Bang Origin' etc.* 6.1. *Flight Conditions.* The basic features of the flight path are shown in Fig. 5. Each run was from north to south commencing at Grave Point and the aircraft usually attained supersonic speed about 5 to 10 miles farther south. Two runs were usually possible in each flight. The aircraft became subsonic at the approximate position shown in order to avoid bangs near the relatively densely populated region around Arcachon. The general flight plan consisted of approximately level runs at selected altitudes with a different average Mach number for each run. Few runs were made at 30,000 ft since the bang pressures are low and data were already available from tests in England.

During the northward flight from Cazaux and before the start of each run, the pilot confirmed with 'Radar' the prospective Mach number and altitude of the run. Then, with the help of Fig. 6 the plan distance of the aircraft from 'Radar' which would give a delay of one minute before arrival of the bangs (assuming no refraction), was known. This distance was marked beforehand on the radar plotting table. The plan distance of the theoretical 'bang origin' was also marked simply so that some guidance could be given to the pilot to enable him, as far as possible, to achieve steady flight conditions while generating the bangs subsequently reaching 'Radar' and 'Tower'.* No such estimates were made for 'Beehive' site since it was so far off-track that refraction effects are important and these invalidate the simple calculations. Thus, when the 'one minute delay' distance did not exceed the 'lock-follow' range of the radar, adequate warning could always be passed to the

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* The aircraft was assumed to fly over 'Radar'. At high flight altitudes the 'bang origin' for 'Radar' and 'Tower' would be almost coincident.

recording sites to start their recorders. When the radar pick-up was successful, bangs usually arrived within ± 5 seconds of the expected time, so that camera running times of 20 seconds or less could be used with confidence.

At flight altitudes of 25,000 ft to 30,000 ft considerable difficulty was experienced in picking up the aircraft on the radar early enough to 'lock-on' and produce a plot including the 'one minute delay' and 'bang origin' positions. Some of this trouble was undoubtedly due to the very small area presented by the *FD* 2 when viewed head on. The situation could have been improved by locating the radar farther north, by using a radar set of greater range or, preferably, by fitting the aircraft with a transponder*. With the experience of these trials it is clear that early consideration should be given to these points before any future trials. To enable the recording of bangs in the absence of radar warning, the pilot passed estimates of his position to 'Radar'. The disposition of several large lakes adjacent to the course and the unusually straight coastline were of advantage for this. A number of points were selected along the course and were given letter references.

At all flight altitudes between 25,000 ft and 5,000 ft the radar coverage was adequate.

Towards the end of the trials when flights were being made at altitudes of 7,000 ft and below, it was found necessary to shift the flight track so that the pilot flew over 'Radar' in order that useful bang measurements could be made. This was because refraction associated with the relatively low flight Mach number so restricted the lateral extent of the bangs that it became of the same order of distance as that between 'Tower' and 'Radar', *see* Section 9. Consequently, when the pilot flew as usual between the sites, only very weak bangs were picked up, *see* run 43, Table 1 and Fig. 20.

Synchronisation between the aircraft and radar records was obtained at the point marked 'TIMING CHECK' by a count down to the pilot from an observer watching the radar plotting table. The position of the timing check usually coincided with the pre-determined 'bang origin' mark so that the selection of the relevant part of the aircraft flight history was facilitated. The radar equipment was provided and operated by French colleagues from C.E.V. Bretigny. The plan distance and altitude of the aircraft were plotted simultaneously (*see* Figs. 10 to 23). These plots were synchronised at the point marked 'SYNC' although, of course, the point 'TIMING CHECK' serves equally well. Since the time interval between the beginning of each dash is one second, one may also derive the forward velocity and acceleration of the aircraft. However, the aircraft records (*see* Figs. 24 to 44) are more accurate over the significant time range.

Although it would have been desirable for the aircraft to maintain steady speed in the region of the 'bang origin' it was, unfortunately, seldom possible to achieve this. The reason for this was that the engine thrust with reheat could not be closely controlled, so that to avoid exceeding any stipulated Mach number below maximum the pilot had to switch reheat on and off. As a result, the motion of the aircraft through the 'bang origin' was usually accelerated or decelerated.

To avoid the possibility of causing excessive damage in Lacanau-Océan, special arrangements were made for the two low-altitude supersonic runs, S50 and S51, at 4,000 ft and 3,750 ft respectively. The recording equipment at 'Radar' and 'Tower' sites was re-sited at a point near the sea about 9 statute miles south of Lacanau-Océan. The runs commenced at Cap Ferret (*see* Fig. 4) and followed the coast northwards as far as Le Gressier. Reheat was applied at a position about 7½ statute miles south of Le Gressier and the aircraft decelerated after passing the recording site. A smoke screen on the beach near the site was used to guide the pilot on his approach. Because of the low altitude there was no radar pick-up until the aircraft had completed the supersonic portion

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* A radio device for intensifying the radar 'echo'.

of its run. However, the required flight history was found by an extrapolation of the incomplete radar plots (not reproduced here), based on the points of commencement of the turns, and the fact that up to these points, the aircraft track was parallel to the coast.

6.2. *Communications.* The essential communication requirements for the trials were:

(*a*) communications between the aircraft and 'Radar',

(*b*) communications between the three recording sites,

(*c*) communications between the base at Cazaux and 'Radar'.

All the equipment used operated on V.H.F. The equipment for the link between the aircraft and 'Radar' was located in the radar plotting room. The pilot passed flight details (Mach number and altitude) before the start of each supersonic run and, if necessary through lack of radar pick-up, details of his position on the run. Finally, at the appropriate instant, the 'timing check' signal was given to the pilot to enable the marking of the auto-pilot records.

The links between the three recording sites were essential for obvious reasons, e.g., passing the recording alert, details of run numbers, general liaison etc. Owing to intervening high ground, messages could not be passed to 'Beehive' from 'Radar' direct so 'Tower' was used as a relay station, its aerial being placed, conveniently, at the top of the tower. Strength and readability were excellent between 'Radar' and 'Tower'. Signals from 'Beehive' were sometimes very weak at 'Tower' but usually readable.

Some difficulties were experienced in obtaining satisfactory communication between Cazaux and 'Radar'. The link was important so that advance details of flight, take-off times etc. could be passed to the recording teams. About two weeks after the trials had started, a telephone connection was made and the situation improved. In the meantime, it was found that very weak but readable signals could be exchanged between Cazaux and 'Tower'.

6.3. *Determination of the Positions of the 'Bang Origins'.* The procedure used for fixing the positions of the 'bang origins' was as follows. Because both speed and altitude of the aircraft were varying a trial and error method was necessary. A trial position for the 'bang origin' was first obtained by choosing a point in time on the flight-history chart and then calculating the plan distance of the 'bang origin' from 'Radar' using the indicated Mach number and altitude as a basis*, and assuming no refraction. This trial position was then marked on the radar plot. Since the time interval between the beginning of each dash on this plot is one second, the position in time of the trial 'bang origin' from the 'timing check' point was easily found and was then inserted on the flight data. The new indicated altitude and Mach number were then compared with the originally selected values. If these values did not agree, the whole procedure was repeated using a new trial point on the flight history until agreement was reached. In this way the altitude and Mach number linked with the bangs received at 'Radar' and 'Tower' recording sites were found.

The aircraft 'indicated altitude' readings were converted to true altitude using the meteorological data in Table 3. The corresponding heights were found also from the radar plots and except in two bad cases (i.e., runs 20 and 40), agreement with the flight values was within 5%.

Although radar plots were not obtained for the whole of runs S.50 and S.51, the 'bang origin' positions were calculated from the following data. For run 50 the track was estimated to be parallel

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* A graphical method facilitated this.

to and about $\frac{3}{4}$ mile inland from the coast, with the timing check occurring at a position 12 miles from 'Radar', while in run 51 the track was estimated to be parallel to and $\frac{1}{4}$ mile inland from the coast, with the timing check occurring at a position $11\frac{3}{4}$ miles from 'Radar'.

7. *Summary and Discussion of the Results.* 7.1. *General Remarks.* For convenience, all the bang pressures deduced from the pressure/time records obtained are summarised in Table 1, which gives also the approximate flight conditions associated therewith. In Table 2 impulse values and durations etc. of the pressure disturbances are given.

During the early part of the trials bangs were missed either owing to no radar pick-up or to cloudy weather conditions when the pilot was unable to give an idea of his position. There were only four occasions when records were missed due to jamming of the film in the recording cameras, and two of these could have been avoided.

As a complete quantitative analysis of the data is not the purpose of this report, copies of the original radar plots (Figs. 10 to 23) and relevant parts of the associated flight histories (Figs. 24 to 44) are included for such analysis if need be. Figs. 45 to 76 are reproductions of the original pressure/time oscilloscope records.

It should be noted that the pressures quoted in Table 1 are *peak* values above and below atmospheric. In the case of the front limb of the N-wave, the positive peak pressure is obviously equal to the strength of the bow shock wave, whereas for the rear limb, the negative peak pressure in theory will be somewhat less than the strength of the stern shock wave because there is some overshoot of pressure to a value slightly above atmospheric*. In practice, however, it was observed that in the majority of cases this overshoot was absent or else very small so making the negative peak pressure approximately equal to the strength of the stern shock wave.

It is noted that the N-waves were rarely symmetrical and the results suggest that an enhanced front limb (bow wave) is linked with accelerated flight, while an enhanced rear limb (stern wave) in general appears with decelerated flight at the 'bang origin'. The two notable exceptions to this, however, are the special low-altitude runs S.50 and S.51 (Figs. 75 and 76).

The largest positive peak pressure observed was $+ 4 \cdot 05$ lb/ft^2 on run 44 at 'Radar' site, whilst the largest negative peak pressure was $- 5 \cdot 25$ lb/ft^2 on run S.50 at the 'special' recording site 9 miles south of 'Radar' (*see* Section 6).

7.2. *Occurrence of Secondary Pressure Disturbances.* It will be seen that several of the pressure/time records show secondary pressure disturbances following the N-wave (the whole of the N-wave is regarded here as comprising the primary disturbance). Although these disturbances occur after the bangs which are the chief concern of this report, a short discussion of the phenomena seems not without interest.

In several cases these later pressure changes were rapid enough and of sufficient amplitude to result in fainter bangs sometimes up to 1 to 3 seconds after the first bang or bangs were heard—*see* Table 2. On one occasion, run 29 at 'Tower', a group of three bangs was heard following the primary double bang (*see* Fig. 57). These secondary pressure changes appear to be a fundamental attribute of the shock wave patterns produced by the aircraft's motion and cannot be regarded as echoes. (It should be mentioned here that some of the secondary bangs may not have been heard due to temporary distraction of the observers, on some occasions, during the recording period.) In some cases, the

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* *See* Ref. 3.

origin of these secondary pressure disturbances may be attributable to the formation of the so-called 'rear wave'[2, 4, 5] associated with acceleration of the aircraft to supersonic speed; this may well apply to all runs except Nos. 18, 24, 29, 33 and 36 (Figs. 51, 56, 57, 61 and 63 respectively), in which the secondary waves seem of much too complex a structure for this explanation. An examination of the flight histories of the latter runs shows that, compared with the remaining runs, there were successive periods of acceleration and deceleration at times and positions on the flight path which might result in several shocks arriving successively at a recording site. These effects will be more important in runs of low average Mach number (i.e., about 1·1) where the corresponding 'bang origin' is, for a given altitude, at large distances from the recording site. Since detailed calculations are clearly necessary to substantiate these ideas, no further discussion of them will be given here, for example it will be necessary to check that the time of arrival of the disturbances is that expected.

7.3. *Form of the Pressure/Time Histories.* Most of the records show the characteristic N-form but in some cases the primary pressure changes were complicated, e.g., runs 18, 23 and 24 at 'Radar' (Figs. 51, 55 and 56), and the explanation may well be the same as that suggested for the peculiar secondary waves. However, it is possible in the case of run 23 that the flight Mach number was sufficiently low for one to expect that refraction would modify the pressure-wave profile. In the case of run 24, the secondary pressure disturbance appears to have interfered with the primary N (*see* Fig. 56).

7.4. *Comparison of the Experimental Peak Pressure and Impulse Values with Theory.* In Figs. 7a, 7b and 8a, 8b, a comparison is made between the observed peak pressures and impulses and those predicted by Warren[3] for straight and level flight. It will be seen that the ordinates of the theoretical curves have been increased by a factor of 1·08 compared with Warren's original curves for the pressure jump ΔP. This is to render the curves applicable to the *FD* 2. The value of S taken (the maximum cross-sectional area of the aircraft) was 22 ft^2, an l (overall length of the aircraft) equal to 45 ft, and a constant term equal to 0·73*. Similarly, the ordinates of the original theoretical impulse curves have been multiplied by 0·96. The comparisons are of a rough nature only as the positions of the 'bang origins' were determined on the assumption that 'Radar' and 'Tower' recording sites could, in effect, be considered as lying on the flight track. In practice, the errors introduced into the correlation by this assumption are not as bad as at first might be supposed since the distances of the 'bang origin', compared with the distance between 'Radar' and 'Tower' sites, were large (due to the low Mach numbers), and for altitudes less than about 10,000 ft the aircraft aimed to fly over 'Radar' anyway, no significant bangs being received at the other sites, i.e., runs 44, 45, 46, S.50 and S.51. Moreover, refraction effects are very difficult to allow for, by calculation, in a correlation of this nature; it is better that such effects be avoided at the outset, as far as possible, in the planning of the flight programme, and by working at the highest practicable Mach number (*see* Section 9). The problem has, therefore, been reduced to two-dimensions to facilitate a rough assessment of the results. For the purpose of any calculations according to the theory of Rao[4, 5], the 'shape factor' of the *FD* 2 is given as:

$$\left[\int_0^{\eta_0} F(\eta')d\eta'\right]^{1/2} = 3\cdot06,$$

with the notation of those references.

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* The value of 0·60 in equations (10) and (12) of Ref. 3 is in error and has been corrected.

In Table 1 the actual flight Mach numbers at the 'bang origins' may be compared with values of the theoretical critical Mach numbers, below which bangs should not be heard on the ground. The theory assumes straight and level flight in the I.C.A.O. atmosphere and no wind—*see* also Fig. 1 and Ref. 3. The meteorological data indicate that the wind was negligible during the trials. It is seen that, in runs 2, 3, 9, 10, 23, 24, 29, 32, 39 and 41, the flight Mach numbers were near the critical values and the bang intensities fall below theoretical expectations or silence resulted. In general, the rate of climb was too small to affect the spread or intensity of the bangs. A rapid supersonic climb will, of course, introduce refraction and weaken the bang intensities.

In conclusion then, if one discounts those results which may reasonably be expected to be influenced by refraction, the correlation of the observed peak pressures with the simple linear theory of Warren[3] is quite satisfactory. Runs 22, 33 (at 'Tower'), 44 (at 'Radar'), 47 (at 'Radar') and 49 (at 'Tower') are associated with bangs in excess of the theoretical values, but this may be accounted for by the aircraft acceleration.

The values of the pressure impulses given in Table 3 were obtained by integration of the positive and negative halves of the pressure/time curves. Except when the latter were complicated, the two areas were equal as expected, within experimental errors. From Fig. 8a and 8b it appears that the observed impulses are in fair agreement with the theoretical estimate of Warren[3] although the bang impulses from low-altitude flight seem greater than expected. Clearly this will arise if the peak pressure or duration of the N-wave is greater than the theoretical estimate, but since the increased pressure alone does not account entirely for the difference, it is concluded that the duration of the N-wave also is greater than the theoretical estimate. The largest impulse measured in the trials occurred on run S.50 and was $+ 0\cdot154\ (\pm\ 0\cdot011)$ lb sec/ft^2. Comparable with this was the impulse of $0\cdot116\ (\pm 0\cdot008)$ lb sec/ft^2 measured at 'Radar' site on run 44.

8. *Physiological Sensations.* During the course of the experiments observers at the recording sites noted the number of bangs they heard and the loudness. From this information and the associated pressure/time histories certain qualitative features of the bangs have been derived and are briefly reported here out of general interest.

A bang was always heard when the rate of rise of pressure was sufficiently large, i.e., not less than about 100 lb/ft^2/sec. In a well-defined N-wave, the minimum peak to peak duration observed was about $0\cdot06$ sec and this was associated with a distinct double bang. In some cases pressure changes in the secondary pressure disturbances following the initial N-wave were sufficiently rapid to produce further bangs a second or two later (*see* Table 3 and the pressure/time histories—Figs. 45 to 76). A triple bang, that is three bangs in quick succession, was recorded on run 24 at 'Radar' site; an examination of the pressure/time history (Fig. 56) shows that there were three occasions when the rates of change of pressure were large. Similarly, in the case of run 29 (Fig. 57) where, at 'Tower' site, three bangs were heard following the primary double bang. It is also possible that at 'Tower' site again on run 33 (Fig. 61), four bangs might have been heard if the disturbance had not been of such unexpected intensity; the bangs were likened to the sensation of heavy artillery firing in close proximity to the site.

On several occasions 'dull booms' were heard; these seem to be associated with a pressure/time wave which has a rounded-off positive peak and a low rate of rise of pressure from the negative peak (e.g., *see* runs 6, 48 and 49 at 'Radar'—Figs. 46, 73 and 74 respectively), or also, it appears, from N-waves which have both peaks rounded off (e.g., *see* runs 16 at 'Radar' and 'Beehive' sites, and runs

32, 33, 43 at 'Radar' site—Figs. 50, 60, 61 and 68 respectively). Although the occurrence of bangs seems linked with some recognizable features in the pressure/time records, it will be noticed from Figs. 52, 53 ('Beehive' site) and 69 that bangs were apparently heard which cannot be explained from the film records. However, in the cases of Figs. 52, 53 and 69 this was probably due to the low recording sensitivity so that weak bangs would not show up; in the case of Fig. 68 ('Tower' site) the microphone was inadvertently left covered during the recording period.

The sharpness of the bangs is definitely determined by the sharpness of the N-profile of the pressure/time curve. The low-altitude high-intensity bangs were like exceedingly-loud double cracks followed by rumblings and then the noise of the engine. When heard, the secondary bangs occurred during or after the rumbling. The very loud bangs at 'Radar' site on run 44 were similar to the sound of close gunfire; a distinct pressure was felt on the chest, the ears were left momentarily 'singing', and there was a distinct impression that the ground shook. Even although arrival of the bangs was expected within a second or two, the occurrence of the high-intensity bangs was still startling.

9. *The Lateral Spread of the Bangs.* The results in Table 1 provide interesting data on the lateral spread of the bangs and show the practical importance of refraction effects in limiting both the lateral extent and intensity of bangs produced by low-altitude level flights at low supersonic Mach numbers.

For example, although the 'Beehive' recording site was only some 8·6 miles east of the average flight track, bangs were only heard there from flights above 20,000 ft and significant bangs for flight Mach numbers greater than about 1·3 (at the 'bang origin'). Thus it seems reasonable to conclude that, in cases of flight below 20,000 ft and with Mach numbers less than 1·3 the affected ground becomes confined, because of refraction, to a strip less than 18 miles wide. The intensities of the few bangs received at 'Beehive' for flights between 20,000 ft and 30,000 ft appear to be somewhat less than theoretically predicted[3]. At high Mach numbers, however, the rays along which the shock waves travel are steeper, they are refracted less, and thus one may expect the observed bang pressures from flights at high Mach number to be higher, and the affected area on the ground to be correspondingly increased in extent; quite measurable bangs were in fact received at 'Beehive' on runs 5, 12, 16 and 49 (*see* Table 1).

Evidence was obtained in run 49 that bangs generated at high supersonic Mach numbers of 1·5 or above, and altitudes greater than 25,000 ft, have a lateral spread of at least 20 miles each side of the flight track. Observers for this run were stationed at the following places (*see* Fig. 5) and gave reports on the occurrence of bangs. At Marcheprimé a triple bang and rumbling were heard. At a point about 2 miles south of Salaunes on the road to Issac a loud triple bang was heard while at Castelnau-de-Médoc and St. Laurent de Médoc, no bangs were heard. It will be seen from Table 1 that a peak pressure of only about 0·9 lb/ft^2 was recorded at 'Tower' even with accelerated flight at the 'bang origin' directly towards the site, and at 'Radar' only a weak bang was received. Since at 'Beehive' the bang pressure was low (about 0·4 lb/ft^2), the bang pressures at 20 miles off track were probably insignificant.

From runs 22 to 36 inclusive, 'Beehive' site was unmanned, so that no reliable deductions can be made regarding the spread of bangs from the flights between about 21,000 ft and 14,000 ft, although run 47 (Table 1) indicates that the spread was not greater than about 9 miles from the track when the flight altitude and Mach number were 20,000 ft and 1·2 respectively. The roughly equal bang pressures simultaneously recorded at 'Radar' and 'Tower' on runs 29, 30 and 31 are because the flight track was straight and passed almost exactly between the sites. The weak bangs recorded on run 29 were almost certainly attenuated due to refraction (*see* Fig. 1) arising from the low flight Mach number.

At flight altitudes between 10,000 ft and 6,000 ft the Mach number at the 'bang origin' did not exceed 1·1 and further restriction of lateral spread of the bangs is evident. At 6,000 ft with a Mach number of about 1·1, the observed lateral spread appeared to be little more than 2·5 miles.

The effect of refraction in limiting the lateral spread of sonic bangs produced in straight and level flight has been investigated theoretically. Of the two possible sources of refraction in the atmosphere, namely wind gradients and temperature gradients, the treatment here is confined to the effect of temperature gradient, since the effect of wind in the present tests was found to be small. Moreover the theoretical lateral spread determined by the temperature gradient is found to be in satisfactory agreement with the observed values. The meteorological data of Table 3 show that the atmospheric temperature gradients (at Bordeaux) throughout the trials followed I.C.A.O. conditions very closely, so this has been adopted for the calculations.

The mathematical theory and procedure is given in Appendix I. Briefly the method consisted of first working out (using a step-by-step method in intervals of 50 ft) the total refraction of a characteristic ray travelling from the aircraft to a point on the ground where its direction became parallel to the ground. The lateral spread of the actual shock waves as a function of Mach number and altitude was then found by the application of elementary 3-dimensional geometry. The results are given in Figs. 9a and 9b.

It is seen that the values of Mach numbers below which no bangs are heard are correctly predicted (*see* Fig. 1) and that there is a rapid increase in lateral spread as the flight Mach number increases above the critical value, but that this tends to a limiting distance at high Mach numbers.

A comparison of the experimental data with Fig. 9 shows that the observed lateral spreads are all less than the theoretical distance, except in the case of run 49 when flight at 27,000 ft and Mach 1·47 gave a spread of over 20 miles as mentioned earlier. The reason for this discrepancy is not clear, since apparently the wind gradients were not excessive and the temperature gradient was not abnormal. It is possible, however, that the local atmospheric conditions over this large 'spread' distance were sufficiently different from the vertical soundings taken at Bordeaux to give a freak effect. Thus, if the data of Figs. 9a and 9b were to be used as a guide to positioning a test flight track, it is recommended that the indicated distances for flight above 20,000 ft be increased by 25% at least. Below this altitude, when the bang pressures become more significant, and therefore knowledge of the probable lateral spread becomes more important, the theoretical distances as given in Fig. 9 should be adequate.

10. *Details of Damage to Property.* The first damage to house property during the trials was reported in Le Moutchic, near 'Tower' site (*see* Fig. 5) during run 33 where a roughly triangular piece of glass was apparently sucked out of a northward-facing verandah window-pane. The dimensions of the pane were 4 ft 9 in. by 2 ft 8 in. In another house there, a lamp fell on to a table. As seen from Tables 1 and 2, the bang pressure and impulse measured at 'Tower' site on run 33 were respectively 1·89 lb/ft^2 and 0·075 lb sec/ft^2. No further damage was reported until after runs 44, 45 and 46, and all reports then were from Lacanau-Océan near 'Radar' site.

Of the three runs 44, 45 and 46, run 44 produced the largest bang pressure and impulse at 'Radar' site. The values there were respectively 4·04 lb/ft^2 and 0·116 lb sec/ft^2 (*see* Tables 1 and 2).

It is seen from Fig. 61 that the pressure/time waveform obtained at 'Tower' site on run 33 is unusual in that the amplitude of the secondary N-wave exceeds that of the first, so resulting in a large effective impulse. Some possible reasons for this peculiarity were suggested in Section 7; it is thought to arise from the alternately accelerated and decelerated nature of the flight, leading possibly to some form of focusing in the neighbourhood of the 'Tower' site. Since no comparable disturbances were received at the same time at 'Radar' site, only 2·6 miles to the west, effects of this nature are probably localised.

The town of Lacanau-Océan was estimated to comprise at least five hundred houses and shops etc., covering an area of about half a square mile, although about two thirds of these were shuttered up and unoccupied. There were six reports of cracked ceilings. In three cases cracks appeared, while in the remaining three, old cracks were extended and some plaster fell down in one of these cases. There were three cases of cracks appearing in glass windows. In two of these cases, the cracks appeared in south-facing windows, and in the remaining one, a crack appeared in a pane of glass in an interior door. There were four cases in which pieces of glass in already cracked panes (not due to sonic bangs), facing north, were slightly displaced only.

Throughout the trials the local inhabitants seemed remarkably indifferent to the bangs. For example, no complaints were made to an observer who visited several shops in Lacanau-Océan immediately after run 44, when the 4 lb/ft^2 bang was recorded at 'Radar', nor were representations made to any of the scientific staff resident in Lacanau-Océan, at any time.

Whilst the damage reported appears of a moderate nature and of limited extent, it is necessary to state that the trial region was rather sparsely populated.

Attention has already been drawn to the unusual pressure/time records shown in Figs. 51, 56, 57 and 61 and their associated flight histories. Some theoretical estimates of the peak pressures on the ground expected from alternately accelerated and decelerated motion have been made[7], and it has been found that over very small areas (about a quarter of a mile square) the peak pressure may be three or four times greater than the general level elsewhere, that is, these regions are subjected to a form of 'superbang'. Since the proportion of 'sensitive' areas (i.e., regions containing microphones or householder's windows etc.) was small and such areas were spread relatively large distances apart, the chance of such 'superbangs' being detected in these areas in the present trials was very remote.

11. *Meteorological Data.* Table 3 is a summary of the meteorological data obtained from soundings at Bordeaux during the course of the trials.

It will be seen that the temperature *versus* height variation does not exactly follow I.C.A.O. conditions so that the values of M_{CRIT} given in Table 1 are slightly in error (nor has wind gradient been allowed for), but this is not of any consequence for present purposes.

The variations of wind velocity and direction with height show that refraction due to excessive wind-velocity gradients, headwinds etc., was not to be expected, the predominant effect being due to the temperature gradient.

12. *Conclusions.* Techniques and instrumentation have been developed for recording ground pressure transients associated with sonic bangs.

The measured pressures and impulses are considered to be accurate to within about ± 7%, this being decided largely by experimental errors in the calibration technique. However, the observed pressure jumps may be greater than the 'free-field' values required for comparison with theoretical estimates, due to non-grazing incidence of the shock wave with respect to the microphone diaphragm and particularly the effect of ground reflection. On the other hand, owing to the use of a static calibration of the pressure transducer (a condenser microphone), the quoted peak pressures may be too low. A satisfactory dynamic-calibrating technique has yet to be developed, but early tests have suggested the correction factor should not exceed about 1·6.

Satisfactory agreement was found in a comparison of the experimental peak-pressure values which had not been significantly affected by refraction, and theoretical estimates for straight and level flight at steady speeds. The corresponding impulse values were also found to be in satisfactory agreement with the theory.

It is necessary to point out that theoretical estimates give the strength of the shock wave in free air whereas what has been measured in the experiment is the shock strength as modified by reflection (or the formation of a Mach stem) and by surface topography, trees, etc., and the measurements are also subject to the aforementioned instrumental and experimental uncertainties. Taking these points into account, agreement to much better than a factor of two is not significant. Within these limits satisfactory agreement was obtained.

Some evidence was obtained that the bang pressures resulting from low-altitude flight may exceed theoretical values, although detailed calculations are first necessary to see if this is accounted for by acceleration of the aircraft.

The pressure/time records showed the usual N-shape but this was frequently followed by additional pressure disturbances thought to arise from successive periods of acceleration while the aircraft was moving at supersonic speed.

The results have also shown that in level flight and within certain Mach number limits, depending on altitude, refraction restricts the lateral spread of the bangs each side of the flight track. The low bang intensities, or even silence, observed on many occasions are attributed to refraction.

The damage observed and reported was confined to cracks in ceilings and windows, and dislodged ornaments etc. in houses but it must be recalled that the region of the trials was sparsely populated.

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Note added in Proof

The presence of a calibration error, discussed in Sections 4 and 12 of this report, has been confirmed recently and consequently *all values given for pressure jump and impulse should be multiplied by* 1·6.

The apparently good correlation with theory shown in Figs. 7a and b and 8a and b is, however, only slightly affected because the ordinates of the theoretical curves should be multiplied by 2·0 to allow for the pressure doubling accompanying reflexion of the shock wave by the ground. This correction applies in the present work and in all cases of correlation with experimental results obtained from microphones on, or very near, the ground.

APPENDIX I

The Limitation of the Lateral Spread of Sonic Bangs from Straight and Level Flight, by Refraction Due to the Atmospheric Temperature Gradient

For simplicity, this discussion is restricted to steady, straight and level flight at supersonic speed in the standard I.C.A.O. atmosphere and wind effects are neglected.

If there were no refraction in the atmosphere, the bow shock wave surrounding an aircraft in straight and level flight would be approximately a cone, each part of which would propagate, in a direction normal to itself, at sonic speed. It is evident therefore that the ultimate lateral spread of the shock wave from the flight path in this case will be very large indeed. At all flight Mach numbers, from 1 upwards, bangs would be heard on the ground.

When discussing the effect of refraction on the direction of propagation of such shock waves, it is found more convenient not to work in terms of the shock waves themselves, but rather their 'rays' which, as described in Section 1, mark the direction of propagation of the shock wave. The treatment of the problem then becomes similar to that of the refraction of light waves, except that the increase of acoustic 'refractive index', causing the bending of the rays away from the ground, arises from the decrease of atmospheric temperature with increasing height above the ground.

First, we may consider the simple two-dimensional problem of a ray initially at an angle α to the vertical and becoming refracted parallel to the ground. It is found that there is a critical flight Mach number, M_{CRIT}, below which bangs (i.e., shock waves) will not be heard on the ground.

In the diagram, Fig. A.1, the aircraft A is at the 'bang origin' (*see* Section 1), and the observer O is at such a position that the ray just touches him on the ground.

The critical condition will be when

$$\sin \alpha_{\mathrm{CRIT}} = \frac{a_h}{a_g}$$

but

$$\sin \mu = \frac{1}{M}, \text{ and } \alpha = \mu$$

$\therefore$

$$M_{\mathrm{CRIT}} = \frac{a_g}{a_h}, \tag{1}$$

i.e., simply the ratio of the speeds of sound at ground level and altitude h at which the aircraft is flying.

This relation was used to plot the middle curve of Fig. 1; the remaining curves for various climb and dive angles are easily derived.

In order to determine the lateral spread of the bangs, the 3-dimensional critical condition is found by considering the refraction of a characteristic ray, in the plane of the ray, and then introducing the effect of flight Mach number.

The first part of this problem involves the calculation of ΔY shown in Fig. A.1. The detailed step-by-step procedure may be explained by reference to Fig. A.2.

From ordinary geometry we have:

$$\Delta y_1 = \Delta h \tan r_1$$

and the refraction law gives

$$\frac{\sin \alpha_{\mathrm{CRIT}}}{\sin r_1} = \frac{a_0}{a_1}$$

where a_0, a_1 etc. are the mean speeds of sound over the height intervals Δh as shown. Similarly

$$\Delta y_2 = \Delta h \tan r_2, \quad \frac{\sin \alpha_{\text{CRIT}}}{\sin r_2} = \frac{a_0}{a_2}$$

and so on. Summing gives

$$\Delta Y = \Delta y_1 + \Delta y_2 + \ldots = \Delta h\,[\Sigma_n \tan r_n] \tag{2}$$

where

$$r_n = \sin^{-1}\left[\frac{a_n}{a_0} \sin \alpha_{\text{CRIT}}\right] \tag{3}$$

Calculations were done with Δh = 50 ft to within 50 ft of the ground. Because of the effect of local variations of ground level, curvature of the earth, etc. for all practical purposes there is little to be gained by approaching the 'cut off' limit more closely. (As has been seen in Section 9, the observed spreads, with one exception, are all less than the theoretical estimates.)

The values of $\sin \alpha_{\text{CRIT}}$ and ΔY obtained at various altitudes are as follows:

Altitude (feet)	$\sin \alpha_{\text{CRIT}}$	ΔY (miles)
30,000	0·8908	23·81
20,000	0·9285	19·36
15,000	0·9470	17·00
10,000	0·9650	13·98
5,000	0·9826	9·79

From an analytical method using $a_h = a_g - kh$ where k is a constant, it may be shown that ΔY is proportional to $h^{1/2}$ where h is the flight altitude.

It will be seen later that ΔY is, in fact, the maximum possible lateral spread occurring for an infinite flight Mach number. The effect of Mach number on the lateral spread may be derived with the help of Fig. A.3.

It may be shown that the relation between the angles α_{CRIT}, μ and γ is given by:

$$\sin \gamma = \frac{\sin \mu}{\sin \alpha_{\text{CRIT}}} \tag{4}$$

In triangle BDC:

$$\cos \gamma = \frac{\Delta y}{\Delta Y}$$

and from (4):

$$\cos \gamma = (\sin^2 \alpha_{\text{CRIT}} - \sin^2 \mu)^{1/2} / \sin \alpha_{\text{CRIT}}$$

$\therefore$

$$\Delta y = \Delta Y (\sin^2 \alpha_{\text{CRIT}} - \sin^2 \mu)^{1/2} / \sin \alpha_{\text{CRIT}} \tag{5}$$

Hence we see from equation (5) that the condition for bangs just to be heard on the track is $\Delta y = 0$, $\gamma = 90°$, or $\sin \alpha_{\mathrm{CRIT}} = \sin \mu$, i.e., $a_h/a_g = 1/M_{\mathrm{CRIT}}$, which is the same as equation (1).

Conversely for $M = \infty$, we have $\mu = 0$, $\sin \alpha \neq 0$, so that $\Delta y = \Delta Y$, and the lateral spread is a maximum.

Using the values of ΔY and $\sin \alpha_{\mathrm{CRIT}}$ given earlier, and appropriate for each flight altitude, equation (5) yields the curves given in Fig. 9a. (Fig. 9b is simply a cross plot.)

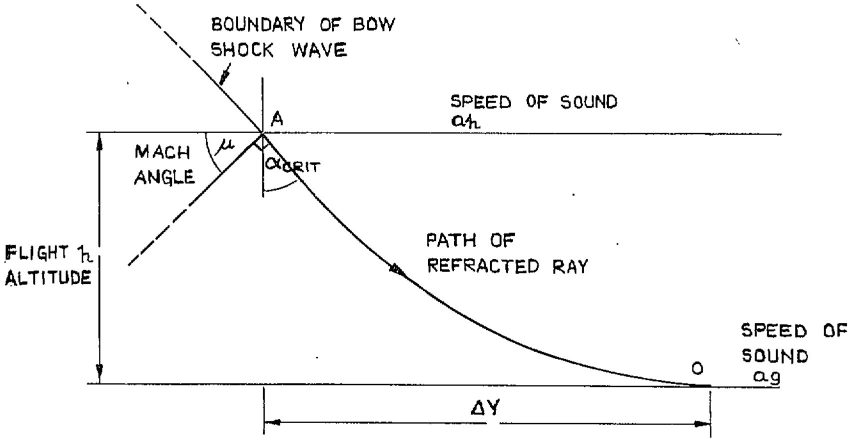

FIG. A.1.

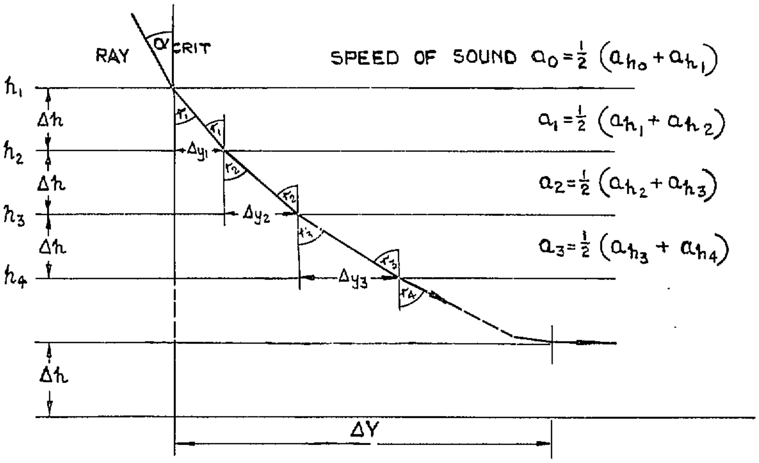

FIG. A.2.

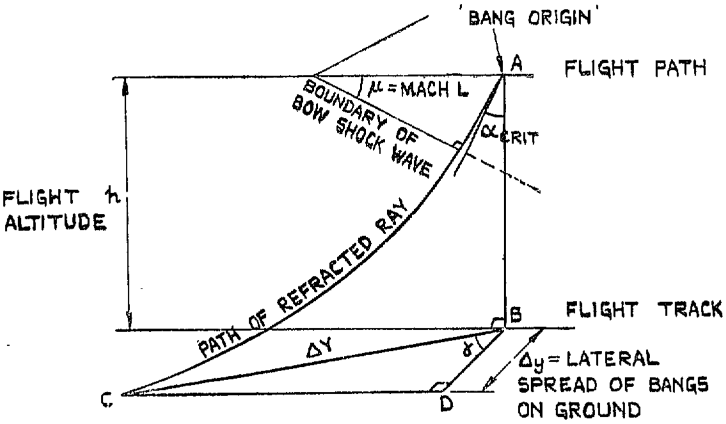

FIG. A.3.

TABLE 1

Summary of Peak Bang Pressures and Flight Data etc.

Date 1956	R.A.E. Run No.	Fairey Flight No.	Approximate flight conditions linked with bangs received at Nos. 1 and 2 sites: Altitude (ft) From radar	Altitude (ft) From aircraft	Indicated Mach No.	Critical Mach No.* (*see* Fig. 1 and Section 7)	Peak bang pressures in pounds per square foot: No. 1 'Radar' site Positive	No. 1 'Radar' site Negative	No. 2 'Tower' site Positive	No. 2 'Tower' site Negative	No. 3 'Beehive' site Positive	No. 3 'Beehive' site Negative	Remarks
15/10	2	—	No plot	31,810	1·1	1·122	—	—	Not recorded		—	—	Results probably affected by refraction.
15/10	3	—	No plot	31,810	1·1	1·122	—	—	—	—	—	—	
15/10	4	—	No plot	31,810	1·25	1·122	Not recorded		Not recorded		Not recorded		
15/10	5	—	No plot	31,810	1·5	1·122	Not recorded		0·51	0·42	0·35	0·55	
15/10	6	—	No plot	31,810	1·5	1·122	0·57	0·49	0·28	0·30	Not recorded		
21/10	9	200	No plot	26,780	1·1	1·100	—	—	Not recorded		—	—	No timing check on A/C record: altitude and Mach number estimated only. Results probably affected by refraction: bangs heard in Cazaux.
21/10	10	200	No plot	26,780	1·1	1·100	Not recorded		Not recorded		Not recorded		
21/10	11	201	No plot	26,780	1·2	1·102	Not recorded		Not recorded		Not recorded		
21/10	12	201	No plot	28,290*C*	1·3*d*	1·106	0·51	0·59	0·25	0·39	0·36	0·31	
21/10	13	202	No plot	27,290*L*	1·36*d*	1·102	0·45	0·61	0·41	0·40	Not recorded		
21/10	14	202	No plot	26,480*L*	1·22*a*	1·100	0·47	0·52	0·29	0·38	—	—	
22/10	16	204	No plot	26,650*C*	1·47*d*	1·100	0·50	0·57	Obscured by wind noise		0·29	0·15	
22/10	18	205	No plot	21,830*C*	1·1	1·078	0·59	1·00	0·49	0·52	Unmeasurable		
22/10	19	205	No plot	22,100*C*	1·15*d*	1·080	Not recorded		0·35	0·37	0·10	0·17	
22/10	20	206	25,000	21,360*L*	1·33*d*	1·077	0·78	0·87	0·35	0·40	0·10	0·13	
26/10	22	209	22,000	20,720*C*	1·3*a*	1·080	1·09	1·12	1·18	0·75	Site unmanned		Set No. 3 in use at 'Tower' site for runs 22, 23, 24.
26/10	23	210	No plot	17,740*L*	1·02*s*	1·065	0·40	0·30	0·91	0·84	Site unmanned		Reduced bang intensity probably due to refraction.
26/10	24	210	No plot	18,210*C*	1·10*d*	1·068	0·49	0·65	0·83	0·68	Site unmanned		

5/11	27	216	—	20,420*L*	1·20max	1·073	—	—	—	—	Site unmanned no power		No flight histories, max *M* values only. A/C off course.
5/11	28	217	No plot	21,360*L*	1·29max	1·077	Not recorded		Not recorded		Site unmanned no power		Bangs heard in Cazaux.
6/11	29	218	17,200	16,910*L*	1·06*d*	1·062	0·52	0·69	0·58	0·79	Site unmanned no power		Reduced bang intensity probably due to refraction.
6/11	30	218	16,900	16,400*L*	1·17*d*	1·060	1·01	1·31	0·90	1·12	Site unmanned no power		
6/11	31	219	15,400	15,070*L*	1·26*a*	1·055	1·28	1·31	1·37	1·20	Site unmanned no power		
6/11	32	219	15,500	14,460*L*	Estimate just sonic *a*	1·053	0·58	0·71	—	—	Site unmanned no power		No synchro between A/C and radar—radio U/S. Reduced bang intensity probably due to refraction.
6/11	33	220	14,300	13,830*L*	1·11*d*	1·050	0·51	0·45	1·89	1·61	Site unmanned no power		Bang pressures for 'Tower' site are for second N-wave (*see* Fig. 61).
6/11	34	220	15,000	14,230*C*	1·18*s*	1·052	1·35	1·63	0·51	0·65	Site unmanned no power		
6/11	35	221	—	—	Subsonic	—	—	—	—	—	Site unmanned no power		A/C just supersonic for only 12 seconds.
6/11	36	221	16,300	15,570*C*	1·14*d*	1·057	0·83	1·05	0·97	1·18	Site unmanned no power		
7/11	37	222	20,000	19,920*C*	1·27*d*	1·072	0·81	1·01	Not recorded		—	—	
7/11	38	223	—	—	Subsonic (0·97)	—	—	—	—	—	—	—	
7/11	39	223	11,850	11,680*C*	1·04*d*	1·040	0·92	1·11	0·95	1·33	—	—	Reduced bang intensity probably due to refraction.
7/11	40	224	13,650	11,890*C*	1·12*d*	1·041	0·92	1·36	0·87	1·15	—	—	
7/11	41	225	9,050	8,870*C*	1·06*a*	1·030	—	—	—	—	—	—	
7/11	42	225	9,650	9,370*C*	1·07*d*	1·031	1·50	1·60	1·14	1·65	—	—	
8/11	43	227	7,350	6,660*C*	1·07*d*	1·023	0·30	0·45	Very weak		—	—	
8/11	44	227	6,330	6,130*C*	1·08*a*	1·021	4·05	3·90	—	—	—	—	Flight conditions apply to bangs received at 'Radar' site only.
8/11	45	228	6,330	5,960*C*	1·09*d*	1·020	2·85	2·80	Very weak		—	—	
8/11	46	228	6,260	5,960*C*	1·10*d*	1·020	2·65	2·73	Very weak		—	—	

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* Mach number (at bang origin) below which bangs should not reach ground.

C = climbing; L = level; a = accelerating; d = decelerating; s = steady.

TABLE 1—*continued*

Date 1956	R.A.E. Run No.	Fairey Flight No.	Approximate flight conditions linked with bangs received at Nos. 1 and 2 sites			Critical Mach No.* (*see* Fig. 1 and Section 7)	Peak bang pressures in pounds per square foot						Remarks
			Altitude (ft)		Indicated Mach No.		No. 1 'Radar' site		No. 2 'Tower' site		No. 3 'Beehive' site		
			From radar	From aircraft			Positive	Negative	Positive	Negative	Positive	Negative	
8/11	47	229	19,650	19,420*C*	1·22*a*	1·072	1·19	1·01	0·82	1·03	—	—	
8/11	48	229	22,000	23,100*C*	1·24*d*	1·088	0·80	0·80	0·67	0·73	—	—	
8/11	49	230	27,300	26,950*C*	1·47*a*	1·106	0·57	0·49	0·87	0·73	0·42	0·47	
9/11	S.50	231	—	4,850*L*	1·05*a*	1·016	3·95	5·25	3·88	5·25	—	—	Special site 9 miles south of Lacanau-Océan, 1 and 2 sets together.
9/11	S.51	232	—	4,020*C*	1·03*a*	1·013	3·40	4·07	3·30	4·10	—	—	

* Mach number (at bang origin) below which bangs should not reach ground.

C = climbing; *L* = level; *a* = accelerating; *d* = decelerating; *s* = steady.

TABLE 2

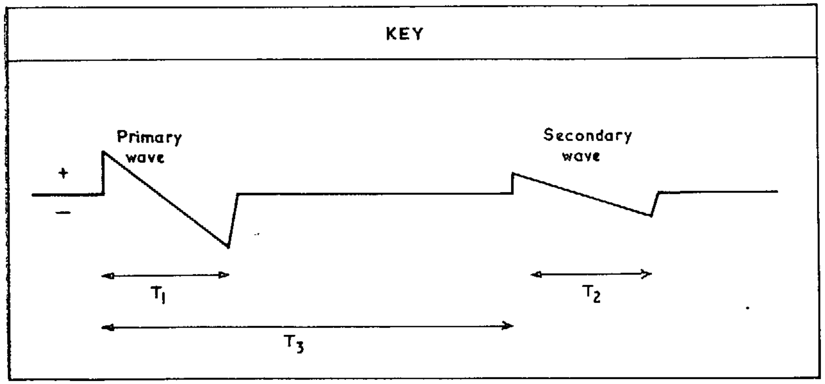
Summary of Bang Impulse Values and Wave Durations etc.

Date 1956	R.A.E. Run No.	Fairey Flight No.	Site	No. of bangs heard	Pressure impulse from primary N-wave in lb sec/ft^2		Pressure impulse from secondary N-wave in lb sec/ft^2		T_1 sec	T_2 sec	T_3 sec	Remarks
					Positive	Negative	Positive	Negative				
15/10	5	—	Tower	2	0·0102	0·0084	—	—	0·0820	—	—	
15/10	5	—	Beehive	2	0·0092	0·0188	—	—	0·0673	—	—	
15/10	6	—	Radar	1	0·0154	0·0135	—	—	0·0618	—	—	
15/10	6	—	Tower	2	0·0074	0·0053	0·0045	0·0078	0·0820	0·0780	0·1110	
21/10	12	201	Radar	2	0·0133	0·0113	—	—	0·0650	—	—	
21/10	12	201	Tower	2	0·0065	0·0080	—	—	0·0745	—	—	
21/10	12	201	Beehive	1	0·0046	0·0068	—	—	0·0718	—	—	No timing wave: estimates made from other films.
21/10	13	202	Radar	2	0·0130	0·0116	—	—	0·0720	—	—	
21/10	13	202	Tower	2	0·0083	0·0064	—	—	0·0673	—	—	
21/10	14	202	Radar	2	0·0154	0·0103	—	—	0·0700	—	—	
21/10	14	202	Tower	2+2	0·0076	0·0067	0·0048	0·0059	0·0591	0·0727	0·1045	
22/10	16	204	Radar	1	0·0137	0·0114	—	—	0·0700	—	—	
22/10	16	204	Beehive	1	—	—	—	—	—	—	—	Pressure wave of very small amplitude—too difficult to measure.
22/10	18	205	Radar	2	0·0230	0·0421	—	—	0·0820	—	—	Unusual pressure waveforms: may be due to refraction. *See* Section 7.
22/10	18	205	Tower	2	—	—	—	—	—	—	—	
22/10	19	205	Tower	2	0·0112	0·0476	—	—	0·0800	—	—	Some wind noise apparent on film.
22/10	19	205	Beehive	2	—	—	—	—	—	—	—	Pressure wave of very small amplitude—too difficult to measure.
22/10	20	206	Radar	2	0·0151	0·0125	—	—	0·0650	—	—	
22/10	20	206	Tower	2	0·0080	0·0070	—	—	0·0700	—	—	
22/10	20	206	Beehive	2	—	—	—	—	—	—	—	Pressure wave of small amplitude —too difficult to measure.
26/10	22	209	Radar	2	0·0198	0·0159	—	—	0·0600	—	—	

TABLE 2—*continued*

Date 1956	R.A.E. Run No.	Fairey Flight No.	Site	No. of bangs heard	Pressure impulse from primary N-wave in lb sec/ft^2		Pressure impulse from secondary N-wave in lb sec/ft^2		T_1 sec	T_2 sec	T_3 sec	Remarks
					Positive	Negative	Positive	Negative				
26/10	22	209	Tower	2+2	0·0109	0·0112	—	—	0·0822	—	—	No secondary pressure disturbance observed on film.
26/10	23	210	Radar	2	0·0105	0·0104	—	—	0·0692	—	—	Unusual pressure waveforms: may be affected by refraction. *See* Section 7.
26/10	23	210	Tower	2	0·0243	0·0227	—	—	0·0822	—	—	
26/10	24	210	Radar	3	—	—	—	—	—	—	—	Very unusual pressure waveform possibly due to interference between 2 N-waves (*see* Section 7).
26/10	24	210	Tower	2	0·0090	0·0106	—	—	0·0933	0·1600	0·3800	*See* Fig. 56.
6/11	29	218	Radar	2	0·0198	0·0190	0·0265	0·0281	0·0921	0·0900	0·3360	Unusual triply-peaked secondary pressure disturbance. *See* Section 7 and Fig. 57.
6/11	29	218	Tower	2+3	0·0216	0·0228	0·0116	0·0191	0·0900	0·1145	0·6764	
6/11	30	218	Radar	2+2	0·0299	0·0280	0·0034	0·0120	0·0920	0·1300	2·7000	Secondary pressure waves much delayed (*see* Fig. 58).
6/11	30	218	Tower	2+2	0·0233	0·0242	0·0041	0·0141	0·0755	0·1127	3·0700	
6/11	31	219	Radar	2	0·0348	0·0283	0·0163	0·0182	0·0840	0·1286	2·9900	Secondary pressure waves much delayed (*see* Fig. 59).
6/11	31	219	Tower	2+2	0·0300	0·0239	0·0078	0·0192	0·0760	0·1109	2·9800	
6/11	32	219	Radar	1 (dull boom)	0·0220	0·0281	—	—	0·0940	—	—	N-wave not sharp.
6/11	33	220	Radar	1 (dull boom)	0·0215	0·0203	—	—	0·0800	—	—	N-wave not sharp.
6/11	33	220	Tower	2	0·0454	0·0360	0·0745	0·0802	0·1427	0·1155	0·1427	Secondary pressure wave of remarkably high amplitude (*see* Fig. 61).
6/11	34	220	Radar	2+2	0·0401	0·0396	—	—	0·0860	—	—	No apparent secondary wave on film.
6/11	34	220	Tower	2+2	0·0176	0·0141	0·0051	0·0211	0·0764	0·1855	1·3700	
6/11	36	221	Radar	2	0·0278	0·0232	0·0197	0·0311	0·0891	0·2000	1·0300	Secondary pressure wave of unusual shape and of high amplitude (*see* Fig. 63).
6/11	36	221	Tower	2+2	0·0345	0·0400	0·0162	0·0330	0·0818	0·1036	0·4336	
7/11	37	222	Radar	2	0·0180	0·0148	—	—	0·0655	—	7·0000	Note remarkably long T_3 (*see* Fig. 64).

7/11	39	223	Radar	2	0·0294	0·0290	0·0105	0·0135	0·1125	0·1625	1·7400	
7/11	39	223	Tower	2+2	0·0336	0·0384	—	—	0·0836	—	—	No secondary wave observed on film.
7/11	40	224	Radar	2	0·0314	0·0291	—	—	0·1000	—	—	
7/11	40	224	Tower	2+2	0·0276	0·0307	—	—	0·0782	—	—	No secondary wave observed on film.
7/11	42	225	Radar	2	0·0470	0·0460	—	—	0·1050	—	—	N-wave with low rate of rise and fall of pressure.
7/11	42	225	Tower	2	0·0346	0·0378	0·0078	0·0157	0·0764	0·0818	0·7927	
8/11	43	227	Radar	1 (dull boom)	0·0230	0·0276	—	—	0·2075	—	—	Pressure wave of low amplitude.
8/11	44	227	Radar	2+2	0·1160	0·1016	—	—	0·1075	—	—	No secondary wave observed on film.
8/11	45	228	Radar	2	0·0760	0·0646	—	—	0·1000	—	—	
8/11	46	228	Radar	2	0·0620	0·0620	—	—	0·0950	—	—	Primary N-wave shows unusual peakiness (*see* Fig. 71).
8/11	47	229	Radar	2	0·0252	0·0230	—	—	0·0900	—	—	
8/11	47	229	Tower	2	0·0255	0·0255	—	—	0·0636	—	—	
8/11	48	229	Radar	1 (dull boom)	0·0217	0·0193	—	—	0·0740	—	—	
8/11	48	229	Tower	2	0·0202	0·0187	—	—	0·0600	—	—	
8/11	49	230	Radar	1 (dull boom)	0·0156	0·0074	—	—	0·0700	—	—	
8/11	49	230	Tower	1 (dull boom)	0·0182	0·0173	—	—	0·0645	—	—	
8/11	49	230	Beehive	2	0·0155	0·0179	—	—	0·0627	—	—	
9/11	S.50/1	231	Special	2	0·1230	0·1484	0·0262	0·0452	0·0746	0·1062	0·1277	
9/11	S.50/2	231	Special	2	0·1540	0·1400	0·0316	0·0396	0·0800	0·1164	0·1530	
9/11	S.51/1	232	Special	2	0·0828	0·0714	0·0084	0·0242	0·0643	0·1200	0·2940	
9/11	S.51/2	232	Special	2	0·0975	0·0964	0·0110	0·0352	0·0782	0·1545	0·3691	



TABLE

Extracts from Sounding

Height \ Date	15 Oct. 1400 h	21 Oct. 1400 h	22 Oct. 1400 h	26 Oct. 1400 h
0	1016 +20·0 12·0 120/04	1030 +17·0 8·0 080/10	1020 +21·0 10·0 120/12	1018 +10·0 5·3 340/08
500	961 +17·0 8·8 180/08	970 +11·6 8·0 060/12	968 +14·0 7·2 120/15	955 +5·0 5·0 340/12
1000	908 +11·9 7·2 200/15	917 +10·0 4·5 050/17	910 +10·2 5·8 130/20	902 +0·5 4·2 330/18
1500	856 +7·5 6·2 210/20	863 +9·5 3·2 030/18	856 +8·0 7·2 140/30	849 −2·0 3·5 330/21
2000	804 +5·5 4·8 210/20	810 +9·0 3·5 030/21	806 +6·0 3·7 130/29	794 −5·6 2·7 330/23
2500	758 +2·3 4·3 200/20	761 +7·0 3·5 030/23	760 +4·4 3·4 120/25	744 −9·0 2·0 330/26
3000	710 −1·1 3·7 200/19	715 +4·5 5·1 030/25	714 +2·7 3·0 110/22	700 −12·0 1·6 330/30
3500	666 −4·5 3·4 200/19	672 +2·7 5·0 030/28	671 +0·8 2·5 110/22	656 −15·5 1·0 330/34
4000	626 −7·2 2·6 200/19	631 +0·7 5·0 030/30	631 −1·6 1·4 110/22	616 −19·2 0·6 330/42
4500	586 −10·0 1·9 200/18	594 −2·0 4·5 030/32	592 −3·7 1·0 100/25	574 −21·6 0·7 330/45
5000	548 −12·5 1·2 210/18	560 −5·5 3·5 040/34	557 −7·2 0·8 100/25	537 −24·6 0·6 340/50
5500	513 −16·4 0·8 210/18	526 −7·8 1·8 040/37	521 −10·8 0·7 100/28	500 −28·3 0·4 360/59
6000	480 −20·0 0·6 200/18	493 −10·5 1·8 040/39	489 −14·2 0·6 100/31	467 −30·4 0·4 340/73
6500	448 −23·6 0·5 200/18	462 −13·7 1·2 030/42	458 −17·6 0·4 100/32	
7000	420 −26·8 0·4 180/19	432 −17·8 0·8 020/50	427 −21·7 0·3 090/32	
7500	390 −30·2 0·3 170/19	403 −21·2 0·6 020/60	398 −25·2 0·2 090/33	
8000	363 −33·5 0·2 170/19	377 −25·7 0·4 020/60		
8500	339 −37·2 0·1 180/18	352 −29·7 0·3 020/60		
9000	316 −41·0 0·1 180/18	326 −34·0 0·2 010/58		

3

Data at Bordeaux

Day and Time

Height in geopotential metres	Pressure in millibars	Temperature in degrees C	Mixing ratio in gm per kg
	Wind: direction in degrees/velocity in kt		

5 Nov. 1400 h	6 Nov. 1400 h	7 Nov. 1400 h	8 Nov. 1400 h	9 Nov. 1400 h
1023 +12·0 5·8 020/04	1022 +13·0 5·8 070/08	1020 +15·2 6·0 150/08	1012 +15·0 7·5 120/12	1018 +17·0 7·0 230/12
969 +8·4 5·2 360/08	955 +6·0 4·2 070/12	968 +13·0 4·0 160/10	960 +15·0 4·7 130/15	960 +12·0 8·8 240/17
911 +3·4 4·5 360/08	900 +3·5 3·0 080/17	910 +13·0 3·1 160/12	900 +12·2 3·2 210/30	907 −8·5 7·2 260/18
859 −1·0 3·7 350/09	850 +0·5 2·0 090/19	858 +9·5 2·8 170/15	850 +9·3 2·3 210/28	853 −5·2 6·3 270/19
810 −3·0 3·7 360/12	795 +4·6 1·1 100/16	808 +7·0 1·9 170/12	795 +6·3 1·8 200/30	
761 +1·2 2·7 360/20	748 +2·2 0·8 120/12	760 +4·0 2·4 180/11	750 +2·7 1·1 190/35	
715 0·0 2·5 010/25	700 −1·0 0·7 090/12	712 +1·1 2·2 190/10	700 +0·8 0·8 180/28	
670 −2·6 2·2 010/24	658 −4·5 0·6 060/10	667 −2·4 0·8 180/10	658 −2·4 0·6 180/25	
628 −7·2 1·8 360/24	618 −8·0 0·4 040/10	626 −6·0 0·7 170/12	618 −6·0 0·4 180/22	
589 −10·7 1·4 350/23	578 −9·3 0·4 030/10	588 −9·3 0·5 160/14	578 −9·0 0·4 180/20	
550 −13·4 1·0 340/24		548 −13·2 0·4 150/15	542 −12·0 0·3 180/18	
515 −16·0 0·7 340/23		516 −16·5 0·3 140/18	505 −16·0 0·2 180/17	
482 −19·0 0·5 340/21			473 −19·8 0·2 180/19	
			442 −29·7 0·1 180/20	
			412 −27·6 0·1 180/20	
			384 −30·0 0·1 180/18	

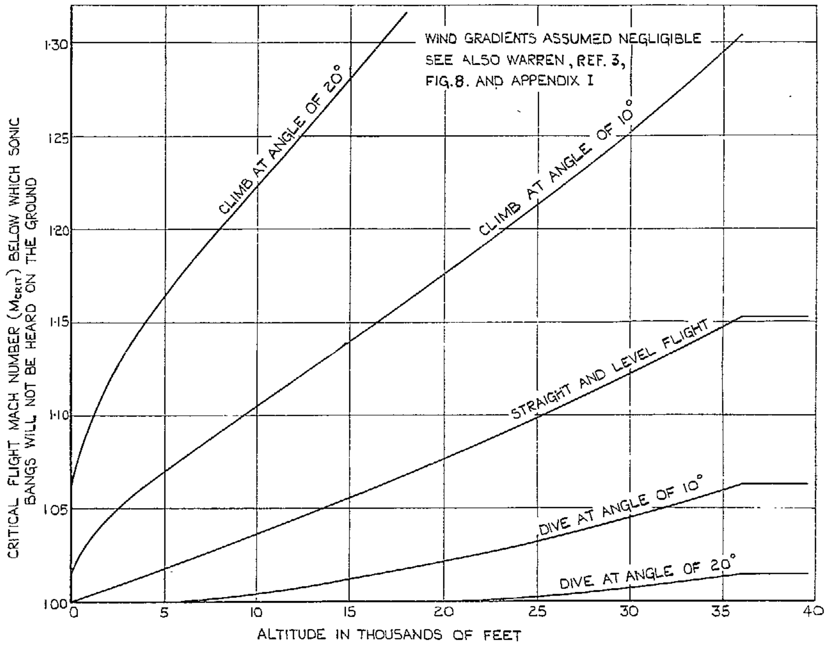


FIG. 1. Effect of refraction on the occurrence of sonic bangs from an aircraft in steady straight flight in the standard atmosphere.

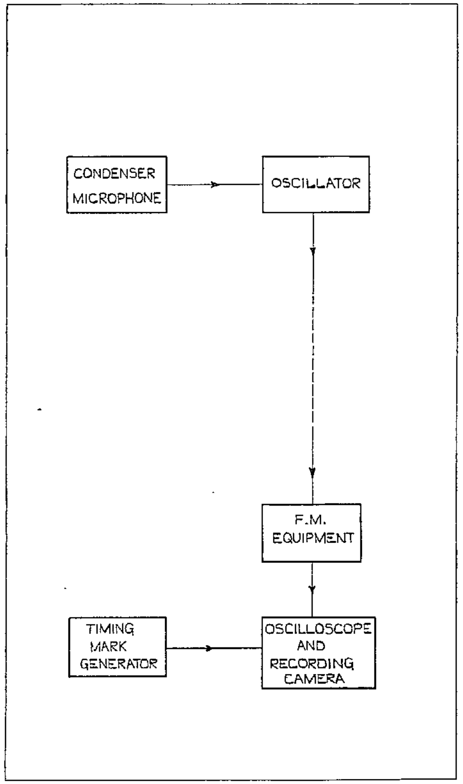


FIG. 2. Schematic diagram of recording apparatus.

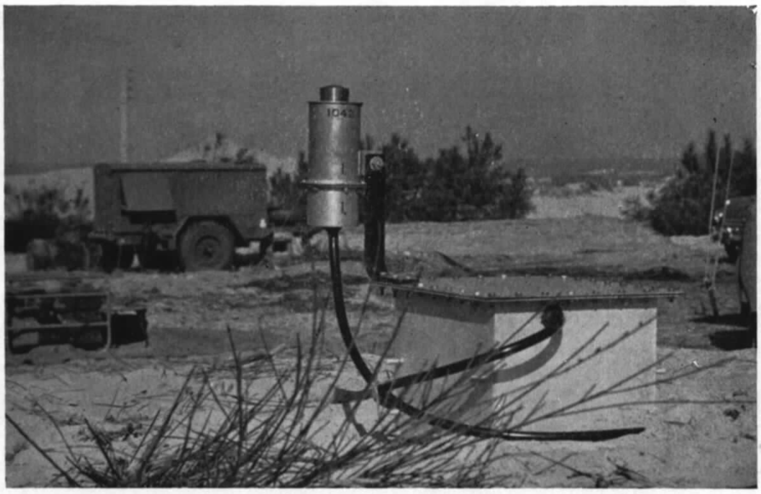


FIG. 3a. Installation of microphone and oscillator box at 'Radar' site.

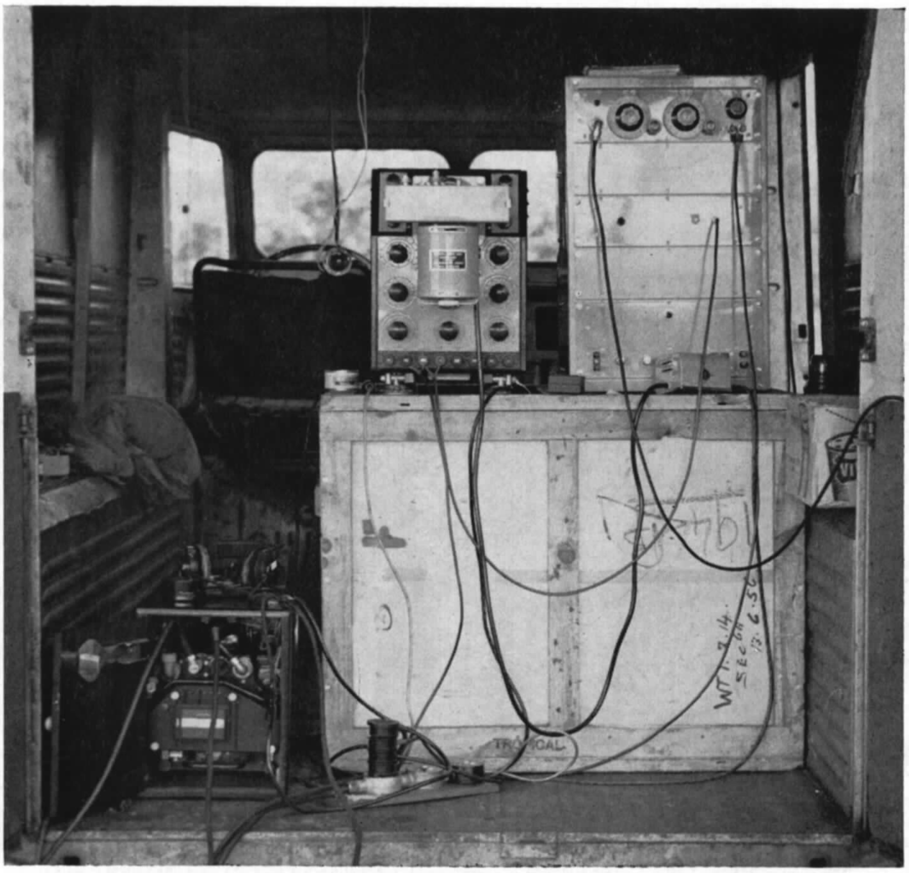


FIG. 3b. Installation of F.M. equipment, recording oscillograph, and intersite communications transceiver in Citroën van at 'Radar' site.

FIG. 3c. Aerial views of No. 1 'Radar' site.

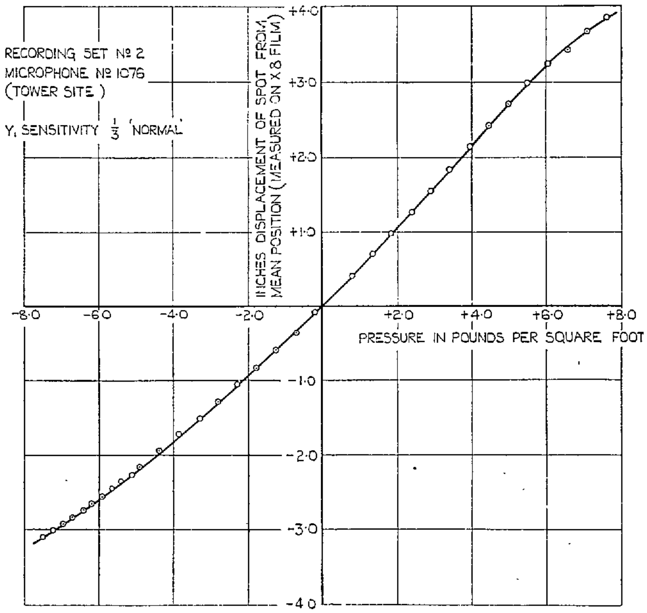


FIG. 4. Example of a static-calibration curve for one of the recording sets.

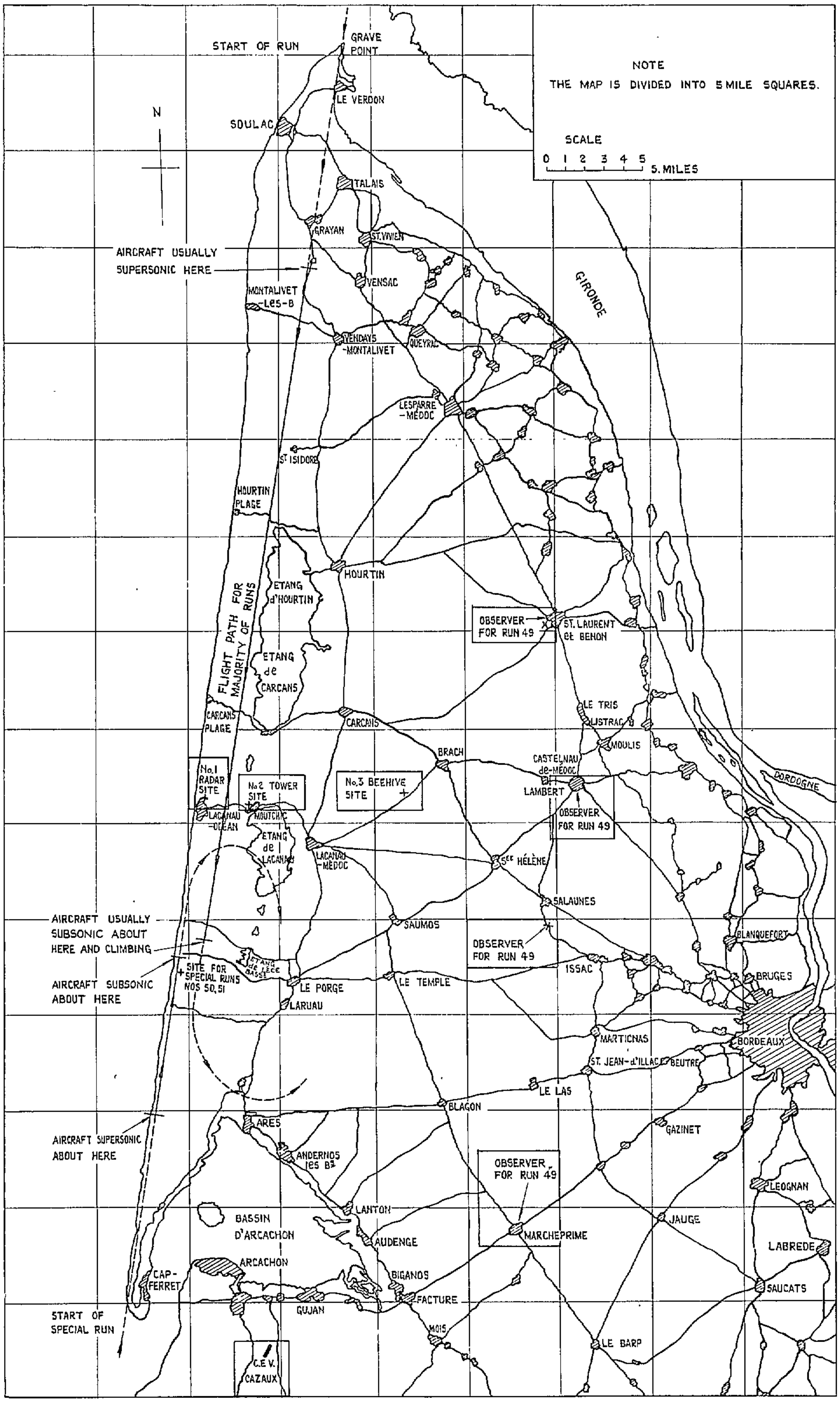


FIG. 5. Sketch map of the trials area.

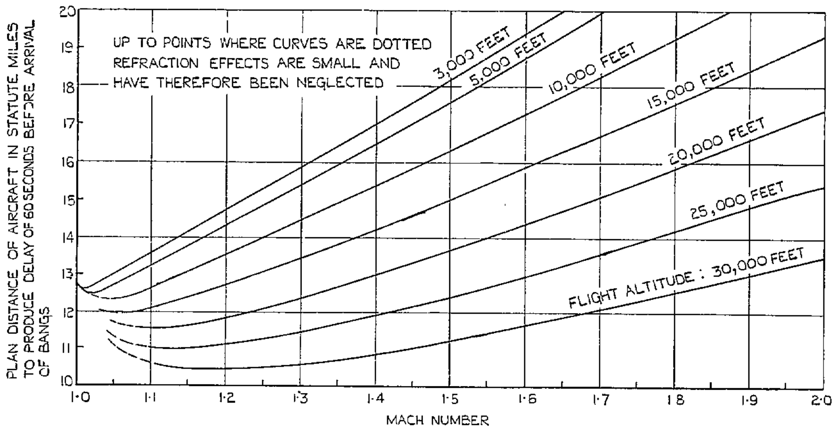
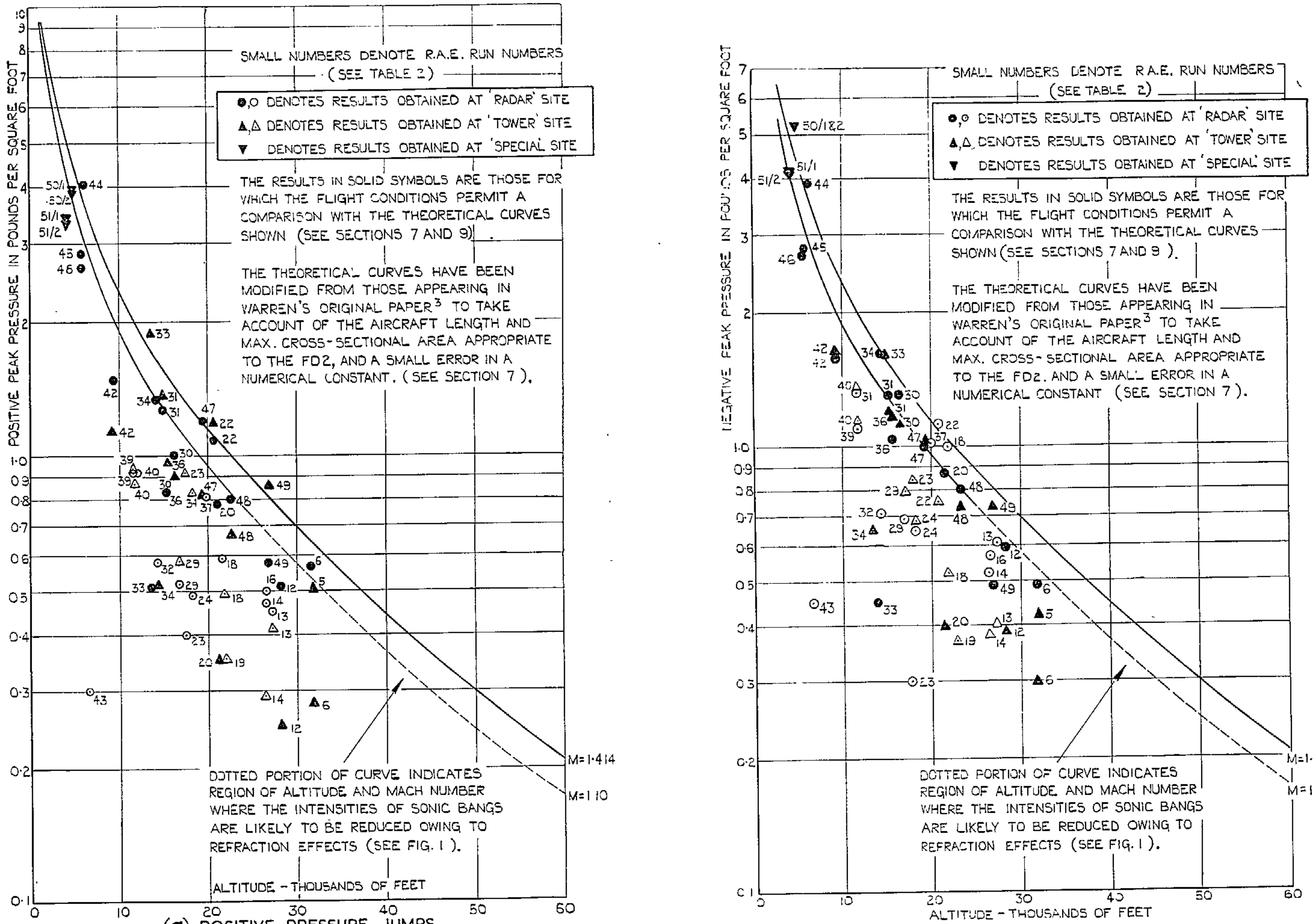


Fig. 6. Curves for determining plan distance of aircraft as a function of Mach number and altitude which will give one minute delay before bangs arrive.



SMALL NUMBERS DENOTE R.A.E. RUN NUMBERS (SEE TABLE 2)

THE RESULTS IN SOLID SYMBOLS ARE THOSE FOR WHICH THE FLIGHT CONDITIONS PERMIT A COMPARISON WITH THE THEORETICAL CURVES SHOWN (SEE SECTIONS 7 AND 9).

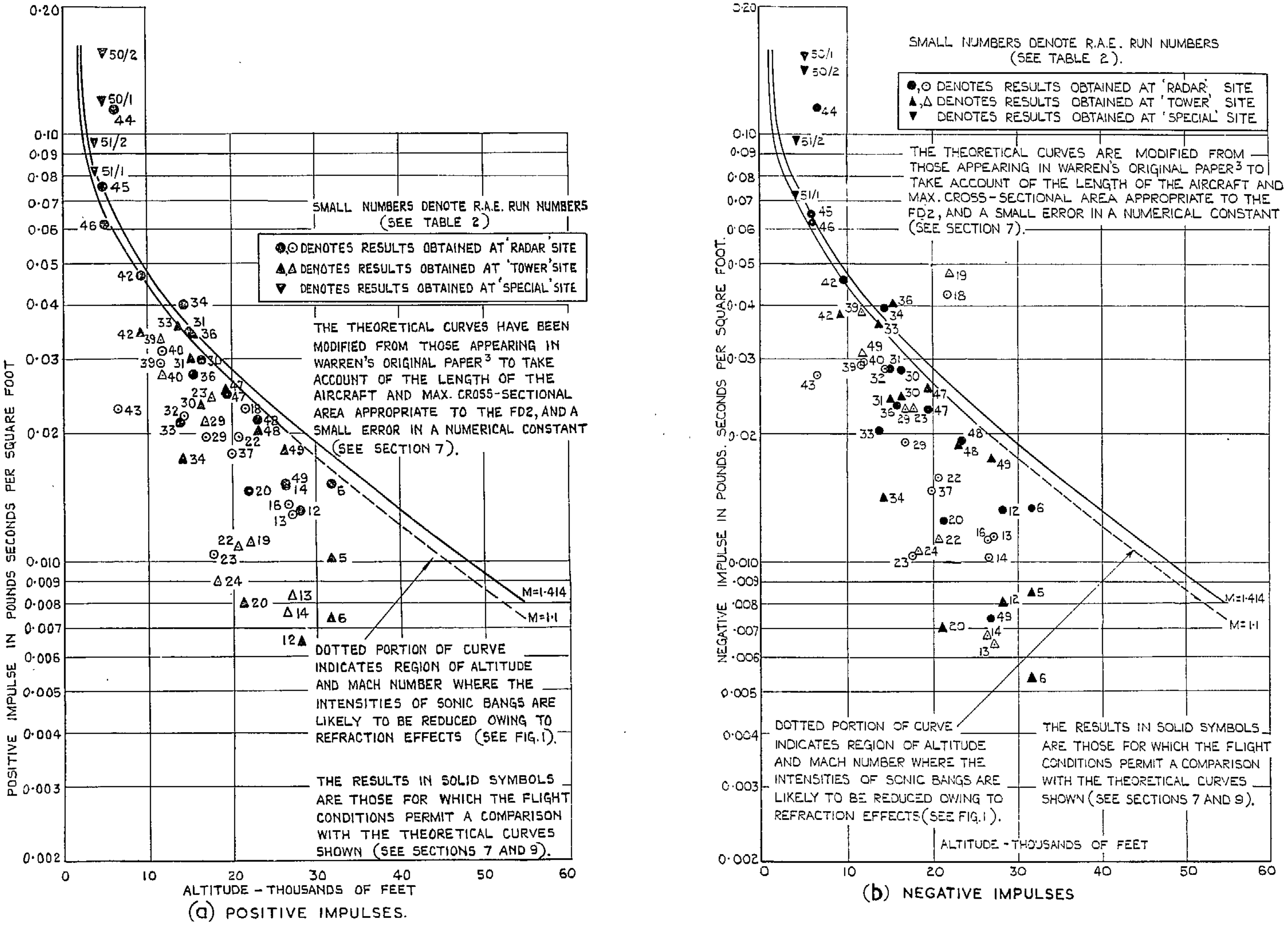
THE THEORETICAL CURVES HAVE BEEN MODIFIED FROM THOSE APPEARING IN WARREN'S ORIGINAL PAPER³ TO TAKE ACCOUNT OF THE AIRCRAFT LENGTH AND MAX. CROSS-SECTIONAL AREA APPROPRIATE TO THE FD2, AND A SMALL ERROR IN A NUMERICAL CONSTANT. (SEE SECTION 7).

DOTTED PORTION OF CURVE INDICATES REGION OF ALTITUDE AND MACH NUMBER WHERE THE INTENSITIES OF SONIC BANGS ARE LIKELY TO BE REDUCED OWING TO REFRACTION EFFECTS (SEE FIG. 1).

(a) POSITIVE PRESSURE JUMPS.

(b) NEGATIVE PRESSURE JUMPS

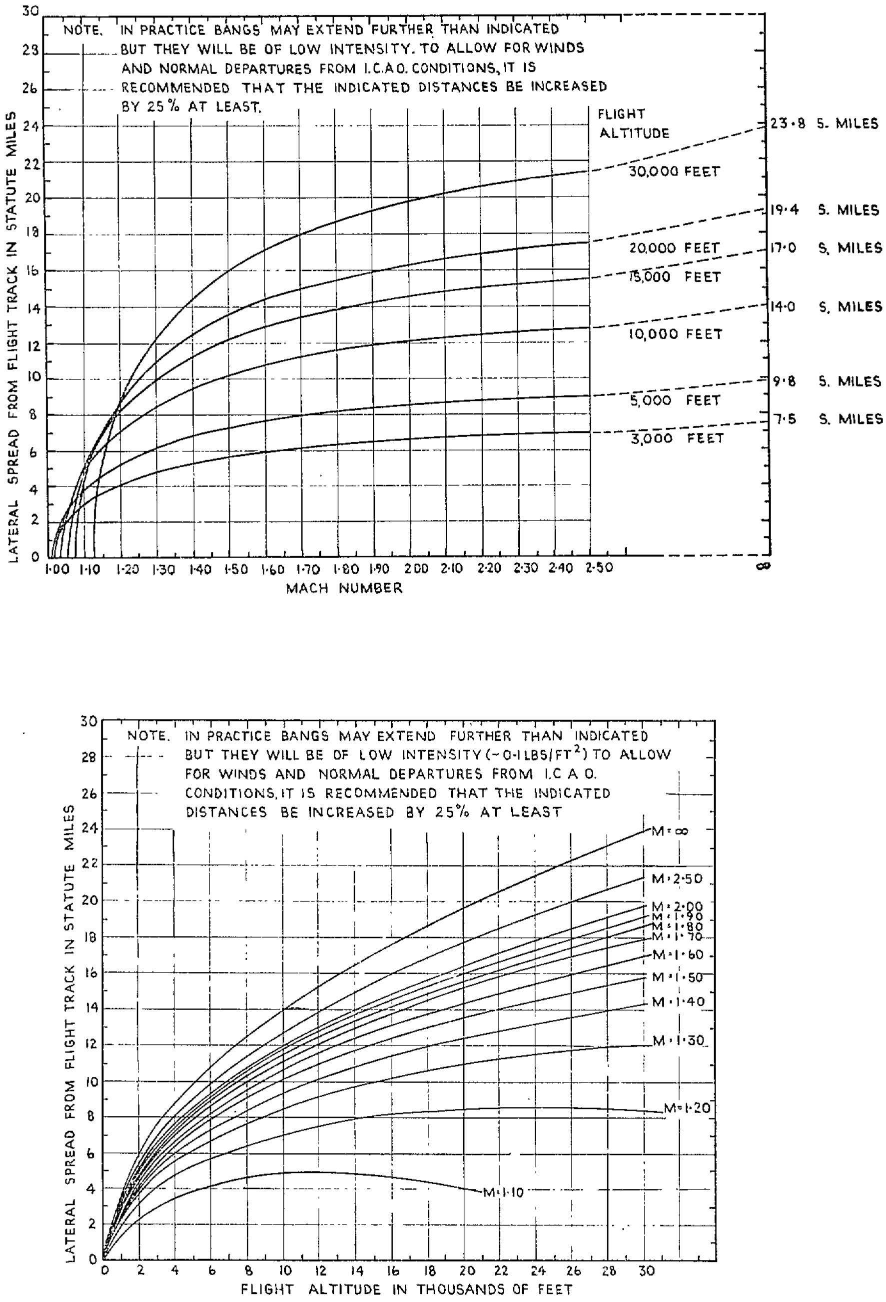
FIGS. 7a and b. A comparison of the observed peak bang pressures with the theory of Warren for steady level flight.



(a) POSITIVE IMPULSES.

(b) NEGATIVE IMPULSES

FIGS. 8a and b. A comparison of the observed bang impulses with the theory of Warren for steady level flight.



FIGS. 9a and b. Theoretical estimates of the lateral spread from the flight track of sonic bangs produced by straight and level flight in the standard atmosphere (no wind).

RADAR PLOTS

Figs. 10 to 23

Notes

1 Points 'SYNC' (or 'TIMING CHECK') serve to synchronise the plan and altitude plots.

2 Calculated positions of the 'bang origins' shown are APPROXIMATE only: analysis was in 2-dimensions only and refraction was neglected. (*See* Section 7.)

3 The time between the beginning of each dash on the plot is one second.

4 The marking 'BANG' on some of the later plots indicates the aircraft's position when the bangs were heard at 'Radar' site.

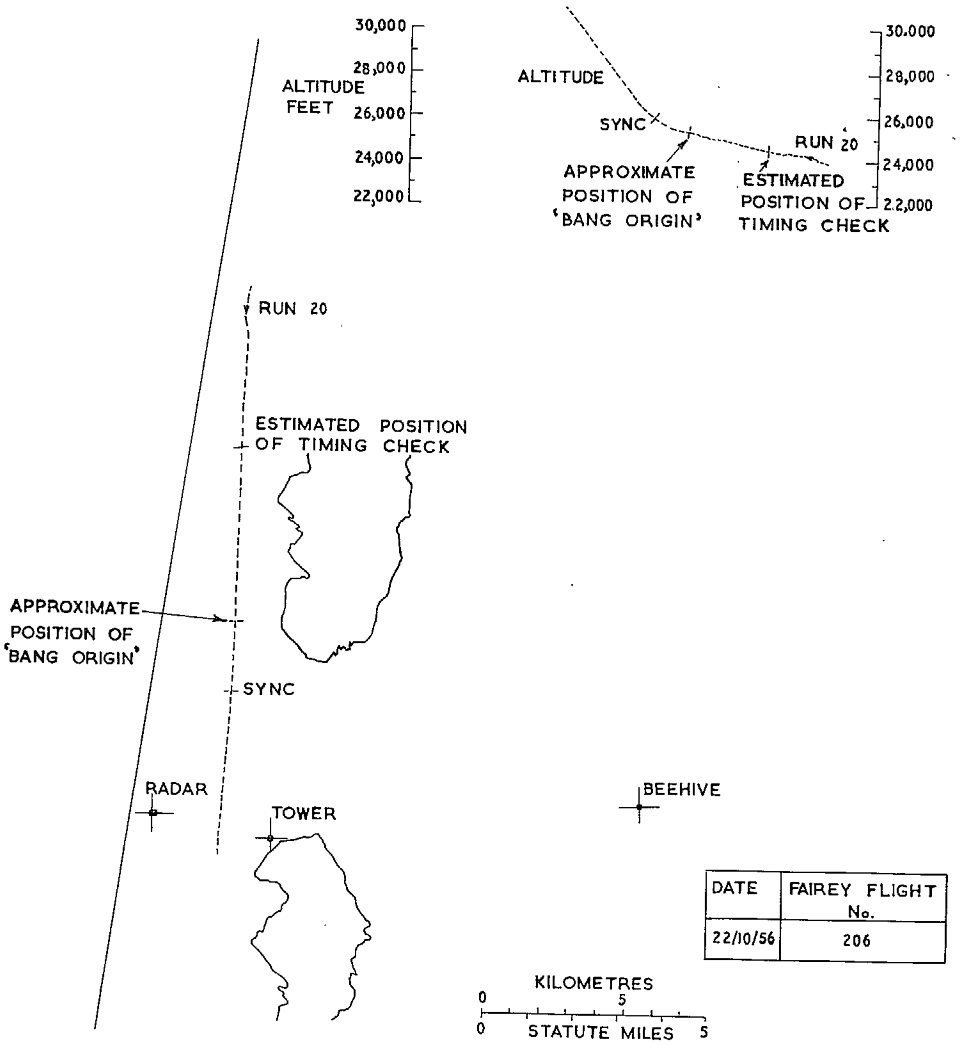


FIG. 10. Radar plot. Run 20.

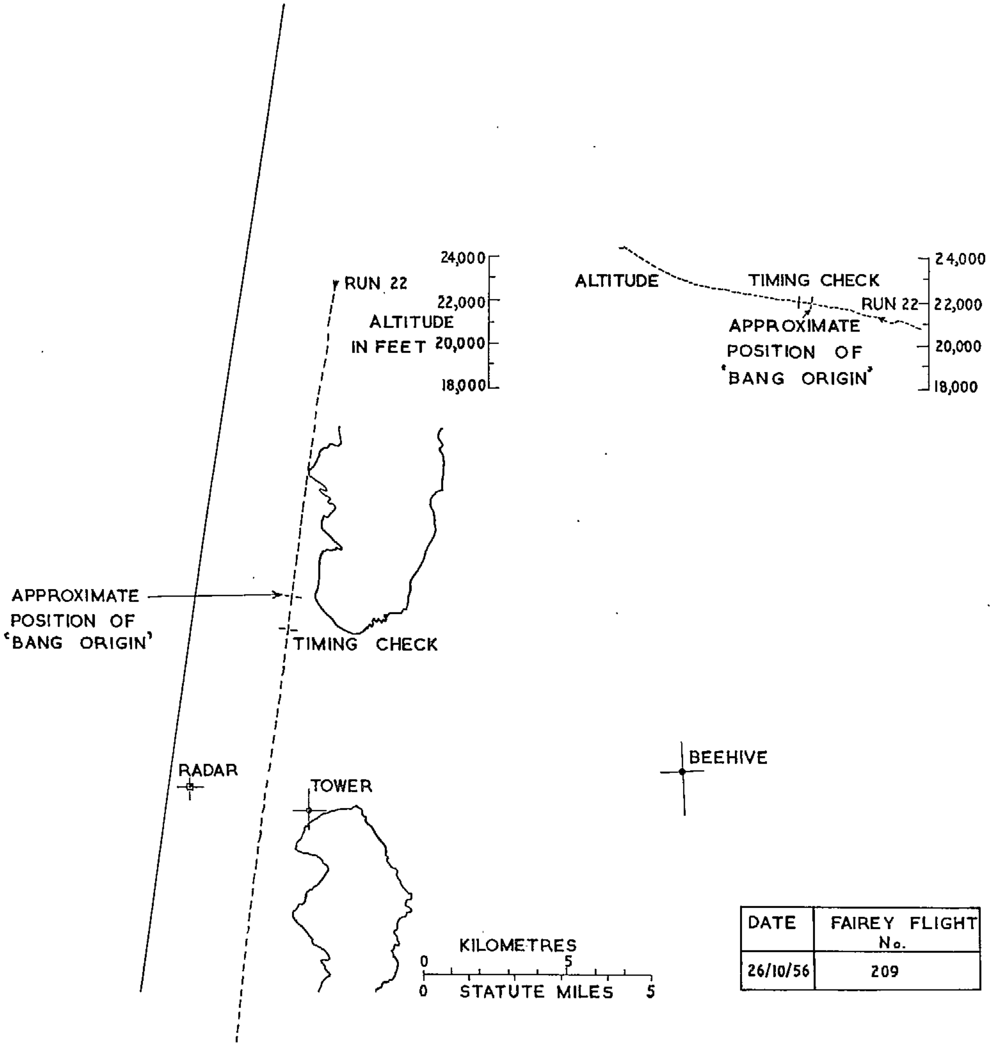


FIG. 11. Radar plot. Run 22.

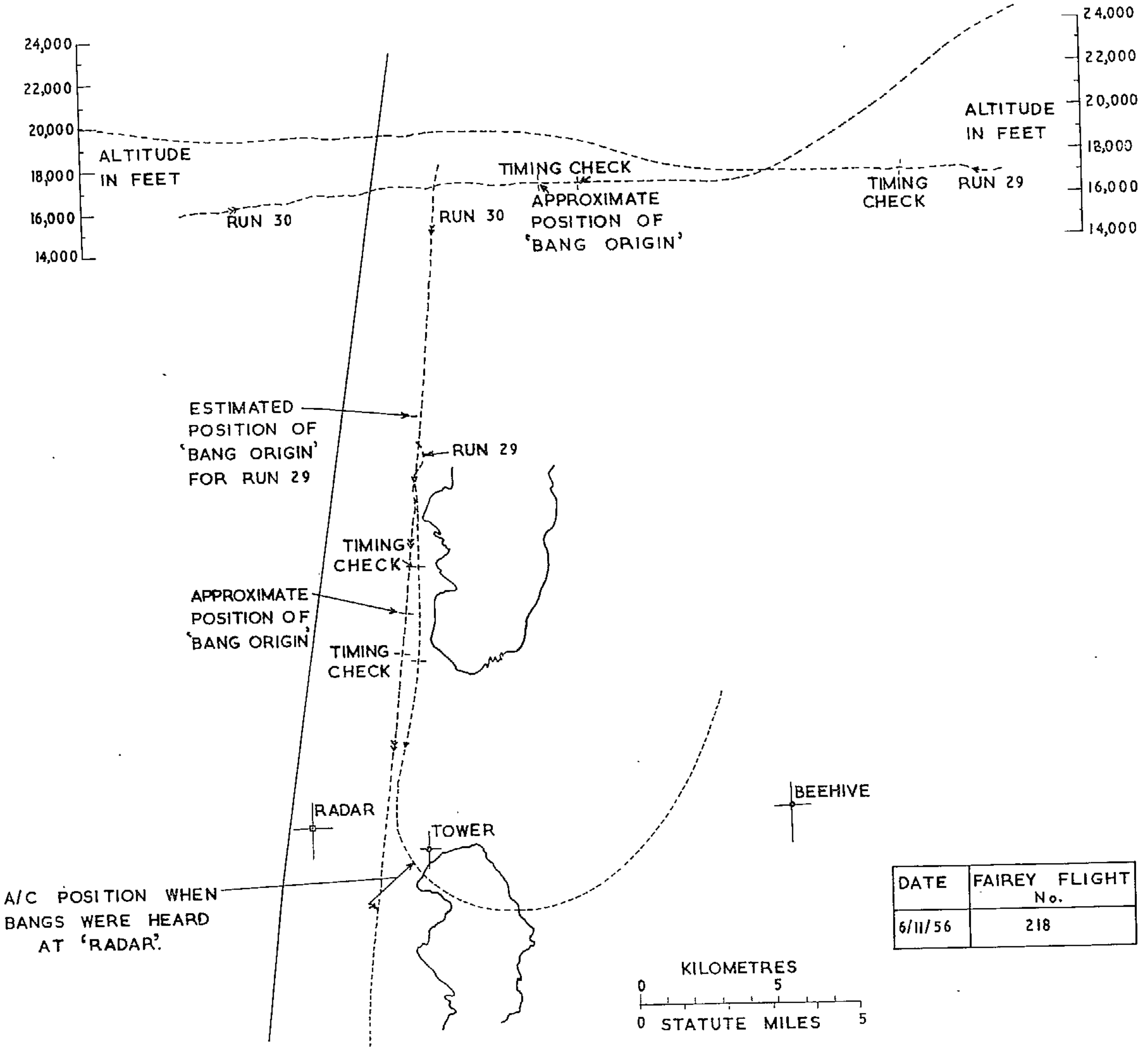


FIG. 12. Radar plot. Runs 29 and 30.

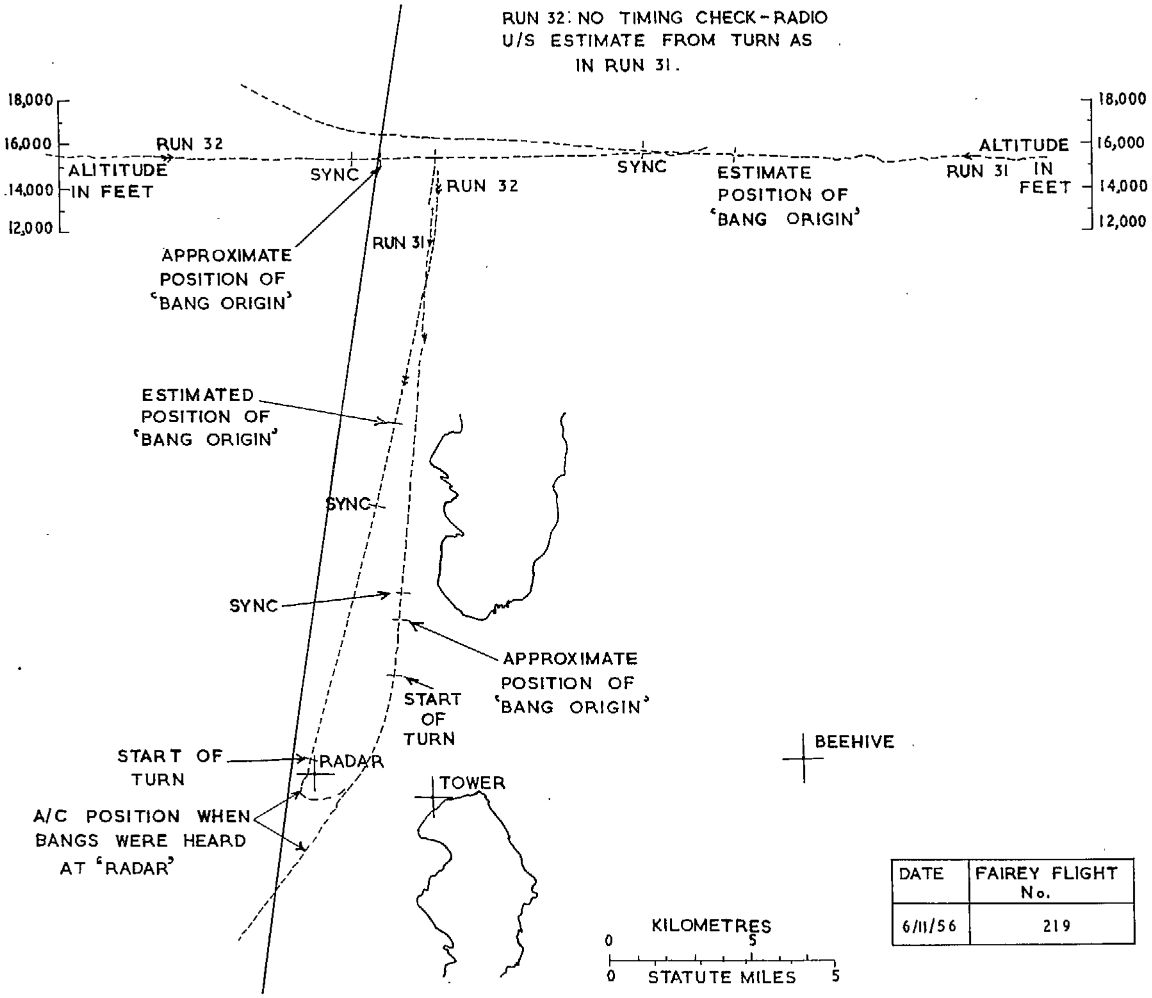


Fig. 13. Radar plot. Runs 31 and 32.

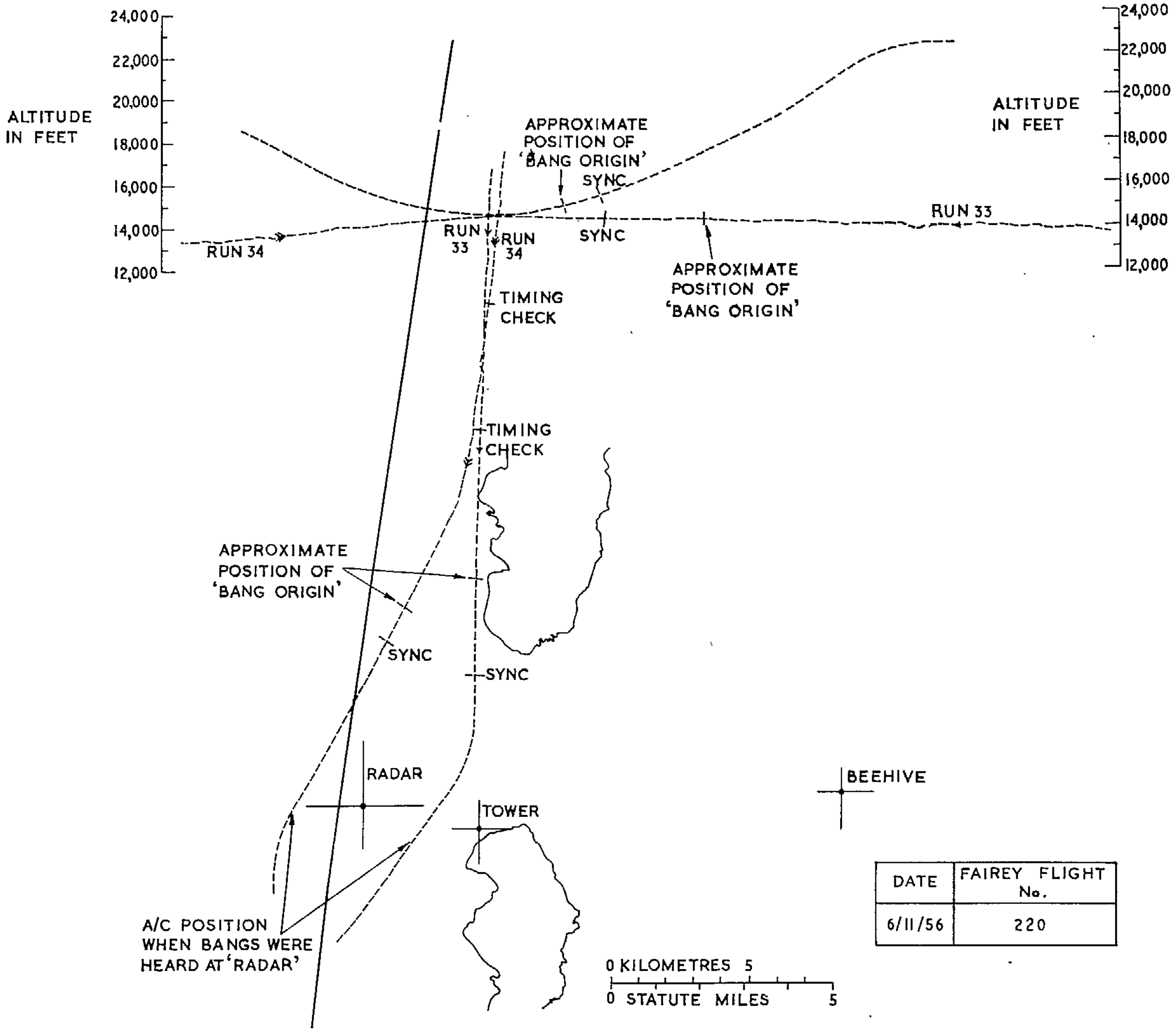


FIG. 14. Radar plot. Runs 33 and 34.

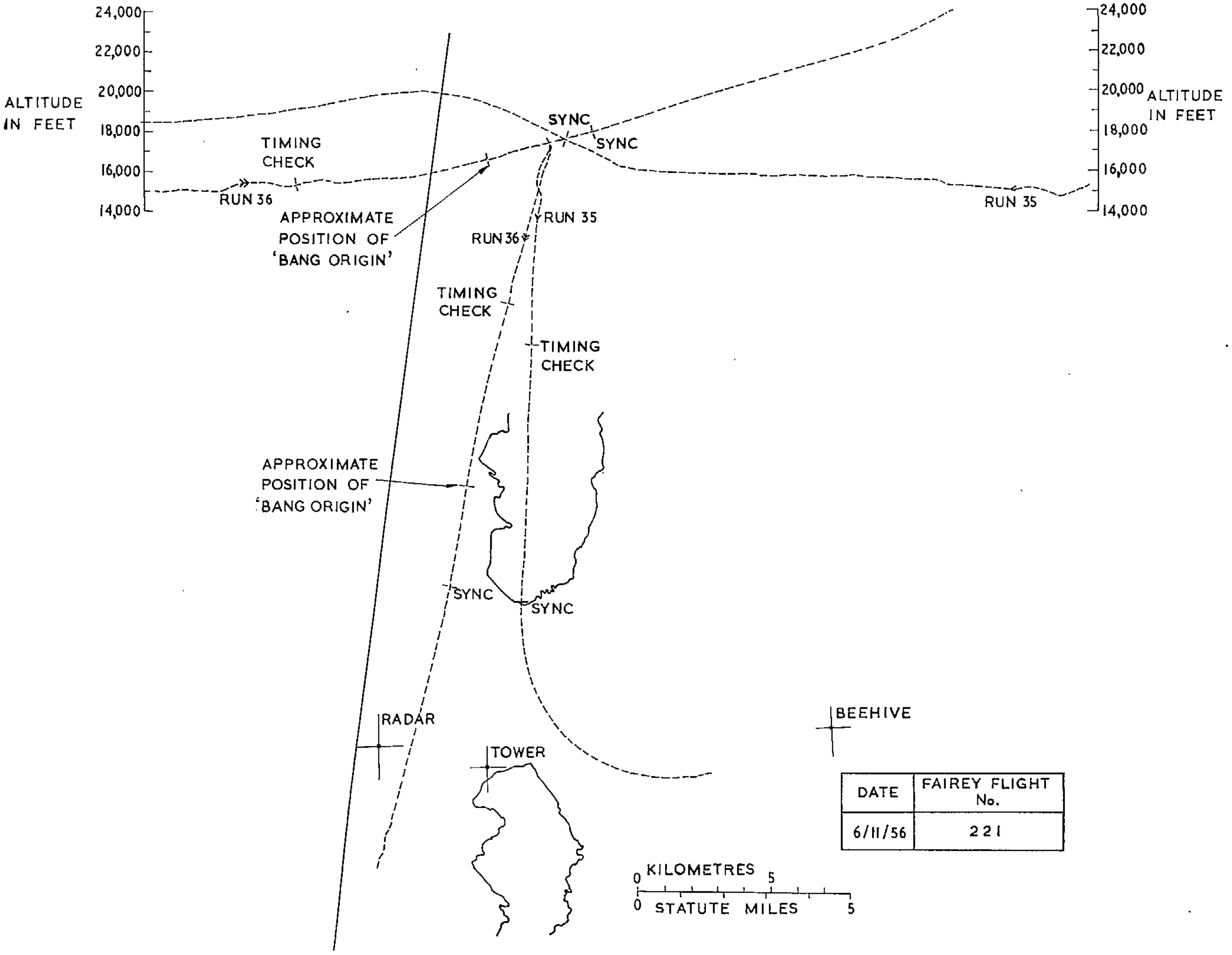


FIG. 15. Radar plot. Runs 35 and 36.

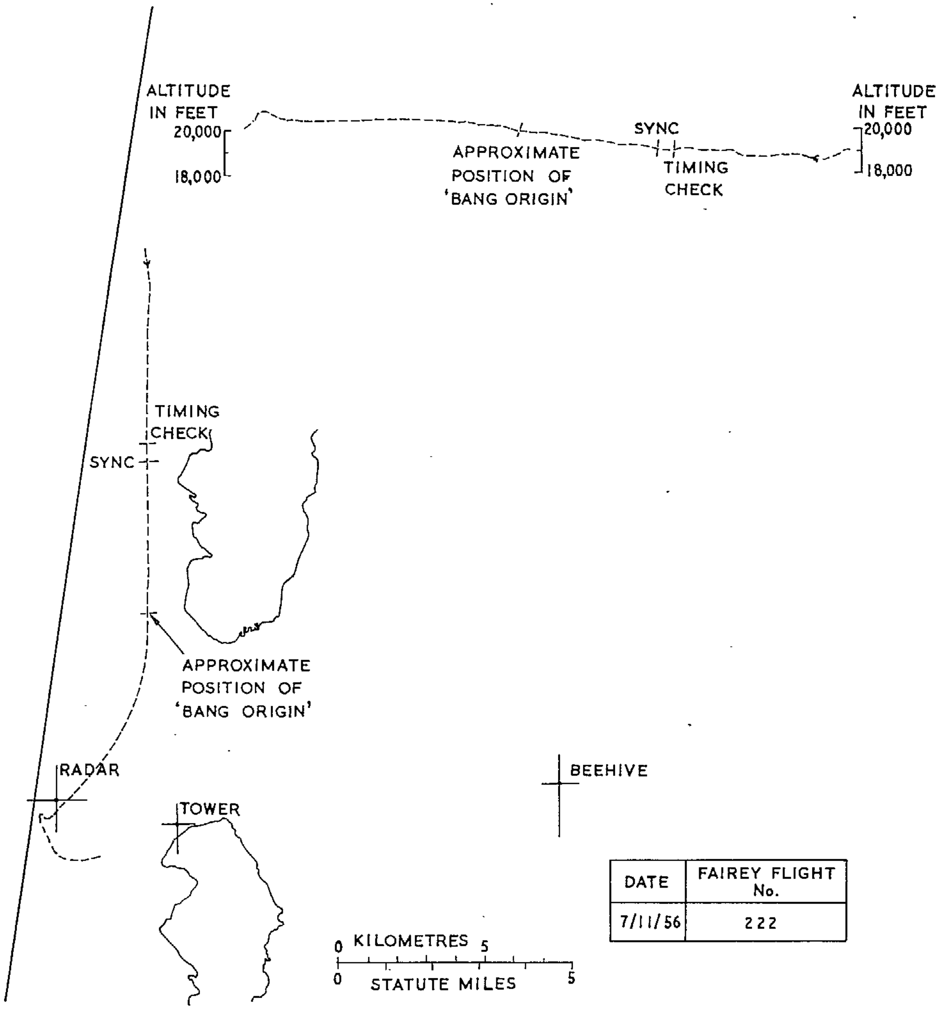


FIG. 16. Radar plot. Run 37.

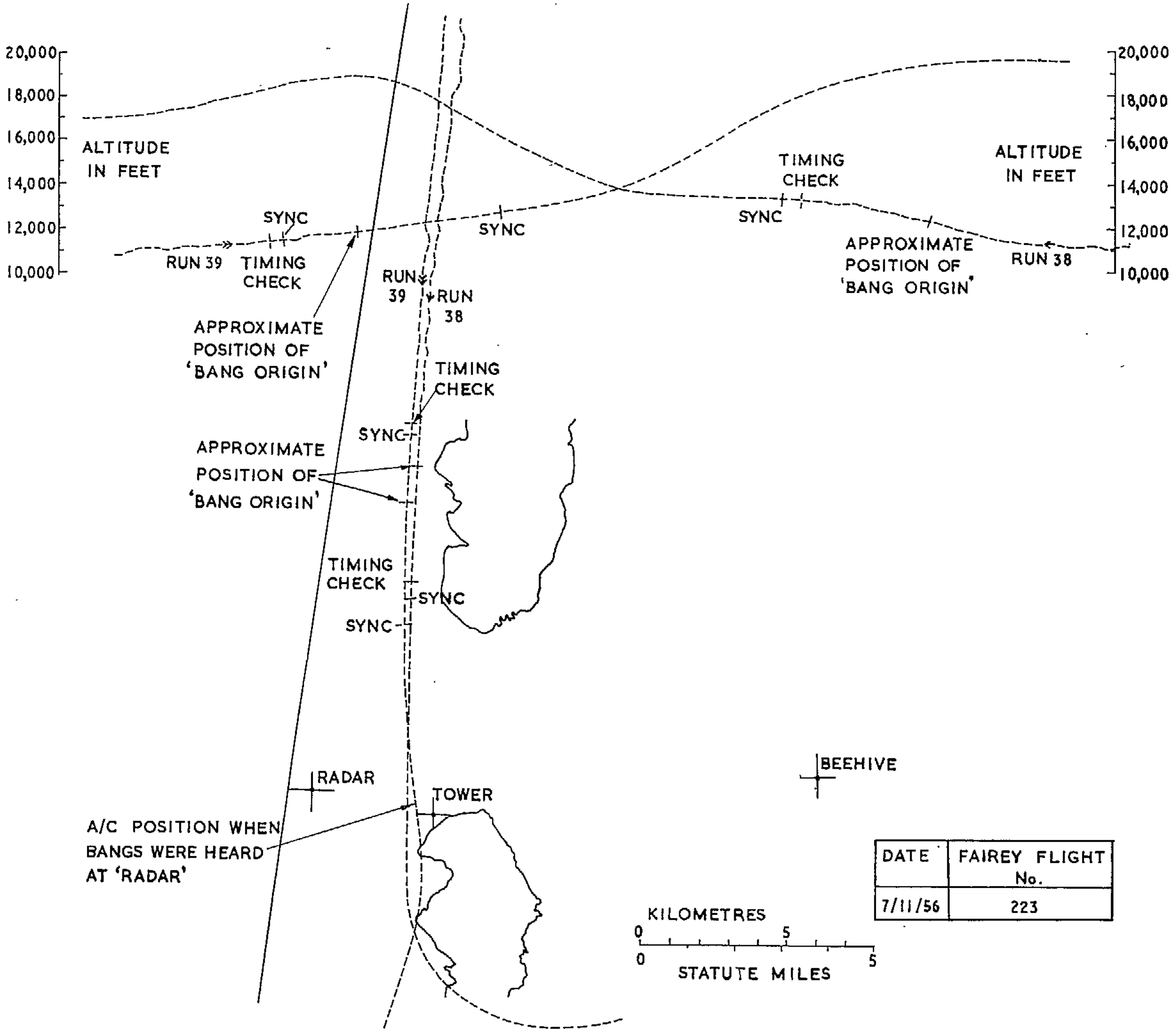


FIG. 17. Radar plot. Runs 38 and 39.

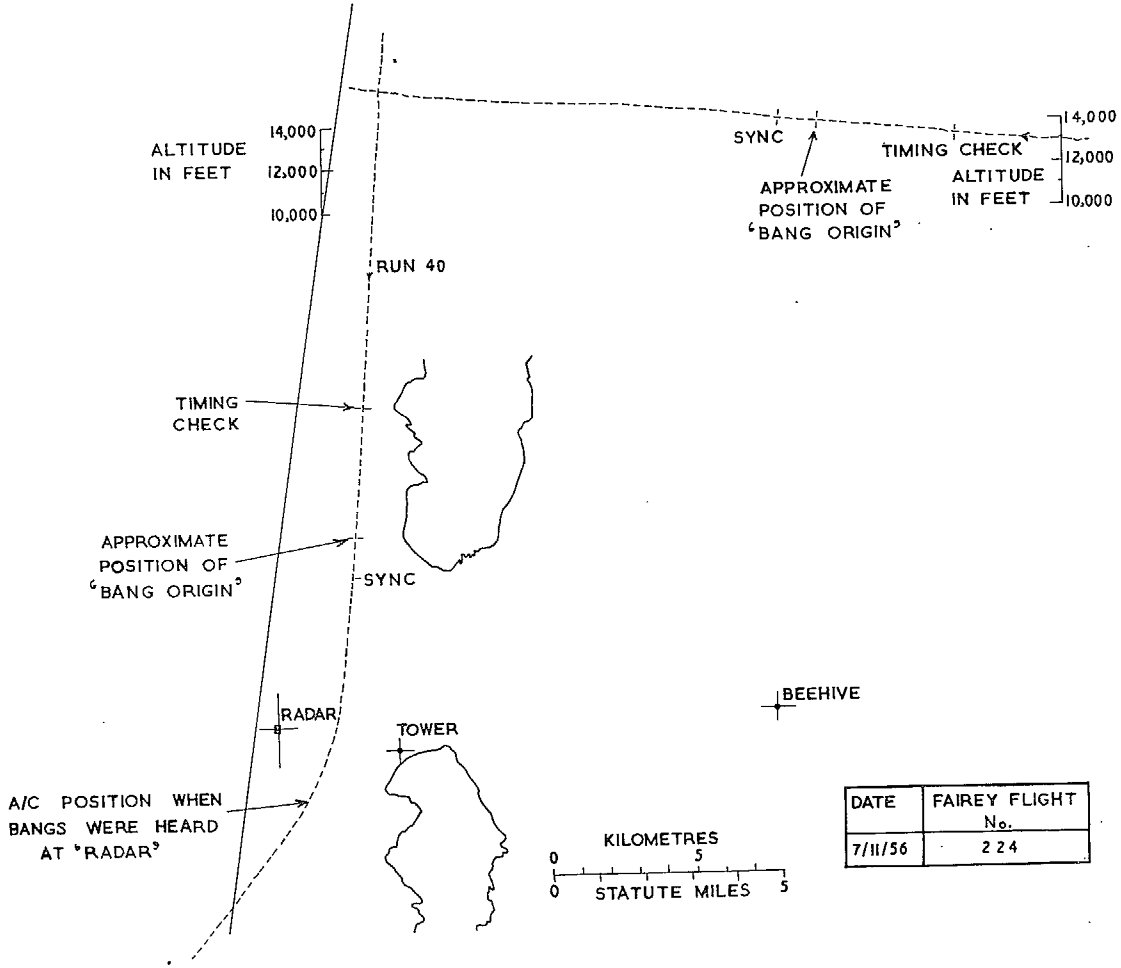


FIG. 18. Radar plot. Run 40.

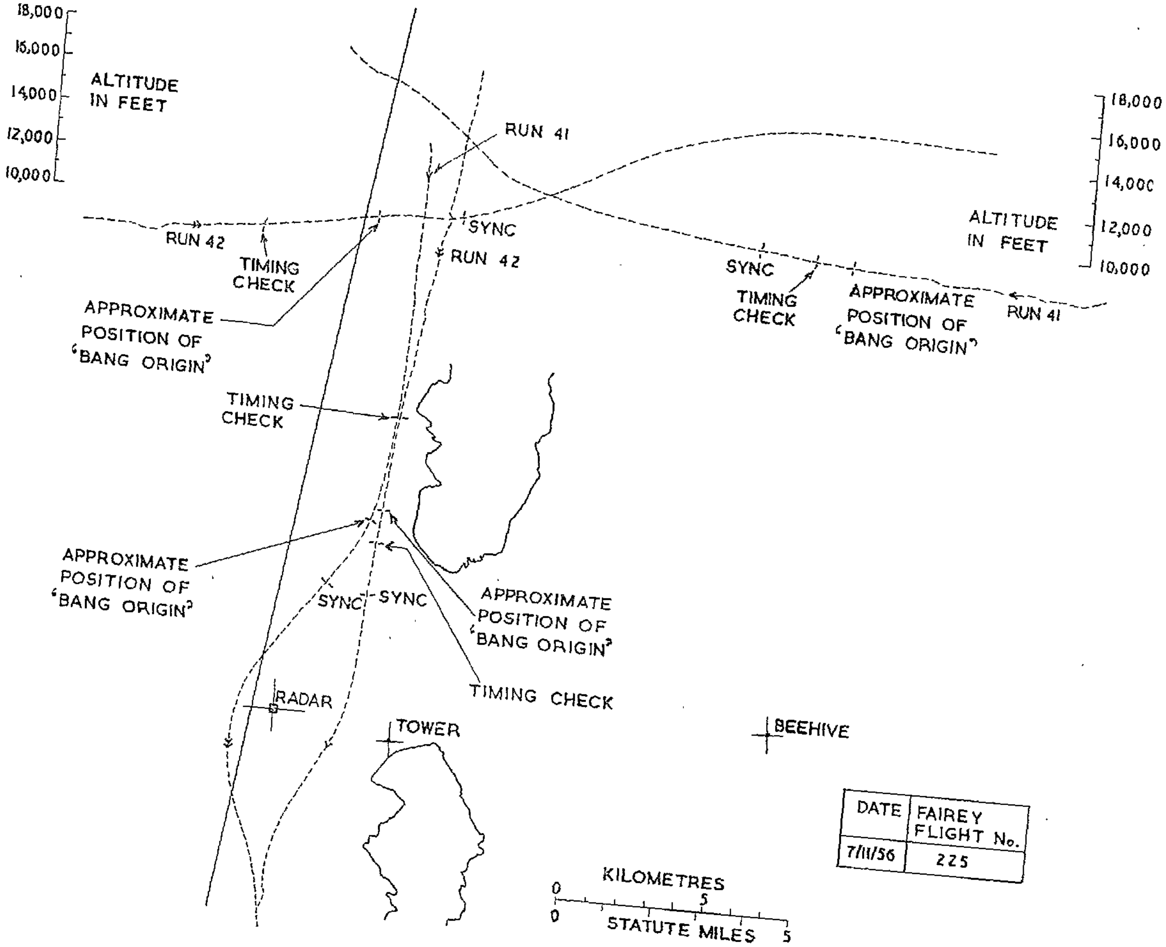


FIG. 19. Radar plot. Runs 41 and 42.

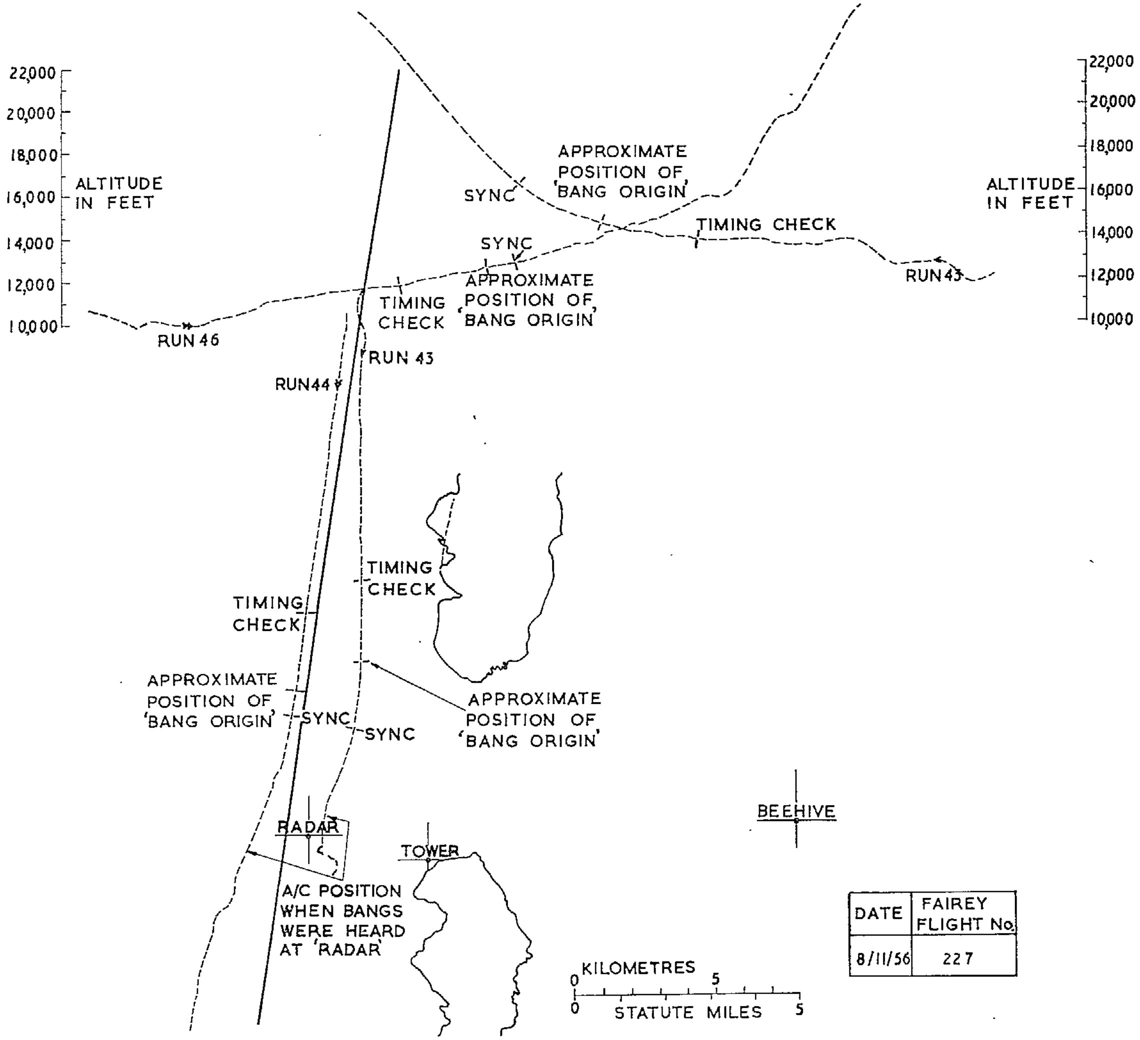


FIG. 20. Radar plot. Runs 43 and 44.

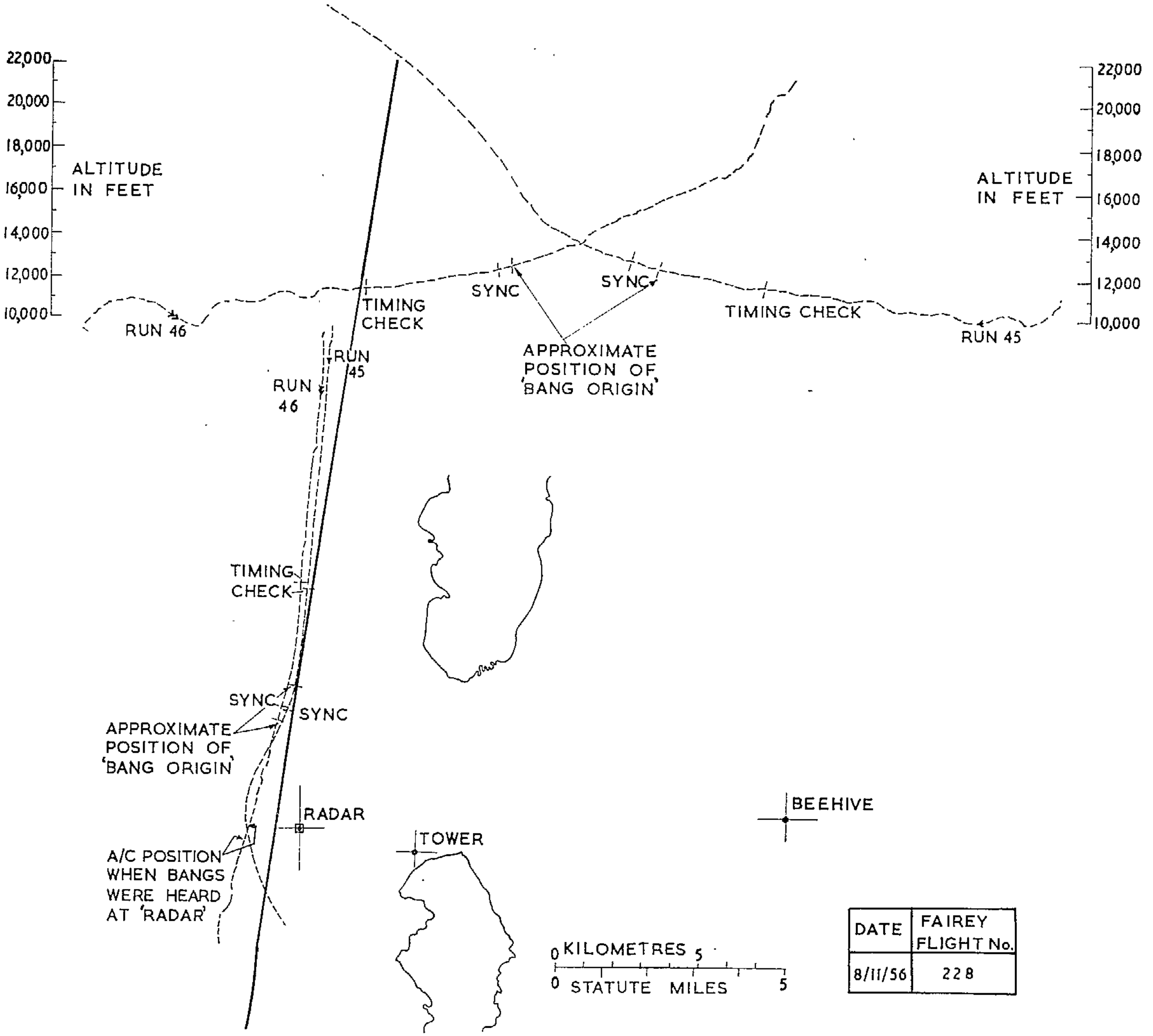


FIG. 21. Radar plot. Runs 45 and 46.

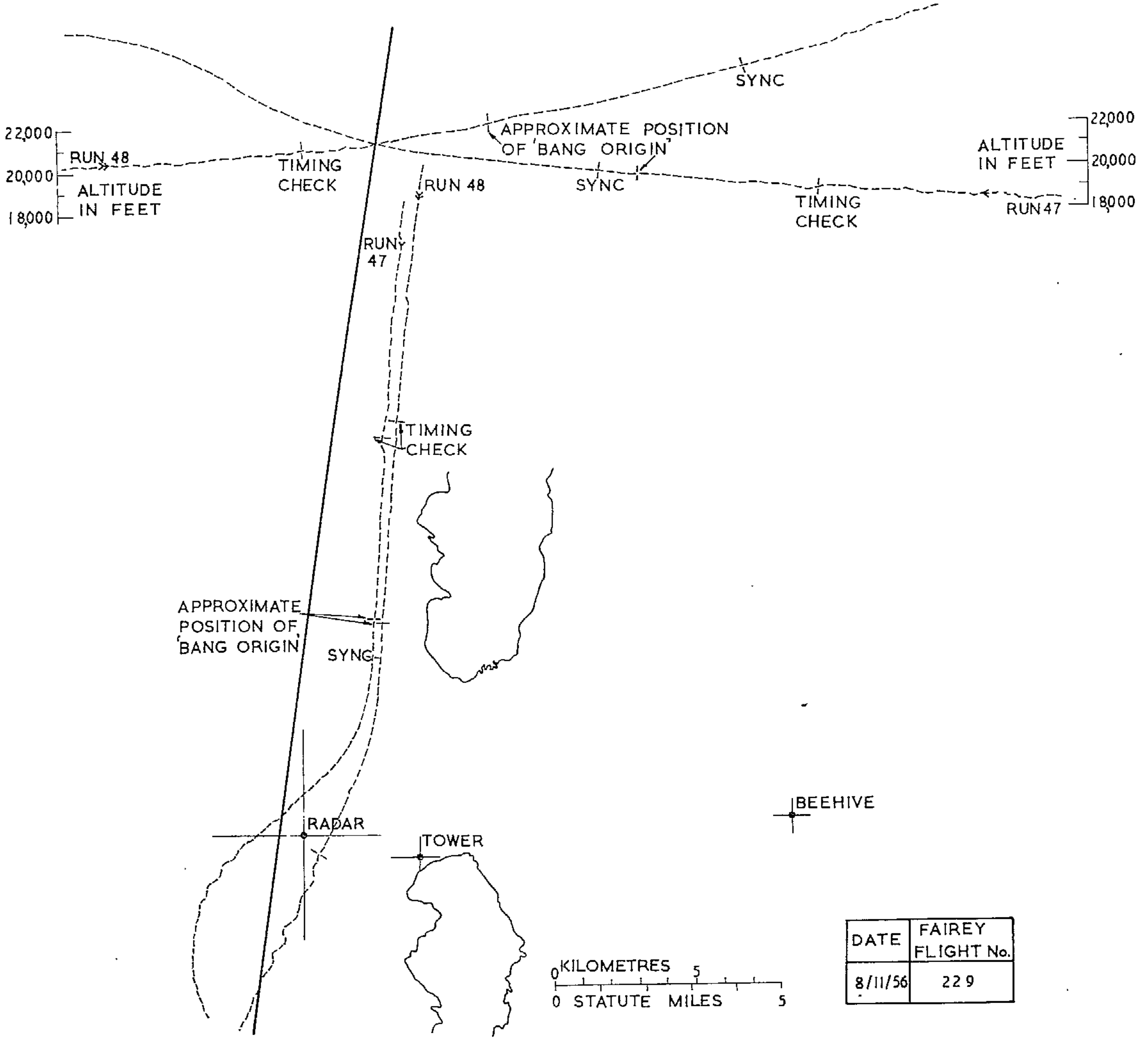


Fig. 22. Radar plot. Runs 47 and 48.

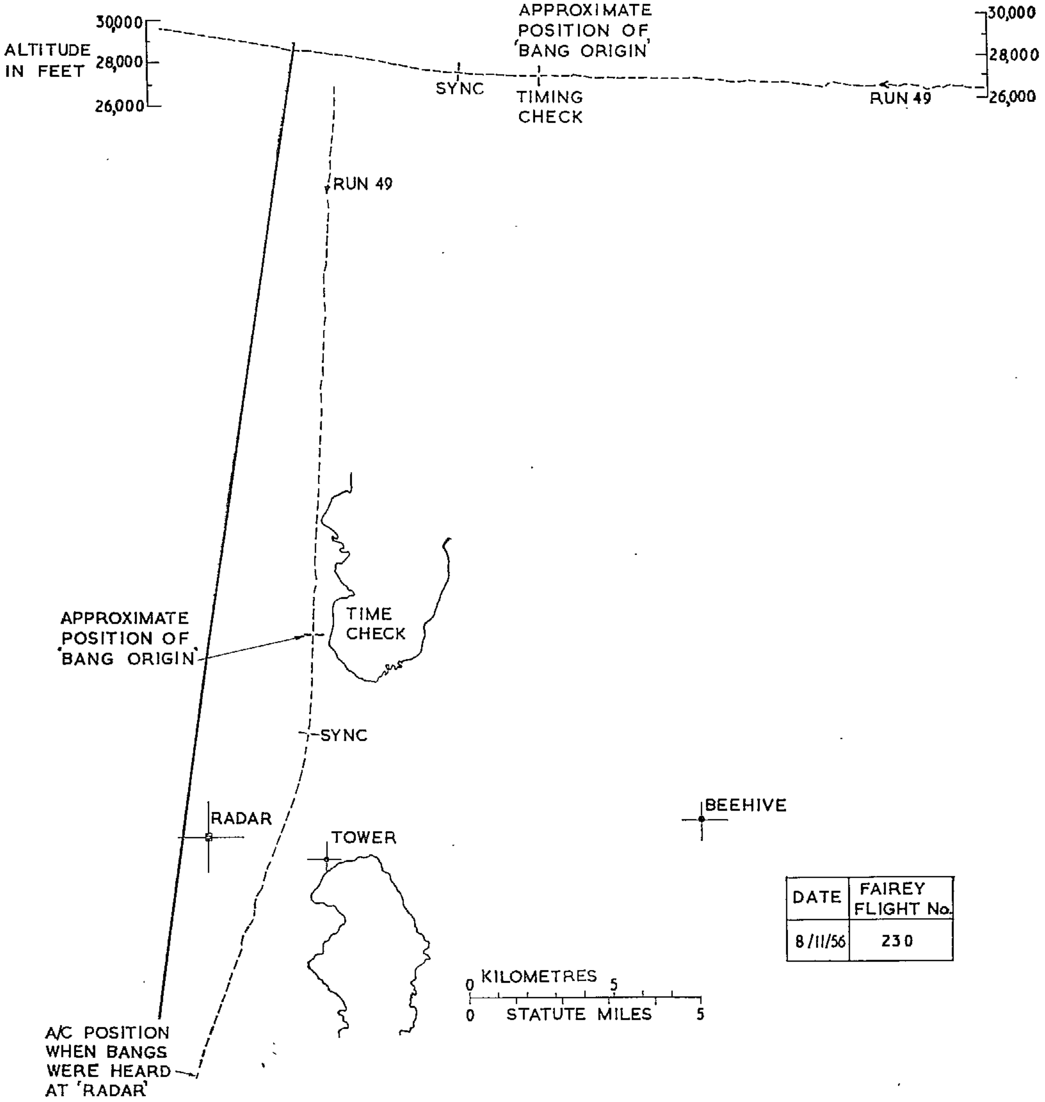


Fig. 23. Radar plot. Run 49.

FLIGHT HISTORIES

Figs. 24 to 44

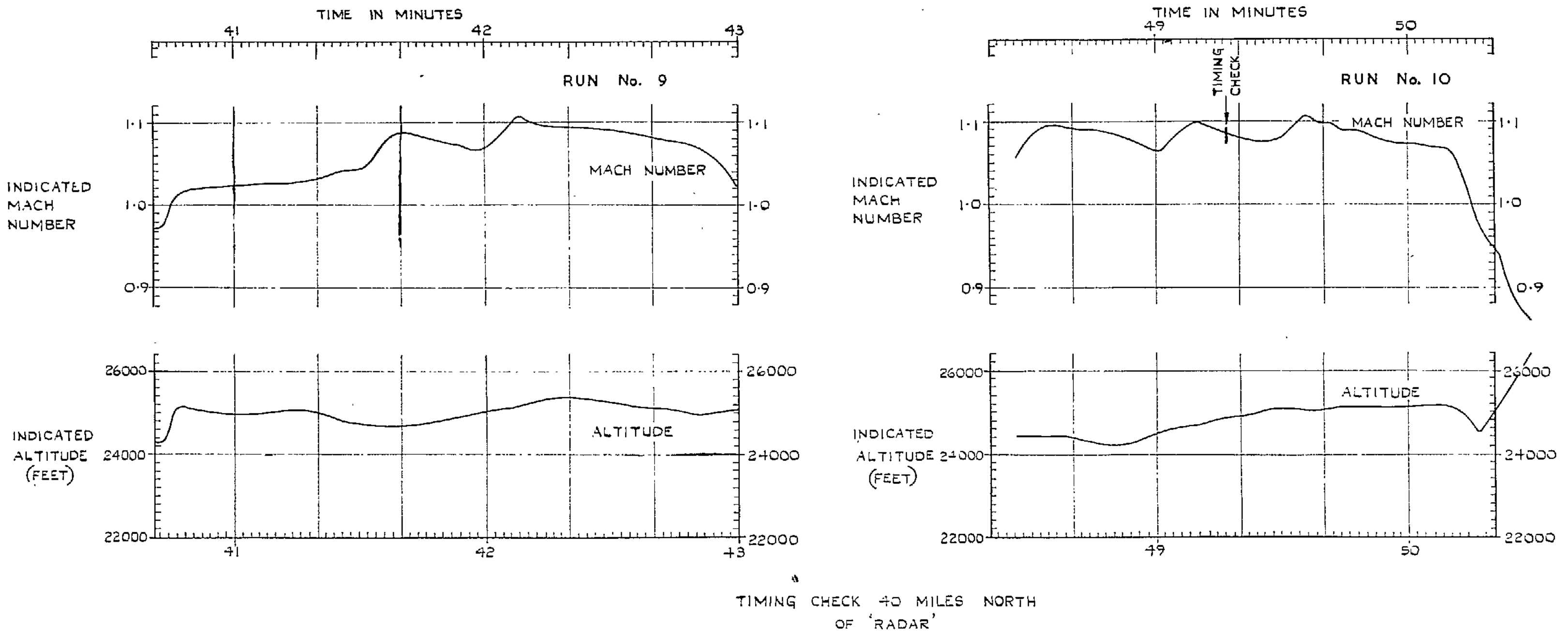


FIG. 24. Flight histories for runs 9 and 10.

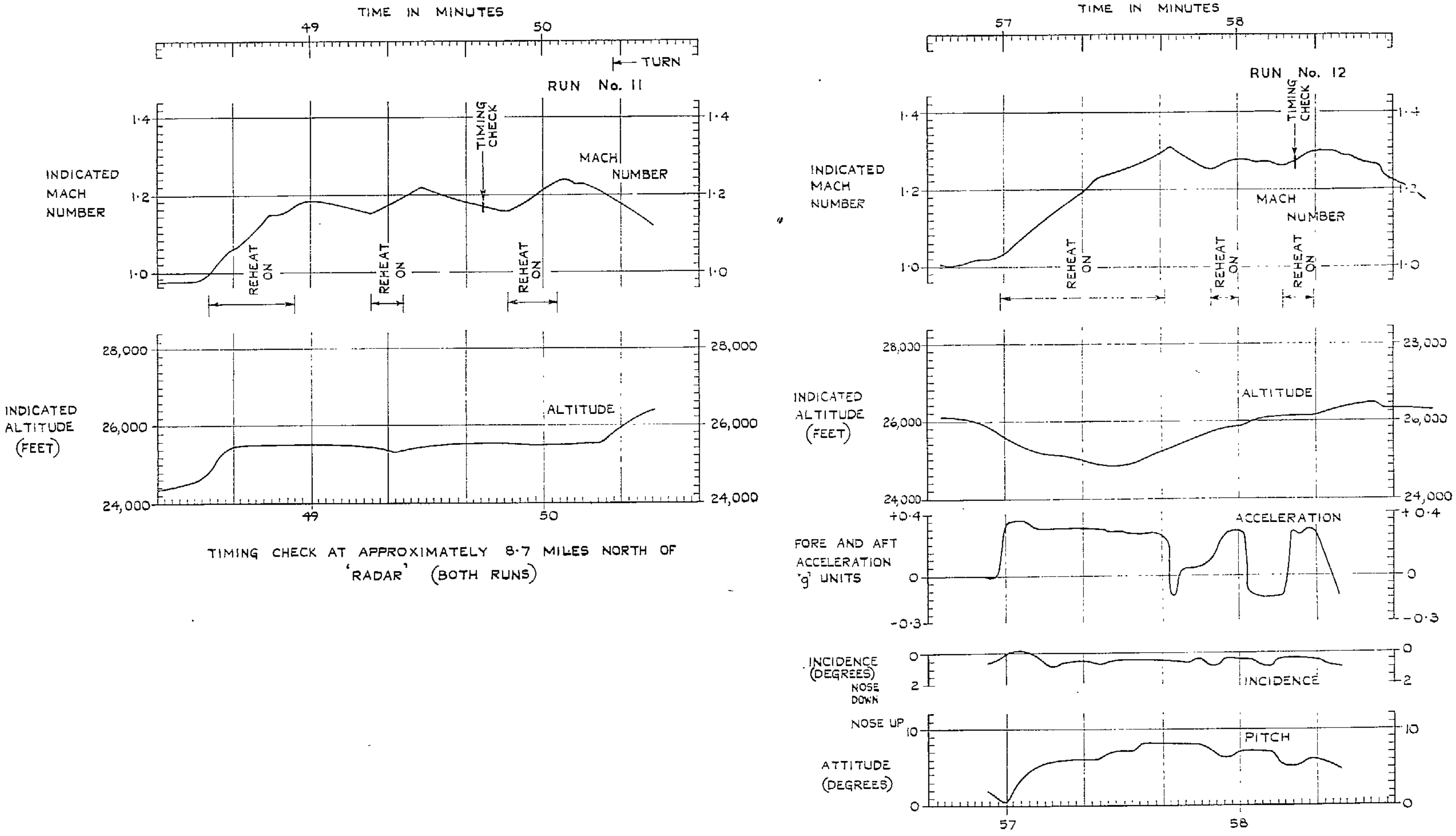
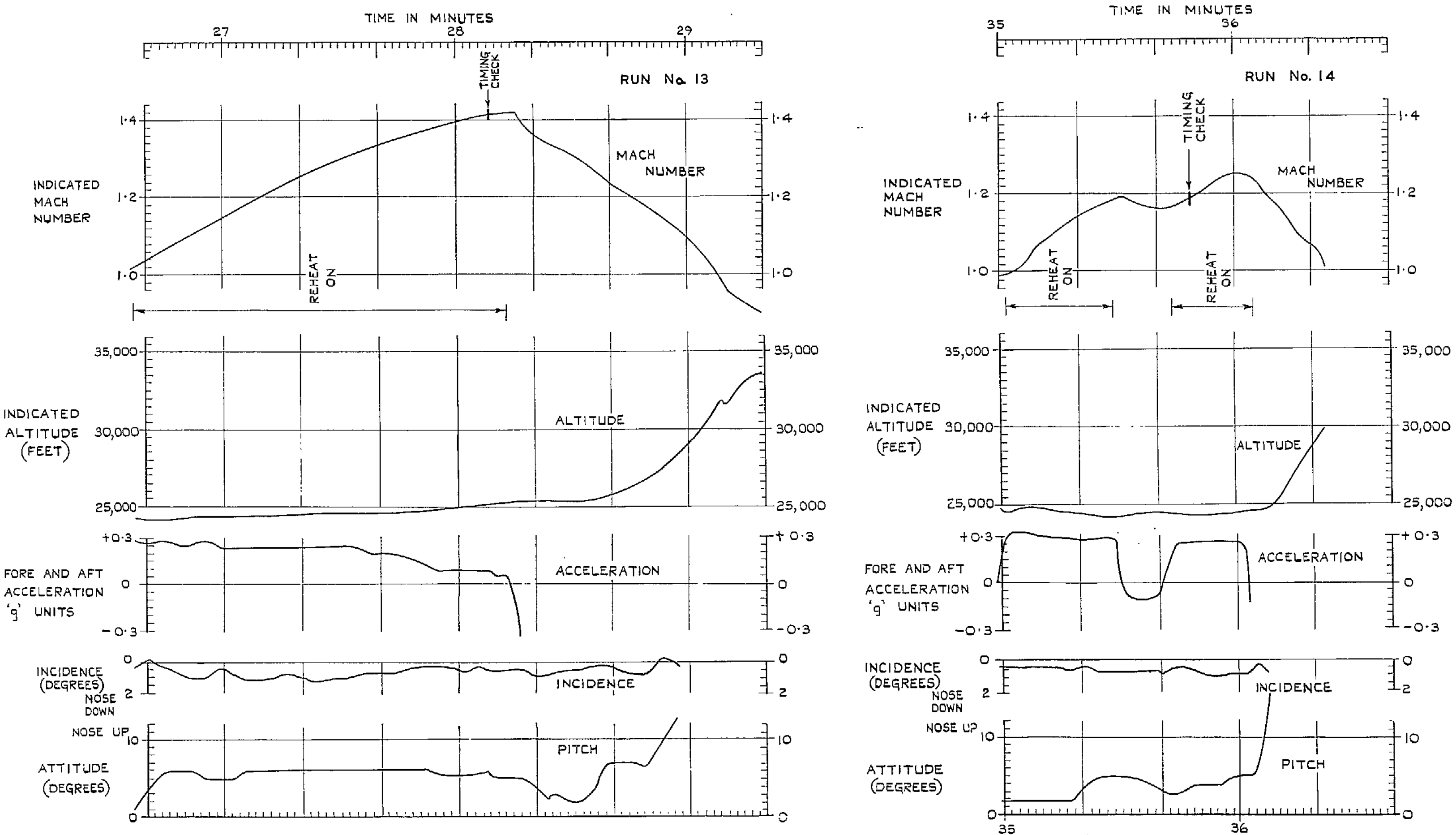
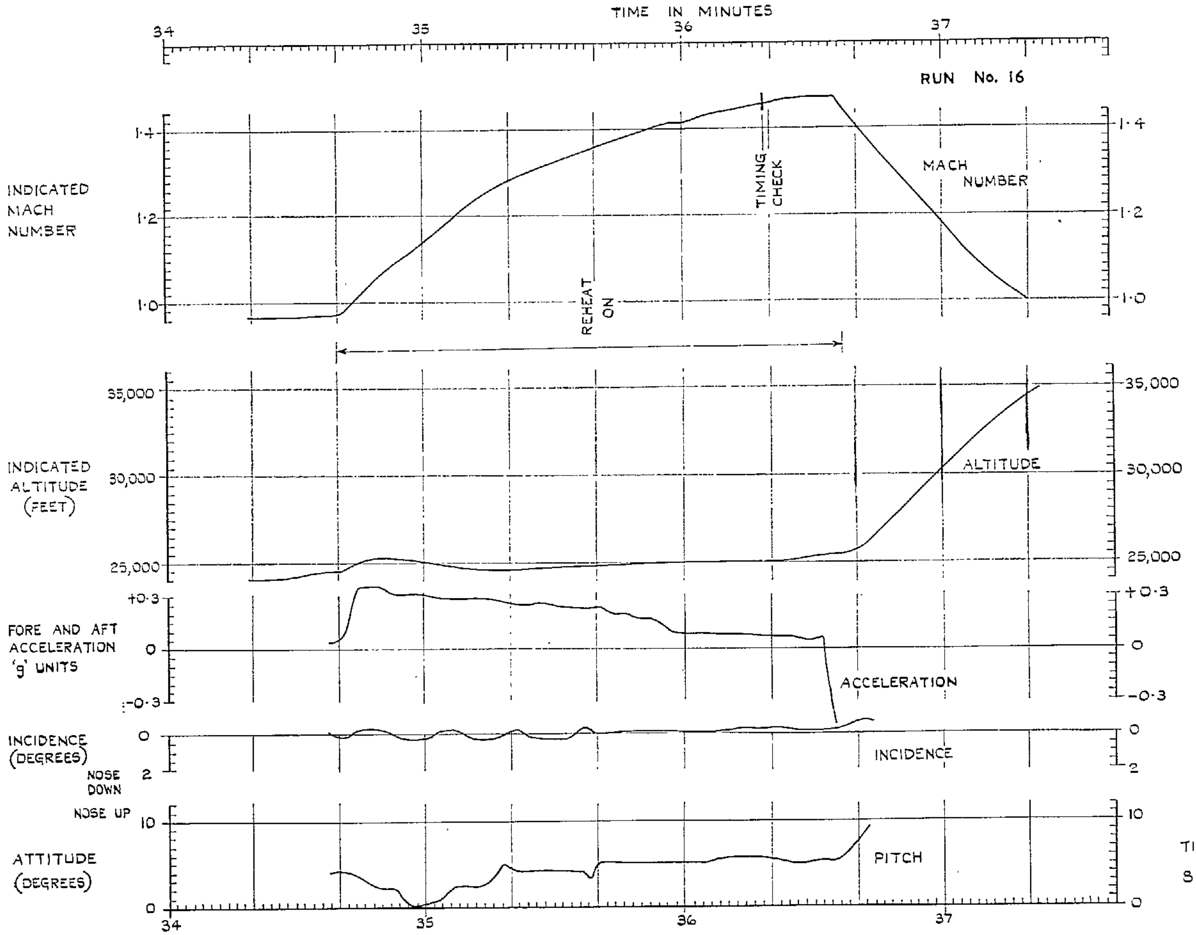


FIG. 25. Flight histories for runs 11 and 12.



TIMING CHECK AT APPROXIMATELY 8·7 MILES NORTH OF 'RADAR' (BOTH RUNS)

FIG. 26. Flight histories for runs 13 and 14.



TIMING CHECK AT APPROXIMATELY 8·7 MILES NORTH OF 'RADAR'

FIG. 27. Flight history for run 16.

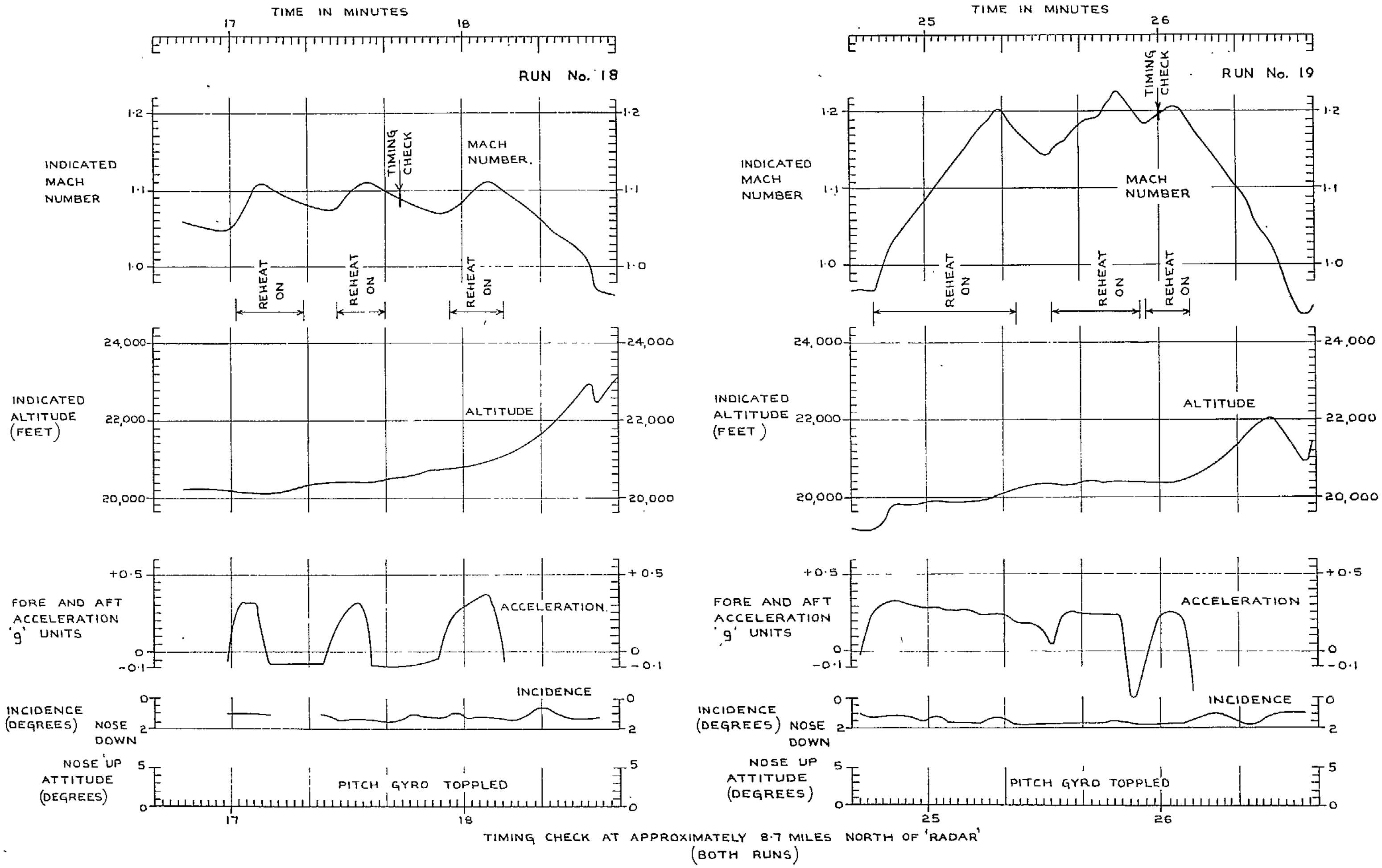


FIG. 28. Flight histories for runs 18 and 19.

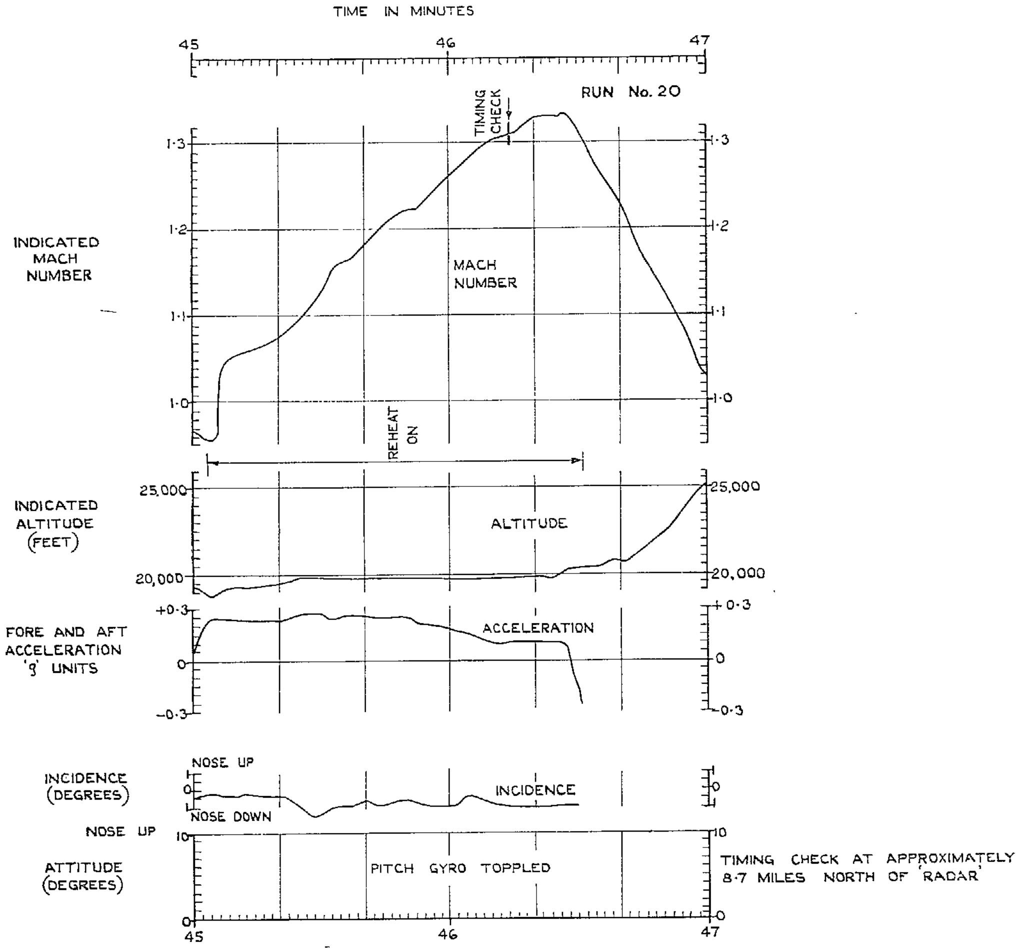


FIG. 29. Flight history for run 20.

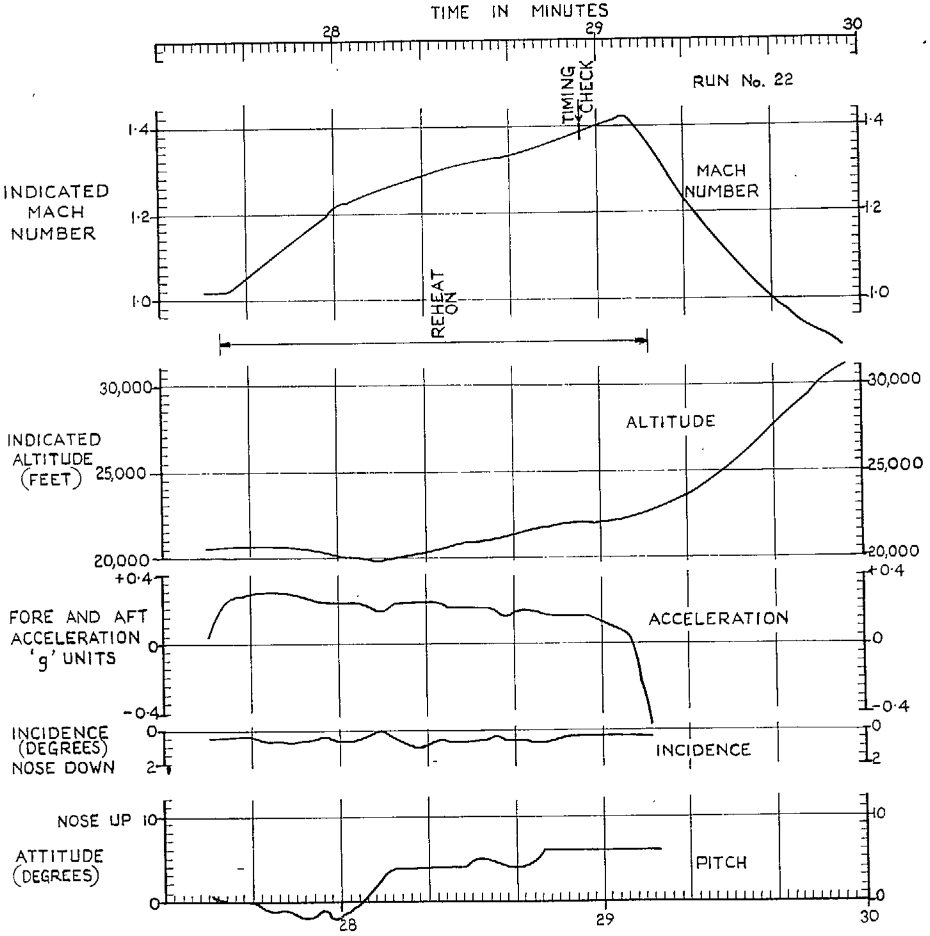
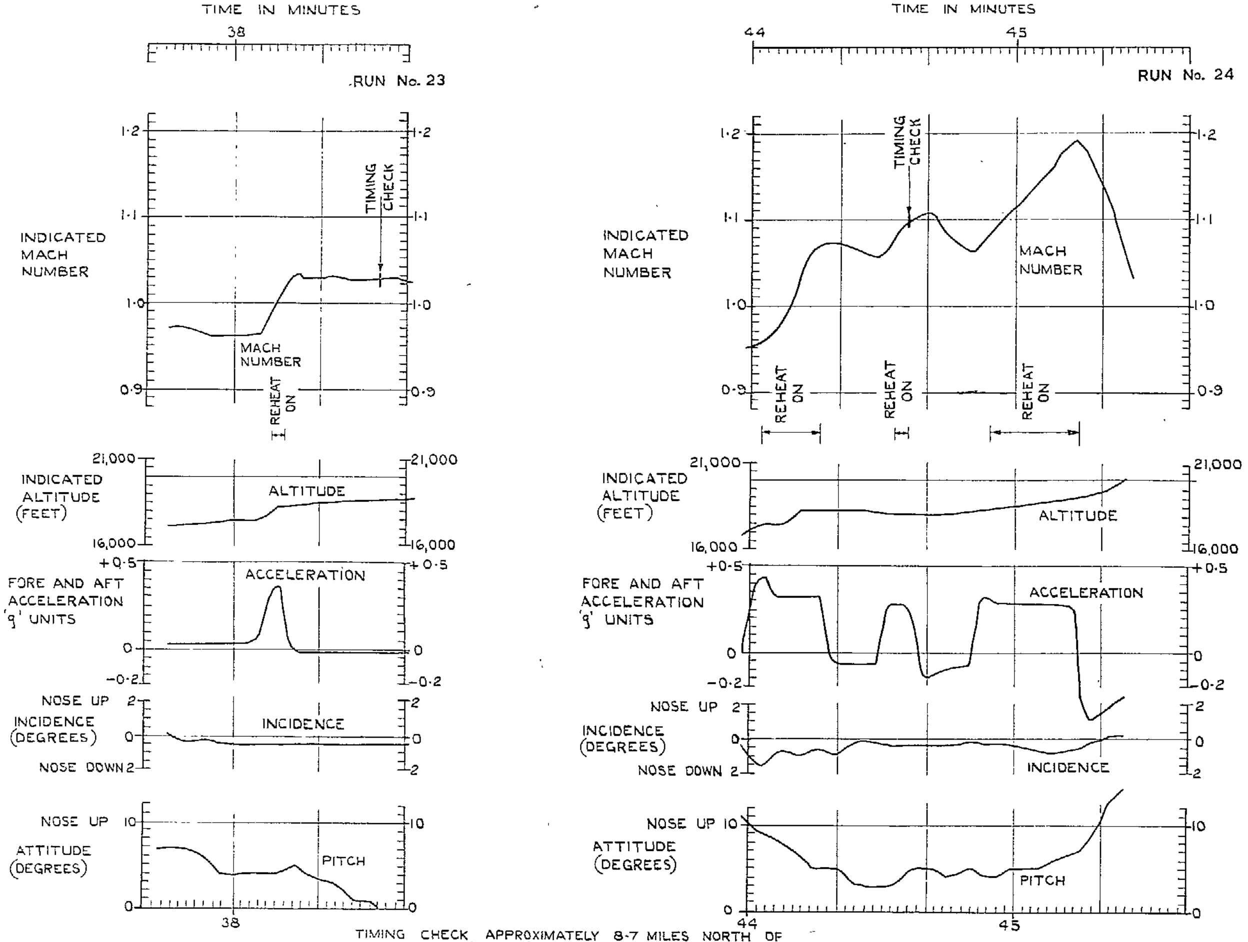


FIG. 30. Flight history for run 22.



TIMING CHECK APPROXIMATELY 8·7 MILES NORTH OF 'RADAR' (BOTH RUNS)

FIG. 31. Flight histories for runs 23 and 24.

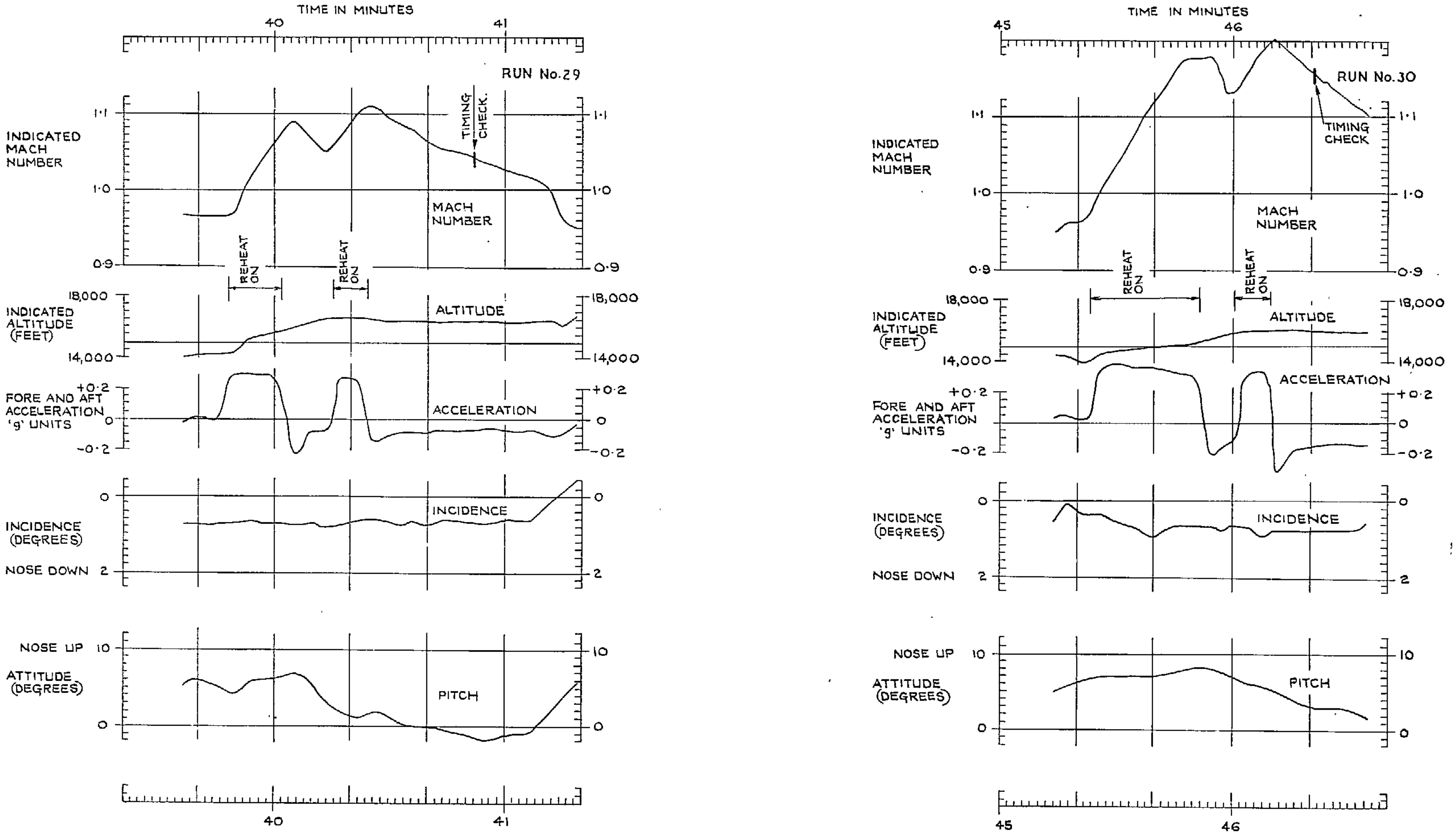


FIG. 32. Flight histories for runs 29 and 30.

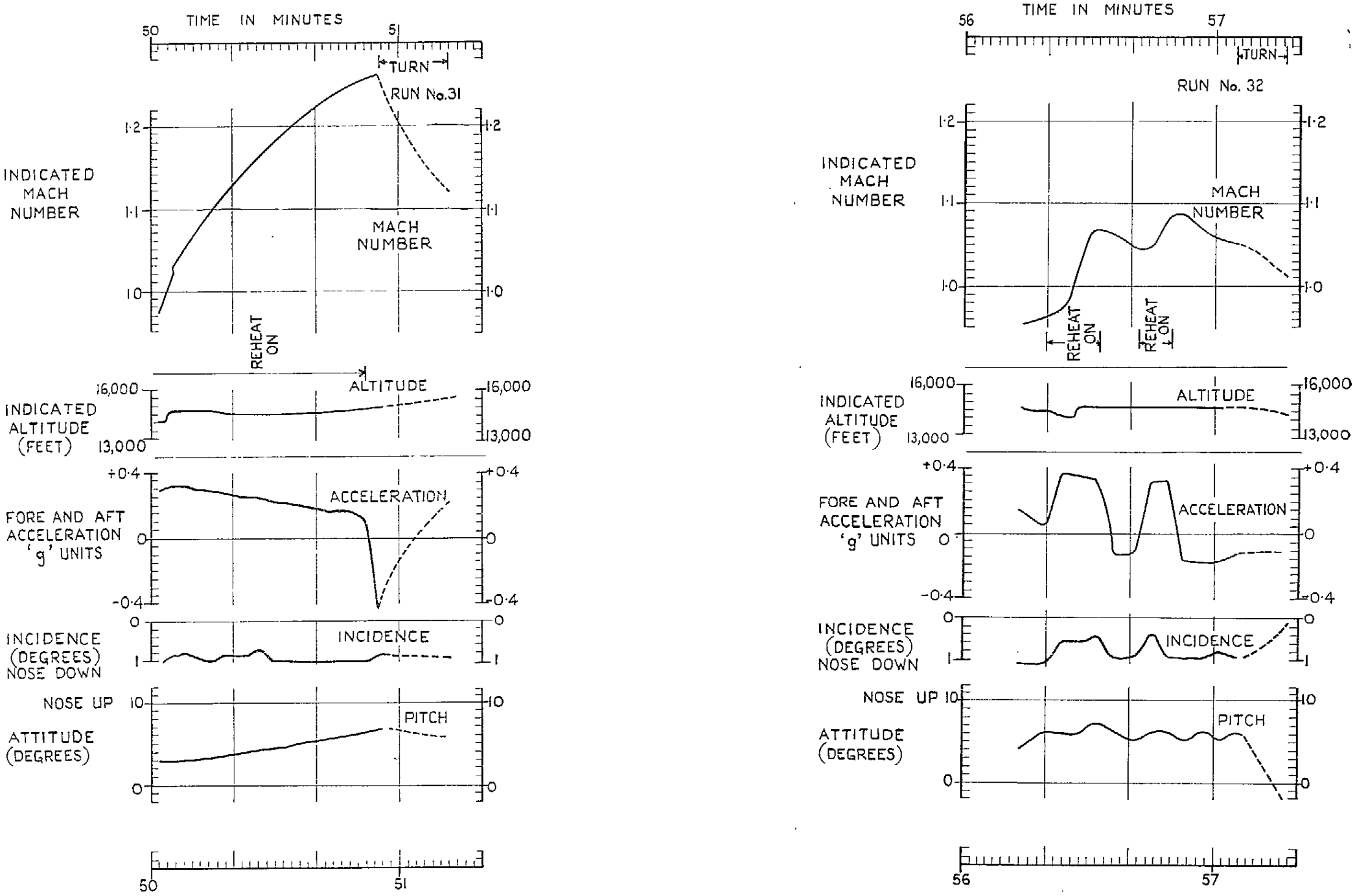


FIG. 33. Flight histories for runs 31 and 32.

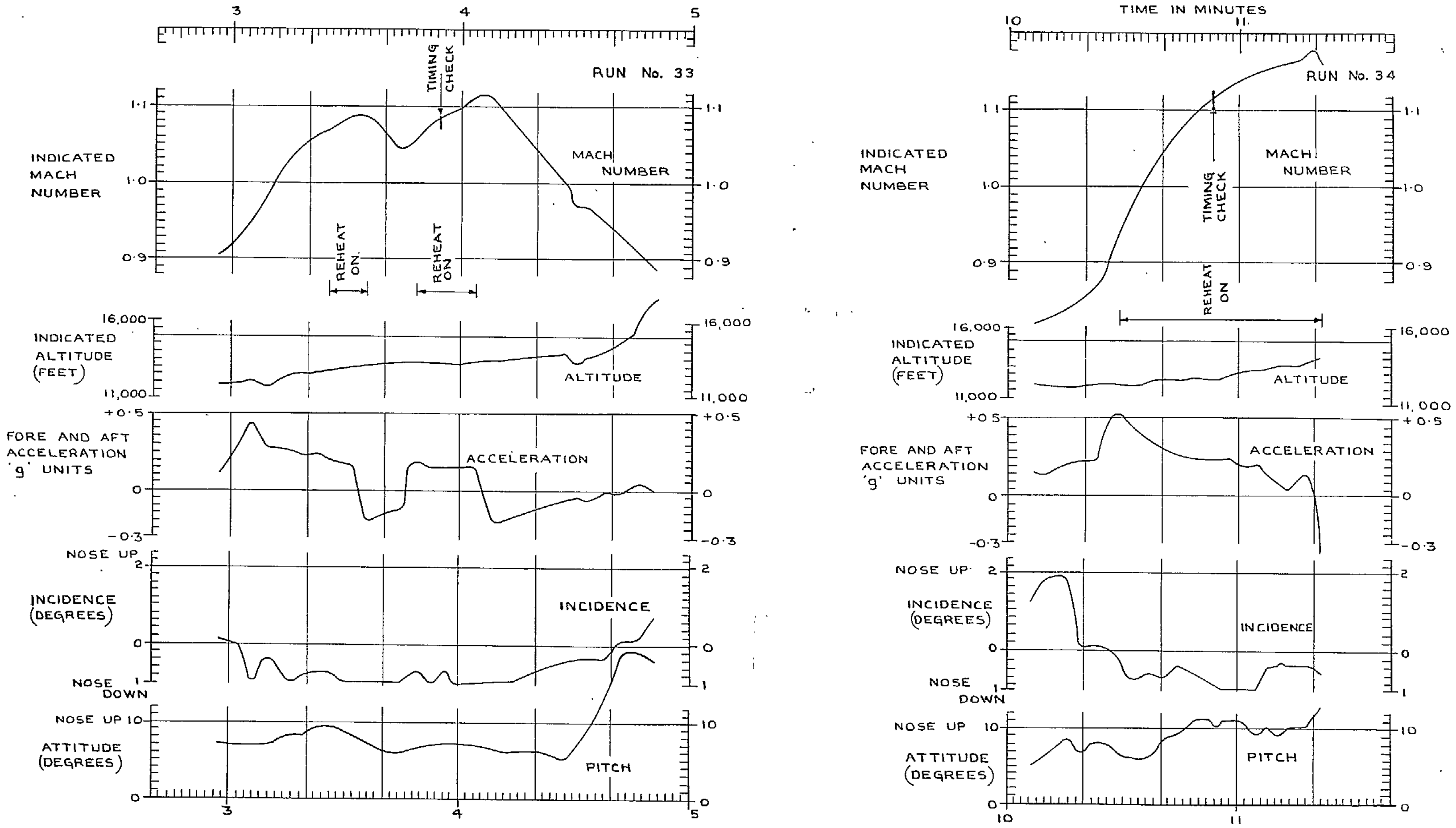


FIG. 34. Flight histories for runs 33 and 34.

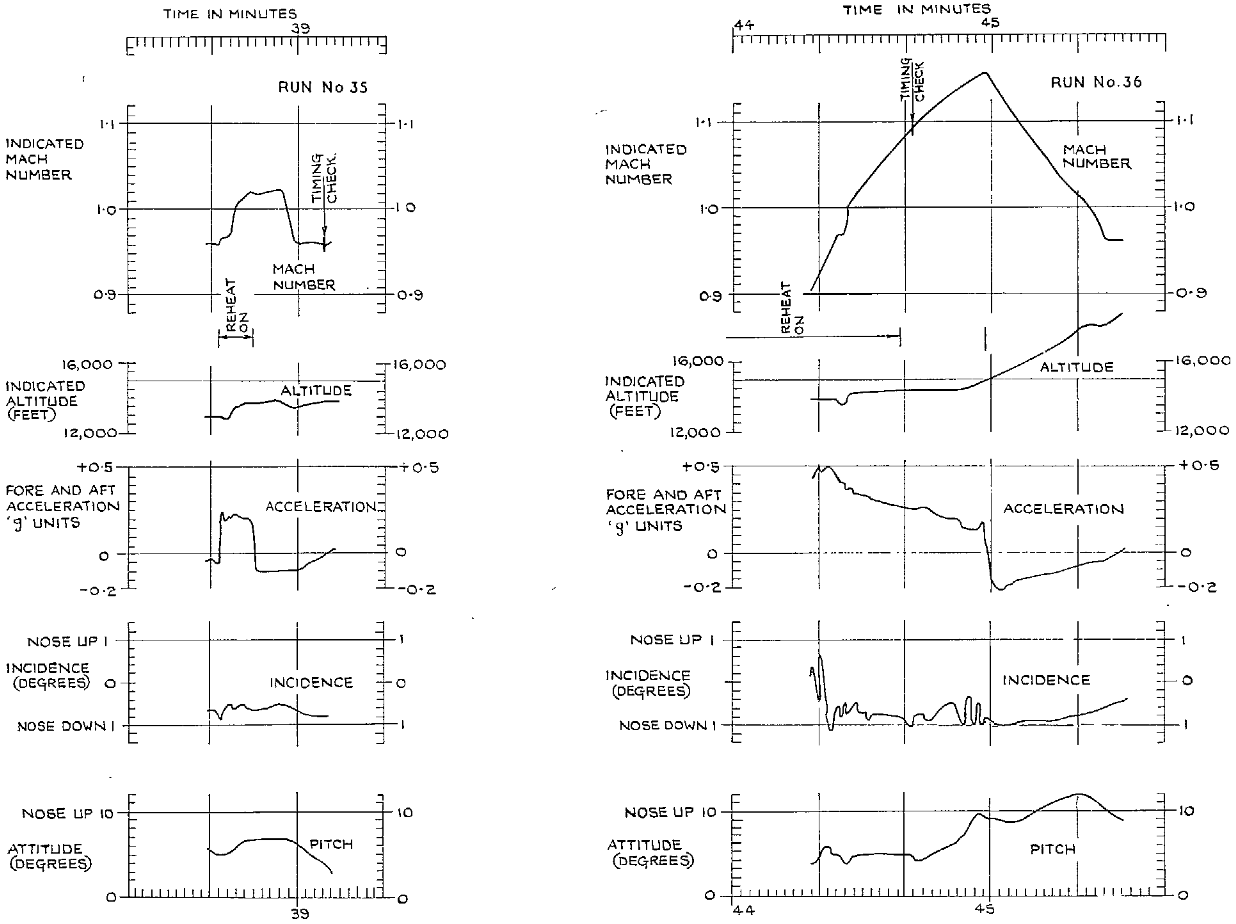


FIG. 35. Flight histories for runs 35 and 36.

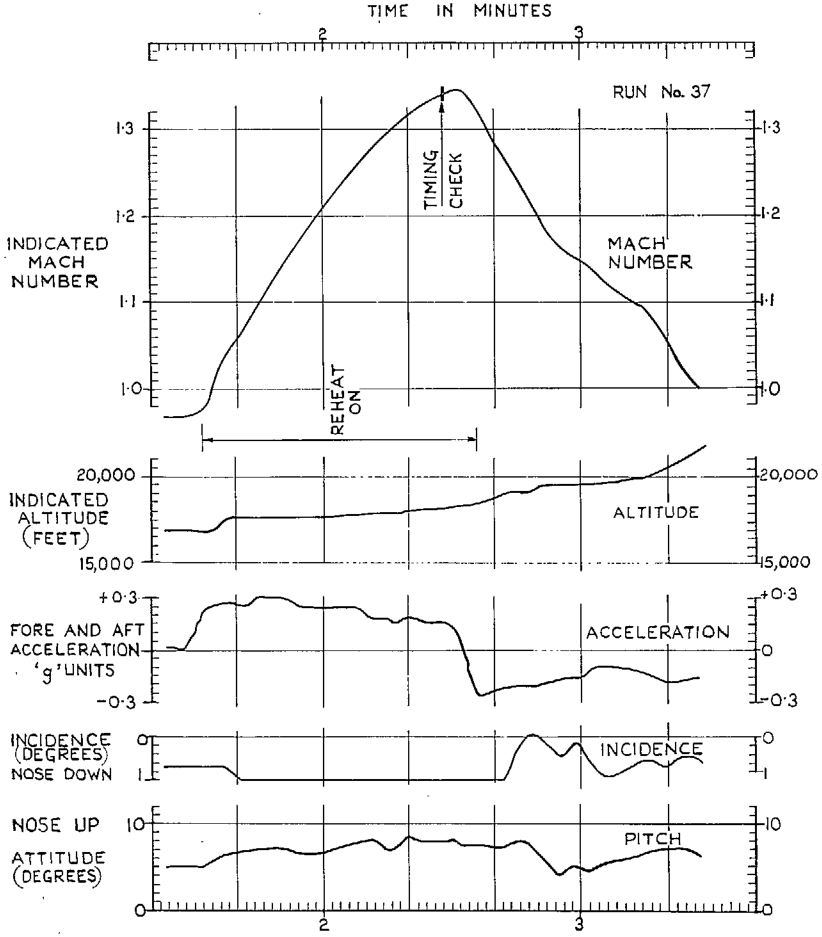


FIG. 36. Flight history for run 37.

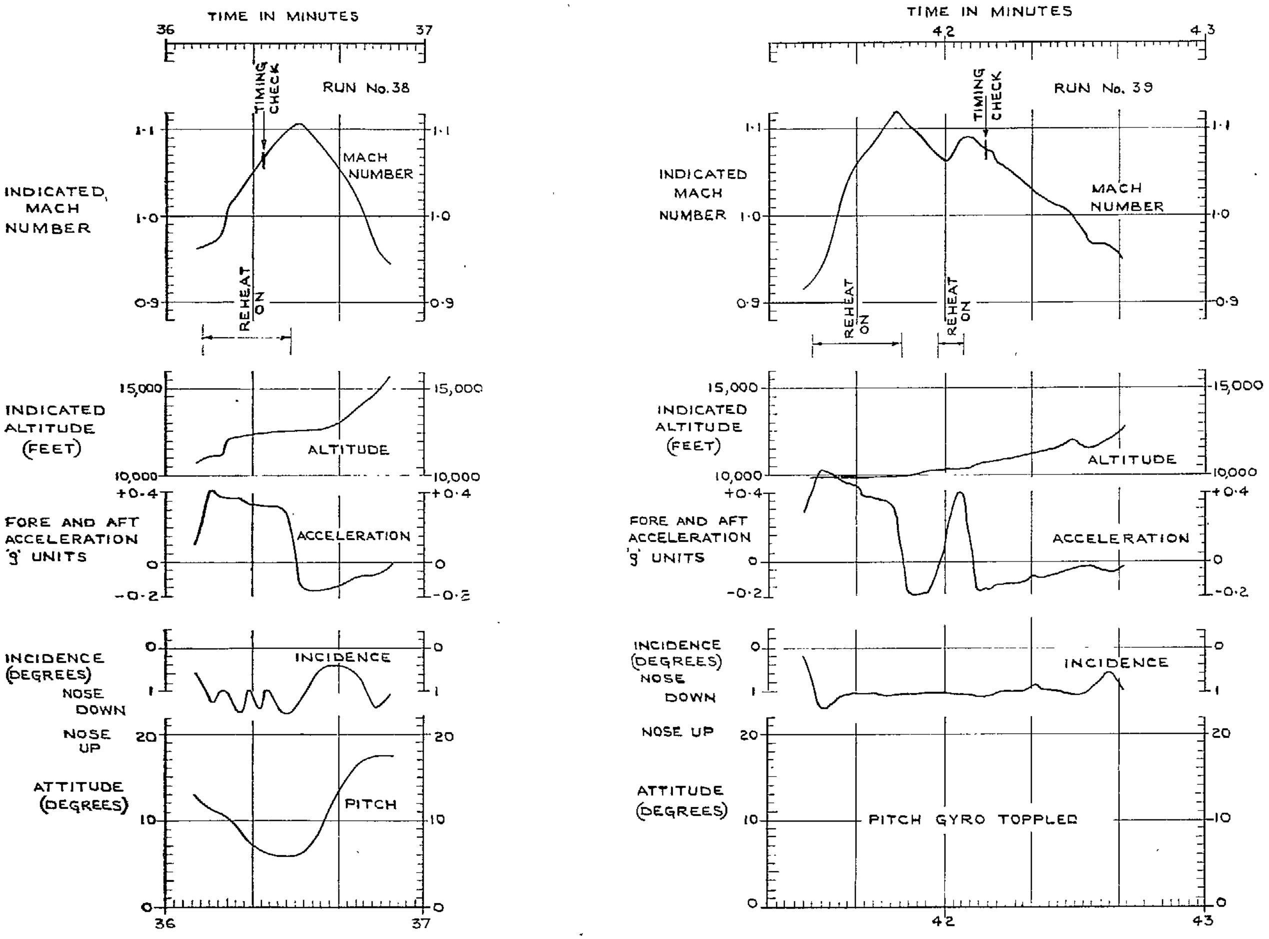


FIG. 37. Flight histories for runs 38 and 39.

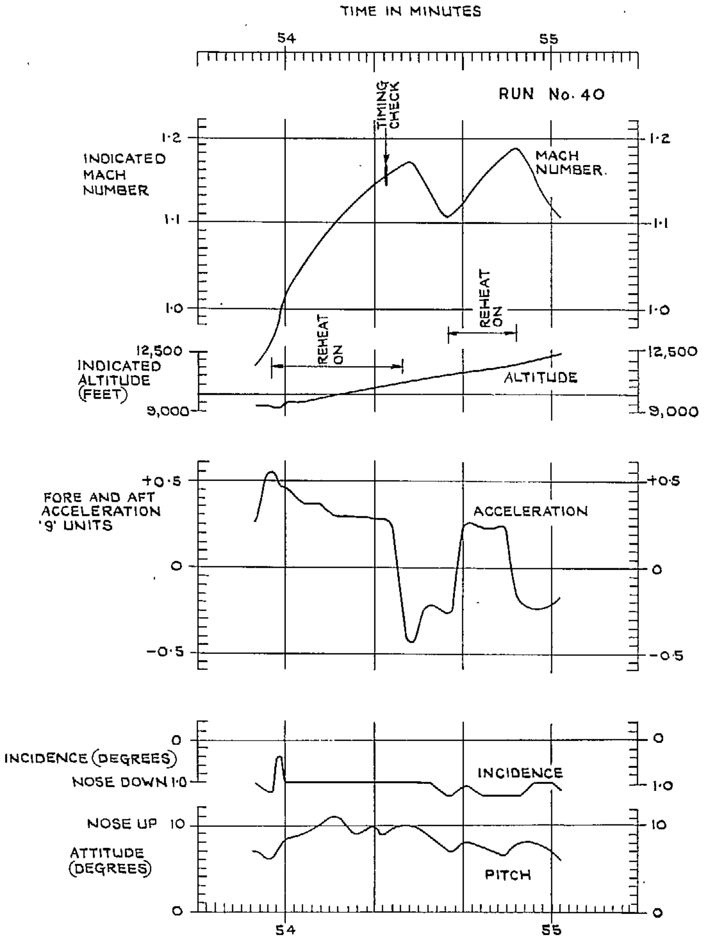


FIG. 38. Flight history for run 40.

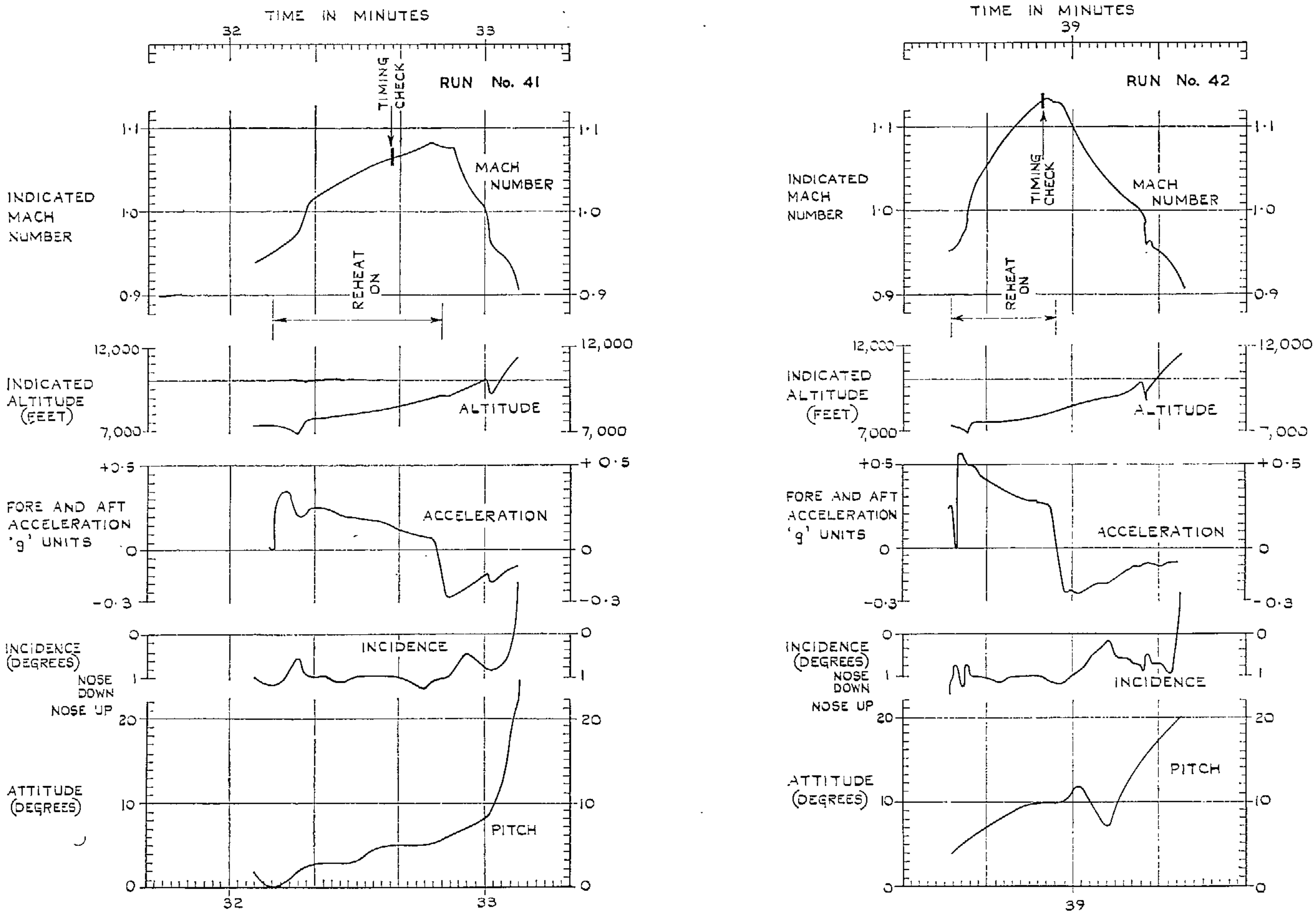


FIG. 39. Flight histories for runs 41 and 42.

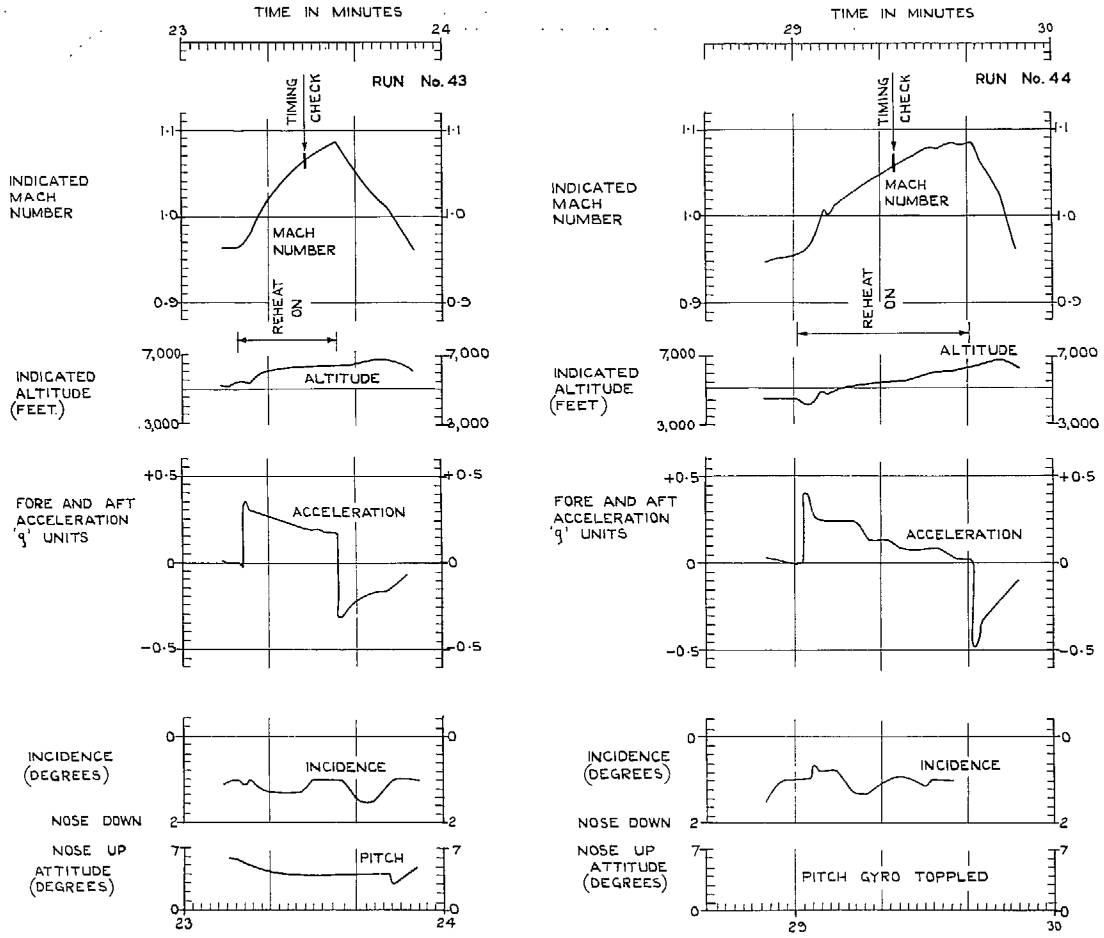


FIG. 40. Flight histories for runs 43 and 44.

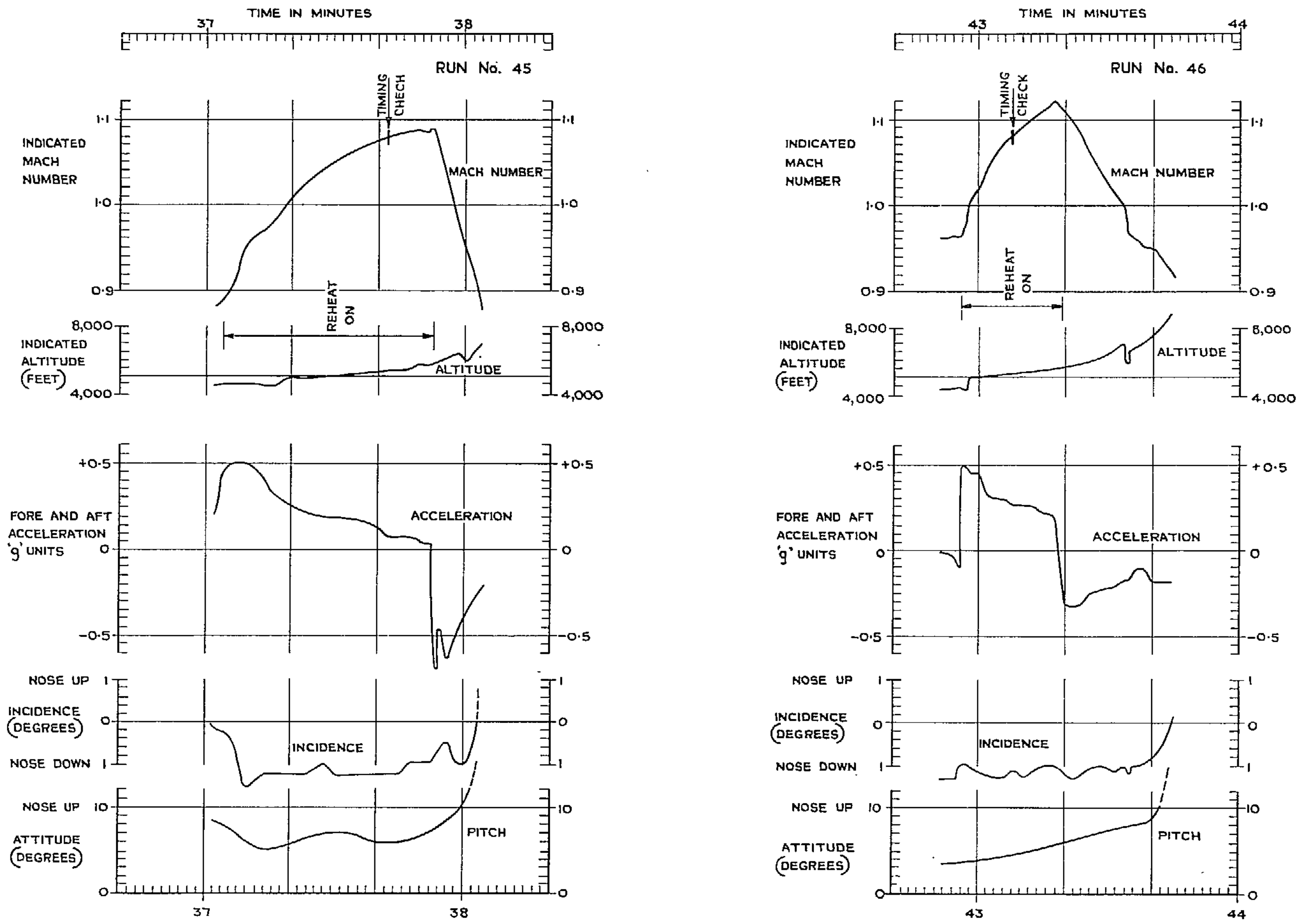


FIG. 41. Flight histories for runs 45 and 46.

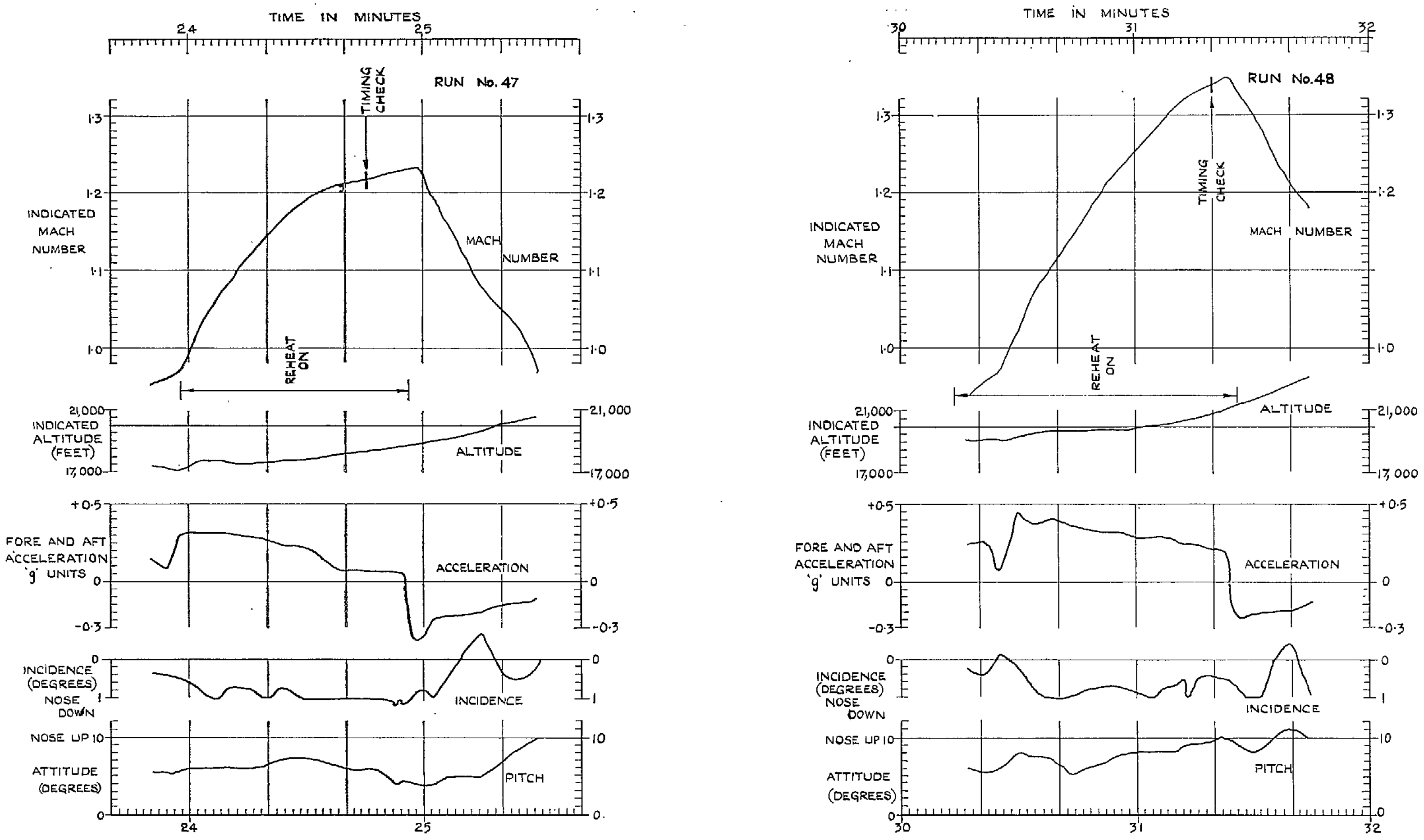


FIG. 42. Flight histories for runs 47 and 48.

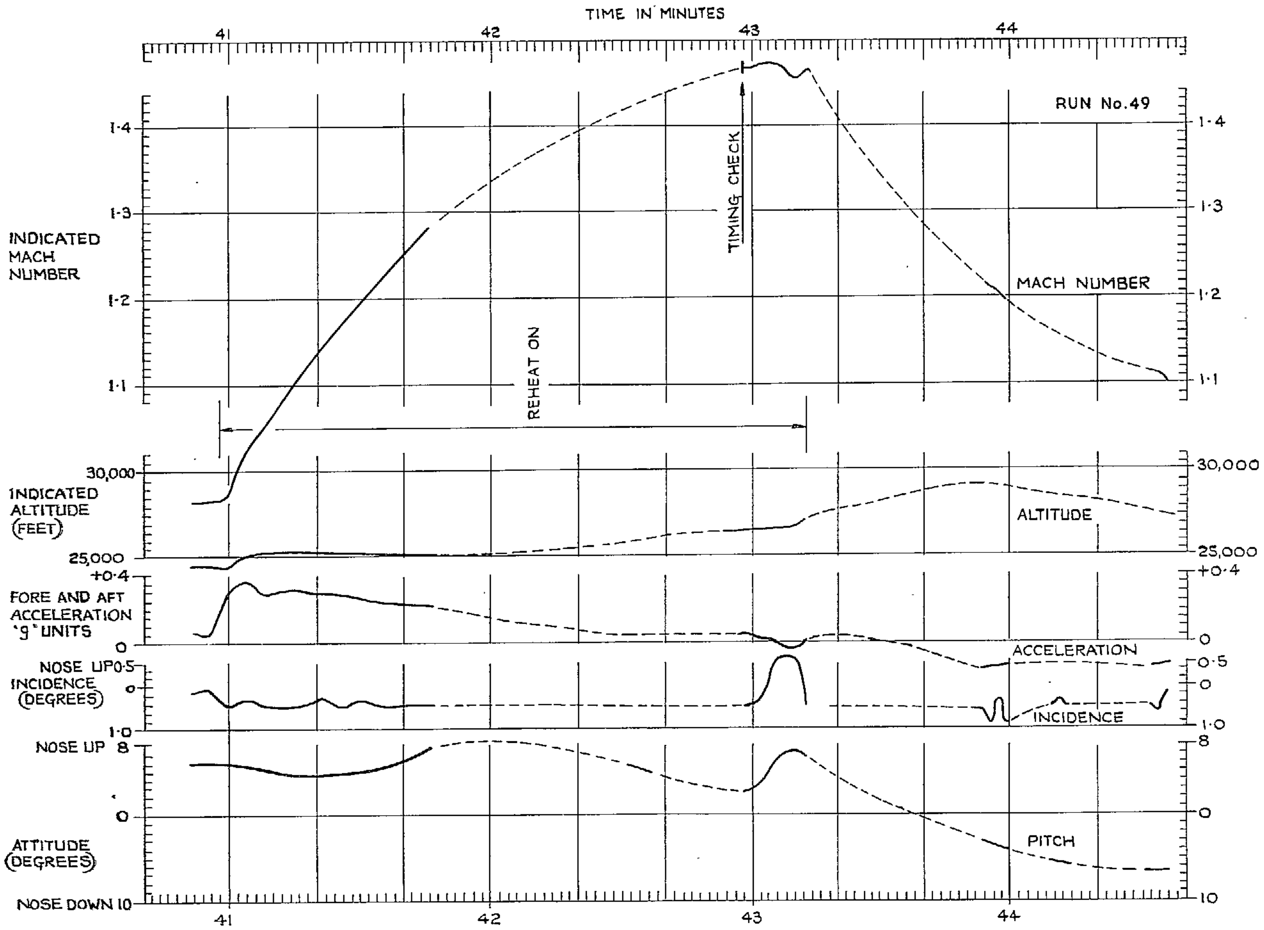


Fig. 43. Flight history for run 49.

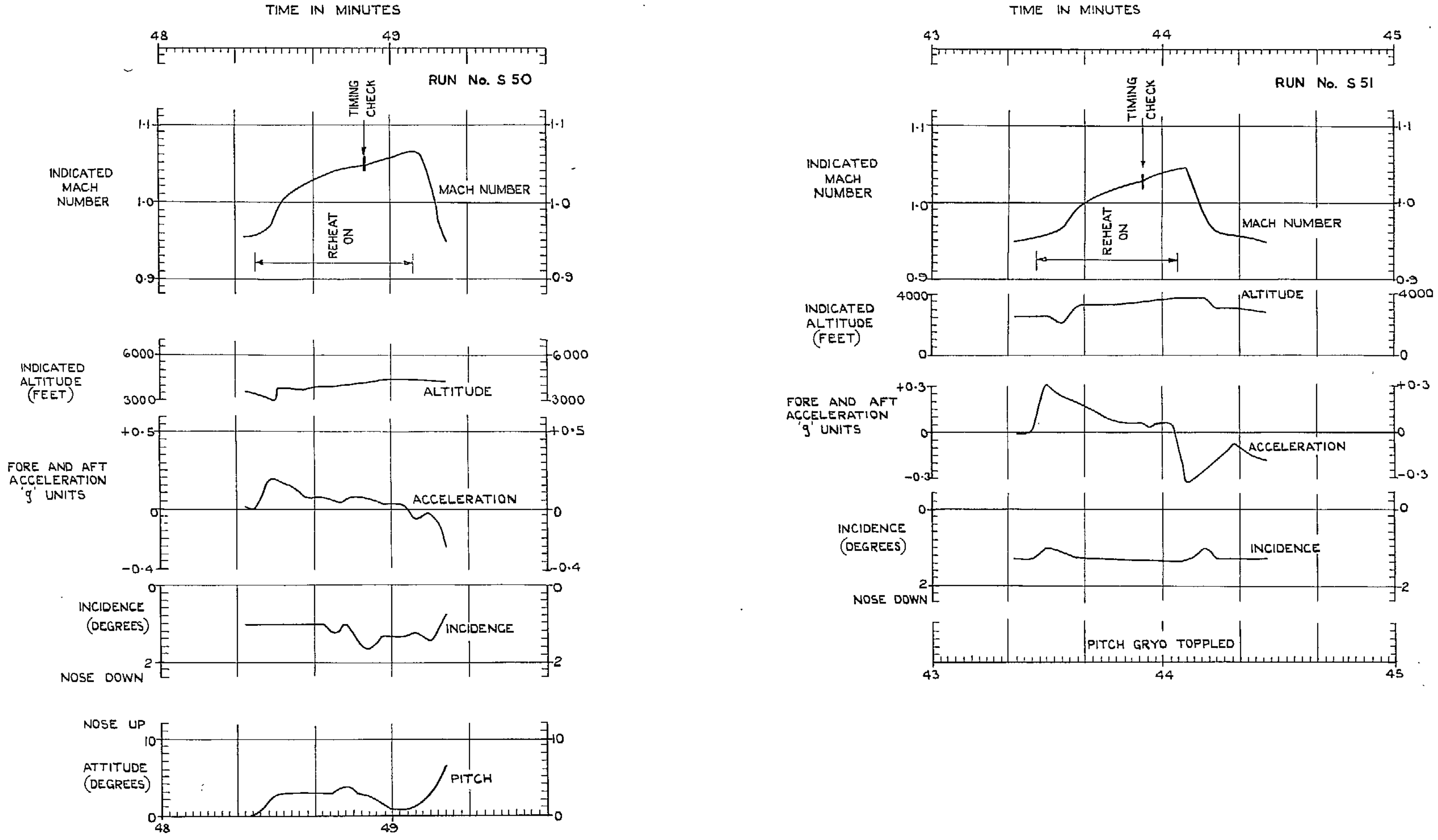


FIG. 44. Flight histories for runs S.50 and S.51.

PRESSURE/TIME RECORDS

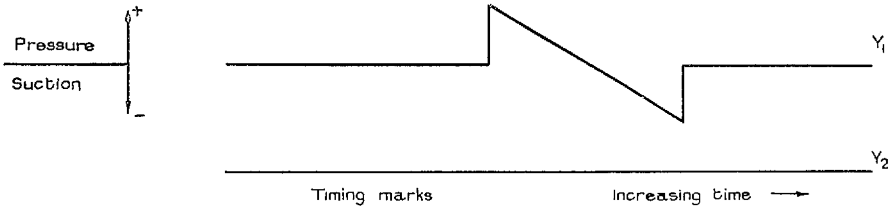
Figs. 45 to 76

Notes

The timing marks occur at intervals of 0·01 sec.

The records are unsynchronised.

The records read from left to right as follows:



KEY TO FLIGHT CONDITIONS

h_0 Aircraft altitude
M_0 Aircraft Mach number
} linked with the bangs received at 'Radar' and 'Tower' recording sites.

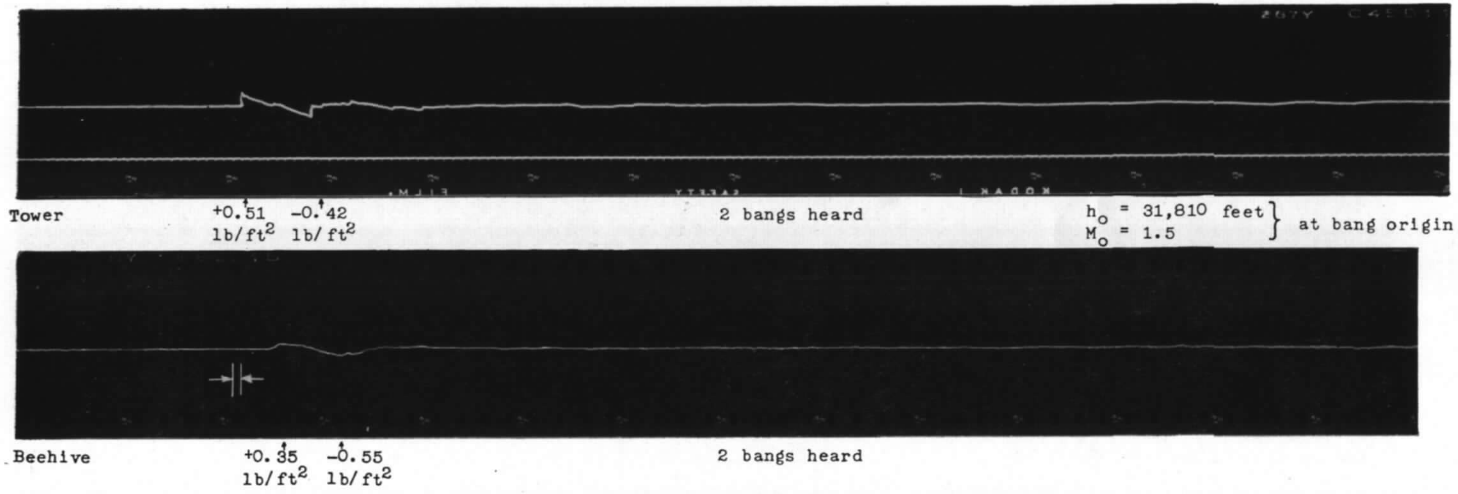


FIG. 45. Pressure/time records—run 5.

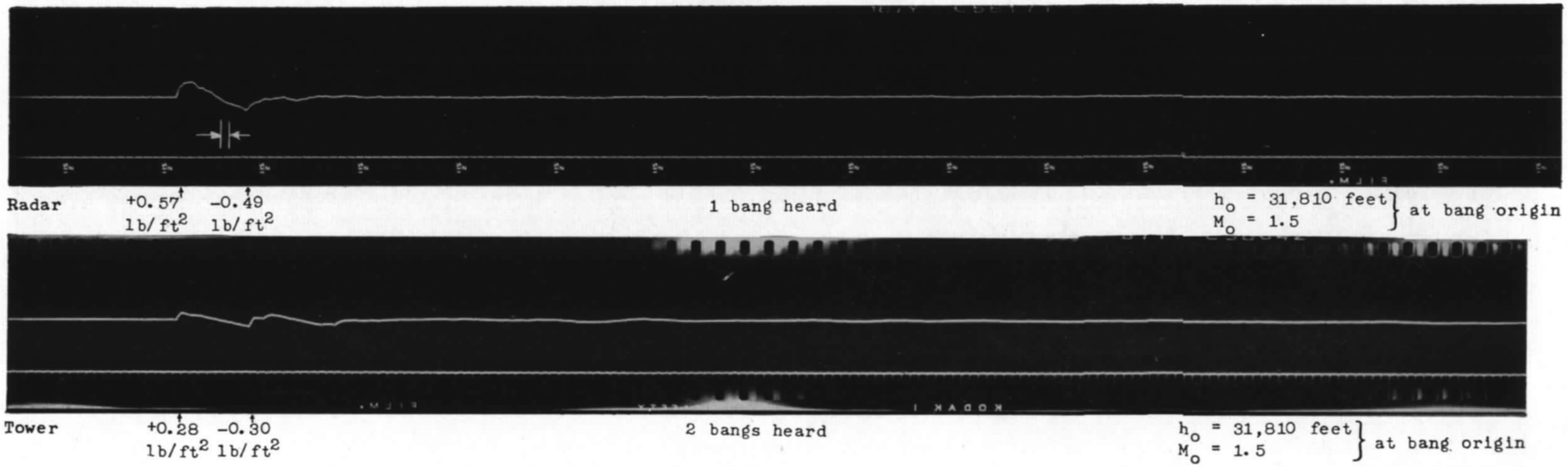
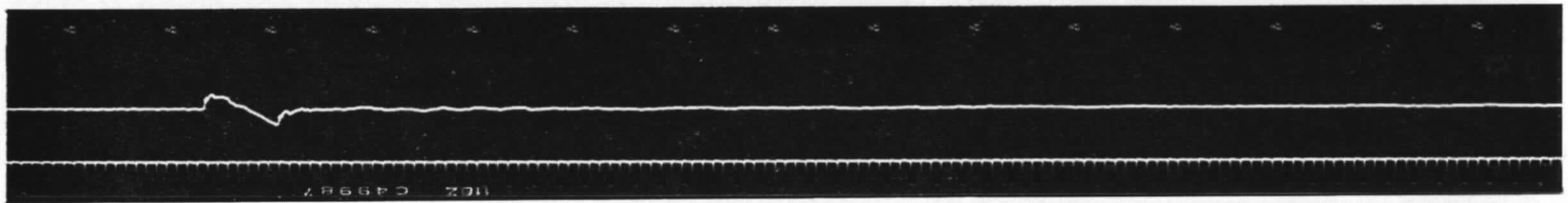
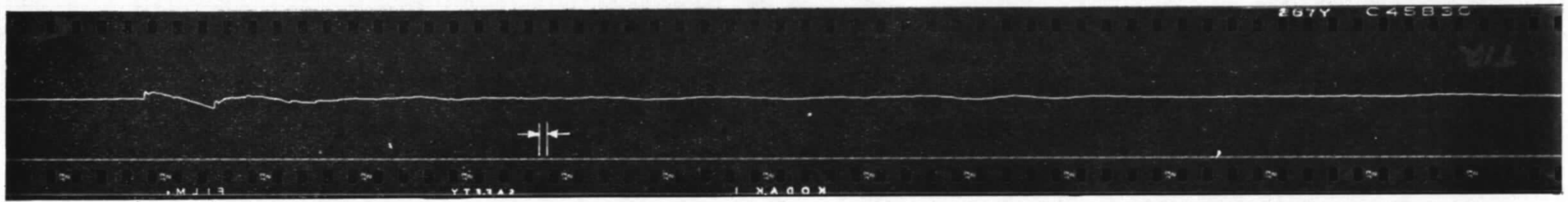


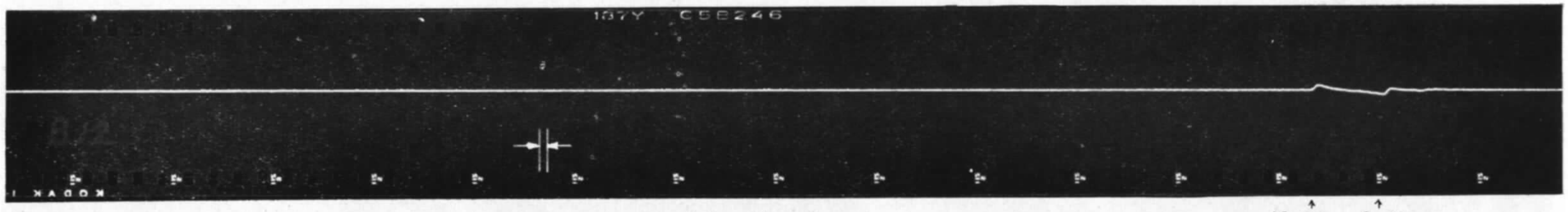
FIG. 46. Pressure/time records—run 6.



Radar ↑+0.51 lb/ft^2 ↑−0.59 lb/ft^2 2 bangs heard h_o = 28,290 feet (climbing), M_o = 1.30 (decelerating) } at bang origin



Tower ↑+0.25 lb/ft^2 ↑−0.39 lb/ft^2 2 bangs heard h_o 28,290 feet (climbing), M_o 1.30 (decelerating) } at bang origin



Beehive 1 bang heard ↑+0.36 lb/ft^2 ↑−0.31 lb/ft^2

FIG. 47. Pressure/time records—run 12.

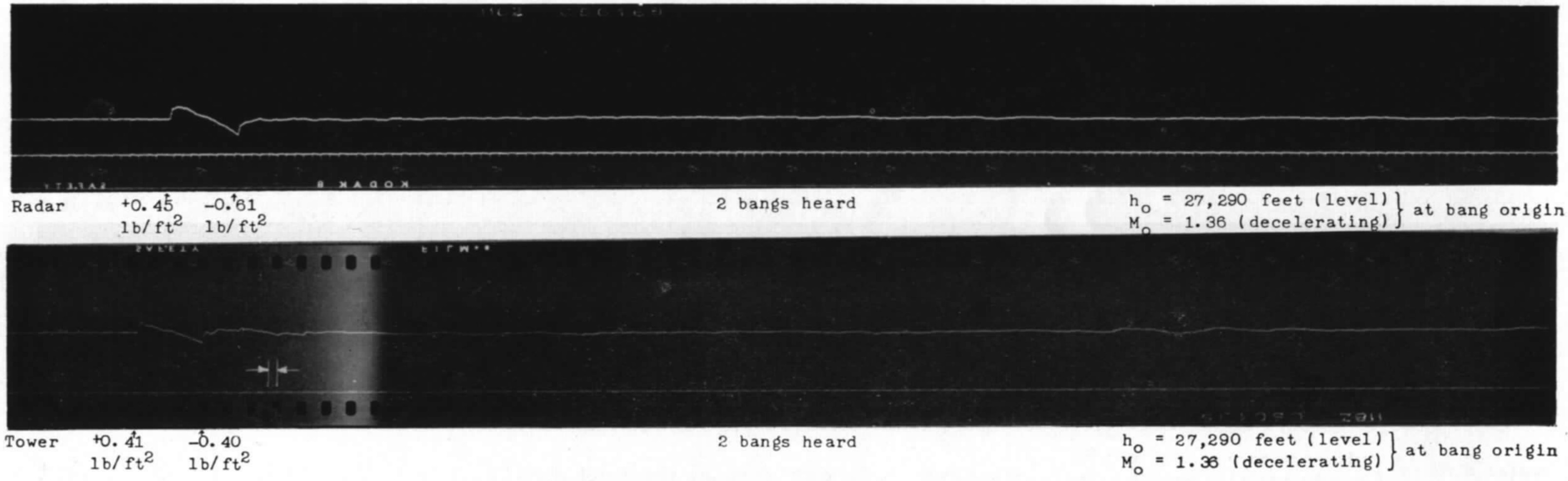


FIG. 48. Pressure/time records—run 13.

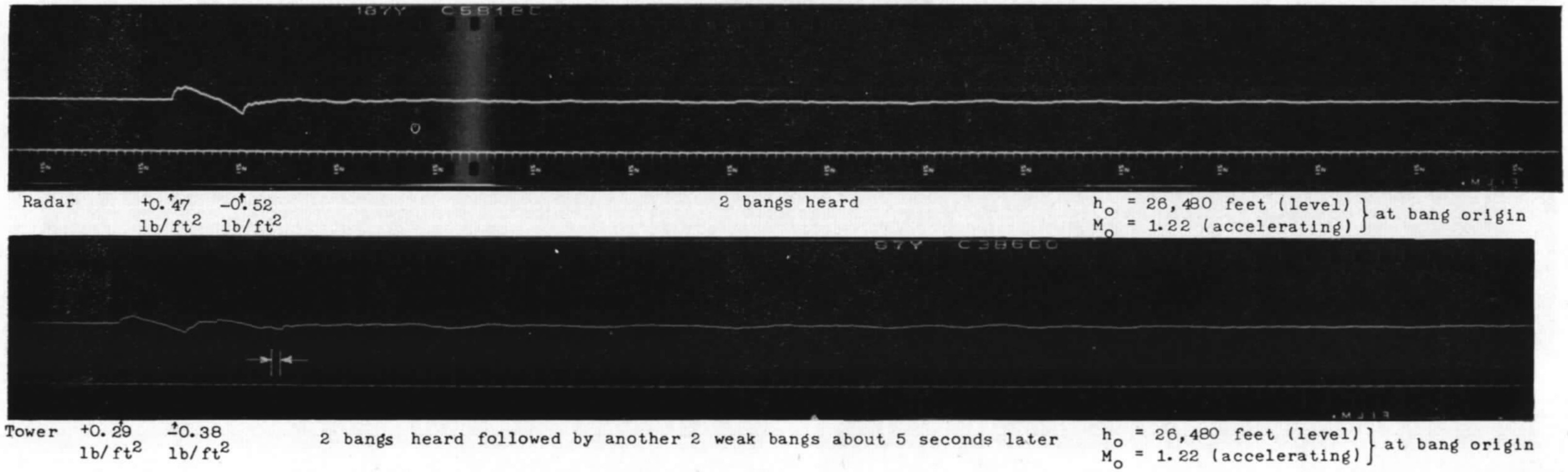


FIG. 49. Pressure/time records—run 14.

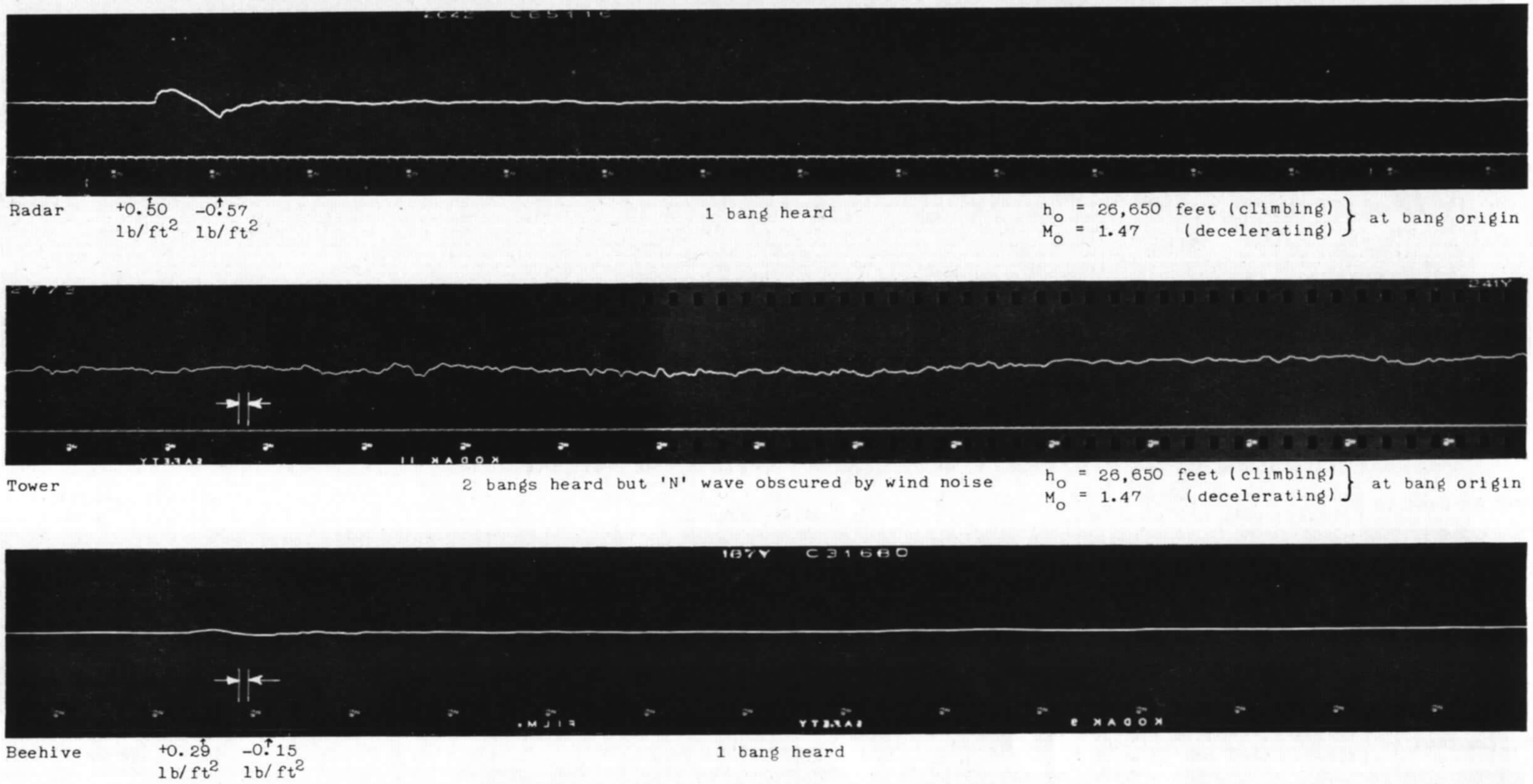


FIG. 50. Pressure/time records—run 16.

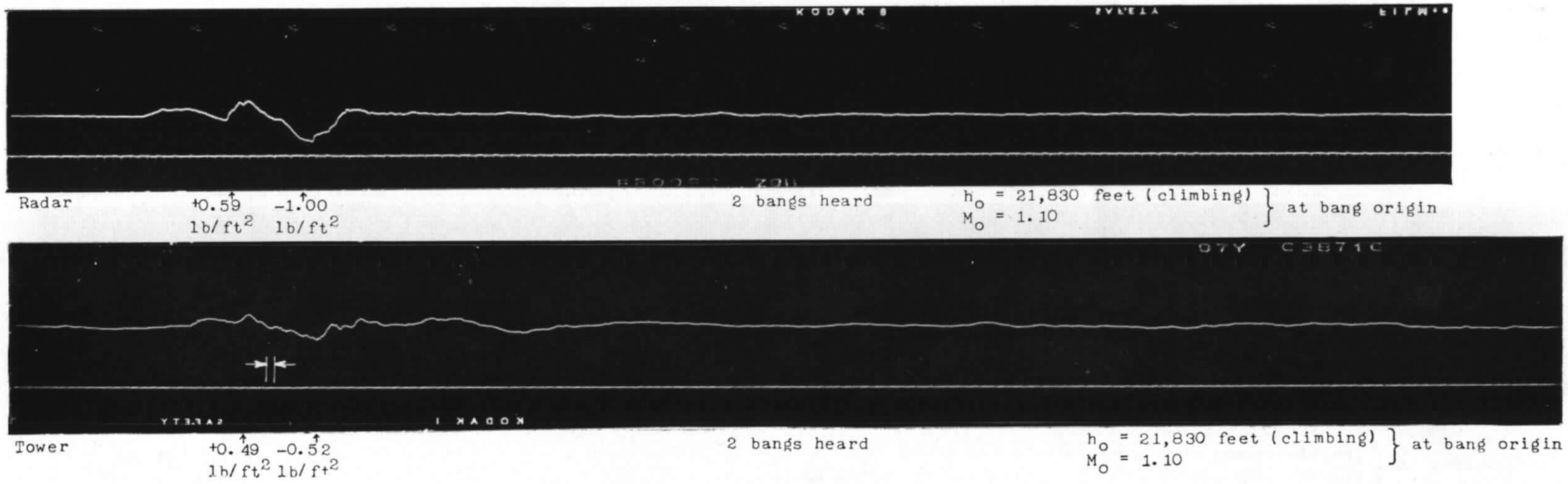


FIG. 51. Pressure/time records—run 18.

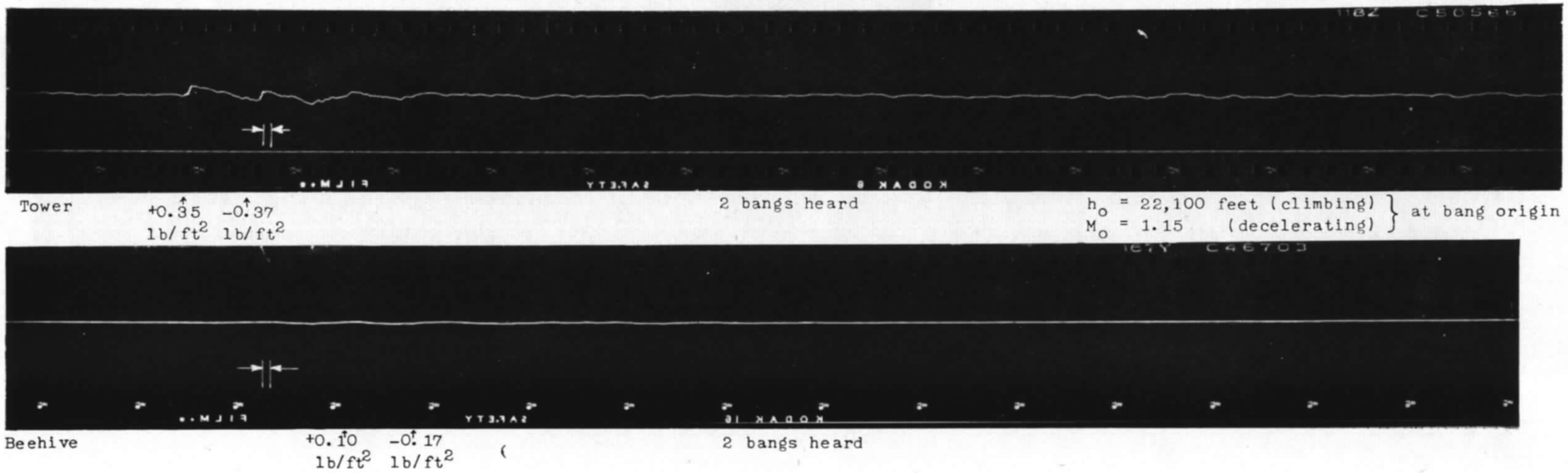


FIG. 52. Pressure/time records—run 19.

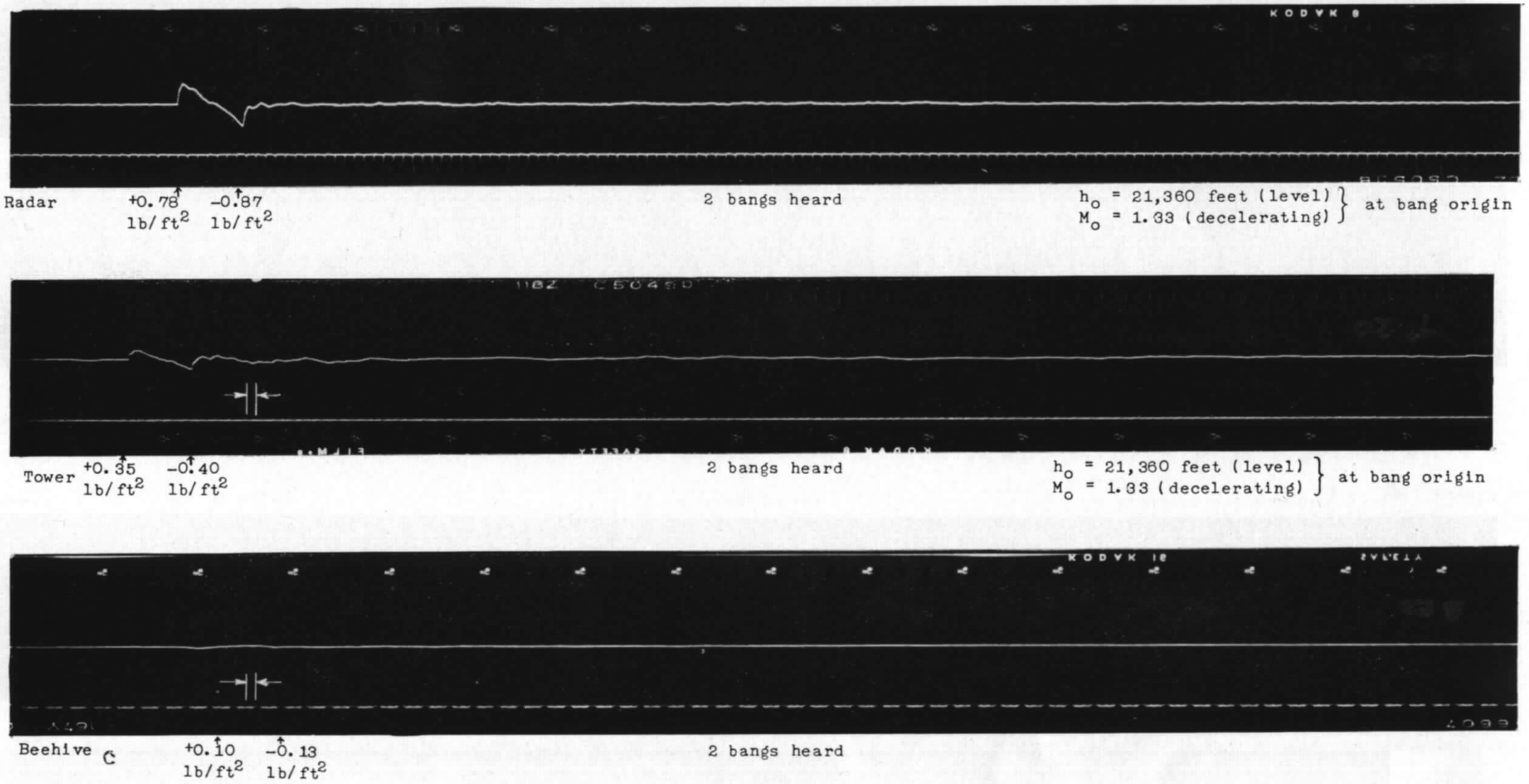


FIG. 53. Pressure/time records—run 20.

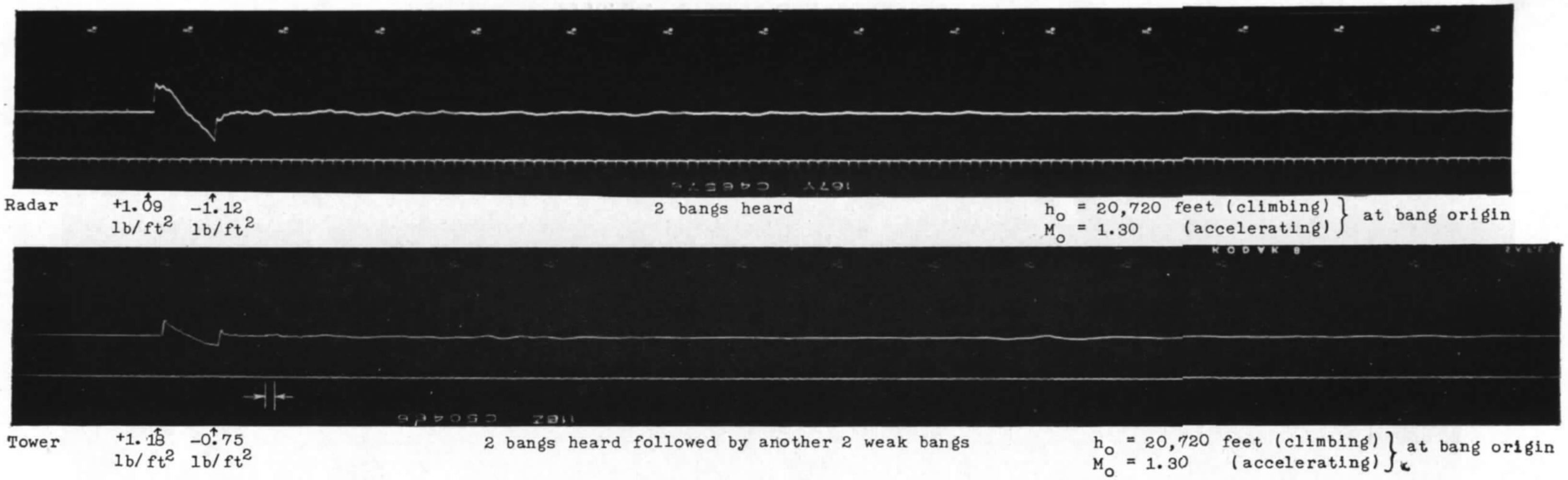


FIG. 54. Pressure/time records—run 22.

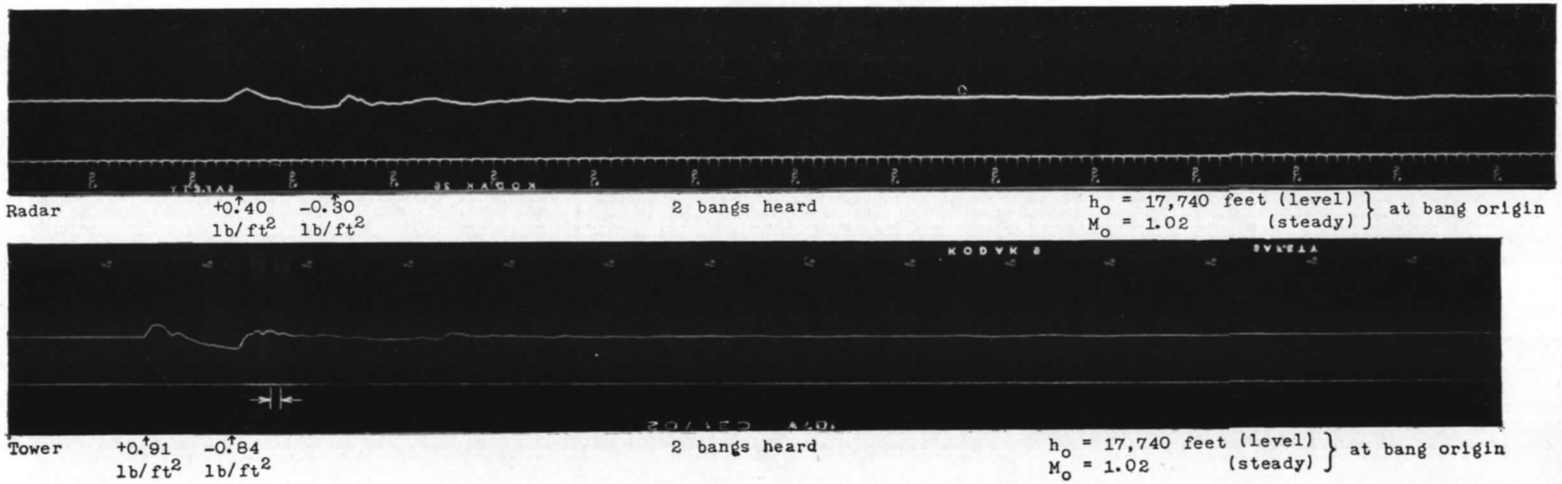


FIG. 55. Pressure/time records—run 23.

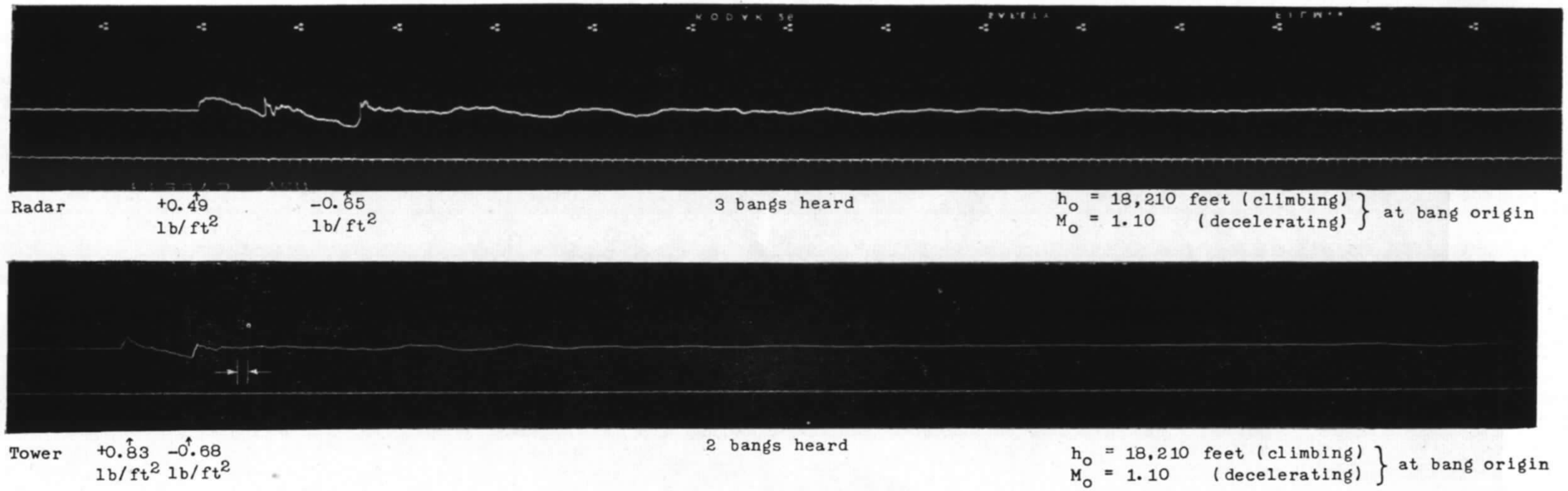


FIG. 56. Pressure/time records—run 24.

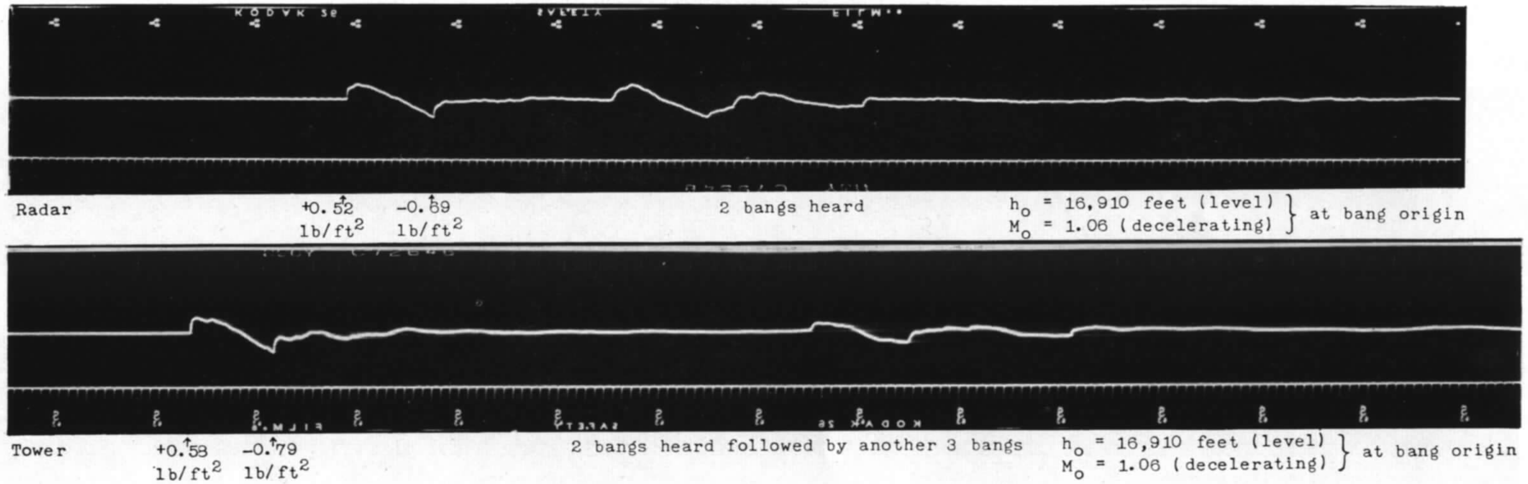


FIG. 57. Pressure/time records—run 29.

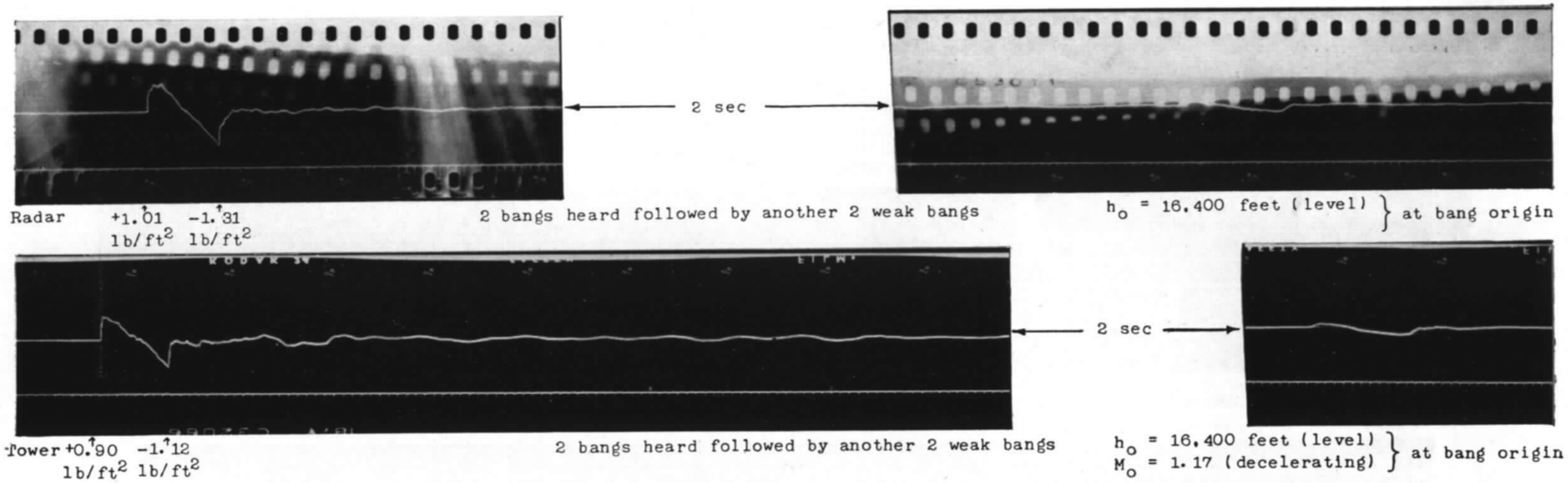


FIG. 58. Pressure/time records—run 30.

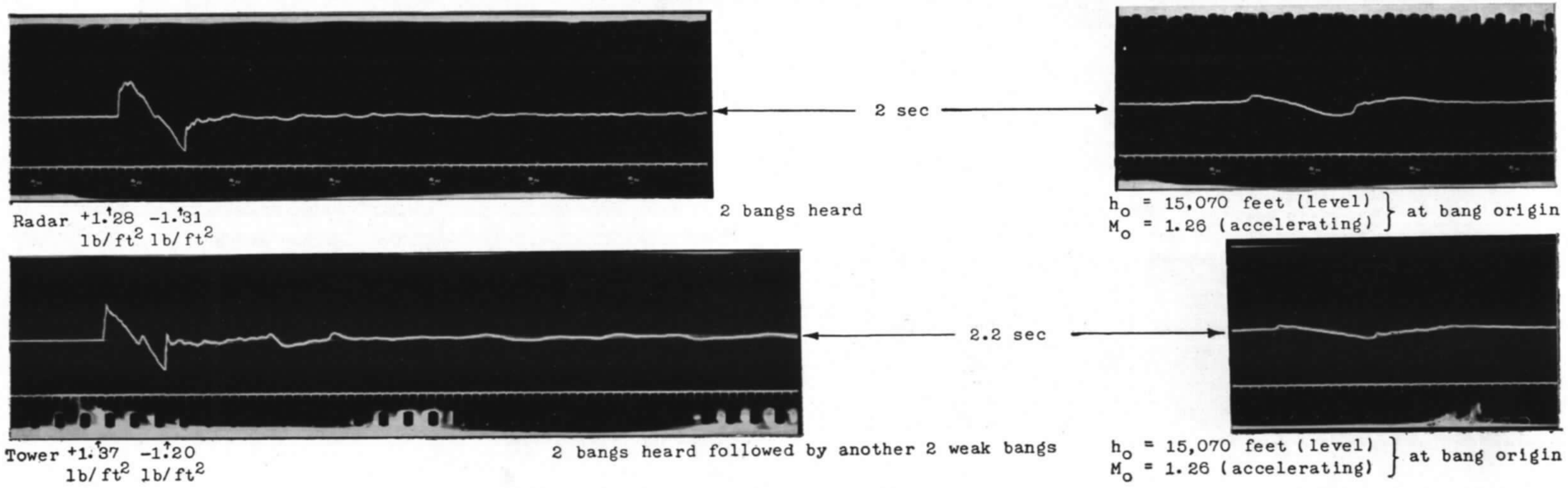


FIG. 59. Pressure/time records—run 31.

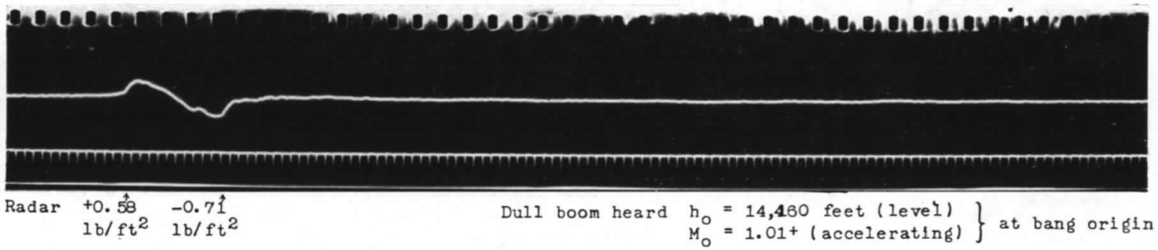


FIG. 60. Pressure/time records—run 32.

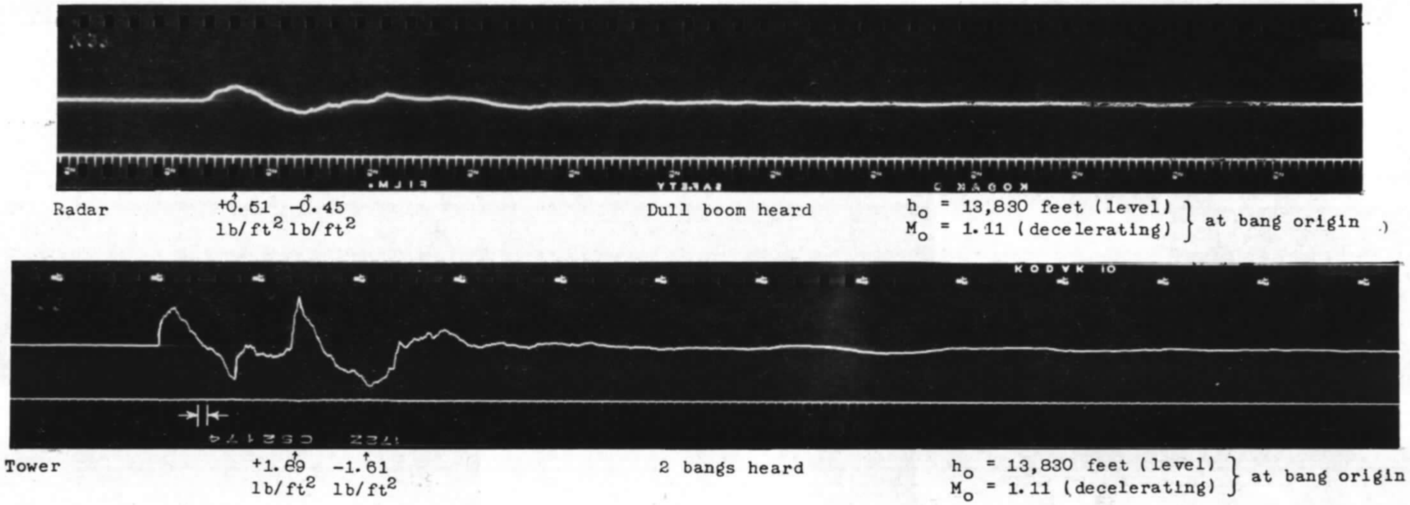


FIG. 61. Pressure/time records—run 33.

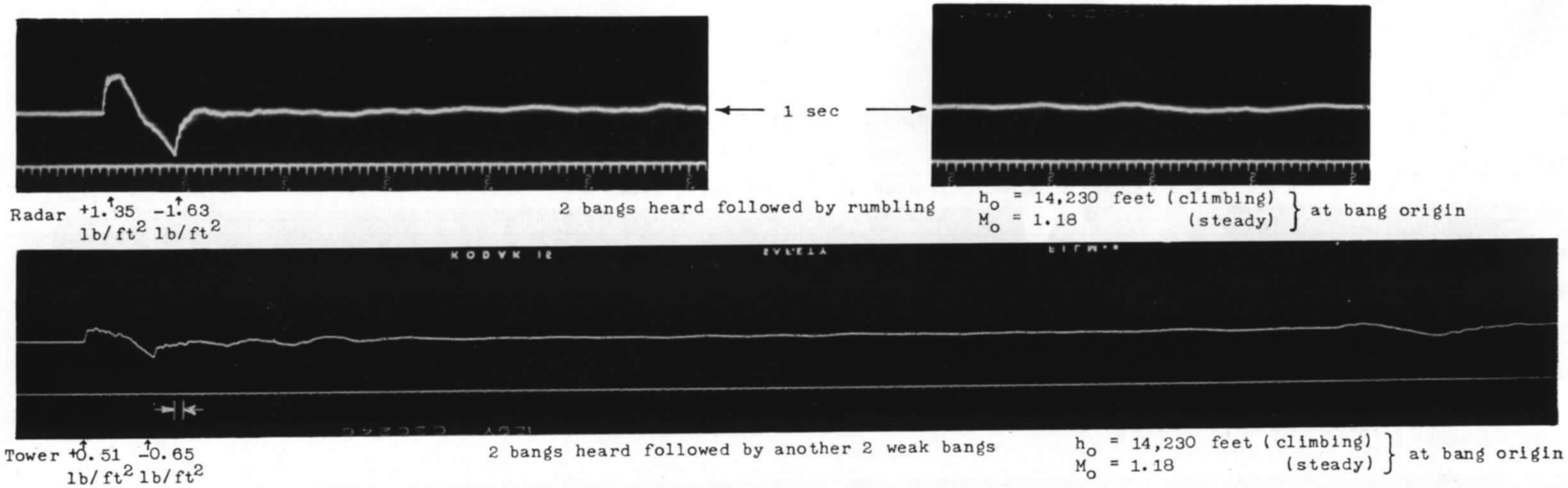


FIG. 62. Pressure/time records—run 34.

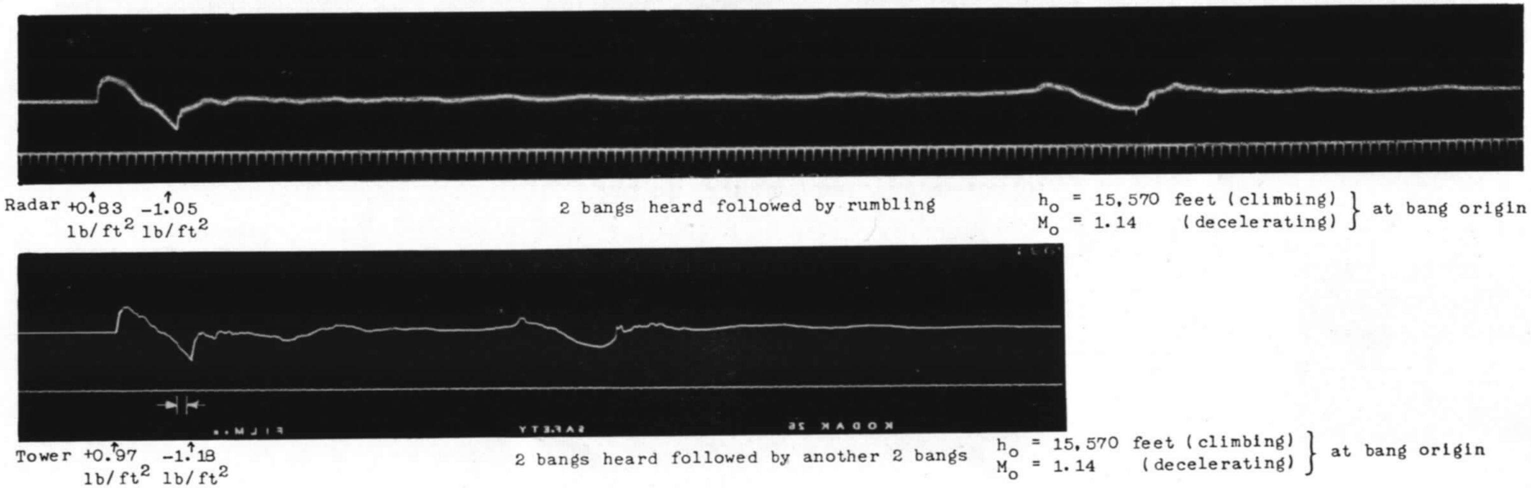
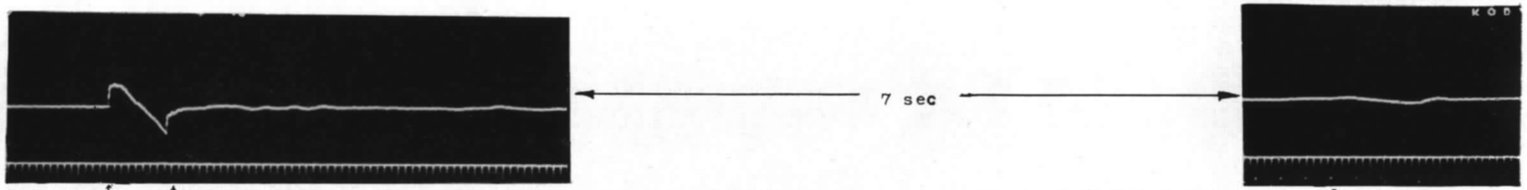


FIG. 63. Pressure/time records—run 36.

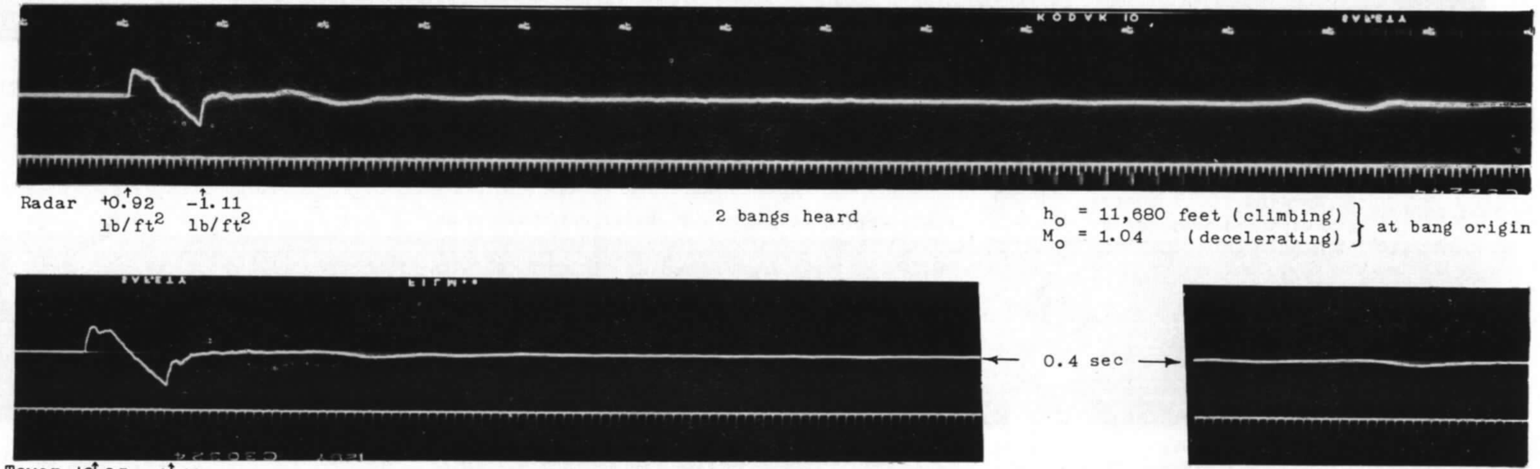


Radar +0.81 lb/ft² −1.01 lb/ft²

2 bangs heard

h_o = 19,920 feet (climbing)
M_o = 1.27 (decelerating)
} at bang origin

FIG. 64. Pressure/time records—run 37.



Radar +0.92 lb/ft² −1.11 lb/ft²

2 bangs heard

h_o = 11,680 feet (climbing)
M_o = 1.04 (decelerating)
} at bang origin

Tower +0.95 lb/ft² −1.33 lb/ft²

2 bangs heard followed by another 2 bangs

h_o = 11,680 feet (climbing)
M_o = 1.04 (decelerating)
} at bang origin

FIG. 65. Pressure/time records—run 39.

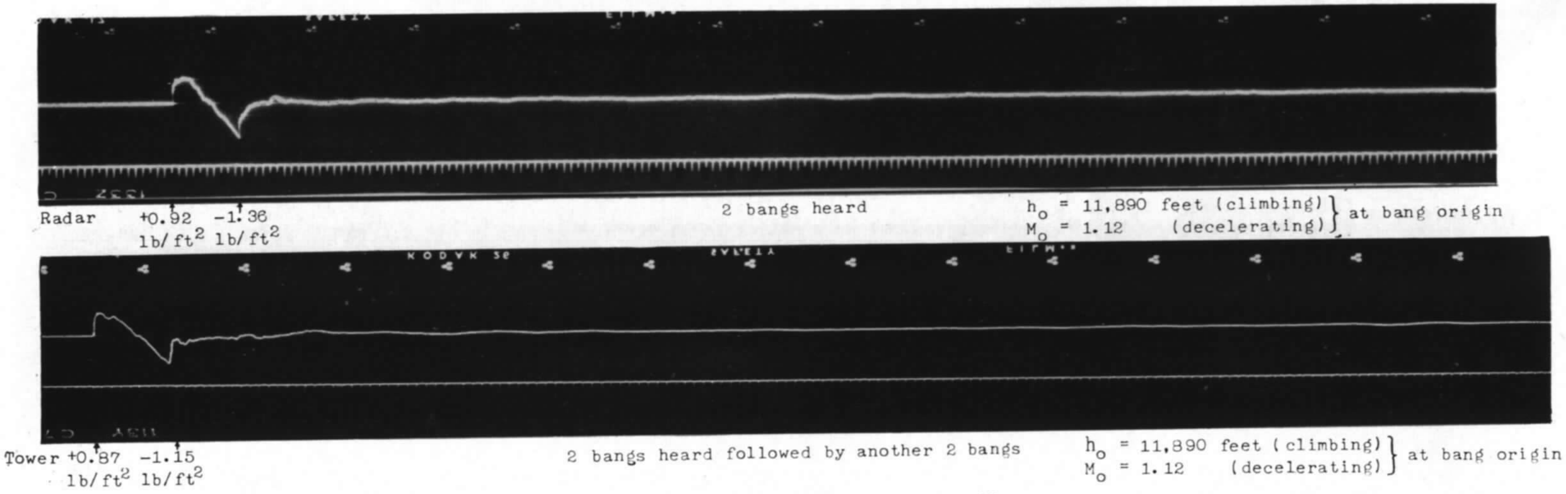


FIG. 66. Pressure/time records—run 40.

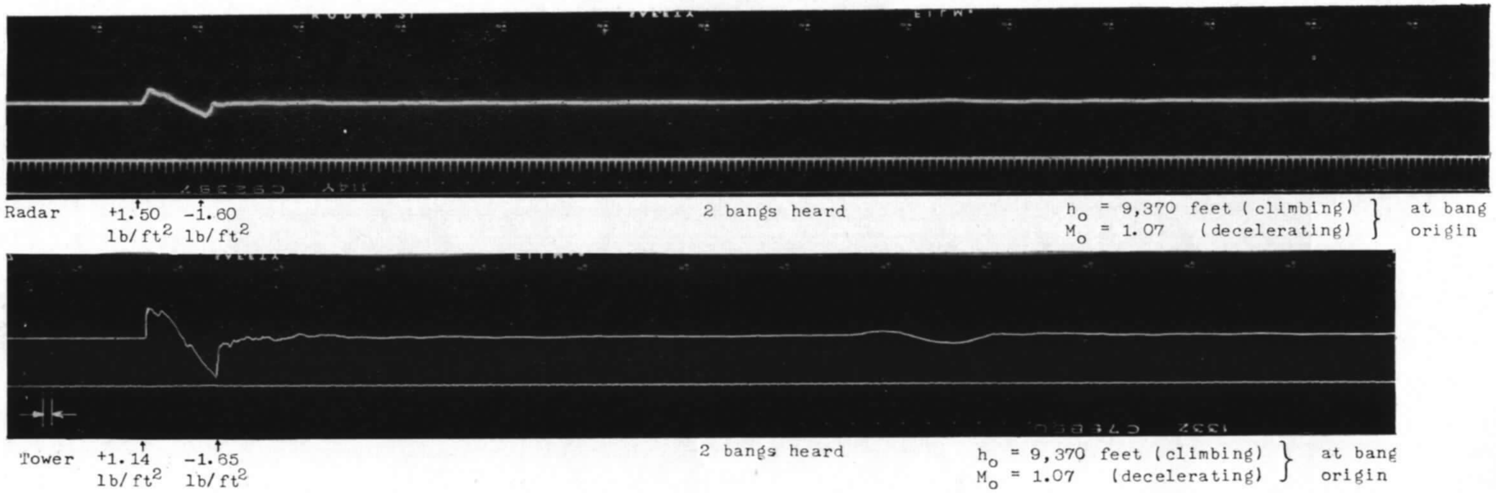
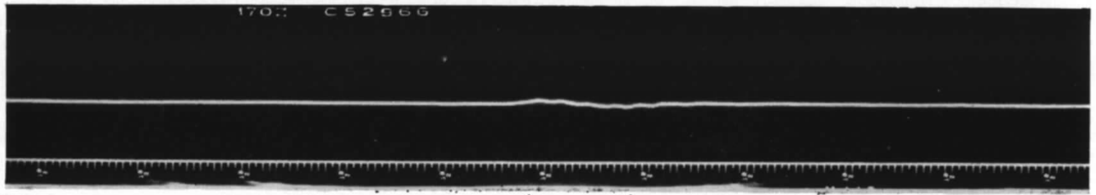


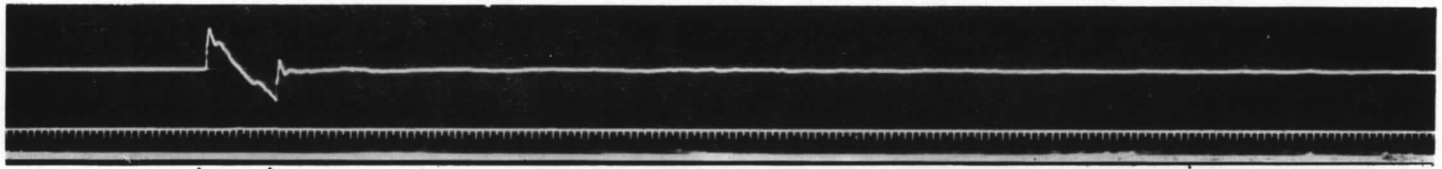
FIG. 67. Pressure/time records—run 42.



Radar   Dull boom heard   +0.30 $\mathrm{lb/ft^2}$ −0.45 $\mathrm{lb/ft^2}$ h_o = 6,660 feet (climbing), M_o = 1.07 (decelerating) } at bang origin

Tower   No recording obtained   2 bangs heard h_o = 6,660 feet (climbing), M_o = 1.07 (decelerating) } at bang origin

FIG. 68. Pressure/time records—run 43.



Radar   +4.05 $\mathrm{lb/ft^2}$ −3.90 $\mathrm{lb/ft^2}$ 2 bangs heard followed by another 2 bangs h_o = 6,130 feet (climbing), M_o = 1.08 (accelerating) } at bang origin

FIG. 69. Pressure/time records—run 44.

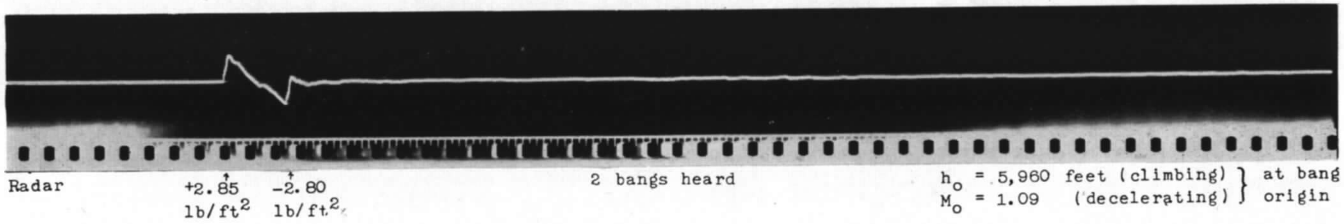


FIG. 70. Pressure/time records—run 45.

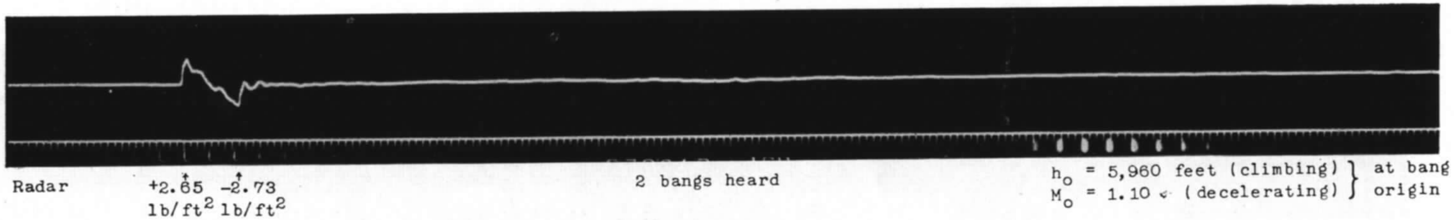
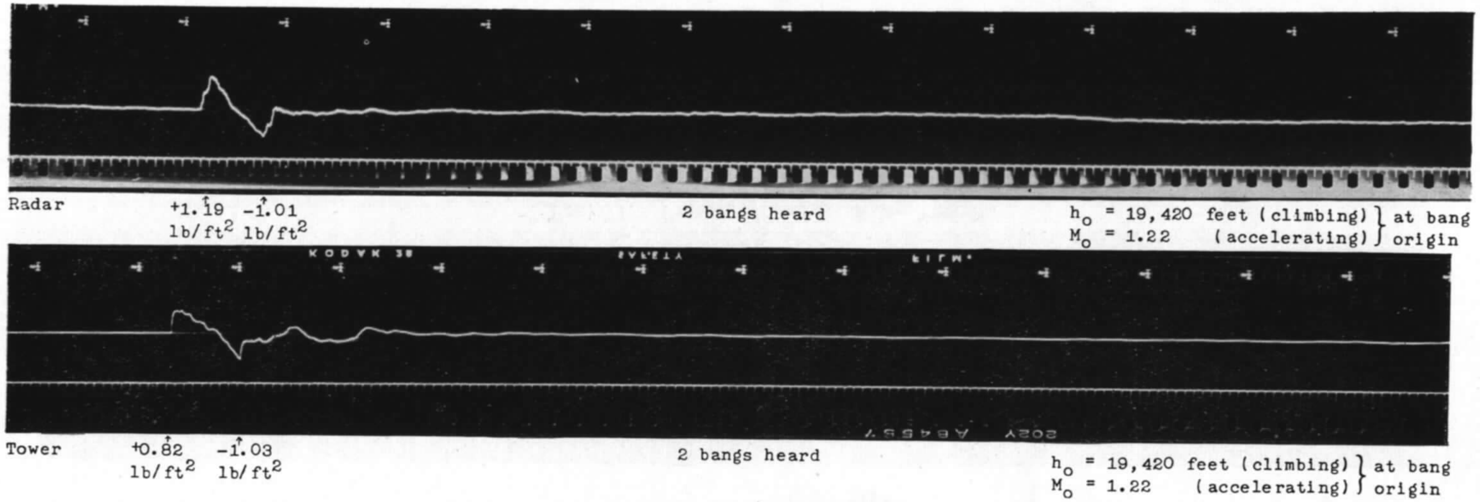


FIG. 71. Pressure/time records—run 46.



Radar +1.19 lb/ft² −1.01 lb/ft²

2 bangs heard

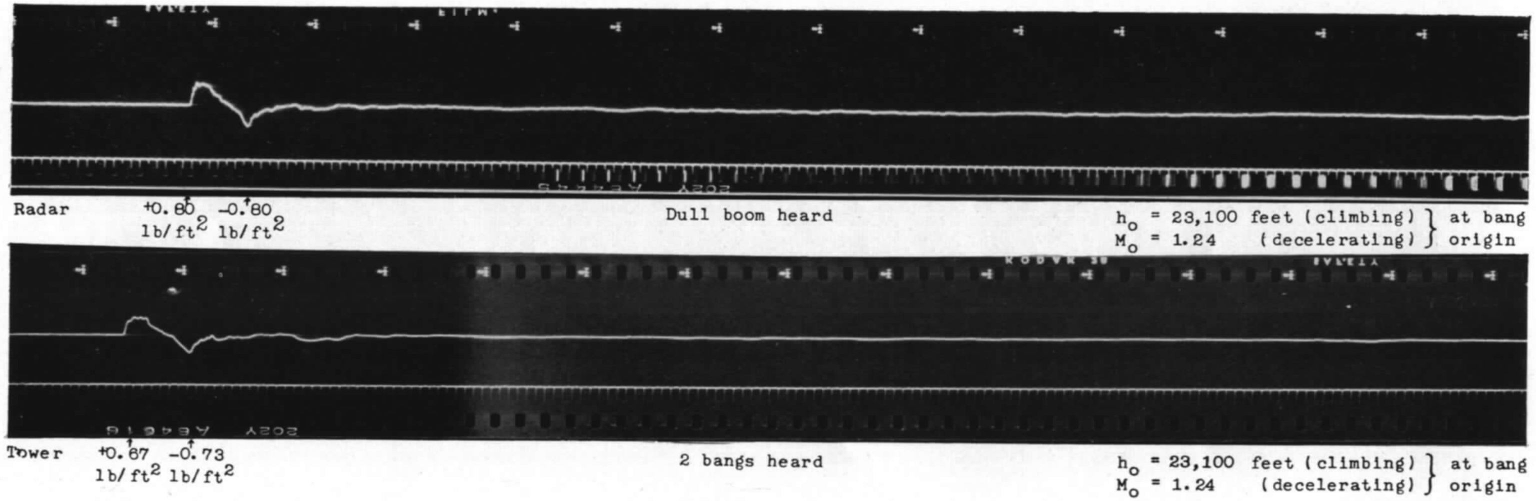
h_o = 19,420 feet (climbing) } at bang origin
M_o = 1.22 (accelerating) }

Tower +0.82 lb/ft² −1.03 lb/ft²

2 bangs heard

h_o = 19,420 feet (climbing) } at bang origin
M_o = 1.22 (accelerating) }

FIG. 72. Pressure/time records—run 47.



Radar +0.80 lb/ft² −0.80 lb/ft²

Dull boom heard

h_o = 23,100 feet (climbing) } at bang origin
M_o = 1.24 (decelerating) }

Tower +0.67 lb/ft² −0.73 lb/ft²

2 bangs heard

h_o = 23,100 feet (climbing) } at bang origin
M_o = 1.24 (decelerating) }

FIG. 73. Pressure/time records—run 48.

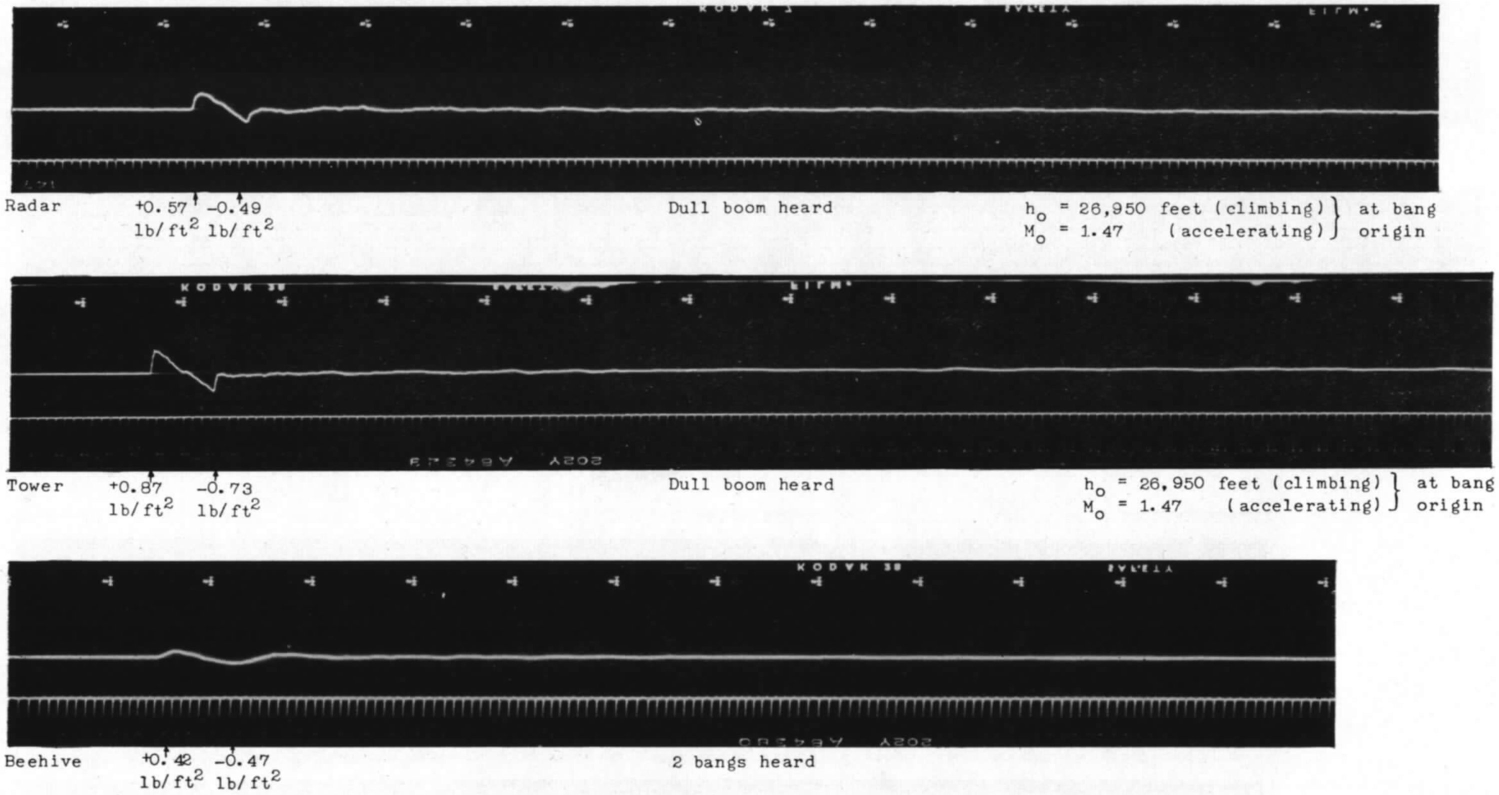


FIG. 74. Pressure/time records—run 49.

(8651) Wt. 64/1857 K.5 3/63 Hw.

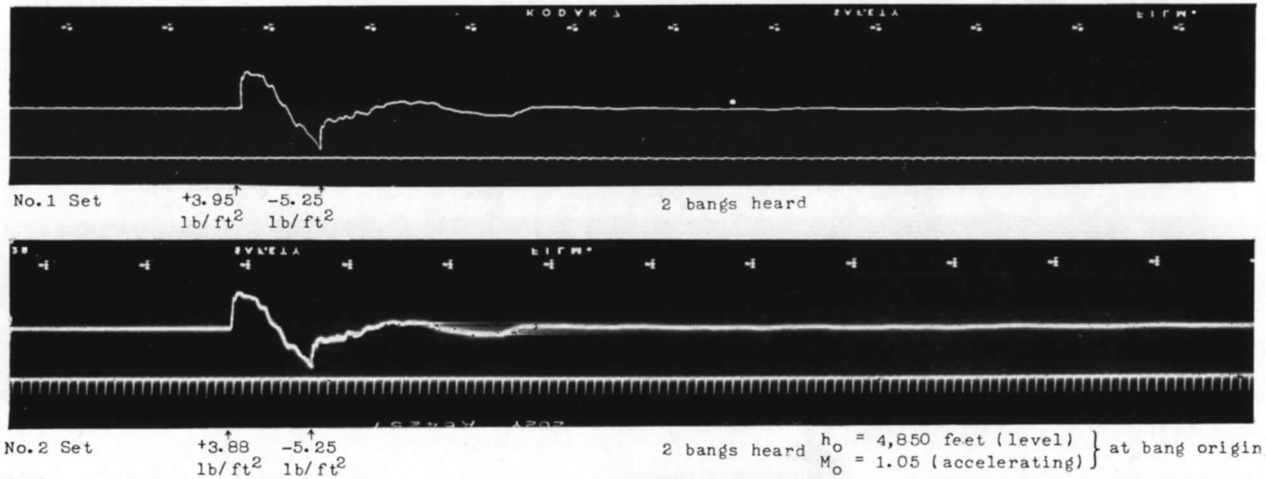


FIG. 75. Pressure/time records—run S.50.

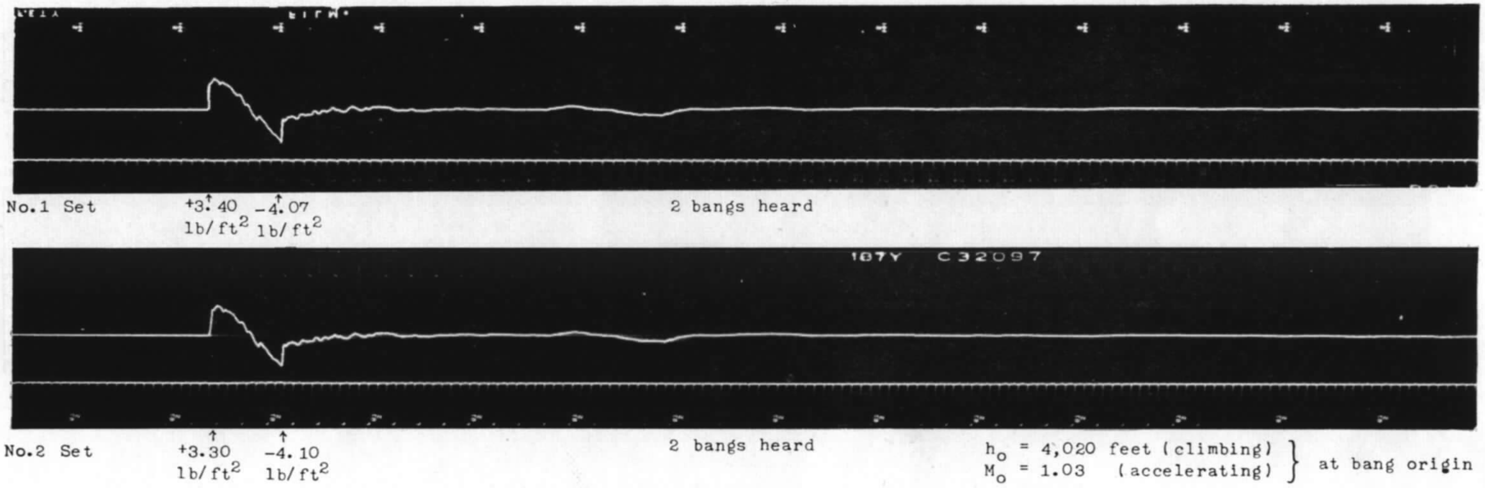


FIG. 76. Pressure/time records—run S.51.

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