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Effects of Wing Stiffness and Inertia Changes on the Modes and Frequencies of a Model Delta Aircraft

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Effects of Wing Stiffness and Inertia Changes on the Modes and Frequencies of a Model Delta Aircraft

By D. R. B. Webb

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Summary. Resonance tests were made on a model delta aircraft to investigate the effect of stiffness changes of the leading- and trailing-edge spars on the frequencies and modes of vibration of the model. The results showed that, whereas considerable frequency changes were apparent, the general shape of the modes of vibration did not change significantly. A criterion of modal orthogonality proved to be very useful in checking the purity of the modes.

1. *Introduction.* A set of resonance tests was carried out on a model delta aircraft with several different inertia loadings and reported in Ref. 1. The present Report describes an extension of this work in which the effect of stiffness changes on the frequencies and modal shapes was investigated by changing the stiffnesses of the leading- and trailing-edge spars. The results given in the present Report have been used in Ref. 4 to compare the change in flutter characteristics of the wing with the change in spar stiffness.

When the flutter calculations were started, using the results from the first series of tests, it was found that the measured resonance modes were far from orthogonal, either to each other or to the body freedoms. An attempt was therefore made in these second series of tests to improve the testing technique, and the orthogonality relation which was used as a check of the results showed that a considerable improvement was in fact achieved. The new technique was used to check the earlier results of Ref. 1 and in a few cases the differences are appreciable.

2. *Details of Model.* 2.1. *General Description.* The delta model has been fully described in Ref. 1 but for convenience a brief description is given here. A plan view and section of the model is shown in Fig. 1 with a general photographic view in Fig. 2.

The model is 11 ft span and 5 ft 1 in. fuselage length and wing-root length, and is constructed of 20 s.w.g. aluminium. The fuselage is hollow and of constant section and carries internally three wooden blocks; the positions of two are variable whilst the centre one is fixed. The fixed block is used as an attachment point for a vibrator, whilst the other two are used as attachment points for the fuselage masses, simulated by lead blocks. The wings are formed by a construction of leading- and trailing-edge spars and chordwise ribs, the inboard ends of the spars being bolted to the fuselage. The leading- and trailing-edge spars are in addition attached to the surface skin which covers the

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whole of the model. The mass distribution of the wings is simulated by lead strips running in a chordwise direction and spaced at each rib station along the wing. The pitching and overall inertia and c.g. positions of the model were varied by adjusting the positions of the lead and wooden blocks along the fuselage. The wing inertia axis was varied by moving the lead strips along the wings in a chordwise direction.

2.2. *Stiffness Variation of the Model.* It was not possible to vary the torsional stiffness of the wings independently of the flexural stiffness because of the manner in which the wing spars were connected to the fuselage. Accordingly the stiffness variations were largely flexural. To change the flexural stiffness, the leading- and trailing-edge spars on both wings were replaced by thicker spars, and to keep the model sensibly representative of a full-scale structure only two variations were decided upon. From Fig. 1 it can be seen that if a torsional couple is applied to the wing with the fuselage encastre, the chief resistance to the couple is supplied by the wing/fuselage in shear, the skin in tension and compression, and a small amount of twist in the wing spars. Any increase in spar stiffness will not greatly change the stiffness at the fuselage/wing junction because of the fuselage construction. Since it was not practical to stiffen the fuselage structure, or to use a thicker skin, any change of the wing spars would not result in any great change in torsional stiffness measured at the section of the applied couple. If a flexural spanwise bending moment is applied to the wing with the fuselage encastre, nearly all the deflection takes the form of a rotation of the wing at the junction of the spars and the fuselage and if the spar/fuselage junction stiffness is not greatly changed when different wing spars are used, there will be no great change apparent in the wing bending stiffness. The overall stiffnesses are therefore similar for each change of wing spars. For this reason the stiffness tests on the model were only qualitative and were carried out by fixing the first six inches of the wing, as well as the fuselage.

When the model is vibrating freely, however, the loading is quite different from that applied in the stiffness tests. The dynamic stiffnesses associated with the modes of deformation involved can in fact be influenced appreciably by a change in spar stiffness, thus leading to significant changes in frequencies and modal shapes.

In addition to the standard condition, two sets of spars were fitted, both sets of which were stiffer than the original standard spars used in the model during the earlier tests. The measured stiffnesses based on the results of stiffness tests on the port wing of the model are given in Table A below. The loads were applied at 0·7 span and the deflections at this section were measured by dial indicators. Some slight differences were found for upwards and downwards readings for each incremental load. The starboard wing, fuselage and the inboard 16 per cent of the port wing were rigidly held for both flexural and torsional loads.

TABLE A

	Standard Spars	Intermediate Spars	Ratio of Stiffnesses	Final Spars	Ratio of Stiffnesses
Flexural Stiffness	683,101 lb in./rad	1,406,250 lb in./rad	2·06	1,567,526 lb in./rad	2·29
Torsional Stiffness	1,476,767 lb in./rad	1,765,200 lb in./rad	1·2	2,100,000 lb in./rad	1·42

The flexural stiffness is referred to the fixed root of the wing.

2.3. *Support of the Model.* The model was supported by helical springs at three points chosen in such a manner that the weight of the model was shared equally between them. Each spring was enclosed within two sliding cylinders, the outer one being connected through an universal coupling to the model and the inner one to a rigid stand resting on the ground. In this condition the natural frequency of the model in vertical translation was 4 c.p.s. By using universal couplings the model was supported in the pitching and rolling planes with little restraint. This type of support was preferred to the 'bungee cord' type of support, since the latter introduces damping and is prone to creep under heavy loads, and would therefore have necessitated frequent adjustment of the pick-up connections.

3. *Instrumentation.* Displacement measurements of the modes were made by using strain-gauge cantilever-type pick-ups. The application of this type of pick-up is basically attractive since:

(*a*) Strain gauges of the type used are cheap and easy to obtain.

(*b*) They can be incorporated in a pick-up which is simple and robust.

(*c*) The pick-up measures amplitude direct and if a d.c. amplifier is used it can be calibrated statically. The pick-up must of course be used well below its natural frequency.

(*d*) There are no phase differences between pick-ups.

A number of pick-up designs were tried, but Fig. 3 shows the model finally adopted in these tests. The working part of the pick-up is a cantilever beam, 5 inches long, which is reduced in thickness for 1 inch from the root. When the free end of the cantilever is deflected nearly all the strain takes place in the reduced section, which is called the gauge length. Strain gauges are attached to the gauge length to form part of a strain bridge. The pick-ups were used in conjunction with McMichael carrier-type amplifiers the outputs of which were switched to two single-beam and one double-beam oscillographs, the amplitudes and phases being determined in the manner described in Ref. 3.

The pick-ups were connected to the model by thin telescopic aluminium tubes whose effective lengths could be varied. The base of the pick-ups was rigidly fastened to scaffolding structure erected around the model. The inconvenience of having to erect any such structure of this type is perhaps an important consideration.

The end of the cantilever was slotted (Fig. 3) so that the distance of the connection point from the cantilever root could be varied, so providing means of adjusting the sensitivity. This was useful in arranging groups of pick-ups to have the same sensitivity. The stiffness and inertia effects of these pick-ups were negligible in relation to the dynamic characteristics of the model.

The model was vibrated by two electro-magnetic vibrators driven by a variable-frequency alternator. These were placed at fore and aft points along the fuselage centre-line and the position of the rear one could easily be varied. The power to each vibrator could be independently varied so that by suitable adjustment of their relative force outputs the best phase relationship between pick-ups could be obtained.

The frequency of excitation was measured by comparing the frequency from a good-quality continuously-variable oscillator with the signal from a velocity pick-up incorporated in one of the exciters. The two signals were conducted to the x and y plates of an oscilloscope, and frequency co-incidence was determined by observation of the Lissajous figure produced.

4. *Range of Tests.* The tests were divided into two main groups:

(i) Measurement of the three natural modes of lowest frequency for three cases of wing flexural stiffness for the standard mass distribution.

(ii) A check on the results given in Ref. 1 using the modified technique.

The three cases in (i) are denoted by:

(*a*) with the original spars (thus providing continuity with the work of Ref. 1).

(*b*) with the stiffer intermediate spars,

(*c*) with the final spars which were stiffer than the intermediate spars.

5. *Procedure for Obtaining Mode—Use of Criterion of Orthogonality.* In making calculations based upon normal modes, a check can be made on the degree of orthogonality of resonance-test modes. The usual criterion adopted in flutter calculations for orthogonality is that the product of inertia between two modes should not exceed 10 per cent of the geometric mean of the two direct moments of inertia[2], *i.e.*:

$$A_{rs} \ngtr 0\cdot 1 \sqrt{(A_{rr}A_{ss})},$$

where

$$A_{rr} = \sum m\theta_r^2,$$

$$A_{ss} = \sum m\theta_s^2,$$

$$A_{rs} = \sum m\theta_r\theta_s,$$

where

m Mass of particle.

θ_i Amplitude of displacement of mass in the i'th mode. It is assigned a $-$ ve or $+$ ve value according to the position of the mass relative to the nodal line.

The modes used in this criterion can be any of the measured resonance modes or rigid-body vertical translation or pitch about the c.g. (symmetric case).

In the particular case when one of the modes is taken to be aircraft pitch the criterion reduces to the form:

$$A_{rs} = \sum m\theta_r\bar{x} \ngtr 0\cdot 1 \sqrt{(A_{rr}A_{ss})},$$

where

$$A_{rr} = \sum m\theta_r^2,$$

$$A_{ss} = \sum m\bar{x}^2$$

and

$\bar{x}$ Distance of particle from a pitch axis passing through the c.g. of the model. It is assigned a $-$ ve or $+$ ve value according to the position of the particle relative to the pitch axis.

The procedure adopted was to plot the particular mode as indicated by the fixed position pick-ups, and then to apply the above criterion. If the products of inertia fell outside this value, the mode was re-examined in detail with a wandering pick-up, particularly in those areas near nodal lines or large masses. If this produced no further improvement, the force ratio of the two vibrators was varied and if necessary the position of the rear vibrator. Using these methods it was possible to obtain modes which satisfied the criterion.

6. *Test Results.* 6.1. *General Remarks.* The modes obtained from the tests described in 4(i) are shown tabulated in Table 1a, and larger-scale plots of these modes are shown in Figs. 4, 5 and 6. The modes obtained for the tests 4(ii) are shown in Table 1b, larger-scale plots of these modes are shown in Figs. 7, 8, 9 and 10. It can be seen that for the stiffness variation the modes are of a similar general pattern to those obtained for inertia variation. It seems surprising, however, that the nodal line in the torsional mode for intermediate spars should be so far forward in the outer wing area in relation to the torsional modes for the other spars.

When some of the tests of Ref. 1 were repeated, significant deviation in modal shape was found in three instances and some of the frequencies differed by as much as 20 per cent. It is difficult to account fully for these differences in the results. Some of the differences may have been due to the increased accuracy of frequency measurement in the second series of tests and moreover the method of locating resonant conditions differed in the two series of tests. In the earlier experiments it was assumed that a resonance occurred at the frequency of maximum amplitude as indicated by a transducer placed at a master station on a wing tip. In the second series of tests however peak amplitude of a master station was used only to indicate proximity to a resonant frequency and further changes to the frequency were then made to obtain the most reasonable phase relationships.

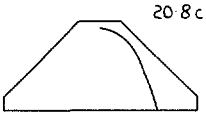
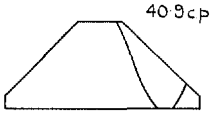
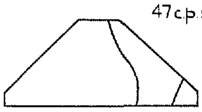
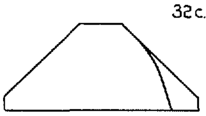
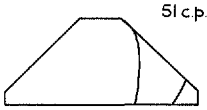
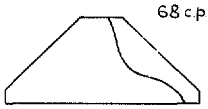
7. *Conclusions.* The modal patterns obtained when the wing spar stiffness of the model was varied considerably, followed the same general pattern as those obtained with standard spars. The criterion of modal orthogonality used was sensitive to small changes in modal shape and proved very useful in checking the validity of the mode. A minimum of two vibrators, whose force output could be independantly varied was found to be necessary to obtain good phase relationships between pick-up stations.

REFERENCES

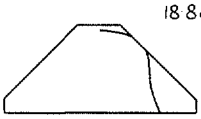
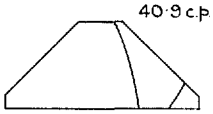
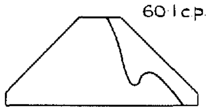
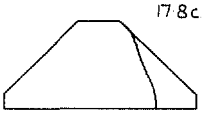
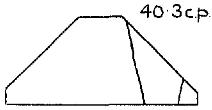
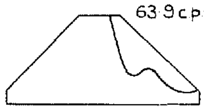
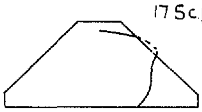
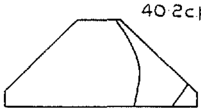
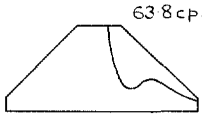
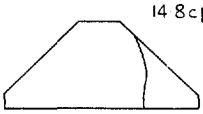
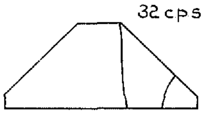
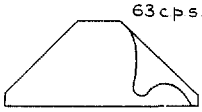
No.	*Author*	*Title, etc.*
1	D. R. Gaukroger	Natural frequencies and modes of a model delta aircraft. A.R.C. R. & M. 2762. June, 1950.
2	E. G. Broadbent	Research on wing and control surface flutter with particular reference to the choice of co-ordinates. A.R.C. 17,872. June, 1955.
3	W. G. Molyneux and E. G. Broadbent	Ground resonance testing of aircraft. A.R.C. R. & M. 2155. July, 1946.
4	H. Hall	Flutter calculations on a resonance model delta wing. A.R.C. C.P. 260. January, 1955.

TABLE 1

Summary of Frequencies and Modes of Vibration of Model Delta Aircraft

	STIFFNESS VARIATION	FUNDAMENTAL MODE	1ST. OVERTONE FLEXURAL MODE	TORSIONAL MODE
WITH 'STANDARD' INERTIA	ORIGINAL SPARS	20·8 c.p.s. 	40·9 c.p.s 	61·8 c.p.s. 
	INTERMEDIATE SPARS	29·4 c.p.s 	47 c.p.s. 	65 c.p.s. 
	FINAL SPARS	32 c.p.s. 	51 c.p.s. 	68 c.p.s. 

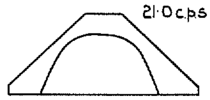
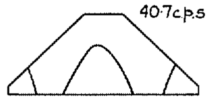
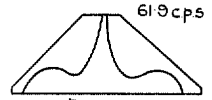
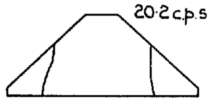
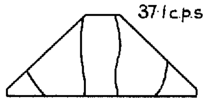
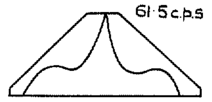
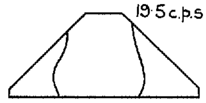
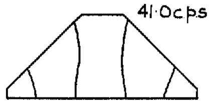
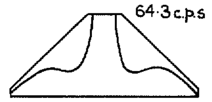
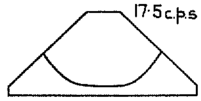
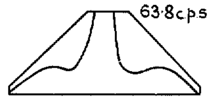
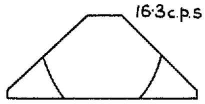
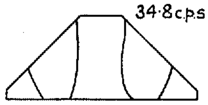
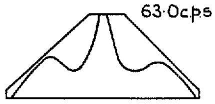
(a)

INERTIA PARAMETERS					FUNDAMENTAL MODE	1ST. OVERTONE FLEXURAL MODE	TORSIONAL MODE
WING INERTIA AXIS	MEAN C.G. POSITION	TOTAL PITCHING M.I.	FUSELAGE PITCHING M.I.				
50	55	99	118		18·8 c.p.s 	40·9 c.p.s 	60·1 c.p.s 
50	54	105	118		17·8 c.p.s 	40·3 c.p.s 	63·9 c.p.s. 
50	54	96	100		17·5 c.p.s 	40·2 c.p.s 	63·8 c.p.s. 
40	50	83	111		14·8 c.p.s 	32 c.p.s. 	63 c.p.s. 

(b)

TABLE 2

Part of a Table Produced in A.R.C. R. & M. 2762 (Produced for Comparison with Table 1)

INERTIA PARAMETER					NATURAL FREQUENCIES AND MODES OF VIBRATION.		
WING INERTIA AXIS	MEAN C.G POSITION	TOTAL PITCHING M.I.	FUSELAGE PITCHING M.I.		**FUNDAMENTAL MODE**	**1st OVERTONE FLEXURAL MODE**	**TORSIONAL MODE**
* 50	STANDARD CASE * 50	100	100		21·0 c.p.s 	40·7 c.p.s 	61·9 c.p.s 
50	* 55	98	118		20·2 c.p.s 	37·1 c.p.s 	61·5 c.p.s 
50	54	105	* 118		19·5 c.p.s 	41·0 c.p.s 	64·3 c.p.s 
50	54	96	* 100		17·5 c.p.s 	40·1 c.p.s 	63·8 c.p.s 
* 40	50	83	111		16·3 c.p.s 	34·8 c.p.s 	63·0 c.p.s 

* DENOTES PARAMETER VARIATION CONCERNED.

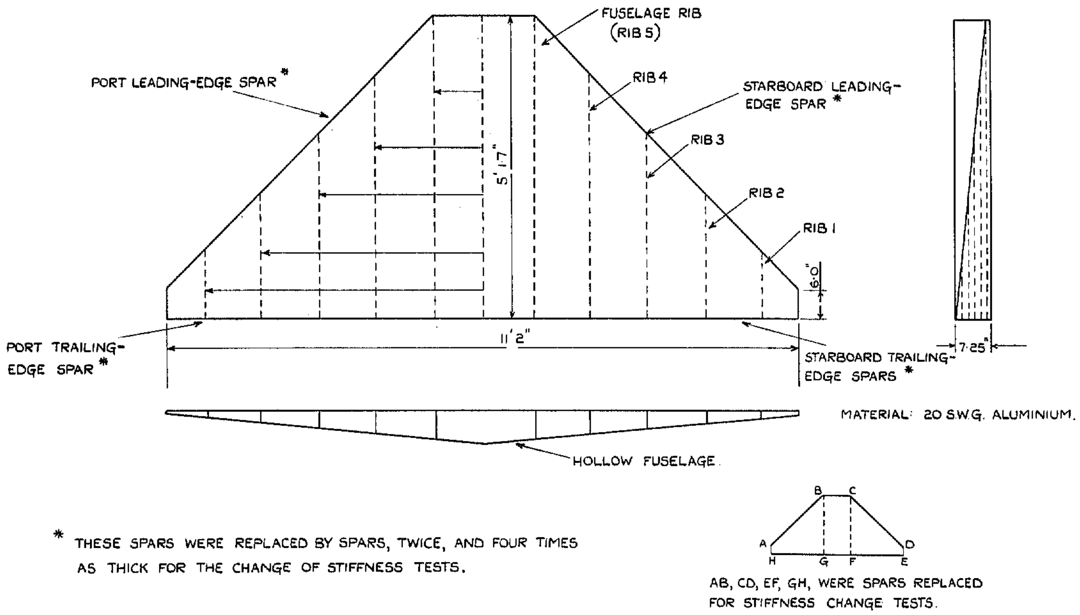


FIG. 1. Model delta aircraft.

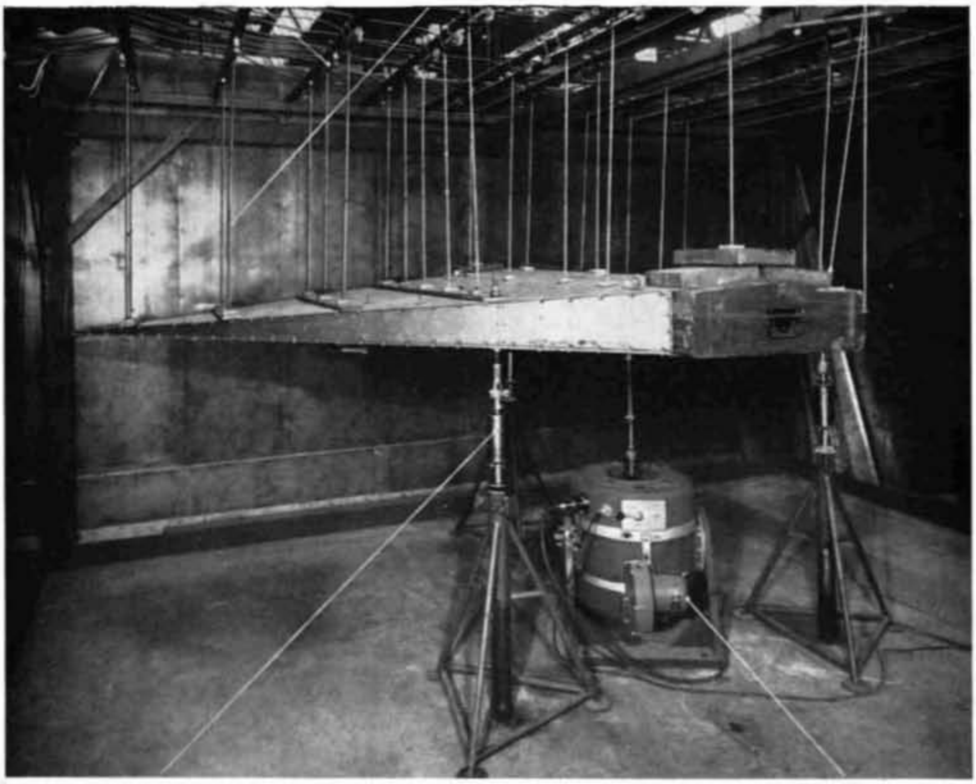


FIG. 2. General view of experimental set-up, model delta.

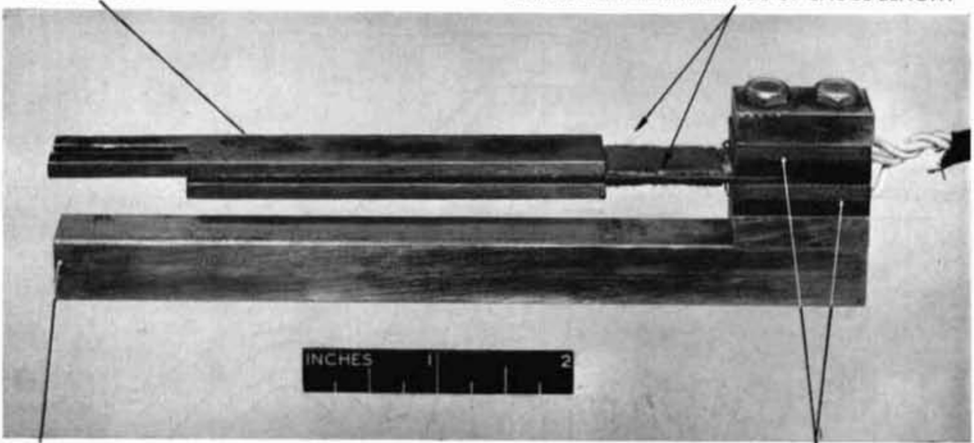


FIG. 3. Cantilever-type strain-gauge pick-up.

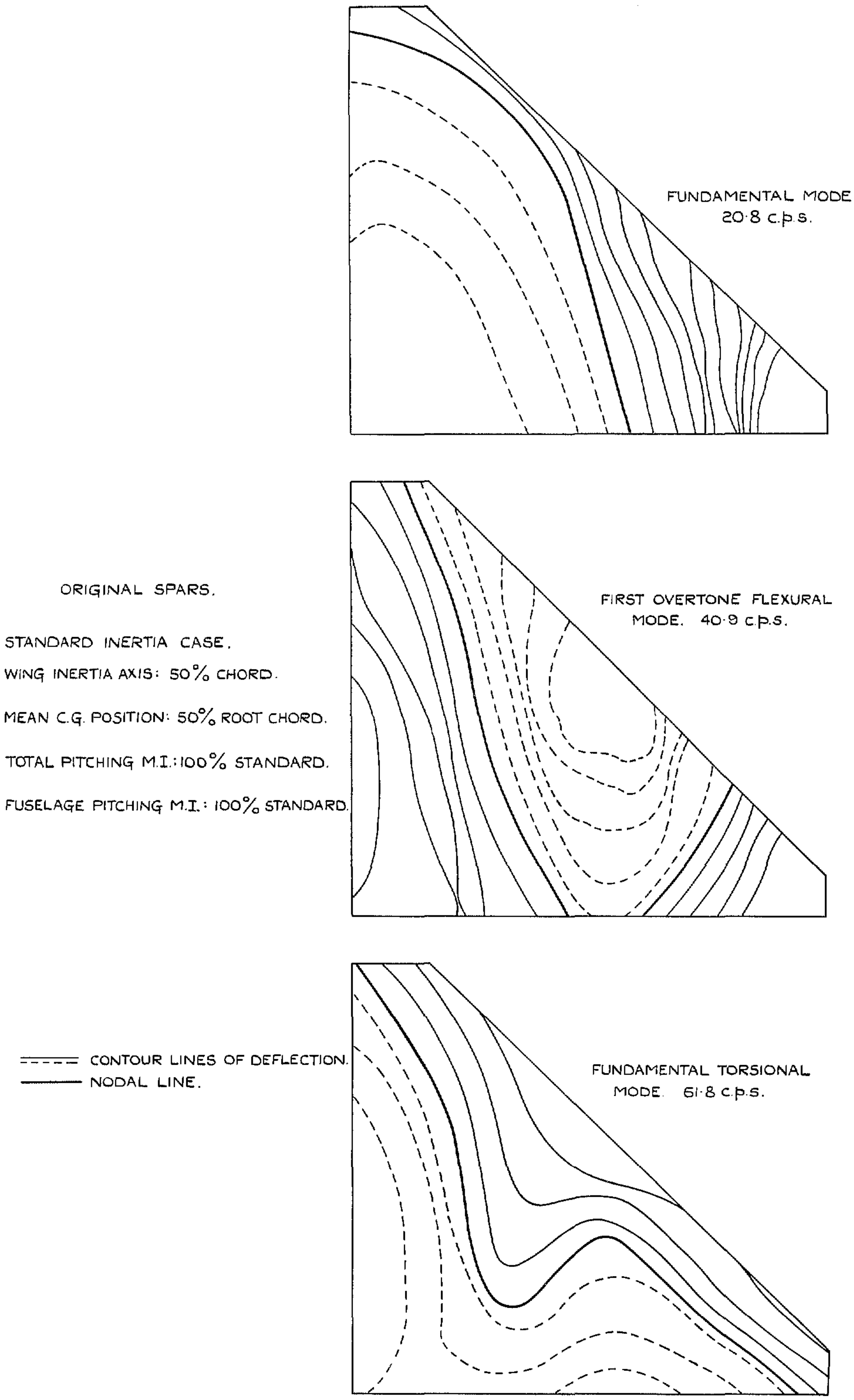


FIG. 4. Modes of vibration of model delta aircraft.

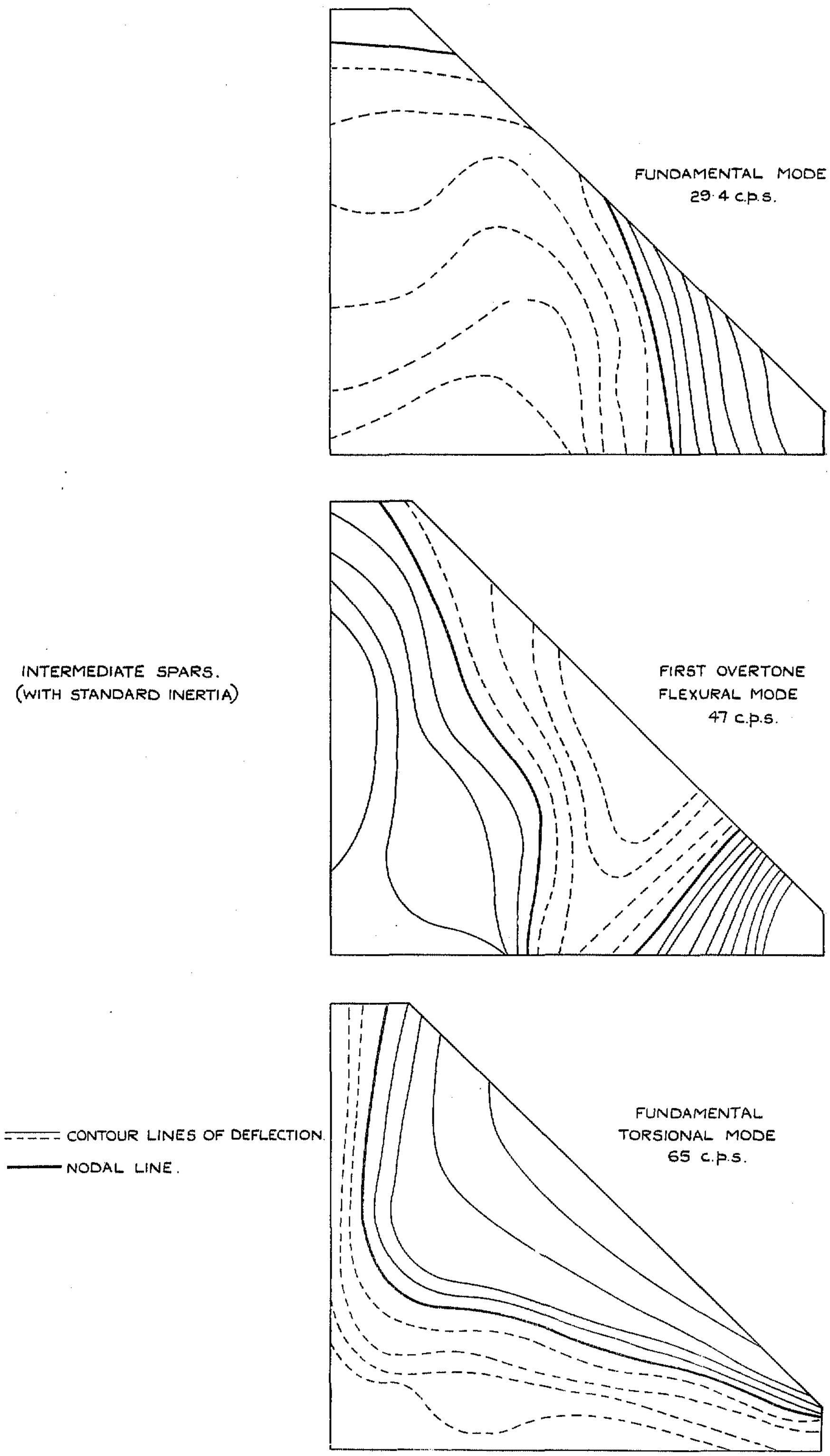


FIG. 5. Modes of vibration of model delta aircraft.

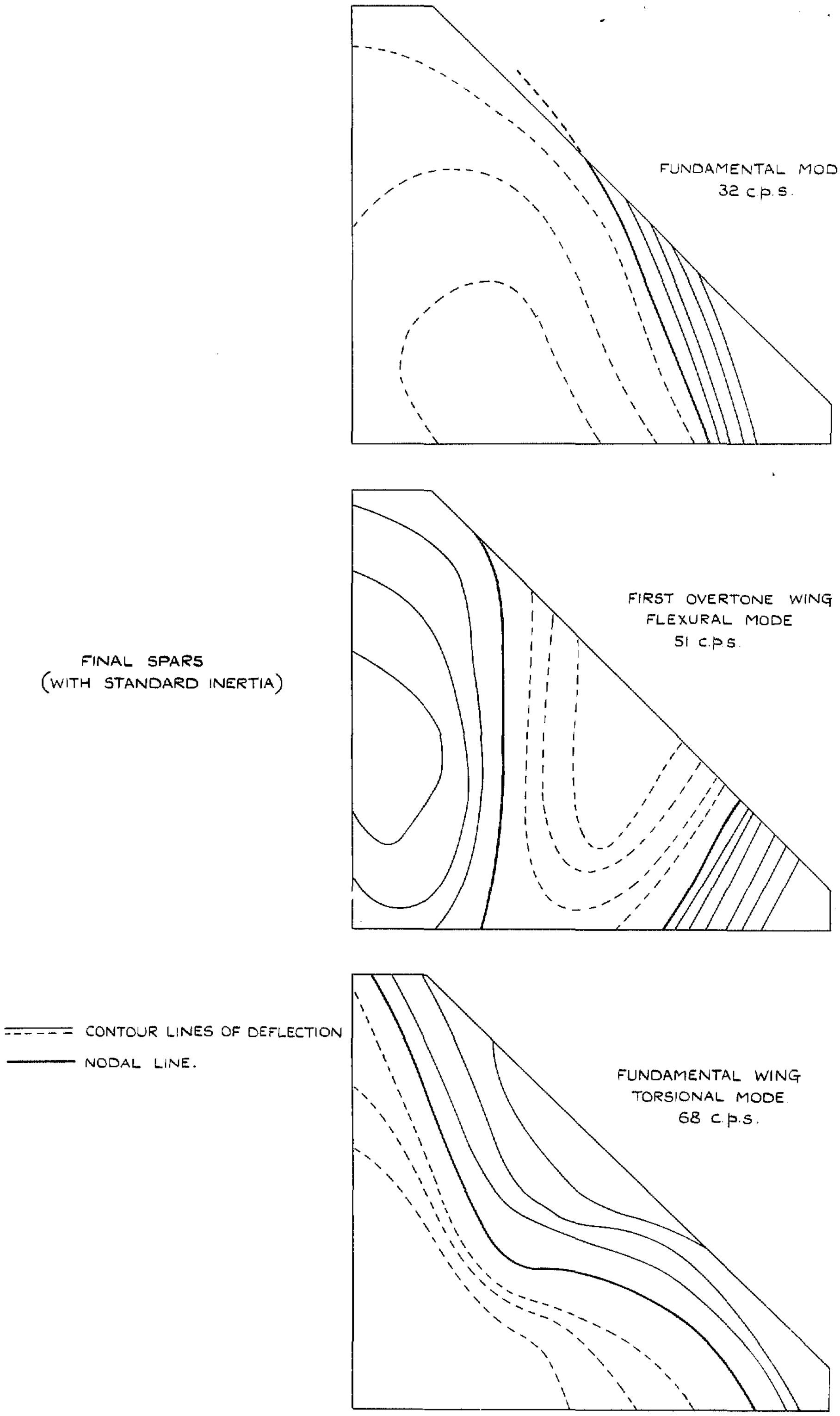


FIG. 6. Modes of vibration of model delta aircraft.

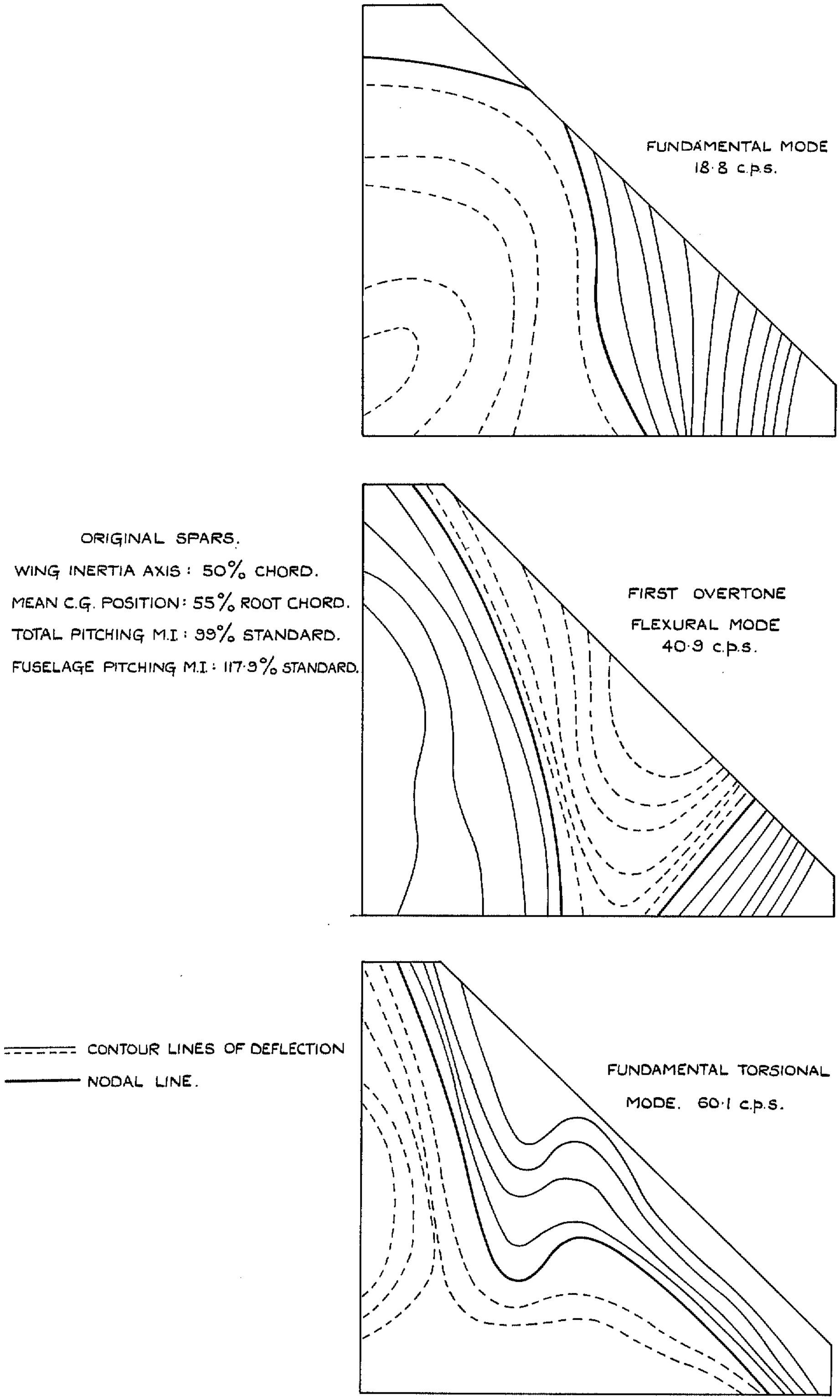


FIG. 7. Modes of vibration of model delta aircraft. Second inertia variation.

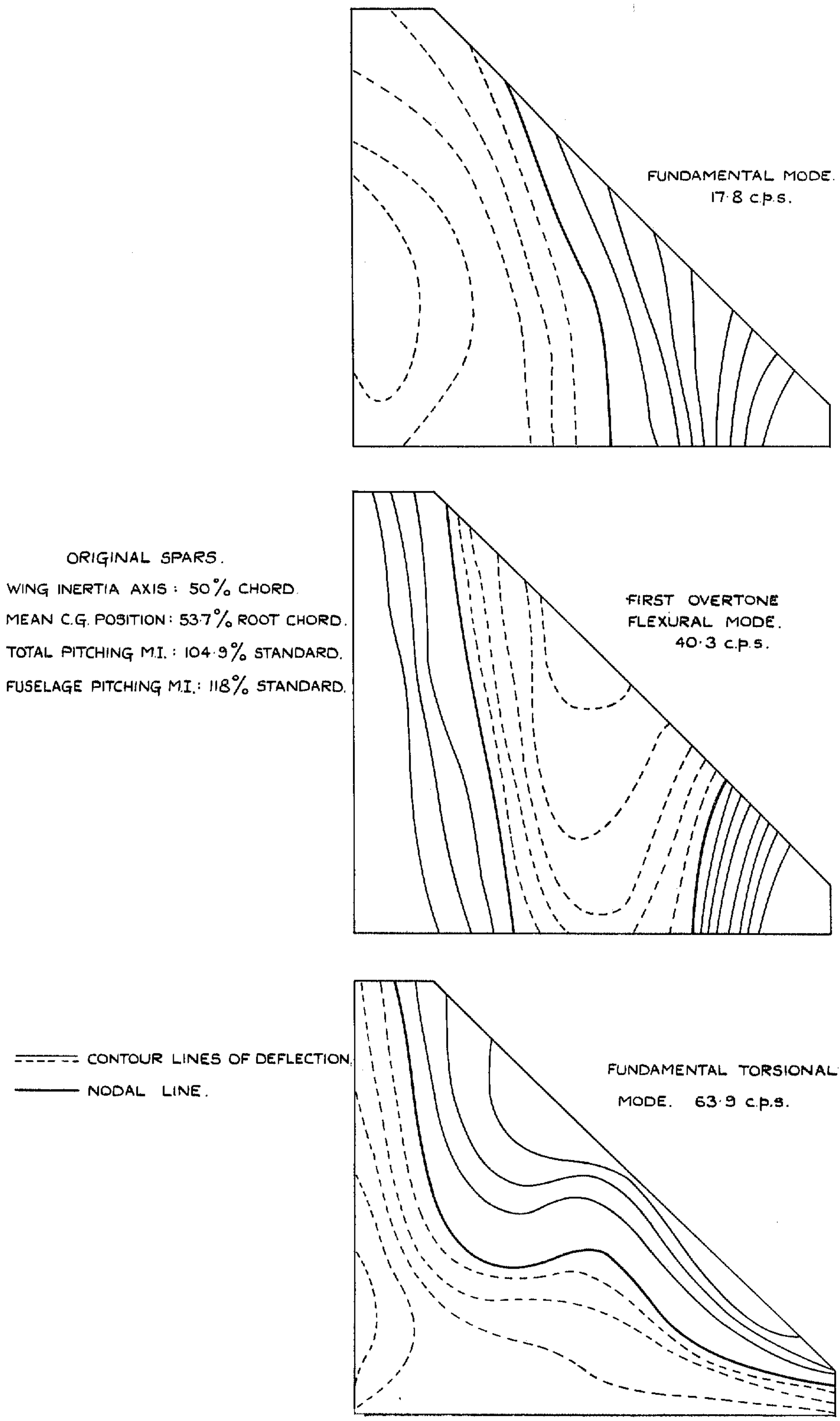


FIG. 8. Modes of vibration of model delta aircraft. Third inertia variation.

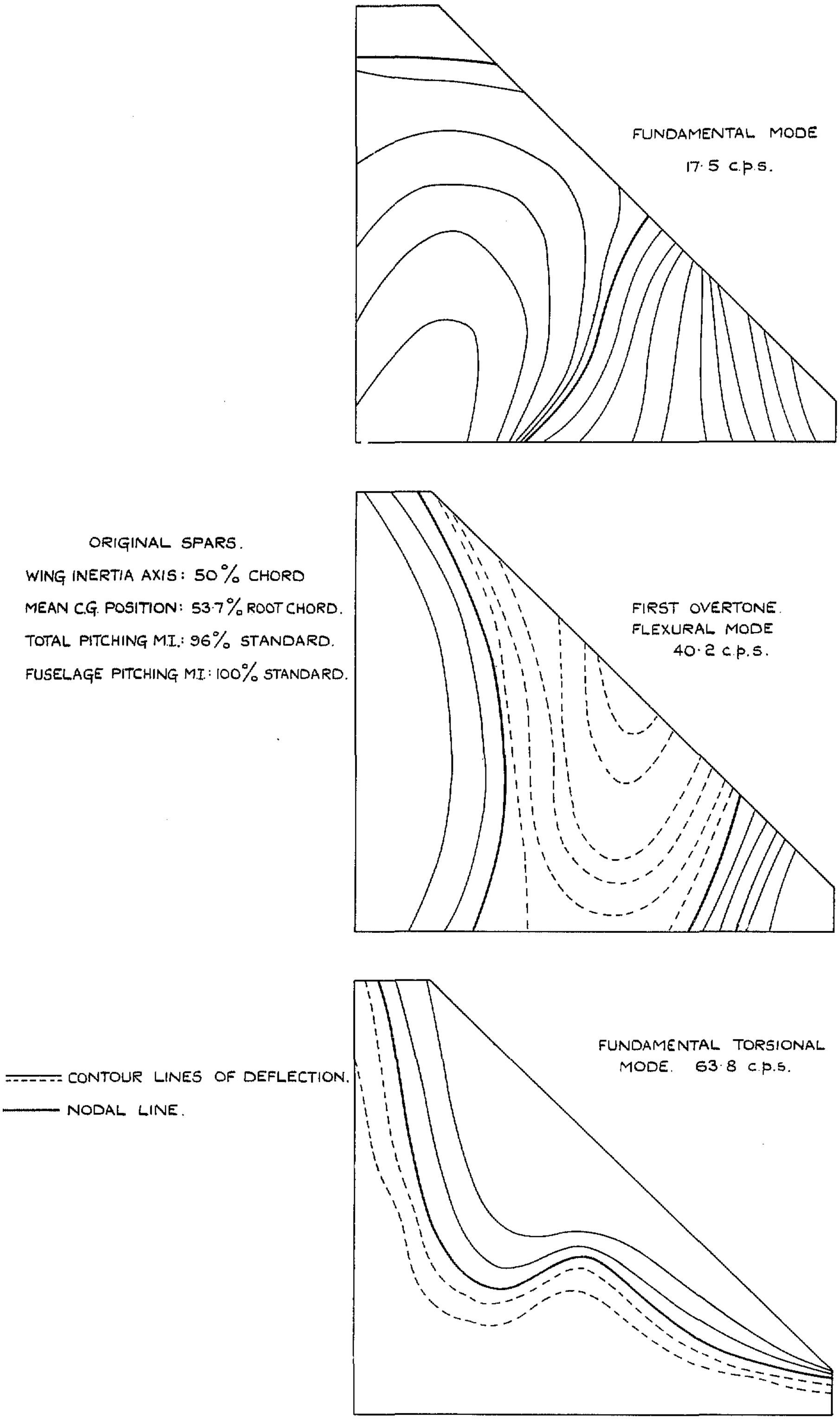


FIG. 9. Modes of vibration of model delta aircraft. Fourth inertia variation.

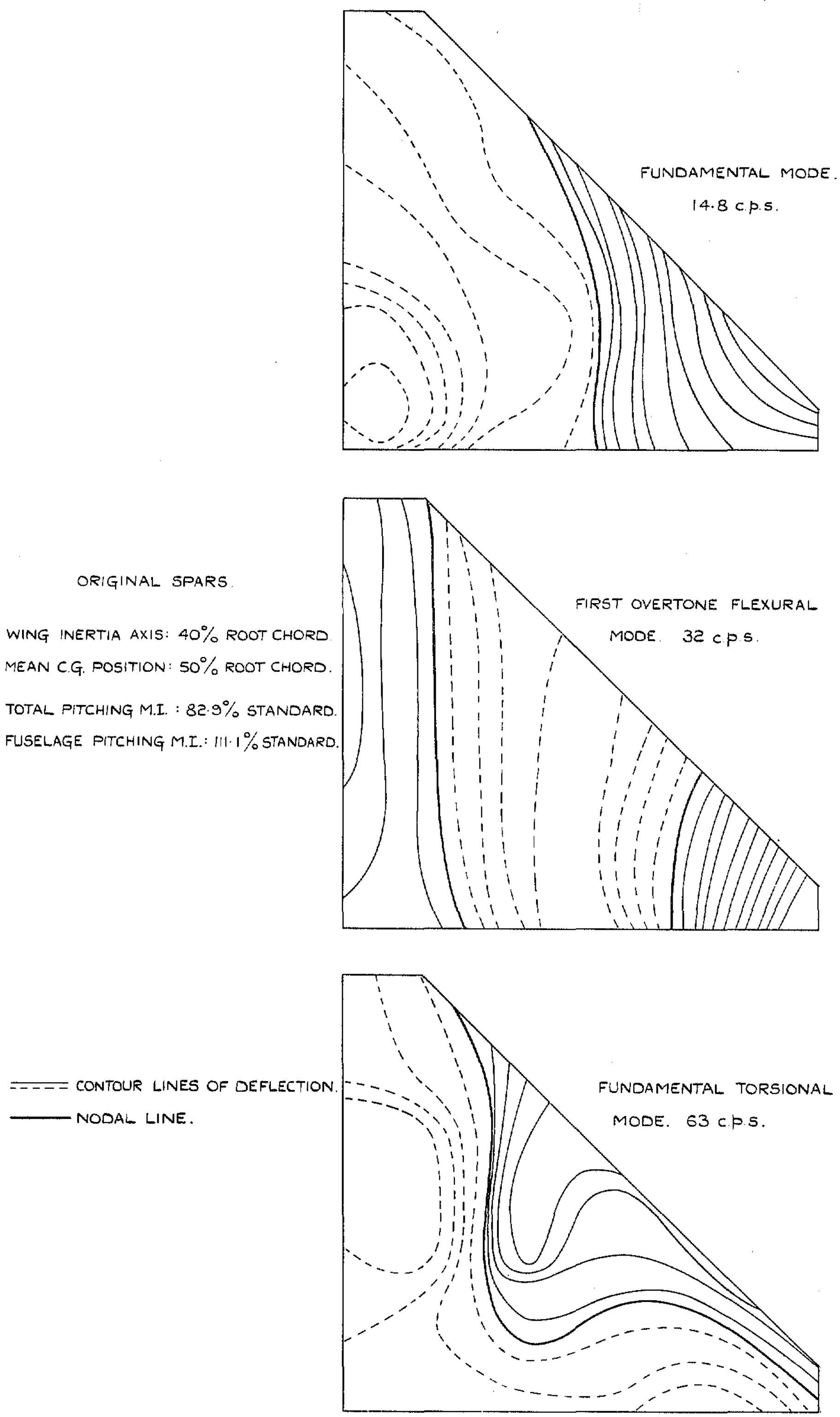


FIG. 10. Modes of vibration of model delta aircraft. First inertia variation.

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