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Influence of the Testing Machine on the Flexural Failure of Panels

By

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Summary.—The pin-ended length of a panel tested flat-ended in a compression testing machine is influenced by the flexibility of the machine. A system of elastic constants is defined to describe this influence (section 2) and equations for the critical load developed (section 3). These constants are determined by stiffness measurements made on the testing machine (section 4) and by calculation of the platten deformation at the area of contact with the panel (section 5). Finally, the degree of fixation achieved in the testing of typical panels is calculated and the results given in graphical form (section 6).

1. *Introduction.*—The testing of panels in compression is important both for research and for the justification of designs. In the case where the failure of the panel is flexural in character, the nature of the support exerted by the machine upon the ends of the panel is of crucial significance for the interpretation of the results. Most panels are tested " flat-ended ", and in this case it is clear that, provided the machine is sufficiently rigid, the ends of the panel will be completely clamped. Real testing machines cannot of course reach this ideal and so an investigation into the precise influence exerted by the machine is of importance, both for the interpretation of test results and to provide information for guidance in the design of testing machines.

The flexibility of a testing machine may conveniently be divided into two parts. In the first place we have flexibility of the machine structure as a whole. In the second place we have flexibility of the platten in the neighbourhood of the area of contact with the panel. Both these forms of flexibility are important and must be allowed for in the calculation of panel buckling loads. The most convenient methods for the determination of these two flexibilities are of quite different character. The first is best determined by experiment (section 4), the second by calculation (section 5).

2. *The Elastic Constants of the Machine.*—We begin by considering the relations existing between the forces applied by the flexurally buckled panel to the testing machine and the resulting relative displacements of the areas of contact of the panel with the plattens. These areas of contact are shown by the heavy black lines in Fig. 1. That on the upper platten is shown in its normal and deflected state, while that on the bottom platten is shown as fixed. The forces exerted upon the top platten consist of the panel end load P, the side force S and the couple M. The displacement of the upper (" moving ") area of contact is denoted by δ and its rotation by θ ; both quantities are relative to the lower area of contact. The forces exerted upon the lower platten are also shown in Fig. 1. They follow from the equilibrium conditions for the panel. All forces and displacements are defined with reference to the normal (undeflected) axes of the testing machine. The length of the panel is denoted by l.

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It is to be remarked that we are assuming that the areas of contact with the plattens are plane. This is consistant with the usual supposition that the bending of the panel follows the assumptions of the beam theory.

The relations between force and displacement may be written

$$\left.\begin{aligned} S &= a_{11}\delta + a_{12}\theta, \\ M &= a_{21}\delta + a_{22}\theta, \\ a_{12} &= a_{21}, \end{aligned}\right\} \qquad (1)$$

where the quantities a_{ij} $(i, j = 1, 2)$ are functions of P, l and the position of the hydraulic rams relative to their cylinders in the case of a hydraulic machine.

Solving (1) we find—

$$\left.\begin{aligned} \delta &= A_{11}\, S + A_{12}\, M, \\ \theta &= A_{21}\, S + A_{22}\, M, \end{aligned}\right\} \qquad (2)$$

where,

$$\left.\begin{aligned} A_{11} &= a_{22}/\Delta \quad A_{12} = A_{21} = -\, a_{12}/\Delta \quad A_{22} = a_{11}/\Delta, \\ \Delta &= a_{11}\, a_{22} - a_{12}^{2}. \end{aligned}\right\} \qquad (3)$$

The a_{ij}'s are "stiffnesses" and the A_{ij}'s "flexibilities". The validity of Hooke's law (equation (1)) for this system is assured by general structural theory for small δ and θ and is fully confirmed by experiment.

3. *The Critical Load for a Panel.*—The calculation of the critical load for a panel or strut whose ends are elastically supported and which buckles flexurally under compressive load is a straightforward exercise in beam deflection theory. We confine ourselves here therefore to a statement of the results. The elastic restraint resisting relative movement of the ends is assumed to be defined by equation (1). The critical load P_{crit} is found to be given by the transcendental equation

$$\frac{l^3 a_{11}}{B}\left(\frac{\sin\zeta - \zeta\cos\zeta}{\zeta^2}\right) + \frac{2l^2 a_{12}}{B}\left(\frac{1-\cos\zeta}{\zeta}\right) + \frac{l a_{22}}{B}\sin\zeta + \frac{l^4\Delta}{B^2}\left(\frac{2 - 2\cos\zeta - \zeta\sin\zeta}{\zeta^3}\right) + \zeta\cos\zeta = 0, \qquad (4)$$

where B = the flexural rigidity of the panel and

$$\zeta = l\sqrt{\frac{P_{crit}}{B}} \qquad (5)$$

A special case of equation (4) is of particular interest in practice. Most testing machines are relatively rigid as regards rotation of the plattens, their weakness, if any, taking the form of excessive flexibility for sideways movement δ.

If we make $a_{22} \to \infty$ in (4) we find

$$\frac{l^3 a_{11}}{B} = \frac{\zeta^3 \sin\zeta}{\zeta\sin\zeta - 2\,(1 - \cos\zeta)} \qquad (6)$$

Equation (6) has the root $\zeta = 2\pi$ corresponding to clamped ended buckling. When a_{11} is small there is a root $\zeta = \pi$, which increases with a_{11} eventually reaching 2π and exceeding it. For this special case clamped ended buckling is therefore guaranteed if

$$a_{11} \geqslant \frac{4\pi^2 B}{l^3} = \frac{P_{\text{clamped}}}{l}, \qquad (7)$$

where $P_{\text{clamped}} = 4\pi^2 B/l^2$ = the clamped ended buckling load.

If $P_{clamped}$ is written equal to the maximum capacity of the testing machine, equation (7) then gives a rough " stiffness requirement " for testing machine design.

4. *Measurement of the Machine Stiffness.*—The overall stiffness of testing machine structures may be measured by experiments illustrated diagrammatically in Fig. 2. A pin-ended strut of length l is compressed in the testing machine with either one end (a) or both ends (b) offset by varying small distances from the centre line of the machine. The relative deflection δ and rotation θ of the plattens is measured for a representative series of values of P and for typical strut lengths. The forces S and M applied to the top platten can be calculated from P, the offsets and the measured deflections. The experimental results can then be analysed to give values of a_{ij} appropriate to the stiffness of the machine structure. The only type of platten surface deformation which can occur in this experiment is that due to the shear forces S. It may be shown by the methods used in section 5 below that these are negligibly small.

The values actually obtained from experiments performed upon the Universal 50-ton Avery Testing Machine in the Bristol Aeroplane Co. Structural Research Laboratory, Filton, are given below.

a_{11} (ton/in.)

	Rams Closed			Rams Extended 13 in.		
l (in.)	30	50	70	30	30	70
P = 10 tons	5·2	3·6	2·5	4·3	2·9	2·0
P = 50 tons	5·6	4·0	2·9	5·0	3·4	2·3

$a_{12} = a_{21}$ (tons)

	Rams Closed			Rams Extended 13 in.		
l (in.)	30	50	70	30	50	70
P = 10 tons	−236	−210	−190	−210	−180	−160
P = 50 tons	−222	−194	−174	−196	−170	−150

a_{22} (ton/in.)

	Rams Closed			Rams Extended 13 in.		
l (in.)	30	50	70	30	50	70
P = 10 tons	19800	21900	24700	16400	17600	19500
P = 50 tons	19800	21900	24700	16400	17600	19500

where l = lengths between plattens and P = load indicated by machine.

5. *Calculation of the Platten Deformation.*—The deformation of the plattens under pressure applied by the ends of the panel may be estimated with fair approximation using the known theory of forces acting upon the surface of a " half-space " or body bounded by a single plane surface. A suitable reference for the appropriate results is Love's Mathematical Theory of

Elasticity, p. 192–3 and p. 243. Applying these results to the case of a rectangular area sides $2a$ and $2b$, where $a >> b$, subjected to uniform pressure p, we find that the deflection at the centre w_o is given by

$$w_o = \frac{4bp\,(1 - \sigma^2)}{\pi\, E}\left(1 + \log\frac{2a}{b}\right), \quad \ldots \quad (8)$$

where E = Young's Modulus and σ = Poisson's Ratio for the " platten " material.

Consider now the case of a panel with Zed-section stringers applying a couple to a platten which is its reaction to the clamping moment assumed exerted by the machine. The pressures applied by the free flanges and the skin can be calculated by beam theory and the deflections of the platten at the centres of these free flanges and skin determined using equation (8). An approximate formula for the rotation of the contact area with the platten follows at once. Similar calculations applied to the effects of the shear force at the panel ends show that the corresponding deformations induced in the platten to be quite unimportant.

The calculation of the flexibilities A_{ij} corresponding to platten deformation is now immediate. Denote the flexibility for a single platten surface by F. F is then the ratio of the rotation of the contact area of the panel to a couple (bending moment) causing this rotation ; the bending moment assumed distributed linearly as in beam theory. For a penel with Zed-section stringers we find, as outlined above,

$$F = \frac{2(1 - \sigma^2)t_s}{\pi E I}\left\{\left(1 - \frac{\bar{y}}{h}\right)\left(1 + \log\frac{2d}{t_s}\right) + \frac{\bar{y}\,t}{h\,t_s}\left(1 + \log\frac{2B}{t}\right)\right\}, \quad \ldots \quad (9)$$

where, t_s = thickness of the stringer.
I = second moment of area of panel section about neutral axis,
$\bar{y}$ = distance of neutral axis from the skin,
h = depth of the stringer,
d = width of the free flange of the stringer,:
t = thickness of the skin,
and B = total width of the skin.

This formula will apply to other section stringers with suitable modification of the meaning of the constants. For top-hats for example we may take d equal to the width of the top of the hat. The bending moments and corresponding deflections are shown in Fig. 1. Using the quantity F we find

$$\left.\begin{aligned} \delta &= lF\,(M + Sl - P\delta) \\ \theta &= F\,(2M + Sl - P\delta) \end{aligned}\right\}, \quad \ldots \quad (10)$$

and comparing (10) with (3) we find,

$$A_{11} = \frac{l^2F}{(1 + PlF)},\; A_{12} = A_{21} = \frac{lF}{(1 + PlF)},\; A_{22} = \frac{2F\left(1 + \frac{PlF}{2}\right)}{(1 + PlF)} \quad \ldots \quad (11)$$

6. *Numerical Results.*—We now apply the theory to a special case, namely, tests upon typical panels tested in a machine with the stiffnesses given in section 4. For panels designed in accordance with the practice of the Bristol Aeroplane Co. in 1945 the quantity F can be taken as given roughly by

$$F = \frac{4 \cdot 1 \times 10^{-5}}{l^2}\,(b^{-1}\,\text{in}^{-1}). \quad \ldots \quad (12)$$

Substituting from (12) into (11) we find flexibilities associated with platten deformation. Converting the numerical results of section 4 for the rams extended case into flexibilities by means of equations (3) we find flexibilities associated with the machine structure. Summing corresponding values we obtain total flexibilities, which may be converted to stiffnesses using equation (3). These stiffnesses may then be substituted in equation (4) and the resulting equation solved for the smallest value of P_{crit}. It is convenient for this purpose to transform equation (4) into an explicit formula for P_{clamp} in terms of P_{crit}/P_{clamp} and l.

The results of the calculations sketched above are given below in tabular form and presented graphically in Fig. 3.

$P_{crit}/P_{clamp} = 0 \cdot 9$

P_{clamp}	45	–	–	–	53
l	10	30	50	70	90

$P_{crit}/P_{clamp} = 0 \cdot 95$

P_{clamp}	30	51	53	47	32
l	10	30	50	70	90

$P_{crit}/P_{clamp} = 0 \cdot 975$

P_{clamp}	18	37	37	29	18
l	10	30	50	70	90

$P_{crit}/P_{clamp} = 0 \cdot 99$

P_{clamp}	10	20	21	16	10
l	10	30	50	70	90

Where P_{clamp} is in tons and l in inches.

We conclude that panels, whose failing load is in the region of 50 tons, the capacity of the Avery machine, will for lengths lying between 30 and 70 in., fail flexurally at loads which differ from the fully clamped load by less than 5 per cent., while longer or shorter panels may fail at as much as 10 per cent below the fully clamped load.

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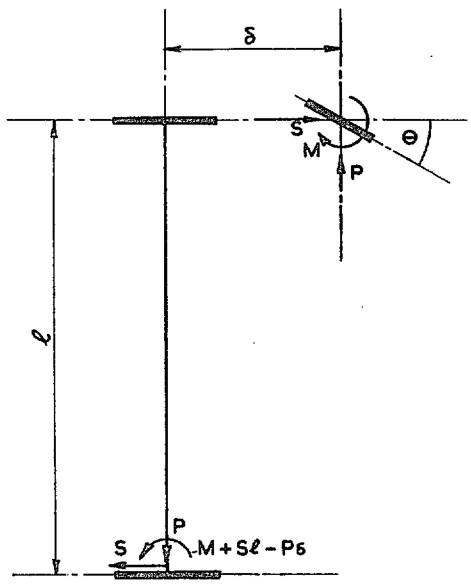

FIG. 1.

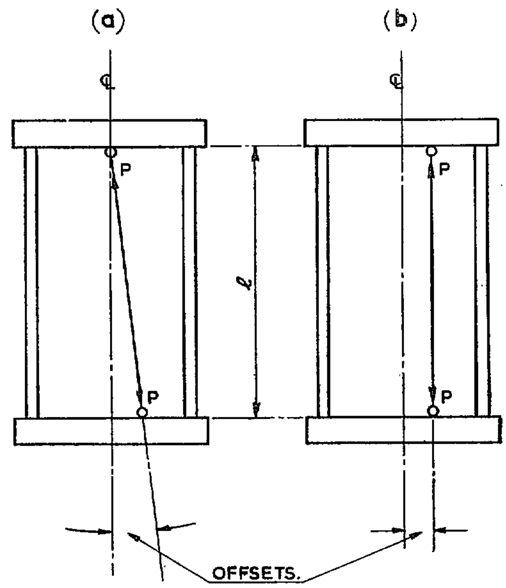

FIG. 2.

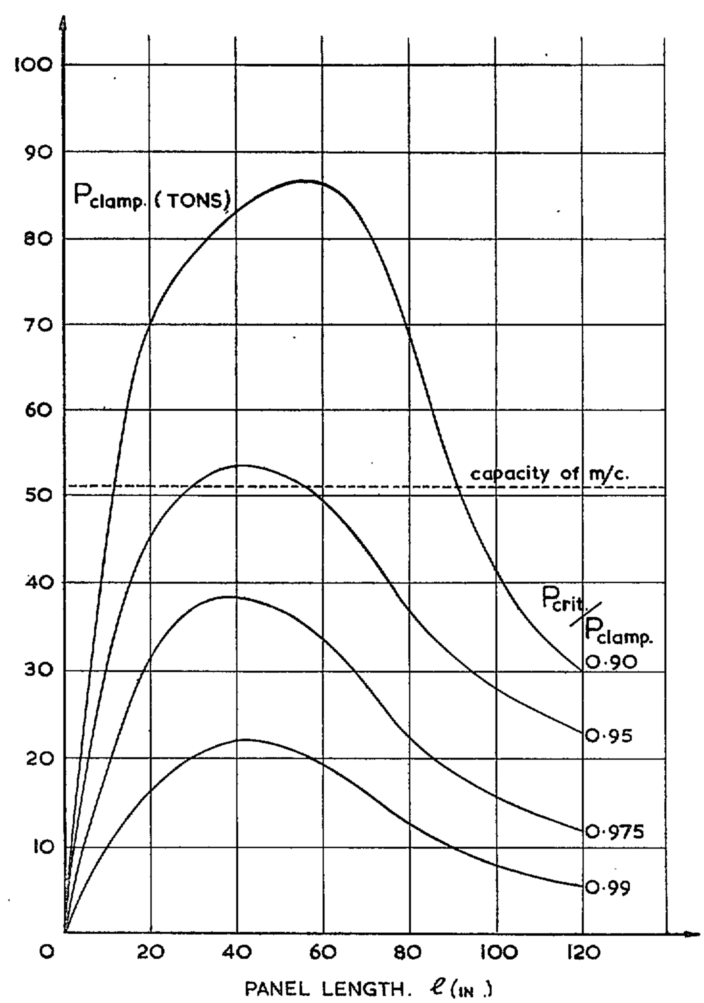

FIG. 3.

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