C.P. No. 361
(17,683)
A.R.C. Technical Report

C.P. No. 361
(17,683)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
CURRENT PAPERS

The Calculation of the Paths of Vortices from a System of Vortex Generators, and a Comparison with Experiment

By

J. P. Jones, B.Sc. (Eng.),
University of Southampton

LONDON : HER MAJESTY'S STATIONERY OFFICE
1957
THREE SHILLINGS NET

C.P. No. 361

The Calculation of the Paths of Vortices from a System of Vortex Generators, and a Comparison with Experiment

- By -
J. P. Jones, B.Sc.(Eng),
University of Southampton*.

March, 1955

SUMMARY

A method is described for the calculation of the paths of the trailing vortices from a system of counter-rotating vortex generators, and a comparison is made with some experimental results obtained in a water tunnel. The method can be adapted to other configurations of generators arranged to produce rows of vortices close to a plane surface.

Introduction

A large body of experimental evidence was available, from both wind and water tunnel tests, on the paths taken by vortices downstream of several types of generator mounted adjacent to a surface. Observation had shown that (usually) the vortices first approached the surface, until a certain minimum height above it was reached, after which they moved steadily away. The general form of the path of any vortex of the system can be confirmed by qualitative reasoning, on the basis that there are always one or two vortices whose influence is predominant[1].

It was found that the minimum height, and the distance behind the generators at which it was reached, were functions of the height, span, spacing and inclination of the generators. Since the efficiency of any system of generators depends upon the vortices remaining close to the surface, it would be an advantage to be able to predict the minimum height with accuracy; and if a simple analytical method could be found, it would be more satisfactory than a lengthy experimental programme. This report is an account of the theory as developed for a particular configuration, but it would be a simple matter to extend it to other arrangements.

2. Scope of the Theory

A complete solution of the problem would be difficult, if not impossible to find. Many factors, such as the effects of viscosity and the strengths of the vortices, cannot be determined with accuracy, and the time required for a full investigation would be prohibitive. The following main assumptions were found to be necessary:-

a/

---

*This paper was written whilst the author was working in the Aerodynamics Division of the National Physical Laboratory.

(a) That three-dimensional effects can be completely neglected, - except when estimating the circulation around the generators, - i.e., at any point downstream of the generators, the flow in a plane perpendicular to the main stream can be regarded as the two-dimensional flow of a perfect fluid. This assumption is commonly used in the evaluation of wind tunnel corrections - which is an analogous problem - but it might be expected to introduce an error in the regions close to the generators. The reason is that a "two-dimensional" vortex extends to infinity in both directions normal to its plane, so that the induced velocity at any point is twice that produced opposite the end of a semi-infinite vortex. Therefore, close to the generators, the induced velocity is only about one-half that predicted by two-dimensional flow. (But see Discussion, Sec. 6.) It is implicit in the assumption of uniform two-dimensional flow that the slopes of the vortex paths must everywhere be small. Otherwise the induced velocity would have an appreciable component in the direction of motion. This condition is satisfied in all the observed examples.

(b) Viscosity has been completely neglected. In practice it has two important effects:-

(i) The vortices decay in strength as they move downstream. It is assumed here that the strength is constant.

(ii) For the greater part of their length the vortices are in the boundary layer close to the surface. Hence the velocity of the "undisturbed" stream (i.e., undisturbed by the vortices) depends upon the distance from the surface. For simplicity this variation was neglected.

(c) It was also assumed that, even in the presence of the vortices, the component of the velocity parallel to the surface remained everywhere uniform and equal to V.

3. Notation

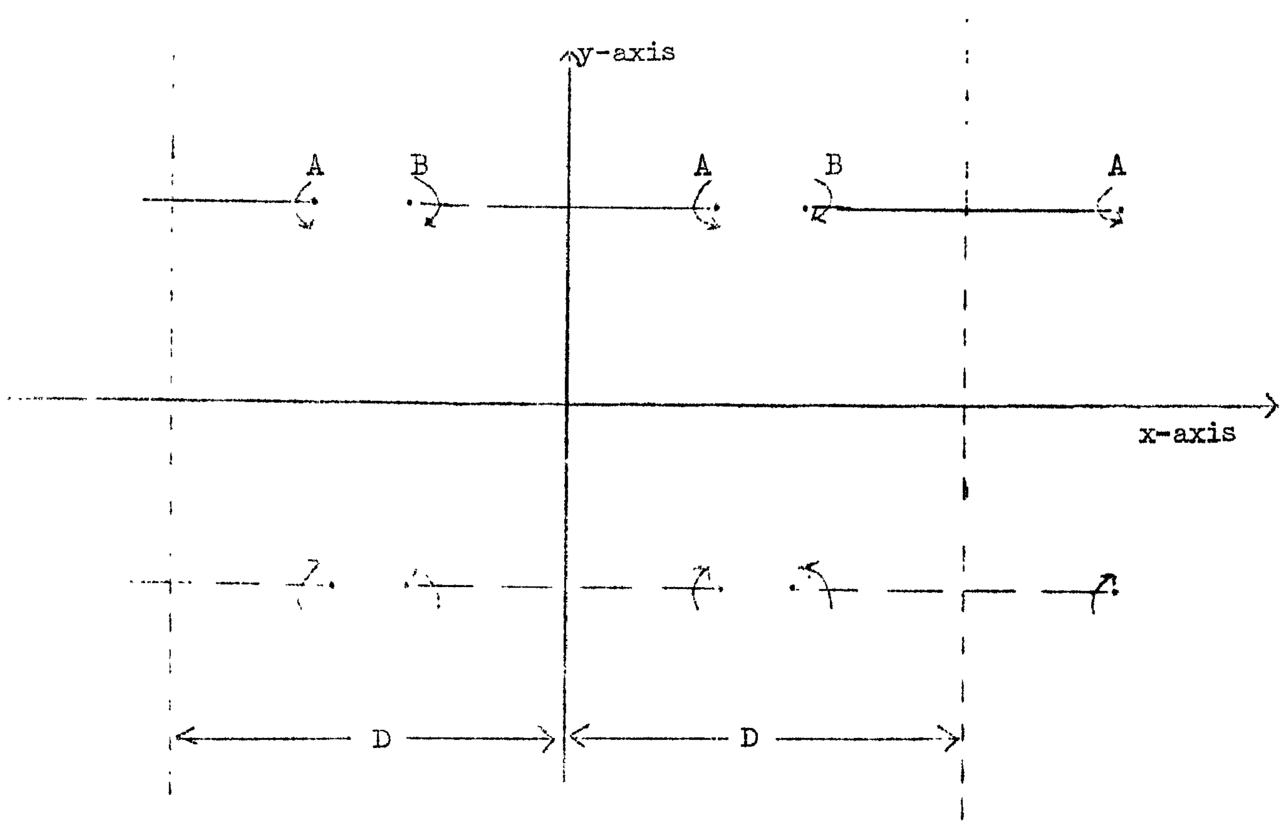
A, B	Rows of vortices
D	Distance between generators (spanwise period)
V	Velocity of the undisturbed stream
ρ	Density
x	Spanwise co-ordinate
y	Distance above surface
s	Distance downstream
x_1, y_1	Co-ordinates of the tip of a generator
x_0, y_0	Co-ordinates of a particular vortex core.
ξ	$2\pi x_0/D$
η	$2\pi y_0/D$
θ	$2\pi s/D$
K	Strength of a vortex
w(z)	Complex potential

k	$\frac{K}{D.V}$
α_0	Inclination of generators
c_0	Chord of generators
ϕ	Effective incidence
c	c_0/D
$w(x')$	Downwash velocity
λ	Dw_m/K
a	Lift slope in two-dimensional flow
w_M	Mean downwash

4. Theory

Consider a system of vortices in counter-rotating pairs (sketch I), such as would be generated by an infinite row of "wing-type" generators (See Ref. 1) mounted close to a plane surface, and inclined at a small positive angle to it.* In the sketch, the direction of the undisturbed stream is assumed to be into the paper. The row of equally-spaced vortices marked A (referred to below as row A) are produced at the right-hand tips of the generators and the row B by the left-hand tips. Each vortex has an image of opposite sign in the surface.

A system of co-ordinates (x,y,s) is chosen with the surface as x-s plane and with the y-s plane bisecting one of the generators. Sketch I represents a section normal to the flow at a distance s downstream of the generators; the dotted vertical lines, spaced a distance D apart, denote the relative positions of the generators.



Sketch I

Consider/

---

*Such a system could also be generated by an infinite row of pairs of divergent vanes protruding from the surface.

Consider the upper rows of vortices A and B. If A_0 is the point $z_0(\equiv x_0 + iy_0)$ then B_0 is the point $-\bar{z}$ ($\equiv -x_0 + iy_0$).

It can be shown that the complex potential $w(z)$ of these rows of vortices is (Ref. 2):-

$$w = \frac{-iK}{2\pi} \log \sin \frac{\pi}{D}(z - z_0) + \frac{iK}{2\pi} \log \sin \frac{\pi}{D}(z + \bar{z}_0) \qquad \ldots(1)$$

Hence the complex potential of these two rows and their images is

$$w = -\frac{iK}{2\pi} \log \sin \frac{\pi}{D}(z - z_0) + \frac{iK}{2\pi} \log \sin \frac{\pi}{D}(z + \bar{z}_0)$$

$$+ \frac{iK}{2\pi} \log \sin \frac{\pi}{D}(z - \bar{z}_0) - \frac{iK}{2\pi} \log \sin \frac{\pi}{D}(z + z_0) \qquad \ldots(2a)$$

We now confine attention to the path of the vortex A_0, i.e., $z = z_0$. (It can be inferred from symmetry that the paths of the vortices A and B are symmetrical about the y-axis.) Now the row of equally-spaced vortices to which A_0 belongs induces no velocity at A_0, hence the motion of A_0 is due to (i) the images of the row A and (ii) the row B and its images.

Thus to determine the motion of A_0, we need only consider the complex potential:-

$$w = \frac{iK}{2\pi} \log \sin \frac{\pi}{D}(z - \bar{z}_0) + \frac{iK}{2\pi} \log \sin \frac{\pi}{D}(z + \bar{z}_0)$$

$$- \frac{iK}{2\pi} \log \sin \frac{\pi}{D}(z + \bar{z}_0) \qquad \ldots(2b)$$

Hence $\frac{dw}{dz} = \frac{dx}{dt} - i\frac{dy}{dt}$

$$= \frac{iK}{2D} \cot \frac{\pi}{D}(z - \bar{z}_0) + \frac{iK}{2D} \cot \frac{\pi}{D}(z + \bar{z}_0)$$

$$- \frac{iK}{2D} \cot \frac{\pi}{D}(z + z_0) \qquad \ldots(3)$$

To calculate the motion of A_0, put $z = z_0$

$$\text{i.e.,} \left(\frac{dw}{dz}\right)_{z=z_0} = \left\{\frac{dx}{dt} - i\frac{dy}{dt}\right\}_A = \frac{iK}{2D} \cot \frac{2i\pi y_0}{D}$$

$$+ \frac{iK}{2D} \cot \frac{2\pi x_0}{D} - \frac{iK}{2D} \cot \frac{2\pi}{D}(x_0 + iy_0) \qquad \ldots(4)$$

By writing $\xi = \frac{2\pi x_0}{D}$, $\eta = \frac{2\pi y_0}{D}$, equating real and imaginary parts and re-arranging we have:-

$\frac{D}{2\pi}$ /

$$\frac{D}{2\pi} \cdot \frac{d\eta}{dt} = \frac{K}{2D} \cdot \frac{\tan^2\xi}{\sinh 2\eta\,(\tan^2\xi + \tanh^2\eta)} \qquad \ldots(5)$$

$$\frac{D}{2\pi} \cdot \frac{d\xi}{dt} = -\frac{K}{2D} \cdot \frac{\tanh^2\eta}{\sin 2\xi(\tan^2\xi + \tanh^2\eta)} \qquad \ldots(6)$$

On dividing (5) by (6) and integrating once we get,

$$\operatorname{cosech}^2\eta + \operatorname{cosec}^2\xi = -2C \qquad \ldots(7)$$

----- a relation between ξ and η which is independent of K. It defines the projection, in a plane normal to the stream, of the path of the vortex A and is determined uniquely if the co-oridnates in this plane are known at any one instant. In a given case the constant C can be determined from the co-ordinates (ξ_1, η_1) of the tips of generators --- the point of formation of the vortices.

The graph of the equation (7) for $-\pi < \xi < \pi$ is shown in Fig. 1. The right-hand curve is the projection of the path of the vortex A_0. It passes through the projection of the tip of the generator and is symmetrical about the line $\xi = \pi/2$, which therefore gives the minimum height. The vortex traces out the portion of the curve - shown as a full line - to the right of the tip. (The same projected path would apply for all generators with tips lying on this curve, the part of the curve traced out in each case being the part to the right of the tip.) The left-hand curve is the projected path of the vortex B_0 and is the mirror image, about the axis $\xi = 0$, of that of the vortex A_0.

It should be noted that the result that the projection of the path of a vortex is independent of the magnitude of K is a direct consequence of the assumption of two-dimensional flow, and is strictly true only for a vortex which is infinitely long and straight. But if the slope of the vortex path is everywhere small, the above conclusion may be generalised to include curved vortex lines.

Now (7) only defines the projection of the path of the vortex. The vorticity shed at the tips is convected downstream with velocity V, i.e., ξ and η are measured with respect to an origin which is moving with a velocity V. Thus the position in space (x,y,s) of the shed vorticity after a time t, will depend upon the rate at which the curve (7) is traced out. This can be determined from eqns. (5) and (6).

After a time t the vorticity will be some distance downstream given by

$$s = V.t.$$

and if we substitute

$$\theta = \frac{2\pi s}{D}$$

eqns. (5) and (6) reduce to the dimensionless form

$\frac{d\xi}{d\theta}$/

$$\left.\begin{aligned} \frac{d\xi}{d\theta} &= \frac{k \tan^2\xi}{\sinh 2\eta\,(\tan^2\xi + \tanh^2\eta)} \\ \frac{d\eta}{d\theta} &= \frac{k \tanh^2\eta}{\sin 2\xi\,(\tan^2\xi + \tanh^2\eta)} \end{aligned}\right\} \qquad \ldots(8)$$

where $k = \frac{K}{DV}$

eqn. (8) may then be solved as follows,

we have $$k\theta = \int_{\xi_1}^{\xi} \left\{1 + \frac{\tanh^2\eta}{\tan^2\xi}\right\} \sinh 2\eta \,.\, d\xi \qquad \ldots(9)$$

which can be integrated in closed form after first substituting for η from (7). The expression is complicated however, and in practice it is easier, and faster, to use a numerical method.

Now from (7), as $\eta \to \infty$, ξ tends to a finite limit (ξ_{max}) since $\mathrm{cosech}^2\eta \to 0$. Thus at a great distance from the generators, the vortices tend to become parallel once more. This is confirmed by experiment. The limits of variation of ξ are ξ_1 and ξ_{max}. Various values of ξ are taken between these limits, and the corresponding values of η determined from (7). It is then possible to plot a graph showing the variation of the integrand of (9) with ξ as in Fig. 2. The integral is evaluated by Simpson's rule.

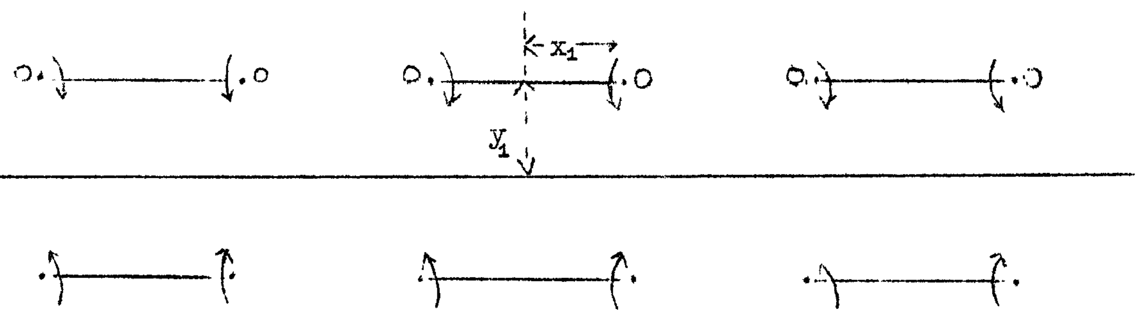
The integral represents $k\theta$, so that the rate at which the path is traced out is directly proportional to the non-dimensional parameter k.

The Evaluation of k

Owing to the complicated nature of the flow and the low aspect ratios common in vortex generators, it is not to be expected that an accurate value of k can be obtained, but an attempt is made here to determine the order of magnitude. As we are not interested in the actual loading distribution over the surface of the generators, but only in the final strength of the vortices, a "Lifting-line" theory will be adequate.

In conventional lifting-line theory, the tip vortices are assumed to lie along straight lines perpendicular to the span. Observation has shown that the vortices are welldeveloped at the trailing edges of the generators, and that the "rolling-up" process is complete in a very short distance; but the vortices do not remain perpendicular to the span. (Fig. 4 shows a typical case.) However, this spanwise movement does not take place immediately aft of the trailing edge, but the vortices remain parallel for several chord-lengths. Therefore, to avoid an unjustifiable complication in the calculation of k, the subsequent span-wise movement is ignored and the vortices are assumed to spring from the tips and to extend downstream to infinity, remaining perpendicular to the span.

Sketch II/



Sketch II

The lines OO in sketch II represent the traces of the trailing edges of the generators, in a plane perpendicular to the free stream.

The strengths of the trailing vortices behind any generator are equal to the total circulation around that generator.

Hence it will be necessary to know the angle of incidence at all spanwise points.

Let the generators be at a height y_1 above the surface, and let each generator be of span $2x_1$. Let $P(x',y_1)$ be any point on the generator, with the origin at mid-span. Then the effective incidence at P is

$$\phi = \alpha_0 - \frac{w(x')}{V} \qquad \ldots(10)$$

But it has been assumed that all the vorticity is shed from the tips, which in turn implies that the circulation is constant across the span. For constant chord generators this implies also that $w(x')$ must be constant across the span, a condition which cannot be realised with a simple horse-shoe vortex system. To avoid this, a mean value-w_m- will be assumed for $w(x')$.

Then the lift on any span-wise element of span dx' and chord c_0 is

$$\delta L = a\rho/2 \left\{\alpha_0 - \frac{w(M)}{V}\right\} c_0 V^2 \, dx' \qquad \ldots(11)$$

But

$$L = \rho V K dx' \qquad \ldots(12)$$

Hence

$$K = a/2 \left\{K - \frac{w_M}{V}\right\} c_0 V \qquad \ldots(13)$$

Put

$$c_0 = cD$$

Then

$$k = \frac{K}{D.V} = \frac{a}{2} \left\{\alpha_0 - \frac{w_M}{V}\right\} c \qquad \ldots(14)$$

There/

There is some uncertainty as to the best value for w_M, but an expression for k will be derived on the basis of an assumption that seems reasonable. Fairly large arbitrary variations from the assumed value are later (§§ 5, 6) shown to produce only fairly small changes in the vortex paths.

The assumption is that w_M has the value of $w(x')$ at the centre of the span.

To calculate the induced velocity at mid-span it is necessary to use the complex potential of equation (2a), i.e., the vortices in row A must now be included.

$$w = -\frac{iK}{2\pi}\log\sin\frac{\pi}{D}(z - z_0) + \frac{iK}{2\pi}\log\sin\frac{\pi}{D}(z + \bar{z}_0)$$

$$+ \frac{iK}{2\pi}\log\sin\frac{\pi}{D}(z - \bar{z}_0) - \frac{iK}{2\pi}\log\sin\frac{\pi}{D}(z + z_0) \quad \ldots(15)$$

The mid-span is the point $(0, y_1)$ and z_0 is assumed to be the point $(x_1 + iy_1)$

Hence

$$\left\{\frac{dx}{dy} - \frac{idy}{dt}\right\}_{iy_1} = \frac{iK}{4D}\left\{2\cot\frac{\pi x_1}{D} - \cot\frac{\pi}{D}(x_1 - 2iy_1) - \cot\frac{\pi}{D}(x_1 + 2iy_1)\right\} \quad \ldots(16)$$

(Note that a factor of $\frac{1}{2}$ has been introduced on the right-hand side, since it must now be assumed that the vortices are semi-infinite.)

On substitution of $\xi_1 = \frac{2\pi x_1}{D}$ and $\eta_1 = \frac{2\pi y_1}{D}$ and further simplification, the induced velocity at mid-span is given by

$$w(x' = 0) = \frac{K}{D} \cdot \frac{\tanh^2\eta_1}{\sin 2\xi_1\left(\tan^2\frac{\xi_1}{2} + \tanh^2\eta_1\right)} \quad \ldots(17)$$

This is the value assumed for w_M, but we write

$$w_M = \frac{K\lambda}{D} \text{ where } \lambda = \frac{\tanh^2\eta_1}{\sin 2\xi_1\left(\tan^2\frac{\xi_1}{2} + \tanh^2\eta_1\right)} \quad \ldots(18)$$

In Section 5, the effect on the solution of arbitrary variations of λ are examined (see also Discussion).

If we substitute $w_M = \frac{K\lambda}{D}$ in (14) we have

$$k = \frac{a_0}{\left\{\frac{2}{ac} + \lambda\right\}} \quad \ldots(19)$$

Using this value for k, the paths of the vortices can then be calculated from (7) and (9).

It/

It will be seen that k is independent of V, and indeed it is a function only of the incidence and configuration of the generators. It follows from (9) that θ, and therefore the paths of the vortices, are independent of the velocity of the undisturbed stream.

5. Comparison with Experiment

To check the accuracy of the preceding analysis a simple experiment was performed in the small water tunnel in the Aerodynamics Division of the National Physical Laboratory.

Three "wing type" generators were mounted on a 0.020" thick brass plate, the spacing between the generators being $4\frac{5}{16}$". The tunnel width was 13", so this arrangement effectively provided an infinite distribution of equally spaced generators. The generators were of 1 sq in. shim brass, with their centres 0.5" above the brass plate, and were soldered to streamline supports at an inclination of 15°. A thin brass base plate was chosen so as to disturb the mainstream as little as possible.

The tunnel was run at 7 ft per sec and the paths of the vortices were made visible by introducing air into the stream from fine jets. The air collected in the regions of low pressure at the cores of the vortices. The vortices were then photographed, and enlargements made to approximately 2/3 full size (Figs. 4 and 5) from which it was possible to measure approximately the positions of the vortices. The plan view photographs were taken at a slightly longer exposure than the elevation view, and more air was delivered from the tubes. The trailing vortices produced by the left-hand generator, Fig. 5, were made visible by air previously discharged, but still circulating in the tunnel.

The dimensions of the generators were such that the co-ordinates of the right-hand tip of one, - (ξ_1, η_1) in the notation used in the preceding section, - were (0.75, 0.75). The value of the constant C in equation (7) is then found to be 1.815. The co-ordinates (ξ, η) of points on the projected path of the vortex are given in Table I.

The limits of integration in equation (9), $\xi_1 . \xi_{max.}$ are 0.75, 2.59 respectively. The values of kθ found by integrating this equation are tabulated in Table II.

The value of λ according to equation (18) is found to be 0.99. Then, if a is assumed to be 2π, equation (19) gives k = 0.113. Arbitrary variations of ±30% in λ, i.e., in mean downwash, give values of k equal to 0.100 and 0.133 respectively.

The values of θ, and hence of s, corresponding to the three values of k are given in Table II. The (x,y) co-ordinates, derived from the (ξ, η) co-ordinates in Table I, are included in Table II, so that the calculated paths can be drawn in the physical plane.

The calculated and observed paths are compared in plan and elevation in Fig. 3.

6. Discussion

In plan-view the agreement between theory and experiment is good, Fig. 3, especially when a large downwash is assumed, but the elevation photographs show that the predicted minimum height is consiberably in excess of the experimental value. It is probable that the discrepancy is due to:-

(i) The presence of the boundary layer. It is obvious that there must be some distortion of the vortex path, which will become more pronounced as the vortex approaches the surface, but it is difficult to see how any allowance could be made for this, since the boundary layer profile will be considerably modified by the vortices.

(ii)/

(ii) It has been assumed that, apart from the downwash of the trailing vortices, the fluid has no component of velocity towards the surface. This is not strictly true, since there is a small component normal to the general direction of flow, produced by the bound vorticity. Perhaps better agreement would have been obtained if allowance had been made for this, but as was pointed out in Sec. 4, the assumption of two-dimensional flow leads to an over-estimate of the downwash from the trailing vortices in the vicinity of the generators, so that to some extent these errors will cancel.

(iii) A decrease in vortex strength due to viscous damping.

In plan-view the agreement is improved slightly by assuming a 30% increase of downwash, but the discrepancy produced by a 30% decrease is not large. It has been pointed out to the writer, by Mr. L. H. Tanner, that the true mean downwash is greater than that at mid-span, because of the rapid increase of induced velocity in the vicinity of the vortices. But the larger proportion of the total circulation is produced over the centre sections, so that it is doubtful if the outer sections contribute more than the assumed 30% increase.

7. Conclusions

The projection of the vortex paths on a plane normal to the stream are shown to be independent of the vortex strength, but the rate at which the path is traced out is directly proportional to the vortex strength. The vortex strength can be determined from the configuration of the generators, and the three-dimensional path may be determined when that strength is known.

The theory is in fairly good agreement with experiment, but the minimum height of the vortices above the plane surface was less in practice than in theory.

The vortex strength was estimated by a form of "lifting-line theory", from which it was shown that large changes in the mean downwash produce only small changes in the arrangement of the vortices. It was shown that for a given configuration the vortex strength increases linearly with speed of the undisturbed stream, and that as a result the paths of the vortices are independent of that speed.

The results are confined to flow over a plane surface; no attempt has been made to estimate the effects of surface curvature.

Acknowledgements

The writer wishes to thank Messrs. H. H. Pearcey and L. H. Tanner and Dr. J. T. Stuart for their kind interest and suggestions throughout the writing of this paper.

References

No.	Author(s)	Title, etc.
1	L. H. Tanner and H. H. Pearcey	The paths of vortices and their relation to the design of vortex generators. (In preparation).
2	H. Lamb	Hydrodynamics. Cambridge University Press.

TABLE I/

TABLE I

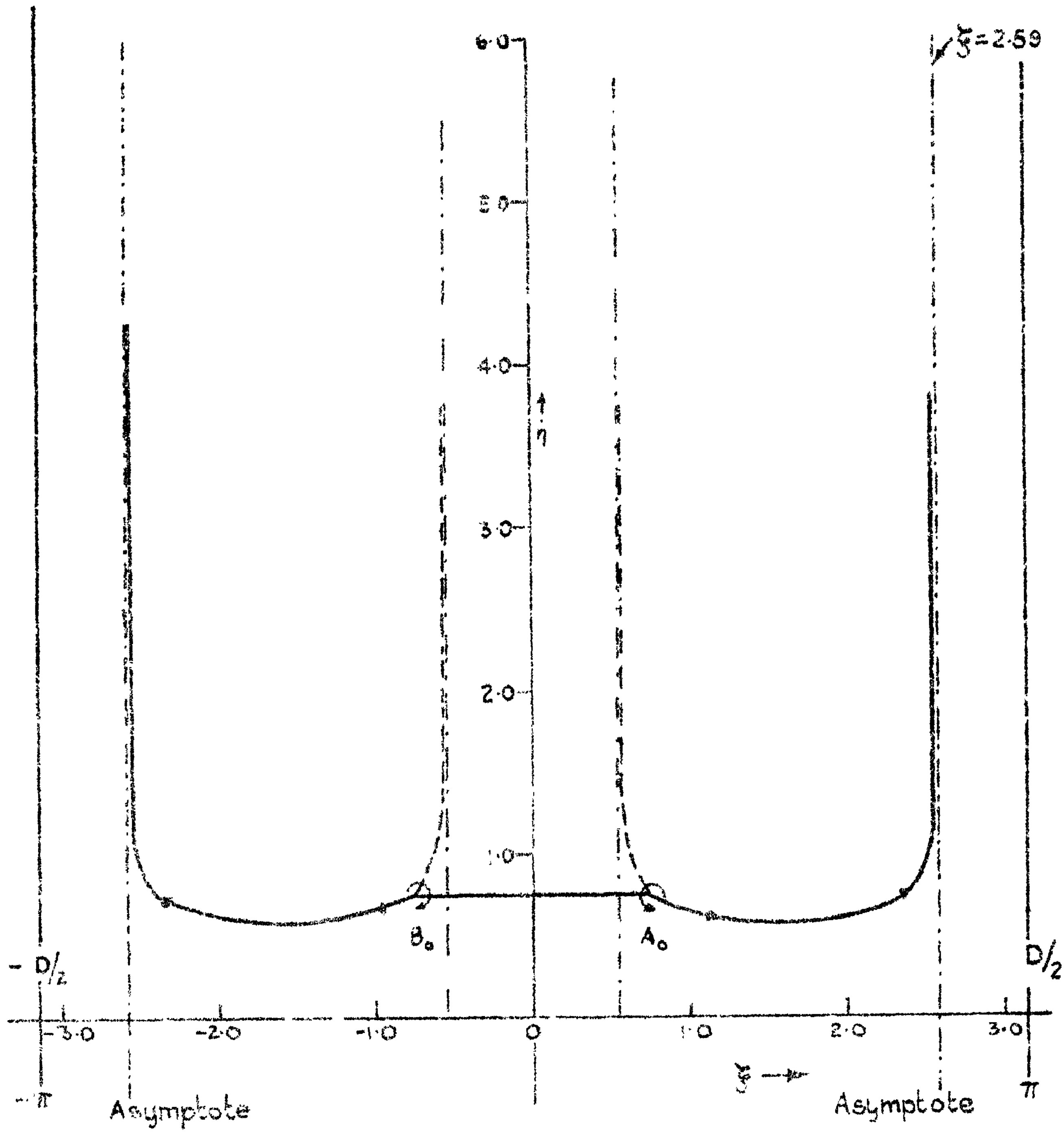
ξ	η	$\tanh^2 \eta$	$\sinh 2\eta$	$\cot^2 \xi$	$k\frac{d\theta}{d\xi}$
0.75	0.75	0.4035	2.13	1.155	3.120
$\pi/4$	0.72	0.3800	1.99	1.000	2.745
50°	0.67	0.3420	1.78	0.703	2.210
$\pi/3$	0.62	0.303	1.58	0.334	1.750
75°	0.59	0.279	1.47	0.0719	1.495
$\pi/2$	0.58	0.276	1.448	0	1.448
105°	0.59	0.279	1.47	0.0719	1.495
$2\pi/3$	0.62	0.303	1.58	0.334	1.750
135°	0.72	0.380	1.99	1.000	2.745

TABLE II

ξ	x in.	y in.	$k\theta$	θ k=0.133	θ k=0.113	θ k=0.100	s in. k=0.133	s in. k=0.113	s in. k=0.100
0.75	0.5	0.5	0	0	0	0	0	0	0
0.85	0.566		0.268	2.02	2.375	2.68	1.35	1.58	1.78
0.95	0.634	0.428	0.483	3.56	4.28	4.85	2.37	2.86	3.23
1.05	0.700		0.667	5.02	5.90	6.66	3.34	3.93	4.44
1.25	0.835	0.393	0.994	7.47	8.80	9.83	4.98	5.86	6.63
1.45	0.967		1.290	9.70	11.41	12.9	6.46	7.61	8.62
1.55	1.033	0.390	1.435	10.78	12.70	14.35	7.19	8.46	9.56
1.65	1.110		1.580	11.89	14.0	15.82	7.91	9.33	10.55
1.85	1.233	0.393	1.870	14.08	16.56	18.8	9.39	11.02	12.45
2.05	1.370		2.190	16.45	19.4	21.9	10.98	12.92	14.6
2.25	1.500	0.448	2.570	19.3	22.7	25.65	12.88	15.22	17.2
2.35	1.570		2.810	21.1	24.9	28.1	14.1	16.6	18.75

SC

FIG. 1.

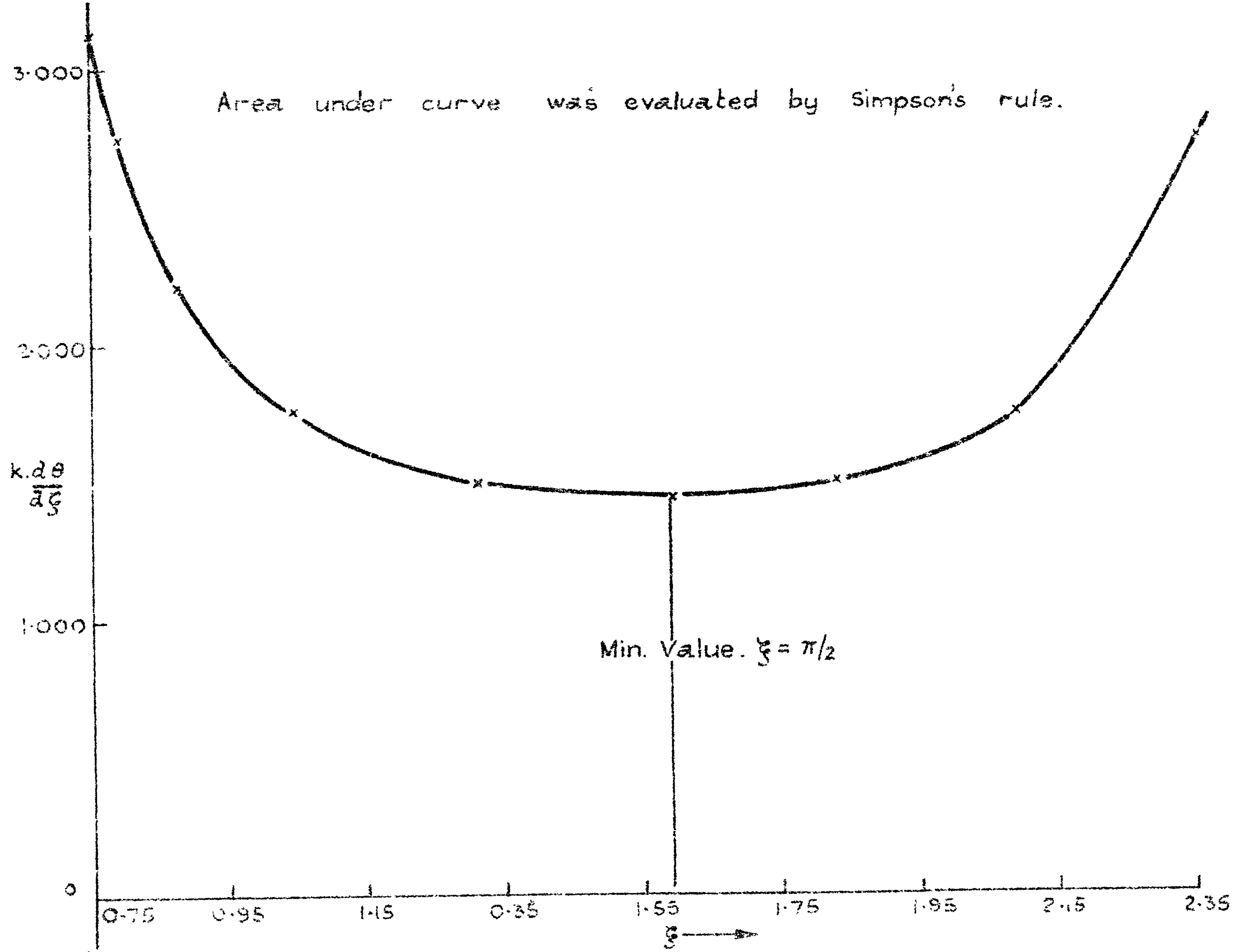


(Curves derived from equation (7), with 2C = 3.63)

Projection of the paths of a pair of vortices.

M.C.

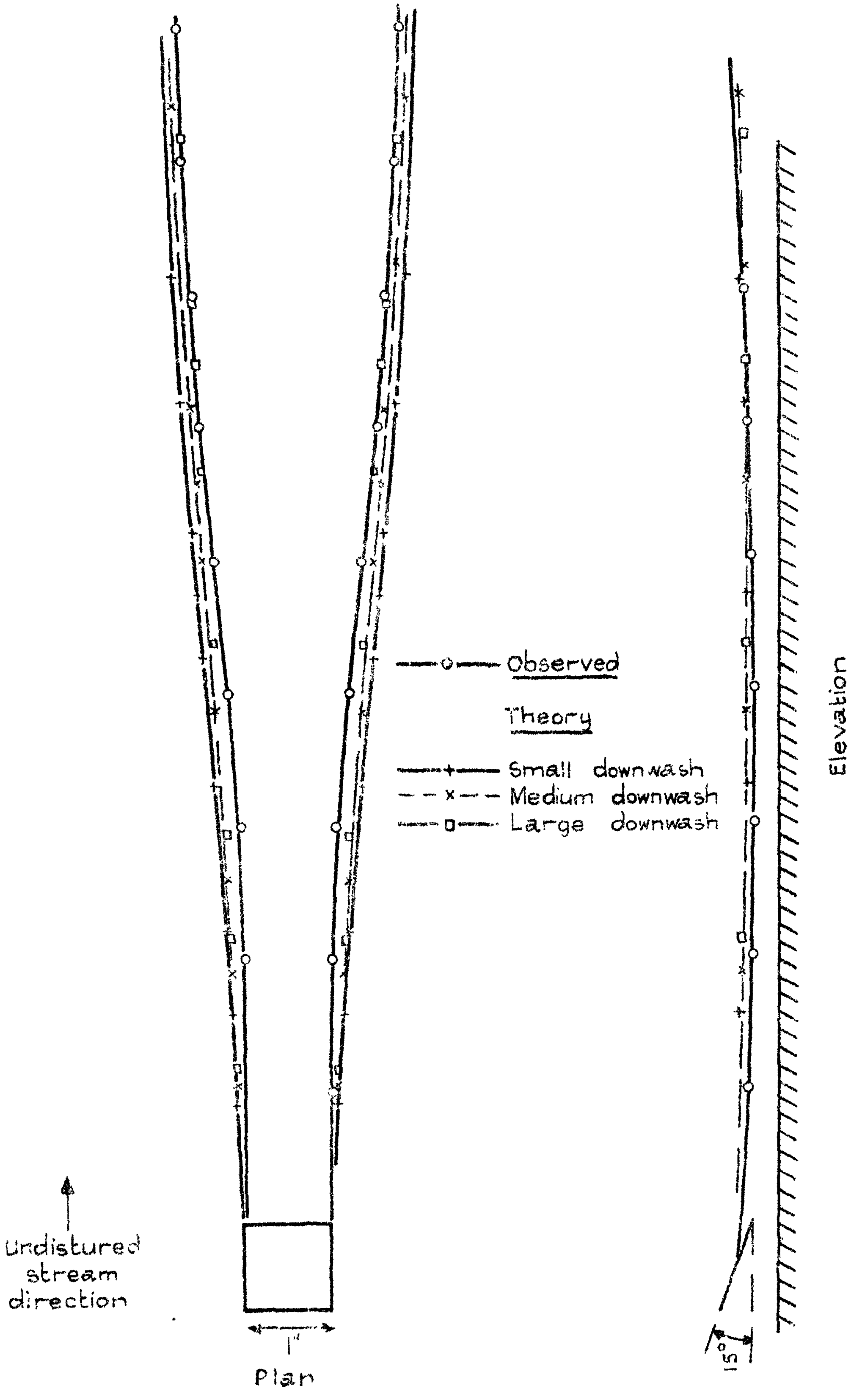
FIG 2



Plot of $k.\frac{d\theta}{d\xi}$ vs. ξ

M.C.

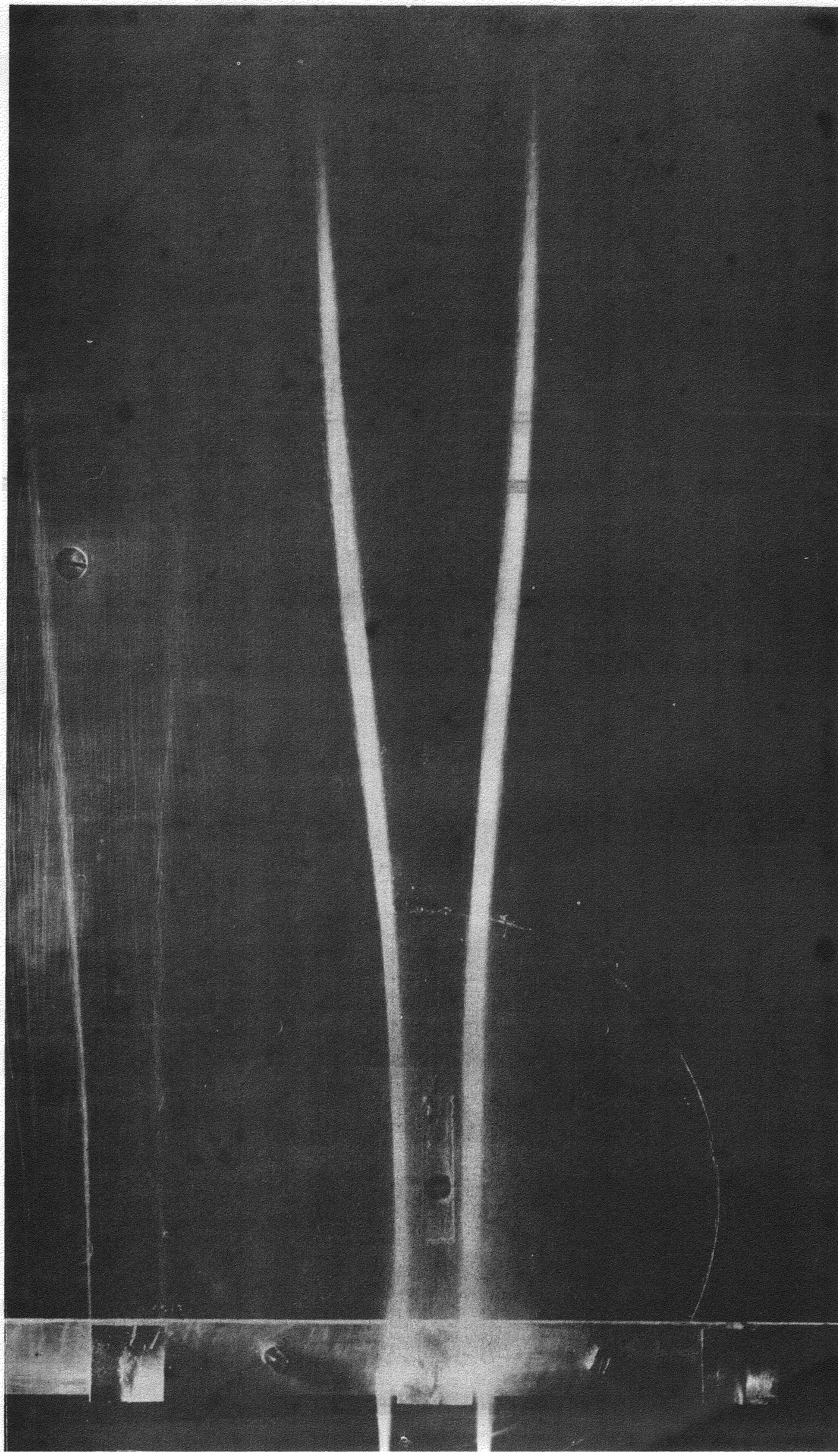
FIG. 3.



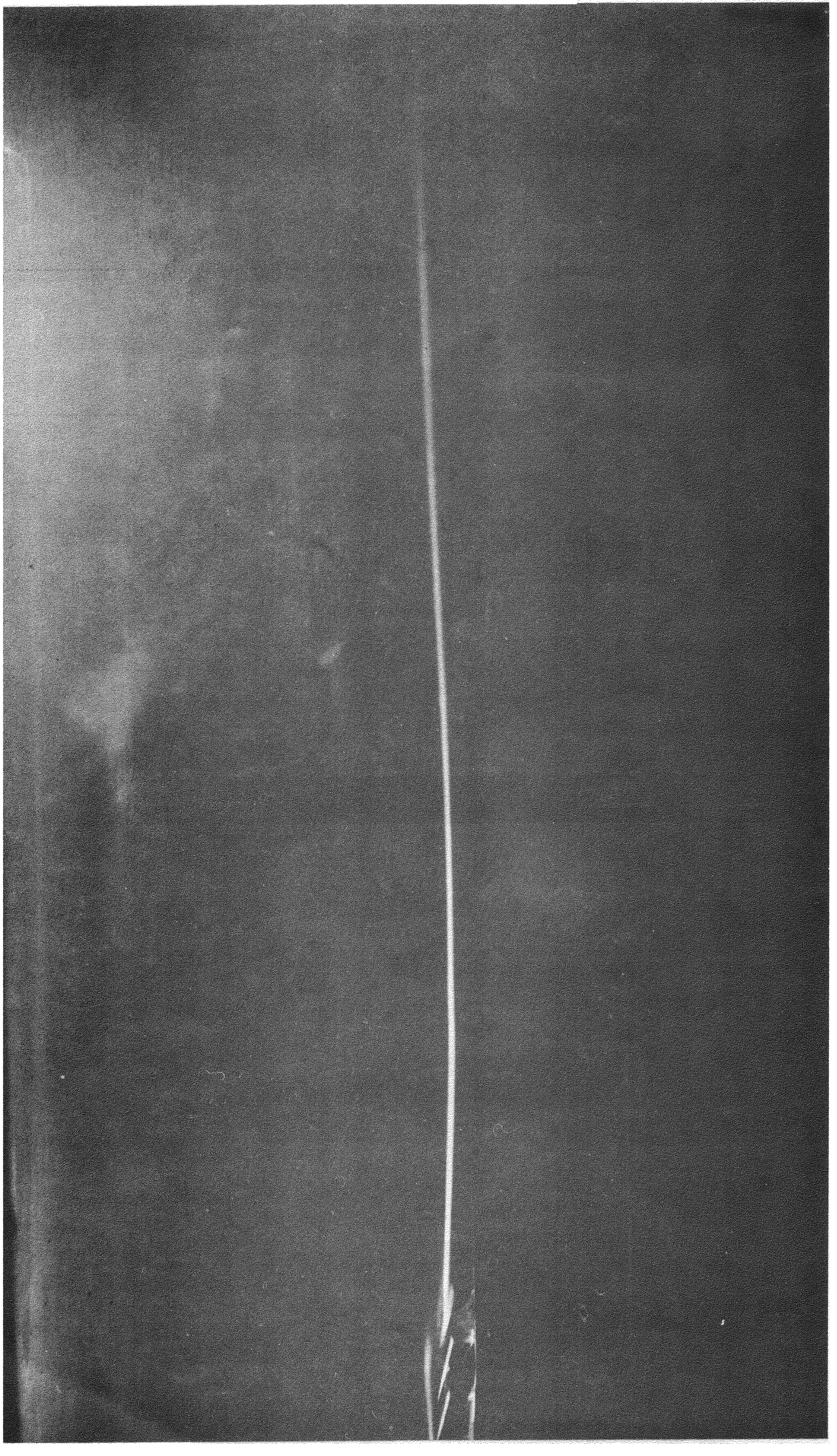
M.C.

Comparison of theory and experiment.

17,683 FIG. 4.



17,683 FIG. 5.



C.P. No. 361
(17,683)
A.R.C. Technical Report

Published by
HER MAJESTY'S STATIONERY OFFICE

To be purchased from
York House, Kingsway, London W.C.2
423 Oxford Street, London W.1
13A Castle Street, Edinburgh 2
109 St. Mary Street, Cardiff
39 King Street, Manchester 2
Tower Lane, Bristol 1
2 Edmund Street, Birmingham 3
80 Chichester Street, Belfast
or through any bookseller

PRINTED IN GREAT BRITAIN

C.P. No. 361
(17,683)
A.R.C. Technical Report