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The Effect of Humidity on Laminar Recovery Temperature Measurements in Supersonic Flow of Air in Wind Tunnels

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Summary. Wind-tunnel tests have been made at a Mach number of 2·92 to measure the effect of humidity on laminar recovery temperature on a cone. Its effect is to increase the recovery temperature above that obtained with dry air.

Consideration of the latent heat addition to the air in the condensation shock allows a theoretical estimate of recovery temperature to be made, based on true total temperature and true Mach number. The results agree with theory and support assumptions of no re-evaporation in the boundary layer and no change in recovery factor. An analysis of results published by Brun and Plan[2] ($M = 1·85$) and Laufer and Marte[3] ($M = 4·02$, 4·43, 4·71) is included.

It is suggested that a device for the continuous measurement of laminar recovery temperature would provide a simple monitor of the degree of condensation in wind tunnels.

1. *Introduction.* At low Mach numbers ($M < 2$) the air in a wind tunnel can usually be dried sufficiently to avoid any condensation of water vapour, but the required dryness becomes excessive as the Mach number is increased. One must then accept the presence of a condensation shock (or shocks) in the nozzle, but seek to dry the air sufficiently to ensure that the resulting disturbances in the working section are 'negligible'. Some years ago we studied this problem, for $M > 2$, by comparing pitot and static pressures in the working section over ranges of humidity and Mach number. Before long one comes up against limiting accuracy of measurement and the results showed a considerable amount of scatter[4], although they agreed broadly with the trends given by a simple theoretical approach.

Then work by Brun and Plan[2] became available; this suggested that temperature recovery factor might be a very convenient indicator of condensation and its degree, and this led to the tests described in the present Report.

The recovery temperature (T_{w0}) at the surface of a body exposed to an airflow is given by the formula

$$T_{w0}/T = 1 + r(\gamma - 1)\, M^2/2 \qquad (1)$$

where T_{w0} is the surface temperature for the condition of zero convective heat transfer between the surface and the boundary layer (no radiation effects), T is the ambient temperature of the air in the local inviscid flow, M is the Mach number of this flow, and r is the temperature recovery factor appropriate to the type of boundary layer, laminar or turbulent.

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In order to minimise effects from the condensation of water vapour, the air supplied to supersonic wind tunnels is usually dried as much as is possible. If the air were completely dry, then the expansion through a fixed nozzle would be along a dry adiabatic line and both the working section Mach number and the temperature ratio T_{w0}/T_0 would be constants, where T_0 is the stagnation temperature upstream of the throat. (Constancy of the temperature ratio follows from equation (1) and the isentropic relation

$$T_0/T = 1 + (\gamma - 1)\, M^2/2\,.) \tag{2}$$

If, however, the air is allowed to become wet then by the process of condensation in the nozzle, described, *e.g.*, by Lukasiewicz[1], some, if not all, of the latent heat of vaporization of water/ice is given up to the air, and the working-section Mach number is reduced. Experimentally it is found that there is an increase in T_{w0} with increase in absolute humidity and hence there is an increase in r_i the indicated recovery factor based on dry air values of M and T_0. This was shown in the report by Brun and Plan[2] in which the measurement of total and recovery temperatures through a condensation shock in a tunnel having a dry Mach number of 1·85 was described. A significant increase in T_{w0} due to humidity was shown but an exhaustive analysis of the effect was not made.

The present tests were made at $M = 2{\cdot}92$ over wide ranges of humidity at several levels of stagnation temperature. Theoretical considerations are given in Section 2 and details of the experiments and results follow in Sections 3 and 4. The results are in good agreement with estimates based on the simple theoretical approach of Section 2.

A further analysis is then made of the results of Brun and Plan[2] ($M = 1{\cdot}85$) and of Laufer and Marte[3] ($M = 4{\cdot}02$, $4{\cdot}43$ and $4{\cdot}71$) and the overall agreement with theory is very good.

Thus it seems that the measurement of recovery temperature on a slender body could be an accurate monitor of condensation in a wind tunnel nozzle. Apart from its implications for boundary-layer research, knowledge of any variation in the degree of condensation could help, for example, in assessing the reliability of overall force measurements. (In this respect it may be noted that, in addition to pressure changes, condensation shocks in the nozzle can produce small variations in flow direction in the working section, *see* Section 4.3 below.)

2. *Theoretical Increase in Recovery Temperature* (T_{w0}) *due to Heat Addition through Condensation of Water Vapour.* Formulae have been derived in Refs. 1 and 4 for the effects of condensation shocks on the pressures and Mach numbers obtained in supersonic wind tunnels. Parameters and formulae of Refs. 1 and 4 relevant to the present problem are listed in Section 2.1 below and are followed in Section 2.2 by the derivation of relations governing the effect of condensation shocks on recovery temperature, T_{w0}.

2.1. *Relevant Parameters and Formulae* (*for derivations see Refs.* 1 *and* 4). (*a*) *Relative humidity,* ϕ, *at a temperature* T.

$$\phi = (P_v/P_{sv})_T \tag{3}$$

where P_v is the partial pressure of the water vapour and P_{sv} is the saturation vapour pressure at temperature T. If the values of ϕ and T in the settling chamber (ϕ_0 and T_0) are known and an assumption is made concerning the amount of supercooling that is possible before a condensation shock will occur, then the Mach number (M_1) at which the first condensation shock appears in the nozzle may be deduced from the tables in Ref. 1. In the present work, excepting Section 5.1, a value of 45 deg C supercooling has been assumed (as in Ref. 4).

(*b*) *Absolute humidity*, Ω. This is the quantity ratio ($\frac{\text{lb water vapour}}{\text{lb air}}$) and from the general gas laws Ω is related to the partial pressures of the mixture by the formula

$$\Omega = \frac{\text{mol. wt of water}}{\text{mol. wt of air}} \times \frac{P_v}{P - P_v}$$

$$= 0 \cdot 622 \frac{P_v}{P - P_v} \tag{4}$$

where P is the total pressure of the mixture, and P_v is the partial pressure of the vapour. The vapour pressure is usually small by comparison with the total pressure so that approximately

$$\Omega = 0 \cdot 622 P_v / P . \tag{5}$$

(*c*) *Heat release in a condensation shock.* The absolute humidity determines the amount of latent heat available for release to the air in a condensation 'shock'. Thus the specific heat addition q can be written

$$q = \Omega_0 n h \tag{6}$$

where h is latent heat of vaporisation

n is fraction of water vapour condensed

Ω_0 is absolute humidity at stagnation conditions.

Dimensionless expressions for this heat release are

$$Q = \frac{q}{C_p T} \tag{7}$$

where $C_p T$ is the local enthalpy of the air (C_p is specific heat of air at constant pressure) and

$$Q_0 = \frac{q}{C_p T_0} \tag{8}$$

where $C_p T_0$ is the enthalpy of the air in the settling chamber.

(*d*) *Maximum heat release in a single condensation shock.* The maximum amount of heat that can be liberated in a single condensation shock is given[1] by the equation

$$(Q_0)_{\max} = \frac{(M_1^2 - 1)^2}{M_1^2 \left(M_1^2 + \frac{2}{\gamma - 1} \right) (\gamma^2 - 1)} \tag{9}$$

with Q_0 defined as in Equation (8) above.

The variation of $(Q_0)_{\max}$ with M_1 is shown in Fig. 10.

(*e*) *Increase in total temperature of air across a condensation shock.* The increase in total enthalpy (of the air) must be equal to the heat input, hence

$$T_{02} = T_{01} + \frac{q}{C_p} \tag{10}$$

or

$$\frac{T_{02}}{T_{01}} = 1 + Q_0 \tag{11}$$

where suffix 1 refers to conditions ahead of, and suffix 2 to conditions behind the shock. (Note:— $T_{01} = T_0$.)

2.2. *Effect of Humidity on Recovery Temperature T_{w0}.* The Mach number in the working section with humid flow (M_c) may be related to the corresponding Mach number with dry air (M) by the approximate formula[4],

$$\frac{M_c}{M} = 1 - \frac{\frac{1}{2}\left(1 + \gamma M_1^2\right)\left(1 + \frac{\gamma - 1}{2} M^2\right)}{(M^2 - 1)} Q_0 \,. \tag{12}$$

This assumes that there is a single, normal condensation shock at Mach number M_1. Values of M_c appropriate to the present tests are plotted in Fig. 6a.

If we assume that there is no re-evaporation of ice crystals and that a boundary layer of air and ice-crystals behaves substantially as a boundary layer of air alone (plausible in view of the weak concentrations that occur in practice) then, following Equation (1), the recovery temperature is given by

$$\frac{T_{w0}}{T_c} = 1 + r\frac{\gamma - 1}{2} M_c^2 \tag{13}$$

where T_c is the static temperature in the working section with humid flow and r is the 'dry' value of recovery factor.

Likewise, following Equation (2),

$$\frac{T_{02}}{T_c} = 1 + \frac{\gamma - 1}{2} M_c^2 \,. \tag{14}$$

Hence, making use of Equation (11),

$$\frac{T_{w0}}{T_{01}} = (1 + Q_0)\frac{1 + r\frac{\gamma - 1}{2} M_c^2}{1 + \frac{\gamma - 1}{2} M_c^2} \,. \tag{15}$$

The function of Mach number on the right-hand side of Equation (15) shows only a slight variation with Mach number. Thus, for Mach numbers around 3, a ten per cent change in M would give a variation of less than a half of one per cent in this function. Therefore it is permissible to replace M_c by M (the dry value) in Equation (15), giving the working equation

$$\frac{T_{w0}}{T_{01}} = (1 + Q_0) \cdot \frac{1 + r\frac{\gamma - 1}{2} M^2}{1 + \frac{\gamma - 1}{2} M^2} \tag{16}$$

$$= \left(\frac{T_{w0}}{T_{01}}\right)_{\text{dry}} \cdot (1 + Q_0)$$

where

$$Q_0 = \frac{q}{C_p T_{01}} \tag{8}$$

and

$$q = \Omega_0 nh \,. \tag{6}$$

Alternatively, the increase in recovery temperature with humidity (ΔT_{w0}) is given (from Equation (16)) by

$$\Delta T_{w0} = \frac{q}{C_p} \cdot \frac{1 + r\frac{\gamma - 1}{2}M^2}{1 + \frac{\gamma - 1}{2}M^2} . \tag{17}$$

Reverting to Equation (9) the maximum amount of heat that can be liberated in a single condensation shock is given by

$$(Q_0)_{\max} = \frac{(M_1^2 - 1)^2}{M_1^2\left(M_1^2 + \frac{2}{\gamma - 1}\right)\left(\gamma^2 - 1\right)} \tag{9}$$

as plotted in Fig. 10.

This means that if Q_0 is greater than $(Q_0)_{\max}$ only a fraction of the water vapour will be condensed in the first condensation shock and the remainder will continue on as supercooled vapour until a second condensation shock occurs. If the nozzle has a low Mach number, less than about 2, a second condensation shock may not occur, and n will be less than unity (*e.g.*, Brun and Plan result at $M_c = 1 \cdot 78$, plotted in Fig. 10).

In the present tests at $M = 2 \cdot 92$ the $(Q_0)_{\max}$ parameter was exceeded on occasions by a large amount, Fig. 10, and multiple shock formations were found, Fig. 7. Within the experimental limitations n was found to be unity.

It may be noted that Equation (16) or (17) does not involve any assumptions regarding the number or nature (*i.e.*, normal or oblique) of the condensation shocks in the nozzle. Hence its predictions might be expected to be more accurate than corresponding results for Mach number (Equation (12)) or pressure, obtained in Ref. 4.

3. *Experimental Apparatus and Techniques.* 3.1. *The No.* 5 *Supersonic Tunnel at R.A.E. Farnborough.* The present tests were made with a 15 deg steel cone in the No. 5 Supersonic Wind Tunnel at R.A.E. Farnborough. This was a continuous, non-return flow tunnel with a 5-inch-square working section and a single-sided nozzle in wood. For these tests a nozzle was chosen which produced a dry free stream Mach number of 3·12 and the corresponding local Mach number for a 15 deg cone model was 2·92.

The distribution of stagnation temperature T_0 in the settling chamber was measured by means of copper/constantan thermocouples mounted on a grid, and after a preliminary survey the centre thermocouple was chosen as being a representative average and was used throughout the tests. A thermostatically and manually controlled electric heater upstream of the dust filter section enabled the stagnation temperature to be maintained to within $\pm \frac{1}{2}$ deg C.

Air drying was accomplished by a Butterley cold-air machine which dries by refrigeration.

The tests were carried out at a stagnation pressure of one atmosphere absolute, but some information obtained at a later date with a stagnation pressure of about 4 atmospheres has been included (*see* Fig. 6).

3.2. *Model and Mounting.* The model used for the measurement of recovery temperatures is shown in Fig. 1. It was a sharp-pointed 15 deg included angle cone in mild steel of wall thickness 0·1 inch. Thermocouple junctions of mild steel versus constantan were positioned at one-inch intervals on the top and bottom generators. Their calibration is described in Ref. 5.

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The mounting of the cone was via a thermally insulating sleeve of Tufnol ($\frac{1}{8}$ inch thick), followed by a hollow Duralumin sting (about 6 inches long) attached to a steel aerofoil beam (5 inches long) spanning the steel mounting box immediately downstream of the working section.

The model was designed for tests[5] on the effect of cooling on boundary-layer transition and so was not designed specifically for the degree of accuracy of measurement of laminar recovery temperature required by the present tests. Hence it was necessary to make an initial analysis (Section 4.1) to sort out the effect of heat conduction from the rear of the model. This analysis led to the thermocouple at the station nearest to the tip (2·6 inches back) being chosen for the subsequent analysis of the effects of humidity. (Under 'dry' conditions the recovery factor obtained at this station was close to the theoretical laminar value.)

3.3. *Humidity Measurement and Control.* Fig. 2a shows the simple air circuit of the dewpoint measuring apparatus. Air from the settling chamber was fed into a Brewer and Dobson dewpoint meter (which had been calibrated against a master instrument) and then exhausted to atmosphere. To keep response time to a minimum the air supply line was made very short, less than a foot in length. In order to isolate the meter chamber from atmospheric air and to keep a check on the pressure inside the instrument, a tapping from the inlet was fed through a glass tube into a beaker of dibutyl phthalate, and the outlet from the instrument was exhausted just below the surface of the liquid. Hence ΔP, the air pressure above atmospheric at the meter inlet, could be measured from the depth of the air/butyl meniscus below the surface. By this means it was possible to operate the dewpoint meter at low air flow rates and at a constant chamber pressure.

Initially, liquid nitrogen was used as a coolant for the thimble but excessive gassing caused large fluctuations of pressure inside the instrument and the dewpoint measurements were found to be unreliable. Alcohol cooled with solid carbon dioxide was found to be much more suitable and was used in all the tests that have been analysed.

The absolute humidity of the air in the tunnel was varied by (*a*) controlling the temperature of the outlet air from the Butterley refrigeration machine by switching the machine on and off, (*b*) starting dry with a very cold Butterley, then allowing the temperature to rise of its own accord with the machine switched off. Method (*b*) required a continuous plot of dewpoint measurements against time, coupled with the appropriate T_w/T_{01} measurements. A typical plot of method (*b*) is shown in Fig. 2b.

3.4. *Temperature measurement.* A Tinsley constant-resistance potentiometer and mirror galvanometer were used to measure thermocouple electro-motive force, from which temperatures T_w and T_{01} were computed. A near-null current technique was employed and the least count on the galvanometer slide corresponded to 0·02 deg C. The fixed temperature junctions of the thermocouples were kept in a thermostatically controlled electrically heated pot (Sunvic), the temperature of which was maintained at 41 deg C $\pm$ 0·1 deg C, and measured with a calibrated mercury thermometer. A close control of T_0 was kept, but because small differences in T_w/T_{01} were involved, it was the common practice to select T_0 and T_w alternately for measurement.

4. *Results of Tests at* $M = 2 \cdot 92$. (The results and relevant test conditions are given in Table 1.)

4.1. *The Effect of Absolute Humidity on Laminar Recovery Temperature* T_{w0}. Fig. 3 gives plots of the temperature ratio T_w/T_{01} for the whole length of the cone for both wet and dry air and for three values of T_0. The effect of humidity on recovery temperature is very noticeable from the separation of the wet and dry plots, but it is also apparent that there is some heat conduction from the rear of the model. The latter point is illustrated more clearly by Fig. 4a wherein the results are plotted on a temperature scale to show the increase in heat conduction from the rear with decreasing temperature T_w. The heat input may be from two sources (*a*) the room, when wall temperatures are lower than room temperature, and (*b*) unknown flow conditions at the rear of the cone, for example, the probable onset of transition over the last half inch. It has been shown[3] that humidity has an effect of delaying transition and this would account for some levelling out of the 'wet' air temperature profiles.

Fig. 4b shows the probable effect of heat conduction from the room on laminar recovery factor measured at the 2·6-inch station. This shows that the measured recovery factor for 'dry' air increases with increasing values of $T_R - T_w$, the room temperature minus the cone wall temperature. The values plotted to obtain this relationship are mean values computed from the results given in Fig. 5.

The thermocouple at station 2·6 was chosen as being the most free from the effects of heat conduction and hence giving the nearest to the true laminar recovery temperature, T_{w0}, and the analysis of the effect of absolute humidity on recovery temperature is based on temperatures measured at this station.

Fig. 5 gives measured values of T_{w0}/T_{01} for station 2·6, plotted against absolute humidity for three values of T_0 and the results support the theory of Section 2.2 quite well. The theoretical curves give the values predicted by Equation (16), with recovery factor $r = 0 \cdot 85$ and Q_0 from Equations (8) and (6), with $n = 1$, *i.e.*, assuming that all of the water vapour is condensed in the nozzle and that there is no re-evaporation either through the tip shock of the cone, or in the boundary layer.

The deviation of the results from the theoretical curve is explained as follows. With dry air the effect of heat conduction is shown by an increase in the temperature ratio T_{w0}/T_{01} with decrease in stagnation temperature. With wet air the slopes of the results in Fig. 5 are slightly smaller than the slopes of the theoretical curves due not to re-evaporation in the boundary layer, but to a probable error in the calibration of the dewpoint meter thermometer. A recent calibration (1958) showed disagreement with the previous calibration (1954) of the order of 2 to 3 deg C at the high dewpoint temperatures. Correction of the humidity values to this new calibration would bring the slopes of the results exactly into line with the slopes of the theoretical curves, but as the tests were made in 1956 it could not be guaranteed that the new calibration would have applied at that time.

The results support the hypothesis that the recovery temperature rise at the surface of a body under a laminar boundary layer in a humid airflow in a supersonic tunnel, includes all the latent heat of water vapour given up in the condensation process, and that no re-evaporation occurs in the boundary layer.

The recovery temperature may be very closely approximated by Equation (16), putting $n = 1$.

Later tests at a higher stagnation pressure and therefore greater convective heat transfer from the boundary layer, Fig. 6b, support this view. Results from Laufer and Marte[3], Fig. 9, are given as further confirmation.

4.2. *Measured Values of M_1, the First Condensation Shock Mach Number, and the Effect of Relative Humidity on Shock Formation.* Three schlieren pictures were taken of condensation shocks at different levels of absolute humidity and at a constant stagnation temperature. By measurement of the Mach angle between the shock and the profile of the curved liner values of

M_1 were calculated and are compared with theoretical values of M_1 from Fig. 10 in the following table.

Ω_0	T_0 (deg K)	M_1 (Theoretical)	M_1 (Measured)
0·0025	314	1·53	1·5
0·0054	314	1·39	1·43
0·0063	314	1·37	1·35

There is good agreement between theoretical and measured M_1 values.

Fig. 10 consists of (*a*) a plot of the relationship between $(Q_0)_{max}$ and M_1 (Equation (9)) and (*b*) a superimposed plot of isothermals (constant T_0) and equi-humid lines (constant Ω_0). The plot (*b*) is geared to a supercooling of 45 deg C since M_1 is very sensitive to the degree of supercooling.

For a given value of absolute humidity, Ω_0, and a given stagnation temperature, T_{01} the intersection of the isothermal and the equi-humid lines gives the values of Q_0 and M_1 (from the vertical and horizontal scales). Also, the position of the intersection with respect to the $(Q_0)_{max}$ curve indicates either complete or partial condensation of the vapour in the first condensation shock. For complete condensation the intersection will lie below the $(Q_0)_{max}$ curve and for partial condensation it will lie above it. In the latter case the ratio of vapour condensed to vapour available is given by the ratio of $(Q_0)_{max}$ to Q_0.

Fig. 7 shows the effect of relative humidity ϕ_0 and the ratio $Q_0/(Q_0)_{max}$ on the condensation shock pattern (shadowgraph). At low values of ϕ_0 and $Q_0/(Q_0)_{max}$ only a single shock was visible, attached to the curved liner a short distance from the throat. With increase in ϕ_0 and $Q_0/(Q_0)_{max}$ the shock was formed nearer the throat, i.e. M_1 was reduced, and at a value of $Q_0/(Q_0)_{max} = 0{\cdot}98$ a reflection of the condensation shock was visible. At higher values of ϕ_0 and $Q_0/(Q_0)_{max}$ the reflected shock grew in strength and moved nearer the throat until the condition was reached when two reflections and re-reflections, apparently from two condensation shocks, were observed. The condensation shocks themselves were close to the throat and outside the field of vision in this case.

4.3. *The Effect of Humidity on Flow Direction.* During a separate series of boundary-layer studies of transition movement on this cone, humidity had distinct effects on the flow direction in the working section. No quantitative measurements were made, but, on occasion, an incidence change of 10 minutes was observed (Ω_0 of the order of 3×10^{-3}), this being the correction applied to the cone model incidence to obtain circumferential similarity of transition position when running with humid air instead of the usual dry air. (It should be noted that the nozzle of this tunnel was 'single-sided', *i.e.*, the bottom liner corresponded to the centre line of a two-dimensional nozzle, *see* Fig. 7.)

5. *Analysis of Results of Humidity Effect on Wall Temperature Published by French and American Authors.* 5.1. *Results of Brun and Plan* ($M = 1{\cdot}85$). A report on the effect of humidity on pitot and recovery temperatures was published in 1954 by Brun and Plan[3]. No values of absolute humidity or of stagnation temperature were given in this report but these have since been obtained by private communication. The tests were made at $M = 1{\cdot}85$ (dry) and measurements were made (*a*) of recovery temperature on the cylindrical portion of an ogive cylinder made from Plexiglas (*see* Fig. 8a and b) of total temperature, using a pitot-thermocouple.

The measurements extended through the condensation shock and from Fig. 8a it can be seen that the effect of humidity on total temperature as measured by the pitot-thermocouple was negligible. On the other hand there was a marked effect of humidity on recovery temperature.

The results for T_{w0}/T_{01} versus Ω_0 for the position 16 cm from the throat, where the Mach number is constant, are plotted in Fig. 8b. They are only two in number, for $\Omega_0 = 0 \cdot 0002$ and $\Omega_0 = 0 \cdot 005$, and therefore do not permit too detailed an analysis. Theoretical curves are drawn for recovery factor values of $0 \cdot 85$ and $0 \cdot 86$ (experimental). The change in value of r merely displaces the curve and does not affect the slope, the latter is determined by n, the fraction of water vapour condensed.

The wet result does not fit either curve for $n = 1$, which suggests that either (*a*) some re-evaporation is occurring through the tip shock since it has been shown (Section 4.1) that no re-evaporation occurs in a laminar boundary layer or (*b*) for this amount of humidity only a fraction of water vapour has condensed, the remainder being supercooled.

Turning to Fig. 8a it is apparent from the recovery temperature traverse that only one condensation shock occurred, since there is only one dip in the curve; this was also confirmed by schlieren observation.

There is a limit, Equation (9) and Fig. 10, to the amount of water that can be condensed in a single condensation shock, and hence in this case it is appropriate to take $n = (Q_0)_{max}/Q_0$ when Q_0 is greater than $(Q_0)_{max}$.

The upper curve in Fig. 8b (for $r = 0 \cdot 86$ and $n = 1$) was therefore carried only as far as the value of Ω_0 for which $Q_0 = (Q_0)_{max}$. For greater values of Ω_0, the curve is extended (full line) by taking $n = (Q_0)_{max}/Q_0$. The location of this extended portion depends on the amount of supercooling and, as shown in Fig. 8b, the 'wet' experimental point can be fitted by assuming a supercooling of 47 deg C (with $r = 0 \cdot 86$).

Calculations from the measured temperature increase at $\Omega_0 = 0 \cdot 005$ show that 69 per cent of the available water vapour was condensed in the shock and that the remainder continued on as supercooled vapour. The relative humidity of this vapour, based on $M_c = 1 \cdot 78$ and $T_c = 188$ deg K, the free-stream condition, was equal to 1990 and the degree of supercooling equal to $67 \cdot 5$ deg C.

It would be of interest to obtain further experimental confirmation of these large degrees of supercooling and relative humidity *after* the first condensation shock. Maximum values previously quoted[1], *before* the first condensation shock, have been $\phi = 300$ and supercooling of 63 deg C.

5.2. *Results of Laufer and Marte*[3] ($M = 4 \cdot 02,\ 4 \cdot 43,\ 4 \cdot 71$). Fig. 9 shows results for the effect of absolute humidity on recovery temperature calculated from information given[3]. They are for laminar boundary layers on a 5 deg included angle insulated cone at local Mach numbers, computed from quoted free stream values of $4 \cdot 02$, $4 \cdot 43$ and $4 \cdot 71$. The value of T_{01} for all tests was 325 deg K.

Theoretical curves, Equation (16), are shown for recovery factors of $0 \cdot 85$ and $0 \cdot 828$ (experimental) with $n = 1$. The plotted results for the two lower Mach numbers support the theoretical curve for $r = 0 \cdot 828$, and show 100 per cent transfer of available latent heat of water vapour to the air with no re-evaporation at the surface. For $M = 4 \cdot 71$ the only two experimental points available do not permit any other conclusion to be drawn.

The choice of $r = 0 \cdot 828$ to tie in with the 'dry' experimental results does not affect the slope of the theoretical curve. For a constant value of n the slope will be constant.

6. *The Effect of Humidity on a Temperature Recovery Factor Based on Dry Air Conditions.* The foregoing analysis has been based on the assumption (Section 2.2, Equation (13)) that recovery

factor does not vary with humidity, provided the correct values of M_c and T_c are used in its definition. It is also of interest to show how an indicated recovery factor, r_i, based throughout on dry values of M and T, would vary with humidity. This is done in Fig. 11 which collects together all the results previously discussed.

The assumption of a constant Mach number has very little influence on the results. Hence the indicated recovery factor r_i can be computed from the experimental temperature ratio T_{w0}/T_{01} using the formula

$$r_i = \frac{[\{1+(\gamma-1)M^2/2\}\, T_{w0}/T_{01} - 1]}{(\gamma-1)M^2/2}\,. \tag{18}$$

Theoretical curves of r_i are shown in Fig. 11 for the various Mach numbers and stagnation temperatures covered in the tests. These were computed from Equation (18), using Equation (16) for T_{w0}/T_{01}, i.e.,

$$r_i = \frac{[(1+Q_0)\{1+r(\gamma-1)M^2/2\} - 1]}{(\gamma-1)M^2/2} \tag{19}$$

where n (in Q_0) is equal to one and r is the appropriate experimental 'dry' recovery factor.

The effect of humidity is to increase the value of r_i and increases of the order of 10 per cent are shown. The scatter in the present $M = 2\cdot92$ results is explained partly by the effect of heat conduction from the base of the model (Section 4.1) and partly by the experimental limitations. For instance a total error of 1 deg C in the recovery and stagnation temperature measurement, due to a slight unsteadiness in the stagnation temperature, can displace a point about 1 per cent on the r_i ordinate and an error of 1 deg C in the dewpoint measurement at the high humidities, say $\Omega_0 = 0\cdot005$, will displace a point about 5 per cent along the abscissa.

The results of Laufer and Marte show some scatter but fit the theoretical curve well and there is little effect of Mach number at this level.

Brun and Plan's result at $M = 1\cdot85$ does not fit the $(n = 1)$ curve, due to incomplete condensation of the available water vapour (Section 5.1). The theoretical curve shows the large rate of increase in r_i to be expected at low supersonic Mach numbers, for values of $Q_0 < (Q_0)_{max}$.

7. *Comments.* The tests described are shown to be in close agreement with theory and with results obtained from other sources[2,3]. They show that the effect of the condensation 'shock' is to increase the enthalpy of the air and, because no re-evaporation takes place in a laminar boundary layer or the weak tip shock of a sharp 15 deg cone or similar slender body*, the recovery temperature may be calculated knowing the absolute humidity.

The true recovery factor is shown not to be affected but an indicated recovery factor, r_i, assuming dry air conditions shows a marked increase with humidity.

On the other hand Brun and Plan have shown (Fig. 8a) that the total temperature of a pitot-thermocouple, which has a strong bow shock, is negligibly affected by humidity.

It seems then that the measurement of recovery temperature at the surface of a model under a laminar boundary layer could be a convenient method of monitoring the degree of condensation in the air in a supersonic wind tunnel. It would not replace the need for the occasional direct measurement of absolute humidity, but would provide a running check of conditions prevailing, for example, during a series of force measurement.

* Later measurements by the author on a sharp-edged flat plate confirm this statement. However, they also indicate that with turbulent boundary layers some re-evaporation may occur.

8. *Conclusions*. The following conclusions may be drawn from the tests described.

(*a*) The results support the hypothesis that the recovery temperature rise at the surface of a slender body under a laminar boundary layer in a humid airflow in a supersonic tunnel, includes all the latent heat of water vapour given up in the condensation process, and that no re-evaporation occurs in the boundary layer.

(*b*) The recovery to stagnation temperature ratio T_{w0}/T_{01} is shown to be given by Equation (16) (Fig. 5) and an approximate increase in recovery temperature due to humidity by Equation (17).

(*c*) There is no effect of humidity on r, the recovery factor, if the true Mach number M_c and the true temperature T_c are used in its calculation, but if an indicated recovery factor r_i is computed, using dry values of M and T, then it will increase with increase in absolute humidity (Fig. 11).

(*d*) For Mach numbers of 2·8 and above, more than one condensation shock may occur, dependent on the initial relative humidity (Fig. 7) and all the latent heat of water/ice is transferred to the air in the process. For Mach numbers below 1·85 and when only one condensation shock occurs the amount of latent heat released appears to be limited by the $(Q_0)_{\max}$ parameter of Lukasiewicz, Equation (9), which is a function of the degree of supercooling before the shock.

(*e*) Small changes in flow direction due to condensation shocks in the nozzle have been observed.

(*N.B.* Conclusions (*a*), (*b*) and (*c*) are applicable only to laminar boundary layers. Some later test results (unpublished), obtained on a flat plate, indicate that with turbulent boundary layers some re-evaporation may occur.)

LIST OF SYMBOLS

C_p	Specific heat of air—0·24 C.H.U./lb deg C
h	Latent heat of vaporization (ice)—690 C.H.U./lb
M	Mach number
n	Fraction of water vapour condensed
p, P	Pressure
Q	Heat of condensation/air enthalpy
q	Heat of condensation ($hn\Omega_0$)—C.H.U./lb
r	Temperature recovery factor
T	Temperature—deg C or deg K
x	Distance along generator from tip of cone—inches
γ	Ratio of specific heats for air (1·4)
Ω	Absolute humidity
ϕ	Relative humidity

SUFFIXES

$_0$	Stagnation conditions
$_1$	Conditions immediately upstream of the first condensation shock
$_2$	Conditions after the condensation shock or shocks
$_c$	Condensation conditions in the working section
$_i$	Indicated, assuming dry conditions
$_v$	Vapour
$_{sv}$	Saturated vapour
$_w$	Wall
$_{w0}$	Wall at zero convective heat transfer

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TABLE 1

Measured Laminar Recovery Temperatures with Humid Flow

(Station 2·6)

	Test No.	$\Omega_0 \times 10^3$	T_0	T_{room}	T_{w0}	$\frac{T_{w0}}{T_{01}}$	r_i
Method (*a*) Section 2.3	222	5·85	289·6	293	278·1	0·9603	0·937
	222	6·0	302·2	293	288·3	0·954	0·927
	222	6·0	313	293	297·3	0·950	0·9205
	222	3·91	289·6	293	273·1	0·943	0·910
	222	3·5	302·1	293	282·7	0·936	0·8985
	222	4·02	304·5	293	287·2	0·943	0·910
	222	3·81	313	293	292·1	0·933	0·894
	223	2·6	289·4	294	270	0·933	0·894
	223	1·95	289·1	294	268·2	0·928	0·886
	223	2·1	289·3	294	270·1	0·934	0·895
	223	1·15	290·3	294	267·3	0·921	0·875
	223	2·2	289·3	294	270·1	0·934	0·895
	224	0·14	289·8	294	264·3	0·912	0·860
	224	0·11	289·4	294	264·2	0·913	0·862
	224	0·11	312·5	294	282·4	0·904	0·847
	224	4·6	313·9	294	294	0·937	0·900
	224	4·95	313·8	294	295·4	0·941	0·9065
	224	5·05	314	294	296·4	0·944	0·911
	228	0·03	313·1	293·5	283·6	0·906	0·85
	228	0·03	283	292	258·4	0·912	0·86
	228	0·09	299·4	292	275·8	0·921	0·875
	229	7·19	313·5	291	299	0·954	0·927
	229	7·2	302·8	291	291·9	0·966	0·946
	230	0·37	288·7	291	266·1	0·922	0·876
	230	0·26	301·3	291	274·1	0·91	0·857
	232	5·0	286	292	273·6	0·957	0·932
Method (*b*) Section 2.3	228	0·15	300·9	292	274·2	0·911	0·858
	228	0·18	302·3	292	275·5	0·911	0·858
	228	0·25	300·9	292	275·4	0·915	0·865
	228	0·38	300·9	292	275·4	0·915	0·865
	228	0·60	302·3	292	276·5	0·915	0·865
	228	0·90	302·1	292	277·5	0·919	0·871
	228	1·3	299·8	292	278·8	0·93	0·889
	228	1·85	301·8	292	278·8	0·924	0·879
	228	2·45	300·1	292	279·2	0·93	0·889
	228	3·1	300·5	292	281·2	0·936	0·898
	228	3·75	303·3	292	283·3	0·934	0·895
	228	4·3	304·8	292	286·45	0·94	0·905
	228	4·85	305·2	292	287·4	0·942	0·908

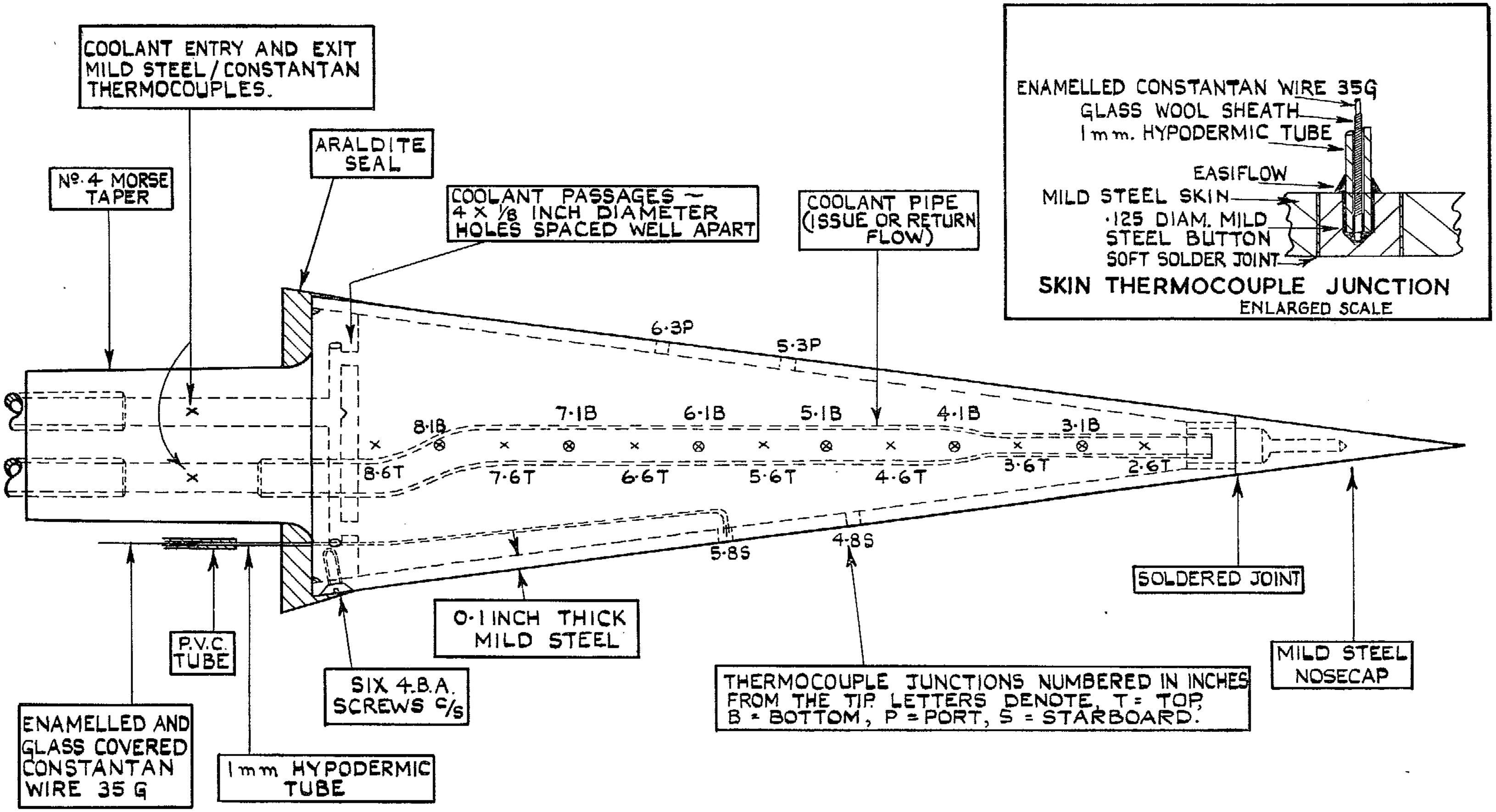

FIG. 1. 15 deg mild steel heat transfer cone.

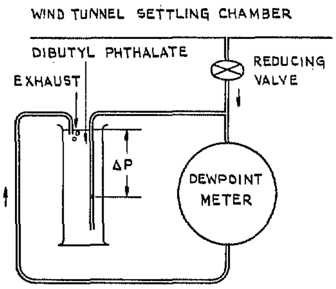

FIG. 2a. Diagram of dewpoint meter circuit.

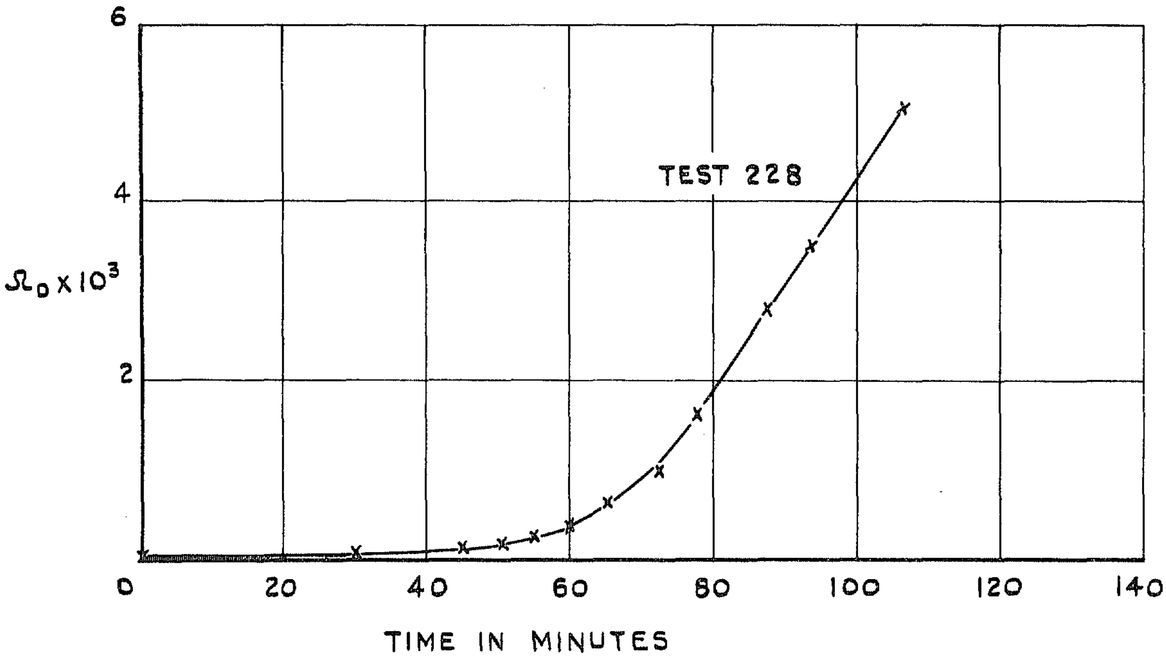

FIG. 2b. A typical plot of absolute humidity versus time. (Method (*b*) sect. 2.3.)

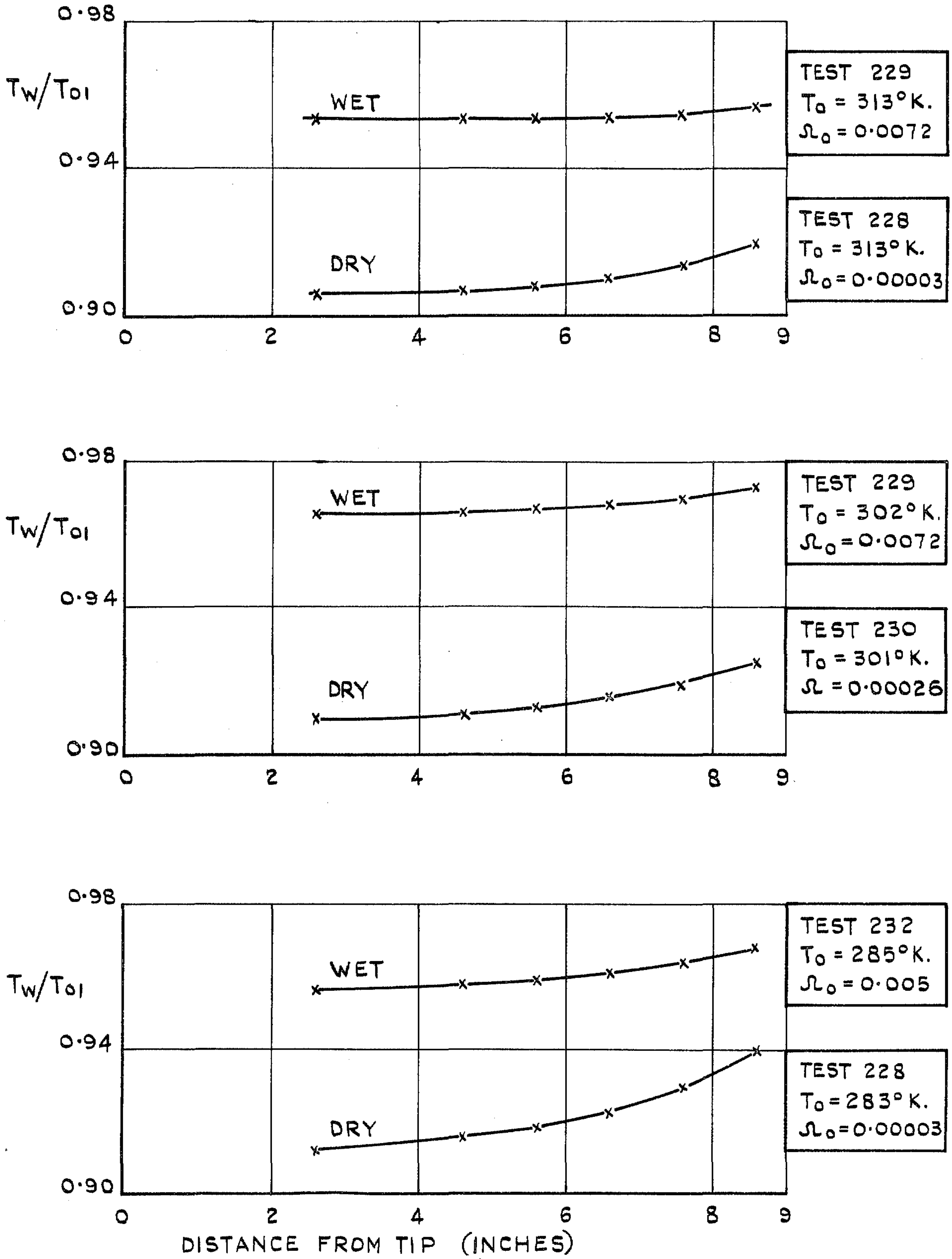

FIG. 3. Temperature distributions along the cone for both wet and dry air at various stagnation temperatures.

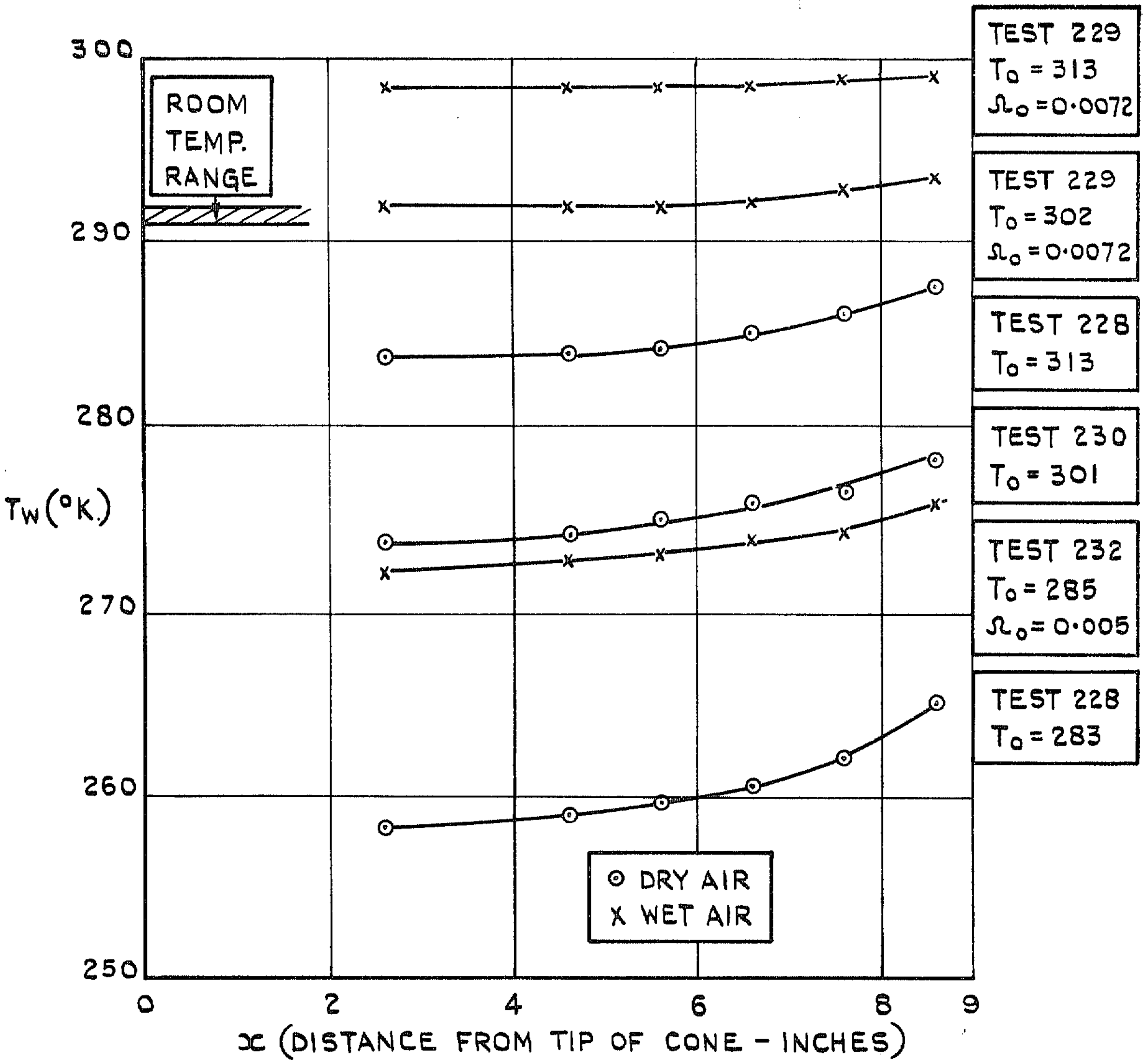

FIG. 4a. Absolute temperature distributions along cone.

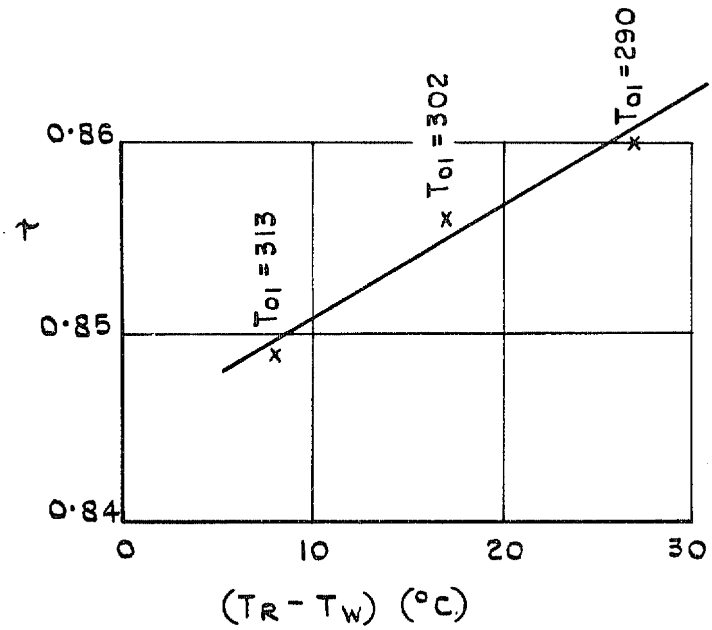

FIG. 4b. The effect of $(T_R - T_W)$, the room temperature minus cone wall temperature on measured recovery factor for dry air. (Mean values from plots in Fig. 5.)

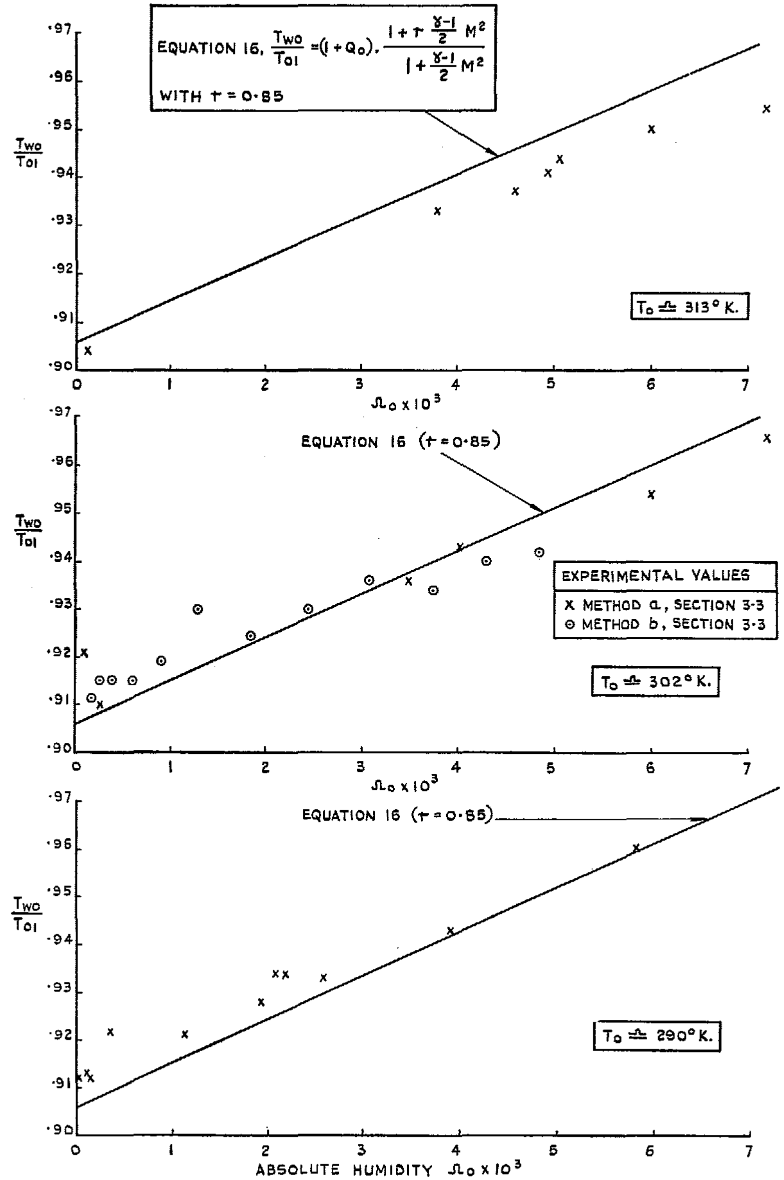

FIG. 5. The effect of absolute humidity on temperature ratio T_{w0}/T_{01} for station 2·6.

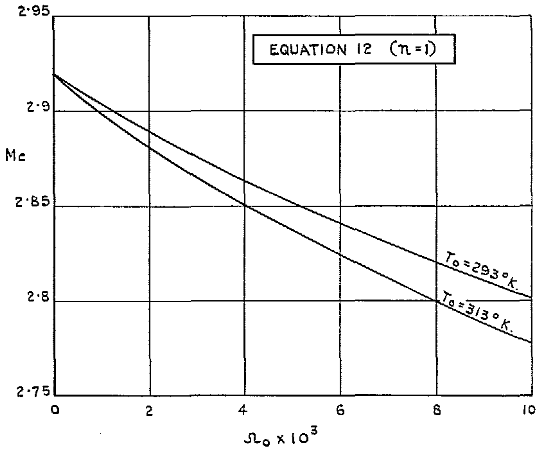

FIG. 6a. The effect of absolute humidity on true Mach number M_c.

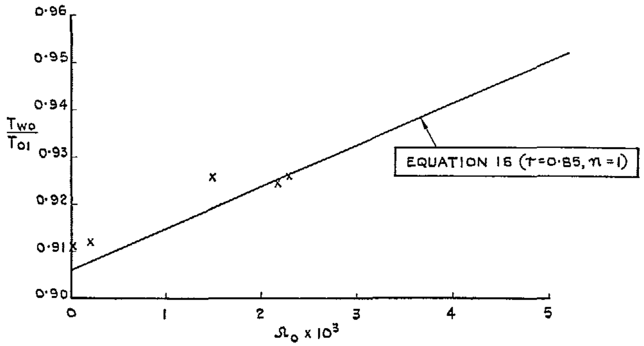

FIG. 6b. The effect of absolute humidity T_{w0}/T_{01} for laminar boundary layers at $P_0 \simeq 4$ atmos. and $T_0 = 313$ deg K.

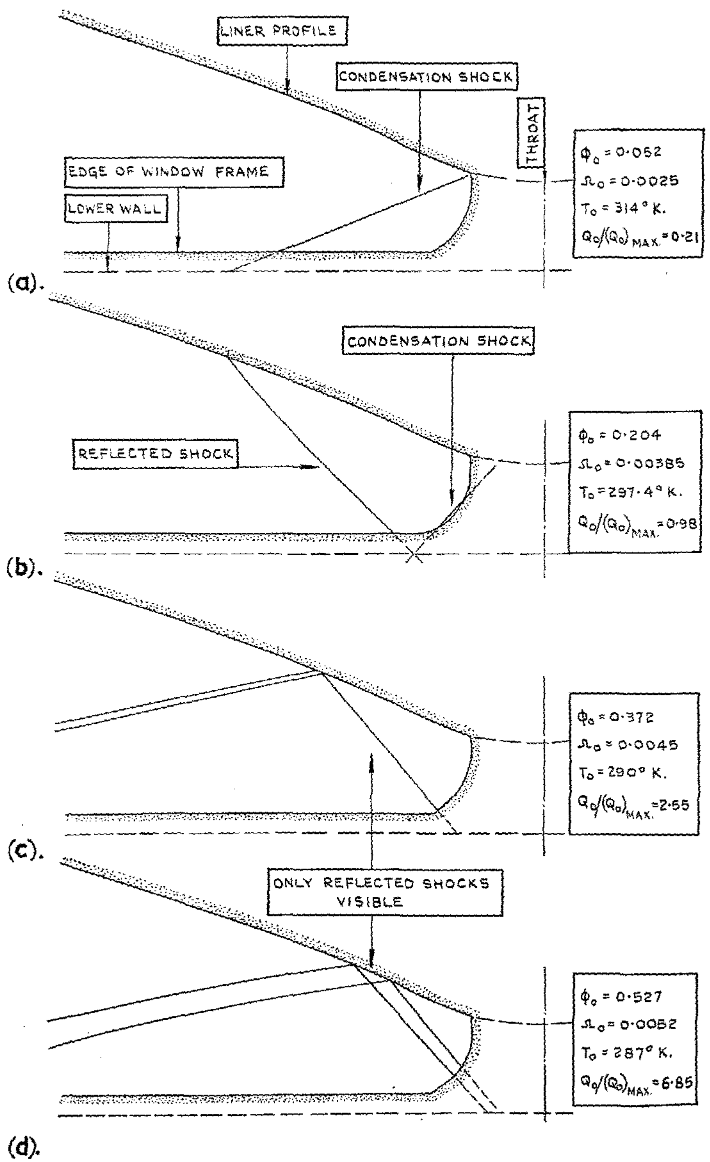

FIG. 7a to d. The effect of relative humidity ϕ_0 and the ratio $Q_0/(Q_0)_{max}$ on condensation shock patterns (shadowgraph).

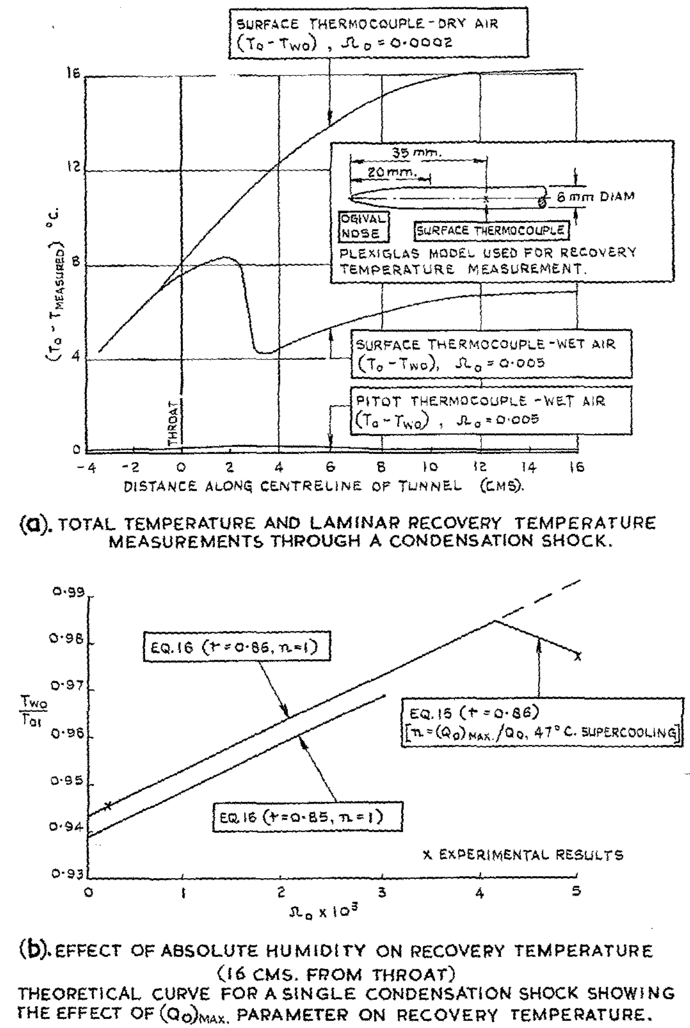

FIG. 8a and b. Results of Brun and Plan, ref. 2. ($M = 1{\cdot}85$, $T_0 = 298$ deg K.)

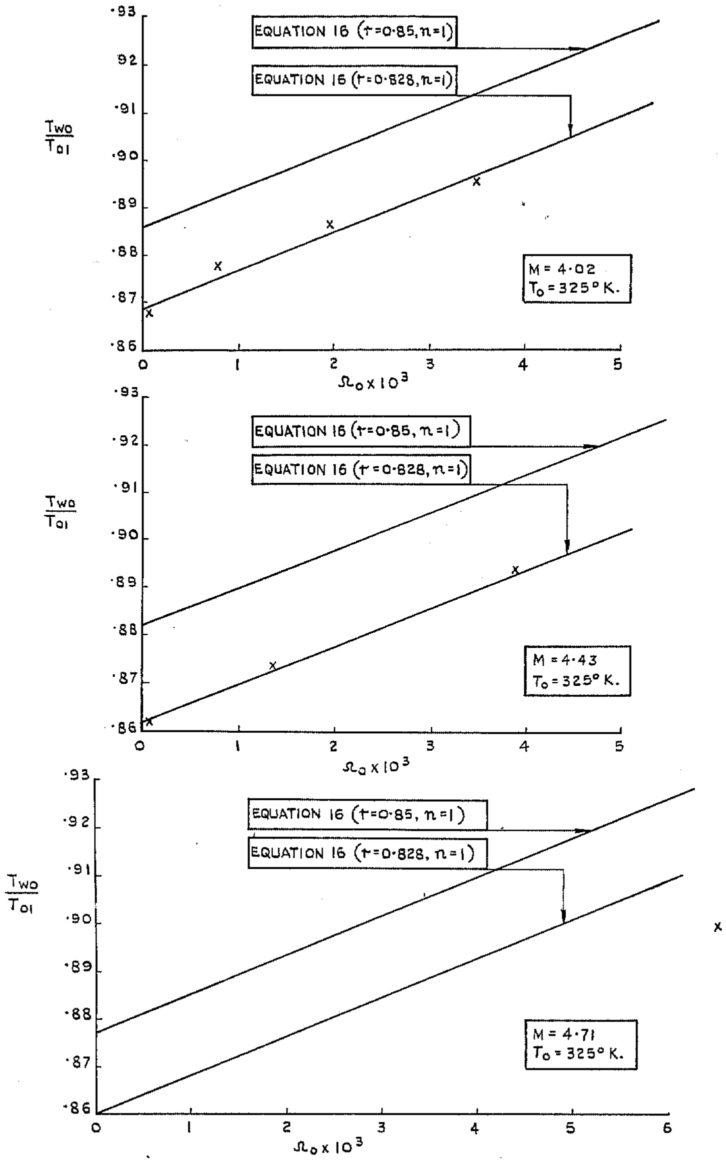

FIG. 9. Results of Laufer and Marte (5 deg insulated cone) ($M = 4 \cdot 02, 4 \cdot 43, 4 \cdot 71$).

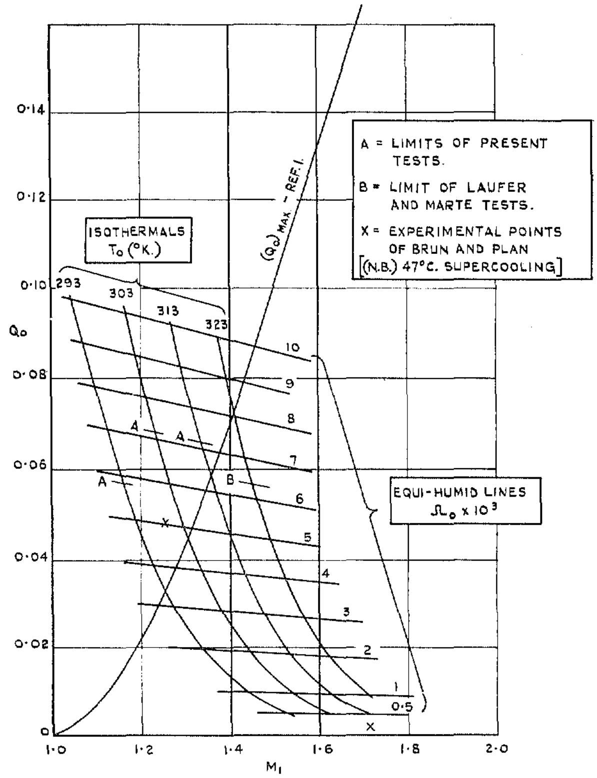

FIG. 10. Theoretical plot of isothermals (T_0) and equi-humid lines (Ω_0), based on 45 deg C. supercooling, giving the first condensation shock Mach number (M_1) and the heat release (Q_0). $\left(Q_0 = \frac{\Omega_0 nh}{C_p T_0}\right)$.

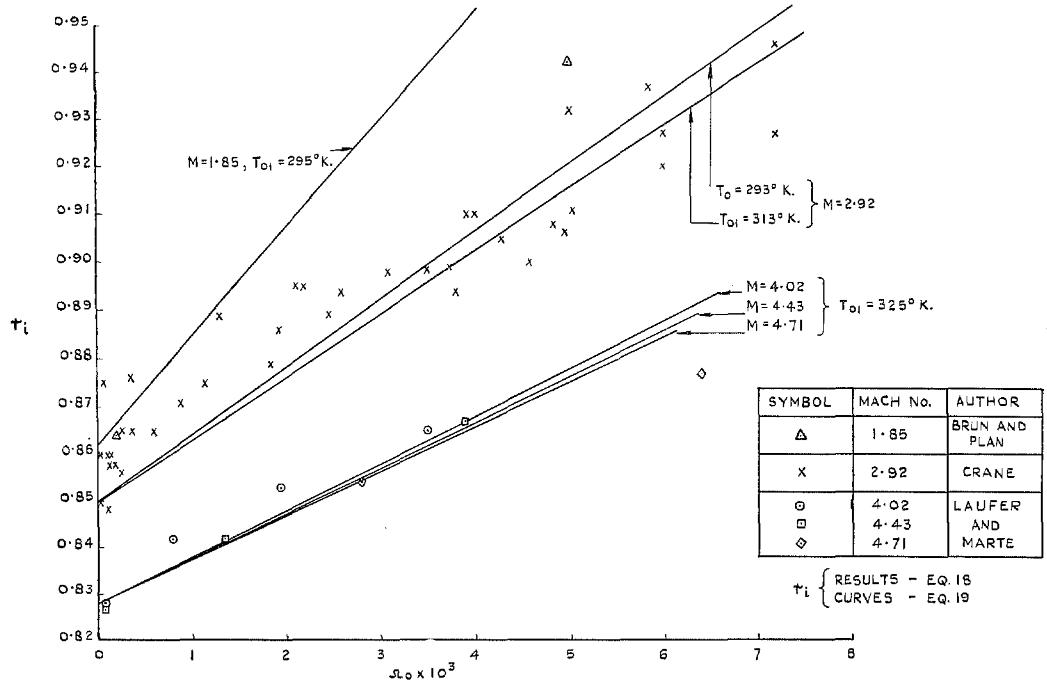

FIG. 11. The effect of absolute humidity on indicated recovery factor (laminar boundary layer).

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