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Summary. The method developed by C. E. Watkins and J. H. Berman in N.A.C.A. Report 1099 and N.A.C.A. Technical Note 3009 is used to obtain the velocity potential on a triangular (or related) wing with subsonic leading edges oscillating in a supersonic flow. A symmetric mode of oscillation is considered which is represented by a polynomial expression in the co-ordinates of the points of the wing. This polynomial expression is of higher degree than those of the above two papers. The results enable the first few terms of a power series in frequency for the velocity potential to be obtained.

1. *Introduction.* It is shown in Refs. 1 and 2 how to obtain theoretically the velocity potential on the surface of a flat plate triangular (or related) wing with subsonic leading edges oscillating harmonically in a supersonic flow when the mode of distortion of the wing can be represented by a polynomial expression in the co-ordinates of the points of the wing. The surface potential is obtained as a power series in the frequency, the coefficients of which are polynomial expressions in the co-ordinates of the points of the wing. The coefficients in the latter polynomial expressions depend on the free stream Mach number and the leading-edge sweepback; they are polynomial expressions in $1/M^2$ with coefficients which are functions of $\rho_0 = \beta\lambda$.

The coefficients of the powers of frequency in the expression for the surface potential are increasingly more laborious to obtain with each increase in the power of frequency. They also become more laborious to obtain when the degree of the polynomial representing the distortion is increased. Only the first few terms of the power series are determined in this paper when the mode of distortion is given by a not very high degree polynomial. Numerical values of the functions of ρ_0 referred to above which are appropriate to these first few terms are given for a set of values of ρ_0.

In Ref. 2 an expression for the surface potential is given to the third power in frequency when the distortion is represented by a polynomial expression of the second degree. A polynomial expression of the second degree does not however in general give an adequate representation of the modes of distortion of actual wings. A polynomial expression which represents the symmetrical distortion of a

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wing rather better than a second degree polynomial expression is considered here and the surface potential may be obtained to the third power of frequency by using the tables given. If the two highest degree terms in the polynomial expression for the distortion are omitted then tables which enable the surface potential to be obtained to the fifth power of frequency may be prepared with very little additional work; these tables have been prepared and are given in the Report.

2. *Discussion.* We consider an infinitely thin symmetrical triangular wing oscillating in a supersonic potential flow. It is assumed that at any instant the displacement of any point on the surface of the wing from the plane of equilibrium of the wing is small in comparison with the linear dimensions of the wing. The projection of the wing onto the equilibrium plane will be a triangle and it will be assumed that the line of symmetry of this triangle is in the direction of the main stream. It is assumed also that the leading edges of the wing are subsonic.

(The velocity potential on the surface of any other thin wing with straight subsonic leading edges swept back symmetrically, and supersonic trailing edge(s) will be the same as if this wing were part of a symmetrical triangular wing since the velocity potential on the surface of the wing is independent of conditions behind it).

A system of rectangular axes is chosen with origin at the apex and x-axis along the axis of symmetry of the wing projection, with z-axis at right angles to the equilibrium plane in a direction we shall call downwards, and y-axis mutually at right angles.

The velocity potential ϕ of the disturbed flow satisfies the linearised condition of tangential flow:

$$\left(\frac{\partial \phi}{\partial z}\right)_{z=0} = W(x, y, t) = \left(V\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right) Z(x, y, t) \tag{1}$$

on the equilibrium plane, where $W(x, y, t)$ and $Z(x, y, t)$ represent respectively the downwash velocity and the downward vertical displacement at a point x, y on the wing at time t.

The displacement of points on the wing in a harmonic oscillation is to be given by a polynomial in x and y. We shall consider here only a symmetric oscillation in which the displacement is given by a polynomial of the third degree in x and y^2:

$$Z(x, y, t) = e^{i\omega t}(p_0+p_1x+p_2y^2+p_3x^2+p_4xy^2+p_5y^4+p_6x^3+p_7x^2y^2+p_8xy^4+p_9y^6) \tag{2}$$

where the coefficients p_0, p_1, . . . , p_9 are in general complex so that phase differences between the different components of the displacement may be taken into consideration. A fairly general symmetrical oscillation may be represented by such a polynomial expression.

By substitution of $Z(x, y, t)$ from Equation (2) into Equation (1) the following expression is obtained for the downwash:

$$W(x, y, t) = e^{i\omega t}(P_0+P_1x+P_2\beta^2y^2+P_3x^2+P_4x\beta^2y^2+P_5\beta^4y^4+ \\ +P_6x^3+P_7\beta^2x^2y^2+P_8\beta^4xy^4+P_9\beta^6y^6) \tag{3}$$

where

$$\left.\begin{aligned} P_0 &= Vp_1 + i\omega p_0\,, \quad P_1 = 2Vp_3 + i\omega p_1\,, \quad P_2 = \frac{Vp_4 + i\omega p_2}{\beta^2}\,, \\ P_3 &= 3Vp_6 + i\omega p_3\,, \quad P_4 = \frac{2Vp_7 + i\omega p_4}{\beta^2}\,, \quad P_5 = \frac{Vp_8 + i\omega p_5}{\beta^4}\,, \\ P_6 &= i\omega p_6\,, \quad P_7 = \frac{i\omega p_7}{\beta^2}\,, \quad P_8 = \frac{i\omega p_8}{\beta^4}\,, \quad P_9 = \frac{i\omega p_9}{\beta^6}\,. \end{aligned}\right\} \tag{4}$$

The governing equations of the flow are linearised and so the potential on the surface may be expressed as the sum of the separate potentials associated with the different terms of the downwash expression (3):

$$\phi = e^{i\omega t}(P_0\phi_0 + P_1\phi_1 + P_2\phi_2 + P_3\phi_3 + P_4\phi_4 + P_5\phi_5 + + P_6\phi_6 + P_7\phi_7 + P_8\phi_8 + P_9\phi_9). \quad (5)$$

There is a difference of sign between the potentials at the upper and lower sides of the wing surface and for this reason it is necessary to stipulate the side on which the potential is obtained. The results in this paper all refer to the upper side of the wing ($z = -0$).

It may be well to remark here that there is some confusion as to which sides the values of the potential are given for in Refs. 1 and 2. If the above convention of taking the z-axis downwards is used the potentials are correct if they are for the upper side of the wing. In particular, the potentials ϕ_0, ϕ_1, ϕ_2 may be obtained directly from Ref. 2, and it is only for completeness that means of obtaining them are given here.

The first few terms of the power series in frequency for the surface potential ϕ_j are given by the following formulae:

$$\begin{aligned}\phi_0 = \sqrt{(\lambda^2x^2 - y^2)}\,[&a_0 + (i\bar{\omega})a_1x + (i\bar{\omega})^2(a_2x^2 + a_3\beta^2y^2) + \\ &+ (i\bar{\omega})^3(a_4x^3 + a_5x\beta^2y^2) + \\ &+ (i\bar{\omega})^4(a_6x^4 + a_7x^2\beta^2y^2 + a_8\beta^4y^4) + \\ &+ (i\bar{\omega})^5(a_9x^5 + a_{10}x^3\beta^2y^2 + a_{11}x\beta^4y^4)]\end{aligned} \quad (6)$$

$$\begin{aligned}\phi_1 = \sqrt{(\lambda^2x^2 - y^2)}\,[&b_0x + (i\bar{\omega})(b_1x^2 + b_2\beta^2y^2) + (i\bar{\omega})^2(b_3x^3 + b_4x\beta^2y^2) + \\ &+ (i\bar{\omega})^3(b_5x^4 + b_6x^2\beta^2y^2 + b_7\beta^4y^4) + \\ &+ (i\bar{\omega})^4(b_8x^5 + b_9x^3\beta^2y^2 + b_{10}x\beta^4y^4) + \\ &+ (i\bar{\omega})^5(b_{11}x^6 + b_{12}x^4\beta^2y^2 + b_{13}x^2\beta^4y^4 + b_{14}\beta^6y^6)]\end{aligned} \quad (7)$$

$$\begin{aligned}\phi_2 = \sqrt{(\lambda^2x^2 - y^2)}\,[&(c_0x^2 + c_1\beta^2y^2) + (i\bar{\omega})(c_2x^3 + c_3x\beta^2y^2) + \\ &+ (i\bar{\omega})^2(c_4x^4 + c_5x^2\beta^2y^2 + c_6\beta^4y^4) + \\ &+ (i\bar{\omega})^3(c_7x^5 + c_8x^3\beta^2y^2 + c_9x\beta^4y^4) + \\ &+ (i\bar{\omega})^4(c_{10}x^6 + c_{11}x^4\beta^2y^2 + c_{12}x^2\beta^4y^4 + c_{13}\beta^6y^6) + \\ &+ (i\bar{\omega})^5(c_{14}x^7 + c_{15}x^5\beta^2y^2 + c_{16}x^3\beta^4y^4 + c_{17}x\beta^6y^6)]\end{aligned} \quad (8)$$

$$\begin{aligned}\phi_3 = \sqrt{(\lambda^2x^2 - y^2)}\,[&(d_0x^2 + d_1\beta^2y^2) + (i\bar{\omega})(d_2x^3 + d_3x\beta^2y^2) + \\ &+ (i\bar{\omega})^2(d_4x^4 + d_5x^2\beta^2y^2 + d_6\beta^4y^4) + \\ &+ (i\bar{\omega})^3(d_7x^5 + d_8x^3\beta^2y^2 + d_9x\beta^4y^4) + \\ &+ (i\bar{\omega})^4(d_{10}x^6 + d_{11}x^4\beta^2y^2 + d_{12}x^2\beta^4y^4 + d_{13}\beta^6y^6) + \\ &+ (i\bar{\omega})^5(d_{14}x^7 + d_{15}x^5\beta^2y^2 + d_{16}x^3\beta^4y^4 + d_{17}x\beta^6y^6)]\end{aligned} \quad (9)$$

$$\begin{aligned}\phi_4 = \sqrt{}(\lambda^2x^2-y^2)\,[&(e_0x^3 + e_1x\beta^2y^2) + (i\varpi)\,(e_2x^4+e_3x^2\beta^2y^2+e_4\beta^4y^4) + \\ &+ (i\varpi)^2(e_5x^5+e_6x^3\beta^2y^2+e_7x\beta^4y^4) + \\ &+ (i\varpi)^3(e_8x^6+e_9x^4\beta^2y^2+e_{10}x^2\beta^4y^4+e_{11}\beta^6y^6) + \\ &+ (i\varpi)^4(e_{12}x^7+e_{13}x^5\beta^2y^2+e_{14}x^3\beta^4y^4+e_{15}x\beta^6y^6) + \\ &+ (i\varpi)^5(e_{16}x^8+e_{17}x^6\beta^2y^2+e_{18}x^4\beta^4y^4+e_{19}x^2\beta^6y^6+e_{20}\beta^8y^8)]\end{aligned} \tag{10}$$

$$\begin{aligned}\phi_5 = \sqrt{}(\lambda^2x^2-y^2)\,[&(f_0x^4 + f_1x^2\beta^2y^2+f_2\beta^4y^4) + \\ &+ (i\varpi)\,(f_3x^5+f_4x^3\beta^2y^2+f_5x\beta^4y^4) + \\ &+ (i\varpi)^2(f_6x^6+f_7x^4\beta^2y^2+f_8x^2\beta^4y^4+f_9\beta^6y^6) + \\ &+ (i\varpi)^3(f_{10}x^7+f_{11}x^5\beta^2y^2+f_{12}x^3\beta^4y^4+f_{13}x\beta^6y^6) + \\ &+ (i\varpi)^4(f_{14}x^8+f_{15}x^6\beta^2y^2+f_{16}x^4\beta^4y^4+f_{17}x^2\beta^6y^6+f_{18}\beta^8y^8) + \\ &+ (i\varpi)^5(f_{19}x^9+f_{20}x^7\beta^2y^2+f_{21}x^5\beta^4y^4+f_{22}x^3\beta^6y^6+f_{23}x\beta^8y^8)]\end{aligned} \tag{11}$$

$$\begin{aligned}\phi_6 = \sqrt{}(\lambda^2x^2-y^2)\,[&(g_0x^3+g_1x\beta^2y^2) + (i\varpi)\,(g_2x^4+g_3x^2\beta^2y^2+g_4\beta^4y^4) + \\ &+ (i\varpi)^2(g_5x^5+g_6x^3\beta^2y^2+g_7x\beta^4y^4) + \\ &+ (i\varpi)^3(g_8x^6+g_9x^4\beta^2y^2+g_{10}x^2\beta^4y^4+g_{11}\beta^6y^6) + \\ &+ (i\varpi)^4(g_{12}x^7+g_{13}x^5\beta^2y^2+g_{14}x^3\beta^4y^4+g_{15}x\beta^6y^6) + \\ &+ (i\varpi)^5(g_{16}x^8+g_{17}x^6\beta^2y^2+g_{18}x^4\beta^4y^4+g_{19}x^2\beta^6y^6+g_{20}\beta^8y^8)]\end{aligned} \tag{12}$$

$$\begin{aligned}\phi_7 = \sqrt{}(\lambda^2x^2-y^2)\,[&(h_0x^4+h_1x^2\beta^2y^2+h_2\beta^4y^4) + \\ &+ (i\varpi)\,(h_3x^5+h_4x^3\beta^2y^2+h_5x\beta^4y^4) + \\ &+ (i\varpi)^2(h_6x^6+h_7x^4\beta^2y^2+h_8x^2\beta^4y^4+h_9\beta^6y^6) + \\ &+ (i\varpi)^3(h_{10}x^7+h_{11}x^5\beta^2y^2+h_{12}x^3\beta^4y^4+h_{13}x\beta^6y^6) + \\ &+ (i\varpi)^4(h_{14}x^8+h_{15}x^6\beta^2y^2+h_{16}x^4\beta^4y^4+h_{17}x^2\beta^6y^6+h_{18}\beta^8y^8) + \\ &+ (i\varpi)^5(h_{19}x^9+h_{20}x^7\beta^2y^2+h_{21}x^5\beta^4y^4+h_{22}x^3\beta^6y^6+h_{23}x\beta^8y^8)]\end{aligned} \tag{13}$$

$$\begin{aligned}\phi_8 = \sqrt{}(\lambda^2x^2-y^2)\,[&(k_0x^5+k_1x^3\beta^2y^2+k_2x\beta^4y^4) + \\ &+ (i\varpi)\,(k_3x^6+k_4x^4\beta^2y^2+k_5x^2\beta^4y^4+k_6\beta^6y^6) + \\ &+ (i\varpi)^2(k_7x^7+k_8x^5\beta^2y^2+k_9x^3\beta^4y^4+k_{10}x\beta^6y^6) + \\ &+ (i\varpi)^3(k_{11}x^8+k_{12}x^6\beta^2y^2+k_{13}x^4\beta^4y^4+k_{14}x^2\beta^6y^6+k_{15}\beta^8y^8)]\end{aligned} \tag{14}$$

$$\begin{aligned}\phi_9 = \sqrt{}(\lambda^2x^2-y^2)\,[&(m_0x^6+m_1x^4\beta^2y^2+m_2x^2\beta^4y^4+m_3\beta^6y^6) + \\ &+ (i\varpi)\,(m_4x^7+m_5x^5\beta^2y^2+m_6x^3\beta^4y^4+m_7x\beta^6y^6) + \\ &+ (i\varpi)^2(m_8x^8+m_9x^6\beta^2y^2+m_{10}x^4\beta^4y^4+m_{11}x^2\beta^6y^6+m_{12}\beta^8y^8) + \\ &+ (i\varpi)^3(m_{13}x^9+m_{14}x^7\beta^2y^2+m_{15}x^5\beta^4y^4+m_{16}x^3\beta^6y^6+m_{17}x\beta^8y^8)]\end{aligned} \tag{15}$$

In these formulae we have:

$$\left.\begin{aligned} a_i &= a_i' + a_i'' \frac{1}{M^2} + a_i''' \frac{1}{M^4} \\ b_i &= b_i' + b_i'' \frac{1}{M^2} + b_i''' \frac{1}{M^4} \end{aligned}\right\} \tag{16}$$

etc. The quantities a_i', a_i'', a_i''', b_i', b_i'', b_i''' etc. are functions of ρ_0 which are tabulated in Table 1 for a set of values of ρ_0 in the range $0 \leqslant \rho_0 \leqslant 1$. This range of ρ_0 covers the supersonic speed range over which the leading edges are subsonic. The values of the quantities a_i', a_i'', a_i''', b_i', b_i'', b_i''' etc. can be obtained for any value of ρ_0 by interpolation in these tables.

If any of the quantities a_i'', a_i''', b_i'', b_i''' etc., which are required for the evaluation of coefficients written down in Equations (6) to (15) are not tabulated in Table 1, then the reason is that the quantity is identically zero.

The theory behind the derivation of these formulae is given in Appendix 1.

3. *Calculation of Generalised Aerodynamic Forces.* For a flutter calculation it is assumed that a wing deforms in a finite number only of independent modes. We therefore assume that the displacement of a point on the wing at time t in any deformation is given by the equation

$$Z(x, y, t) = \sum_{j=1}^{n} f_j(x, y)\, q_j(t) \tag{17}$$

where $f_j(x, y)$ gives the shape of the jth mode and $q_j(t)$ is the jth generalised co-ordinate. If all the $f_j(x, y)$ are linearly independent then the n generalised co-ordinates are independent.

The potential of a wing which deforms according to the equation

$$Z_j(x, y, t) = f_j(x, y)\, q_j(t) \tag{18}$$

will be denoted by

$$\Phi_j(x, y, t)\,, \quad j = 1, 2, \ldots, n\,.$$

When the wing deforms according to Equation (17) we shall write:

$$\phi(x, y, t)$$

for the potential on its upper surface and

$$l(x, y, t)$$

for the lift distribution over it, the lift being assumed positive when it acts upwards.

Since the governing equations of the airflow are linearized, the principle of superposition is applicable and we may write

$$\phi(x, y, t) = \sum_{j=1}^{n} \Phi_j(x, y, t)\,. \tag{19}$$

In a small displacement with δq_j small and arbitrary

$$\delta Z = \sum_{j=1}^{n} f_j(x, y)\, \delta q_j$$

the virtual work done by the air forces is

$$-\iint_{\text{wing}} l(x, y, t)\, \delta Z\, dx\, dy = -\sum_{j=1}^{n} \delta q_j \iint_{\text{wing}} l(x, y, t) f_j(x, y)\, dx\, dy \tag{20}$$

the minus sign being introduced because the directions of positive lift distribution and displacement are opposite.

In such a small displacement, however, the virtual work is given by

$$\sum_{j=1}^{n} Q_j(t)\, \delta q_j \tag{21}$$

where $Q_j(t)$ is the generalised aerodynamic force in the jth mode at time t.

Comparing (20) and (21) we get, since the δq_j are independent

$$Q_j(t) = -\iint\limits_{\text{wing}} l(x, y, t) f_j(x, y)\, dx\, dy\,. \tag{22}$$

When the wing is deforming harmonically we may write, as is common in linear problems, $q_i = q_{i0}e^{i\omega t}$ and $\Phi_j(x, y, t) = \hat{\phi}_j(x, y)\, q_{j0} e^{i\omega t}$, $j = 1, 2, \ldots, n$. The potentials on the upper and lower surfaces of the wing are equal but of opposite signs, so that applying the linearised Bernoulli equation we obtain the following equation for the lift distribution:

$$\begin{aligned} l(x, y, t) &= 2\rho \left(V \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial t} \right) \\ &= 2\rho \sum_{k=1}^{n} \left(V \frac{\partial \hat{\phi}_k}{\partial x} + i\omega \hat{\phi}_k \right) q_{k0} e^{i\omega t}. \end{aligned} \tag{23}$$

Substituting for $l(x, y, t)$ from (23) into (22) we get

$$Q_j(t) = -2\rho e^{i\omega t} \sum_{k=1}^{n} q_{k0} \iint\limits_{\text{wing}} \left(V \frac{\partial \hat{\phi}_k}{\partial x} + i\omega \hat{\phi}_k \right) f_j(x, y)\, dx\, dy \tag{24}$$

$$= e^{i\omega t} \sum_{k=1}^{n} Q_{j,k} q_{k0} \tag{25}$$

where

$$Q_{j,k} = -2\rho \iint\limits_{\text{wing}} \left(V \frac{\partial \hat{\phi}_k}{\partial x} + i\omega \hat{\phi}_k \right) f_j(x, y)\, dx\, dy\,. \tag{26}$$

The $Q_{j,k}$ are to be obtained for j, $k = 1, 2, \ldots, n$.

If each of the $f_j(x, y)$ is a polynomial of at most the third degree in x and y^2, and the wing under consideration has symmetric subsonic leading edges and supersonic trailing edge(s), then the upper surface potentials $\hat{\phi}_k(x, y)$ may be obtained from the results of this paper. The surface potentials $\hat{\phi}_k(x, y)$ will take the form

$$\hat{\phi}_k(x, y) = \sqrt{(\lambda^2 x^2 - y^2)} \times \{\text{a polynomial in } x \text{ and } y^2\}$$

and therefore

$$V \frac{\partial \hat{\phi}_k}{\partial x} + i\omega \hat{\phi}_k = \frac{1}{\sqrt{(\lambda^2 x^2 - y^2)}} \times \{\text{a polynomial in } x \text{ and } y^2\}.$$

In order to determine the $Q_{j,k}$ it will therefore be necessary to evaluate the double integrals

$$I_{p,q} = \iint\limits_{\text{wing}} \frac{x^p y^{2q}}{\sqrt{(\lambda^2 x^2 - y^2)}}\, dx\, dy \tag{27}$$

for different values of the positive integers p and q.

We show here how to evaluate the integral over the wing with straight leading and trailing edges shown in Fig. 1.

Evidently

$$I_{p,q} = 2 \iint\limits_{\text{half-wing}} \frac{x^p y^{2q}}{\sqrt{(\lambda^2 x^2 - y^2)}} \, dx \, dy. \tag{28}$$

If the change of variables

$$\left.\begin{aligned} \xi &= \lambda x + y \\ \eta &= \lambda x - y \end{aligned}\right\} \tag{29}$$

is made, the half-wing shown shaded in Fig. 1 is transformed to the area of the (ξ, η) plane shown shaded in Fig. 2.

If the slopes of the leading and trailing edges are λ and γ respectively, and the root chord of the wing is c, then the integral (28) becomes

$$I_{p,q} = \frac{1}{(2\lambda)^{p+1} 2^{2q-1}} \int_0^{c\lambda} \frac{d\eta}{\sqrt{\eta}} \int_\eta^{a-b\eta} (\xi+\eta)^p (\xi-\eta)^{2q} \frac{d\xi}{\sqrt{\xi}} \tag{30}$$

where

$$\left.\begin{aligned} a &= \frac{2}{1-\left(\frac{\lambda}{\gamma}\right)} (\lambda c) \\ b &= \frac{1+\left(\frac{\lambda}{\gamma}\right)}{1-\left(\frac{\lambda}{\gamma}\right)} \end{aligned}\right\}. \tag{31}$$

The value of the integral $I_{p,q}$ can now be obtained quite easily from (30). An illustration with $p = 1$, $q = 0$ is given as an example:

$$\begin{aligned} I_{1,0} &= \frac{2}{(2\lambda)^2} \int_0^{c\lambda} \frac{d\eta}{\sqrt{\eta}} \int_\eta^{a-b\eta} (\xi+\eta) \frac{d\xi}{\sqrt{\xi}} \\ &= \frac{1}{2\lambda^2} \int_0^{c\lambda} \left\{ \frac{2}{3}(a-b\eta)^{3/2} + 2(a-b\eta)^{1/2} \right\} \frac{d\eta}{\sqrt{\eta}} - \frac{2}{3} c^2. \end{aligned}$$

With the substitution

$$\eta = \frac{a}{b} \sin^2\theta,$$

this becomes

$$I_{1,0} = \frac{a^2}{\lambda^2 \sqrt{b}} \int_0^{\psi} \left\{ \frac{2}{3} \cos^4\theta + \frac{2}{b} \cos^2\theta \sin^2\theta \right\} d\theta - \frac{2}{3} c^2$$

$$= 8c^2 \left[\frac{1}{\sqrt{\left\{ \left(1+\frac{\lambda}{\gamma}\right) \left(1-\frac{\lambda}{\gamma}\right)^3 \right\}}} \int_0^{\psi} \left\{ \frac{1}{3} \cos^4\theta + \frac{\left(1-\frac{\lambda}{\gamma}\right)}{\left(1+\frac{\lambda}{\gamma}\right)} \cos^2\theta \sin^2\theta \right\} d\theta - \frac{1}{12} \right] \tag{32}$$

where

$$\sin\psi = \sqrt{\left(\frac{1+\frac{\lambda}{\gamma}}{2} \right)}. \tag{33}$$

The integrals occurring in (32) are incomplete Beta functions, the values of which can be found either from tables or by direct evaluation. The procedure for other values of p, q is similar.

If the wing considered has a tip of finite chord length then the expressions derived for the potential will not be correct over the whole of the wing. They may in some cases be correct over most of the wing and the error introduced by assuming that they are correct over the whole wing may be insignificant. In this case the area of integration is not triangular and the evaluation of the integrals (27) is a little more involved. However, in this case too, the transformation of variables (29) may be carried out with advantage.

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LIST OF SYMBOLS

V		Free stream velocity
M		Free stream Mach number
a		Speed of sound
β	=	$\sqrt{(M^2-1)}$
x, y, z		Rectangular co-ordinates fixed relative to the equilibrium position of the wing with z positive downwards
t		Time
$Z(x, y, t)$		Displacement of a point on the wing
$W(x, y, t)$		Downwash velocity on wing surface
ω		Circular frequency of oscillation
$\bar{\omega}$	=	$M^2\omega/V\beta^2$
λ		Tangent of half-apex angle
ρ_0	=	$\beta\lambda$
$(\rho_0')^2$	=	$1 - \rho_0^2$
$\phi(x, y, z, t)$		Disturbance velocity potential
K'		Complete elliptic integral of the first kind
	=	$\int_0^{\pi/2} \frac{d\theta}{\sqrt{(1-(\rho_0')^2 \sin^2\theta)}}$
E'		Complete elliptic integral of the second kind
	=	$\int_0^{\pi/2} \sqrt{(1-(\rho_0')^2 \sin^2\theta)}\, d\theta$
ρ		Air density
γ		Slope of trailing edge (*see* Fig. 1)
c		Root chord

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APPENDIX I

The Theory of the Method

The velocity potential of the disturbance flow about a thin oscillating wing satisfies the partial differential equation

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=\frac{1}{a^2}\left(\frac{\partial}{\partial t}+V\frac{\partial}{\partial x}\right)^2\phi \tag{34}$$

where a is the speed of propagation of small disturbances. The main boundary condition satisfied by the velocity potential is

$$\left(\frac{\partial\phi}{\partial z}\right)_{z=0}=W(x,y,t)=\left(V\frac{\partial Z}{\partial x}+\frac{\partial Z}{\partial t}\right) \tag{35}$$

where $Z(x, y, t)$ is the downward vertical displacement of any point of the wing. In a harmonic oscillation we write

$$W(x,y,t)=w(x,y)e^{i\omega t}. \tag{36}$$

We shall consider the potential about a wing for which

$$w(x,y)=x^p y^q. \tag{37}$$

When $w(x, y)$ is given by a polynomial expression the potential is obtained by simple superposition of results obtained using $w(x, y)$ from Equation (37) with different values of p and q.

The function

$$\phi_D(x,y,z)e^{i\omega t}=\left.\begin{array}{ll}\dfrac{1}{\pi}m(\xi,\eta)\dfrac{\partial}{\partial z}\left[e^{-i\bar{\omega}x}\dfrac{\cos\left(\dfrac{\bar{\omega}}{M}R\right)}{R}\right]e^{i\omega t}, & x-\xi>\sqrt{(\beta^2(y-\eta)^2+\beta^2z^2)}\\ 0, & x-\xi<\sqrt{(\beta^2(y-\eta)^2+\beta^2z^2)}\end{array}\right\} \tag{38}$$

where

$$\bar{\omega}=\frac{M\omega}{a\beta^2}=\frac{M^2\omega}{V\beta^2}$$

and

$$R=\sqrt{\{(x-\xi)^2-\beta^2(y-\eta)^2-\beta^2z^2\}}$$

satisfies the differential equation (34) and corresponds to the potential about a pulsating doublet located at the point $(\xi, \eta, 0)$ of magnitude $m(\xi, \eta)e^{i\omega t}$ pointing in the direction of positive z.

The differential equation (34) is satisfied if the wing area is regarded as a pulsating doublet layer. The strength distribution of this layer is to be determined from the condition that the boundary condition (35) is also satisfied. The velocity potential of the disturbance flow about this doublet layer will then be the velocity potential of the disturbance flow about the oscillating wing.

Let us consider a doublet layer covering the wing and of strength

$$\sum_{n=0}^{\infty} a_n D_n(\xi,\eta)e^{i\omega t} \tag{39}$$

at the point $(\xi, \eta, 0)$. Here a_n is independent of ξ, η but depends on ω.

With the aid of Equation (38) we can write down the velocity potential function corresponding to flow about this doublet layer

$$\phi_1(x, y, z)e^{i\omega t} = \frac{e^{i\omega t}}{\pi} \sum_{n=0}^{\infty} a_n \iint D_n(\xi, \eta) \frac{\partial}{\partial z} \left[e^{-i\bar{\omega}x} \frac{\cos\left(\frac{\bar{\omega}}{M} R\right)}{R} \right] d\xi\, d\eta, \tag{40}$$

the area of integration r being the part of the wing cut off by the forward Mach cone from the point x, y, z.

We can write $\phi_1(x, y, z)$ after expansion of $\cos\left(\frac{\bar{\omega}}{M} R\right)$ in powers of $\bar{\omega}$ in the form

$$\phi_1(x, y, z) = \frac{1}{\pi} \sum_{n=0}^{\infty} \left[a_n e^{-i\bar{\omega}x} \frac{\partial}{\partial z} \iint_r D_n(\xi, \eta) \frac{1}{R}\, d\xi\, d\eta \right.$$

$$\left. + a_n e^{-i\bar{\omega}x} \sum_{m=1}^{\infty} \frac{(-1)^m}{(2m)!} \left(\frac{\bar{\omega}}{M}\right)^{2m} \frac{\partial}{\partial z} \iint_r D_n(\xi, \eta) R^{2m-1}\, d\xi\, d\eta \right] \tag{41}$$

The downwash velocity at $z = 0$ due to this layer is

$$w_1(x, y, z)e^{i\omega t}$$

where

$$w_1 = \left(\frac{\partial \phi_1}{\partial z}\right)_{z=0} = \frac{1}{\pi} \lim_{z\to 0} \sum_{n=0}^{\infty} \left[a_n e^{-i\bar{\omega}x} \frac{\partial^2}{\partial z^2} \iint_r D_n(\xi, \eta) \frac{1}{R}\, d\xi\, d\eta \right.$$

$$\left. + a_n e^{-i\bar{\omega}x} \sum_{m=1}^{\infty} \frac{(-1)^m}{(2m)!} \left(\frac{\bar{\omega}}{M}\right)^{2m} \frac{\partial^2}{\partial z^2} \iint_r D_n(\xi, \eta) R^{2m-1}\, d\xi\, d\eta \right] \tag{42}$$

We take the functions $D_n(\xi, \eta)$ to be the functions which satisfy the integral equations

$$\frac{1}{\pi} \lim_{z\to 0} \frac{\partial^2}{\partial z^2} \iint_r D_n(\xi, \eta) \frac{1}{R}\, d\xi\, d\eta = x^{n+p} y^q \tag{43}$$

for $n = 0, 1, 2, \ldots$. We show in Appendix II how the Equations (43) may be solved for a triangular wing.

We have therefore

$$\frac{1}{\pi} \lim_{z\to 0} \sum_{n=0}^{\infty} a_n e^{-i\bar{\omega}x} \frac{\partial^2}{\partial z^2} \iint_r D_n(\xi, \eta) \frac{1}{R}\, d\xi\, d\eta = \sum_{n=0}^{\infty} a_n e^{-i\bar{\omega}x} x^{n+p} y^q$$

$$= e^{-i\bar{\omega}x} x^p y^q \sum_{n=0}^{\infty} a_n x^n$$

$$= x^p y^q \tag{44}$$

if we choose

$$a_n = \frac{(i\bar{\omega})^n}{n!}. \tag{45}$$

We may now write Equation (42) in the form

$$w_1 = x^p y^q + w_1' \tag{46}$$

where

$$w_1' = \frac{1}{\pi} \lim_{z\to 0} \sum_{n=0}^{\infty} a_n e^{-i\bar{\omega}x} \sum_{m=1}^{\infty} \frac{(-1)^m}{(2m)!} \left(\frac{\bar{\omega}}{M}\right)^{2m} \frac{\partial^2}{\partial z^2} \iint_r D_n(\xi, \eta) R^{2m-1}\, d\xi\, d\eta. \tag{47}$$

It is to be observed that w_1' may be written as a power series in $\bar{\omega}$ beginning with the term in $\bar{\omega}^2$.

Let us superimpose on the doublet layer (39) a doublet layer of strength

$$\sum_{n=0}^{\infty} \bar{a}_n \bar{D}_n(\xi, \eta) e^{i\omega t} \tag{48}$$

at the point $(\xi, \eta, 0)$. The downwash velocity $\bar{w}_1 e^{i\omega t}$ at $z = 0$ due to this layer is obtained from Equation (42) by replacing a_n and $D_n(\xi, \eta)$ by $\bar{a}_n$ and $\bar{D}_n(\xi, \eta)$ respectively. This gives

$$\bar{w}_1 = \frac{1}{\pi} \lim_{z\to 0} \sum_{n=0}^{\infty} \left[\bar{a}_n e^{-i\bar{\omega}x} \frac{\partial^2}{\partial z^2} \iint_r \bar{D}_n(\xi, \eta) \frac{1}{R} \, d\xi \, d\eta \right.$$
$$\left. + \bar{a}_n e^{-i\bar{\omega}x} \frac{\partial^2}{\partial z^2} \sum_{m=1}^{\infty} \frac{(-1)^m}{(2m)!} \left(\frac{\bar{\omega}}{M}\right)^{2m} \frac{\partial^2}{\partial z^2} \iint_r \bar{D}_n(\xi, \eta) \, R^{2m-1} \, d\xi \, d\eta \right] \tag{49}$$

If we arrange that

$$\frac{1}{\pi} \lim_{z\to 0} \sum_{n=0}^{\infty} \bar{a}_n e^{-i\bar{\omega}x} \frac{\partial^2}{\partial z^2} \iint_r \bar{D}_n(\xi, \eta) \frac{1}{R} \, d\xi \, d\eta =$$
$$= \frac{1}{\pi} \lim_{z\to 0} \sum_{n=0}^{\infty} \frac{\bar{\omega}^2}{2M^2} a_n e^{-i\bar{\omega}x} \frac{\partial^2}{\partial z^2} \iint_r D_n(\xi, \eta) R \, d\xi \, d\eta \tag{50}$$

then we shall cancel the terms in $\bar{\omega}^2$ and $\bar{\omega}^3$ in the Expression (47) for w_1'.

If we take the $\bar{D}_n(\xi, \eta)$ to be the functions which satisfy the integral equations

$$\frac{1}{\pi} \lim_{z\to 0} \frac{\partial^2}{\partial z^2} \iint_r \bar{D}_n(\xi, \eta) \frac{1}{R} \, d\xi \, d\eta = \frac{1}{\pi} \lim_{z\to 0} \frac{\partial^2}{\partial z^2} \iint_r D_n(\xi, \eta) R \, d\xi \, d\eta \tag{51}$$

for $n = 0, 1, 2, \ldots,$ and take

$$\bar{a}_n = \frac{\bar{\omega}^2}{2M^2} a_n \tag{52}$$

then Equation (50) will be satisfied. The downwash velocity due to the doublet layer of strength

$$\sum_{n=0}^{\infty} a_n \left[D_n(\xi, \eta) + \frac{\bar{\omega}^2}{2M^2} \bar{D}_n(\xi, \eta) \right] \tag{53}$$

is then

$$w_2 e^{i\omega t}$$

where

$$w_2 = x^p y^q + w_2' \tag{54}$$

and

$$w_2' = \frac{1}{\pi} \lim_{z\to 0} \sum_{n=0}^{\infty} a_n e^{-i\bar{\omega}x} \sum_{m=2}^{\infty} \frac{(-1)^m}{(2m)!} \left(\frac{\bar{\omega}}{M}\right)^{2m} \frac{\partial^2}{\partial z^2} \iint_r D_n(\xi, \eta) R^{2m-1} \, d\xi \, d\eta$$
$$+ \frac{1}{\pi} \lim_{z\to 0} \sum_{n=0}^{\infty} \frac{\bar{\omega}^2}{2M^2} a_n e^{-i\bar{\omega}x} \sum_{m=1}^{\infty} \frac{(-1)^m}{(2m)!} \left(\frac{\bar{\omega}}{M}\right)^{2m} \frac{\partial^2}{\partial z^2} \iint_r \bar{D}_n(\xi, \eta) R^{2m-1} \, d\xi \, d\eta \tag{55}$$

It is to be observed that w_2' may be written as a power series in $\bar{\omega}$ beginning with the term in $\bar{\omega}^4$.

Let us superimpose on the doublet layer (53) a doublet layer of strength

$$\sum_{n=0}^{\infty} \bar{\bar{a}}_n \bar{\bar{D}}_n(\xi, \eta) e^{i\omega t} \tag{56}$$

at the point $(\xi, \eta, +0)$. The downwash velocity at $z = 0$ due to this layer is $\bar{\bar{w}}_1 e^{i\omega t}$ where

$$\bar{\bar{w}}_1 = \frac{1}{\pi} \lim_{z\to 0} \sum_{n=0}^{\infty} \left[\bar{\bar{a}}_n e^{-i\bar{\omega}x} \frac{\partial^2}{\partial z^2} \iint_r \bar{\bar{D}}_n(\xi, \eta) \frac{1}{R}\, d\xi\, d\eta \right.$$
$$\left. + \bar{\bar{a}}_n e^{-i\bar{\omega}x} \frac{\partial^2}{\partial z^2} \sum_{m=1}^{\infty} \frac{(-1)^m}{(2m)!} \left(\frac{\bar{\omega}}{M}\right)^{2m} \frac{\partial^2}{\partial z^2} \iint_r \bar{\bar{D}}_n(\xi, \eta) R^{2m-1}\, d\xi\, d\eta \right] \tag{57}$$

If we arrange that

$$\frac{1}{\pi} \lim_{z\to 0} \sum_{n=0}^{\infty} \bar{\bar{a}}_n e^{-i\bar{\omega}x} \frac{\partial^2}{\partial z^2} \iint_r \bar{\bar{D}}_n(\xi, \eta) \frac{1}{R}\, d\xi\, d\eta =$$
$$= \frac{1}{\pi} \lim_{z\to 0} \sum_{n=0}^{\infty} \left(\frac{\bar{\omega}^2}{2M^2}\right)^2 a_n e^{-i\bar{\omega}x} \frac{\partial^2}{\partial z^2} \iint_r \bar{D}_n(\xi, \eta) R\, d\xi\, d\eta$$
$$- \frac{1}{6\pi} \lim_{z\to 0} \sum_{n=0}^{\infty} \left(\frac{\bar{\omega}^2}{2M^2}\right)^2 a_n e^{-i\bar{\omega}x} \frac{\partial^2}{\partial z^2} \iint_r D_n(\xi, \eta) R^3\, d\xi\, d\eta \tag{58}$$

then we shall cancel the terms in $\bar{\omega}^4$ and $\bar{\omega}^5$ in the expression (55) for w_2'.

If we take the $\bar{\bar{D}}_n(\xi, \eta)$ to be the functions which satisfy the integral equations

$$\frac{1}{\pi} \lim_{z\to 0} \frac{\partial^2}{\partial z^2} \iint_r \bar{\bar{D}}_n(\xi, \eta) \frac{1}{R}\, d\xi\, d\eta = \frac{1}{\pi} \lim_{z\to 0} \frac{\partial^2}{\partial z^2} \iint_r \bar{D}_n(\xi, \eta) R\, d\xi\, d\eta$$
$$- \frac{1}{6\pi} \lim_{z\to 0} \frac{\partial^2}{\partial z^2} \iint_r D_n(\xi, \eta) R^3\, d\xi\, d\eta \tag{59}$$

for $n = 0, 1, 2, \ldots.$, and take

$$\bar{\bar{a}}_n = \left(\frac{\bar{\omega}^2}{2M^2}\right)^2 \tag{60}$$

the Equation (58) will be satisfied. The downwash velocity due to the doublet layer of strength

$$\sum_{n=0}^{\infty} a_n \left[D_n(\xi, \eta) + \frac{\bar{\omega}^2}{2M^2} \bar{D}_n(\xi, \eta) + \left(\frac{\bar{\omega}^2}{2M^2}\right)^2 \bar{\bar{D}}_n(\xi, \eta) \right] \tag{61}$$

is then

$$w_3 e^{i\omega t}$$

where

$$w_3 = x^p y^q + w_3' \tag{62}$$

and w_3' can be expressed as a power series in $\bar{\omega}$ beginning with the term in $\bar{\omega}^6$.

By superimposing doublet layers in this manner we can satisfy the boundary condition (46) and (47) to any finite power of frequency that we desire. The strength of the corresponding doublet layer will be expressible as a power series in frequency and it will not be necessary to retain any powers of frequency greater than this finite power in the expression for the doublet strength. If for example we are interested in powers of frequency only up to the fifth the only terms which must be retained in the expression for the strength of the doublet layer are

$$\sum_{n=0}^{5} a_n D_n(\xi, \eta) + \frac{\varpi^2}{2M^2} \sum_{n=0}^{3} a_n \bar{D}_n(\xi, \eta) + \left(\frac{\varpi^2}{2M^2}\right)^2 \sum_{n=0}^{1} a_n \bar{\bar{D}}_n(\xi, \eta) \tag{63}$$

The velocity potential corresponding to the flow about the doublet layer of strength (63) is obtained by using Equation (40). It may be shown that (*see*, for example, Ref. 3)

$$\left.\begin{aligned} &\lim_{z \to -0} \frac{1}{\pi} \frac{\partial}{\partial z} \iint_r K(\xi, \eta) \frac{1}{R} d\xi\, d\eta = K(x, y) \\ \text{and} \quad &\lim_{z \to -0} \frac{1}{\pi} \frac{\partial}{\partial z} \iint_r K(\xi, \eta) R^{2m-1} d\xi\, d\eta = 0 \quad m \geqslant 1 \end{aligned}\right\} \tag{64}$$

Thus from (41) we get by using the Formulae (64) that the potential on the surface of the wing correct as far as terms of the fifth power in frequency is given by

$$\begin{aligned} \phi(x, y, -0) = \Bigg[&\sum_{n=0}^{5} \frac{(i\varpi)^n}{n!} D_n(x, y) + \frac{\varpi^2}{2M^2} \sum_{n=0}^{3} \frac{(i\varpi)^n}{n!} \bar{D}_n(x, y) \\ &+ \left(\frac{\varpi^2}{2M^2}\right)^2 \sum_{n=0}^{1} \frac{(i\varpi)^n}{n!} \bar{\bar{D}}_n(x, y) \Bigg] e^{-i\varpi x} \end{aligned} \tag{65}$$

APPENDIX II

The Solution of The Integral Equations

We consider now the triangular wings whose leading edges have equations

$$y = \pm \lambda x. \tag{66}$$

We seek the solution of the integral equation

$$\frac{1}{\pi} \lim_{z \to 0} \frac{\partial^2}{\partial z^2} \iint_{\tau} D_n(\xi, \eta) \frac{1}{R} \, d\xi \, d\eta = x^{n+p} y^q \tag{67}$$

which is the Equation (43) obtained in Appendix I.

We make the change of variables

$$\left.\begin{aligned} \sigma &= \eta/\xi \\ \zeta &= \xi \end{aligned}\right\} \tag{68}$$

in the integral and obtain the integral equation

$$x^{n+p} y^q = \frac{1}{\pi} \lim_{z \to 0} \frac{\partial^2}{\partial z^2} \int_{-\lambda}^{\lambda} d\sigma \int_0^{\zeta_1} \frac{\zeta D_n(\zeta, \sigma\zeta) \, d\zeta}{\sqrt{\{(x-\zeta)^2 - \beta^2(y-\sigma\zeta)^2 - \beta^2 z^2\}}} \tag{69}$$

where ζ_1 is the smallest root of

$$(x-\zeta)^2 - \beta^2(y-\sigma\zeta)^2 - \beta^2 z^2 = 0. \tag{70}$$

We shall seek a function $D_n(\zeta, \sigma\zeta)$ of the form

$$D_n(\zeta, \sigma\zeta) = \zeta^{n+p+q+1} F_n(\sigma). \tag{71}$$

Now

$$(x-\zeta)^2 - \beta^2(y-\sigma\zeta)^2 - \beta^2 z^2 =$$

$$= (1-\beta^2\sigma^2) \left[\left\{ \zeta - \left(\frac{x-\beta^2 y\sigma}{1-\beta^2\sigma^2} \right) \right\}^2 - \frac{\beta^2}{(1-\beta^2\sigma^2)^2} \left\{ (\sigma x - y)^2 + z^2(1-\beta^2\sigma^2) \right\} \right]$$

$$= (1-\beta^2\sigma^2) [(S-\zeta)^2 - N^2] \tag{72}$$

where

$$\left.\begin{aligned} S &= \frac{x}{1-\beta^2\sigma^2}(1-\beta^2\sigma\theta) \\ N &= \frac{\beta x}{1-\beta^2\sigma^2} \sqrt{\left\{ (\theta-\sigma)^2 + \frac{z^2}{x^2}(1-\beta^2\sigma^2) \right\}} \\ \theta &= \frac{y}{x} \end{aligned}\right\} \tag{73}$$

From (72) we see that

$$\zeta_1 = S - N. \tag{74}$$

By substituting

$$S - \zeta = N \cosh \tau \tag{75}$$

into the inner integral of (70) a form of integrand which does not become infinite at the upper limit ζ_1 is obtained. The integral equation then becomes

$$x^{n+p+q}\theta^q = \frac{1}{\pi}\lim_{z\to 0}\frac{\partial^2}{\partial z^2}\int_{-\lambda}^{+\lambda}\frac{F_n(\sigma)\,d\sigma}{(1-\beta^2\sigma^2)^{1/2}}\int_0^{\cosh^{-1}\frac{S}{N}}(S-N\cosh\tau)^{n+p+q+2}\,d\tau$$

$$= \frac{1}{\pi}\lim_{z\to 0}\Bigg\{-\beta^2(n+p+q+2)\int_{-\lambda}^{\lambda}\frac{F_n(\sigma)\,d\sigma}{(1-\beta^2\sigma^2)^{3/2}}\times$$

$$\times\int_0^{\cosh^{-1}\frac{S}{N}}N^{n+p+q}\left(\frac{S}{N}-\cosh\tau\right)^{n+p+q+1}\cosh\tau\,d\tau$$

$$+\beta^4 z^2(n+p+q+2)\int_{-\lambda}^{\lambda}\frac{F_n(\sigma)\,d\sigma}{(1-\beta^2\sigma^2)^{5/2}}\int_0^{\cosh^{-1}\frac{S}{N}}N^{n+p+q+2}\left(\frac{S}{N}-\cosh\tau\right)^{n+p+q}\times$$

$$\times\left[\frac{S}{N}+(n+p+q)\cosh\tau\right]\cosh\tau\,d\tau\Bigg\} \tag{76}$$

We shall go to the limit $z = 0$ before performing the integrations and treat the resulting integrals as principal value integrals. The result is

$$x^{n+p+q}\theta^q = \frac{-\beta^2(n+p+q+2)}{\pi}\int_{-\lambda}^{+\lambda}\frac{F_n(\sigma)\,d\sigma}{(1-\beta^2\sigma^2)^{3/2}}\int_0^{\cosh^{-1}\frac{S}{N_0}}N_0^{\,n+p+q}\times$$

$$\times\left(\frac{S}{N_0}-\cosh\tau\right)^{n+p+q+1}\cosh\tau\,d\tau \tag{77}$$

where

$$N_0 = \lim_{z\to 0} N$$

$$= \frac{\beta x}{1-\beta^2\sigma^2}\,|\theta-\sigma|\,. \tag{78}$$

By means of straightforward differentiation it is easy to show that

$$\frac{\partial^{n+p+q+1}}{\partial\theta^{n+p+q+1}}\left[N_0^{\,n+p+q}\left(\frac{S}{N_0}-\cosh\tau\right)^{n+p+q+1}\right] =$$

$$= \left.\begin{array}{ll} -\dfrac{(n+p+q+1)\,!}{(-N_0)^{n+p+q+2}}\left(\dfrac{\beta x^2}{1-\beta^2\sigma^2}\right)^{n+p+q+1} & \text{for } \theta > \sigma \\[2ex] \dfrac{(n+p+q+1)\,!}{(N_0)^{n+p+q+2}}\left(\dfrac{\beta x^2}{1-\beta^2\sigma^2}\right)^{n+p+q+1} & \text{for } \theta < \sigma \end{array}\right\} \tag{79}$$

The behaviour at $\theta = \sigma$ will be accounted for automatically by the principal value integrals which occur.

Also

$$\left[\frac{\partial^m}{\partial\theta^m}N_0^{\,n+p+q}\left(\frac{S}{N_0}-\cosh\tau\right)^{n+p+q+1}\right]_{\tau=\cosh^{-1}\frac{S}{N_0}} = 0 \tag{80}$$

for $m = 0, 1, 2, \ldots, n+p+q$.

Hence if we differentiate Equation (77) $(n+p+q+1)$ times with respect to θ we obtain

$$0 = \frac{-\beta^2(n+p+q+2)}{\pi} \int_{-\lambda}^{\lambda} \frac{F_n(\sigma)\, d\sigma}{(1-\beta^2\sigma^2)^{3/2}} \int_0^{\cosh^{-1}\frac{S}{N_0}} \frac{\partial^{n+p+q+1}}{\partial\theta^{n+p+q+1}} \left[N_0^{\,n+p+q} \left(\frac{S}{N_0} - \cosh\tau \right)^{n+p+q+1} \times \right.$$

$$\times \cosh\tau\, d\tau$$

$$= \frac{-\beta^2(n+p+q+2)!}{\pi} (\beta x^2)^{n+p+q+1} \left[\int_{-\lambda}^{\theta} \frac{F_n(\sigma)\, d\sigma}{-(-N_0)^{n+p+q+2}(1-\beta^2\sigma^2)^{n+p+q+5/2}} \int_0^{\cosh^{-1}\frac{S}{N_0}} \cosh\tau\, d\tau \right.$$

$$\left. + \int_{\theta}^{\lambda} \frac{F_n(\sigma)\, d\sigma}{(N_0)^{n+p+q+2}(1-\beta^2\sigma^2)^{n+p+q+5/2}} \int_0^{\cosh^{-1}\frac{S}{N_0}} \cosh\tau\, d\tau \right]$$

$$= \frac{(-1)^{n+p+q+2}}{\pi} (n+p+q+2)!\, x^{n+p+q} \sqrt{(1-\beta^2\theta^2)} \int_{-\lambda}^{\lambda} \frac{F_n(\sigma)}{(\theta-\sigma)^{n+p+q+3}}\, d\sigma \tag{81}$$

Therefore $F_n(\sigma)$ must satisfy the following principal value integral equation

$$\int_{-\lambda}^{+\lambda} \frac{F_n(\sigma)\, d\sigma}{(\theta-\sigma)^{n+p+q+3}} = 0. \tag{82}$$

The integral Equation (82) is satisfied by the function

$$F_n(\sigma) = \sum_{j=0}^{n+p+q} B_j f_j \tag{83}$$

where

$$f_j = \sigma^j \sqrt{(\lambda^2-\sigma^2)} \qquad j = 0, 1, 2, \ldots, n+p+q \tag{84}$$

and the B_j, $j = 0, 1, 2, \ldots, n+p+q$ are constants.

With $F_n(\sigma)$ given by Equation (83) the result of further differentiation of the right-hand side of Equation (77) is zero. It then follows by Taylor's Theorem that the right-hand side of Equation (77) is a polynomial in θ of degree $(n+p+q)$ when $F_n(\sigma)$ is given by the Equation (83). The coefficients B_j must be determined so that this polynomial is $x^{n+p+q}\theta^q$. A convenient way of doing this is to differentiate Equation (77) and express the fact that

$$\left[\frac{\partial^m}{\partial\theta^m} (x^{n+p+q}\theta^q) \right]_{\theta=0} = \left. \begin{matrix} 0 & m \neq q \\ q!\,x^{n+p+q} & m = q \end{matrix} \right\} \tag{85}$$

for $m = 0, 1, 2, \ldots, n+p+q$. This leads to $(n+p+q+1)$ linear simultaneous equations for the $(n+p+q+1)$ constants B_j. The solution of Equation (67) is then found after solving these equations.

The simultaneous equations corresponding to particular values of $n+p$ and q are given in Appendix III.

Having determined $D_n(\xi, \eta)$ we have to determine $\bar{D}_n(\xi, \eta)$ as the solution of

$$\frac{1}{\pi} \lim_{z\to 0} \frac{\partial^2}{\partial z^2} \iint_r \bar{D}_n(\xi, \eta) \frac{1}{R}\, d\xi\, d\eta = \frac{1}{\pi} \lim_{z\to 0} \frac{\partial^2}{\partial z^2} \iint_r D_n(\xi, \eta)\, R\, d\xi\, d\eta \tag{86}$$

which is the Equation (51) obtained in Appendix I.

We can show, in the same way as we did for Equation (88), that the left-hand side of Equation (97) is a polynomial of degree $(n+p+q+4)$ in θ. By differentiating Equation (97) $(n+p+q+5)$ times with respect to θ we find that $\bar{\bar{F}}_n(\sigma)$ must satisfy

$$\int_{-\lambda}^{+\lambda} \frac{\bar{\bar{F}}_n(\sigma)\, d\sigma}{(\theta-\sigma)^{n+p+q+7}} = 0 \tag{98}$$

and this equation is satisfied by

$$\bar{\bar{F}}_n(\sigma) = \sum_{j=0}^{n+p+q+4} \bar{\bar{B}}_j f_j \tag{99}$$

where the $\bar{\bar{B}}_j$, $j = 0, 1, 2, \ldots, n+p+q+4$ are constants.

The $\bar{\bar{B}}_j$ may be determined by expressing the fact that Equation (97) and the results of differentiating it up to $(n+p+q+4)$ times hold when $\theta = 0$. This leads to $(n+p+q+5)$ linear simultaneous equations for the $(n+p+q+5)$ constants $\bar{\bar{B}}_j$.

The simultaneous equations corresponding to different values of n, p and q are given in Appendix III.

The functions $D_n(\xi, \eta)$, $\bar{D}_n(\xi, \eta)$, $\bar{\bar{D}}_n(\xi, \eta)$ can now be found by the above described procedure. Any further functions $\bar{\bar{\bar{D}}}_n(\xi, \eta)$ etc. necessary for determining the velocity potential correct to higher powers of frequency than the fifth can be determined in the same way but the numerical process rapidly becomes very tedious.

APPENDIX III

Some Subsidiary Equations

The expressions for the potential functions ϕ_j which occur in Equation (5) may, with the aid of Equation (65) and the results of Appendix II, be written as

$$\phi_j = \sqrt{(\lambda^2x^2 - y^2)}\Big\{P_{0,j} + (i\bar{\omega})(P_{1,j} - xP_{0,j}) + \frac{(i\bar{\omega})^2}{2}\left(P_{2,j} - 2xP_{1,j} + x^2P_{0,j} - \frac{1}{M^2}\bar{P}_{0,j}\right)$$

$$+ \frac{(i\bar{\omega})^3}{6}\left(P_{3,j} - 3xP_{2,j} + 3x^2P_{1,j} - x^3P_{0,j} - \frac{3}{M^2}\bar{P}_{1,j} + \frac{3x}{M^2}\bar{P}_{0,j}\right)$$

$$+ \frac{(i\bar{\omega})^4}{24}\Big(P_{4,j} - 4xP_{3,j} + 6x^2P_{2,j} - 4x^3P_{1,j} + x^4P_{0,j}, - \frac{6}{M^2}\bar{P}_{2,j} + \frac{12x}{M^2}\bar{P}_{1,j}$$

$$- \frac{6x^2}{M^2}\bar{P}_{0,j} + \frac{6}{M^4}\bar{\bar{P}}_{0,j}\Big)$$

$$+ \frac{(i\bar{\omega})^5}{120}\Big(P_{5,j} - 5xP_{4,j} + 10x^2P_{3,j} - 10x^3P_{2,j} + 5x^4P_{1,j} - x^5P_{0,j} - \frac{10}{M^2}\bar{P}_{3,j}$$

$$+ \frac{30x}{M^2}\bar{P}_{2,j} - \frac{30x^2}{M^2}\bar{P}_{1,j} + \frac{10x^3}{M^2}\bar{P}_{0,j} + \frac{30}{M^4}\bar{\bar{P}}_{1,j} - \frac{30x}{M^4}\bar{\bar{P}}_{0,j}\Big) + \dots\Big\} \qquad (100)$$

$$j = 0, 1, 2, 3, \dots, 9.$$

where the $P_{k,j}$, $\bar{P}_{k,j}$ and $\bar{\bar{P}}_{k,j}$ are polynomials in $\beta y/x$ and x as expressed below. The coefficients of these polynomials are obtained by solving sets of linear equations with coefficients $W_{n,m}{}^P$, $\bar{W}_{n,m}{}^P$ and $\bar{\bar{W}}_{n\,m}{}^P$ which are functions of ρ_0. These sets of simultaneous equations are written down immediately after the polynomials $P_{k,j}$, $\bar{P}_{k,j}$ and $\bar{\bar{P}}_{k,j}$ and the expressions for $W_{n,m}{}^P$, $\bar{W}_{n,m}{}^P$, $\bar{\bar{W}}_{n,m}{}^P$ are then given.

The expressions for the $P_{k,j}$, $\bar{P}_{k,j}$, $\bar{\bar{P}}_{k,j}$, with $\psi = \beta y/x$ are:

$$\text{(i) } j = 0: \left.\begin{aligned}
P_{0,0} &= A_0\\
P_{1,0} &= xA_1\\
P_{2,0} &= x^2(B_{11} + B_{21}\psi^2)\\
P_{3,0} &= x^3(C_{11} + C_{21}\psi^2)\\
P_{4,0} &= x^4(E_{11} + E_{21}\psi^2 + E_{31}\psi^4)\\
P_{5,0} &= x^5(F_{11} + F_{21}\psi^2 + F_{31}\psi^4)\\
\bar{P}_{0,0} &= x^2(\bar{B}_{11} + \bar{B}_{21}\psi^2)\\
\bar{P}_{1,0} &= x^3(\bar{C}_{11} + \bar{C}_{21}\psi^2)\\
\bar{P}_{2,0} &= x^4(\bar{E}_{11} + \bar{E}_{21}\psi^2 + \bar{E}_{31}\psi^4)\\
\bar{P}_{3,0} &= x^5(\bar{F}_{11} + \bar{F}_{21}\psi^2 + \bar{F}_{31}\psi^4)\\
\bar{\bar{P}}_{0,0} &= x^4(\bar{\bar{E}}_{11} + \bar{\bar{E}}_{21}\psi^2 + \bar{\bar{E}}_{31}\psi^4)\\
\bar{\bar{P}}_{1,0} &= x^5(\bar{\bar{F}}_{11} + \bar{\bar{F}}_{21}\psi^2 + \bar{\bar{F}}_{31}\psi^4)
\end{aligned}\right\} \qquad (101)$$

$$
\left.\begin{aligned}
\text{(ii) } j = 1 : P_{0,1} &= xA_1 \\
P_{1,1} &= x^2(B_{11} + B_{21}\psi^2) \\
P_{2,1} &= x^3(C_{11} + C_{21}\psi^2) \\
P_{3,1} &= x^4(E_{11} + E_{21}\psi^2 + E_{31}\psi^4) \\
P_{4,1} &= x^5(F_{11} + F_{21}\psi^2 + F_{31}\psi^4) \\
P_{5,1} &= x^6(G_{11} + G_{21}\psi^2 + G_{31}\psi^4 + G_{41}\psi^6) \\
\bar{P}_{0,1} &= x^3(\bar{C}_{11} + \bar{C}_{21}\psi^2) \\
\bar{P}_{1,1} &= x^4(\bar{E}_{11} + \bar{E}_{21}\psi^2 + \bar{E}_{31}\psi^4) \\
\bar{P}_{2,1} &= x^5(\bar{F}_{11} + \bar{F}_{21}\psi^2 + \bar{F}_{31}\psi^4) \\
\bar{P}_{3,1} &= x^6(\bar{G}_{11} + \bar{G}_{21}\psi^2 + \bar{G}_{31}\psi^4 + \bar{G}_{41}\psi^6) \\
\bar{\bar{P}}_{0,1} &= x^5(\bar{\bar{F}}_{11} + \bar{\bar{F}}_{21}\psi^2 + \bar{\bar{F}}_{31}\psi^4) \\
\bar{\bar{P}}_{1,1} &= x^6(\bar{\bar{G}}_{11} + \bar{\bar{G}}_{21}\psi^2 + \bar{\bar{G}}_{31}\psi^4 + \bar{\bar{G}}_{41}\psi^6)
\end{aligned}\right\} \qquad (102)
$$

$$
\left.\begin{aligned}
\text{(iii) } j = 2 : P_{0,2} &= x^2(B_{12} + B_{22}\psi^2) \\
P_{1,2} &= x^3(C_{12} + C_{22}\psi^2) \\
P_{2,2} &= x^4(E_{12} + E_{22}\psi^2 + E_{32}\psi^4) \\
P_{3,2} &= x^5(F_{12} + F_{22}\psi^2 + F_{32}\psi^4) \\
P_{4,2} &= x^6(G_{12} + G_{22}\psi^2 + G_{32}\psi^4 + G_{42}\psi^6) \\
P_{5,2} &= x^7(H_{12} + H_{22}\psi^2 + H_{32}\psi^4 + H_{42}\psi^6) \\
\bar{P}_{0,2} &= x^4(\bar{E}_{12} + \bar{E}_{22}\psi^2 + \bar{E}_{32}\psi^4) \\
\bar{P}_{1,2} &= x^5(\bar{F}_{12} + \bar{F}_{22}\psi^2 + \bar{F}_{32}\psi^4) \\
\bar{P}_{2,2} &= x^6(\bar{G}_{12} + \bar{G}_{22}\psi^2 + \bar{G}_{32}\psi^4 + \bar{G}_{42}\psi^6) \\
\bar{P}_{3,2} &= x^7(\bar{H}_{12} + \bar{H}_{22}\psi^2 + \bar{H}_{32}\psi^4 + \bar{H}_{42}\psi^6) \\
\bar{\bar{P}}_{0,2} &= x^6(\bar{\bar{G}}_{12} + \bar{\bar{G}}_{22}\psi^2 + \bar{\bar{G}}_{32}\psi^4 + \bar{\bar{G}}_{42}\psi^6) \\
\bar{\bar{P}}_{1,2} &= x^7(\bar{\bar{H}}_{12} + \bar{\bar{H}}_{22}\psi^2 + \bar{\bar{H}}_{32}\psi^4 + \bar{\bar{H}}_{42}\psi^6)
\end{aligned}\right\} \qquad (103)
$$

$$
\left.\begin{aligned}
\text{(iv) } j = 3 : P_{0,3} &= x^2(B_{11} + B_{21}\psi^2) \\
P_{1,3} &= x^3(C_{11} + C_{21}\psi^2) \\
P_{2,3} &= x^4(E_{11} + E_{21}\psi^2 + E_{31}\psi^4) \\
P_{3,3} &= x^5(F_{11} + F_{21}\psi^2 + F_{31}\psi^4) \\
P_{4,3} &= x^6(G_{11} + G_{21}\psi^2 + G_{31}\psi^4 + G_{41}\psi^6) \\
P_{5,3} &= x^7(H_{11} + H_{21}\psi^2 + H_{31}\psi^4 + H_{41}\psi^6) \\
\bar{P}_{0,3} &= x^4(\bar{E}_{11} + \bar{E}_{21}\psi^2 + \bar{E}_{31}\psi^4) \\
\bar{P}_{1,3} &= x^5(\bar{F}_{11} + \bar{F}_{21}\psi^2 + \bar{F}_{31}\psi^4) \\
\bar{P}_{2,3} &= x^6(\bar{G}_{11} + \bar{G}_{21}\psi^2 + \bar{G}_{31}\psi^4 + \bar{G}_{41}\psi^6) \\
\bar{P}_{3,3} &= x^7(\bar{H}_{11} + \bar{H}_{21}\psi^2 + \bar{H}_{31}\psi^4 + \bar{H}_{41}\psi^6) \\
\bar{\bar{P}}_{0,3} &= x^6(\bar{\bar{G}}_{11} + \bar{\bar{G}}_{21}\psi^2 + \bar{\bar{G}}_{31}\psi^4 + \bar{\bar{G}}_{41}\psi^6) \\
\bar{\bar{P}}_{1,3} &= x^7(\bar{\bar{H}}_{11} + \bar{\bar{H}}_{21}\psi^2 + \bar{\bar{H}}_{31}\psi^4 + \bar{\bar{H}}_{41}\psi^6)
\end{aligned}\right\} \qquad (104)
$$

$$
\left.
\begin{aligned}
\text{(v) } j = 4 : P_{0,4} &= x^3(C_{12} + C_{22}\psi^2) \\
P_{1,4} &= x^4(E_{12} + E_{22}\psi^2 + E_{32}\psi^4) \\
P_{2,4} &= x^5(F_{12} + F_{22}\psi^2 + F_{32}\psi^4) \\
P_{3,4} &= x^6(G_{12} + G_{22}\psi^2 + G_{32}\psi^4 + G_{42}\psi^6) \\
P_{4,4} &= x^7(H_{12} + H_{22}\psi^2 + H_{32}\psi^4 + H_{42}\psi^6) \\
P_{5,4} &= x^8(J_{12} + J_{22}\psi^2 + J_{32}\psi^4 + J_{42}\psi^6 + J_{52}\psi^8) \\
\bar{P}_{0,4} &= x^5(\bar{F}_{12} + \bar{F}_{22}\psi^2 + \bar{F}_{32}\psi^4) \\
\bar{P}_{1,4} &= x^6(\bar{G}_{12} + \bar{G}_{22}\psi^2 + \bar{G}_{32}\psi^4 + \bar{G}_{42}\psi^6) \\
\bar{P}_{2,4} &= x^7(\bar{H}_{12} + \bar{H}_{22}\psi^2 + \bar{H}_{32}\psi^4 + \bar{H}_{42}\psi^6) \\
\bar{P}_{3,4} &= x^8(\bar{J}_{12} + \bar{J}_{22}\psi^2 + \bar{J}_{32}\psi^4 + \bar{J}_{42}\psi^6 + \bar{J}_{52}\psi^8) \\
\bar{\bar{P}}_{0,4} &= x^7(\bar{\bar{H}}_{12} + \bar{\bar{H}}_{22}\psi^2 + \bar{\bar{H}}_{32}\psi^4 + \bar{\bar{H}}_{42}\psi^6) \\
\bar{\bar{P}}_{1,4} &= x^8(\bar{\bar{J}}_{12} + \bar{\bar{J}}_{22}\psi^2 + \bar{\bar{J}}_{32}\psi^4 + \bar{\bar{J}}_{42}\psi^6 + \bar{\bar{J}}_{52}\psi^8)
\end{aligned}
\right\} \qquad (105)
$$

$$
\left.
\begin{aligned}
\text{(vi) } j = 5 : P_{0,5} &= x^4(E_{13} + E_{23}\psi^2 + E_{33}\psi^4) \\
P_{1,5} &= x^5(F_{13} + F_{23}\psi^2 + F_{33}\psi^4) \\
P_{2,5} &= x^6(G_{13} + G_{23}\psi^2 + G_{33}\psi^4 + G_{43}\psi^6) \\
P_{3,5} &= x^7(H_{13} + H_{23}\psi^2 + H_{33}\psi^4 + H_{43}\psi^6) \\
P_{4,5} &= x^8(J_{13} + J_{23}\psi^2 + J_{33}\psi^4 + J_{43}\psi^6 + J_{53}\psi^8) \\
P_{5,5} &= x^9(K_{13} + K_{23}\psi^2 + K_{33}\psi^4 + K_{43}\psi^6 + K_{53}\psi^8) \\
\bar{P}_{0,5} &= x^6(\bar{G}_{13} + \bar{G}_{23}\psi^2 + \bar{G}_{33}\psi^4 + \bar{G}_{43}\psi^6) \\
\bar{P}_{1,5} &= x^7(\bar{H}_{13} + \bar{H}_{23}\psi^2 + \bar{H}_{33}\psi^4 + \bar{H}_{43}\psi^6) \\
\bar{P}_{2,5} &= x^8(\bar{J}_{13} + \bar{J}_{23}\psi^2 + \bar{J}_{33}\psi^4 + \bar{J}_{43}\psi^6 + \bar{J}_{53}\psi^8) \\
\bar{P}_{3,5} &= x^9(\bar{K}_{13} + \bar{K}_{23}\psi^2 + \bar{K}_{33}\psi^4 + \bar{K}_{43}\psi^6 + \bar{K}_{53}\psi^8) \\
\bar{\bar{P}}_{0,5} &= x^8(\bar{\bar{J}}_{13} + \bar{\bar{J}}_{23}\psi^2 + \bar{\bar{J}}_{33}\psi^4 + \bar{\bar{J}}_{43}\psi^6 + \bar{\bar{J}}_{53}\psi^8) \\
\bar{\bar{P}}_{1,5} &= x^9(\bar{\bar{K}}_{13} + \bar{\bar{K}}_{23}\psi^2 + \bar{\bar{K}}_{33}\psi^4 + \bar{\bar{K}}_{43}\psi^6 + \bar{\bar{K}}_{53}\psi^8)
\end{aligned}
\right\} \qquad (106)
$$

$$
\left.
\begin{aligned}
\text{(vii) } j = 6 : P_{0,6} &= x^3(C_{11} + C_{21}\psi^2) \\
P_{1,6} &= x^4(E_{11} + E_{21}\psi^2 + E_{31}\psi^4) \\
P_{2,6} &= x^5(F_{11} + F_{21}\psi^2 + F_{31}\psi^4) \\
P_{3,6} &= x^6(G_{11} + G_{21}\psi^2 + G_{31}\psi^4 + G_{41}\psi^6) \\
P_{4,6} &= x^7(H_{11} + H_{21}\psi^2 + H_{31}\psi^4 + H_{41}\psi^6) \\
P_{5,6} &= x^8(J_{11} + J_{21}\psi^2 + J_{31}\psi^4 + J_{41}\psi^6 + J_{51}\psi^8) \\
\bar{P}_{0,6} &= x^5(\bar{F}_{11} + \bar{F}_{21}\psi^2 + \bar{F}_{31}\psi^4) \\
\bar{P}_{1,6} &= x^6(\bar{G}_{11} + \bar{G}_{21}\psi^2 + \bar{G}_{31}\psi^4) \\
\bar{P}_{2,6} &= x^7(\bar{H}_{11} + \bar{H}_{21}\psi^2 + \bar{H}_{31}\psi^4 + \bar{H}_{41}\psi^6) \\
\bar{P}_{3,6} &= x^8(\bar{J}_{11} + \bar{J}_{21}\psi^2 + \bar{J}_{31}\psi^4 + \bar{J}_{41}\psi^6 + \bar{J}_{51}\psi^8) \\
\bar{\bar{P}}_{0,6} &= x^7(\bar{\bar{H}}_{11} + \bar{\bar{H}}_{21}\psi^2 + \bar{\bar{H}}_{31}\psi^4 + \bar{\bar{H}}_{41}\psi^6) \\
\bar{\bar{P}}_{1,6} &= x^8(\bar{\bar{J}}_{11} + \bar{\bar{J}}_{21}\psi^2 + \bar{\bar{J}}_{31}\psi^4 + \bar{\bar{J}}_{41}\psi^6 + \bar{\bar{J}}_{51}\psi^8)
\end{aligned}
\right\} \qquad (107)
$$

$$
\left.\begin{aligned}
\text{(viii) } j = 7 : P_{0,7} &= x^4(E_{12}+E_{22}\psi^2+E_{32}\psi^4)\\
P_{1,7} &= x^5(F_{12}+F_{22}\psi^2+F_{32}\psi^4)\\
P_{2,7} &= x^6(G_{12}+G_{22}\psi^2+G_{32}\psi^4+G_{42}\psi^6)\\
P_{3,7} &= x^7(H_{12}+H_{22}\psi^2+H_{32}\psi^4+H_{42}\psi^6)\\
P_{4,7} &= x^8(J_{12}+J_{22}\psi^2+J_{32}\psi^4+J_{42}\psi^6+J_{52}\psi^8)\\
P_{5,7} &= x^9(K_{12}+K_{22}\psi^2+K_{32}\psi^4+K_{42}\psi^6+K_{52}\psi^8)\\
\bar{P}_{0,7} &= x^6(\bar{G}_{12}+\bar{G}_{22}\psi^2+\bar{G}_{32}\psi^4+\bar{G}_{42}\psi^6)\\
\bar{P}_{1,7} &= x^7(\bar{H}_{12}+\bar{H}_{22}\psi^2+\bar{H}_{32}\psi^4+\bar{H}_{42}\psi^6)\\
\bar{P}_{2,7} &= x_8(\bar{J}_{12}+\bar{J}_{22}\psi^2+\bar{J}_{32}\psi^4+\bar{J}_{42}\psi^6+\bar{J}_{52}\psi^8)\\
\bar{P}_{3,7} &= x^9(\bar{K}_{12}+\bar{K}_{22}\psi^2+\bar{K}_{32}\psi^4+\bar{K}_{42}\psi^6+\bar{K}_{52}\psi^8)\\
\bar{\bar{P}}_{0,7} &= x^8(\bar{\bar{J}}_{12}+\bar{\bar{J}}_{22}\psi^2+\bar{\bar{J}}_{32}\psi^4+\bar{\bar{J}}_{42}\psi^6+\bar{\bar{J}}_{52}\psi^8)\\
\bar{\bar{P}}_{1,7} &= x^9(\bar{\bar{K}}_{12}+\bar{\bar{K}}_{22}\psi^2+\bar{\bar{K}}_{32}\psi^4+\bar{\bar{K}}_{42}\psi^6+\bar{\bar{K}}_{52}\psi^8)
\end{aligned}\right\} \quad (108)
$$

$$
\left.\begin{aligned}
\text{(ix) } j = 8 : P_{0,8} &= x^5(F_{13}+F_{23}\psi^2+F_{33}\psi^4)\\
P_{1,8} &= x^6(G_{13}+G_{23}\psi^2+G_{33}\psi^4+G_{43}\psi^6)\\
P_{2,8} &= x^7(H_{13}+H_{23}\psi^2+H_{33}\psi^4+H_{43}\psi^6)\\
P_{3,8} &= x^8(J_{13}+J_{23}\psi^2+J_{33}\psi^4+J_{43}\psi^6+J_{53}\psi^8)\\
\bar{P}_{0,8} &= x^7(\bar{H}_{13}+\bar{H}_{23}\psi^2+\bar{H}_{33}\psi^4+\bar{H}_{43}\psi^6)\\
\bar{P}_{1,8} &= x^8(\bar{J}_{13}+\bar{J}_{23}\psi^2+\bar{J}_{33}\psi^4+\bar{J}_{43}\psi^6+\bar{J}_{53}\psi^8)
\end{aligned}\right\} \quad (109)
$$

$$
\left.\begin{aligned}
\text{(x) } j = 9 : P_{0,9} &= x^6(G_{14}+G_{24}\psi^2+G_{34}\psi^4+G_{44}\psi^6)\\
P_{1,9} &= x^7(H_{14}+H_{24}\psi^2+H_{34}\psi^4+H_{44}\psi^6)\\
P_{2,9} &= x^8(J_{14}+J_{24}\psi^2+J_{34}\psi^4+J_{44}\psi^6+J_{54}\psi^8)\\
P_{3,9} &= x^9(K_{14}+K_{24}\psi^2+K_{34}\psi^4+K_{44}\psi^6+K_{54}\psi^8)\\
\bar{P}_{0,9} &= x^8(\bar{J}_{14}+\bar{J}_{24}\psi^2+\bar{J}_{34}\psi^4+\bar{J}_{44}\psi^6+\bar{J}_{54}\psi^8)\\
\bar{P}_{1,9} &= x^9(\bar{K}_{14}+\bar{K}_{24}\psi^2+\bar{K}_{34}\psi^4+\bar{K}_{44}\psi^6+\bar{K}_{54}\psi^8)
\end{aligned}\right\} \quad (110)
$$

It is to be noticed that we have given only enough results in the last two cases to determine the potential to the third power of frequency. The potential may be determined to the fifth power of frequency in all the other cases from our results.

The simultaneous linear equations for the coefficients in the above polynomial expressions may be expressed in matrix form.

The square matrix with element $W_{n,2p}{}^{2q}$ in the $(q+1)$th row and $(p+1)$th column will be written $[W_n]$. The order of this matrix will be $(\frac{1}{2}n+1) \times (\frac{1}{2}n+1)$ or $\frac{1}{2}(n+1) \times \frac{1}{2}(n+1)$ according as n is even or odd.

The equations may then be written*, with I denoting the unit matrix of appropriate order, as:

(I) $W_{0,0}{}^0 A_0 = 1$

$W_{1,0}{}^0 A_1 = 1$

* To clarify the notation, the matrices $[W_2]$ and $[W_4]$ are written out in full.

$$[W_2]\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} W_{2,0}{}^0 & W_{2,2}{}^0 \\ W_{2,0}{}^2 & W_{2,2}{}^2 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = I$$

$$[W_3]\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = I$$

$$[W_4]\begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix} = \begin{bmatrix} W_{4,0}{}^0 & W_{4,2}{}^0 & W_{4,4}{}^0 \\ W_{4,0}{}^2 & W_{4,2}{}^2 & W_{4,4}{}^2 \\ W_{4,0}{}^4 & W_{4,2}{}^4 & W_{4,4}{}^4 \end{bmatrix} \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix} = I$$

$$[W_5]\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} = I$$

$$[W_6]\begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} = I$$

$$[W_7]\begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{bmatrix} = I$$

$$[W_8]\begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} \\ J_{51} & J_{52} & J_{53} & J_{54} & J_{55} \end{bmatrix} = I$$

$$[W_9]\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} \end{bmatrix} = I$$

Note: The last column in the J matrix and the first and last in the K matrix will not be required.

(II) $$[W_2]\begin{bmatrix}\bar{B}_{11}\\ \bar{B}_{21}\end{bmatrix} = A_0\begin{bmatrix}\overline{W}_{0,0}{}^{0}\\ \overline{W}_{0,0}{}^{2}\end{bmatrix}$$

$$[W_3]\begin{bmatrix}\bar{C}_{11}\\ \bar{C}_{21}\end{bmatrix} = A_1\begin{bmatrix}\overline{W}_{1,0}{}^{0}\\ \overline{W}_{1,0}{}^{2}\end{bmatrix}$$

$$[W_4]\begin{bmatrix}\bar{E}_{11} & \bar{E}_{12}\\ \bar{E}_{21} & \bar{E}_{22}\\ \bar{E}_{31} & \bar{E}_{32}\end{bmatrix} = \begin{bmatrix}\overline{W}_{2,0}{}^{0} & \overline{W}_{2,2}{}^{0}\\ \overline{W}_{2,0}{}^{2} & \overline{W}_{2,2}{}^{2}\\ \overline{W}_{2,0}{}^{4} & \overline{W}_{2,2}{}^{4}\end{bmatrix}\begin{bmatrix}B_{11} & B_{12}\\ B_{21} & B_{22}\end{bmatrix}$$

$$[W_5]\begin{bmatrix}\bar{F}_{11} & \bar{F}_{12}\\ \bar{F}_{21} & \bar{F}_{22}\\ \bar{F}_{31} & \bar{F}_{32}\end{bmatrix} = \begin{bmatrix}\overline{W}_{3,0}{}^{0} & \overline{W}_{3,2}{}^{0}\\ \overline{W}_{3,0}{}^{2} & \overline{W}_{3,2}{}^{2}\\ \overline{W}_{3,0}{}^{4} & \overline{W}_{3,2}{}^{4}\end{bmatrix}\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}$$

$$[W_6]\begin{bmatrix}\bar{G}_{11} & \bar{G}_{12} & \bar{G}_{13}\\ \bar{G}_{21} & \bar{G}_{22} & \bar{G}_{23}\\ \bar{G}_{31} & \bar{G}_{32} & \bar{G}_{33}\\ \bar{G}_{41} & \bar{G}_{42} & \bar{G}_{43}\end{bmatrix} = \begin{bmatrix}\overline{W}_{4,0}{}^{0} & \overline{W}_{4,2}{}^{0} & \overline{W}_{4,4}{}^{0}\\ \overline{W}_{4,0}{}^{2} & \overline{W}_{4,2}{}^{2} & \overline{W}_{4,4}{}^{2}\\ \overline{W}_{4,0}{}^{4} & \overline{W}_{4,2}{}^{4} & \overline{W}_{4,4}{}^{4}\\ \overline{W}_{4,0}{}^{6} & \overline{W}_{4,2}{}^{6} & \overline{W}_{4,4}{}^{6}\end{bmatrix}\begin{bmatrix}E_{11} & E_{12} & E_{13}\\ E_{21} & E_{22} & E_{23}\\ E_{31} & E_{32} & E_{33}\end{bmatrix}$$

$$[W_7]\begin{bmatrix}\bar{H}_{11} & \bar{H}_{12} & \bar{H}_{13}\\ \bar{H}_{21} & \bar{H}_{22} & \bar{H}_{23}\\ \bar{H}_{31} & \bar{H}_{32} & \bar{H}_{33}\\ \bar{H}_{41} & \bar{H}_{42} & \bar{H}_{43}\end{bmatrix} = \begin{bmatrix}\overline{W}_{5,0}{}^{0} & \overline{W}_{5,2}{}^{0} & \overline{W}_{5,4}{}^{0}\\ \overline{W}_{5,0}{}^{2} & \overline{W}_{5,2}{}^{2} & \overline{W}_{5,4}{}^{2}\\ \overline{W}_{5,0}{}^{4} & \overline{W}_{5,2}{}^{4} & \overline{W}_{5,4}{}^{4}\\ \overline{W}_{5,0}{}^{6} & \overline{W}_{5,2}{}^{6} & \overline{W}_{5,4}{}^{6}\end{bmatrix}\begin{bmatrix}F_{11} & F_{12} & F_{13}\\ F_{21} & F_{22} & F_{23}\\ F_{31} & F_{32} & F_{33}\end{bmatrix}$$

$$[W_8]\begin{bmatrix}\bar{J}_{11} & \bar{J}_{12} & \bar{J}_{13} & \bar{J}_{14}\\ \bar{J}_{21} & \bar{J}_{22} & \bar{J}_{23} & \bar{J}_{24}\\ \bar{J}_{31} & \bar{J}_{32} & \bar{J}_{33} & \bar{J}_{34}\\ \bar{J}_{41} & \bar{J}_{42} & \bar{J}_{43} & \bar{J}_{44}\\ \bar{J}_{51} & \bar{J}_{52} & \bar{J}_{53} & \bar{J}_{54}\end{bmatrix} = \begin{bmatrix}\overline{W}_{6,0}{}^{0} & \overline{W}_{6,2}{}^{0} & \overline{W}_{6,4}{}^{0} & \overline{W}_{6,6}{}^{0}\\ \overline{W}_{6,0}{}^{2} & \overline{W}_{6,2}{}^{2} & \overline{W}_{6,4}{}^{2} & \overline{W}_{6,6}{}^{2}\\ \overline{W}_{6,0}{}^{4} & \overline{W}_{6,2}{}^{4} & \overline{W}_{6,4}{}^{4} & \overline{W}_{6,6}{}^{4}\\ \overline{W}_{6,0}{}^{6} & \overline{W}_{6,2}{}^{6} & \overline{W}_{6,4}{}^{6} & \overline{W}_{6,6}{}^{6}\\ \overline{W}_{6,0}{}^{8} & \overline{W}_{6,2}{}^{8} & \overline{W}_{6,4}{}^{8} & \overline{W}_{6,6}{}^{8}\end{bmatrix}\begin{bmatrix}G_{11} & G_{12} & G_{13} & G_{14}\\ G_{21} & G_{22} & G_{23} & G_{24}\\ G_{31} & G_{32} & G_{33} & G_{34}\\ G_{41} & G_{42} & G_{43} & G_{44}\end{bmatrix}$$

$$[W_9]\begin{bmatrix}\bar{K}_{12} & \bar{K}_{13} & \bar{K}_{14}\\ \bar{K}_{22} & \bar{K}_{23} & \bar{K}_{24}\\ \bar{K}_{32} & \bar{K}_{33} & \bar{K}_{34}\\ \bar{K}_{42} & \bar{K}_{43} & \bar{K}_{44}\\ \bar{K}_{52} & \bar{K}_{53} & \bar{K}_{54}\end{bmatrix} = \begin{bmatrix}\overline{W}_{7,0}{}^{0} & \overline{W}_{7,2}{}^{0} & \overline{W}_{7,4}{}^{0} & \overline{W}_{7,6}{}^{0}\\ \overline{W}_{7,0}{}^{2} & \overline{W}_{7,2}{}^{2} & \overline{W}_{7,4}{}^{2} & \overline{W}_{7,6}{}^{2}\\ \overline{W}_{7,0}{}^{4} & \overline{W}_{7,2}{}^{4} & \overline{W}_{7,4}{}^{4} & \overline{W}_{7,6}{}^{4}\\ \overline{W}_{7,0}{}^{6} & \overline{W}_{7,2}{}^{6} & \overline{W}_{7,4}{}^{6} & \overline{W}_{7,6}{}^{6}\\ \overline{W}_{7,0}{}^{8} & \overline{W}_{7,2}{}^{8} & \overline{W}_{7,4}{}^{8} & \overline{W}_{7,6}{}^{8}\end{bmatrix}\begin{bmatrix}H_{12} & H_{13} & H_{14}\\ H_{22} & H_{23} & H_{24}\\ H_{32} & H_{33} & H_{34}\\ H_{42} & H_{43} & H_{44}\end{bmatrix}$$

$$
\text{(III)}\quad [W_4]\begin{bmatrix}\bar{\bar{E}}_{11}\\ \bar{\bar{E}}_{21}\\ \bar{\bar{E}}_{31}\end{bmatrix}=\begin{bmatrix}\bar{\bar{W}}_{2,0}^{0} & \bar{\bar{W}}_{2,2}^{0}\\ \bar{\bar{W}}_{2,0}^{2} & \bar{\bar{W}}_{2,2}^{2}\\ \bar{\bar{W}}_{2,0}^{4} & \bar{\bar{W}}_{2,2}^{4}\end{bmatrix}\begin{bmatrix}\bar{B}_{11}\\ \bar{B}_{21}\end{bmatrix}-\frac{1}{6}\begin{bmatrix}\bar{\bar{\bar{W}}}_{0,0}^{0}\\ \bar{\bar{\bar{W}}}_{0,0}^{2}\\ \bar{\bar{\bar{W}}}_{0,0}^{4}\end{bmatrix}A_0
$$

$$
[W_5]\begin{bmatrix}\bar{\bar{F}}_{11}\\ \bar{\bar{F}}_{21}\\ \bar{\bar{F}}_{31}\end{bmatrix}=\begin{bmatrix}\bar{\bar{W}}_{3,0}^{0} & \bar{\bar{W}}_{3,2}^{0}\\ \bar{\bar{W}}_{3,0}^{2} & \bar{\bar{W}}_{3,2}^{2}\\ \bar{\bar{W}}_{3,0}^{4} & \bar{\bar{W}}_{3,2}^{4}\end{bmatrix}\begin{bmatrix}\bar{C}_{11}\\ \bar{C}_{21}\end{bmatrix}-\frac{1}{6}\begin{bmatrix}\bar{\bar{\bar{W}}}_{1,0}^{0}\\ \bar{\bar{\bar{W}}}_{1,0}^{2}\\ \bar{\bar{\bar{W}}}_{1,0}^{4}\end{bmatrix}A_1
$$

$$
[W_6]\begin{bmatrix}\bar{\bar{G}}_{11} & \bar{\bar{G}}_{12}\\ \bar{\bar{G}}_{21} & \bar{\bar{G}}_{22}\\ \bar{\bar{G}}_{31} & \bar{\bar{G}}_{32}\\ \bar{\bar{G}}_{41} & \bar{\bar{G}}_{42}\end{bmatrix}=\begin{bmatrix}\bar{\bar{W}}_{4,0}^{0} & \bar{\bar{W}}_{4,2}^{0} & \bar{\bar{W}}_{4,4}^{0}\\ \bar{\bar{W}}_{4,0}^{2} & \bar{\bar{W}}_{4,2}^{2} & \bar{\bar{W}}_{4,4}^{2}\\ \bar{\bar{W}}_{4,0}^{4} & \bar{\bar{W}}_{4,2}^{4} & \bar{\bar{W}}_{4,4}^{4}\\ \bar{\bar{W}}_{4,0}^{6} & \bar{\bar{W}}_{4,2}^{6} & \bar{\bar{W}}_{4,4}^{6}\end{bmatrix}\begin{bmatrix}\bar{E}_{11} & \bar{E}_{12}\\ \bar{E}_{21} & \bar{E}_{22}\\ \bar{E}_{31} & \bar{E}_{32}\end{bmatrix}-\frac{1}{6}\begin{bmatrix}\bar{\bar{\bar{W}}}_{2,0}^{0} & \bar{\bar{\bar{W}}}_{2,2}^{0}\\ \bar{\bar{\bar{W}}}_{2,0}^{2} & \bar{\bar{\bar{W}}}_{2,2}^{2}\\ \bar{\bar{\bar{W}}}_{2,0}^{4} & \bar{\bar{\bar{W}}}_{2,2}^{4}\\ \bar{\bar{\bar{W}}}_{2,0}^{6} & \bar{\bar{\bar{W}}}_{2,2}^{6}\end{bmatrix}\begin{bmatrix}B_{11} & B_{12}\\ B_{21} & B_{22}\end{bmatrix}
$$

$$
[W_7]\begin{bmatrix}\bar{\bar{H}}_{11} & \bar{\bar{H}}_{12}\\ \bar{\bar{H}}_{21} & \bar{\bar{H}}_{22}\\ \bar{\bar{H}}_{31} & \bar{\bar{H}}_{32}\\ \bar{\bar{H}}_{41} & \bar{\bar{H}}_{42}\end{bmatrix}=\begin{bmatrix}\bar{\bar{W}}_{5,0}^{0} & \bar{\bar{W}}_{5,2}^{0} & \bar{\bar{W}}_{5,4}^{0}\\ \bar{\bar{W}}_{5,0}^{2} & \bar{\bar{W}}_{5,2}^{2} & \bar{\bar{W}}_{5,4}^{2}\\ \bar{\bar{W}}_{5,0}^{4} & \bar{\bar{W}}_{5,2}^{4} & \bar{\bar{W}}_{5,4}^{4}\\ \bar{\bar{W}}_{5,0}^{6} & \bar{\bar{W}}_{5,2}^{6} & \bar{\bar{W}}_{5,4}^{6}\end{bmatrix}\begin{bmatrix}\bar{F}_{11} & \bar{F}_{12}\\ \bar{F}_{21} & \bar{F}_{22}\\ \bar{F}_{31} & \bar{F}_{32}\end{bmatrix}-\frac{1}{6}\begin{bmatrix}\bar{\bar{\bar{W}}}_{3,0}^{0} & \bar{\bar{\bar{W}}}_{3,2}^{0}\\ \bar{\bar{\bar{W}}}_{3,0}^{2} & \bar{\bar{\bar{W}}}_{3,2}^{2}\\ \bar{\bar{\bar{W}}}_{3,0}^{4} & \bar{\bar{\bar{W}}}_{3,2}^{4}\\ \bar{\bar{\bar{W}}}_{3\ 0}^{6} & \bar{\bar{\bar{W}}}_{3,2}^{6}\end{bmatrix}\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}
$$

$$
[W_8]\begin{bmatrix}\bar{\bar{J}}_{11} & \bar{\bar{J}}_{12} & \bar{\bar{J}}_{13}\\ \bar{\bar{J}}_{21} & \bar{\bar{J}}_{22} & \bar{\bar{J}}_{23}\\ \bar{\bar{J}}_{31} & \bar{\bar{J}}_{32} & \bar{\bar{J}}_{33}\\ \bar{\bar{J}}_{41} & \bar{\bar{J}}_{42} & \bar{\bar{J}}_{43}\\ \bar{\bar{J}}_{51} & \bar{\bar{J}}_{52} & \bar{\bar{J}}_{53}\end{bmatrix}=\begin{bmatrix}\bar{\bar{W}}_{6,0}^{0} & \bar{\bar{W}}_{6,2}^{0} & \bar{\bar{W}}_{6,4}^{0} & \bar{\bar{W}}_{6,6}^{0}\\ \bar{\bar{W}}_{6,0}^{2} & \bar{\bar{W}}_{6,2}^{2} & \bar{\bar{W}}_{6,4}^{2} & \bar{\bar{W}}_{6,6}^{2}\\ \bar{\bar{W}}_{6,0}^{4} & \bar{\bar{W}}_{6,2}^{4} & \bar{\bar{W}}_{6,4}^{4} & \bar{\bar{W}}_{6,6}^{4}\\ \bar{\bar{W}}_{6,0}^{6} & \bar{\bar{W}}_{6,2}^{6} & \bar{\bar{W}}_{6,4}^{6} & \bar{\bar{W}}_{6,6}^{6}\\ \bar{\bar{W}}_{6,0}^{8} & \bar{\bar{W}}_{6,2}^{8} & \bar{\bar{W}}_{6,4}^{8} & \bar{\bar{W}}_{6,6}^{8}\end{bmatrix}\begin{bmatrix}\bar{G}_{11} & \bar{G}_{12} & \bar{G}_{13}\\ \bar{G}_{21} & \bar{G}_{22} & \bar{G}_{23}\\ \bar{G}_{31} & \bar{G}_{32} & \bar{G}_{33}\\ \bar{G}_{41} & \bar{G}_{42} & \bar{G}_{43}\end{bmatrix}-\frac{1}{6}\begin{bmatrix}\bar{\bar{\bar{W}}}_{4,0}^{0} & \bar{\bar{\bar{W}}}_{4,2}^{0} & \bar{\bar{\bar{W}}}_{4,4}^{0}\\ \bar{\bar{\bar{W}}}_{4,0}^{2} & \bar{\bar{\bar{W}}}_{4,2}^{2} & \bar{\bar{\bar{W}}}_{4,4}^{2}\\ \bar{\bar{\bar{W}}}_{4,0}^{4} & \bar{\bar{\bar{W}}}_{4,2}^{4} & \bar{\bar{\bar{W}}}_{4,4}^{4}\\ \bar{\bar{\bar{W}}}_{4,0}^{6} & \bar{\bar{\bar{W}}}_{4,2}^{6} & \bar{\bar{\bar{W}}}_{4,4}^{6}\\ \bar{\bar{\bar{W}}}_{4,0}^{8} & \bar{\bar{\bar{W}}}_{4,2}^{8} & \bar{\bar{\bar{W}}}_{4,4}^{8}\end{bmatrix}\begin{bmatrix}E_{11} & E_{12} & E_{13}\\ E_{21} & E_{22} & E_{23}\\ E_{31} & E_{32} & E_{33}\end{bmatrix}
$$

$$
[W_9]\begin{bmatrix}\bar{\bar{K}}_{12} & \bar{\bar{K}}_{13}\\ \bar{\bar{K}}_{22} & \bar{\bar{K}}_{23}\\ \bar{\bar{K}}_{32} & \bar{\bar{K}}_{33}\\ \bar{\bar{K}}_{42} & \bar{\bar{K}}_{43}\\ \bar{\bar{K}}_{52} & \bar{\bar{K}}_{53}\end{bmatrix}=\begin{bmatrix}\bar{\bar{W}}_{7,0}^{0} & \bar{\bar{W}}_{7,2}^{0} & \bar{\bar{W}}_{7,4}^{0} & \bar{\bar{W}}_{7,6}^{0}\\ \bar{\bar{W}}_{7,0}^{2} & \bar{\bar{W}}_{7,2}^{2} & \bar{\bar{W}}_{7,4}^{2} & \bar{\bar{W}}_{7,6}^{2}\\ \bar{\bar{W}}_{7,0}^{4} & \bar{\bar{W}}_{7,2}^{4} & \bar{\bar{W}}_{7,4}^{4} & \bar{\bar{W}}_{7,6}^{4}\\ \bar{\bar{W}}_{7,0}^{6} & \bar{\bar{W}}_{7,2}^{6} & \bar{\bar{W}}_{7,4}^{6} & \bar{\bar{W}}_{7,6}^{6}\\ \bar{\bar{W}}_{7,0}^{8} & \bar{\bar{W}}_{7,2}^{8} & \bar{\bar{W}}_{7,4}^{8} & \bar{\bar{W}}_{7,6}^{8}\end{bmatrix}\begin{bmatrix}\bar{H}_{12} & \bar{H}_{13}\\ \bar{H}_{22} & \bar{H}_{23}\\ \bar{H}_{32} & \bar{H}_{33}\\ \bar{H}_{42} & \bar{H}_{43}\end{bmatrix}-\frac{1}{6}\begin{bmatrix}\bar{\bar{\bar{W}}}_{5,0}^{0} & \bar{\bar{\bar{W}}}_{5,2}^{0} & \bar{\bar{\bar{W}}}_{5,4}^{0}\\ \bar{\bar{\bar{W}}}_{5,0}^{2} & \bar{\bar{\bar{W}}}_{5,2}^{2} & \bar{\bar{\bar{W}}}_{5,4}^{2}\\ \bar{\bar{\bar{W}}}_{5,0}^{4} & \bar{\bar{\bar{W}}}_{5,2}^{4} & \bar{\bar{\bar{W}}}_{5,4}^{4}\\ \bar{\bar{\bar{W}}}_{5,0}^{6} & \bar{\bar{\bar{W}}}_{5,2}^{6} & \bar{\bar{\bar{W}}}_{5,4}^{6}\\ \bar{\bar{\bar{W}}}_{5.0}^{8} & \bar{\bar{\bar{W}}}_{5,2}^{8} & \bar{\bar{\bar{W}}}_{5,4}^{8}\end{bmatrix}\begin{bmatrix}F_{12} & F_{13}\\ F_{22} & F_{23}\\ F_{32} & F_{33}\end{bmatrix}
$$

The expressions for $W_{n,m}^P$, $\overline{W}_{n,m}^P$, $\overline{\overline{W}}_{n,m}^P$, will now be given:

$$W_{0,0}^0 = E'$$

$$W_{1,0}^0 = \frac{1}{(1-\rho_0^2)}\left[\rho_0^2 K' + (1-2\rho_0^2)E'\right]$$

$$W_{2,0}^0 = \frac{1}{2(1-\rho_0^2)^2}\left[(5\rho_0^2-3\rho_0^4)K' + (2-10\rho_0^2+6\rho_0^4)E'\right]$$

$$W_{2,0}^2 = \frac{1}{2(1-\rho_0^2)^2}\left[2\rho_0^2 K' - (1+\rho_0^2)E'\right]$$

$$W_{2,2}^0 = \rho_0^2 W_{2,0}^2$$

$$W_{2,2}^2 = \frac{1}{2(1-\rho_0^2)^2}\left[(5\rho_0^4-3\rho_0^2)K' + (6-10\rho_0^2+2\rho_0^4)E'\right]$$

$$W_{3,0}^0 = \frac{1}{6(1-\rho_0^2)^3}\left[(27\rho_0^2-31\rho_0^4+12\rho_0^6)K' + (6-55\rho_0^2+65\rho_0^4-24\rho_0^6)E'\right]$$

$$W_{3,0}^2 = \frac{1}{2(1-\rho_0^2)^3}\left[(9\rho_0^2-\rho_0^4)K' - (3+7\rho_0^2-2\rho_0^4)E'\right]$$

$$W_{3,2}^0 = \frac{1}{3}\rho_0^2 W_{3,0}^2$$

$$W_{3,2}^2 = \frac{1}{2(1-\rho_0^2)^3}\left[(2\rho_0^6+9\rho_0^4-3\rho_0^2)K' + (6-15\rho_0^2+5\rho_0^4-4\rho_0^6)E'\right]$$

$$W_{4,0}^0 = \frac{1}{8(1-\rho_0^2)^4}\left[(56\rho_0^2-92\rho_0^4+72\rho_0^6-20\rho_0^8)K' + (8-117\rho_0^2+202\rho_0^4-149\rho_0^6+40\rho_0^8)E'\right]$$

$$W_{4,0}^2 = \frac{3}{4(1-\rho_0^2)^4}\left[(17\rho_0^2-2\rho_0^4+\rho_0^6)K' - (4+18\rho_0^2-8\rho_0^4+2\rho_0^6)E'\right]$$

$$W_{4,0}^4 = \frac{1}{8(1-\rho_0^2)^4}\left[(8\rho_0^2+8\rho_0^4)K' - (1+14\rho_0^2+\rho_0^4)E'\right]$$

$$W_{4,2}^0 = \frac{1}{6}\rho_0^2 W_{4,0}^2$$

$$W_{4,2}^2 = \frac{3}{4(1-\rho_0^2)^4}\left[-(2\rho_0^2-10\rho_0^4-10\rho_0^6+2\rho_0^8)K' + (4-13\rho_0^2+2\rho_0^4-13\rho_0^6+4\rho_0^8)E'\right]$$

$$W_{4,2}^4 = \frac{1}{8(1-\rho_0^2)^4}\left[(\rho_0^2-2\rho_0^4+17\rho_0^6)K' - (2-8\rho_0^2+18\rho_0^4+4\rho_0^6)E'\right]$$

$$W_{4,4}^0 = \rho_0^4 W_{4,0}^4$$

$$W_{4,4}^2 = 6\rho_0^2 W_{4,2}^4$$

$$W_{4,4}^4 = \frac{1}{8(1-\rho_0^2)^4}\left[-(20\rho_0^2-72\rho_0^4+92\rho_0^6-56\rho_0^8)K' + (40-149\rho_0^2+202\rho_0^4-117\rho_0^6+8\rho_0^8)E'\right]$$

$$W_{5,0}{}^0 = \frac{1}{40(1-\rho_0{}^2)^5}\,[(400\rho_0{}^2-833\rho_0{}^4+994\rho_0{}^6-553\rho_0{}^8+120\rho_0{}^{10})K'$$
$$+(40-859\rho_0{}^2+1{,}910\rho_0{}^4-2{,}115\rho_0{}^6+1{,}136\rho_0{}^8-240\rho_0{}^{10})E']$$

$$W_{5,0}{}^2 = \frac{1}{4(1-\rho_0{}^2)^5}\,[(115\rho_0{}^2-2\rho_0{}^4+19\rho_0{}^6-4\rho_0{}^8)K' - (20+151\rho_0{}^2-74\rho_0{}^4+39\rho_0{}^6-8\rho_0{}^8)E']$$

$$W_{5,0}{}^4 = \frac{1}{8(1-\rho_0{}^2)^5}\,[(55\rho_0{}^2+74\rho_0{}^4-\rho_0{}^6)K' - (5+108\rho_0{}^2+17\rho_0{}^4-2\rho_0{}^6)E']$$

$$W_{5,2}{}^0 = \frac{1}{10}\,\rho_0{}^2W_{5,0}{}^2$$

$$W_{5,2}{}^2 = \frac{1}{4(1-\rho_0{}^2)^5}\,[-(6\rho_0{}^2-49\rho_0{}^4-112\rho_0{}^6+35\rho_0{}^8-8\rho_0{}^{10})K'$$
$$+(12-47\rho_0{}^2-22\rho_0{}^4-127\rho_0{}^6+72\rho_0{}^8-16\rho_0{}^{10})E']$$

$$W_{5,2}{}^4 = \frac{1}{8(1-\rho_0{}^2)^5}\,[(3\rho_0{}^2-2\rho_0{}^4+131\rho_0{}^6-4\rho_0{}^8)K' - (6-31\rho_0{}^2+108\rho_0{}^4+53\rho_0{}^6-8\rho_0{}^8)E']$$

$$W_{5,4}{}^0 = \frac{1}{5}\,\rho_0{}^4W_{5,0}{}^4$$

$$W_{5,4}{}^2 = \frac{1}{4(1-\rho_0{}^2)^5}\,[(3\rho_0{}^4-2\rho_0{}^6+131\rho_0{}^8-4\rho_0{}^{10})K' - (6\rho_0{}^2-31\rho_0{}^4+108\rho_0{}^6+53\rho_0{}^8-8\rho_0{}^{10})E']$$

$$W_{5,4}{}^4 = \frac{1}{8(1-\rho_0{}^2)^5}\,[-(20\rho_0{}^2-91\rho_0{}^4+154\rho_0{}^6-203\rho_0{}^8-8\rho_0{}^{10})K'$$
$$+(40-187\rho_0{}^2+342\rho_0{}^4-323\rho_0{}^6+16\rho_0{}^8-16\rho_0{}^{10})E']$$

$$W_{6,0}{}^0 = \frac{1}{240(1-\rho_0{}^2)^6}\,[(3{,}240\rho_0{}^2-7{,}989\rho_0{}^4+13{,}005\rho_0{}^6-10{,}851\rho_0{}^8+4{,}715\rho_0{}^{10}-840\rho_0{}^{12})K'$$
$$+(240-7{,}152\rho_0{}^2+19{,}074\rho_0{}^4-28{,}284\rho_0{}^6+22{,}802\rho_0{}^8-9{,}640\rho_0{}^{10}+1{,}680\rho_0{}^{12})E']$$

$$W_{6,0}{}^2 = \frac{1}{16(1-\rho_0{}^2)^6}\,[(900\rho_0{}^2+174\rho_0{}^4+300\rho_0{}^6-114\rho_0{}^8+20\rho_0{}^{10})K'$$
$$+(-120-1{,}383\rho_0{}^2+591\rho_0{}^4-561\rho_0{}^6+233\rho_0{}^8-40\rho_0{}^{10})E']$$

$$W_{6,0}{}^4 = \frac{1}{16(1-\rho_0{}^2)^6}\,[(435\rho_0{}^2+797\rho_0{}^4+45\rho_0{}^6+3\rho_0{}^8)K' + (-30-934\rho_0{}^2-352\rho_0{}^4+42\rho_0{}^6-6\rho_0{}^8)E']$$

$$W_{6,0}{}^6 = \frac{1}{240(1-\rho_0{}^2)^6}\,[(270\rho_0{}^2+740\rho_0{}^4+270\rho_0{}^6)K' + (-15-625\rho_0{}^2-625\rho_0{}^4-15\rho_0{}^6)E']$$

$$W_{6,2}{}^0 = \frac{1}{15}\,\rho_0{}^2W_{6,0}{}^2$$

$$W_{6,2}{}^2 = \frac{1}{16(1-\rho_0{}^2)^6}\,[(-24\rho_0{}^2+315\rho_0{}^4+1{,}245\rho_0{}^6-435\rho_0{}^8+219\rho_0{}^{10}-40\rho_0{}^{12})K'$$
$$+(48-216\rho_0{}^2-366\rho_0{}^4-1{,}404\rho_0{}^6+1{,}026\rho_0{}^8-448\rho_0{}^{10}+80\rho_0{}^{12})E']$$

$$W_{6,2}^{4} = \frac{1}{16(1-\rho_0^2)^6}[(12\rho_0^2+30\rho_0^4+1{,}196\rho_0^6+30\rho_0^8+12\rho_0^{10})K'$$
$$+(-24+153\rho_0^2-769\rho_0^4-769\rho_0^6+153\rho_0^8-24\rho_0^{10})E']$$

$$W_{6,2}^{6} = \frac{1}{240(1-\rho_0^2)^6}[(3\rho_0^2+45\rho_0^4+797\rho_0^6+435\rho_0^8)K' + (-6+42\rho_0^2-352\rho_0^4-934\rho_0^6-30\rho_0^8)E']$$

$$W_{6,4}^{0} = \frac{1}{15}\rho_0^4 W_{6,0}^{4}$$

$$W_{6,4}^{2} = \rho_0^2 W_{6,2}^{4}$$

$$W_{6,4}^{4} = \frac{1}{16(1-\rho_0^2)^6}[(-40\rho_0^2+219\rho_0^4-435\rho_0^6+1{,}245\rho_0^8+315\rho_0^{10}-24\rho_0^{12})K'$$
$$+(80-448\rho_0^2+1{,}026\rho_0^4-1{,}404\rho_0^6-366\rho_0^8-216\rho_0^{10}+48\rho_0^{12})E']$$

$$W_{6,4}^{6} = \frac{1}{240(1-\rho_0^2)^6}[(20\rho_0^2-114\rho_0^4+300\rho_0^6+174\rho_0^8+900\rho_0^{10})K'$$
$$+(-40+233\rho_0^2-561\rho_0^4+591\rho_0^6-1{,}383\rho_0^8-120\rho_0^{10})E']$$

$$W_{6,6}^{0} = \rho_0^6 W_{6,0}^{6}$$

$$W_{6,6}^{2} = 15\rho_0^4 W_{6,2}^{6}$$

$$W_{6,6}^{4} = 15\rho_0^2 W_{6,4}^{6}$$

$$W_{6,6}^{6} = \frac{1}{240(1-\rho_0^2)^6}[(-840\rho_0^2+4{,}715\rho_0^4-10{,}851\rho_0^6+13{,}005\rho_0^8-7{,}989\rho_0^{10}+3{,}240\rho_0^{12})K'$$
$$+(1{,}680-9{,}640\rho_0^2+22{,}802\rho_0^4-28{,}284\rho_0^6+19{,}074\rho_0^8-7{,}152\rho_0^{10}+240\rho_0^{12})E']$$

$$W_{7,0}^{0} = \frac{1}{1{,}680(1-\rho_0^2)^7}[(29{,}400\rho_0^2-82{,}071\rho_0^4+172{,}161\rho_0^6-191{,}049\rho_0^8+124{,}659\rho_0^{10}-44{,}460\rho_0^{12}$$
$$+6{,}720\rho_0^{14})K' + (1{,}680-66{,}648\rho_0^2+203{,}499\rho_0^4-380{,}697\rho_0^6+409{,}449\rho_0^8-259{,}803\rho_0^{10}$$
$$+90{,}600\rho_0^{12}-13{,}440\rho_0^{14})E']$$

$$W_{7,0}^{2} = \frac{1}{80(1-\rho_0^2)^7}[(7{,}980\rho_0^2+4{,}131\rho_0^4+4{,}839\rho_0^6-2{,}271\rho_0^8+801\rho_0^{10}-120\rho_0^{12})K'$$
$$+(-840-13{,}827\rho_0^2+3{,}561\rho_0^4-7{,}593\rho_0^6+4{,}731\rho_0^8-1{,}632\rho_0^{10}+240\rho_0^{12})E']$$

$$W_{7,0}^{4} = \frac{1}{48(1-\rho_0^2)^7}[(3{,}885\rho_0^2+9{,}573\rho_0^4+1{,}827\rho_0^6+87\rho_0^8-12\rho_0^{10})K'$$
$$+(-210-8{,}961\rho_0^2-6{,}537\rho_0^4+501\rho_0^6-177\rho_0^8+24\rho_0^{10})E']$$

$$W_{7,0}^{6} = \frac{1}{240(1-\rho_0^2)^7}[(2{,}415\rho_0^2+8{,}595\rho_0^4+4{,}365\rho_0^6-15\rho_0^8)K'$$
$$+(-105-5{,}955\rho_0^2-8{,}865\rho_0^4-465\rho_0^6+30\rho_0^8)E']$$

$$W_{7,2}^{0} = \frac{1}{21}\rho_0^2 W_{7,0}^{2}$$

$$W_{7,2}{}^{2} = \frac{1}{80(1-\rho_0{}^2)^7}\,[(-120\rho_0{}^2+2{,}475\rho_0{}^4+14{,}589\rho_0{}^6-4{,}569\rho_0{}^8+4{,}311\rho_0{}^{10}-1{,}566\rho_0{}^{12}+240\rho_0{}^{14})K'$$
$$+(240-1{,}200\rho_0{}^2-4{,}473\rho_0{}^4-17{,}421\rho_0{}^6+13{,}773\rho_0{}^8-8{,}991\rho_0{}^{10}+3{,}192\rho_0{}^{12}-480\rho_0{}^{14})E']$$

$$W_{7,2}{}^{4} = \frac{1}{48(1-\rho_0{}^2)^7}\,[(60\rho_0{}^2+495\rho_0{}^4+12{,}507\rho_0{}^6+2{,}013\rho_0{}^8+333\rho_0{}^{10}-48\rho_0{}^{12})K'$$
$$+(-120+915\rho_0{}^2-6{,}339\rho_0{}^4-11{,}193\rho_0{}^6+1{,}959\rho_0{}^8-678\rho_0{}^{10}+96\rho_0{}^{12})E']$$

$$W_{7,2}{}^{6} = \frac{1}{240(1-\rho_0{}^2)^7}\,[(15\rho_0{}^2+465\rho_0{}^4+8{,}595\rho_0{}^6+6{,}315\rho_0{}^8-30\rho_0{}^{10})K'$$
$$+(-30+255\rho_0{}^2-3{,}045\rho_0{}^4-11{,}775\rho_0{}^6-825\rho_0{}^8+60\rho_0{}^{10})E']$$

$$W_{7,4}{}^{0} = \frac{1}{35}\,\rho_0{}^4 W_{7,0}{}^{4}$$

$$W_{7,4}{}^{2} = \frac{3}{5}\,\rho_0{}^2 W_{7,2}{}^{4}$$

$$W_{7,4}{}^{4} = \frac{1}{48(1-\rho_0{}^2)^7}\,[(-120\rho_0{}^2+765\rho_0{}^4-1{,}575\rho_0{}^6+10{,}551\rho_0{}^8+6{,}219\rho_0{}^{10}-576\rho_0{}^{12}+96\rho_0{}^{14})K'$$
$$+(240-1{,}560\rho_0{}^2+4{,}275\rho_0{}^4-8{,}097\rho_0{}^6-8{,}079\rho_0{}^8-3{,}123\rho_0{}^{10}+1{,}176\rho_0{}^{12}-192\rho_0{}^{14})E']$$

$$W_{7,4}{}^{6} = \frac{1}{240(1-\rho_0{}^2)^7}\,[(60\rho_0{}^2-405\rho_0{}^4+1{,}455\rho_0{}^6+3{,}705\rho_0{}^8+10{,}665\rho_0{}^{10}-120\rho_0{}^{12})K'$$
$$+(-120+825\rho_0{}^2-2{,}415\rho_0{}^4+2{,}775\rho_0{}^6-14{,}205\rho_0{}^8-2{,}460\rho_0{}^{10}+240\rho_0{}^{12})E']$$

$$W_{7,6}{}^{0} = \frac{1}{7}\,\rho_0{}^6 W_{7,0}{}^{6}$$

$$W_{7,6}{}^{2} = 3\rho_0{}^4 W_{7,2}{}^{6}$$

$$W_{7,6}{}^{4} = 5\rho_0{}^2 W_{7,4}{}^{6}$$

$$W_{7,6}{}^{6} = \frac{1}{240(1-\rho_0{}^2)^7}\,[(-840\rho_0{}^2+5{,}535\rho_0{}^4-15{,}435\rho_0{}^6+23{,}655\rho_0{}^8-16{,}785\rho_0{}^{10}+18{,}990\rho_0{}^{12}$$
$$+240\rho_0{}^{14})K' + (1{,}680-11{,}280\rho_0{}^2+32{,}175\rho_0{}^4-50{,}325\rho_0{}^6+45{,}525\rho_0{}^8-32{,}535\rho_0{}^{10}$$
$$-120\rho_0{}^{12}-480\rho_0{}^{14})E']$$

$$W_{8,0}{}^{0} = \frac{1}{13{,}440(1-\rho_0{}^2)^8}\,[(295{,}680\rho_0{}^2-904{,}500\rho_0{}^4+2{,}367{,}540\rho_0{}^6-3{,}265{,}320\rho_0{}^8+2{,}844{,}360\rho_0{}^{10}$$
$$-1{,}522{,}980\rho_0{}^{12}+460{,}740\rho_0{}^{14}-60{,}480\rho_0{}^{16})K' + (13{,}440-687{,}480\rho_0{}^2+2{,}324{,}115\rho_0{}^4$$
$$-5{,}298{,}420\rho_0{}^6+7{,}122{,}930\rho_0{}^8-6{,}029{,}460\rho_0{}^{10}+3{,}155{,}475\rho_0{}^{12}-936{,}600\rho_0{}^{14}$$
$$+120{,}960\rho_0{}^{16})E']$$

$$W_{8,0}{}^{2} = \frac{1}{480(1-\rho_0{}^2)^8}\,[(78{,}960\rho_0{}^2+75{,}645\rho_0{}^4+83{,}700\rho_0{}^6-39{,}090\rho_0{}^8+21{,}420\rho_0{}^{10}-6{,}435\rho_0{}^{12}$$
$$+840\rho_0{}^{14})K' + (-6{,}720-150{,}450\rho_0{}^2-2{,}880\rho_0{}^4-107{,}580\rho_0{}^6+85{,}560\rho_0{}^8-44{,}370\rho_0{}^{10}$$
$$+13{,}080\rho_0{}^{12}-1{,}680\rho_0{}^{14})E']$$

$$W_{8,0}{}^4 = \frac{1}{192(1-\rho_0{}^2)^8}\,[(38{,}640\rho_0{}^2+125{,}340\rho_0{}^4+47{,}760\rho_0{}^6+3{,}720\rho_0{}^8-480\rho_0{}^{10}+60\rho_0{}^{12})K'$$
$$+(-1{,}680-94{,}545\rho_0{}^2-114{,}900\rho_0{}^4-1{,}350\rho_0{}^6-3{,}420\rho_0{}^8+975\rho_0{}^{10}-120\rho_0{}^{12})E']$$

$$W_{8,0}{}^6 = \frac{1}{480(1-\rho_0{}^2)^8}\,[(24{,}045\rho_0{}^2+109{,}260\rho_0{}^4+78{,}030\rho_0{}^6+3{,}660\rho_0{}^8+45\rho_0{}^{10})K'$$
$$+(-840-62{,}490\rho_0{}^2-132{,}840\rho_0{}^4-19{,}740\rho_0{}^6+960\rho_0{}^8-90\rho_0{}^{10})E']$$

$$W_{8,0}{}^8 = \frac{1}{13{,}440(1-\rho_0{}^2)^8}\,[(16{,}800\rho_0{}^2+90{,}720\rho_0{}^4+90{,}720\rho_0{}^6+16{,}800\rho_0{}^8)K'$$
$$+(-525-44{,}940\rho_0{}^2-124{,}110\rho_0{}^4-44{,}940\rho_0{}^6-525\rho_0{}^8)E']$$

$$W_{8,2}{}^0 = \frac{1}{28}\,\rho_0{}^2 W_{8,0}{}^2$$

$$W_{8,2}{}^2 = \frac{1}{480(1-\rho_0{}^2)^8}\,[(-720\rho_0{}^2+22{,}710\rho_0{}^4+182{,}970\rho_0{}^6-37{,}620\rho_0{}^8+78{,}180\rho_0{}^{10}-41{,}490\rho_0{}^{12}$$
$$+12{,}690\rho_0{}^{14}-1{,}680\rho_0{}^{16})K'+(1{,}440-7{,}800\rho_0{}^2-53{,}205\rho_0{}^4-237{,}060\rho_0{}^6+179{,}970\rho_0{}^8$$
$$-161{,}940\rho_0{}^{10}+85{,}995\rho_0{}^{12}-25{,}800\rho_0{}^{14}+3{,}360\rho_0{}^{16})E']$$

$$W_{8,2}{}^4 = \frac{1}{192(1-\rho_0{}^2)^8}\,[(360\rho_0{}^2+6{,}285\rho_0{}^4+146{,}220\rho_0{}^6+54{,}510\rho_0{}^8+9{,}300\rho_0{}^{10}-1{,}875\rho_0{}^{12}$$
$$+240\rho_0{}^{14})K'+(-720+6{,}420\rho_0{}^2-59{,}310\rho_0{}^4-165{,}840\rho_0{}^6+14{,}220\rho_0{}^8-13{,}140\rho_0{}^{10}$$
$$+3{,}810\rho_0{}^{12}-480\rho_0{}^{14})E']$$

$$W_{8,2}{}^6 = \frac{1}{480(1-\rho_0{}^2)^8}\,[(90\rho_0{}^2+5{,}010\rho_0{}^4+102{,}420\rho_0{}^6+102{,}420\rho_0{}^8+5{,}010\rho_0{}^{10}+90\rho_0{}^{12})K'$$
$$+(-180+1{,}815\rho_0{}^2-29{,}640\rho_0{}^4-159{,}030\rho_0{}^6-29{,}640\rho_0{}^8+1{,}815\rho_0{}^{10}-180\rho_0{}^{12})E']$$

$$W_{8,2}{}^8 = \frac{1}{13{,}440(1-\rho_0{}^2)^8}\,[(45\rho_0{}^2+3{,}660\rho_0{}^4+78{,}030\rho_0{}^6+109{,}260\rho_0{}^8+24{,}045\rho_0{}^{10})K'$$
$$+(-90+960\rho_0{}^2-19{,}740\rho_0{}^4-132{,}840\rho_0{}^6-62{,}490\rho_0{}^8-840\rho_0{}^{10})E']$$

$$W_{8,4}{}^0 = \frac{1}{70}\,\rho_0{}^4 W_{8,0}{}^4$$

$$W_{8,4}{}^2 = \frac{2}{5}\,\rho_0{}^2 W_{8,2}{}^4$$

$$W_{8,4}{}^4 = \frac{1}{192(1-\rho_0{}^2)^8}\,[(-480\rho_0{}^2+3{,}480\rho_0{}^4-6{,}120\rho_0{}^6+110{,}640\rho_0{}^8+110{,}640\rho_0{}^{10}-6{,}120\rho_0{}^{12}$$
$$+3{,}480\rho_0{}^{14}-480\rho_0{}^{16})K'+(960-7{,}080\rho_0{}^2+22{,}515\rho_0{}^4-58{,}020\rho_0{}^6-131{,}790\rho_0{}^8$$
$$-58{,}020\rho_0{}^{10}+22{,}515\rho_0{}^{12}-7{,}080\rho_0{}^{14}+960\rho_0{}^{16})E']$$

$$W_{8,4}{}^6 = \frac{1}{480(1-\rho_0{}^2)^8}\,[(240\rho_0{}^2-1{,}875\rho_0{}^4+9{,}300\rho_0{}^6+54{,}510\rho_0{}^8+146{,}220\rho_0{}^{10}+6{,}285\rho_0{}^{12}$$
$$+360\rho_0{}^{14})K'+(-480+3{,}810\rho_0{}^2-13{,}140\rho_0{}^4+14{,}220\rho_0{}^6-165{,}840\rho_0{}^8-59{,}310\rho_0{}^{10}$$
$$+6{,}420\rho_0{}^{12}-720\rho_0{}^{14})E']$$

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$$W_{8,4}^{8} = \frac{1}{13{,}440(1-\rho_0^2)^8}\,[(60\rho_0^2 - 480\rho_0^4 + 3{,}720\rho_0^6 + 47{,}760\rho_0^8 + 125{,}340\rho_0^{10} + 38{,}640\rho_0^{12})K' + (-120 + 975\rho_0^2 - 3{,}420\rho_0^4 - 1{,}350\rho_0^6 - 114{,}900\rho_0^8 - 94{,}545\rho_0^{10} - 1{,}680\rho_0^{12})E']$$

$$W_{8,6}^{0} = \frac{1}{28}\,\rho_0^6 W_{8,0}^{6}$$

$$W_{8,6}^{2} = \rho_0^4 W_{8,2}^{6}$$

$$W_{8,6}^{4} = \frac{5}{2}\,\rho_0^2 W_{8,4}^{6}$$

$$W_{8,6}^{6} = \frac{1}{480(1-\rho_0^2)^8}\,[(-1{,}680\rho_0^2 + 12{,}690\rho_0^4 - 41{,}490\rho_0^6 + 78{,}180\rho_0^8 - 37{,}620\rho_0^{10} + 182{,}970\rho_0^{12} + 22{,}710\rho_0^{14} - 720\rho_0^{16})K' + (3{,}360 - 25{,}800\rho_0^2 + 85{,}995\rho_0^4 - 161{,}940\rho_0^6 + 179{,}970\rho_0^8 - 237{,}060\rho_0^{10} - 53{,}205\rho_0^{12} - 7{,}800\rho_0^{14} + 1{,}440\rho_0^{16})E']$$

$$W_{8,6}^{8} = \frac{1}{13{,}440(1-\rho_0^2)^8}\,[(840\rho_0^2 - 6{,}435\rho_0^4 + 21{,}420\rho_0^6 - 39{,}090\rho_0^8 + 83{,}700\rho_0^{10} + 75{,}645\rho_0^{12} + 78{,}960\rho_0^{14})K' + (-1{,}680 + 13{,}080\rho_0^2 - 44{,}370\rho_0^4 + 85{,}560\rho_0^6 - 107{,}580\rho_0^8 - 2{,}880\rho_0^{10} - 150{,}450\rho_0^{12} - 6{,}720\rho_0^{14})E']$$

$$W_{8,8}^{0} = \rho_0^8 W_{8,0}^{8}$$

$$W_{8,8}^{2} = 28\rho_0^6 W_{8,2}^{8}$$

$$W_{8,8}^{4} = 70\rho_0^4 W_{8,4}^{8}$$

$$W_{8,8}^{6} = 28\rho_0^2 W_{8,6}^{8}$$

$$W_{8,8}^{8} = \frac{1}{13{,}440(1-\rho_0^2)^8}\,[(-60{,}480\rho_0^2 + 460{,}740\rho_0^4 - 1{,}522{,}980\rho_0^6 + 2{,}844{,}360\rho_0^8 - 3{,}265{,}320\rho_0^{10} + 2{,}367{,}540\rho_0^{12} - 904{,}500\rho_0^{14} + 295{,}680\rho_0^{16})K' + (120{,}960 - 936{,}600\rho_0^2 + 3{,}155{,}475\rho_0^4 - 6{,}029{,}460\rho_0^6 + 7{,}122{,}930\rho_0^8 - 5{,}298{,}420\rho_0^{10} + 2{,}324{,}115\rho_0^{12} - 687{,}480\rho_0^{14} + 13{,}440\rho_0^{16})E']$$

$$W_{9,0}^{0} = \frac{1}{120{,}960(1-\rho_0^2)^9}\,[(3{,}265{,}920\rho_0^2 - 10{,}671{,}900\rho_0^4 + 34{,}219{,}215\rho_0^6 - 56{,}073{,}780\rho_0^8 + 61{,}165{,}770\rho_0^{10} - 43{,}700{,}760\rho_0^{12} + 19{,}844{,}415\rho_0^{14} - 5{,}213{,}040\rho_0^{16} + 604{,}800\rho_0^{18})K' + (120{,}960 - 7{,}777{,}320\rho_0^2 + 28{,}368{,}075\rho_0^4 - 77{,}195{,}610\rho_0^6 + 124{,}355{,}130\rho_0^8 - 131{,}649{,}960\rho_0^{10} + 91{,}905{,}795\rho_0^{12} - 40{,}935{,}390\rho_0^{14} + 10{,}577{,}280\rho_0^{16} - 1{,}209{,}600\rho_0^{18})E']$$

$$W_{9,0}^{2} = \frac{1}{3{,}360(1-\rho_0^2)^9}\,[(861{,}840\rho_0^2 + 1{,}306{,}695\rho_0^4 + 1{,}555{,}200\rho_0^6 - 604{,}710\rho_0^8 + 492{,}540\rho_0^{10} - 222{,}345\rho_0^{12} + 58{,}140\rho_0^{14} - 6{,}720\rho_0^{16})K' + (-60{,}480 - 1{,}774{,}230\rho_0^2 - 714{,}885\rho_0^4 - 1{,}703{,}160\rho_0^6 + 1{,}493{,}610\rho_0^8 - 1{,}035{,}570\rho_0^{10} + 458{,}595\rho_0^{12} - 117{,}960\rho_0^{14} + 13{,}440\rho_0^{16})E']$$

$$W_{9,0}^{4} = \frac{1}{960(1-\rho_0^2)^9}\,[(423{,}360\rho_0^2 + 1{,}766{,}715\rho_0^4 + 1{,}102{,}860\rho_0^6 + 157{,}410\rho_0^8 - 12{,}540\rho_0^{10} + 3{,}195\rho_0^{12} - 360\rho_0^{14})K' + (-15{,}120 - 1{,}088{,}715\rho_0^2 - 1{,}997{,}220\rho_0^4 - 297{,}810\rho_0^6 - 61{,}860\rho_0^8 + 25{,}845\rho_0^{10} - 6{,}480\rho_0^{12} + 720\rho_0^{14})E']$$

$$W_{9,0}^{6} = \frac{1}{1{,}440(1-\rho_0^2)^9}\,[(263{,}655\rho_0^2 + 1{,}502{,}160\rho_0^4 + 1{,}474{,}650\rho_0^6 + 198{,}540\rho_0^8 + 1{,}815\rho_0^{10} - 180\rho_0^{12})K' + (-7{,}560 - 716{,}475\rho_0^2 - 2{,}086{,}140\rho_0^4 - 639{,}810\rho_0^6 + 12{,}660\rho_0^8 - 3{,}675\rho_0^{10} + 360\rho_0^{12})E']$$

$$W_{9,0}^{8} = \frac{1}{13{,}440(1-\rho_0^2)^9}\,[(184{,}275\rho_0^2 + 1{,}235{,}220\rho_0^4 + 1{,}599{,}570\rho_0^6 + 422{,}100\rho_0^8 - 525\rho_0^{10})K' + (-4{,}725 - 514{,}290\rho_0^2 - 1{,}883{,}910\rho_0^4 - 1{,}013{,}040\rho_0^6 - 25{,}725\rho_0^8 + 1{,}050\rho_0^{10})E']$$

$$W_{9,2}^{0} = \frac{1}{36}\rho_0^2 W_{9,0}^{2}$$

$$W_{9,2}^{2} = \frac{1}{3{,}360(1-\rho_0^2)^9}\,[(-5{,}040\rho_0^2 + 236{,}250\rho_0^4 + 2{,}461{,}665\rho_0^6 - 69{,}480\rho_0^8 + 1{,}433{,}790\rho_0^{10} - 950{,}250\rho_0^{12} + 435{,}465\rho_0^{14} - 115{,}200\rho_0^{16} + 13{,}440\rho_0^{18})K' + (10{,}080 - 57{,}960\rho_0^2 - 654{,}765\rho_0^4 - 3{,}477{,}570\rho_0^6 + 2{,}263{,}050\rho_0^8 - 2{,}831{,}160\rho_0^{10} + 1{,}999{,}275\rho_0^{12} - 898{,}470\rho_0^{14} + 233{,}760\rho_0^{16} - 26{,}880\rho_0^{18})E']$$

$$W_{9,2}^{4} = \frac{1}{960(1-\rho_0^2)^9}\,[(2{,}520\rho_0^2 + 78{,}435\rho_0^4 + 1{,}879{,}260\rho_0^6 + 1{,}236{,}690\rho_0^8 + 281{,}340\rho_0^{10} - 48{,}765\rho_0^{12} + 12{,}600\rho_0^{14} - 1{,}440\rho_0^{16})K' + (-5{,}040 + 51{,}660\rho_0^2 - 620{,}175\rho_0^4 - 2{,}539{,}380\rho_0^6 - 172{,}890\rho_0^8 - 232{,}680\rho_0^{10} + 100{,}545\rho_0^{12} - 25{,}560\rho_0^{14} + 2{,}880\rho_0^{16})E']$$

$$W_{9,2}^{6} = \frac{1}{1{,}440(1-\rho_0^2)^9}\,[(630\rho_0^2 + 57{,}645\rho_0^4 + 1{,}331{,}880\rho_0^6 + 1{,}786{,}950\rho_0^8 + 260{,}370\rho_0^{10} + 3{,}525\rho_0^{12} - 360\rho_0^{14})K' + (-1{,}260 + 14{,}805\rho_0^2 - 320{,}160\rho_0^4 - 2{,}288{,}370\rho_0^6 - 865{,}080\rho_0^8 + 25{,}845\rho_0^{10} - 7{,}140\rho_0^{12} + 720\rho_0^{14})E']$$

$$W_{9,2}^{8} = \frac{1}{13{,}440(1-\rho_0^2)^9}\,[(315\rho_0^2 + 41{,}580\rho_0^4 + 1{,}016{,}610\rho_0^6 + 1{,}818{,}180\rho_0^8 + 564{,}795\rho_0^{10} - 840\rho_0^{12})K' + (-630 + 7{,}875\rho_0^2 - 215{,}040\rho_0^4 - 1{,}883{,}910\rho_0^6 - 1{,}312{,}290\rho_0^8 - 38{,}325\rho_0^{10} + 1{,}680\rho_0^{12})E']$$

$$W_{9,4}^{0} = \frac{1}{126}\rho_0^4 W_{9,0}^{4}$$

$$W_{9,4}^{2} = \frac{2}{7}\rho_0^2 W_{9,2}^{4}$$

$$W_{9,4}^{4} = \frac{1}{960(1-\rho_0^2)^9}\,[(-2{,}400\rho_0^2 + 19{,}440\rho_0^4 - 16{,}965\rho_0^6 + 1{,}343{,}580\rho_0^8 + 1{,}970{,}370\rho_0^{10} + 59{,}220\rho_0^{12} + 88{,}635\rho_0^{14} - 24{,}120\rho_0^{16} + 2{,}880\rho_0^{18})K' + (4{,}800 - 39{,}480\rho_0^2 + 142{,}515\rho_0^4 - 497{,}010\rho_0^6 - 2{,}055{,}990\rho_0^8 - 1{,}244{,}520\rho_0^{10} + 388{,}875\rho_0^{12} - 183{,}030\rho_0^{14} + 48{,}960\rho_0^{16} - 5{,}760\rho_0^{18})E']$$

$$W_{9,4}^{6} = \frac{1}{1{,}440(1-\rho_0^2)^9}\,[(1{,}200\rho_0^2 - 10{,}665\rho_0^4 + 74{,}160\rho_0^6 + 778{,}650\rho_0^8 + 2{,}245{,}500\rho_0^{10} + 339{,}975\rho_0^{12} + 13{,}260\rho_0^{14} - 1{,}440\rho_0^{16})K' + (-2{,}400 + 21{,}630\rho_0^2 - 85{,}905\rho_0^4 + 68{,}280\rho_0^6 - 2{,}153{,}550\rho_0^8 - 1{,}367{,}550\rho_0^{10} + 102{,}855\rho_0^{12} - 26{,}880\rho_0^{14} + 2{,}880\rho_0^{16})E']$$

$$W_{9,4}{}^{8} = \frac{1}{13{,}440(1-\rho_0{}^2)^9}\,[(300\rho_0{}^2 - 2{,}745\rho_0{}^4 + 33{,}660\rho_0{}^6 + 641{,}850\rho_0{}^8 + 1{,}956{,}240\rho_0{}^{10} + 813{,}015\rho_0{}^{12} - 1{,}680\rho_0{}^{14})K' + (-600 + 5{,}565\rho_0{}^2 - 22{,}500\rho_0{}^4 - 39{,}090\rho_0{}^6 - 1{,}546{,}860\rho_0{}^8 - 1{,}773{,}315\rho_0{}^{10} - 67{,}200\rho_0{}^{12} + 3{,}360\rho_0{}^{14})E']$$

$$W_{9,6}{}^{0} = \frac{1}{84}\,\rho_0{}^6 W_{9,0}{}^{6}$$

$$W_{9,6}{}^{2} = \frac{3}{7}\,\rho_0{}^4 W_{9,2}{}^{6}$$

$$W_{9,6}{}^{4} = \frac{3}{2}\,\rho_0{}^2 W_{9,4}{}^{6}$$

$$W_{9,6}{}^{6} = \frac{1}{1{,}440(1-\rho_0{}^2)^9}\,[(-5{,}040\rho_0{}^2 + 42{,}870\rho_0{}^4 - 160{,}515\rho_0{}^6 + 366{,}120\rho_0{}^8 + 159{,}990\rho_0{}^{10} + 2{,}357{,}010\rho_0{}^{12} + 698{,}805\rho_0{}^{14} - 21{,}480\rho_0{}^{16} + 2{,}880\rho_0{}^{18})K' + (10{,}080 - 87{,}000\rho_0{}^2 + 331{,}275\rho_0{}^4 - 728{,}010\rho_0{}^6 + 929{,}850\rho_0{}^8 - 2{,}324{,}280\rho_0{}^{10} - 1{,}455{,}165\rho_0{}^{12} - 155{,}310\rho_0{}^{14} + 43{,}680\rho_0{}^{16} - 5{,}760\rho_0{}^{18})E']$$

$$W_{9,6}{}^{8} = \frac{1}{13{,}440(1-\rho_0{}^2)^9}\,[(2{,}520\rho_0{}^2 - 21{,}885\rho_0{}^4 + 84{,}060\rho_0{}^6 - 173{,}070\rho_0{}^8 + 745{,}980\rho_0{}^{10} + 1{,}466{,}595\rho_0{}^{12} + 1{,}343{,}160\rho_0{}^{14} - 6{,}720\rho_0{}^{16})K' + (-5{,}040 + 44{,}400\rho_0{}^2 - 173{,}355\rho_0{}^4 + 393{,}660\rho_0{}^6 - 630{,}930\rho_0{}^8 - 551{,}460\rho_0{}^{10} - 2{,}338{,}155\rho_0{}^{12} - 193{,}200\rho_0{}^{14} + 13{,}440\rho_0{}^{16})E']$$

$$W_{9,8}{}^{0} = \frac{1}{9}\,\rho_0{}^8 W_{9,0}{}^{8}$$

$$W_{9,8}{}^{2} = 4\rho_0{}^6 W_{9,2}{}^{8}$$

$$W_{9,8}{}^{4} = 14\rho_0{}^4 W_{9,4}{}^{8}$$

$$W_{9,8}{}^{6} = \frac{28}{3}\,\rho_0{}^2 W_{9,6}{}^{8}$$

$$W_{9,8}{}^{8} = \frac{1}{13{,}440(1-\rho_0{}^2)^9}\,[(-60{,}480\rho_0{}^2 + 520{,}380\rho_0{}^4 - 1{,}976{,}505\rho_0{}^6 + 4{,}339{,}980\rho_0{}^8 - 6{,}040{,}710\rho_0{}^{10} + 5{,}887{,}680\rho_0{}^{12} - 1{,}773{,}225\rho_0{}^{14} + 2{,}530{,}080\rho_0{}^{16} + 13{,}440\rho_0{}^{18})K' + (120{,}960 - 1{,}055{,}880\rho_0{}^2 + 4{,}077{,}435\rho_0{}^4 - 9{,}128{,}490\rho_0{}^6 + 13{,}026{,}330\rho_0{}^8 - 12{,}279{,}720\rho_0{}^{10} + 6{,}652{,}275\rho_0{}^{12} - 4{,}759{,}470\rho_0{}^{14} - 67{,}200\rho_0{}^{16} - 26{,}880\rho_0{}^{18})E']$$

$$\overline{W}_{0,0}{}^{0} = -\frac{1}{2(1-\rho_0{}^2)}\,(\rho_0{}^2K' - \rho_0{}^2E')$$

$$\overline{W}_{0,0}{}^{2} = -\frac{1}{2(1-\rho_0{}^2)}\,(\rho_0{}^2K' - E')$$

$$\overline{W}_{1,0}{}^{0} = -\frac{1}{6(1-\rho_0{}^2)^2}\,[(3\rho_0{}^2 - \rho_0{}^4)K' - (4\rho_0{}^2 - 2\rho_0{}^4)E']$$

$$\overline{W}_{1,0}{}^{2} = -\,W_{2,0}{}^{2}$$

$$\overline{W}_{2,0}{}^{0} = -\frac{1}{24(1-\rho_0{}^2)^3}[(12\rho_0{}^2-7\rho_0{}^4+3\rho_0{}^6)K' - (19\rho_0{}^2-17\rho_0{}^4+6\rho_0{}^6)E']$$

$$\overline{W}_{2,0}{}^{2} = -\frac{1}{4(1-\rho_0{}^2)^3}[(7\rho_0{}^2+\rho_0{}^4)K' - (2+7\rho_0{}^2-\rho_0{}^4)E']$$

$$\overline{W}_{2,0}{}^{4} = -\frac{1}{24(1-\rho_0{}^2)^3}[(5\rho_0{}^2+3\rho_0{}^4)K' - (1+7\rho_0{}^2)E']$$

$$\overline{W}_{2,2}{}^{0} = -\frac{1}{24(1-\rho_0{}^2)^3}[(3\rho_0{}^4+5\rho_0{}^6)K' - (7\rho_0{}^4+\rho_0{}^6)E']$$

$$\overline{W}_{2,2}{}^{2} = -\frac{1}{4(1-\rho_0{}^2)^3}[(\rho_0{}^4+7\rho_0{}^6)K' + (\rho_0{}^2-7\rho_0{}^4-2\rho_0{}^6)E']$$

$$\overline{W}_{2,2}{}^{4} = -\frac{1}{24(1-\rho_0{}^2)^3}[(3\rho_0{}^2-7\rho_0{}^4+12\rho_0{}^6)K' - (6-17\rho_0{}^2+19\rho_0{}^4)E']$$

$$\overline{W}_{3,0}{}^{0} = -\frac{1}{120(1-\rho_0{}^2)^4}[(60\rho_0{}^2-44\rho_0{}^4+44\rho_0{}^6-12\rho_0{}^8)K' - (107\rho_0{}^2-126\rho_0{}^4+91\rho_0{}^6-24\rho_0{}^8)E']$$

$$\overline{W}_{3,0}{}^{2} = -\frac{1}{12(1-\rho_0{}^2)^4}[(33\rho_0{}^2+14\rho_0{}^4+\rho_0{}^6)K' - (6+46\rho_0{}^2-6\rho_0{}^4+2\rho_0{}^6)E']$$

$$\overline{W}_{3,0}{}^{4} = -W_{4,0}{}^{4}$$

$$\overline{W}_{3,2}{}^{0} = -\frac{1}{120(1-\rho_0{}^2)^4}[(15\rho_0{}^4+34\rho_0{}^6-\rho_0{}^8)K' - (38\rho_0{}^4+12\rho_0{}^6-2\rho_0{}^8)E']$$

$$\overline{W}_{3,2}{}^{2} = -\frac{1}{12(1-\rho_0{}^2)^4}[(6\rho_0{}^4+44\rho_0{}^6-2\rho_0{}^8)K' + (3\rho_0{}^2-34\rho_0{}^4-21\rho_0{}^6+4\rho_0{}^8)E']$$

$$\overline{W}_{3,2}{}^{4} = -W_{4,2}{}^{4}$$

$$\overline{W}_{4,0}{}^{0} = -\frac{1}{240(1-\rho_0{}^2)^5}[(120\rho_0{}^2-93\rho_0{}^4+174\rho_0{}^6-93\rho_0{}^8+20\rho_0{}^{10})K'$$
$$+(-234\rho_0{}^2+315\rho_0{}^4-360\rho_0{}^6+191\rho_0{}^8-40\rho_0{}^{10})E']$$

$$\overline{W}_{4,0}{}^{2} = -\frac{1}{16(1-\rho_0{}^2)^5}[(64\rho_0{}^2+55\rho_0{}^4+10\rho_0{}^6-\rho_0{}^8)K' + (-8-109\rho_0{}^2-4\rho_0{}^4-9\rho_0{}^6+2\rho_0{}^8)E']$$

$$\overline{W}_{4,0}{}^{4} = -\frac{1}{16(1-\rho_0{}^2)^5}[(47\rho_0{}^2+74\rho_0{}^4+7\rho_0{}^6)K' + (-4-95\rho_0{}^2-30\rho_0{}^4+\rho_0{}^6)E']$$

$$\overline{W}_{4,0}{}^{6} = -\frac{1}{240(1-\rho_0{}^2)^5}[(39\rho_0{}^2+74\rho_0{}^4+15\rho_0{}^6)K' + (-3-82\rho_0{}^2-43\rho_0{}^4)E']$$

$$\overline{W}_{4,2}{}^{0} = -\frac{1}{240(1-\rho_0{}^2)^5}[(30\rho_0{}^4+93\rho_0{}^6+4\rho_0{}^8+\rho_0{}^{10})K' + (-81\rho_0{}^4-56\rho_0{}^6+11\rho_0{}^8-2\rho_0{}^{10})E']$$

$$\overline{W}_{4,2}{}^{2} = -\frac{1}{16(1-\rho_0{}^2)^5}[(13\rho_0{}^4+112\rho_0{}^6+\rho_0{}^8+2\rho_0{}^{10})K' + (4\rho_0{}^2-67\rho_0{}^4-82\rho_0{}^6+21\rho_0{}^8-4\rho_0{}^{10})E']$$

$$\overline{W}_{4,2}{}^{4} = -\frac{1}{16(1-\rho_0{}^2)^5}[(2\rho_0{}^2+\rho_0{}^4+112\rho_0{}^6+13\rho_0{}^8)K' + (-4+21\rho_0{}^2-82\rho_0{}^4-67\rho_0{}^6+4\rho_0{}^8)E']$$

$$\overline{W}_{4,2}{}^{6} = -\frac{1}{240(1-\rho_0{}^2)^5}\left[(\rho_0{}^2+4\rho_0{}^4+93\rho_0{}^6+30\rho_0{}^8)K' + (-2+11\rho_0{}^2-56\rho_0{}^4-81\rho_0{}^6)E'\right]$$

$$\overline{W}_{4,4}{}^{0} = -\frac{1}{240(1-\rho_0{}^2)^5}\left[(15\rho_0{}^6+74\rho_0{}^8+39\rho_0{}^{10})K' + (-43\rho_0{}^6-82\rho_0{}^8-3\rho_0{}^{10})E'\right]$$

$$\overline{W}_{4,4}{}^{2} = -\frac{1}{16(1-\rho_0{}^2)^5}\left[(7\rho_0{}^6+74\rho_0{}^8+47\rho_0{}^{10})K' + (\rho_0{}^4-30\rho_0{}^6-95\rho_0{}^8-4\rho_0{}^{10})E'\right]$$

$$\overline{W}_{4,4}{}^{4} = -\frac{1}{16(1-\rho_0{}^2)^5}\left[(-\rho_0{}^4+10\rho_0{}^6+55\rho_0{}^8+64\rho_0{}^{10})K' + (2\rho_0{}^2-9\rho_0{}^4-4\rho_0{}^6-109\rho_0{}^8-8\rho_0{}^{10})E'\right]$$

$$\overline{W}_{4,4}{}^{6} = -\frac{1}{240(1-\rho_0{}^2)^5}\left[(20\rho_0{}^2-93\rho_0{}^4+174\rho_0{}^6-93\rho_0{}^8+120\rho_0{}^{10})K' + (-40+191\rho_0{}^2-360\rho_0{}^4+315\rho_0{}^6-234\rho_0{}^8)E'\right]$$

$$\overline{W}_{5,0}{}^{0} = -\frac{1}{1{,}680(1-\rho_0{}^2)^6}\left[(840\rho_0{}^2-591\rho_0{}^4+2{,}043\rho_0{}^6-1{,}569\rho_0{}^8+677\rho_0{}^{10}-120\rho_0{}^{12})K' + (-1{,}758\rho_0{}^2+2{,}460\rho_0{}^4-4{,}134\rho_0{}^6+3{,}296\rho_0{}^8-1{,}384\rho_0{}^{10}+240\rho_0{}^{12})E'\right]$$

$$\overline{W}_{5,0}{}^{2} = -\frac{1}{80(1-\rho_0{}^2)^6}\left[(440\rho_0{}^2+642\rho_0{}^4+216\rho_0{}^6-22\rho_0{}^8+4\rho_0{}^{10})K' + (-40-859\rho_0{}^2-309\rho_0{}^4-109\rho_0{}^6+45\rho_0{}^8-8\rho_0{}^{10})E'\right]$$

$$\overline{W}_{5,0}{}^{4} = -\frac{1}{48(1-\rho_0{}^2)^6}\left[(325\rho_0{}^2+759\rho_0{}^4+195\rho_0{}^6+\rho_0{}^8)K' + (-20-728\rho_0{}^2-534\rho_0{}^4+4\rho_0{}^6-2\rho_0{}^8)E'\right]$$

$$\overline{W}_{5,0}{}^{6} = -W_{6,0}{}^{6}$$

$$\overline{W}_{5,2}{}^{0} = -\frac{1}{1{,}680(1-\rho_0{}^2)^6}\left[(210\rho_0{}^4+876\rho_0{}^6+174\rho_0{}^8+24\rho_0{}^{10}-4\rho_0{}^{12})K' + (-597\rho_0{}^4-759\rho_0{}^6+117\rho_0{}^8-49\rho_0{}^{10}+8\rho_0{}^{12})E'\right]$$

$$\overline{W}_{5,2}{}^{2} = -\frac{1}{80(1-\rho_0{}^2)^6}\left[(95\rho_0{}^4+993\rho_0{}^6+153\rho_0{}^8+47\rho_0{}^{10}-8\rho_0{}^{12})K' + (20\rho_0{}^2-466\rho_0{}^4-984\rho_0{}^6+230\rho_0{}^8-96\rho_0{}^{10}+16\rho_0{}^{12})E'\right]$$

$$\overline{W}_{5,2}{}^{4} = -\frac{1}{48(1-\rho_0{}^2)^6}\left[(6\rho_0{}^2+40\rho_0{}^4+930\rho_0{}^6+300\rho_0{}^8+4\rho_0{}^{10})K' + (-12+79\rho_0{}^2-491\rho_0{}^4-879\rho_0{}^6+31\rho_0{}^8-8\rho_0{}^{10})E'\right]$$

$$\overline{W}_{5,2}{}^{6} = -W_{6,2}{}^{6}$$

$$\overline{W}_{5,4}{}^{0} = -\frac{1}{1{,}680(1-\rho_0{}^2)^6}\left[(105\rho_0{}^6+683\rho_0{}^8+495\rho_0{}^{10}-3\rho_0{}^{12})K' + (-316\rho_0{}^6-898\rho_0{}^8-72\rho_0{}^{10}+6\rho_0{}^{12})E'\right]$$

$$\overline{W}_{5,4}{}^{2} = -\frac{1}{80(1-\rho_0{}^2)^6}\left[(50\rho_0{}^6+664\rho_0{}^8+570\rho_0{}^{10}-4\rho_0{}^{12})K' + (5\rho_0{}^4-213\rho_0{}^6-989\rho_0{}^8-91\rho_0{}^{10}+8\rho_0{}^{12})E'\right]$$

$$\overline{W}_{5,4}{}^{4} = -\frac{1}{48(1-\rho_0{}^2)^6}[(-3\rho_0{}^4+55\rho_0{}^6+531\rho_0{}^8+705\rho_0{}^{10}-8\rho_0{}^{12})K'$$
$$+(6\rho_0{}^2-32\rho_0{}^4-74\rho_0{}^6-1{,}044\rho_0{}^8-152\rho_0{}^{10}+16\rho_0{}^{12})E']$$

$$\overline{W}_{5,4}{}^{6} = -W_{6,4}{}^{6}$$

$$\overline{W}_{6,0}{}^{0} = -\frac{1}{13{,}440(1-\rho_0{}^2)^7}[(6{,}720\rho_0{}^2-3{,}468\rho_0{}^4+25{,}203\rho_0{}^6-24{,}057\rho_0{}^8+15{,}697\rho_0{}^{10}-5{,}575\rho_0{}^{12}$$
$$+840\rho_0{}^{14})K'+(-14{,}904\rho_0{}^2+19{,}917\rho_0{}^4-49{,}191\rho_0{}^6+51{,}847\rho_0{}^8-32{,}709\rho_0{}^{10}+11{,}360\rho_0{}^{12}$$
$$-1{,}680\rho_0{}^{14})E']$$

$$\overline{W}_{6,0}{}^{2} = -\frac{1}{480(1-\rho_0{}^2)^7}[(3{,}480\rho_0{}^2+7{,}761\rho_0{}^4+4{,}209\rho_0{}^6-201\rho_0{}^8+131\rho_0{}^{10}-20\rho_0{}^{12})K'$$
$$+(-240-7{,}512\rho_0{}^2-6{,}309\rho_0{}^4-1{,}833\rho_0{}^6+761\rho_0{}^8-267\rho_0{}^{10}+40\rho_0{}^{12})E']$$

$$\overline{W}_{6,0}{}^{4} = -\frac{1}{192(1-\rho_0{}^2)^7}[(2{,}580\rho_0{}^2+8{,}487\rho_0{}^4+4{,}083\rho_0{}^6+213\rho_0{}^8-3\rho_0{}^{10})K'$$
$$+(-120-6{,}249\rho_0{}^2-8{,}283\rho_0{}^4-681\rho_0{}^6-33\rho_0{}^8+6\rho_0{}^{10})E']$$

$$\overline{W}_{6,0}{}^{6} = -\frac{1}{480(1-\rho_0{}^2)^7}[(2{,}145\rho_0{}^2+8{,}125\rho_0{}^4+4{,}835\rho_0{}^6+255\rho_0{}^8)K'$$
$$+(-90-5{,}345\rho_0{}^2-8{,}865\rho_0{}^4-1{,}075\rho_0{}^6+15\rho_0{}^8)E']$$

$$\overline{W}_{6,0}{}^{8} = -\frac{1}{13{,}440(1-\rho_0{}^2)^7}[(1{,}875\rho_0{}^2+7{,}655\rho_0{}^4+5{,}305\rho_0{}^6+525\rho_0{}^8)K'$$
$$+(-75-4{,}735\rho_0{}^2-8{,}865\rho_0{}^4-1{,}685\rho_0{}^6)E']$$

$$\overline{W}_{6,2}{}^{0} = -\frac{1}{13{,}440(1-\rho_0{}^2)^7}[(1{,}680\rho_0{}^4+9{,}213\rho_0{}^6+3{,}957\rho_0{}^8+627\rho_0{}^{10}-137\rho_0{}^{12}+20\rho_0{}^{14})K'$$
$$+(-4{,}986\rho_0{}^4-10{,}257\rho_0{}^6+471\rho_0{}^8-827\rho_0{}^{10}+279\rho_0{}^{12}-40\rho_0{}^{14})E']$$

$$\overline{W}_{6,2}{}^{2} = -\frac{1}{480(1-\rho_0{}^2)^7}[(780\rho_0{}^4+9{,}939\rho_0{}^6+3{,}831\rho_0{}^8+1{,}041\rho_0{}^{10}-271\rho_0{}^{12}+40\rho_0{}^{14})K'$$
$$+(120\rho_0{}^2-3{,}723\rho_0{}^4-12{,}231\rho_0{}^6+1{,}623\rho_0{}^8-1{,}621\rho_0{}^{10}+552\rho_0{}^{12}-80\rho_0{}^{14})E']$$

$$\overline{W}_{6,2}{}^{4} = -\frac{1}{192(1-\rho_0{}^2)^7}[(24\rho_0{}^2+441\rho_0{}^4+9{,}009\rho_0{}^6+5{,}511\rho_0{}^8+387\rho_0{}^{10}-12\rho_0{}^{12})K'$$
$$+(-48+384\rho_0{}^2-3{,}573\rho_0{}^4-11{,}193\rho_0{}^6-807\rho_0{}^8-147\rho_0{}^{10}+24\rho_0{}^{12})E']$$

$$\overline{W}_{6,2}{}^{6} = -\frac{1}{480(1-\rho_0{}^2)^7}[(12\rho_0{}^2+423\rho_0{}^4+7{,}843\rho_0{}^6+6{,}677\rho_0{}^8+405\rho_0{}^{10})K'$$
$$+(-24+207\rho_0{}^2-2{,}651\rho_0{}^4-11{,}193\rho_0{}^6-1{,}729\rho_0{}^8+30\rho_0{}^{10})E']$$

$$\overline{W}_{6,2}{}^{8} = -\frac{1}{13{,}440(1-\rho_0{}^2)^7}[(9\rho_0{}^2+381\rho_0{}^4+7{,}091\rho_0{}^6+7{,}039\rho_0{}^8+840\rho_0{}^{10})K'$$
$$+(-18+159\rho_0{}^2-2{,}257\rho_0{}^4-10{,}611\rho_0{}^6-2{,}633\rho_0{}^8)E']$$

$$\overline{W}_{6,4}{}^{0} = -\frac{1}{13{,}440(1-\rho_0{}^2)^7}\,[(840\rho_0{}^6+7{,}039\rho_0{}^8+7{,}091\rho_0{}^{10}+381\rho_0{}^{12}+9\rho_0{}^{14})K'$$
$$+(-2{,}633\rho_0{}^6-10{,}611\rho_0{}^8-2{,}257\rho_0{}^{10}+159\rho_0{}^{12}-18\rho_0{}^{14})E']$$

$$\overline{W}_{6,4}{}^{2} = -\frac{1}{480(1-\rho_0{}^2)^7}\,[(405\rho_0{}^6+6{,}677\rho_0{}^8+7{,}843\rho_0{}^{10}+423\rho_0{}^{12}+12\rho_0{}^{14})K'$$
$$+(30\rho_0{}^4-1{,}729\rho_0{}^6-11{,}193\rho_0{}^8-2{,}651\rho_0{}^{10}+207\rho_0{}^{12}-24\rho_0{}^{14})E']$$

$$\overline{W}_{6,4}{}^{4} = -\frac{1}{192(1-\rho_0{}^2)^7}\,[(-12\rho_0{}^4+387\rho_0{}^6+5{,}511\rho_0{}^8+9{,}009\rho_0{}^{10}+441\rho_0{}^{12}+24\rho_0{}^{14})K'$$
$$+(24\rho_0{}^2-147\rho_0{}^4-807\rho_0{}^6-11{,}193\rho_0{}^8-3{,}573\rho_0{}^{10}+384\rho_0{}^{12}-48\rho_0{}^{14})E']$$

$$\overline{W}_{6,4}{}^{6} = -\frac{1}{480(1-\rho_0{}^2)^7}\,[(40\rho_0{}^2-271\rho_0{}^4+1{,}041\rho_0{}^6+3{,}831\rho_0{}^8+9{,}939\rho_0{}^{10}+780\rho_0{}^{12})K'$$
$$+(-80+552\rho_0{}^2-1{,}621\rho_0{}^4+1{,}623\rho_0{}^6-12{,}231\rho_0{}^8-3{,}723\rho_0{}^{10}+120\rho_0{}^{12})E']$$

$$\overline{W}_{6,4}{}^{8} = -\frac{1}{13{,}440(1-\rho_0{}^2)^7}\,[(20\rho_0{}^2-137\rho_0{}^4+627\rho_0{}^6+3{,}957\rho_0{}^8+9{,}213\rho_0{}^{10}+1{,}680\rho_0{}^{12})K'$$
$$+(-40+279\rho_0{}^2-827\rho_0{}^4+471\rho_0{}^6-10{,}257\rho_0{}^8-4{,}986\rho_0{}^{10})E']$$

$$\overline{W}_{6,6}{}^{0} = -\frac{1}{13{,}440(1-\rho_0{}^2)^7}\,[(525\rho_0{}^8+5{,}305\rho_0{}^{10}+7{,}655\rho_0{}^{12}+1{,}875\rho_0{}^{14})K'$$
$$+(-1{,}685\rho_0{}^8-8{,}865\rho_0{}^{10}-4{,}735\rho_0{}^{12}-75\rho_0{}^{14})E']$$

$$\overline{W}_{6,6}{}^{2} = -\frac{1}{480(1-\rho_0{}^2)^7}\,[(255\rho_0{}^8+4{,}835\rho_0{}^{10}+8{,}125\rho_0{}^{12}+2{,}145\rho_0{}^{14})K'$$
$$+(15\rho_0{}^6-1{,}075\rho_0{}^8-8{,}865\rho_0{}^{10}-5{,}345\rho_0{}^{12}-90\rho_0{}^{14})E']$$

$$\overline{W}_{6,6}{}^{4} = -\frac{1}{192(1-\rho_0{}^2)^7}\,[(-3\rho_0{}^6+213\rho_0{}^8+4{,}083\rho_0{}^{10}+8{,}487\rho_0{}^{12}+2{,}580\rho_0{}^{14})K'$$
$$+(6\rho_0{}^4-33\rho_0{}^6-681\rho_0{}^8-8{,}283\rho_0{}^{10}-6{,}249\rho_0{}^{12}-120\rho_0{}^{14})E']$$

$$\overline{W}_{6,6}{}^{6} = -\frac{1}{480(1-\rho_0{}^2)^7}\,[(-20\rho_0{}^4+131\rho_0{}^6-201\rho_0{}^8+4{,}209\rho_0{}^{10}+7{,}761\rho_0{}^{12}+3{,}480\rho_0{}^{14})K'$$
$$+(40\rho_0{}^2-267\rho_0{}^4+761\rho_0{}^6-1{,}833\rho_0{}^8-6{,}309\rho_0{}^{10}-7{,}512\rho_0{}^{12}-240\rho_0{}^{14})E']$$

$$\overline{W}_{6,6}{}^{8} = -\frac{1}{13{,}440(1-\rho_0{}^2)^7}\,[(840\rho_0{}^2-5{,}575\rho_0{}^4+15{,}697\rho_0{}^6-24{,}057\rho_0{}^8+25{,}203\rho_0{}^{10}-3{,}468\rho_0{}^{12}$$
$$+6{,}720\rho_0{}^{14})K'+(-1{,}680+11{,}360\rho_0{}^2-32{,}709\rho_0{}^4+51{,}847\rho_0{}^6-49{,}191\rho_0{}^8+19{,}917\rho_0{}^{10}$$
$$-14{,}904\rho_0{}^{12})E']$$

$$\overline{W}_{7,0}{}^{0} = -\frac{1}{120{,}960(1-\rho_0{}^2)^8}\,[(60{,}480\rho_0{}^2-12{,}732\rho_0{}^4+333{,}684\rho_0{}^6-359{,}640\rho_0{}^8+318{,}696\rho_0{}^{10}$$
$$-170{,}028\rho_0{}^{12}+51{,}300\rho_0{}^{14}-6{,}720\rho_0{}^{16})K'+(-140{,}856\rho_0{}^2+162{,}939\rho_0{}^4-624{,}852\rho_0{}^6$$
$$+801{,}762\rho_0{}^8-675{,}444\rho_0{}^{10}+352{,}251\rho_0{}^{12}-104{,}280\rho_0{}^{14}+13{,}440\rho_0{}^{16})E']$$

$$\overline{W}_{7,0}^{2} = -\frac{1}{3{,}360(1-\rho_0^2)^8}[(31{,}080\rho_0^2+98{,}739\rho_0^4+79{,}452\rho_0^6+3{,}570\rho_0^8+2{,}988\rho_0^{10}-909\rho_0^{12}$$
$$+120\rho_0^{14})K' + (-1{,}680-72{,}528\rho_0^2-107{,}208\rho_0^4-40{,}656\rho_0^6+11{,}616\rho_0^8-6{,}192\rho_0^{10}$$
$$+1{,}848\rho_0^{12}-240\rho_0^{14})E']$$

$$\overline{W}_{7,0}^{4} = -\frac{1}{960(1-\rho_0^2)^8}[(23{,}100\rho_0^2+102{,}588\rho_0^4+78{,}744\rho_0^6+10{,}680\rho_0^8-84\rho_0^{10}+12\rho_0^{12})K'$$
$$+(-840-59{,}541\rho_0^2-124{,}596\rho_0^4-29{,}502\rho_0^6-708\rho_0^8+171\rho_0^{10}-24\rho_0^{12})E']$$

$$\overline{W}_{7,0}^{6} = -\frac{1}{1{,}440(1-\rho_0^2)^8}[(19{,}215\rho_0^2+96{,}900\rho_0^4+86{,}490\rho_0^6+12{,}420\rho_0^8+15\rho_0^{10})K'$$
$$+(-630-50{,}790\rho_0^2-127{,}020\rho_0^4-36{,}540\rho_0^6-30\rho_0^8-30\rho_0^{10})E']$$

$$\overline{W}_{7,0}^{8} = -W_{8\ 0}^{8}$$

$$\overline{W}_{7,2}^{0} = -\frac{1}{120{,}960(1-\rho_0^2)^8}[(15{,}120\rho_0^4+106{,}437\rho_0^6+78{,}036\rho_0^8+17{,}790\rho_0^{10}-3{,}156\rho_0^{12}+933\rho_0^{14}$$
$$-120\rho_0^{16})K' + (-46{,}554\rho_0^4-141{,}984\rho_0^6-18{,}348\rho_0^8-13{,}032\rho_0^{10}+6{,}534\rho_0^{12}-1{,}896\rho_0^{14}$$
$$+240\rho_0^{16})E']$$

$$\overline{W}_{7,2}^{2} = -\frac{1}{3{,}360(1-\rho_0^2)^8}[(7{,}140\rho_0^4+110{,}286\rho_0^6+77{,}328\rho_0^8+24{,}900\rho_0^{10}-6{,}228\rho_0^{12}+1{,}854\rho_0^{14}$$
$$-240\rho_0^{16})K' + (840\rho_0^2-33{,}567\rho_0^4-159{,}372\rho_0^6-7{,}194\rho_0^8-25{,}356\rho_0^{10}+12{,}897\rho_0^{12}$$
$$-3{,}768\rho_0^{14}+480\rho_0^{16})E']$$

$$\overline{W}_{7,2}^{4} = -\frac{1}{960(1-\rho_0^2)^8}[(120\rho_0^2+4{,}545\rho_0^4+98{,}172\rho_0^6+96{,}486\rho_0^8+16{,}020\rho_0^{10}-351\rho_0^{12}+48\rho_0^{14})K'$$
$$+(-240+2{,}280\rho_0^2-30{,}294\rho_0^4-146{,}424\rho_0^6-38{,}388\rho_0^8-2{,}592\rho_0^{10}+714\rho_0^{12}-96\rho_0^{14})E']$$

$$\overline{W}_{7,2}^{6} = -\frac{1}{1{,}440(1-\rho_0^2)^8}[(60\rho_0^2+4{,}110\rho_0^4+86{,}160\rho_0^6+106{,}980\rho_0^8+17{,}700\rho_0^{10}+30\rho_0^{12})K'$$
$$+(-120+1{,}245\rho_0^2-23{,}040\rho_0^4-141{,}570\rho_0^6-51{,}540\rho_0^8+45\rho_0^{10}-60\rho_0^{12})E']$$

$$\overline{W}_{7,2}^{8} = -W_{8,2}^{8}$$

$$\overline{W}_{7,4}^{0} = -\frac{1}{120{,}960(1-\rho_0^2)^8}[(7{,}560\rho_0^6+79{,}836\rho_0^8+109{,}728\rho_0^{10}+17{,}640\rho_0^{12}+312\rho_0^{14}-36\rho_0^{16})K'$$
$$+(-24{,}537\rho_0^6-134{,}292\rho_0^8-57{,}654\rho_0^{10}+2{,}004\rho_0^{12}-633\rho_0^{14}+72\rho_0^{16})E']$$

$$\overline{W}_{7,4}^{2} = -\frac{1}{3{,}360(1-\rho_0^2)^8}[(3{,}675\rho_0^6+74{,}148\rho_0^8+117{,}474\rho_0^{10}+19{,}380\rho_0^{12}+411\rho_0^{14}-48\rho_0^{16})K'$$
$$+(210\rho_0^4-15{,}786\rho_0^6-136{,}716\rho_0^8-64{,}692\rho_0^{10}+2{,}682\rho_0^{12}-834\rho_0^{14}+96\rho_0^{16})E']$$

$$\overline{W}_{7,4}^{4} = -\frac{1}{960(1-\rho_0^2)^8}[(-60\rho_0^4+3{,}240\rho_0^6+62{,}136\rho_0^8+127{,}968\rho_0^{10}+21{,}060\rho_0^{12}+792\rho_0^{14}$$
$$-96\rho_0^{16})K' + (120\rho_0^2-825\rho_0^4-8{,}532\rho_0^6-131{,}862\rho_0^8-77{,}844\rho_0^{10}+5{,}319\rho_0^{12}-1{,}608\rho_0^{14}$$
$$+192\rho_0^{16})E']$$

$$\overline{W}_{7,4}{}^{6} = -\frac{1}{1{,}440(1-\rho_0{}^2)^8}[(120\rho_0{}^2-945\rho_0{}^4+5{,}580\rho_0{}^6+50{,}010\rho_0{}^8+132{,}300\rho_0{}^{10}+27{,}855\rho_0{}^{12}$$
$$+120\rho_0{}^{14})K' + (-240+1{,}920\rho_0{}^2-6{,}660\rho_0{}^4+3{,}840\rho_0{}^6-131{,}880\rho_0{}^8-82{,}800\rho_0{}^{10}$$
$$+1{,}020\rho_0{}^{12}-240\rho_0{}^{14})E']$$

$$\overline{W}_{7,4}{}^{8} = -W_{8,4}{}^{8}$$

$$\overline{W}_{7,6}{}^{0} = -\frac{1}{120{,}960(1-\rho_0{}^2)^8}[(4{,}725\rho_0{}^8+59{,}820\rho_0{}^{10}+111{,}870\rho_0{}^{12}+38{,}700\rho_0{}^{14}-75\rho_0{}^{16})K'$$
$$+(-15{,}690\rho_0{}^8-109{,}560\rho_0{}^{10}-86{,}940\rho_0{}^{12}-3{,}000\rho_0{}^{14}+150\rho_0{}^{16})E']$$

$$\overline{W}_{7,6}{}^{2} = -\frac{1}{3{,}360(1-\rho_0{}^2)^8}[(2{,}310\rho_0{}^8+53{,}640\rho_0{}^{10}+116{,}100\rho_0{}^{12}+43{,}080\rho_0{}^{14}-90\rho_0{}^{16})K'$$
$$+(105\rho_0{}^6-9{,}840\rho_0{}^8-106{,}650\rho_0{}^{10}-95{,}340\rho_0{}^{12}-3{,}495\rho_0{}^{14}+180\rho_0{}^{16})E']$$

$$\overline{W}_{7,6}{}^{4} = -\frac{1}{960(1-\rho_0{}^2)^8}[(-15\rho_0{}^6+1{,}860\rho_0{}^8+45{,}510\rho_0{}^{10}+118{,}380\rho_0{}^{12}+49{,}425\rho_0{}^{14}-120\rho_0{}^{16})K'$$
$$+(30\rho_0{}^4-180\rho_0{}^6-6{,}540\rho_0{}^8-97{,}920\rho_0{}^{10}-106{,}290\rho_0{}^{12}-4{,}380\rho_0{}^{14}+240\rho_0{}^{16})E']$$

$$\overline{W}_{7,6}{}^{6} = -\frac{1}{1{,}440(1-\rho_0{}^2)^8}[(-60\rho_0{}^4+450\rho_0{}^6+0\rho_0{}^8+43{,}260\rho_0{}^{10}+111{,}420\rho_0{}^{12}+60{,}210\rho_0{}^{14}$$
$$-240\rho_0{}^{16})K' + (120\rho_0{}^2-915\rho_0{}^4+3{,}060\rho_0{}^6-11{,}730\rho_0{}^8-80{,}940\rho_0{}^{10}-118{,}035\rho_0{}^{12}$$
$$-7{,}080\rho_0{}^{14}+480\rho_0{}^{16})E']$$

$$\overline{W}_{7,6}{}^{8} = -W_{8,6}{}^{8}$$

$$\overline{\overline{W}}_{0,0}{}^{0} = \frac{1}{4}W_{2,2}{}^{0}$$

$$\overline{\overline{W}}_{0,0}{}^{2} = -\frac{3}{4(1-\rho_0{}^2)^2}[(-\rho_0{}^2-\rho_0{}^4)K' + 2\rho_0{}^2E']$$

$$\overline{\overline{W}}_{0,0}{}^{4} = \frac{1}{4}W_{2,0}{}^{2}$$

$$\overline{\overline{W}}_{1,0}{}^{0} = \frac{3}{20}W_{3,2}{}^{0}$$

$$\overline{\overline{W}}_{1,0}{}^{2} = -6\frac{1}{\rho_0{}^2}\overline{W}_{2,2}{}^{0}$$

$$\overline{\overline{W}}_{1,0}{}^{4} = -3\overline{W}_{2,0}{}^{4}$$

$$\overline{\overline{W}}_{2,0}{}^{0} = \frac{1}{60}\rho_0{}^2W_{4,0}{}^{2}$$

$$\overline{\overline{W}}_{2,0}{}^{2} = \frac{1}{16(1-\rho_0{}^2)^4}[(12\rho_0{}^2+32\rho_0{}^4+4\rho_0{}^6)K' + (-31\rho_0{}^2-18\rho_0{}^4+\rho_0{}^6)E']$$

$$\overline{\overline{W}}_{2,0}{}^{4} = \frac{1}{16(1-\rho_0{}^2)^4}[(19\rho_0{}^2+26\rho_0{}^4+3\rho_0{}^6)K' + (-2-36\rho_0{}^2-10\rho_0{}^4)E']$$

$$\overline{\overline{\overline{W}}}_{2,0}{}^{6} = \frac{1}{10} W_{4,0}{}^{4}$$

$$\overline{\overline{\overline{W}}}_{2,2}{}^{0} = \frac{1}{10} \rho_0{}^4 W_{4,0}{}^{4}$$

$$\overline{\overline{\overline{W}}}_{2,2}{}^{2} = \frac{1}{16(1-\rho_0{}^2)^4} [(3\rho_0{}^4 + 26\rho_0{}^6 + 19\rho_0{}^8)K' + (-10\rho_0{}^4 - 36\rho_0{}^6 - 2\rho_0{}^8)E']$$

$$\overline{\overline{\overline{W}}}_{2,2}{}^{4} = \frac{1}{16(1-\rho_0{}^2)^4} [(4\rho_0{}^4 + 32\rho_0{}^6 + 12\rho_0{}^8)K' + (\rho_0{}^2 - 18\rho_0{}^4 - 31\rho_0{}^6)E']$$

$$\overline{\overline{\overline{W}}}_{2,2}{}^{6} = \frac{1}{10} W_{4,2}{}^{4}$$

$$\overline{\overline{\overline{W}}}_{3,0}{}^{0} = \frac{1}{140} \rho_0{}^2 W_{5,0}{}^{2}$$

$$\overline{\overline{\overline{W}}}_{3,0}{}^{2} = \frac{1}{80(1-\rho_0{}^2)^5} [(60\rho_0{}^2 + 241\rho_0{}^4 + 82\rho_0{}^6 + \rho_0{}^8)K' + (-167\rho_0{}^2 - 220\rho_0{}^4 + 5\rho_0{}^6 - 2\rho_0{}^8)E']$$

$$\overline{\overline{\overline{W}}}_{3,0}{}^{4} = \frac{1}{16(1-\rho_0{}^2)^5} [(31\rho_0{}^2 + 74\rho_0{}^4 + 23\rho_0{}^6)K' + (-2 - 69\rho_0{}^2 - 56\rho_0{}^4 - \rho_0{}^6)E']$$

$$\overline{\overline{\overline{W}}}_{3,0}{}^{6} = -3\overline{\overline{W}}_{4,0}{}^{6}$$

$$\overline{\overline{\overline{W}}}_{3,2}{}^{0} = \frac{1}{70} \rho_0{}^4 W_{5,0}{}^{4}$$

$$\overline{\overline{\overline{W}}}_{3,2}{}^{2} = \frac{1}{80(1-\rho_0{}^2)^5} [(15\rho_0{}^4 + 184\rho_0{}^6 + 187\rho_0{}^8 - 2\rho_0{}^{10})K' + (-53\rho_0{}^4 - 298\rho_0{}^6 - 37\rho_0{}^8 + 4\rho_0{}^{10})E']$$

$$\overline{\overline{\overline{W}}}_{3,2}{}^{4} = -\frac{1}{\rho_0{}^2} \overline{\overline{W}}_{4,4}{}^{2}$$

$$\overline{\overline{\overline{W}}}_{3,2}{}^{6} = -3\overline{\overline{W}}_{4,2}{}^{6}$$

$$\overline{\overline{\overline{W}}}_{4,0}{}^{0} = \frac{1}{280} \rho_0{}^2 W_{6,0}{}^{2}$$

$$\overline{\overline{\overline{W}}}_{4,0}{}^{2} = \frac{1}{160(1-\rho_0{}^2)^6} [(120\rho_0{}^2 + 687\rho_0{}^4 + 441\rho_0{}^6 + 33\rho_0{}^8 - \rho_0{}^{10})K' \\ + (-354\rho_0{}^2 - 834\rho_0{}^4 - 84\rho_0{}^6 - 10\rho_0{}^8 + 2\rho_0{}^{10})E']$$

$$\overline{\overline{\overline{W}}}_{4,0}{}^{4} = \frac{1}{192(1-\rho_0{}^2)^6} [(552\rho_0{}^2 + 2{,}034\rho_0{}^4 + 1{,}188\rho_0{}^6 + 66\rho_0{}^8)K' \\ + (-24 - 1{,}365\rho_0{}^2 - 2{,}187\rho_0{}^4 - 267\rho_0{}^6 + 3\rho_0{}^8)E']$$

$$\overline{\overline{\overline{W}}}_{4,0}{}^{6} = \frac{1}{160(1-\rho_0{}^2)^6} [(231\rho_0{}^2 + 705\rho_0{}^4 + 329\rho_0{}^6 + 15\rho_0{}^8)K' + (-12 - 546\rho_0{}^2 - 664\rho_0{}^4 - 58\rho_0{}^6)E']$$

$$\overline{\overline{\overline{W}}}_{4,0}{}^{8} = \frac{3}{56} W_{6\,0}{}^{6}$$

$$\overline{\overline{W}}_{4,2}{}^0 = \frac{1}{4{,}480(1-\rho_0{}^2)^6}\,[(435\rho_0{}^6+797\rho_0{}^8+45\rho_0{}^{10}+3\rho_0{}^{12})K' + (-30\rho_0{}^4-934\rho_0{}^6-352\rho_0{}^8+42\rho_0{}^{10}-6\rho_0{}^{12})E']$$

$$\overline{\overline{W}}_{4,2}{}^2 = \frac{1}{160(1-\rho_0{}^2)^6}\,[(30\rho_0{}^4+498\rho_0{}^6+708\rho_0{}^8+42\rho_0{}^{10}+2\rho_0{}^{12})K' + (-111\rho_0{}^4-909\rho_0{}^6-285\rho_0{}^8+29\rho_0{}^{10}-4\rho_0{}^{12})E']$$

$$\overline{\overline{W}}_{4,2}{}^4 = \frac{1}{192(1-\rho_0{}^2)^6}\,[(129\rho_0{}^4+1{,}791\rho_0{}^6+1{,}791\rho_0{}^8+129\rho_0{}^{10})K' + (12\rho_0{}^2-546\rho_0{}^4-2{,}772\rho_0{}^6-546\rho_0{}^8+12\rho_0{}^{10})E']$$

$$\overline{\overline{W}}_{4,2}{}^6 = \frac{1}{160(1-\rho_0{}^2)^6}\,[(2\rho_0{}^2+42\rho_0{}^4+708\rho_0{}^6+498\rho_0{}^8+30\rho_0{}^{10})K' + (-4+29\rho_0{}^2-285\rho_0{}^4-909\rho_0{}^6-111\rho_0{}^8)E']$$

$$\overline{\overline{W}}_{4,2}{}^8 = \frac{3}{56}\,W_{6,2}{}^6$$

$$\overline{\overline{W}}_{4,4}{}^0 = \frac{1}{4{,}480(1-\rho_0{}^2)^6}\,[(270\rho_0{}^8+740\rho_0{}^{10}+270\rho_0{}^{12})K' + (-15\rho_0{}^6-625\rho_0{}^8-625\rho_0{}^{10}-15\rho_0{}^{12})E']$$

$$\overline{\overline{W}}_{4,4}{}^2 = \frac{1}{160(1-\rho_0{}^2)^6}\,[(15\rho_0{}^6+329\rho_0{}^8+705\rho_0{}^{10}+231\rho_0{}^{12})K' + (-58\rho_0{}^6-664\rho_0{}^8-546\rho_0{}^{10}-12\rho_0{}^{12})E']$$

$$\overline{\overline{W}}_{4,4}{}^4 = \frac{1}{192(1-\rho_0{}^2)^6}\,[(66\rho_0{}^6+1{,}188\rho_0{}^8+2{,}034\rho_0{}^{10}+552\rho_0{}^{12})K' + (3\rho_0{}^4-267\rho_0{}^6-2{,}187\rho_0{}^8-1{,}365\rho_0{}^{10}-24\rho_0{}^{12})E']$$

$$\overline{\overline{W}}_{4,4}{}^6 = \frac{1}{160(1-\rho_0{}^2)^6}\,[(-\rho_0{}^4+33\rho_0{}^6+441\rho_0{}^8+687\rho_0{}^{10}+120\rho_0{}^{12})K' + (2\rho_0{}^2-10\rho_0{}^4-84\rho_0{}^6-834\rho_0{}^8-354\rho_0{}^{10})E']$$

$$\overline{\overline{W}}_{4,4}{}^8 = \frac{3}{56}\,W_{6,4}{}^6$$

$$\overline{\overline{W}}_{5,0}{}^0 = \frac{1}{504}\,\rho_0{}^2 W_{7,0}{}^2$$

$$\overline{\overline{W}}_{5,0}{}^2 = \frac{1}{1{,}120(1-\rho_0{}^2)^7}\,[(840\rho_0{}^2+6{,}549\rho_0{}^4+6{,}765\rho_0{}^6+1{,}227\rho_0{}^8-25\rho_0{}^{10}+4\rho_0{}^{12})K' + (-2{,}598\rho_0{}^2-9{,}609\rho_0{}^4-3{,}033\rho_0{}^6-163\rho_0{}^8+51\rho_0{}^{10}-8\rho_0{}^{12})E']$$

$$\overline{\overline{W}}_{5,0}{}^4 = \frac{1}{960(1-\rho_0{}^2)^7}\,[(3{,}840\rho_0{}^2+20{,}253\rho_0{}^4+19{,}017\rho_0{}^6+2{,}967\rho_0{}^8+3\rho_0{}^{10})K' + (-120-10{,}251\rho_0{}^2-27{,}177\rho_0{}^4-8{,}499\rho_0{}^6-27\rho_0{}^8-6\rho_0{}^{10})E']$$

$$\overline{\overline{W}}_{5,0}{}^6 = \frac{1}{480(1-\rho_0{}^2)^7}\,[(1{,}605\rho_0{}^2+7{,}185\rho_0{}^4+5{,}775\rho_0{}^6+795\rho_0{}^8)K' + (-60-4{,}125\rho_0{}^2-8{,}865\rho_0{}^4-2{,}295\rho_0{}^6-15\rho_0{}^8)E']$$

$$\overline{\overline{W}}_{5,0}{}^{8} = -3\overline{\overline{W}}_{6,0}{}^{8}$$

$$\overline{\overline{W}}_{5,2}{}^{0} = \frac{1}{840}\rho_0{}^4 W_{7,0}{}^{4}$$

$$\overline{\overline{W}}_{5,2}{}^{2} = \frac{1}{1{,}120(1-\rho_0{}^2)^7}[(210\rho_0{}^4+4{,}551\rho_0{}^6+8{,}871\rho_0{}^8+1{,}677\rho_0{}^{10}+59\rho_0{}^{12}-8\rho_0{}^{14})K'$$
$$+(-807\rho_0{}^4-9{,}123\rho_0{}^6-5{,}661\rho_0{}^8+335\rho_0{}^{10}-120\rho_0{}^{12}+16\rho_0{}^{14})E']$$

$$\overline{\overline{W}}_{5,2}{}^{4} = \frac{1}{960(1-\rho_0{}^2)^7}[(915\rho_0{}^4+16{,}347\rho_0{}^6+24{,}093\rho_0{}^8+4{,}713\rho_0{}^{10}+12\rho_0{}^{12})K'$$
$$+(60\rho_0{}^2-3{,}879\rho_0{}^4-28{,}923\rho_0{}^6-13{,}341\rho_0{}^8+27\rho_0{}^{10}-24\rho_0{}^{12})E']$$

$$\overline{\overline{W}}_{5,2}{}^{6} = \frac{1}{480(1-\rho_0{}^2)^7}[(6\rho_0{}^2+339\rho_0{}^4+6{,}339\rho_0{}^6+7{,}401\rho_0{}^8+1{,}275\rho_0{}^{10})K'$$
$$+(-12+111\rho_0{}^2-1{,}863\rho_0{}^4-10{,}029\rho_0{}^6-3{,}537\rho_0{}^8-30\rho_0{}^{10})E']$$

$$\overline{\overline{W}}_{5,2}{}^{8} = -3\overline{\overline{W}}_{6,2}{}^{8}$$

$$\overline{\overline{W}}_{5,4}{}^{0} = \frac{1}{168}\rho_0{}^6 W_{7,0}{}^{6}$$

$$\overline{\overline{W}}_{5,4}{}^{2} = \frac{1}{1{,}120(1-\rho_0{}^2)^7}[(105\rho_0{}^6+2{,}993\rho_0{}^8+8{,}407\rho_0{}^{10}+3{,}867\rho_0{}^{12}-12\rho_0{}^{14})K'$$
$$+(-421\rho_0{}^6-6{,}537\rho_0{}^8-8{,}039\rho_0{}^{10}-387\rho_0{}^{12}+24\rho_0{}^{14})E']$$

$$\overline{\overline{W}}_{5,4}{}^{4} = \frac{1}{960(1-\rho_0{}^2)^7}[(465\rho_0{}^6+10{,}821\rho_0{}^8+24{,}939\rho_0{}^{10}+9{,}879\rho_0{}^{12}-24\rho_0{}^{14})K'$$
$$+(15\rho_0{}^4-1{,}917\rho_0{}^6-21{,}939\rho_0{}^8-21{,}423\rho_0{}^{10}-864\rho_0{}^{12}+48\rho_0{}^{14})E']$$

$$\overline{\overline{W}}_{5,4}{}^{6} = -\frac{2}{5}\frac{1}{\rho_0{}^2}\overline{\overline{W}}_{6,6}{}^{4}$$

$$\overline{\overline{W}}_{5,4}{}^{8} = -3\overline{\overline{W}}_{6,4}{}^{8}$$

Some of the $W_{n,m}{}^{P}$, $\overline{W}_{n,m}{}^{P}$, $\overline{\overline{W}}_{n,m}{}^{P}$ given here are also given in Refs. 1 and 2. It is to be noted however that our expressions differ by constant factors from those in Refs. 1 and 2. These factors have been introduced here to provide some simplification of the matrix equations for A_0, A_1, etc., at the beginning of this Appendix. The method of evaluation of the $W_{n,m}{}^{P}$, $\overline{W}_{n,m}{}^{P}$, $\overline{\overline{W}}_{n,m}{}^{P}$ is explained in Appendix IV.

It is possible to solve these matrix equations analytically so as to obtain the ϕ_j of Equation (100) as analytical functions of x, y, M and ρ_0. This process is very tedious, as it involves the inversion and multiplication of matrices the elements of which are functions of ρ_0. The matrices obtained from these matrix operations have elements which are more complicated functions of ρ_0 than are those of the original matrices. The final results are rather complicated functions and in order that these be of practical value numerical values for them, at least in part, must be given.

If the end results are to be numerical values it is just as well to solve these matrix equations for a set of values of ρ_0. All the matrices on which operations have to be performed are then numerical matrices and advantage may be taken of the matrix routines worked out for the DEUCE digital computer for performing these operations. The solution of these equations in the present work has been obtained by this means.

APPENDIX IV

The Evaluation of Some Integrals

The integral expressions for the $W_{n,m}{}^p$, $\overline{W}_{n,m}{}^p$ and $\overline{\overline{W}}_{n,m}{}^p$ occurring in Appendix III are:

$$W_{n,m}{}^p = -\frac{(n+2)\beta^{2+m-p}}{\pi x^n p\,!}\int_{-\lambda}^{+\lambda}\frac{\sigma^m\sqrt{(\lambda^2-\sigma^2)}}{(1-\beta^2\sigma^2)^{3/2}}\,d\sigma \lim_{\theta\to 0}\frac{\partial^p}{\partial\theta^p}\int_0^{\cosh^{-1}\frac{S}{N_0}} N_0{}^n \times$$
$$\times\left(\frac{S}{N_0}-\cosh\tau\right)^{n+1}\cosh\tau\,d\tau \tag{111}$$

$$\overline{W}_{n,m}{}^p = \frac{\beta^{2+m-p}}{\pi x^{n+2}p\,!}\int_{-\lambda}^{+\lambda}\frac{\sigma^m\sqrt{(\lambda^2-\sigma^2)}}{(1-\beta^2\sigma^2)^{1/2}}\,d\sigma \lim_{\theta\to 0}\frac{\partial^p}{\partial\theta^p}\int_0^{\cosh^{-1}\frac{S}{N_0}}\left(S-N_0\cosh\tau\right)^{n+1}\times$$
$$\times\,[2S-(n+4)N_0\cosh\tau]\sinh^2\tau\,d\tau \tag{112}$$

$$\overline{\overline{W}}_{n,m}{}^p = \frac{\beta^{2+m-p}}{\pi x^{n+4}p\,!}\int_{-\lambda}^{+\lambda}\sigma^m\sqrt{(\lambda^2-\sigma^2)}\,(1-\beta^2\sigma^2)^{1/2}\,d\sigma \lim_{\theta\to 0}\frac{\partial^p}{\partial\theta^p}\int_0^{\cosh^{-1}\frac{S}{N_0}} N_0{}^2\left(S-N_0\cosh\tau\right)^{n+1}\times$$
$$\times\,[4S-(n+6)N_0\cosh\tau]\sinh^4\tau\,d\tau \tag{113}$$

The procedure of Appendix II leads directly to these integrals. The values of these integrals are zero whenever $(m-p)$ is odd, for the inner integral is an odd function of σ when p is odd, and an even function of σ when p is even.

The inner integrals in the above expressions are quite simple to evaluate. Integrals of the form

$$I_n = \int_{-\lambda}^{+\lambda}\frac{\sqrt{(\lambda^2-\sigma^2)}}{(1-\beta^2\sigma^2)^n}\frac{d\sigma}{\sigma^2} \tag{114}$$

$$L_n = \beta^2\int_{-\lambda}^{+\lambda}\frac{\sqrt{(\lambda^2-\sigma^2)}}{(1-\beta^2\sigma^2)^n}\,d\sigma \tag{115}$$

and

$$J_n = \frac{1}{\lambda^2}\int_{-\lambda}^{+\lambda}\frac{\sqrt{(\lambda^2-\sigma^2)}}{(1-\beta^2\sigma^2)^{n+1/2}}\cosh^{-1}\left(\frac{1}{\beta\sigma}\right)d\sigma \tag{116}$$

then have to be evaluated in order to obtain the outer integrals.

Now

$$I_n = I_{n-1}+L_n \tag{117}$$

and

$$L_n = M_{n-1}-\rho_0'^2 M_n \tag{118}$$

where

$$M_n = \int_{-\lambda}^{+\lambda}\frac{1}{(1-\beta^2\sigma^2)^n}\frac{d\sigma}{\sqrt{(\lambda^2-\sigma^2)}}, \tag{119}$$

$$\rho_0 = \beta\lambda, \tag{120}$$

and

$$\rho_0' = \sqrt{(1-\rho_0^2)}. \tag{121}$$

The M_n satisfy the recurrence relationship:

$$(2n-2)\rho_0'^2 M_n = (2n-3)(2-\rho_0^2)M_{n-1} - (2n-4)M_{n-2} \tag{122}$$

Then knowing that

$$\left.\begin{aligned} I_0 &= -\pi \\ M_0 &= \pi \\ M_1 &= \pi \frac{1}{\rho_0'}, \end{aligned}\right\} \tag{123}$$

we can find I_n and L_n for any n using the above formulae in a step by step procedure.

By substitution of $\quad \sigma = \lambda snu$ (124)

into integral (116) we obtain

$$J_n = \int_{-K(\rho_0)}^{K(\rho_0)} \log\left(\frac{1+dn\,u}{1-dn\,u}\right)\frac{cn^2 u}{dn^{2n} u}\,du \tag{125}$$

where $K(\rho_0)$ is the complete elliptic integral of the first kind.

The J_n satisfy the recurrence relationship:

$$(2n-1)\rho_0'^2 J_n = [4n-6-(2n-2)\rho_0^2]\,J_{n-1} - (2n-5)J_{n-2} - 2\rho_0^2 P_n \tag{126}$$

where

$$P_n = \int_{-K(\rho_0)}^{K(\rho_0)} \frac{cn^4 u}{dn^{2n-1} u}\,du\,. \tag{127}$$

The P_n satisfy the recurrence relationship:

$$(2n-2)\rho_0'^2 P_n = [4n-10-(2n-3)\rho_0^2]P_{n-1} - (2n-8)P_{n-2} \tag{128}$$

Then knowing that

$$\left.\begin{aligned} J_0 &= 2\pi K(\rho_0') - \frac{2\pi}{\rho_0^2}[E(\rho_0')-1] \\ J_1 &= \frac{2\pi}{\rho_0^2}[E(\rho_0')-\rho_0'] \end{aligned}\right\} \tag{129}$$

$$\left.\begin{aligned} P_0 &= \frac{3\pi}{8} \\ P_1 &= +\frac{\pi\rho_0'^3}{\rho_0^4} - \frac{\pi}{2\rho_0^4}(2-3\rho_0^2) \end{aligned}\right\} \tag{130}$$

where $K(\rho_0')$ and $E(\rho_0')$ are complete elliptic integrals of the first and second kinds respectively, we can find J_n for any n using the above formulae in a step by step procedure. (*Note*: the Formulae (129) can be obtained quite easily from the formulae 804.01 and 804.02 of Ref. 4.)

We shall illustrate the process by evaluating $W_{2,2}{}^2$.

We must first of all evaluate

$$\lim_{\theta\to 0}\frac{\partial^2}{\partial\theta^2}\int_0^{\cosh^{-1}\frac{S}{N_0}} N_0^2\left(\frac{S}{N_0} - \cosh\tau\right)^3 \cosh\tau\,d\tau \tag{131}$$

and this according to Appendix II is

$$\lim_{\theta\to 0}\int_0^{\cosh^{-1}\frac{S}{N_0}} \frac{\partial^2}{\partial\theta^2}\left[N_0^2\left(\frac{S}{N_0}-\cosh\tau\right)^3\right]\cosh\tau\,d\tau =$$

$$= \frac{\beta^2x^2}{(1-\beta^2\sigma^2)^{3/2}}\left(7-\frac{19}{4}\frac{1}{\beta^2\sigma^2}+\frac{3}{2}\frac{1}{\beta^4\sigma^4}\right)$$

$$-\frac{\beta^2x^2}{(1-\beta^2\sigma^2)^2}(3\beta^2\sigma^2+\tfrac{3}{4})\cosh^{-1}\left(\frac{1}{\beta\sigma}\right). \tag{132}$$

Therefore

$$W_{2,2}^{\,2} = \frac{-4\beta^2}{\pi 2!}\int_{-\lambda}^{+\lambda}\frac{\beta^2\sigma^2\sqrt{(\lambda^2-\sigma^2)}}{(1-\beta^2\sigma^2)^3}\left[7-\frac{19}{4}\frac{1}{\beta^2\sigma^2}+\frac{3}{2}\frac{1}{\beta^4\sigma^4}\right]d\sigma$$

$$+\frac{4\beta^2}{\pi 2!}\int_{-\lambda}^{+\lambda}\frac{\beta^2\sigma^2\sqrt{(\lambda^2-\sigma^2)}}{(1-\beta^2\sigma^2)^{3+1/2}}(3\beta^2\sigma^2+\tfrac{3}{4})\cosh^{-1}\frac{1}{\beta\sigma}\,d\sigma$$

$$= -\frac{2}{\pi}\left[\frac{9}{4}L_3-7L_2+\frac{3}{2}I_3-\frac{15}{4}\rho_0^2J_3+\frac{27}{4}\rho_0^2J_2-3\rho_0^2J_1\right] \tag{133}$$

Then using formulae of this Appendix we obtain

$$\left.\begin{aligned}
L_2 &= \frac{\pi\rho_0^2}{2\rho_0'}\\
L_3 &= \frac{\pi\rho_0^2}{8\rho_0'^3}[4-3\rho_0^2]\\
I_3 &= -\frac{\pi}{8\rho_0'^3}[8-24\rho_0^2+15\rho_0^4]\\
J_1 &= \frac{2\pi}{\rho_0^2}[E(\rho_0')-\rho_0']\\
J_2 &= \frac{2\pi}{3\rho_0'^2}K(\rho_0')+\frac{2\pi}{3\rho_0^2\rho_0'^2}E(\rho_0')[1-2\rho_0^2]+\frac{\pi}{3\rho_0^2\rho_0'}[-2+5\rho_0^2]\\
J_3 &= \frac{4\pi(3-2\rho_0^2)}{15\rho_0'^4}+\frac{2\pi}{15\rho_0^2\rho_0'^4}E(\rho_0')[3-13\rho_0^2+8\rho_0^4]-\frac{\pi}{12\rho_0^2\rho_0'}[24-128\rho_0^2+84\rho_0^4]
\end{aligned}\right\} \tag{134}$$

and on substitution into (133) we obtain

$$W_{2,2}^{\,2} = \frac{1}{\rho_0'^4}[(-3\rho_0^2+5\rho_0^4)K(\rho_0')+(6-10\rho_0^2-2\rho_0^4)E(\rho_0')] \tag{135}$$

The Expression (135) for $W_{2,2}^{\,2}$ contains no terms which are not multiplied by either $K(\rho_0')$ or $E(\rho_0')$ even though such terms occur in the Expressions (134). It is observed that the expressions for all the $W_{n,m}^{\,p}$, $\overline{W}_{n,m}^{\,p}$ and $\overline{\overline{W}}_{n,m}^{\,p}$ contain no terms which are not multiplied by either $K(\rho_0')$ or $E(\rho_0')$. In deriving these expressions therefore such terms can be ignored completely and it is possible to obtain the correct expressions by considering only the J_n in equations like (133) and assuming that the recurrence relationship for it is given by (126) with the term $-2\rho_0^2P_n$ omitted.

TABLE 1

ρ_0	a_0'	a_1'	a_2'	a_3'	a_4'	a_5'
0·00	1·0	0	0	0·083333	0	0
0·05	0·995166	−0·008317	−0·001263	0·080536	+0·000088	−0·001508
0·10	0·984258	−0·025541	−0·002774	0·075042	+0·000392	−0·003954
0·15	0·969438	−0·046638	−0·003110	0·068639	+0·000795	−0·006167
0·20	0·951926	−0·068847	−0·001912	0·062167	+0·001108	−0·007819
0·30	0·912011	−0·110681	+0·004279	0·050426	+0·001016	−0·009534
0·40	0·869069	−0·144886	+0·013095	0·040903	−0·000058	−0·009859
0·50	0·825726	−0·170508	+0·022207	0·033434	−0·001713	−0·009502
0·60	0·783484	−0·188515	+0·030380	0·027605	−0·003536	−0·008860
0·70	0·743167	−0·200386	+0·037161	0·023031	−0·005271	−0·008126
0·80	0·705177	−0·207540	+0·042507	0·019405	−0·006795	−0·007392
0·90	0·669662	−0·211168	+0·046559	0·016500	−0·008068	−0·006699
1·00	0·636620	−0·212207	+0·049515	0·014147	−0·009095	−0·006063

ρ_0	a_6'	a_7'	a_8'	a_9'	a_{10}'	a_{11}'
0·00	0	0	0·003125	0	0	0
0·05	−0·000004	−0·000089	0·002883	0·000000	+0·000007	−0·000074
0·10	−0·000029	−0·000076	0·002469	+0·000002	+0·000023	−0·000165
0·15	−0·000084	+0·000104	0·002056	+0·000005	+0·000027	−0·000222
0·20	−0·000159	+0·000371	0·001696	+0·000015	+0·000015	−0·000246
0·30	−0·000263	+0·000899	0·001155	+0·000037	−0·000044	−0·000240
0·40	−0·000209	+0·001261	0·000800	+0·000044	−0·000107	−0·000207
0·50	−0·000005	+0·001459	0·000568	+0·000027	−0·000157	−0·000172
0·60	+0·000284	+0·001545	0·000411	−0·000009	−0·000191	−0·000140
0·70	+0·000597	+0·001560	0·000304	−0·000054	−0·000213	−0·000114
0·80	+0·000899	+0·001533	0·000228	−0·000101	−0·000226	−0·000092
0·90	+0·001168	+0·001481	0·000174	−0·000146	−0·000233	−0·000075
1·00	+0·001398	+0·001415	0·000135	−0·000186	−0·000235	−0·000061

ρ_0	a_2''	a_3''	a_4''	a_5''	a_6''	a_7''
0·00	0	−0·083333	0	0	0	0
0·05	0·001955	−0·082291	−0·000235	0·001741	0·000025	+0·000246
0·10	0·005813	−0·080105	−0·001042	0·004956	0·000120	+0·000420
0·15	0·010258	−0·077353	−0·002403	0·008431	0·000325	+0·000278
0·20	0·014649	−0·074329	−0·004159	0·011665	0·000662	−0·000145
0·30	0·022188	−0·068076	−0·008120	0·016797	0·001650	−0·001439
0·40	0·027667	−0·062026	−0·011847	0·020128	0·002814	−0·002832
0·50	0·031332	−0·056407	−0·014902	0·022029	0·003926	−0·004033
0·60	0·033615	−0·051282	−0·017247	0·022895	0·004879	−0·004956
0·70	0·034902	−0·046648	−0·018899	0·023043	0·005643	−0·005611
0·80	0·035490	−0·042476	−0·020049	0·022710	0·006230	−0·006034
0·90	0·035559	−0·038730	−0·020790	0·022063	0·006664	−0·006270
1·00	0·035368	−0·035368	−0·021221	0·021221	0·006973	−0·006366

TABLE 1—*continued*

ρ_0	a_8''	a_9''	a_{10}''	a_{11}''	a_6'''	a_7'''
0·00	−0·00625	0	0	0	0	0
0·05	−0·005933	−0·000003	−0·000021	0·000166	0·000015	−0·000212
0·10	−0·005353	−0·000013	−0·000088	0·000404	0·000066	−0·000592
0·15	−0·004724	−0·000034	−0·000157	0·000589	0·000150	−0·000988
0·20	−0·004129	−0·000076	−0·000173	0·000707	0·000252	−0·001342
0·30	−0·003130	−0·000239	−0·000072	0·000787	0·000469	−0·001874
0·40	−0·002386	−0·000477	+0·000160	0·000756	0·000657	−0·002188
0·50	−0·001840	−0·000740	+0·000432	0·000684	0·000802	−0·002345
0·60	−0·001438	−0·000988	+0·000688	0·000602	0·000905	−0·002394
0·70	−0·001139	−0·001204	+0·000905	0·000523	0·000976	−0·002374
0·80	−0·000913	−0·001380	+0·001077	0·000452	0·001021	−0·002312
0·90	−0·000740	−0·001520	+0·001207	0·000390	0·001048	−0·002224
1·00	−0·000606	−0·001628	+0·001291	0·000337	0·001061	−0·002122

ρ_0	a_8'''	a_9'''	a_{10}'''	a_{11}'''		
0·00	0·003125	0	0	0		
0·05	0·003065	−0·000006	0·000016	−0·000100		
0·10	0·002946	−0·000026	0·000127	−0·000264		
0·15	0·002803	−0·000064	0·000282	−0·000429		
0·20	0·002653	−0·000117	0·000461	−0·000570		
0·30	0·002361	−0·000249	0·000817	−0·000767		
0·40	0·002096	−0·000383	0·001108	−0·000871		
0·50	0·001862	−0·000497	0·001313	−0·000913		
0·60	0·001657	−0·000587	0·001444	−0·000915		
0·70	0·001477	−0·000654	0·001516	−0·000891		
0·80	0·001320	−0·000703	0·001544	−0·000853		
0·90	0·001182	−0·000736	0·001541	−0·000808		
1·00	0·001061	−0·000758	0·001516	−0·000758		

ρ_0	b_0'	b_1'	b_2'	b_3'	b_4'	b_5'
0·00	1·0	0	0·166667	0	+0·083333	0
0·05	0·986849	−0·010843	0·161071	−0·000999	+0·076011	+0·000073
0·10	0·958717	−0·031089	0·150084	−0·001597	+0·063180	+0·000277
0·15	0·922800	−0·052859	0·137277	−0·000726	+0·050138	+0·000459
0·20	0·883079	−0·072671	0·124333	+0·001413	+0·038709	+0·000474
0·30	0·801330	−0·102123	0·100852	+0·007328	+0·021823	−0·000035
0·40	0·724183	−0·118697	0·081806	+0·012920	+0·011325	−0·000894
0·50	0·655218	−0·126094	0·066868	+0·017069	+0·004926	−0·001732
0·60	0·594969	−0·127755	0·055211	+0·019774	+0·001027	−0·002401
0·70	0·542781	−0·126064	0·046062	+0·021348	−0·001346	−0·002881
0·80	0·497637	−0·122525	0·038811	+0·022122	−0·002771	−0·003201
0·90	0·458494	−0·118050	0·033000	+0·022355	−0·003597	−0·003397
1·00	0·424413	−0·113177	0·028294	+0·022231	−0·004042	−0·003503

TABLE 1—*continued*

ρ_0	b_6'	b_7'	b_8'	b_9'	b_{10}'	b_{11}'
0·00	0	0·0125	0	0	+0·003125	0
0·05	−0·001866	0·011530	−0·000004	−0·000054	+0·002514	0·000000
0·10	−0·004259	0·009876	−0·000020	+0·000039	+0·001645	+0·000001
0·15	−0·005753	0·008225	−0·000057	+0·000240	+0·000948	+0·000007
0·20	−0·006337	0·006784	−0·000082	+0·000447	+0·000468	+0·000010
0·30	−0·005938	0·004619	−0·000075	+0·000680	−0·000044	+0·000013
0·40	−0·004815	0·003202	+0·000012	+0·000724	−0·000237	+0·000007
0·50	−0·003665	0·002270	+0·000129	+0·000676	−0·000292	−0·000005
0·60	−0·002680	0·001644	+0·000238	+0·000591	−0·000290	−0·000019
0·70	−0·001885	0·001215	+0·000326	+0·000496	−0·000265	−0·000032
0·80	−0·001260	0·000913	+0·000391	+0·000403	−0·000233	−0·000042
0·90	−0·000776	0·000697	+0·000437	+0·000317	−0·000201	−0·000049
1·00	−0·000404	0·000539	+0·000466	+0·000241	−0·000171	−0·000054

ρ_0	b_{12}'	b_{13}'	b_{14}'	b_3''	b_4''	b_5''
0·00	0	0	0·000372	0	−0·083333	0
0·05	+0·000005	−0·000086	0·000324	0·001720	−0·080550	−0·000185
0·10	+0·000012	−0·000154	0·000252	0·004771	−0·075148	−0·000802
0·15	+0·000010	−0·000163	0·000190	0·007856	−0·068922	−0·001752
0·20	−0·000013	−0·000145	0·000143	0·010490	−0·062664	−0·002835
0·30	−0·000050	−0·000087	0·000082	0·014068	−0·051279	−0·004820
0·40	−0·000072	−0·000040	0·000049	0·015820	−0·041898	−0·006219
0·50	−0·000080	−0·000010	0·000030	0·016430	−0·034378	−0·007050
0·60	−0·000080	+0·000009	0·000019	0·016382	−0·028387	−0·007466
0·70	−0·000075	+0·000019	0·000013	0·016003	−0·023604	−0·007612
0·80	−0·000068	+0·000024	0·000008	0·015442	−0·019766	−0·007588
0·90	−0·000059	+0·000025	0·000006	0·014805	−0·016666	−0·007461
1·00	−0·000050	+0·000026	0·000004	0·014147	−0·014147	−0·007276

ρ_0	b_6''	b_7''	b_8''	b_9''	b_{10}''	b_{11}''
0·00	0	−0·0125	0	0	−0·006250	0
0·05	0·002233	−0·011866	0·000016	+0·000183	−0·005434	−0·000001
0·10	0·005796	−0·010706	0·000081	+0·000156	−0·004140	−0·000007
0·15	0·008979	−0·009449	0·000222	−0·000194	−0·002957	−0·000021
0·20	0·011375	−0·008258	0·000433	−0·000663	−0·002008	−0·000048
0·30	0·013918	−0·006261	0·000933	−0·001654	−0·000771	−0·000131
0·40	0·014464	−0·004771	0·001381	−0·002352	−0·000118	−0·000223
0·50	0·013964	−0·003679	0·001706	−0·002737	+0·000211	−0·000300
0·60	0·012982	−0·002875	0·001915	−0·002892	+0·000361	−0·000356
0·70	0·011822	−0·002277	0·002033	−0·002894	+0·000430	−0·000393
0·80	0·010643	−0·001825	0·002089	−0·002803	+0·000443	−0·000416
0·90	0·009524	−0·001479	0·002104	−0·002648	+0·000431	−0·000429
1·00	0·008488	−0·001213	0·002088	−0·002493	+0·000404	−0·000432

TABLE 1—*continued*

ρ_0	b_{12}''	b_{13}''	b_{14}''	b_8'''	b_9'''	b_{10}'''
0·00	0	0	−0·000744	0	0	0·003125
0·05	−0·000022	+0·000199	−0·000670	0·000009	−0·000200	0·002965
0·10	−0·000050	+0·000412	−0·000551	0·000040	−0·000465	0·002681
0·15	−0·000051	+0·000509	−0·000440	0·000086	−0·000706	0·002374
0·20	−0·000047	+0·000512	−0·000349	0·000135	−0·000881	0·002083
0·30	+0·000087	+0·000411	−0·000221	0·000219	−0·001056	0·001594
0·40	+0·000233	+0·000283	−0·000144	0·000274	−0·001080	0·001225
0·50	+0·000349	+0·000176	−0·000096	0·000305	−0·001031	0·000949
0·60	+0·000427	+0·000097	−0·000065	0·000318	−0·000950	0·000742
0·70	+0·000473	+0·000041	−0·000046	0·000321	−0·000858	0·000586
0·80	+0·000492	+0·000002	−0·000033	0·000318	−0·000768	0·000467
0·90	+0·000490	−0·000025	−0·000024	0·000312	−0·000683	0·000375
1·00	+0·000488	−0·000039	−0·000018	0·000303	−0·000606	0·000303

ρ_0	b_{11}'''	b_{12}'''	b_{13}'''	b_{14}'''
0·00	0	0	0	0·000372
0·05	−0·000003	0·000038	−0·000121	0·000347
0·10	−0·000014	0·000098	−0·000298	0·000305
0·15	−0·000033	0·000194	−0·000435	0·000261
0·20	−0·000058	0·000292	−0·000521	0·000222
0·30	−0·000108	0·000442	−0·000582	0·000160
0·40	−0·000147	0·000520	−0·000563	0·000117
0·50	−0·000172	0·000547	−0·000510	0·000086
0·60	−0·000187	0·000541	−0·000448	0·000065
0·70	−0·000194	0·000517	−0·000387	0·000050
0·80	−0·000196	0·000483	−0·000332	0·000038
0·90	−0·000195	0·000447	−0·000282	0·000030
1·00	−0·000193	0·000409	−0·000241	0·000024

ρ_0	c_0'	c_1'	c_2'	c_3'	c_4'	c_5'
0·00	0	0·333333	0	0	0	0
0·05	0·000411	0·333331	−0·000016	−0·000013	0·000000	+0·000004
0·10	0·001501	0·333296	−0·000079	−0·000126	−0·000002	−0·000029
0·15	0·003089	0·333188	−0·000278	−0·000462	+0·000005	−0·000140
0·20	0·004973	0·332969	−0·000626	−0·001097	+0·000030	−0·000331
0·30	0·009077	0·332104	−0·001716	−0·003335	+0·000162	−0·000859
0·40	0·013089	0·330617	−0·003155	−0·006735	+0·000404	−0·001395
0·50	0·016717	0·328521	−0·004751	−0·011009	+0·000730	−0·001765
0·60	0·019876	0·325875	−0·006379	−0·015853	+0·001112	−0·001873
0·70	0·022570	0·322751	−0·007963	−0·021001	+0·001529	−0·001687
0·80	0·024839	0·319225	−0·009462	−0·026238	+0·001962	−0·001219
0·90	0·026730	0·315365	−0·010852	−0·031403	+0·002399	−0·000506
1·00	0·028294	0·311236	−0·012126	−0·036378	+0·002829	+0·000404

TABLE 1—*continued*

ρ_0	c_6'	c_7'	c_8'	c_9'	c_{10}'	c_{11}'
0·00	0·008333	0	0	0	0	0
0·05	0·008332	−0·000001	−0·000001	−0·000002	0·000000	0·000000
0·10	0·008316	0·000000	−0·000008	−0·000018	0·000000	0·000000
0·15	0·008273	+0·000001	−0·000019	−0·000059	0·000000	+0·000002
0·20	0·008195	+0·000001	−0·000024	−0·000124	0·000000	+0·000004
0·30	0·007936	−0·000008	0·000000	−0·000307	0·000000	+0·000009
0·40	0·007568	−0·000034	+0·000079	−0·000518	0·000002	+0·000008
0·50	0·007131	−0·000078	+0·000197	−0·000719	0·000006	+0·000001
0·60	0·006661	−0·000137	+0·000323	−0·000891	0·000013	−0·000015
0·70	0·006183	−0·000209	+0·000429	−0·001026	0·000022	−0·000034
0·80	0·005716	−0·000289	+0·000495	−0·001124	0·000034	−0·000053
0·90	0·005270	−0·000377	+0·000511	−0·001188	0·000047	−0·000069
1·00	0·004850	−0·000470	+0·000470	−0·001225	0·000062	−0·000076

ρ_0	c_{12}'	c_{13}'	c_{14}'	c_{15}'	c_{16}'	c_{17}'
0·00	0	0·000149	0	0	0	0
0·05	0·000000	0·000149	0·000000	0·000000	0·000000	0·000000
0·10	−0·000004	0·000148	0·000000	0·000000	0·000000	−0·000001
0·15	−0·000011	0·000144	0·000000	0·000000	0·000000	−0·000002
0·20	−0·000019	0·000140	0·000000	0·000000	0·000000	−0·000004
0·30	−0·000030	0·000127	0·000000	−0·000001	+0·000003	−0·000008
0·40	−0·000028	0·000112	0·000000	−0·000002	+0·000005	−0·000012
0·50	−0·000012	0·000096	0·000000	−0·000002	+0·000007	−0·000014
0·60	+0·000015	0·000082	−0·000001	−0·000001	+0·000007	−0·000016
0·70	+0·000047	0·000069	−0·000002	0·000000	+0·000005	−0·000016
0·80	+0·000080	0·000058	−0·000003	+0·000002	+0·000002	−0·000015
0·90	+0·000110	0·000049	−0·000005	+0·000005	−0·000001	−0·000014
1·00	+0·000138	0·000041	−0·000007	+0·000007	−0·000006	−0·000013

ρ_0	c_4''	c_5''	c_6''	c_7''	c_8''	c_9''
0·00	0	0	−0·008333	0	0	0
0·05	0·000001	−0·000008	−0·008333	0·000000	+0·000006	0·000002
0·10	0·000009	−0·000011	−0·008329	−0·000002	+0·000013	0·000013
0·15	0·000032	+0·000019	−0·008319	−0·000007	+0·000039	0·000045
0·20	0·000070	+0·000101	−0·008298	−0·000019	+0·000074	0·000101
0·30	0·000185	+0·000438	−0·008223	−0·000063	+0·000139	0·000281
0·40	0·000333	+0·000983	−0·008103	−0·000130	+0·000150	0·000525
0·50	0·000494	+0·001679	−0·007947	−0·000213	+0·000080	0·000801
0·60	0·000656	+0·002464	−0·007762	−0·000304	−0·000081	0·001084
0·70	0·000812	+0·003287	−0·007556	−0·000400	−0·000320	0·001356
0·80	0·000958	+0·004110	−0·007335	−0·000493	−0·000622	0·001606
0·90	0·001092	+0·004909	−0·007105	−0·000583	−0·000947	0·001831
1·00	0·001213	+0·005659	−0·006871	−0·000674	−0·001347	0·002021

TABLE 1—*continued*

ρ_0	c_{10}''	c_{11}''	c_{12}''	c_{13}''	c_{14}''	c_{15}''
0·00	0	0	0	−0·000298	0	0
0·05	0·000000	0·000000	−0·000001	−0·000297	0·000000	0·000000
0·10	0·000000	0·000000	+0·000005	−0·000296	0·000000	0·000000
0·15	0·000001	−0·000003	+0·000018	−0·000293	0·000000	0·000000
0·20	0·000003	−0·000009	+0·000035	−0·000288	0·000000	0·000000
0·30	0·000012	−0·000033	+0·000075	−0·000273	−0·000002	0·000003
0·40	0·000028	−0·000062	+0·000102	−0·000252	−0·000004	0·000008
0·50	0·000050	−0·000087	+0·000106	−0·000230	−0·000008	0·000016
0·60	0·000075	−0·000096	+0·000085	−0·000207	−0·000013	0·000023
0·70	0·000104	−0·000082	+0·000044	−0·000185	−0·000019	0·000028
0·80	0·000134	−0·000048	−0·000015	−0·000164	−0·000026	0·000030
0·90	0·000163	−0·000013	−0·000085	−0·000146	−0·000031	0·000027
1·00	0·000197	+0·000089	−0·000157	−0·000129	−0·000040	0·000017

ρ_0	c_{16}''	c_{17}''	c_{10}'''	c_{11}'''	c_{12}'''	c_{13}'''
0·00	0	0	0	0	0	0·000149
0·05	0·000000	0·000000	0·000000	0·000000	0·000000	0·000149
0·10	+0·000001	0·000001	0·000000	−0·000001	−0·000001	0·000149
0·15	+0·000001	0·000004	0·000000	−0·000002	−0·000004	0·000148
0·20	+0·000001	0·000008	0·000001	−0·000004	−0·000009	0·000148
0·30	−0·000004	0·000018	0·000002	−0·000008	−0·000028	0·000145
0·40	−0·000014	0·000028	0·000005	−0·000009	−0·000053	0·000142
0·50	−0·000024	0·000036	0·000008	−0·000006	−0·000081	0·000137
0·60	−0·000033	0·000042	0·000011	+0·000002	−0·000109	0·000132
0·70	−0·000038	0·000046	0·000014	+0·000013	−0·000135	0·000127
0·80	−0·000037	0·000048	0·000018	+0·000028	−0·000160	0·000122
0·90	−0·000031	0·000048	0·000021	+0·000045	−0·000180	0·000116
1·00	−0·000024	0·000047	0·000024	+0·000063	−0·000197	0·000111

ρ_0	c_{14}'''	c_{15}'''	c_{16}'''	c_{17}'''		
0·00	0	0	0	0		
0·05	0·000000	0·000000	0·000000	0·000000		
0·10	0·000000	0·000000	−0·000001	0·000000		
0·15	0·000000	+0·000001	−0·000002	−0·000002		
0·20	0·000000	+0·000002	−0·000003	−0·000003		
0·30	−0·000001	+0·000005	−0·000003	−0·000009		
0·40	−0·000002	+0·000008	0·000000	−0·000015		
0·50	−0·000004	+0·000009	+0·000008	−0·000022		
0·60	−0·000006	+0·000009	+0·000019	−0·000029		
0·70	−0·000008	+0·000006	+0·000032	−0·000034		
0·80	−0·000011	+0·000001	+0·000047	−0·000039		
0·90	−0·000013	−0·000001	+0·000062	−0·000043		
1·00	−0·000015	−0·000015	+0·000077	−0·000046		

TABLE 1—*continued*

ρ_0	d_0'	d_1'	d_2'	d_3'	d_4'	d_5'
0·00	1·0	0·166667	0	0·333333	0	+0·083333
0·05	0·976006	0·161071	−0·012840	0·313092	−0·000779	+0·070413
0·10	0·927628	0·150084	−0·034283	0·276445	−0·000766	+0·050404
0·15	0·869941	0·137277	−0·054311	0·237552	+0·000651	+0·032879
0·20	0·810408	0·124333	−0·069844	0·201751	+0·002834	+0·019699
0·30	0·699207	0·100852	−0·087467	0·144498	+0·007224	+0·004008
0·40	0·605486	0·081806	−0·092857	0·104457	+0·010239	−0·003119
0·50	0·529124	0·066868	−0·091955	0·076720	+0·011873	−0·006068
0·60	0·467214	0·055211	−0·088208	0·057264	+0·012570	−0·007013
0·70	0·416717	0·046062	−0·083368	0·043369	+0·012704	−0·007002
0·80	0·375112	0·038811	−0·078281	0·033269	+0·012519	−0·006552
0·90	0·340444	0·033000	−0·073340	0·025807	+0·012162	−0·005926
1·00	0·311236	0·028294	−0·068714	0·020210	+0·011722	−0·005255

ρ_0	d_6'	d_7'	d_8'	d_9'	d_{10}'	d_{11}'
0·00	0·0375	0	0	+0·025	0	0
0·05	0·034591	+0·000058	−0·002082	+0·021586	−0·000002	−0·000027
0·10	0·029629	+0·000195	−0·004101	+0·016455	−0·000010	+0·000099
0·15	0·024675	+0·000231	−0·004793	+0·012017	−0·000020	+0·000292
0·20	0·020352	+0·000144	−0·004548	+0·008657	−0·000040	+0·000382
0·30	0·013857	−0·000337	−0·003219	+0·004444	−0·000009	+0·000429
0·40	0·009607	−0·000845	−0·001918	+0·002255	+0·000049	+0·000366
0·50	0·006811	−0·001217	−0·000962	+0·001102	+0·000102	+0·000277
0·60	0·004934	−0·001449	−0·000316	+0·000486	+0·000141	+0·000192
0·70	0·003644	−0·001577	+0·000097	+0·000155	+0·000167	+0·000120
0·80	0·002738	−0·001636	+0·000350	−0·000018	+0·000182	+0·000065
0·90	0·002090	−0·001650	+0·000492	−0·000106	+0·000191	+0·000023
1·00	0·001617	−0·001637	+0·000559	−0·000147	+0·000195	−0·000007

ρ_0	d_{12}'	d_{13}'	d_{14}'	d_{15}'	d_{16}'	d_{17}'
0·00	+0·003125	0·001860	0	0	0	+0·000744
0·05	+0·002086	0·001618	0·000000	+0·000004	−0·000089	+0·000578
0·10	+0·000876	0·001260	+0·000001	+0·000005	−0·000125	+0·000372
0·15	−0·000131	0·000951	+0·000002	−0·000014	−0·000105	+0·000226
0·20	−0·000258	0·000715	+0·000005	−0·000022	−0·000066	+0·000133
0·30	−0·000481	0·000411	+0·000004	−0·000036	−0·000009	+0·000041
0·40	−0·000439	0·000245	−0·000001	−0·000039	+0·000019	+0·000007
0·50	−0·000340	0·000151	−0·000007	−0·000036	+0·000030	−0·000005
0·60	−0·000246	0·000096	−0·000012	−0·000031	+0·000031	−0·000009
0·70	−0·000170	0·000063	−0·000015	−0·000025	+0·000028	−0·000009
0·80	−0·000113	0·000042	−0·000018	−0·000018	+0·000022	−0·000008
0·90	−0·000072	0·000029	−0·000019	−0·000012	+0·000018	−0·000006
1·00	−0·000044	0·000020	−0·000020	−0·000008	+0·000013	−0·000005

TABLE 1—*continued*

ρ_0	d_4''	d_5''	d_6''	d_7''	d_8''	d_9''
0·00	0	−0·083333	−0·0125	0	0	−0·025000
0·05	0·001534	−0·078317	−0·011866	−0·000154	0·002598	−0·022735
0·10	0·003969	−0·069352	−0·010706	−0·000639	0·006109	−0·018987
0·15	0·006103	−0·059935	−0·009449	−0·001307	0·008599	−0·015363
0·20	0·007656	−0·051289	−0·008258	−0·001969	0·010049	−0·012273
0·30	0·009247	−0·037361	−0·006261	−0·002954	0·010610	−0·007802
0·40	0·009601	−0·027434	−0·004771	−0·003456	0·009760	−0·005008
0·50	0·009380	−0·020414	−0·003679	−0·003637	0·008489	−0·003257
0·60	0·008923	−0·015405	−0·002875	−0·003638	0·007198	−0·002142
0·70	0·008391	−0·011782	−0·002277	−0·003546	0·006034	−0·001418
0·80	0·007854	−0·009123	−0·001825	−0·003410	0·005036	−0·000940
0·90	0·007344	−0·007142	−0·001479	−0·003253	0·004228	−0·000617
1·00	0·006871	−0·005659	−0·001213	−0·003099	0·003503	−0·000404

ρ_0	d_{10}''	d_{11}''	d_{12}''	d_{13}''	d_{14}''	d_{15}''
0·00	0	0	−0·006250	−0·002232	0	0
0·05	0·000011	+0·000115	−0·004836	−0·002009	−0·000001	−0·000009
0·10	0·000060	−0·000013	−0·002905	−0·001653	−0·000005	−0·000035
0·15	0·000161	−0·000348	−0·001429	−0·001321	−0·000015	−0·000044
0·20	0·000289	−0·000804	−0·000471	−0·001048	−0·000031	+0·000016
0·30	0·000541	−0·001394	+0·000463	−0·000664	−0·000073	+0·000108
0·40	0·000713	−0·001652	+0·000731	−0·000431	−0·000108	+0·000181
0·50	0·000806	−0·001691	+0·000740	−0·000287	−0·000131	+0·000222
0·60	0·000846	−0·001611	+0·000658	−0·000196	−0·000144	+0·000238
0·70	0·000853	−0·001480	+0·000552	−0·000137	−0·000150	+0·000238
0·80	0·000842	−0·001327	+0·000450	−0·000098	−0·000152	+0·000232
0·90	0·000817	−0·001179	+0·000357	−0·000071	−0·000149	+0·000218
1·00	0·000794	−0·001027	+0·000286	−0·000052	−0·000148	+0·000198

ρ_0	d_{16}''	d_{17}''	d_{10}'''	d_{11}'''	d_{12}'''	d_{13}'''
0·00	0	−0·001488	0	0	0·003125	0·000372
0·05	+0·000219	−0·001229	0·000006	−0·000158	0·002844	0·000347
0·10	+0·000372	−0·000873	0·000026	−0·000367	0·002383	0·000305
0·15	+0·000379	−0·000591	0·000052	−0·000512	0·001939	0·000261
0·20	+0·000319	−0·000394	0·000077	−0·000589	0·001563	0·000222
0·30	+0·000163	−0·000168	0·000112	−0·000614	0·001012	0·000160
0·40	+0·000049	−0·000066	0·000128	−0·000560	0·000662	0·000117
0·50	−0·000018	−0·000019	0·000132	−0·000484	0·000439	0·000086
0·60	−0·000053	+0·000002	0·000131	−0·000409	0·000294	0·000065
0·70	−0·000068	+0·000010	0·000127	−0·000341	0·000199	0·000050
0·80	−0·000071	+0·000013	0·000122	−0·000284	0·000135	0·000038
0·90	−0·000066	+0·000013	0·000116	−0·000236	0·000092	0·000030
1·00	−0·000062	+0·000012	0·000111	−0·000197	0·000063	0·000024

TABLE 1—*continued*

ρ_0	d_{14}'''	d_{15}'''	d_{16}'''	d_{17}'''		
0·00	0	0	0	0·000744		
0·05	−0·000002	0·000019	−0·000140	0·000657		
0·10	−0·000008	0·000074	−0·000303	0·000523		
0·15	−0·000019	0·000137	−0·000395	0·000403		
0·20	−0·000031	0·000189	−0·000429	0·000307		
0·30	−0·000051	0·000247	−0·000404	0·000177		
0·40	−0·000063	0·000258	−0·000335	0·000103		
0·50	−0·000068	0·000246	−0·000265	0·000060		
0·60	−0·000070	0·000223	−0·000205	0·000035		
0·70	−0·000070	0·000197	−0·000156	0·000020		
0·80	−0·000068	0·000172	−0·000119	0·000011		
0·90	−0·000066	0·000150	−0·000090	0·000005		
1·00	−0·000063	0·000129	−0·000068	0·000002		

ρ_0	e_0'	e_1'	e_2'	e_3'	e_4'	e_5'
0·00	0	0·333333	0	0	0·016667	0
0·05	0·000395	0·333319	−0·000009	−0·000005	0·016663	0·000000
0·10	0·001422	0·333170	−0·000082	−0·000184	0·016632	0·000000
0·15	0·002811	0·332726	−0·000268	−0·000742	0·016545	0·000008
0·20	0·004348	0·331872	−0·000566	−0·001759	0·016389	0·000033
0·30	0·007361	0·328769	−0·001393	−0·005053	0·015871	0·000138
0·40	0·009934	0·323882	−0·002348	−0·009524	0·015135	0·000301
0·50	0·011966	0·317512	−0·003292	−0·014538	0·014262	0·000495
0·60	0·013497	0·310022	−0·004154	−0·019598	0·013322	0·000700
0·70	0·014607	0·301750	−0·004905	−0·024375	0·012367	0·000903
0·80	0·015377	0·292986	−0·005537	−0·028675	0·011432	0·001094
0·90	0·015878	0·283962	−0·006055	−0·032414	0·010539	0·001268
1·00	0·016168	0·274858	−0·006467	−0·035570	0·009701	0·001421

ρ_0	e_6'	e_7'	e_8'	e_9'	e_{10}'	e_{11}'
0·00	0	0·008333	0	0	0	0·000595
0·05	0·000000	0·008326	0·000000	−0·000001	−0·000003	0·000595
0·10	−0·000054	0·008262	0·000000	−0·000007	−0·000033	0·000590
0·15	−0·000196	0·008096	+0·000001	−0·000012	−0·000101	0·000579
0·20	−0·000404	0·007822	0·000000	−0·000008	−0·000199	0·000561
0·30	−0·000859	0·007015	−0·000009	+0·000035	−0·000427	0·000509
0·40	−0·001157	0·006015	−0·000027	+0·000114	−0·000629	0·000448
0·50	−0·001175	0·004973	−0·000053	+0·000200	−0·000766	0·000385
0·60	−0·000905	0·003987	−0·000084	+0·000264	−0·000831	0·000328
0·70	−0·000401	0·003104	−0·000119	+0·000292	−0·000839	0·000277
0·80	+0·000268	0·002344	−0·000154	+0·000283	−0·000806	0·000233
0·90	+0·001027	0·001706	−0·000189	+0·000234	−0·000747	0·000196
1·00	+0·001813	0·001176	−0·000223	+0·000164	−0·000671	0·000165

(82496) F

TABLE 1—*continued*

ρ_0	e_{12}'	e_{13}'	e_{14}'	e_{15}'	e_{16}'	e_{17}'
0·00	0	0	0	+0·000149	0	0
0·05	0·000000	0·000000	−0·000001	+0·000148	0·000000	0·000000
0·10	0·000000	0·000000	−0·000005	+0·000144	0·000000	0·000000
0·15	0·000000	+0·000001	−0·000011	+0·000134	0·000000	0·000000
0·20	0·000000	+0·000003	−0·000017	+0·000120	0·000000	0·000000
0·30	0·000000	+0·000004	−0·000017	+0·000085	0·000000	0·000000
0·40	0·000002	0·000000	−0·000001	+0·000052	0·000000	−0·000001
0·50	0·000004	−0·000010	+0·000025	+0·000024	0·000000	0·000000
0·60	0·000008	−0·000022	+0·000052	+0·000001	−0·000001	+0·000001
0·70	0·000012	−0·000033	+0·000074	−0·000009	−0·000001	+0·000003
0·80	0·000017	−0·000041	+0·000092	−0·000018	−0·000002	+0·000003
0·90	0·000022	−0·000043	+0·000103	−0·000022	−0·000002	+0·000004
1·00	0·000028	−0·000044	+0·000107	−0·000025	−0·000003	+0·000005

ρ_0	e_{18}'	e_{19}'	e_{20}'	e_5''	e_6''	e_7''
0·00	0	0	0·000010	0	0	−0·008333
0·05	0·000000	0·000000	0·000010	0·000001	0·000001	−0·008331
0·10	0·000000	−0·000001	0·000010	0·000008	0·000002	−0·008316
0·15	0·000000	−0·000006	0·000010	0·000025	0·000058	−0·008273
0·20	+0·000001	−0·000007	0·000009	0·000051	0·000175	−0·008197
0·30	+0·000003	−0·000009	0·000008	0·000122	0·000577	−0·007942
0·40	+0·000004	−0·000011	0·000006	0·000203	0·001133	−0·007578
0·50	+0·000003	−0·000011	0·000005	0·000281	0·001759	−0·007146
0·60	+0·000001	−0·000009	0·000003	0·000352	0·002383	−0·006678
0·70	0·000000	−0·000007	0·000003	0·000414	0·002967	−0·006200
0·80	−0·000004	−0·000005	0·000002	0·000465	0·003488	−0·005729
0·90	−0·000008	−0·000003	0·000002	0·000509	0·003962	−0·005275
1·00	−0·000009	−0·000001	0·000001	0·000539	0·004312	−0·004850

ρ_0	e_8''	e_9''	e_{10}''	e_{11}''	e_{12}''	e_{13}''
0·00	0	0	0	−0·000595	0	0
0·05	0·000000	0·000000	0·000001	−0·000595	0·000000	0·000000
0·10	−0·000001	+0·000013	0·000024	−0·000593	0·000000	−0·000001
0·15	−0·000005	+0·000034	0·000081	−0·000587	0·000001	−0·000003
0·20	−0·000013	+0·000056	0·000172	−0·000577	0·000002	−0·000008
0·30	−0·000039	+0·000074	0·000430	−0·000546	0·000007	−0·000024
0·40	−0·000075	+0·000027	0·000729	−0·000505	0·000015	−0·000038
0·50	−0·000114	−0·000095	0·001013	−0·000459	0·000025	−0·000039
0·60	−0·000153	−0·000272	0·001255	−0·000413	0·000036	−0·000026
0·70	−0·000191	−0·000485	0·001443	−0·000369	0·000047	+0·000001
0·80	−0·000225	−0·000717	0·001577	−0·000329	0·000058	+0·000042
0·90	−0·000257	−0·000973	0·001660	−0·000292	0·000068	+0·000100
1·00	−0·000280	−0·001169	0·001708	−0·000259	0·000077	+0·000141

TABLE 1—*continued*

ρ_0	e_{14}''	e_{15}''	e_{16}''	e_{17}''	e_{18}''	e_{19}''
0·00	0	−0·000298	0	0	0	0
0·05	+0·000001	−0·000297	0·000000	0·000000	0·000000	0·000000
0·10	+0·000007	−0·000293	0·000000	0·000000	+0·000001	0·000002
0·15	+0·000021	−0·000282	0·000000	0·000000	0·000000	0·000007
0·20	+0·000037	−0·000265	0·000000	0·000000	−0·000001	0·000012
0·30	+0·000062	−0·000219	−0·000001	+0·000003	−0·000006	0·000023
0·40	+0·000060	−0·000168	−0·000002	+0·000006	−0·000013	0·000031
0·50	+0·000033	−0·000121	−0·000004	+0·000009	−0·000018	0·000034
0·60	−0·000013	−0·000080	−0·000006	+0·000011	−0·000019	0·000034
0·70	−0·000070	−0·000047	−0·000008	+0·000011	−0·000016	0·000030
0·80	−0·000127	−0·000021	−0·000010	+0·000007	−0·000010	0·000026
0·90	−0·000180	−0·000002	−0·000013	−0·000003	−0·000003	0·000022
1·00	−0·000230	+0·000012	−0·000015	−0·000006	+0·000007	0·000017

ρ_0	e_{20}''	e_{12}'''	e_{13}'''	e_{14}'''	e_{15}'''	e_{16}'''
0·00	−0·000021	0	0	0	0·000149	0
0·05	−0·000021	0·000000	0·000000	0·000000	0·000149	0·000000
0·10	−0·000020	0·000000	−0·000001	−0·000001	0·000148	0·000000
0·15	−0·000020	0·000000	−0·000002	−0·000005	0·000147	0·000000
0·20	−0·000019	0·000000	−0·000003	−0·000012	0·000144	0·000000
0·30	−0·000017	0·000001	−0·000004	−0·000031	0·000137	−0·000001
0·40	−0·000014	0·000002	−0·000002	−0·000053	0·000126	−0·000001
0·50	−0·000011	0·000004	+0·000003	−0·000073	0·000115	−0·000002
0·60	−0·000009	0·000005	+0·000011	−0·000090	0·000104	−0·000003
0·70	−0·000007	0·000006	+0·000019	−0·000103	0·000093	−0·000003
0·80	−0·000006	0·000007	+0·000029	−0·000112	0·000082	−0·000004
0·90	−0·000005	0·000008	+0·000039	−0·000118	0·000073	−0·000005
1·00	−0·000004	0·000009	+0·000047	−0·000121	0·000065	−0·000005

ρ_0	e_{17}'''	e_{18}'''	e_{19}'''	e_{20}'''		
0·00	0	0	0	0·000010		
0·05	0·000000	0·000000	0·000000	0·000010		
0·10	0·000000	−0·000001	−0·000001	0·000010		
0·15	0·000000	−0·000001	−0·000003	0·000010		
0·20	+0·000001	−0·000002	−0·000006	0·000010		
0·30	+0·000002	0·000000	−0·000013	0·000009		
0·40	+0·000003	+0·000005	−0·000020	0·000008		
0·50	+0·000003	+0·000013	−0·000026	0·000007		
0·60	+0·000001	+0·000021	−0·000030	0·000006		
0·70	−0·000002	+0·000030	−0·000033	0·000005		
0·80	−0·000007	+0·000039	−0·000034	0·000005		
0·90	−0·000011	+0·000046	−0·000034	0·000004		
1·00	−0·000016	+0·000051	−0·000034	0·000004		

TABLE 1—*continued*

ρ_0	f_0'	f_1'	f_2'	f_3'	f_4'	f_5'
0·00	0	0	0·2	0	0	0
0·05	0·000000	0·000250	0·200000	0·000000	0	0·000000
0·10	0·000006	0·000998	0·200000	0·000000	−0·000002	−0·000001
0·15	0·000025	0·002234	0·199999	−0·000003	−0·000016	−0·000006
0·20	0·000065	0·003933	0·199995	−0·000009	−0·000059	−0·000024
0·30	0·000224	0·008570	0·199967	−0·000047	−0·000331	−0·000138
0·40	0·000492	0·014530	0·199890	−0·000127	−0·000994	−0·000429
0·50	0·000851	0·021393	0·199736	−0·000258	−0·002158	−0·000958
0·60	0·001279	0·028774	0·199482	−0·000436	−0·003850	−0·001756
0·70	0·001750	0·036357	0·199113	−0·000655	−0·006035	−0·002823
0·80	0·002243	0·043897	0·198619	−0·000906	−0·008631	−0·004134
0·90	0·002742	0·051219	0·197998	−0·001181	−0·011549	−0·005656
1·00	0·003234	0·058205	0·197251	−0·001470	−0·014698	−0·007349

ρ_0	f_6'	f_7'	f_8'	f_9'	f_{10}'	f_{11}'
0·00	0	0	0	0·002381	0	0
0·05	0·000000	0·000000	+0·000003	0·002381	0·000000	0·000000
0·10	0·000000	−0·000001	+0·000011	0·002381	0·000000	0·000000
0·15	0·000000	−0·000003	+0·000021	0·002380	0·000000	0·000000
0·20	0·000001	−0·000009	+0·000027	0·002379	0·000000	0·000000
0·30	0·000005	−0·000033	+0·000003	0·002371	0·000000	+0·000003
0·40	0·000017	−0·000054	−0·000093	0·002351	−0·000002	+0·000010
0·50	0·000041	−0·000035	−0·000264	0·002317	−0·000004	+0·000022
0·60	0·000077	+0·000065	−0·000487	0·002270	−0·000009	+0·000032
0·70	0·000127	+0·000272	−0·000731	0·002209	−0·000017	+0·000035
0·80	0·000190	+0·000611	−0·000957	0·002139	−0·000028	+0·000019
0·90	0·000264	+0·001074	−0·001145	0·002061	−0·000041	−0·000016
1·00	0·000347	+0·001660	−0·001272	0·001978	−0·000057	−0·000075

ρ_0	f_{12}'	f_{13}'	f_{14}'	f_{15}'	f_{16}'	f_{17}'
0·00	0	0	0	0	0	0
0·05	0·000000	0·000000	0·000000	0·000000	0·000000	0·000000
0·10	0·000000	0·000000	0·000000	0·000000	0·000000	0·000000
0·15	−0·000002	−0·000001	0·000000	0·000000	0·000000	0·000000
0·20	−0·000005	−0·000003	0·000000	0·000000	0·000000	−0·000001
0·30	−0·000019	−0·000013	0·000000	0·000000	0·000000	−0·000003
0·40	−0·000038	−0·000034	0·000000	−0·000001	0·000003	−0·000007
0·50	−0·000053	−0·000065	0·000000	−0·000002	0·000007	−0·000010
0·60	−0·000054	−0·000104	0·000001	−0·000004	0·000012	−0·000013
0·70	−0·000041	−0·000146	0·000002	−0·000007	0·000020	−0·000013
0·80	−0·000005	−0·000188	0·000003	−0·000009	0·000019	−0·000011
0·90	+0·000041	−0·000228	0·000005	−0·000009	0·000017	−0·000009
1·00	+0·000094	−0·000264	0·000007	−0·000006	0·000017	−0·000007

ρ_0	f_{18}'	f_{19}'	f_{20}'	f_{21}'	f_{22}'	f_{23}'
0·00	0·000025	0	0	0	0	0
0·05	0·000025	0·000000	0·000000	0·000000	0·000000	0·000000
0·10	0·000025	0·000000	0·000000	0·000000	0·000000	0·000000
0·15	0·000025	0·000000	0·000000	0·000000	0·000000	0·000000
0·20	0·000025	0·000000	0·000000	0·000000	0·000000	0·000000
0·30	0·000024	0·000000	0·000000	0·000000	0·000000	0·000000
0·40	0·000023	0·000000	0·000000	0·000000	0·000000	−0·000001
0·50	0·000022	0·000000	0·000000	0·000000	0·000000	−0·000001
0·60	0·000021	0·000000	0·000000	−0·000001	0·000001	−0·000002
0·70	0·000019	0·000000	0·000000	−0·000003	0·000002	−0·000002
0·80	0·000017	0·000000	0·000001	−0·000003	0·000002	−0·000003
0·90	0·000016	0·000000	0·000002	−0·000003	0·000003	−0·000003
1·00	0·000014	−0·000001	0·000001	−0·000004	0·000003	−0·000003

ρ_0	f_6''	f_7''	f_8''	f_9''	f_{10}''	f_{11}''
0·00	0	0	0	−0·002381	0	0
0·05	0·000000	0·000000	−0·000003	−0·002381	0·000000	0·000000
0·10	0·000000	0·000000	−0·000012	−0·002381	0·000000	0·000000
0·15	0·000000	0·000001	−0·000025	−0·002381	0·000000	0·000000
0·20	0·000001	0·000006	−0·000042	−0·002381	0·000000	+0·000001
0·30	0·000003	0·000034	−0·000073	−0·002379	−0·000001	0·000000
0·40	0·000009	0·000101	−0·000085	−0·002375	−0·000003	−0·000008
0·50	0·000019	0·000219	−0·000060	−0·002367	−0·000008	−0·000031
0·60	0·000032	0·000389	+0·000009	−0·002354	−0·000014	−0·000076
0·70	0·000047	0·000606	+0·000124	−0·002337	−0·000022	−0·000148
0·80	0·000065	0·000861	+0·000283	−0·002312	−0·000032	−0·000244
0·90	0·000085	0·001145	+0·000480	−0·002290	−0·000044	−0·000366
1·00	0·000105	0·001449	+0·000707	−0·002261	−0·000057	−0·000509

ρ_0	f_{12}''	f_{13}''	f_{14}''	f_{15}''	f_{16}''	f_{17}''
0·00	0	0	0	0	0	0
0·05	0·000000	0·000000	0·000000	0·000000	0·000000	0·000000
0·10	0·000000	0·000000	0·000000	0·000000	0·000000	0·000000
0·15	0·000001	0·000000	0·000000	0·000000	0·000000	0·000000
0·20	0·000005	0·000002	0·000000	0·000000	+0·000001	0·000000
0·30	0·000023	0·000008	0·000000	−0·000001	+0·000001	0·000003
0·40	0·000059	0·000023	0·000001	−0·000001	0·000000	0·000010
0·50	0·000109	0·000049	0·000002	0·000000	−0·000007	0·000018
0·60	0·000166	0·000084	0·000003	+0·000004	−0·000020	0·000027
0·70	0·000218	0·000127	0·000005	+0·000014	−0·000038	0·000035
0·80	0·000258	0·000176	0·000008	+0·000031	−0·000060	0·000040
0·90	0·000281	0·000228	0·000012	+0·000055	−0·000083	0·000041
1·00	0·000283	0·000283	0·000016	+0·000087	−0·000105	0·000039

TABLE 1—*continued*

ρ_0	f_{18}''	f_{19}''	f_{20}''	f_{21}''	f_{22}''	f_{23}''
0·00	−0·000050	0	0	0	0	0
0·05	−0·000050	0·000000	0·000000	0·000000	0·000000	0·000000
0·10	−0·000050	0·000000	0·000000	0·000000	0·000000	0·000000
0·15	−0·000050	0·000000	0·000000	0·000000	0·000000	0·000000
0·20	−0·000050	0·000000	0·000000	0·000000	0·000000	0·000000
0·30	−0·000049	0·000000	0·000000	0·000000	+0·000001	0·000001
0·40	−0·000048	0·000000	0·000000	−0·000001	+0·000001	0·000001
0·50	−0·000047	0·000000	+0·000001	−0·000001	+0·000001	0·000003
0·60	−0·000045	−0·000001	+0·000001	−0·000001	0·000000	0·000004
0·70	−0·000043	−0·000001	0·000000	+0·000001	−0·000002	0·000005
0·80	−0·000041	−0·000001	−0·000002	+0·000004	−0·000005	0·000006
0·90	−0·000038	−0·000002	−0·000004	+0·000009	−0·000008	0·000007
1·00	−0·000034	−0·000003	−0·000009	+0·000014	−0·000010	0·000008

ρ_0	f_{14}'''	f_{15}'''	f_{16}'''	f_{17}'''	f_{18}'''	f_{19}'''
0·00	0	0	0	0	0·000025	0
0·05	0·000000	0·000000	0·000000	0·000000	0·000025	0·000000
0·10	0·000000	0·000000	0·000000	0·000000	0·000025	0·000000
0·15	0·000000	0·000000	0·000000	0·000000	0·000025	0·000000
0·20	0·000000	0·000000	0·000000	0·000000	0·000025	0·000000
0·30	0·000000	0·000000	−0·000001	0·000000	0·000025	0·000000
0·40	0·000000	0·000000	−0·000003	−0·000001	0·000025	0·000000
0·50	0·000000	+0·000001	−0·000006	−0·000003	0·000024	0·000000
0·60	0·000000	+0·000003	−0·000010	−0·000006	0·000024	0·000000
0·70	0·000001	+0·000005	−0·000013	−0·000010	0·000024	0·000000
0·80	0·000001	+0·000009	−0·000016	−0·000014	0·000024	−0·000001
0·90	0·000001	+0·000013	−0·000017	−0·000019	0·000023	−0·000001
1·00	0·000002	+0·000018	−0·000018	−0·000024	0·000023	−0·000001

ρ_0	f_{20}'''	f_{21}'''	f_{22}'''	f_{23}'''		
0·00	0	0	0	0		
0·05	0·000000	0·000000	0·000000	0·000000		
0·10	0·000000	0·000000	0·000000	0·000000		
0·15	0·000000	0·000000	0·000000	0·000000		
0·20	0·000000	0·000000	0·000000	0·000000		
0·30	0·000000	0·000000	−0·000001	0·000000		
0·40	0·000000	0·000001	−0·000001	0·000000		
0·50	0·000000	0·000002	−0·000002	−0·000001		
0·60	−0·000001	0·000004	−0·000003	−0·000001		
0·70	−0·000002	0·000006	−0·000003	−0·000002		
0·80	−0·000003	0·000008	−0·000002	−0·000003		
0·90	−0·000005	0·000011	−0·000001	−0·000004		
1·00	−0·000008	0·000013	0·000000	−0·000005		

TABLE 1—*continued*

ρ_0	g_0'	g_1'	g_2'	g_3'	g_4'	g_5
0·00	1·0	0·5	0	0·5	0·075	0
0·05	0·963166	0·474164	−0·014397	0·453919	0·069182	−0·000605
0·10	0·893345	0·426529	−0·035815	0·377252	0·059258	−0·000180
0·15	0·815631	0·374829	−0·053009	0·303311	0·049349	+0·001342
0·20	0·740564	0·326084	−0·064176	0·241148	0·040703	+0·003267
0·30	0·611740	0·245351	−0·073019	0·152514	0·027714	+0·006214
0·40	0·512629	0·186263	−0·072379	0·098220	0·019214	+0·007704
0·50	0·437167	0·143588	−0·068209	0·064584	0·013622	+0·008221
0·60	0·379006	0·112474	−0·063068	0·043239	0·009868	+0·008222
0·70	0·333349	0·089430	−0·057961	0·029364	0·007287	+0·007971
0·80	0·296831	0·072080	−0·053244	0·020166	0·005476	+0·007611
0·90	0·267104	0·058806	−0·049012	0·013955	0·004179	+0·007212
1·00	0·242522	0·048504	−0·045271	0·009701	0·003234	+0·006810

ρ_0	g_6'	g_7'	g_8'	g_9'	g_{10}'	g_{11}'
0·00	+0·083333	0·1125	0	0	+0·0375	0·007441
0·05	+0·064168	0·099348	+0·000049	−0·002192	+0·029928	0·006472
0·10	+0·038101	0·078995	+0·000134	−0·003703	+0·019958	0·005039
0·15	+0·018499	0·060727	+0·000162	−0·003627	+0·012543	0·003805
0·20	+0·006055	0·046323	−0·000015	−0·003022	+0·007626	0·002860
0·30	−0·005648	0·027188	−0·000373	−0·001501	+0·002521	0·001645
0·40	−0·008873	0·016373	−0·000650	−0·000453	+0·000500	0·000980
0·50	−0·008953	0·010118	−0·000809	+0·000148	−0·000258	0·000605
0·60	−0·007961	0·006392	−0·000885	+0·000454	−0·000496	0·000385
0·70	−0·006711	0·004109	−0·000910	+0·000578	−0·000527	0·000251
0·80	−0·005500	0·002683	−0·000906	+0·000609	−0·000470	0·000169
0·90	−0·004449	0·001770	−0·000886	+0·000582	−0·000396	0·000116
1·00	−0·003577	0·001176	−0·000857	+0·000532	−0·000321	0·000081

ρ_0	g_{12}'	g_{13}'	g_{14}'	g_{15}'	g_{16}'	g_{17}'
0·00	0	0	+0·003125	+0·005580	0	0
0·05	−0·000002	−0·000007	+0·001640	+0·004509	0·000000	+0·000004
0·10	−0·000011	+0·000125	+0·000248	+0·003118	+0·000001	+0·000008
0·15	−0·000043	+0·000210	−0·000394	+0·002079	+0·000001	+0·000002
0·20	−0·000017	+0·000288	−0·000588	+0·001380	+0·000002	−0·000017
0·30	+0·000011	+0·000251	−0·000525	+0·000615	+0·000001	−0·000023
0·40	+0·000044	+0·000169	−0·000341	+0·000278	−0·000002	−0·000021
0·50	+0·000067	+0·000094	−0·000192	+0·000124	−0·000005	−0·000016
0·60	+0·000082	+0·000037	−0·000093	+0·000053	−0·000007	−0·000011
0·70	+0·000089	−0·000003	−0·000031	+0·000020	−0·000008	−0·000004
0·80	+0·000093	−0·000026	−0·000001	+0·000004	−0·000008	−0·000002
0·90	+0·000094	−0·000039	+0·000017	−0·000003	−0·000009	−0·000001
1·00	+0·000093	−0·000045	+0·000024	−0·000005	−0·000009	+0·000002

TABLE 1—*continued*

ρ_0	g_{18}'	g_{19}'	g_{20}'	g_5''	g_6''	g_7''
0·00	0	+0·001116	0·000253	0	−0·083333	−0·0375
0·05	−0·000085	+0·000758	0·000206	0·001381	−0·075719	−0·034601
0·10	−0·000093	+0·000386	0·000145	0·003329	−0·063243	−0·029693
0·15	−0·000052	+0·000155	0·000099	0·004796	−0·051335	−0·024812
0·20	−0·000017	+0·000062	0·000068	0·005686	−0·041240	−0·020532
0·30	+0·000023	−0·000016	0·000033	0·006293	−0·026751	−0·014063
0·40	+0·000032	−0·000028	0·000017	0·006145	−0·017674	−0·009779
0·50	+0·000028	−0·000023	0·000009	0·005743	−0·011925	−0·006936
0·60	+0·000021	−0·000016	0·000005	0·005286	−0·008207	−0·005017
0·70	+0·000014	−0·000011	0·000003	0·004845	−0·005748	−0·003695
0·80	+0·000007	−0·000007	0·000002	0·004444	−0·004087	−0·002765
0·90	+0·000004	−0·000004	0·000001	0·004091	−0·002914	−0·002097
1·00	+0·000002	−0·000002	0·000001	0·003773	−0·002156	−0·001617

ρ_0	g_8''	g_9''	g_{10}''	g_{11}''	g_{12}''	g_{13}''
0·00	0	0	−0·0375	−0·004464	0	0
0·05	−0·000131	0·002828	−0·032406	−0·004017	0·000008	+0·000087
0·10	−0·000519	0·006083	−0·024797	−0·003306	0·000047	−0·000118
0·15	−0·000986	0·007904	−0·018221	−0·002642	0·000116	−0·000479
0·20	−0·001391	0·008442	−0·013216	−0·002096	0·000196	−0·000757
0·30	−0·001872	0·007821	−0·006876	−0·001328	0·000323	−0·001071
0·40	−0·002031	0·006455	−0·003545	−0·000862	0·000389	−0·001111
0·50	−0·002024	0·005107	−0·001777	−0·000575	0·000412	−0·001025
0·60	−0·001945	0·003976	−0·000826	−0·000393	0·000413	−0·000896
0·70	−0·001839	0·003081	−0·000312	−0·000275	0·000403	−0·000762
0·80	−0·001726	0·002382	−0·000040	−0·000196	0·000387	−0·000632
0·90	−0·001620	0·001818	+0·000097	−0·000142	0·000370	−0·000506
1·00	−0·001512	0·001449	+0·000168	−0·000105	0·000350	−0·000433

ρ_0	g_{14}''	g_{15}''	g_{16}''	g_{17}''	g_{18}''	g_{19}''
0·00	−0·00625	−0·006696	0	0	0	−0·002232
0·05	−0·004180	−0·005695	0·000000	−0·000013	+0·000218	−0·001665
0·10	−0·001788	−0·004272	−0·000003	−0·000020	+0·000314	−0·000995
0·15	−0·000293	−0·003095	−0·000010	+0·000003	+0·000257	−0·000544
0·20	+0·000485	−0·002229	−0·000020	+0·000031	+0·000171	−0·000282
0·30	+0·000953	−0·001168	−0·000041	+0·000093	+0·000027	−0·000043
0·40	+0·000879	−0·000628	−0·000055	+0·000125	−0·000045	+0·000029
0·50	+0·000686	−0·000345	−0·000062	+0·000135	−0·000071	+0·000043
0·60	+0·000499	−0·000192	−0·000065	+0·000132	−0·000074	+0·000040
0·70	+0·000348	−0·000107	−0·000065	+0·000124	−0·000066	+0·000031
0·80	+0·000237	−0·000059	−0·000064	+0·000109	−0·000055	+0·000024
0·90	+0·000158	−0·000032	−0·000063	+0·000090	−0·000045	+0·000017
1·00	+0·000099	−0·000016	−0·000060	+0·000083	−0·000034	+0·000012

TABLE 1—*continued*

ρ_0	g_{20}''	g_{12}'''	g_{13}'''	g_{14}'''	g_{15}'''	g_{16}'''
0·00	−0·000362	0	0	+0·003125	0·001116	0
0·05	−0·000305	0·000005	−0·000139	+0·002704	0·001005	+0·000002
0·10	−0·000228	0·000018	−0·000293	+0·002080	0·000828	−0·000005
0·15	−0·000165	0·000034	−0·000375	+0·001545	0·000664	−0·000011
0·20	−0·000119	0·000046	−0·000400	+0·001133	0·000529	−0·000017
0·30	−0·000064	0·000061	−0·000368	+0·000608	0·000337	−0·000026
0·40	−0·000035	0·000065	−0·000302	+0·000327	0·000219	−0·000029
0·50	−0·000021	0·000064	−0·000239	+0·000174	0·000146	−0·000030
0·60	−0·000012	0·000061	−0·000186	+0·000090	0·000100	−0·000030
0·70	−0·000008	0·000058	−0·000144	+0·000043	0·000070	−0·000029
0·80	−0·000005	0·000054	−0·000112	+0·000017	0·000050	−0·000028
0·90	−0·000003	0·000051	−0·000087	+0·000002	0·000037	−0·000026
1·00	−0·000002	0·000047	−0·000068	−0·000005	0·000026	−0·000025

ρ_0	g_{17}'''	g_{18}'''	g_{19}'''	g_{20}'''		
0·00	0	0	+0·001116	0·000109		
0·05	0·000028	−0·000138	+0·000926	0·000096		
0·10	0·000058	−0·000290	+0·000658	0·000076		
0·15	0·000098	−0·000342	+0·000449	0·000059		
0·20	0·000124	−0·000336	+0·000301	0·000045		
0·30	0·000142	−0·000267	+0·000130	0·000027		
0·40	0·000134	−0·000191	+0·000052	0·000017		
0·50	0·000117	−0·000130	+0·000016	0·000010		
0·60	0·000099	−0·000086	0·000000	0·000007		
0·70	0·000081	−0·000056	−0·000006	0·000005		
0·80	0·000066	−0·000035	−0·000009	0·000003		
0·90	0·000054	−0·000021	−0·000009	0·000002		
1·00	0·000044	−0·000013	−0·000008	0·000002		

ρ_0	h_0'	h_1'	h_2'	h_3'	h_4'	h_5'
0·00	0	0·333333	0·016667	0	0	0·033333
0·05	0·000387	0·333313	0·016663	−0·000009	−0·000006	0·033315
0·10	0·001340	0·332986	0·016632	−0·000082	−0·000293	0·033156
0·15	0·002543	0·331983	0·016545	−0·000252	−0·001135	0·032738
0·20	0·003782	0·330113	0·016389	−0·000500	−0·002567	0·032034
0·30	0·005968	0·323716	0·015871	−0·001116	−0·006771	0·029902
0·40	0·007586	0·314358	0·015135	−0·001747	−0·011838	0·027164
0·50	0·008674	0·302974	0·014262	−0·002303	−0·016888	0·024208
0·60	0·009343	0·290424	0·013322	−0·002754	−0·021408	0·021295
0·70	0·009702	0·277375	0·012366	−0·003099	−0·025178	0·018574
0·80	0·009840	0·264311	0·011432	−0·003349	−0·028140	0·016120
0·90	0·009823	0·251548	0·010539	−0·003520	−0·030359	0·013951
1·00	0·009701	0·239288	0·009701	−0·003626	−0·031944	0·012053

TABLE 1—*continued*

ρ_0	h_6'	h_7'	h_8'	h_9'	h_{10}'	h_{11}'
0·00	0	0	+0·008333	0·001786	0	0
0·05	0·000000	−0·000004	+0·008316	0·001784	0·000000	−0·000001
0·10	0·000001	−0·000075	+0·008162	0·001770	0·000000	−0·000006
0·15	0·000010	−0·000231	+0·007793	0·001737	0·000000	−0·000006
0·20	0·000032	−0·000429	+0·007226	0·001682	−0·000001	+0·000003
0·30	0·000112	−0·000755	+0·005734	0·001528	−0·000008	+0·000051
0·40	0·000220	−0·000815	+0·004127	0·001343	−0·000020	+0·000113
0·50	0·000337	−0·000576	+0·002677	0·001156	−0·000035	+0·000161
0·60	0·000448	−0·000111	+0·001493	0·000983	−0·000052	+0·000178
0·70	0·000547	+0·000475	+0·000586	0·000830	−0·000069	+0·000158
0·80	0·000632	+0·001116	−0·000073	0·000698	−0·000085	+0·000119
0·90	0·000700	+0·001730	−0·000536	0·000587	−0·000100	+0·000061
1·00	0·000753	+0·002304	−0·000838	0·000494	−0·000112	−0·000011

ρ_0	h_{12}'	h_{13}'	h_{14}'	h_{15}'	h_{16}'	h_{17}'
0·00	0	0·001190	0	0	0	+0·000149
0·05	−0·000006	0·001188	0·000000	0·000000	−0·000001	+0·000147
0·10	−0·000052	0·001165	0·000000	0·000000	−0·000005	+0·000137
0·15	−0·000147	0·001114	0·000000	+0·000001	−0·000011	+0·000103
0·20	−0·000266	0·001039	0·000000	+0·000002	−0·000013	+0·000090
0·30	−0·000494	0·000849	0·000000	+0·000001	−0·000003	+0·000035
0·40	−0·000632	0·000654	0·000001	−0·000004	+0·000019	−0·000006
0·50	−0·000666	0·000483	0·000003	−0·000012	+0·000042	−0·000030
0·60	−0·000625	0·000345	0·000005	−0·000019	+0·000059	−0·000040
0·70	−0·000544	0·000240	0·000007	−0·000023	+0·000073	−0·000042
0·80	−0·000439	0·000163	0·000009	−0·000024	+0·000070	−0·000040
0·90	−0·000336	0·000106	0·000011	−0·000024	+0·000065	−0·000035
1·00	−0·000244	0·000066	0·000013	−0·000019	+0·000061	−0·000030

ρ_0	h_{18}'	h_{19}'	h_{20}'	h_{21}'	h_{22}'	h_{23}'
0·00	0·000052	0	0	0	0	+0·000021
0·05	0·000052	0·000000	0·000000	−0·000005	0·000000	+0·000020
0·10	0·000051	0·000000	0·000000	0·000000	−0·000002	+0·000020
0·15	0·000048	0·000000	0·000000	0·000000	+0·000009	+0·000018
0·20	0·000045	0·000000	0·000000	+0·000001	−0·000007	+0·000015
0·30	0·000037	0·000000	0·000000	+0·000002	−0·000009	+0·000010
0·40	0·000030	0·000000	0·000000	+0·000002	−0·000008	+0·000006
0·50	0·000023	0·000000	0·000000	0·000000	−0·000005	+0·000003
0·60	0·000017	0·000000	+0·000001	−0·000001	−0·000002	+0·000001
0·70	0·000013	−0·000001	+0·000003	−0·000006	0·000000	0·000000
0·80	0·000010	−0·000001	+0·000003	−0·000006	+0·000002	0·000000
0·90	0·000008	−0·000001	+0·000005	−0·000006	+0·000003	0·000000
1·00	0·000006	−0·000001	+0·000003	−0·000007	+0·000003	−0·000001

TABLE 1—*continued*

ρ_0	h_6''	h_7''	h_8''	h_9''	h_{10}''	h_{11}''
0·00	0	0	−0·008333	−0·000595	0	0
0·05	0·000001	−0·000005	−0·008330	−0·000595	0·000000	+0·000002
0·10	0·000007	+0·000015	−0·008292	−0·000593	−0·000001	+0·000012
0·15	0·000019	+0·000092	−0·008193	−0·000587	−0·000004	+0·000027
0·20	0·000038	+0·000231	−0·008024	−0·000577	−0·000009	+0·000039
0·30	0·000083	+0·000651	−0·007511	−0·000546	−0·000025	+0·000026
0·40	0·000128	+0·001160	−0·006850	−0·000505	−0·000045	−0·000049
0·50	0·000167	+0·001664	−0·006133	−0·000459	−0·000064	−0·000173
0·60	0·000199	+0·002111	−0·005423	−0·000413	−0·000082	−0·000324
0·70	0·000223	+0·002482	−0·004757	−0·000369	−0·000097	−0·000483
0·80	0·000240	+0·002772	−0·004153	−0·000329	−0·000110	−0·000633
0·90	0·000252	+0·002989	−0·003615	−0·000292	−0·000119	−0·000769
1·00	0·000259	+0·003143	−0·003143	−0·000259	−0·000127	−0·000887

ρ_0	h_{12}''	h_{13}''	h_{14}''	h_{15}''	h_{16}''	h_{17}''
0·00	0	−0·001190	0	0	0	−0·000298
0·05	0·000004	−0·001189	0·000000	0·000000	+0·000001	−0·000296
0·10	0·000039	−0·001178	0·000000	0·000000	+0·000009	−0·000285
0·15	0·000123	−0·001150	0·000000	−0·000003	+0·000022	−0·000262
0·20	0·000247	−0·001106	0·000001	−0·000007	+0·000035	−0·000228
0·30	0·000554	−0·000984	0·000004	−0·000016	+0·000042	−0·000149
0·40	0·000850	−0·000841	0·000009	−0·000019	+0·000020	−0·000075
0·50	0·001079	−0·000700	0·000013	−0·000012	−0·000022	−0·000018
0·60	0·001228	−0·000573	0·000018	+0·000006	−0·000072	+0·000021
0·70	0·001303	−0·000463	0·000023	+0·000032	−0·000118	+0·000045
0·80	0·001323	−0·000371	0·000027	+0·000062	−0·000158	+0·000057
0·90	0·001300	−0·000296	0·000030	+0·000093	−0·000188	+0·000062
1·00	0·001249	−0·000235	0·000033	+0·000124	−0·000208	+0·000063

ρ_0	h_{18}''	h_{19}''	h_{20}''	h_{21}''	h_{22}''	h_{23}''
0·00	−0·000062	0	0	0	0	−0·000041
0·05	−0·000062	0·000000	0·000000	0·000000	0·000000	−0·000041
0·10	−0·000061	0·000000	0·000000	0·000000	+0·000004	−0·000040
0·15	−0·000059	0·000000	0·000000	0·000000	+0·000010	−0·000037
0·20	−0·000057	0·000000	+0·000001	−0·000002	+0·000016	−0·000034
0·30	−0·000050	−0·000001	+0·000002	−0·000007	+0·000025	−0·000026
0·40	−0·000042	−0·000001	+0·000004	−0·000011	+0·000028	−0·000018
0·50	−0·000034	−0·000002	+0·000005	−0·000011	+0·000025	−0·000012
0·60	−0·000028	−0·000003	+0·000004	−0·000008	+0·000019	−0·000007
0·70	−0·000022	−0·000004	+0·000002	−0·000003	+0·000013	−0·000004
0·80	−0·000018	−0·000005	−0·000001	+0·000004	+0·000007	−0·000002
0·90	−0·000014	−0·000005	−0·000005	+0·000010	+0·000002	−0·000001
1·00	−0·000012	−0·000006	−0·000010	+0·000017	−0·000001	0·000000

TABLE 1—*continued*

ρ_0	h_{14}'''	h_{15}'''	h_{16}'''	h_{17}'''	h_{18}'''	h_{19}'''
0·00	0	0	0	0·000149	0·000010	0
0·05	0·000000	0·000000	0·000000	0·000149	0·000010	0·000000
0·10	0·000000	0·000000	−0·000002	0·000147	0·000010	0·000000
0·15	0·000000	−0·000001	−0·000007	0·000144	0·000010	0·000000
0·20	0·000000	−0·000002	−0·000014	0·000139	0·000010	0·000000
0·30	0·000001	−0·000001	−0·000031	0·000124	0·000009	0·000000
0·40	0·000001	+0·000002	−0·000048	0·000106	0·000008	−0·000001
0·50	0·000002	+0·000006	−0·000061	0·000089	0·000007	−0·000001
0·60	0·000002	+0·000011	−0·000069	0·000073	0·000006	−0·000001
0·70	0·000003	+0·000017	−0·000073	0·000060	0·000005	−0·000001
0·80	0·000003	+0·000022	−0·000074	0·000048	0·000005	−0·000002
0·90	0·000003	+0·000027	−0·000072	0·000039	0·000004	−0·000002
1·00	0·000004	+0·000031	−0·000069	0·000031	0·000004	−0·000002

ρ_0	h_{20}'''	h_{21}'''	h_{22}'''	h_{23}'''		
0·00	0	0	0	0·000021		
0·05	0·000000	0·000000	0·000000	0·000021		
0·10	0·000000	−0·000001	−0·000001	0·000020		
0·15	0·000000	−0·000001	−0·000004	0·000020		
0·20	+0·000001	−0·000001	−0·000008	0·000019		
0·30	+0·000001	+0·000002	−0·000016	0·000016		
0·40	+0·000001	+0·000007	−0·000022	0·000013		
0·50	+0·000001	+0·000013	−0·000026	0·000010		
0·60	−0·000002	+0·000019	−0·000027	0·000008		
0·70	−0·000004	+0·000024	−0·000026	0·000006		
0·80	−0·000006	+0·000028	−0·000025	0·000004		
0·90	−0·000009	+0·000030	−0·000022	0·000003		
1·00	−0·000011	+0·000031	−0·000020	0·000002		

ρ_0	k_0'	k_1'	k_2'	k_3'	k_4'	k_5'
0·00	0	0	0·2	0	0	0
0·05	0·000000	0·000250	0·200000	0·000000	0·000000	+0·000006
0·10	0·000006	0·000996	0·199999	−0·000003	−0·000003	+0·000021
0·15	0·000022	0·002218	0·199992	−0·000002	−0·000022	+0·000036
0·20	0·000056	0·003874	0·199971	−0·000008	−0·000078	+0·000030
0·30	0·000178	0·008239	0·199829	−0·000037	−0·000396	−0·000132
0·40	0·000364	0·013536	0·199461	−0·000093	−0·001101	−0·000615
0·50	0·000594	0·019235	0·198778	−0·000177	−0·002227	−0·001486
0·60	0·000843	0·024924	0·197726	−0·000281	−0·003721	−0·002729
0·70	0·001094	0·030321	0·196290	−0·000401	−0·005492	−0·004285
0·80	0·001337	0·035266	0·194485	−0·000526	−0·007410	−0·006048
0·90	0·001561	0·039670	0·192342	−0·000652	−0·009401	−0·007946
1·00	0·001764	0·043507	0·189902	−0·000775	−0·011377	−0·009894

TABLE 1—*continued*

ρ_0	k_6'	k_7'	k_8'	k_9'	k_{10}'	k_{11}'
0·00	0·004762	0	0	0	0·002381	0
0·05	0·004762	0·000000	0·000000	+0·000003	0·002381	0·000000
0·10	0·004762	0·000000	0·000000	+0·000010	0·002381	0·000000
0·15	0·004761	0·000000	−0·000003	+0·000016	0·002378	0·000000
0·20	0·004758	0·000001	−0·000008	+0·000012	0·002371	0·000000
0·30	0·004741	0·000004	−0·000024	−0·000054	0·002332	0·000000
0·40	0·004702	0·000012	−0·000022	−0·000208	0·002249	−0·000001
0·50	0·004635	0·000027	+0·000031	−0·000423	0·002122	−0·000003
0·60	0·004540	0·000049	+0·000160	−0·000649	0·001959	−0·000006
0·70	0·004419	0·000077	+0·000377	−0·000853	0·001772	−0·000010
0·80	0·004278	0·000108	+0·000667	−0·000972	0·001576	−0·000015
0·90	0·004122	0·000142	+0·001026	−0·001022	0·001378	−0·000021
1·00	0·003956	0·000178	+0·001434	−0·000989	0·001187	−0·000028

ρ_0	k_{12}'	k_{13}'	k_{14}'	k_{15}'	k_7''	k_8''
0·00	0	0	0	0·000099	0	0
0·05	0·000000	0·000000	0·000000	0·000099	0·000000	0·000000
0·10	0·000000	0·000000	0·000000	0·000099	0·000000	0·000000
0·15	0·000000	−0·000002	−0·000001	0·000099	0·000000	0·000002
0·20	0·000000	−0·000006	−0·000005	0·000099	0·000001	0·000007
0·30	+0·000003	−0·000018	−0·000025	0·000097	0·000002	0·000034
0·40	+0·000008	−0·000029	−0·000061	0·000094	0·000006	0·000093
0·50	+0·000014	−0·000026	−0·000107	0·000089	0·000011	0·000188
0·60	+0·000015	−0·000007	−0·000155	0·000083	0·000018	0·000313
0·70	+0·000007	+0·000041	−0·000196	0·000076	0·000025	0·000458
0·80	−0·000016	+0·000073	−0·000230	0·000069	0·000033	0·000617
0·90	−0·000053	+0·000108	−0·000256	0·000062	0·000041	0·000779
1·00	−0·000100	+0·000161	−0·000267	0·000056	0·000048	0·000940

ρ_0	k_9''	k_{10}''	k_{11}''	k_{12}''	k_{13}''	k_{14}''
0·00	0	−0·002381	0	0	0	0
0·05	−0·000003	−0·002381	0·000000	0·000000	0·000000	0·000000
0·10	−0·000012	−0·002381	0·000000	0·000000	0·000000	0·000000
0·15	−0·000024	−0·002380	0·000000	0·000000	0·000002	0·000000
0·20	−0·000037	−0·002379	0·000000	0·000000	0·000006	0·000002
0·30	−0·000050	−0·002371	−0·000001	−0·000001	0·000026	0·000015
0·40	−0·000026	−0·002351	−0·000002	−0·000010	0·000058	0·000043
0·50	+0·000049	−0·002318	−0·000004	−0·000032	0·000095	0·000085
0·60	+0·000174	−0·002271	−0·000008	−0·000068	0·000126	0·000138
0·70	+0·000342	−0·002211	−0·000011	−0·000119	0·000141	0·000196
0·80	+0·000541	−0·002140	−0·000016	−0·000183	0·000138	0·000255
0·90	+0·000761	−0·002062	−0·000020	−0·000256	0·000114	0·000310
1·00	+0·000990	−0·001978	−0·000025	−0·000336	0·000072	0·000360

TABLE 1—*continued*

ρ_0	k_{15}''					
0·00	−0·000099					
0·05	−0·000099					
0·10	−0·000099					
0·15	−0·000099					
0·20	−0·000099					
0·30	−0·000098					
0·40	−0·000097					
0·50	−0·000094					
0·60	−0·000090					
0·70	−0·000086					
0·80	−0·000081					
0·90	−0·000076					
1·00	−0·000071					

ρ_0	m_0'	m_1'	m_2'	m_3'	m_4'	m_5'
0·00	0	0	0	0·142857	0	0
0·05	0·000000	0·000000	0·000179	0·142857	0·000000	0·000000
0·10	0·000000	0·000005	0·000714	0·142857	0·000000	0·000000
0·15	0·000000	0·000026	0·001607	0·142857	0·000000	0·000000
0·20	0·000001	0·000081	0·002855	0·142857	0·000000	−0·000002
0·30	0·000007	0·000371	0·006401	0·142856	−0·000002	−0·000025
0·40	0·000024	0·001031	0·011285	0·142851	−0·000006	−0·000111
0·50	0·000057	0·002168	0·017381	0·142839	−0·000018	−0·000324
0·60	0·000108	0·003822	0·024514	0·142812	−0·000038	−0·000723
0·70	0·000177	0·005974	0·032477	0·142763	−0·000068	−0·001350
0·80	0·000266	0·008582	0·041071	0·142687	−0·000110	−0·002233
0·90	0·000369	0·011559	0·050085	0·142577	−0·000163	−0·003370
1·00	0·000485	0·014828	0·059342	0·142428	−0·000226	−0·004749

ρ_0	m_6'	m_7'	m_8'	m_9'	m_{10}'	m_{11}'
0·00	0	0	0	0	0	0
0·05	0·000000	0·000000	0·000000	0·000000	0·000000	+0·000001
0·10	0·000000	0·000000	0·000000	0·000000	0·000000	+0·000005
0·15	−0·000001	0·000000	0·000000	0·000000	0·000000	+0·000011
0·20	−0·000003	−0·000001	0·000000	0·000000	−0·000001	+0·000019
0·30	−0·000035	−0·000006	0·000000	−0·000001	−0·000008	+0·000038
0·40	−0·000164	−0·000030	0·000001	+0·000001	−0·000032	+0·000049
0·50	−0·000490	−0·000090	0·000003	+0·000014	−0·000078	+0·000041
0·60	−0·001120	−0·000210	0·000007	+0·000051	−0·000138	+0·000003
0·70	−0·002143	−0·000409	0·000013	+0·000129	−0·000185	−0·000066
0·80	−0·003604	−0·000700	0·000023	+0·000262	−0·000204	−0·000165
0·90	−0·005533	−0·001090	0·000037	+0·000459	−0·000172	−0·000288
1·00	−0·007914	−0·001583	0·000054	+0·000733	−0·000023	−0·000419

TABLE 1—*continued*

ρ_0	m_{12}'	m_{13}'	m_{14}'	m_{15}'	m_{16}'	m_{17}'
0·00	0·000992	0	0	0	0	0
0·05	0·000992	0·000000	0·000000	0·000000	0·000000	0·000000
0·10	0·000992	0·000000	0·000000	0·000000	0·000000	0·000000
0·15	0·000992	0·000000	0·000000	0·000000	0·000000	0·000000
0·20	0·000992	0·000000	0·000000	0·000000	0·000000	0·000000
0·30	0·000992	0·000000	0·000000	−0·000001	−0·000002	−0·000001
0·40	0·000991	0·000000	+0·000001	−0·000001	−0·000009	−0·000002
0·50	0·000988	0·000000	+0·000001	+0·000002	−0·000021	−0·000006
0·60	0·000983	−0·000001	0·000000	+0·000011	−0·000036	−0·000013
0·70	0·000974	−0·000002	−0·000005	+0·000023	−0·000051	−0·000022
0·80	0·000963	−0·000003	−0·000014	+0·000046	−0·000069	−0·000033
0·90	0·000949	−0·000006	−0·000033	+0·000086	−0·000068	−0·000045
1·00	0·000932	−0·000009	−0·000069	+0·000092	−0·000068	−0·000059

ρ_0	m_8''	m_9''	m_{10}''	m_{11}''	m_{12}''	m_{13}''
0·00	0	0	0	0	−0·000992	0
0·05	0·000000	0·000000	0·000000	−0·000001	−0·000992	0·000000
0·10	0·000000	0·000000	0·000000	−0·000005	−0·000992	0·000000
0·15	0·000000	0·000000	0·000000	−0·000011	−0·000992	0·000000
0·20	0·000000	0·000000	0·000000	−0·000020	−0·000992	0·000000
0·30	0·000000	0·000002	0·000001	−0·000043	−0·000992	0·000000
0·40	0·000000	0·000008	0·000011	−0·000072	−0·000992	0·000000
0·50	0·000001	0·000024	0·000038	−0·000102	−0·000991	0·000000
0·60	0·000002	0·000052	0·000094	−0·000126	−0·000990	−0·000001
0·70	0·000004	0·000097	0·000187	−0·000141	−0·000989	−0·000002
0·80	0·000006	0·000160	0·000321	−0·000142	−0·000986	−0·000003
0·90	0·000009	0·000240	0·000500	−0·000126	−0·000983	−0·000005
1·00	0·000013	0·000337	0·000721	−0·000093	−0·000978	−0·000007

ρ_0	m_{14}''	m_{15}''	m_{16}''	m_{17}''		
0·00	0	0	0	0		
0·05	0·000000	0·000000	0·000000	0·000000		
0·10	0·000000	0·000000	0·000000	0·000000		
0·15	0·000000	0·000000	0·000001	0·000000		
0·20	0·000000	0·000000	0·000001	0·000000		
0·30	0·000000	+0·000001	0·000002	0·000000		
0·40	−0·000001	+0·000004	0·000008	0·000001		
0·50	−0·000005	+0·000009	0·000020	0·000004		
0·60	−0·000013	+0·000011	0·000041	0·000008		
0·70	−0·000029	+0·000007	0·000071	0·000014		
0·80	−0·000053	−0·000008	0·000108	0·000023		
0·90	−0·000088	−0·000045	0·000145	0·000033		
1·00	−0·000133	−0·000093	0·000186	0·000046		

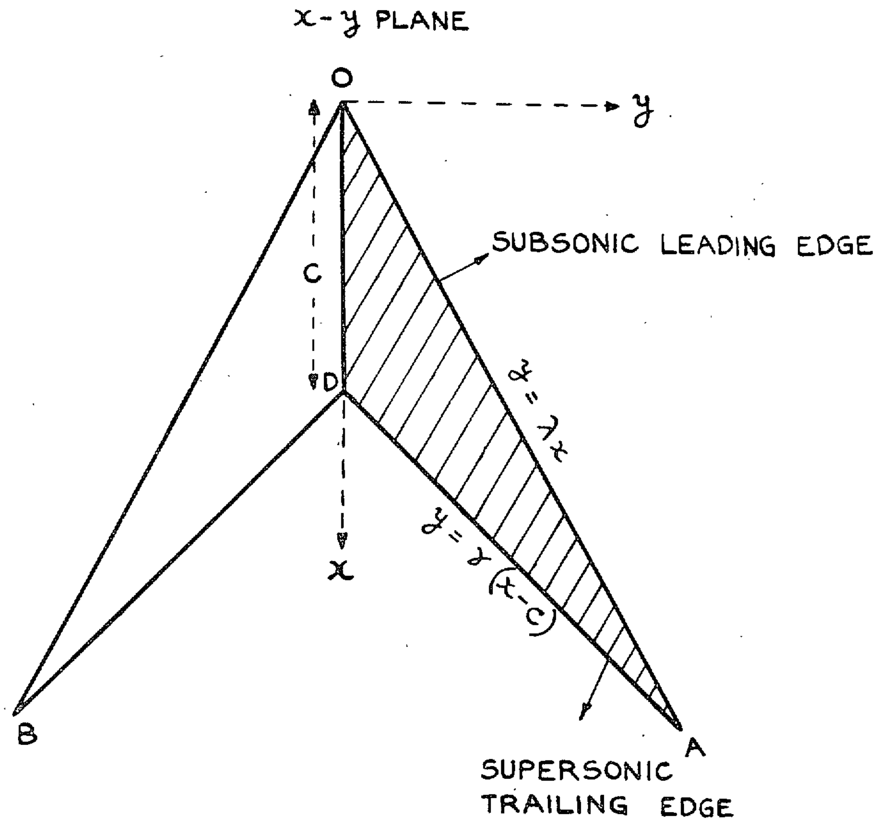

FIG. 1.

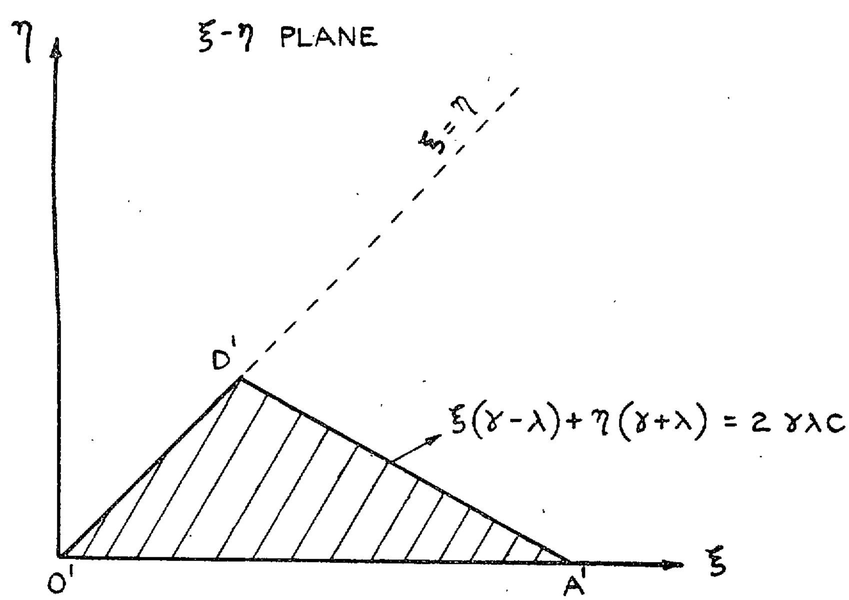

FIG. 2.

(82496) Wt. 66/991 K.5 12/61 Hw.

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