C.P. No. 360
(18,567)
A.R.C Technical Report

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MINISTRY OF SUPPLY

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CURRENT PAPERS

Formulae and Approximations for Aerodynamic Heating Rates in high Speed Flight

By

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1957

PRICE 7s. 6d. NET

Corrigenda and Addenda

Fig. 3(b). The straight line $\frac{k_H}{k_{Hi}} = \left(\frac{T_1}{T_w}\right)^{\frac{1}{2}}$ should pass through the point $\frac{T_w}{T_1} = 4$, $\frac{k_H}{k_{Hi}} = 0.5$ instead of $\frac{T_w}{T_1} = 3$, $\frac{k_H}{k_{Hi}} = 0.5$.

At the end of the sixth paragraph on page 13 (concluding with the words ".... shown in Fig. 3b"), add the sentence "An approximation to intermediate enthalpy results for Mach numbers up to ten, is given by:

$$\frac{k_H}{k_{Hi}} = \left(\frac{T_1}{T_w}\right)^{1/3} \left(\frac{T_i}{T_{wo}}\right)^{4/15}$$

After second paragraph of section 3, page 14, add "More recent tests (1957) are indicating that the curves in Fig. 5 for $M = 3$, 4 and 5 may overestimate the favourable effect of surface cooling. For example, for $M = 3$ and $T_w = 150°C$, R_T may not exceed 6 millions instead of 13 millions as given by Fig. 5. The following numerical example should, therefore, be treated with caution".

Add the words "when M is constant" to the end of the last sentence on page 20.

Page 24, third paragraph. Add the words "as mean values of h over the temperature ranges in question" to the end of the sentence beginning "From Fig. 7b we then take ....".

C.P. No. 360

U.D.C. No. 533.6.011.5 : 533.6.011.6 : 532.526

Technical Note No. Aero 2407

October, 1955

ROYAL AIRCRAFT ESTABLISHMENT

Formulae and approximations for aerodynamic heating rates in high speed flight

by

R. J. Monaghan, M.A.

SUMMARY

This note gives formulae and approximations suitable for making preliminary estimates of aerodynamic heating rates in high speed flight. The formulae are based on the "intermediate enthalpy" approximation which has given good agreement with theoretical and experimental evidence. In the general flight case they could be used in conjunction with an analogue computer or a step-by-step method of integration to predict the variations of heat flow and skin temperature with time.

In the restricted case of flight at constant altitude and Mach number, simple analytical methods and results are given which include the effects of radiation and can be applied to "thick" as well as "thin" skins. In dealing with "thick" skins, the important parameters are $\frac{h}{G}$ and $\frac{hd}{k_s}$, where h is the aerodynamic heat transfer factor, and G, d and k_s are the heat capacity, thickness and thermal conductivity of the skin. If $\frac{hd}{k_s} < 0.1$ the skin is approximately "thin", i.e. temperature gradients across its thickness may be neglected.

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1 Introduction

Several years ago RAE Report Aero 2454 by F. V. Davies and R. J. Monaghan[3] gave methods for estimating skin temperatures in high speed flight and included data on the heat transfer coefficients, zero heat transfer temperatures etc, necessary for the calculations. At that time the state of the art made it advisable to restrict the scope to calculations of mean temperatures over the length of a body or wing and to uniform skins thin enough for temperature gradients through their thickness to be neglected. In addition, no account was taken of longitudinal temperature or pressure gradients, and the heat transfer formulae were strictly applicable only to flat plates or cones.

Since that time additional theoretical and experimental evidence has become available, so that much of Aero 2454[3] is now dated, both in concept and content. Some of this evidence has been reviewed in Ref.2, which includes a section on the effects of temperature gradients, and more recently the latest evidence has been summarised in Refs. 4 and 1 (the last reference includes an approximate method for estimating the effects of pressure gradients in laminar boundary layers).

These reports could be used in conjunction with the calculation methods of Ref.1, or with an analogue computer, to make better estimates of heat transfer rates and skin temperatures in high speed flight, particularly of local rather than mean values, and of variations along the length of the body. However, as the ability to deal with more complicated cases increases, so, unfortunately, does the complexity of the calculations, and there is probably still a need for the rough approximation which will give a general idea of the extent of the problem in any particular case before detailed calculations are started.

The "rough approximation" of the present report is that longitudinal temperature and pressure gradients are neglected and "flat plate" formulae are used throughout. However, this approximation may not be too bad, since pressure gradients are mostly small on the thin wings and slender bodies appropriate to supersonic flight and temperature gradients may not have too serious an effect except near the leading edge or nose. (As for shape, transformations are available between flat plates and bodies of revolution with laminar boundary layers[16] and recent experimental work[17] has shown that similar transformations may be applicable with turbulent boundary layers.)

Thus the formulae and methods to be described may give reasonable estimates of heat flow rates, skin temperatures etc. along a body or wing, but in particular cases it would be advisable to make more detailed calculations in regions where temperature or pressure gradients are found to be severe. This may involve consideration of longitudinal conduction within a skin and structure which is by no means the uniform shell with constant thermal properties assumed in the preliminary calculations, so that caution should be exercised to ensure that the refinement of the aerodynamic calculations is not nullified by inaccuracies in the solution of the conduction problem within the structure, and vice versa. These difficulties are considered by E. H. Bateman in Ref.18.

In the present paper, section 2 considers the nature of the aerodynamic heat transfer problem in high speed flow and goes on to give the latest formulae for estimating heat flow rates, local and mean, with laminar or with turbulent boundary layers. (As mentioned above, these are strictly applicable only to a flat plate.) Section 3 considers very briefly the present state of knowledge concerning transition from laminar to turbulent flow.

Finally, section 4 considers the application of the formulae of section 2, including local heat flow rates and how they are affected by Mach number, surface temperature, altitude and Reynolds number, the effects of radiation and the resulting equilibrium temperatures and lastly considers the times required for a skin to heat up in flight at constant Mach number and altitude, and gives charts for estimating the heating rates of "thick" as well as of "thin" skins. Numerical examples are given in section 4.5. Most of the numerical results in section 4 are particular to a given body station, but the methods are of general application.

The aim of the present paper is to give formulae and methods for use in calculations and to show the relative importance of the various parameters involved rather than to give a broad description of the problems of aerodynamic heating. This latter need has probably been met already by Refs. 12, 18 and 19.

2 Heat transfer formulae

When dealing with heat transfer by convection and conduction between a fluid and a solid it is usual to introduce a heat transfer factor "h", defined by

$$q = h\Delta T \tag{2.1}$$

where q is the heat transferred per unit time per unit area $\left(\frac{\text{CHU}}{\text{ft}^2\ \text{sc}}\right)$

and ΔT is a representative temperature difference between the fluid and the solid ($^\circ$C).

Equation (2.1) is an expression of Newton's cooling "law", but except over limited ranges, h is by no means independent of temperature as was assumed originally. Its main usefulness arises from the fact that h has the same dimensions as the factor k/d appearing in the formula for heat transfer by pure conduction

$$q = \frac{k}{d}\Delta T \tag{2.2}$$

where k is thermal conductivity

and d is the length of the heat path.

Thus when dealing with the aerodynamic heating of thick skins in section 3.4, it is necessary to express the aerodynamic heating rates in the form of equation (2.1).

Now in low speed flow $\left(\frac{\gamma-1}{2}M^2 << 1\right)$ the temperature difference ΔT in equation (2.1) is taken to be that existing between the stream outside the boundary layer (temperature T_1) and the surface of the body (temperature T_W), i.e.

$$q_1 = h\,(T_1 - T_W) \tag{2.3}$$

if heat is flowing from the airstream to the body. By definition, T_1 is independent of T_W (or q)*. This is illustrated in Fig.1a which shows temperature distributions across the boundary layer corresponding to several values of T_W.

* This is so since, as far as the air is concerned, the effects of the heat transfer to the body are felt only within the boundary layer. This layer will, however, thicken or thin depending on the direction and amount of the heat flow, note Fig.1.

In high speed flow $\left(\text{when } \frac{\gamma-1}{2} M_1^2 \text{ is not negligible}\right)$ the picture becomes more complicated. Fig.1b shows some typical temperature distributions for this case. If no heat is being transferred to the body, then the surface is at a temperature (T_{wo}) which is near to the stagnation or total temperature of the airstream. When heat is flowing into the body the temperatures within the boundary layer are everywhere less than T_{wo}, but it is convenient to use this temperature T_{wo} in forming the temperature difference ΔT, i.e. to put

$$q = h\,(T_{wo} - T_w)\,. \tag{2.4}$$

This procedure has some theoretical support and obviously satisfies the requirement that $q \to 0$ as $T_w \to T_{wo}$, but the nature of the curves in Fig.1b may help to illustrate both that $(T_{wo}-T_w)$ in high speed flow has not the same physical significance as (T_1-T_w) in low speed flow and also that h may now be dependent on T_w and on M (the latter influences the magnitude of the temperature peaks within the boundary layer). Indeed recent theoretical solutions for heat transfer with a laminar boundary layer have shown that it also becomes necessary to replace T_{wo} in equation (2.4) by an "effective" temperature which is equal to T_{wo} only at small rates of heat transfer. Thus equation (2.4) should not be regarded as being more than a convenient expression for q.

The zero heat transfer temperature T_{wo} is related to T_1 and M_1 by

$$\frac{T_{wo}}{T_1} = 1 + r_T \frac{\gamma-1}{2} M_1^2 \tag{2.5}$$

where r_T is temperature recovery factor. The values of r are considered in sections 2.1 and 2.2 below.

The heat transfer factor h is dimensional and it varies considerably with flight conditions (cf. Fig.7), so in aerodynamic work it is more convenient to use the non-dimensional heat transfer coefficient k_H, defined by

$$k_H = \frac{h}{\rho_1 u_1 c_p} \tag{2.6}$$

where ρ_1 and u_1 are the density and velocity in the stream outside the boundary layer

and c_p is the specific heat of air at constant pressure* (assumed constant in this instance).

Part of the benefit of using k_H is that it is closely linked with the skin friction coefficient c_f by

$$\frac{k_H}{\frac{1}{2} c_f} = s \tag{2.7}$$

where s is the "Reynolds analogy factor" which in approximation depends only on the physical properties of the fluid through the Prandtl number σ, where

* For k_H to be non-dimensional it is essential for ρ and c_p to be expressed in consistent units. Thus if ρ is $\frac{\text{lb}}{\text{ft}^3}$ then c_p is $\frac{\text{CHU}}{\text{lb}\ {}^\circ\text{C}}$ (= 0.24), but if ρ is $\frac{\text{slugs}}{\text{ft}^3}$ then c_p is $\frac{\text{CHU}}{\text{slug}\ {}^\circ\text{C}}$ (= 7.73).

$$\sigma = \frac{c_p \mu}{k} \tag{2.8}$$

where μ is viscosity.

Thus k_H and c_f will show the same variation with Reynolds number. (Strictly speaking, k_H is a function of Peclet number Pe rather than of Reynolds number Re, where

$$Pe = \frac{\rho c_p ux}{k} .$$

However it is easily shown that

$$Pe = \sigma Re \qquad \text{since} \qquad Re = \frac{\rho ux}{\mu}$$

and hence it is permissible to speak of its "variation with Reynolds number".)

Up to this stage it has been assumed that the specific heat of air (c_p) is constant, which is a good approximation to reality up to temperatures around 400°K. Recent developments in boundary layer analysis have removed this restriction (and also the restriction that σ is constant) and the resulting effects are discussed in section 2.1 below.

2.1 Heat transfer with laminar boundary layers, allowing for the variation of all the physical properties of air

E. R. G. Eckert has given a good summary[4] of the various stages of refinement in the solution of the equations for the laminar boundary layer on a flat plate with zero temperature and pressure gradient along its length, culminating in the numerical solutions of Young and Janssen[5], Klunker and McLean[6], and Van Driest[7], which allowed for the unrestricted variation of all the physical properties of air.

If the specific heat of air (c_p) varies, then temperature is no longer a realistic quantity for insertion in heat transfer formulae. By their very nature we should think instead of the heat content or enthalpy "i" of the air in the boundary layer, where

$$i = \int_0^T c_p \, dT \tag{2.9}$$

and Fig.2a shows the relationship between enthalpy and temperature (values taken from Keenan and Kaye[8]) over the range of temperatures appropriate to the present paper.

Corresponding to equation (2.5) for zero heat transfer temperature T_{wo}, we now have

$$\frac{i_{wo}}{i_1} = 1 + r \frac{\gamma-1}{2} M_1^2 \tag{2.10}$$

for the zero heat transfer enthalpy i_{wo}, where r is now the enthalpy recovery factor. Alteration of equation (2.4) for h is not so straightforward since it is dimensional, but we may define a new quantity h' given by

$$q = h' (i_{wo} - i_w) \tag{2.11}$$

and with this definition equation (2.6) becomes

$$k_H = \frac{h'}{\rho_1 u_1} . \tag{2.12}$$

This may appear to be a highly inconvenient alteration since in the end we are interested in skin temperatures, etc, which will involve transforming back from enthalpy to temperature, and even before that stage is reached it is necessary to make use of the physical properties of air and these are tabulated against temperature. Indeed in the lower temperature range there might appear to be no virtue in thinking in terms of enthalpy rather than temperature (cf. Fig.2a). However, the justification for the change rests on the fact that once it is made it is possible to apply the subsequent formulae without further alteration at least up to Mach numbers when the air inside the boundary layer becomes dissociated (see Ref.1) and possibly beyond this stage if correct assumptions can be made about the physical properties of the air in its dissociated state.

The numerical solutions[5,6,7] referred to above cover a wide range of Mach numbers and temperatures and Eckert has shown[4] that a close approximation to them can be obtained in the majority of cases if the physical properties of air appearing in the well known "incompressible" flow formulae for heat transfer etc. are evaluated at a temperature corresponding to an "intermediate" enthalpy $i^{\times}$ which is given by the simple formula

$$i^{\times} = i_1 + 0.5(i_w - i_1) + 0.22(i_{wo} - i_1) \tag{2.13}$$

where subscripts 1, w and wo refer to conditions in the stream outside the boundary layer, at the wall (body surface) and at the wall under zero heat transfer conditions. i_1 and i_w will be known and i_{wo} is given by equation (2.10)

$$\frac{i_{wo}}{i_1} = 1 + r\frac{\gamma-1}{2} M_1^{\,2} \tag{2.10}$$

where the enthalpy recovery factor r is given approximately by the Pohlhausen formula

$$r \simeq (\sigma^{\times})^{\frac{1}{2}} \tag{2.14}$$

and Prandtl number $\sigma^{\times}$ is evaluated at $T^{\times}$ corresponding to $i^{\times}$ from equation (2.13). The particular variation of Prandtl number with temperature now in favour is given in the NBS-NACA tables[9] and is reproduced in Fig.2b. (Alternatively r may be read from the graphs of Ref.7, for example, if these are available.)

Zero heat transfer and "effective" enthalpies

The enthalpy for zero heat transfer is obtained from equations (2.10) and (2.14) in combination with equation (2.13) with $i_w = i_{wo}$. When $i_w \neq i_{wo}$ these same equations would indicate a different value of $(i_{wo} - i_1)$. The significance of this variation of $(i_{wo} - i_1)$ with i_w appears when evaluating the heat transfer rate from the heat transfer coefficient h' through

$$q = h'(i_{wo} - i_w) \tag{2.11}$$

since $i_w \neq i_{wo}$ implies (through equations (2.13), (2.10) and (2.14)) that i_{wo} no longer retains the value which it had under zero heat transfer conditions $i_w = i_{wo}$. To avoid confusion it might be better to call the value of i_{wo} in equation (2.11) an "effective" or a "potential" enthalpy and replace i_{wo} by i_{eff} (say) in equations (2.10), (2.11), (2.13) and (2.14). The alternative is to retain i_{wo} and forget pre-conceived notions about its constancy. This avoids multiplicity of symbols and is adopted in the remainder of this paper.

Local Heat Transfer Coefficients

In incompressible flow, the Reynolds analogy factor is given approximately by

$$\frac{k_H}{\frac{1}{2}c_f} = s \qquad (2.7)$$

$$\simeq \sigma^{-2/3} \qquad (2.15)$$

and for local skin friction we have

$$c_f = 0.664\,Re_x^{-\frac{1}{2}}$$

so that local heat transfer coefficient is given by

$$k_{H_1} = 0.332\,\sigma^{-2/3}\,Re_x^{-\frac{1}{2}}\,. \qquad (2.16)$$

Following Eckert's procedure we then have

$$k_H^{\times} = 0.332\,(\sigma^{\times})^{-2/3}\,(Re_x^{\times})^{-\frac{1}{2}} \qquad (2.17)$$

in compressible flow, where

$$k_H^{\times} = \frac{h'}{\rho^{\times} u_1}$$

$$= \frac{h'}{\rho_1 u_1} \cdot \frac{\rho_1}{\rho^{\times}}$$

$$= k_H \frac{T^{\times}}{T_1} \qquad (2.18)$$

(since static pressure p is constant across the boundary layer, the equation of state gives $\frac{\rho_1}{\rho^{\times}} = \frac{T^{\times}}{T_1}$).

Likewise

$$Re_x^{\times} = \frac{\rho^{\times} u_1 x}{\mu^{\times}}$$

$$= \frac{\rho_1 u_1 x}{\mu_1} \cdot \frac{\rho^{\times}}{\rho_1} \cdot \frac{\mu_1}{\mu^{\times}}$$

$$= Re_x \frac{T_1}{T^{\times}} \cdot \frac{\mu_1}{\mu^{\times}} . \qquad (2.19)$$

Hence, substituting from equations (2.18) and (2.19) in (2.17) we obtain

$$k_H = 0.332 \, (\sigma_1)^{-2/3} \, Re_x^{-\frac{1}{2}} \left(\frac{T_1}{T^{\times}} \cdot \frac{\mu^{\times}}{\mu_1}\right)^{\frac{1}{2}} \left(\frac{\sigma^{\times}}{\sigma_1}\right)^{-2/3} \qquad (2.20)$$

and comparing this with equation (2.16) we see that for the same free stream temperature (defining σ_1) and Reynolds number, we have

$$\frac{k_H}{k_{H_i}} = \left(\frac{\sigma^{\times}}{\sigma_1}\right)^{-2/3} \left(\frac{T_1}{T^{\times}} \cdot \frac{\mu^{\times}}{\mu_1}\right)^{\frac{1}{2}} \qquad (2.21)$$

for the ratio of the coefficients in compressible and in incompressible flow.

$\frac{\mu^{\times}}{\mu_1}$ should be evaluated from Sutherland's formula

$$10^5 \mu = 9.800 \times 10^{-2} \left(\frac{T^{3/2}}{T + 110.4}\right) \left(\frac{lb}{ft \; sc}\right) \qquad (2.22a)$$

giving

$$\frac{\mu^{\times}}{\mu_1} = \left(\frac{T^{\times}}{T_1}\right)^{3/2} \frac{1 + \frac{110.4}{T_1}}{\frac{T^{\times}}{T_1} + \frac{110.4}{T_1}} . \qquad (2.22b)$$

Fig.3a shows the effects of Mach number and surface temperature on laminar heat transfer coefficient as given by these equations, for flight in the stratosphere when $T_1 = 216.5^{\circ}K$. In this case $\sigma_1 = 0.733$ (Fig.2b) and hence

$$k_{H_i} = 0.408 \, Re_x^{-\frac{1}{2}} .$$

(Ref.1 considers the effects of changes in ambient temperature and in particular the differences which may occur between wind-tunnels and flight.)

Since in the aircraft case there is the possibility of sustained flight at a given Mach number, there is an interest in how the heat transfer coefficient will vary as the surface warms up, so the plot in Fig.3 is of lines of constant Mach number against T_w/T_1. The full line gives the limiting values under zero heat transfer conditions and the appropriate values of T_{wo}/T_1 are indicated on this line by M = 1, 2 ... etc.

It is interesting to note that the total variation of k_H/k_{H_1} up to $M \simeq 5$ and zero heat transfer is only 14 per cent. (The vertical scale has been kept the same as in the corresponding plot, Fig.3b, for the turbulent boundary layer to make it easier to compare the relative effects of compressibility in the two cases.)

Mean Heat Transfer Coefficients

The compressibility variation given by equation (2.22) does not involve Reynolds number, so it (and Fig.3a) applies equally to mean heat transfer coefficients, i.e. to coefficients based on the overall amount of heat being transferred between stations 0 and x, as distinct from the local heat transfer at the station x. In this case the "incompressible" flow formula is

$$k_{H_1} = 0.664\,\sigma_1^{-2/3}\,Re_x^{-\frac{1}{2}} . \tag{2.23}$$

2.11 Significance of the "intermediate" enthalpy, i^x

The constants in the formula for intermediate enthalpy

$$i^x = i_1 + 0.5(i_w - i_1) + 0.22(i_{wo} - i_1) \tag{2.13}$$

are empirical and were chosen by Eckert[4] to give the best fit with the numerical results for skin friction, etc.[5,6,7]

The analysis in Ref.1 has shown, however, that this intermediate enthalpy is close to the mean enthalpy ($\bar{i}$) taken with respect to velocity (u) across the boundary layer, the latter being given by

$$\bar{i} = i_1 + 0.54(i_w - i_1) + 0.16(i_{wo} - i_1) \tag{2.24}$$

for values of σ between 0.75 and 0.70.

For zero heat transfer conditions ($i_w = i_{wo}$) there is little difference between the two formulae, but the emphasis alters when heat is being transferred. To check the significance of this the numerical results of Refs. 5-7 were re-analysed in Ref.1 on the basis of mean enthalpy and an equally good correlation was obtained (as compared with the correlation on the basis of intermediate enthalpy).

The significance of this analogy arises when considering turbulent boundary layers in section 2.2 below.

2.2 Heat transfer formulae for turbulent boundary layers in compressible flow

In view of the success of the intermediate enthalpy method with laminar boundary layers and also because of its apparent relation to mean enthalpy with respect to velocity, it seemed worthwhile to make an estimate of the mean enthalpy of a turbulent boundary layer. This was done in Ref.1, and gave

$$(\bar{i})_{turb} = i_1 + 0.54(i_w - i_1) + 0.16(i_{wo} - i_1) \tag{2.25}$$

when r and s (equations (2.10) and (2.7)) were given the values

$$r = 0.89$$

and $$s = 1.22$$

both of which are derived from experimental results.

The constants in equation (2.25) are identical with those in equation (2.24) and this may go some way towards explaining why application of the intermediate enthalpy formula (equation (2.13)) to the turbulent boundary layer has been successful in correlating experimental results[4].

Either intermediate or mean enthalpies give good correlations of the numerous experimental skin friction and heat transfer results now available for turbulent boundary layers up to $M = 8$ under conditions close to zero heat transfer. Only a few results are available under conditions of large heat transfer, and these do not agree amongst themselves. (For further discussion see Ref.1.) Further experimental programmes on the effect of T_w/T_1 are under way in the United States and in this country it is hoped to make similar measurements in the Hypersonic Wind Tunnel at the R.A.E. Meanwhile it seems likely that the effects of T_w/T_1 may not be larger than would be given by the intermediate enthalpy method, and for this reason it is suggested as being the most suitable for use in design calculations.

The application is similar to that already described in the case of laminar boundary layers. The factors r and s must be obtained from experimental results. From these Seiff[10] has shown that the mean value of s is 1.22 and no consistent variations with M or T_w/T_1 could be discovered. Hence, from the Prandtl-Schlichting type formula for local skin friction*

$$c_{f_i} = 0.288 \, (\log_{10} Re_x)^{-2.45} \tag{2.26}$$

we obtain

$$k_{H_i} = 0.176 \, (\log_{10} Re_x)^{-2.45} \tag{2.27}$$

which in compressible flow becomes

$$k_H^{\times} = 0.176 \, (\log_{10} Re_x^{\times})^{-2.45} \tag{2.28}$$

with density and viscosity evaluated at the temperature corresponding to the intermediate enthalpy

$$i^{\times} = i_1 + 0.5(i_w - i_1) + 0.22(i_{wo} - i_1) \, . \tag{2.13}$$

Concerning recovery factor, the vast majority of experimental results have been obtained in wind tunnels and have been analysed in terms of temperature recovery factor r_T (equation (2.5)). These results indicate that r_T may vary with Reynolds number as shown in Fig.4. No systematic effect of Mach number was noticed among the results collected in Ref.2, but when

* The various formulae for turbulent skin friction in incompressible flow are considered in Ref.1 and equation (2.26) was chosen as the most suitable and convenient for application.

combined with Brevoort and Rashis results[11] in Fig.4 it seems possible that differences ascribed to change in model (flat plate to bodies of revolution) in Ref.2 may have been a Mach number effect in disguise. However, the accuracy of measurement of zero heat transfer conditions is insufficient to allow such a conclusion to be drawn from the amount of evidence available.

On the other hand the results give a definite indication of a reduction in temperature recovery factor from 0.89-0.90 through 0.88 to 0.87 as Reynolds number is increased from 10^6 through 10^7 to 10^8.

In extending these results to the flight case, we shall make the arbitrary assumption that temperature is replaced by enthalpy and that the enthalpy recovery factors r of equation (2.10) will be given by the curves of Fig.4. (Under wind tunnel conditions of low temperature there would be little difference between temperature and enthalpy recovery factors, cf.Fig.2.)

In addition to a variation of r, it is found that equation (2.28) implies a small Reynolds number effect on k_H/k_{H_i} (see Ref.1). Therefore in plotting k_H/k_{H_i} it is necessary to specify the associated Reynolds number and Fig.3b shows the variation of k_H/k_{H_i} with M and T_w/T_1 for $Re_x = 10^7$ and $r = 0.88$.

This variation is much larger than was found for laminar boundary layers in Fig.3a.

Earlier work at the R.A.E. had suggested that c_f/c_{f_i} or k_H/k_{H_i} for turbulent boundary layers was a function only of the temperature ratio T_w/T_1 and would not be affected by Mach number. This seemed to be supported by the correlations of Ref.2, but since that time, further experimental results have become available and it begins to appear that the variations may be more as shown in Fig.3b. From the designer's point of view this is all to the good. Dependence only on T_w/T_1 would give the solid line labelled "zero heat transfer" under all conditions, but it now seems that this would progressively overestimate the heating at low values of T_w/T_1 as Mach number is increased. It is possible that the reduced values shown in Fig.3b may still be overestimates, since the N.O.L. tunnel wall results quoted in Ref.1 stayed close to the limiting value for zero heat transfer at the Mach number in question as T_w/T_1 was reduced. Further experimental results are obviously required: meanwhile (as mentioned above) it seems likely that the variations with T_w/T_1 at given Mach number may not exceed those shown in Fig.3b. An approximation to intermediate enthalpy results for Mach numbers up to ten, is given by:

$$\frac{k_H}{k_{H_i}} = \cancel{\left(\frac{T_1}{T_w}\right)^{\frac{1}{2}}} \left(\frac{T_1}{T_w}\right)^{\frac{1}{3}} \left(\frac{T_1}{T_{wo}}\right)^{\frac{4}{15}}$$

has been added for comparison, since this is used as a limiting case in the applications of section 4.4.

Finally the overall heat transfer between station 0 and station x will be given by the mean heat transfer coefficient

$$k_H^x = 0.28 \left(\log_{10} Re_x^x\right)^{-2.6} \tag{2.29}$$

with density and viscosity evaluated as before at the temperature corresponding to the intermediate enthalpy

$$i^x = i_1 + 0.5(i_w - i_1) + 0.22(i_{wo} - i_1) \,. \tag{2.13}$$

2.3 Heat transfer by radiation

In addition to an inwards flow of heat by convection as considered above, the surface will also be losing heat by radiation to its surroundings of magnitude

$$q_r = 2.78\ \varepsilon \left(\frac{T_w}{1000}\right)^4 \frac{\text{CHU}}{\text{ft}^2\ \text{sec}} \qquad (2.30)$$

where ε is emissivity factor (values in Ref.3)

and T_w is $^{\circ}$K.

[It will also be receiving back heat by radiation from the surrounding air of amount

$$2.78\ \varepsilon\varepsilon_G \left(\frac{T_\infty}{1000}\right)^4$$

where ε_G is the gas emissivity. This is usually taken to be small: values of the order of $3 \times 10^{-3} \frac{\text{CHU}}{\text{ft}^2\ \text{sec}}$ have been quoted.]

Heat will also be received by solar radiation to surfaces exposed to the sun's rays (and also by reflection from the earth or clouds, see Ref.3) and this would have a maximum value of $7 \times 10^{-2} \frac{\text{CHU}}{\text{ft}^2\ \text{sec}}$, but it can be reduced very considerably by suitable choice of surface finish.

3 Transition from laminar to turbulent flow

So far, formulae have been given for heat transfer with fully laminar or with fully turbulent boundary layers and it is of interest to know how far back a laminar boundary layer may extend on a body or wing. Appendix I lists various factors which will influence transition position but in most cases it is not possible at present to give more than a qualitative answer as to their effects.

However it is beginning to appear that the curves presented in Ref.2 for the quantitative effects of Mach number and heat transfer may not be over-optimistic and Fig.5 shows the combined effect of these two factors (curves derived from Figs. 27 and 31 of Ref.2). The curves give possible transition Reynolds numbers on a smooth surface, uninfluenced by pressure gradients, roughness or shock waves, and show the marked effect which surface temperature (heat transfer) can have. The curves for $M = 4$ and $M = 5$ are less definite than those for $M = 2$ and $M = 3$ and so they are given as broken lines.

The transition Reynolds numbers can be translated into distances from nose or leading edge when the flight conditions are known and Fig.6 (Fig.1a of Ref.1) gives Reynolds number per foot run in terms of Mach number and altitude. Thus $M = 4$ at 60,000 ft gives $\frac{Re}{ft} = 3 \times 10^6$ (Fig.6) so that a surface temperature of 250°C, giving $Re_T = 18 \times 10^6$ (Fig.5), would give 6 ft of laminar boundary layer.

See Addendum

These figures could be improved by favourable pressure gradients but might be much reduced by roughness, so it would be unwise to place too much reliance on them until actual flight experience is available.

4 Applications of the heat transfer formulae

Section 2 has given the present ideas about the most suitable and convenient formulae for use in calculations of aerodynamic heating and it might be wise to close the paper at this stage and refer to Ref.3 for details of calculation procedures, since all applications are bound to be particular in some respect and their results may not agree with those of other applications which at first sight appear closely similar. However, even particular applications are helpful in giving an idea of the magnitudes involved and it is in this spirit that much of the following section is presented.

Where necessary, the standard atmosphere as given in Ref.3 has been used, and the results throughout are strictly applicable only to a flat plate.

4.1 Zero convective heat transfer or "recovery" temperatures

Aerodynamic heating rates would become zero at the surface temperatures given in the following table (and plotted later on as the solid lines in Fig.11). The turbulent values correspond to a recovery factor of 0.88, appropriate to $Re_x = 10^7$ (Fig.4).

M_1	1	2	3	4	5
Altitude: 0 ft, $T_1 = 15°C$					
T_{wo} °C Laminar	64	206	428	725	1086
T_{wo} °C Turbulent	66	216	456	774	1160
Altitude: 25,000 ft, $T_1 = -34.5°C$					
T_{wo} °C Laminar	6	124	313	565	878
T_{wo} °C Turbulent	7	132	336	607	937
Altitude: 36,000 ft to 100,000, $T_1 = -56.5°C$					
T_{wo} °C Laminar	-20	88	260	493	780
T_{wo} °C Turbulent	-18	95	282	532	834

These show a considerable effect of altitude, caused by changes in the ambient temperature T_1. It should be emphasised that the temperatures quoted are the maxima to which any structure could be subjected. In practice radiation away from the surface (and internal cooling if present) would hold the surface temperatures down to the equilibrium temperatures considered in section 4.3 and the reduction can be large at the higher altitudes.

(The temperatures quoted in the above table are lower than those given by Refs. 3 or 12. This is because of the improved values of recovery factor used in the present paper.)

4.2 Local heat transfer factors "h"

These factors are defined by

$$q = h(T_{wo} - T_w) \qquad (2.4)$$

and the values of T_{wo} quoted in section 4.1. They have been obtained from the formulae of section 3, giving

$$q = h'(i_{wo} - i_w) \qquad (2.11)$$

and the relation

$$h = h' \frac{i_{wo} - i_w}{T_{wo} - T_w}. \qquad (4.1)$$

4.21 Effect of M, $^{T}w/T_1$ and altitude at given Re_x

Fig.7 gives values of the local heat transfer factor h in terms of $\frac{CHU}{ft^2\ sc\ ^oC}$ for $Re_x = 10^7$ and over ranges of Mach number, surface temperature and altitude.

Altitude is seen to have the most pronounced effect, there being an 11 to 1 variation between 50,000 ft and 100,000 ft. There is a slightly greater variation with Mach number for laminar than for turbulent boundary layers, but on the other hand the positions are reversed as regards effects of surface temperature.

Comparison of Figs. 7a and 7b shows that there is considerable benefit to be gained from maintaining a laminar boundary layer to this Reynolds number, there being a factor of about 8 between corresponding laminar and turbulent values. Effects of variations in Reynolds number are considered in section 4.22 below.

Two general points should be made about Fig.7. First, the comparison is at constant Reynolds number and comparison with Fig.6 will show that for the same Mach number this corresponds to different distances back along the body at different altitudes. If the values were for given x rather than Re_x, the variation with Mach number at given altitude would be reduced, particularly in the case of the laminar boundary layer. Second, all the altitudes are within the stratosphere for which an ambient temperature of -56.5°C was assumed. Variation in ambient temperature would require the plot to be against $^{T}w/T_1$ rather than T_w.

The marked effect of altitude is illustrated further by Fig.8, which compares the value of h at any altitude to that at 50,000 ft and the same Reynolds number, Mach number, and ratio $^{T}w/T_1$. Under these conditions the variation in the stratosphere (full line) is equivalent to the variation of air density. In the troposphere the variation also includes the speed of sound and with a laminar boundary layer should also include small variations in recovery factor due to changes in temperature level. The last named item was neglected in preparing Fig.8 and hence this portion of the curve is shown as a broken line.

4.22 Effect of variation in Reynolds number

The effect on h of variations in Reynolds number at constant Mach number, altitude and surface temperature is illustrated by Fig.9. The conditions taken correspond to the case taken by N. J. Hoff in his paper to the 3rd Anglo-American Aeronautical Conference[13] (for which he assumed a value of $90 \frac{CHU}{ft^2\ hr\ ^oC}$ for a turbulent boundary layer, this value being based on an incompressible flow formula for h).

Surface temperatures (uniform) of 15°C and 250°C are assumed in Fig.9 to show the effect of this parameter. Also the laminar curves are continued to the point where transition would occur from Fig.5: this is much later at the lower temperature.

The turbulent heat transfer factors are well below Hoff's value of $90 \frac{\text{CHU}}{\text{ft}^2 \text{ hr } ^\circ\text{C}}$. This is caused by the effects of compressibility.

With a laminar boundary layer the variation of h with x (or Re_x) is simply

$$h \sim x^{-\frac{1}{2}} \qquad (4.1)$$

and thus high values would be reached near a sharp leading edge, x = 0. (The variation of equation (4.1) would no longer apply in the neighbourhood of a blunt leading edge.)

With a turbulent boundary layer the variation is more complex. Approximate values are given in the following table, where n is the index in an assumed variation

$$h \sim x^{-1/n} . \qquad (4.2)$$

(The values of n are given to the nearest 0.5 and are based on equation (2.28).)

Values of n

T_w / Re_x / M	Incompressible		160°C		760°C	
	10^6–10^7	10^7–10^8	10^6–10^7	10^7–10^8	10^6–10^7	10^7–10^8
0 (inc.)	6	7	-	-	-	-
3	-	-	5.5	6.5	-	-
5	-	-	5.5	6.5	5	6

4.3 Heat flow rates "q" and influence of radiation

Fig.10 shows the actual heat flow rates "q" which would be experienced in flight at 50,000 ft at the station corresponding to $Re_x = 10^7$. These are given in terms of kW/ft² at different surface temperatures and Mach numbers and represent the heating loads that would have to be applied at this station during structural tests in a laboratory.

The solid lines give the aerodynamic heating rates inwards to the body both for laminar boundary layers (Fig.10a) and for turbulent boundary layers (Fig.10b). Values read from these curves would vary with Reynolds number (distance along body) as discussed in section 4.22 and illustrated in Fig.9 and would vary with altitude in accordance with Fig.8. Thus there would be about a 10:1 factor in either direction in going from 50,000 ft to sea level (increase in q) or to 100,000 ft (decrease in q).

The broken lines give the amount of heat which could be radiated away from the surface, assuming two values of surface emissivity factor, $\varepsilon = 0.9$ and $\varepsilon = 0.1$. This heat loss is independent of altitude.

For a given surface temperature and Mach number, the net flow of heat into the surface will be given by the difference between the aerodynamic heat input (solid lines) and radiative heat loss (broken lines). Inspection shows the benefit of a high surface emissivity, particularly since the surface temperature will be stabilised when the net heat input becomes zero and this equilibrium temperature can be considerably less than the zero heat transfer temperature.

(Since Fig.10 is an illustration confined to one altitude, the benefits of high emissivity are not very apparent if the boundary layer is turbulent. However, they become more noticeable as the altitude is increased, because of the substantial decrease in aerodynamic heating rate as mentioned above. Illustrations of this effect are given in Ref.1.)

4.31 Equilibrium temperatures

Values are given in Fig.11 of the equilibrium temperatures (T_{we}) reached when the aerodynamic heat input is balanced by the radiative heat loss. Fig.11a is for a laminar boundary layer, Fig.11b is for a turbulent boundary layer, and both give local values corresponding to $Re_x = 10^7$. (Thus different values would be reached at other stations along the body because of variations in heat transfer coefficient, and the variations would be greater for laminar than for turbulent boundary layers.)

The solid lines give the zero heat transfer temperatures discussed in section 4.1. These increase rapidly with Mach number and would correspond to the very worst case of zero emissivity.

The broken lines correspond to the very good case of $\varepsilon = 0.9$ and show substantial reductions in temperature particularly at the higher altitudes. For example at $M = 5$ and 100,000 ft, zero heat transfer and equilibrium temperatures are as given in the following table (for $Re_x = 10^7$).

	Laminar	Turbulent
T_{wo} ($\varepsilon = 0$) °C	780	834
T_{we} ($\varepsilon = 0.9$) °C	170	355

The importance of a high emissivity is emphasised by the fact that the combination of a turbulent boundary layer and high emissivity may well give a lower equilibrium temperature than a laminar boundary layer and low emissivity.

The question as to the Mach number at which emissivity becomes important cannot be answered directly since Fig.11 will show that it is also necessary to specify an altitude. The dotted line across Fig.11a and Fig.11b gives equilibrium temperatures corresponding to the Mach number-altitude relation of a constant equivalent air speed (constant dynamic head) of 660 knots. In this case one might say that emissivity effects begin to be noticeable at Mach numbers above 2. However, this conclusion would be altered by taking a different E.A.S. or a different station on the body.

4.32 Heat transfer factors relative to equilibrium temperature

Continuing consideration of the case $Re_x = 10^7$, it is interesting to deduce heat transfer factors relative to equilibrium rather than to zero heat transfer temperature, since it is these which will determine the time taken for the skin to heat up to the equilibrium temperature. Taking $\varepsilon = 0.9$, we define

$$h_{net} = \frac{q_{net}}{T_{we} - T_w} \tag{4.2}$$

where q_{net} is the difference between convection and radiation and can be obtained from curves such as in Fig.10.

Values of h_{net} appropriate to Figs. 10 and 11 are given in Fig.12. These are intended only to give an illustration of the magnitudes involved and it will be noticed that the values do not extend right up to the appropriate equilibrium temperatures as would be given by Fig.11. This restricted range was dictated solely by considerations of convenience and economy in computing time, since reference to Fig.10 will indicate that increased accuracy would become necessary if deriving values close to the equilibrium temperature.

Comparing the values of h_{net} in Fig.12 with those of h in Fig.7 shows that the two are of similar magnitude, but altitude has less effect on h_{net} than it had on h (the values of h_{net} become relatively larger as altitude is increased). Also the effect of surface temperature is reversed in most cases: h_{net} increases with T_w, particularly at the higher altitudes.

It should be emphasised that Fig.12 is included only as an illustration and variations from it might occur if either Reynolds number or emissivity factor were altered.

4.4 Time taken to reach equilibrium conditions

The heat transfer coefficients and factors discussed in the previous sections can be used in combination with a known skin material and thickness and a known flight plan to compute the variation of skin temperature with time. The basic equation for a skin thin enough for the temperature across it to be sensibly uniform (and neglecting internal cooling) is

$$G \frac{dT_w}{dt} = h\,(T_{wo} - T_w) - 2.78\,\varepsilon \left(\frac{T_w}{1000}\right)^4 \tag{4.3}$$

where t is time

and G is the heat capacity of the skin

($= \rho_s c_s d$ where ρ_s is the density, c_s is the specific heat and d is the thickness of the skin).

An alternative expression would be

$$G \frac{dT_w}{dt} = h_{net}\,(T_{we} - T_w) \tag{4.4}$$

with values of h_{net} derived as in section 4.3.

If temperature gradients across the skin are significant then the right hand sides of equations (4.3) and (4.4) become the boundary conditions of a pure conduction problem.

In either case there is no general solution which will cover all flight plans, so the equations have to be solved in individual cases either by a step-by-step numerical integration (Ref.3) or by means of an analogue computer.

However, analytical solutions are available if the variation of h is restricted and radiation is neglected (alternatively radiation can be included and the variation of h_{net} is restricted). The commonest of these is to take h constant which is roughly the case for constant altitude and Mach number (see Fig.7 or 12) and this has been a popular assumption when considering the effects of aerodynamic heating. This assumption of constant altitude and Mach number may be reasonable over considerable portions of a given flight plan, but unfortunately it is necessary to specify how the aircraft reached this altitude and Mach number before the initial skin temperature conditions can be postulated. It is highly artificial to assume that the whole surface of the aircraft will be at uniform temperature at the beginning of the cruise (as is usually done) and likewise the effects of non-uniform temperature distribution[2] should be included when calculating its subsequent history, particularly for stations near the leading edge or nose where the temperature gradients may be large.

However, when this is done, one should also take into account the fact that the structure is by no means the uniform shell assumed in equations 4.3 and 4.4, so that the complications multiply themselves and in the end one is forced to the conclusion that there may be something to be said for the artificial approach, provided one recognises that it is artificial and applies it only to gain general impressions of what may happen. This is the philosophy of the present section.

The two main assumptions used throughout are that the flight is

(a) at constant Mach number

and (b) at constant altitude.

The effects of assuming h = const are considered below and this is followed by consideration of "thin" and of "thick" skins. In all cases there is the further assumption of

(c) no heat flow from the skin into the interior of the body.

Analysis by Davies[14] of the relative magnitudes of external and internal heat transfer coefficients, assuming the skin and structure to be a uniform shell, has indicated that this last assumption should not lead to over-serious errors, provided there is no forced cooling of the inner wall.

4.41 Aerodynamic heating of thin skins. Effect of assuming h = constant

Fig.3 shows that if M is constant then k_H is a function only of T_W/T_1. If the altitude remains constant then the same can be said of h (cf. equation (2.6) and Fig.7 or 12).

This variation of h with T_W is larger for turbulent than for laminar boundary layers, so the following discussion is confined to the turbulent case.

Fig.3 shows that the turbulent variation of h (k_H) with T_W according to the intermediate enthalpy formula lies between

$$h = h_o = \text{constant} \tag{4.5}$$

(where h_o is the value under zero heat transfer conditions) and the line

$$\frac{h}{h_o} = \left(\frac{T_{WO}}{T_W}\right)^{\frac{1}{2}} \tag{4.6}$$

when M is constant.

Substitution of these relations in equation (4.3) and neglecting the radiation term yields the following solutions for the variation of skin temperature with time:

$$\frac{h_o}{G} t = \Big[- \log(T_{wo} - T) \Big]_{T_i}^{T_w} \qquad (4.7)$$

if $h_o = \text{const}$, where T_i is the value of T_w at $t = 0$, and

$$\frac{h_o}{G} t = \left[\tanh^{-1} \left(\frac{T}{T_{wo}}\right)^{\frac{1}{2}} - \left(\frac{T}{T_{wo}}\right)^{\frac{1}{2}} \right]_{T_i}^{T_w} \qquad (4.8)$$

if

$$\frac{h}{h_o} = \left(\frac{T_{wo}}{T_w}\right)^{\frac{1}{2}} .$$

The variations of $\frac{T_w}{T_{wo}}$ with $\frac{h_o}{G} t$ given by equations (4.7) and (4.8), assuming $T_i = 0$ are shown in Fig.13. (If $T_i \neq 0$ then heating times can still be read from Fig.13 by subtracting the value of $\frac{h_o}{G} t$ corresponding to $\frac{T_i}{T_{wo}}$ from the final value of $\frac{h_o}{G} t$.)

As might be expected, heating is more rapid if h follows the variation given by equation (4.6) instead of remaining constant at its zero heat transfer value, but percentagewise the difference in times to reach a given skin temperature decreases as $\frac{T_w}{T_{wo}}$ tends to unity. This is illustrated further by Fig.15 which gives a plot of

$$\frac{T_{wo} - T_w}{T_{wo} - T_i} \qquad \text{against} \qquad \frac{h_o}{G} t .$$

There is a single curve for the case $h_o = \text{const}$ (equation (4.7)) but when h varies (equation (4.8)) there are individual curves for individual values of $\frac{T_i}{T_{wo}}$.

These two figures (13 and 15) give an idea of the errors introduced by taking h = const and it should be emphasised that the true variations may be somewhere about half way between the extremes given by equations (4.7) and (4.8). In practice therefore and at least in preliminary calculations it may be sufficiently accurate to use mean values of h taken off curves such as in Figs. 7 and 12*.

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* This applies to estimates of times taken to reach a given temperature. If individual heat flow rates are being considered (for estimating temperature gradients etc) it would be preferable to use the values of h appropriate to the temperature of the surface at the time in question.

A corollary of this result is that if tests are being planned wherein aerodynamic heating rates are to be deduced from measured variations of skin temperature with time and it is desired to be able to differentiate between various "theoretical" estimates then it would be preferable to choose a skin material of large heat capacity so that h_o/G is small and hence the time scale is extended as much as possible.

4.42 Effects of skin thickness and thermal properties h = constant

A solution is given in Ref.15 for the linear flow of heat in a solid bounded by two parallel planes, where one face is heated (and h is constant) and the other face is insulated (assumption (c) above). For simplicity we shall assume that the solid (skin) is at uniform temperature throughout. Then apart from distance into the skin, the solution depends on the two parameters

$$\frac{k_s t}{\rho_s c_s d^2} \quad \text{and} \quad \frac{hd}{k_s} .$$

The solutions in section 4.41 for thin skins depended on the parameter

$$\frac{h}{G} t = \frac{h}{\rho_s c_s d} t$$

which is the product of the two parameters listed above, so in considering the extension of the results of section 4.41 to include "thick" skins it is convenient to replot the results of Ref.15 in terms of

$$\frac{h}{G} t \quad \text{and} \quad \frac{hd}{k_s} .$$

This separates the effects of heat capacity G and thermal conductivity k_s.

Fig.14 illustrates the effect of $\frac{hd}{k_s}$ by plotting $\frac{T}{T_{wo}}$ against $\frac{h}{G} t$ for values of $\frac{hd}{k_s}$ of zero and unity. The former corresponds to the ideal "thin" skin of Fig.13 since there will be no temperature differential across it. In the latter case it is necessary to specify position across the skin and curves are given for the outer surface, the inner surface and the mean temperature.

The results in Fig.14 illustrate the following general trends which accompany an increase in $\frac{hd}{k_s}$ while keeping $\frac{h}{G}$ constant.

1. Both the mean temperature and the inner surface temperature rise more slowly than would be predicted by the "thin skin analysis" of section 4.41.

2. Initially the outer surface temperature rises more rapidly but later it lags behind the results from the "thin skin analysis". (This is a consequence of the fact that initially the aerodynamic heat flows swamp the capability of the thermal conductivity to take heat away from the surface, but as the surface temperature rises the aerodynamic heat flow decreases and the heat is able to get away from the surface more easily.)

The definition of a "thin" skin and the suitability of the analysis in section 4.41 therefore depends on the value of the parameter hd/k_s. Fig.16 gives plots of $\frac{T_{wo} - T}{T_{wo} - T_i}$ against $\frac{h}{G}\, t$ for a range of values of $\frac{hd}{k_s}$. Fig.16a considers mean temperature, Fig.16b inner surface temperature and Fig.16c outer surface temperature, and the trends are as discussed above. Finally Fig.17 gives a cross-plot in terms of $\frac{hd}{k_s}$ and shows how the times to a given temperature ratio vary as $\frac{hd}{k_s}$ is increased.

Arbitrarily one might say that a skin could be considered "thin" if $\frac{hd}{k_s} < 0.1$. The physical thickness will therefore depend both on h and on k_s. However if heating times are worked out on the basis of the "thin skin analysis", then the curves of Fig.17 might be used to correct for skin thickness. Alternatively, the answers may be obtained directly from Fig.16.

Fig.17a displays the interesting feature that if mean temperatures are considered, then if

$$\frac{T_{wo} - \bar{T}}{T_{wo} - T_i} < 0.3$$

the values of the time parameter $\frac{h}{G}\, t$ increase linearly with $\frac{hd}{k_s}$, at least up to $\frac{hd}{k_s} = 2$, according to

$$\frac{\left(\frac{h}{G}\, t\right)}{\left(\frac{h}{G}\, t\right)_{\frac{hd}{k_s} = 0}} = 1 + 0.35\, \frac{hd}{k_s} . \qquad (4.9)$$

In practice it would be unusual to be able to vary the thermal conductivity of the skin while keeping its thermal capacity constant. The more usual situation is that of varying the skin thickness and this affects both $\frac{h}{G}\, t$ and $\frac{hd}{k_s}$. Increasing the skin thickness decreases $\frac{h}{G}$ and hence increases the time to a given mean temperature, and the foregoing results show that this time will be increased still further by the increase in $\frac{hd}{k_s}$. The same applies to inner surface temperature, but the outer surface may not always show this happy trend, i.e. in the early stages of the heating, an increase in skin thickness will not give a commensurate increase in time to a given temperature.

4.5 Numerical examples

The use of the foregoing solutions can be illustrated by some numerical examples.

In these we shall assume that an aircraft arrives at an altitude of 75,000 ft with a skin temperature of 15°C and is travelling at $M = 5$. The problem is to determine the subsequent variations in skin temperature at a station $5\frac{1}{2}$ feet from the nose (corresponding to $Re_x = 10^7$, Fig.6), neglecting effects of temperature gradients and pressure gradients and using "flat plate formulae".

In the first case the skin is taken to be $^1/10$ inch thick and have the thermal properties of mild steel*. For comparison, results are then given for a transparency of fused silica of the same thickness and in both cases the effects of radiation are considered (assuming $\varepsilon = 0.9$, which assumes a suitable surface finish on the steel and is the value quoted for polished glass at low temperatures). Finally the effects of skin thickness are illustrated by increasing the thickness of the fused silica to $\frac{1}{2}$ inch.

In all cases we assume a turbulent boundary layer. The zero heat transfer temperature (Fig.11b) is then

$$T_{wo} = 834^{\circ}C$$

and the equilibrium temperature with $\varepsilon = 0.9$ is

$$T_{we} = 490^{\circ}C \qquad \text{(Fig.11b)}$$

From Fig.7b we then take

$$h = \frac{q}{T_{wo} - T_w}$$

$$\simeq 3.3 \times 10^{-3} \frac{CHU}{ft^2 \, sc \, ^{\circ}C}$$

and from Fig.12b

$$h_{net} = \frac{q_{net}}{T_{we} - T_w}$$

$$\simeq 7 \times 10^{-3} .$$

as mean values of h over the temperature ranges in question

The final general result is that the maximum heat input during the cruise will occur at its start when $T_w = 15^{\circ}C$, and from Figs. 10 and 8 this is

$$q_{max} = 5.4 \frac{kw}{ft^2}$$

$$= 2.85 \frac{CHU}{ft^2 \, sc} .$$

(Larger values than this may easily have been reached during the climb to 75,000 ft.)

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* The choice of mild steel is purely a matter of convenience since the thermal properties are easily available. In the examples to follow it will be assumed that these properties do not vary with temperature.

Case 1. $^1/_{10}$" mild steel

The thermal properties are assumed to be[3]

$$G = 4.5 \frac{CHU}{ft^2\ ^oC} \quad \text{per inch thickness}$$

and

$$k_s = 8.65 \times 10^{-2} \frac{CHU\ in.}{ft^2\ sc\ ^oC}$$

hence

$$\frac{h}{k_s} = 3.82 \times 10^{-2} \frac{1}{in} \quad \text{and} \quad \frac{h_{net}}{k_s} = 8.1 \times 10^{-2} \frac{1}{in}$$

and these show that under the present flight conditions a steel skin up to 1 inch thick could still be regarded as thin (assuming $^{hd}/_{k_s} < 0.1$ as the criterion).

Fig.18 shows the variation of skin temperature with time, obtained from Fig.16a with $\frac{hd}{k_s} = 0$, and

$$\frac{h}{G} = 0.44 \frac{1}{min} \qquad \frac{h_{net}}{G} = 0.933 \frac{1}{min} .$$

In either case Fig.17 indicates that the corrections for finite skin thickness would be negligible.

Considering the effects of radiation, Fig.18 shows that these become important around 300°C, and the equilibrium temperature is reached in about 5 minutes. Neglecting radiation ($\varepsilon = 0$) the skin temperature is still rising after 6 minutes. It would reach 800°C in 7 minutes and 826°C $\left(\frac{T_{wo} - T}{T_{wo} - T_i} = 0.99\right)$ in $10\frac{1}{2}$ minutes.

The times shown in Fig.18 would vary directly with thickness if the skin thickness were changed (at least up to 1 inch thickness).

Finally the maximum temperature gradient through the skin would be at the surface at $t = 0$ and is

$$\frac{q_{max}}{k_s} = 33 \frac{^oC}{in} .$$

If the cruising altitude were changed, then the times to a given temperature neglecting radiation would vary inversely as the change in h given in Fig.8 provided the new value of $^{hd}/_{k_s}$ was less than 0.1. For example at 50,000 ft the $^1/_{10}$" skin would reach 800°C in about 2 minutes. Obviously skin thickness can be altered as well as altitude provided the conductivity condition $^{hd}/_{k_s} < 0.1$ is satisfied.

The results with radiation are particular to 75,000 ft since T_{we} varies with altitude, and h_{net} does not follow the same variation as h.

Case 2. $^1/_{10}$" fused silica

In this case we shall take

$$G = 2.28 \frac{\text{CHU}}{\text{ft}^2\ {}^\circ\text{C}} \quad \text{per inch thickness}$$

and
$$k_s = 2.74 \times 10^{-3} \frac{\text{CHU in.}}{\text{ft}^2\ \text{sec}\ {}^\circ\text{C}}$$

so that by comparison with mild steel, the heat capacity is about halved and there is a factor of about 30 on the conductivity.

$$\frac{h}{k_s} = 1.20 \frac{1}{\text{in}} \qquad \text{and} \qquad \frac{h_{net}}{k_s} = 2.56 \frac{1}{\text{in}}$$

so that effects of skin thickness will be more marked.

For a thickness of $^1/_{10}$",

$$\frac{hd}{k_s} = 0.121 \qquad \text{and} \qquad \frac{h_{net}d}{k_s} = 0.256$$

of which the former is just above the value 0.1, and Fig.18b shows the resulting variations in outer surface, inner surface and mean temperatures. The combination of scales (the same as in Fig.18a) does not show up very well the variations in temperature across the skin, but in fact there is 40°-60°C difference between the outer and inner surfaces in the early stages of the cruise. However, later on these differences are much reduced and the variation becomes essentially that of a thin skin. By comparison with an ideal thin skin $\left(\frac{hd}{k_s} = 0\right)$ the times to a given mean temperature in the later stages would be greater by about 4 per cent in the case neglecting radiation (from equation (4.9)).

Considering mean temperatures and neglecting radiation, 800°C is reached in just under 4 minutes and 826°C in $5\frac{1}{2}$ minutes. These reductions, compared with steel, are primarily the effect of reduced heat capacity. Including radiation, the equilibrium temperature is reached in about 3 minutes.

The maximum temperature gradient through the skin still occurs at the surface at $t = 0$, but now has the value $1040 \frac{{}^\circ\text{C}}{\text{in}}$.

These results can not be generalised to other thicknesses and altitudes because of the relatively large value of h/k.

Case 3. $\frac{1}{2}$" fused silica

In this case

$$\frac{hd}{k_s} = 0.603$$

which is well above the value 0.1, and Fig.19 compares the heating rates of $^1/_{10}$" and $\frac{1}{2}$" fused silica, neglecting radiation (to avoid confusion of curves). The differences in temperature across the $\frac{1}{2}$" silica are comparatively large.

For example, after 5 minutes the outer, mean and inner temperatures are 505, 430 and 390°C respectively. An alternative comparison is the time taken to reach a given temperature. The 1/10 in. silica reaches 600°C in about $1\frac{1}{2}$ minutes, whereas the outer surface and mean values for the $\frac{1}{2}$ in. silica do not reach this temperature until 7.3 and 8.8 minutes respectively.

These increases in time compared with the 1/10 in. silica are not entirely an effect of increased heat capacity. This alone would give a factor of 5 on time and result in the dotted line labelled "thin skin approximation". (Note that the thin skin approximation crosses the outer surface curve at about 7 minutes. This is the effect noted in Fig.14.) According to this approximation a temperature of 826°C would be reached in $26\frac{1}{2}$ minutes, whereas the more accurate value allowing for conductivity effects would be about 32 minutes.

These three examples may help to illustrate the roles played by heat transfer coefficient, heat capacity and thermal conductivity. It should be emphasised in conclusion that the numerical values are mostly particular to the altitude, speed, body station and initial conditions chosen.

List of symbols

c_p specific heat of air at constant pressure

c_s specific heat of skin material

d skin thickness

G $= \rho_s c_s d$, heat capacity of skin material per unit surface area

h Newtonian heat transfer factor $= \dfrac{q}{T_{wo} - T_w}$

h' modified heat transfer factor $= \dfrac{q}{i_{wo} - i_w}$

i enthalpy $\left(= \int_0^T c_p \, dT\right)$

T temperature (degrees absolute)

k thermal conductivity of air

k_s thermal conductivity of skin material

q heat flow per unit time per unit surface area

r enthalpy recovery factor

r_T temperature recovery factor

s "Reynolds analogy factor" $\left(= \dfrac{k_H}{\frac{1}{2}c_f}\right)$

u velocity

x distance from leading edge or nose

M Mach number

List of symbols (Contd)

γ ratio of specific heats of air

ε emissivity factor (radiation)

μ viscosity of air

ρ density of air

ρ_s density of skin material

σ Prandtl number $\left(= \frac{c_p \mu}{k}\right)$

c_f local skin friction coefficient

k_H heat transfer coefficient $\left(= \frac{h}{\rho_1 u_1 c_p} \text{ or } \frac{h'}{\rho_1 u_1}\right)$

Re Reynolds number $\left(= \frac{\rho u x}{\mu}\right)$

Superscript

$\times$ refers to intermediate temperature (enthalpy)

Suffixes

1 conditions in stream outside boundary layer

w surface of body (thin skin)

wo zero convective heat transfer conditions

we equilibrium conditions (allowing for radiation)

s1 outer surface of skin (thick skin)

s2 inner surface of skin (thick skin)

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3	F.V. Davies R.J. Monaghan	The estimation of skin temperatures attained in high speed flight. ARC Current Paper No.123. January 1953
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16	K.W. Mangler	Compressible boundary layers on bodies of revolution. M.A.P. Volkenrode VG 83 (Rep. and Transl. 47T) 1946.
17	F.V. Davies J.R. Cooke	Boundary layer measurements on 10° and 20° cones at $M = 2.45$ and zero heat transfer. C.P. 264. November 1954.
18	E.H. Bateman	An engineering approach to some structural problems arising from kinetic heating. August 1955. Pubd. in revised form in J.R.Ae.S. Vol. 60, p.402. June, 1956.
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APPENDIX I

Factors influencing transition

Item	Favourable Effect	Unfavourable Effect
1. Mach Number (zero heat transfer conditions)	Possibly for $M > 4$-5	In range $1 < M < 3$-4 (Note Fig.31, Ref.2)
2. Heat Transfer	For heat flow from airstream to body ($T_W < T_{WO}$) (Note Fig.27,Ref.2)	For heat flow from body to airstream ($T_W > T_{WO}$) (Note Fig.27,Ref.2)
3. Pressure Gradients	If favourable (and if combined with 2 may have very pronounced effect, Note G.M. Low. NACA TN 3103)	If adverse. (Transition may occur soon after the start of an adverse pressure gradient.)
4. Roughness	Never.	Relevant parameter may be k/δ. δ increases with M so the problem may become easier as speeds increase.
5. Shock waves striking boundary layer	-	Almost certain to cause transition. (Ref. N.P.L. work on shock-wave boundary layer interaction.)
6. Incidence	Wings: Should not be noticed by boundary layer on straight wings. Bodies: Forward movement of transition on leeward side. Backward movement of transition on windward side. (See Davies, RAE Tech Note Aero to be issued.)	

Wind tunnel test results may be affected by tunnel turbulence, and there has also been reported an effect of ambient pressure (increased pressure giving increased transition Reynolds number), but a consistent correlation has yet to be made of this latter effect.

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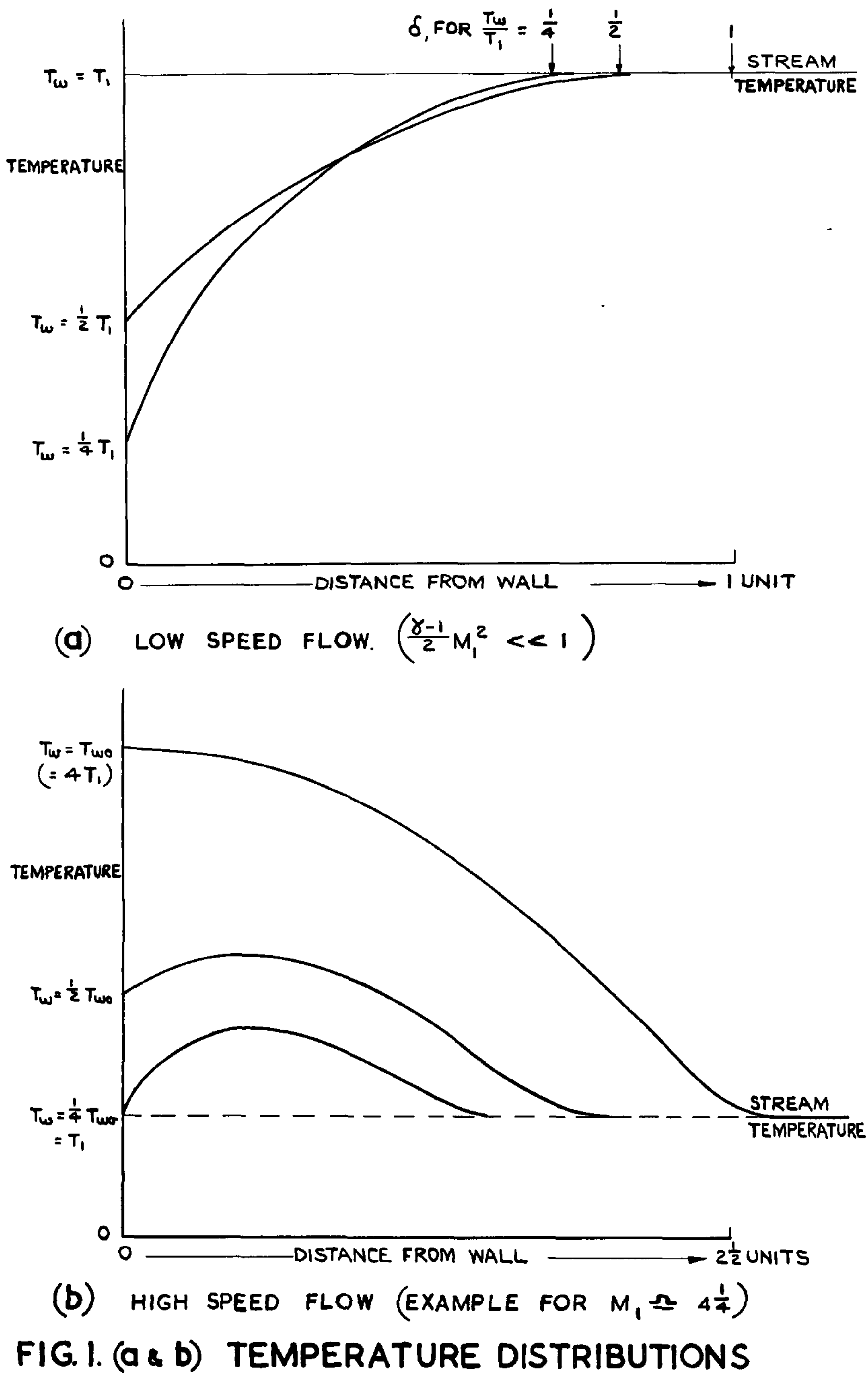
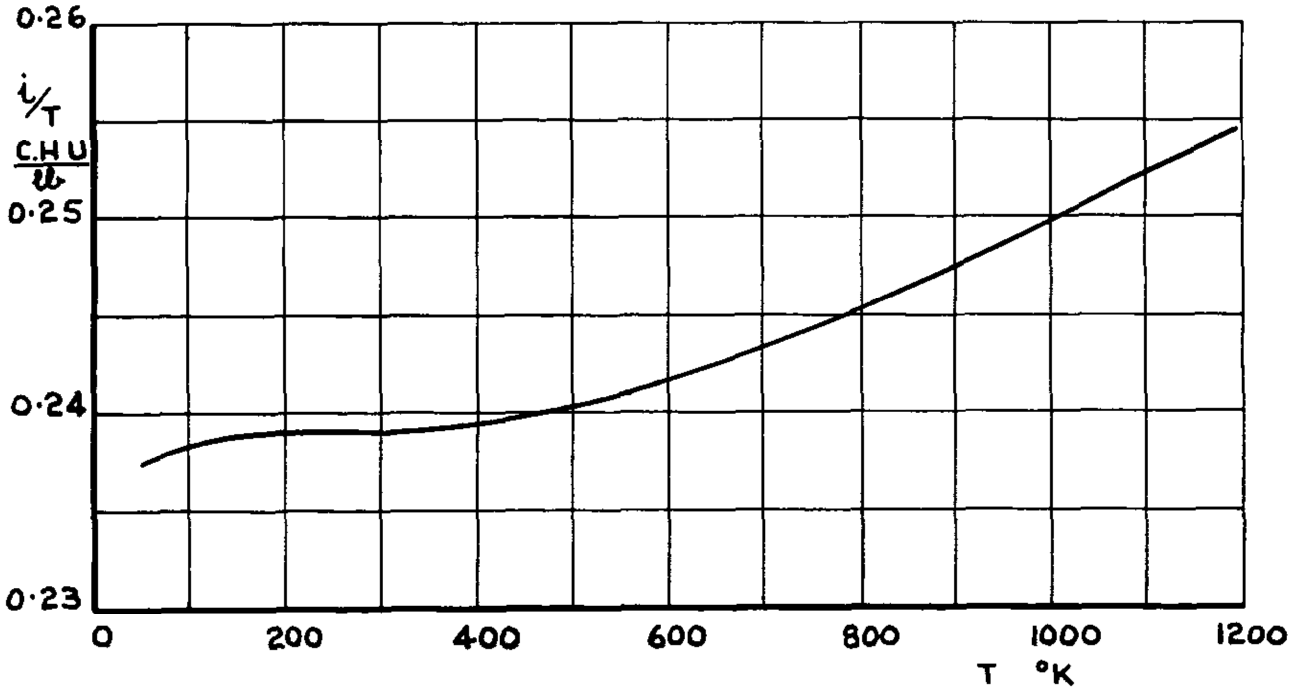
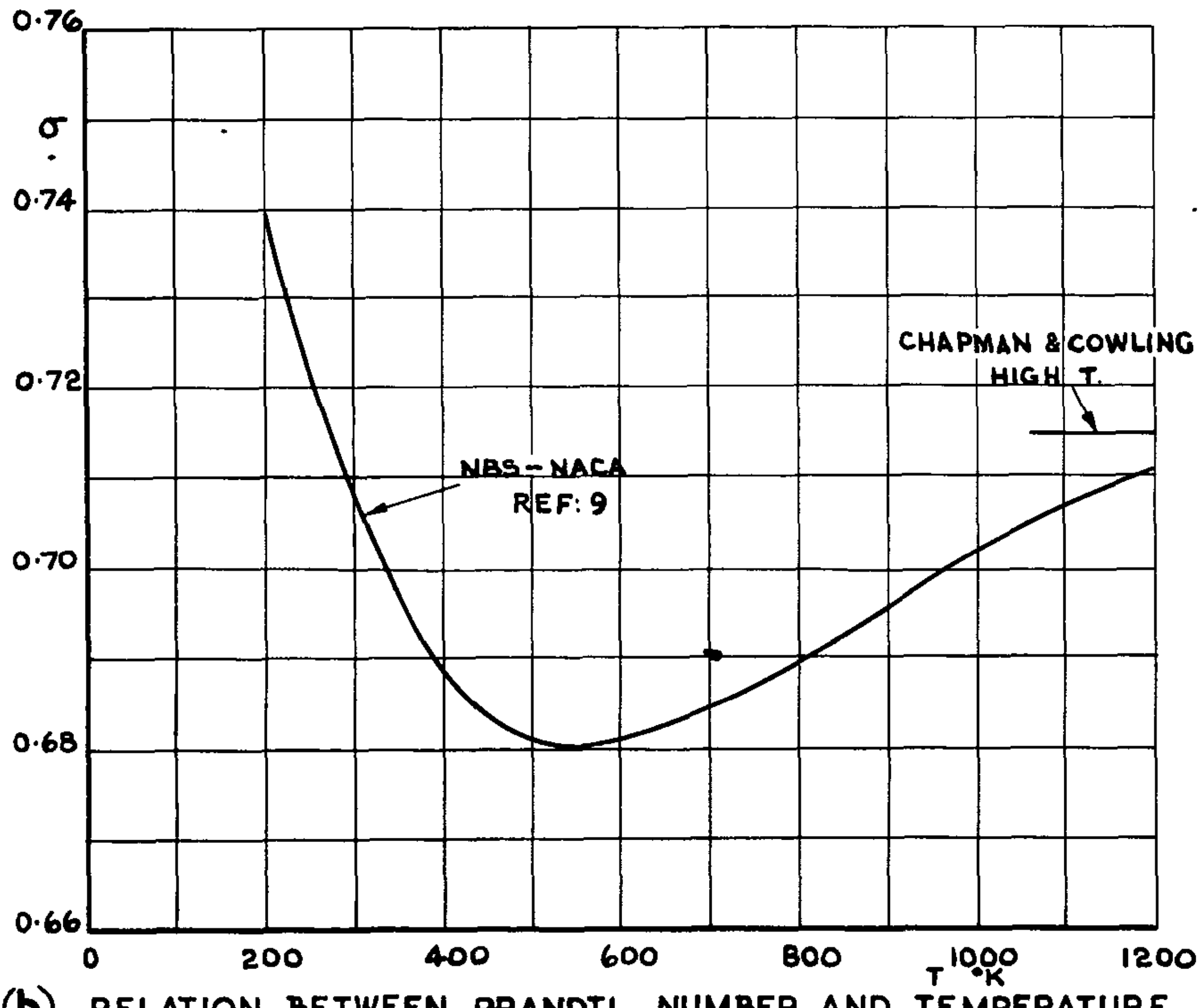


FIG. 1. (a & b) TEMPERATURE DISTRIBUTIONS ACROSS LAMINAR BOUNDARY LAYERS. EFFECTS OF CHANGES IN SURFACE TEMPERATURE.

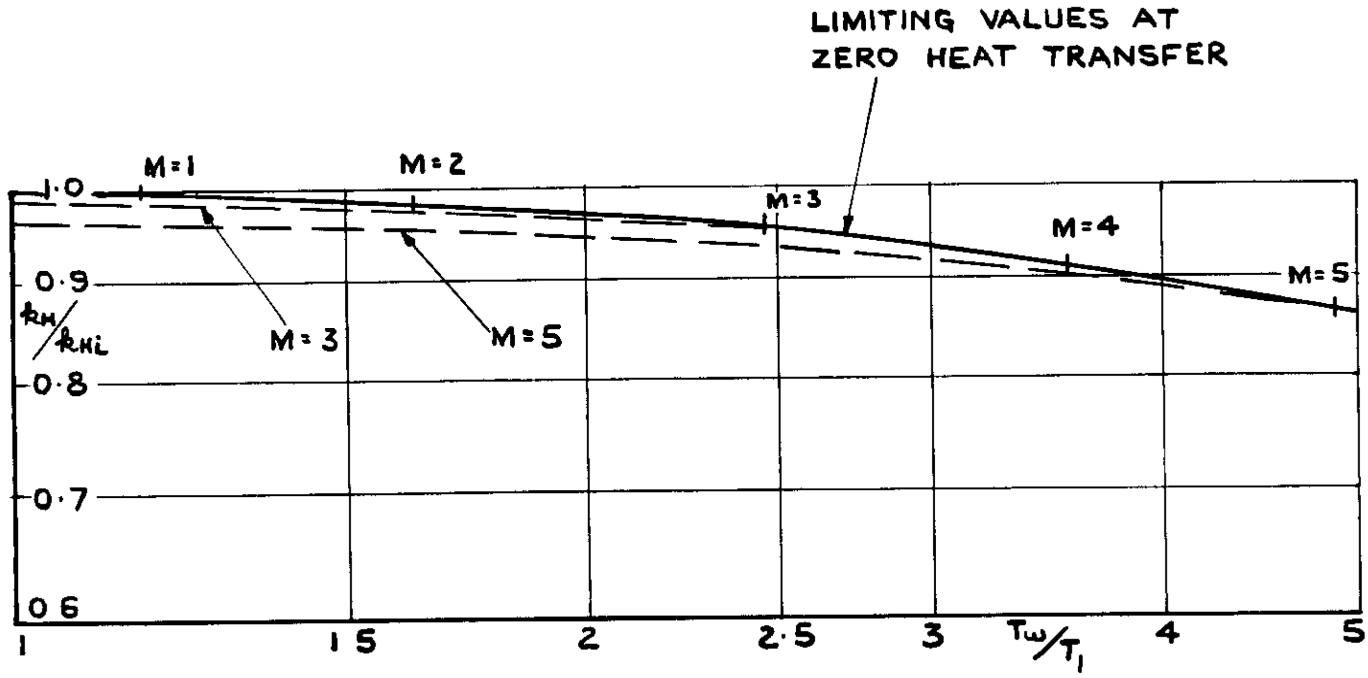


(a) RELATION BETWEEN ENTHALPY AND TEMPERATURE. (FROM REF: 8.)

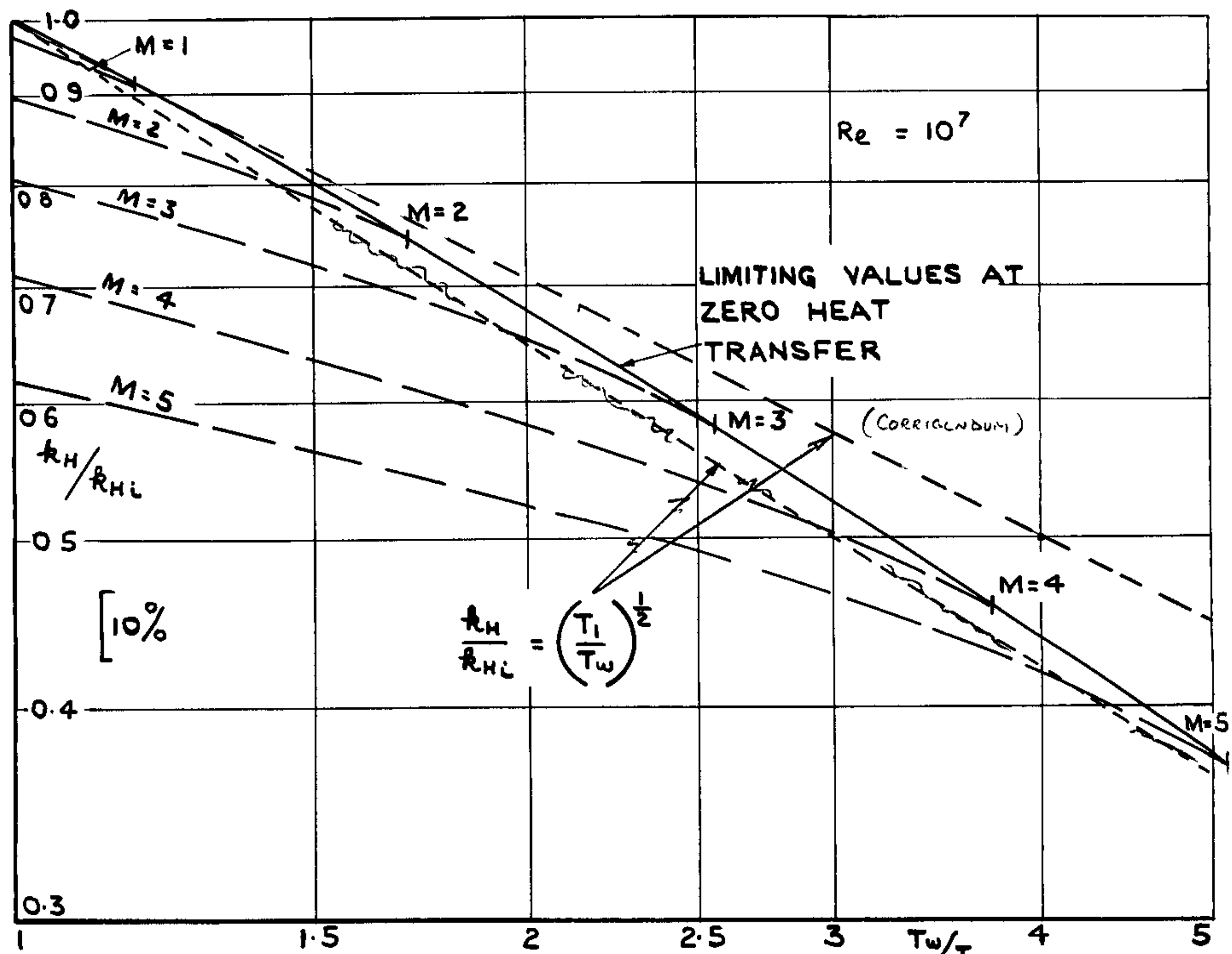


(b) RELATION BETWEEN PRANDTL NUMBER AND TEMPERATURE.

FIG. 2 (a&b) RELEVANT PHYSICAL PROPERTIES OF AIR.



(a) LAMINAR BOUNDARY LAYER. ALL Re, T_1 = 216.5°K



(b) TURBULENT BOUNDARY LAYER. T_1 = 216.5°K.

FIG 3(a & b) VARIATION OF LOCAL HEAT TRANSFER COEFFICIENT IN COMPRESSIBLE FLOW. (k_{Hi} = INCOMPRESSIBLE FLOW VALUE)

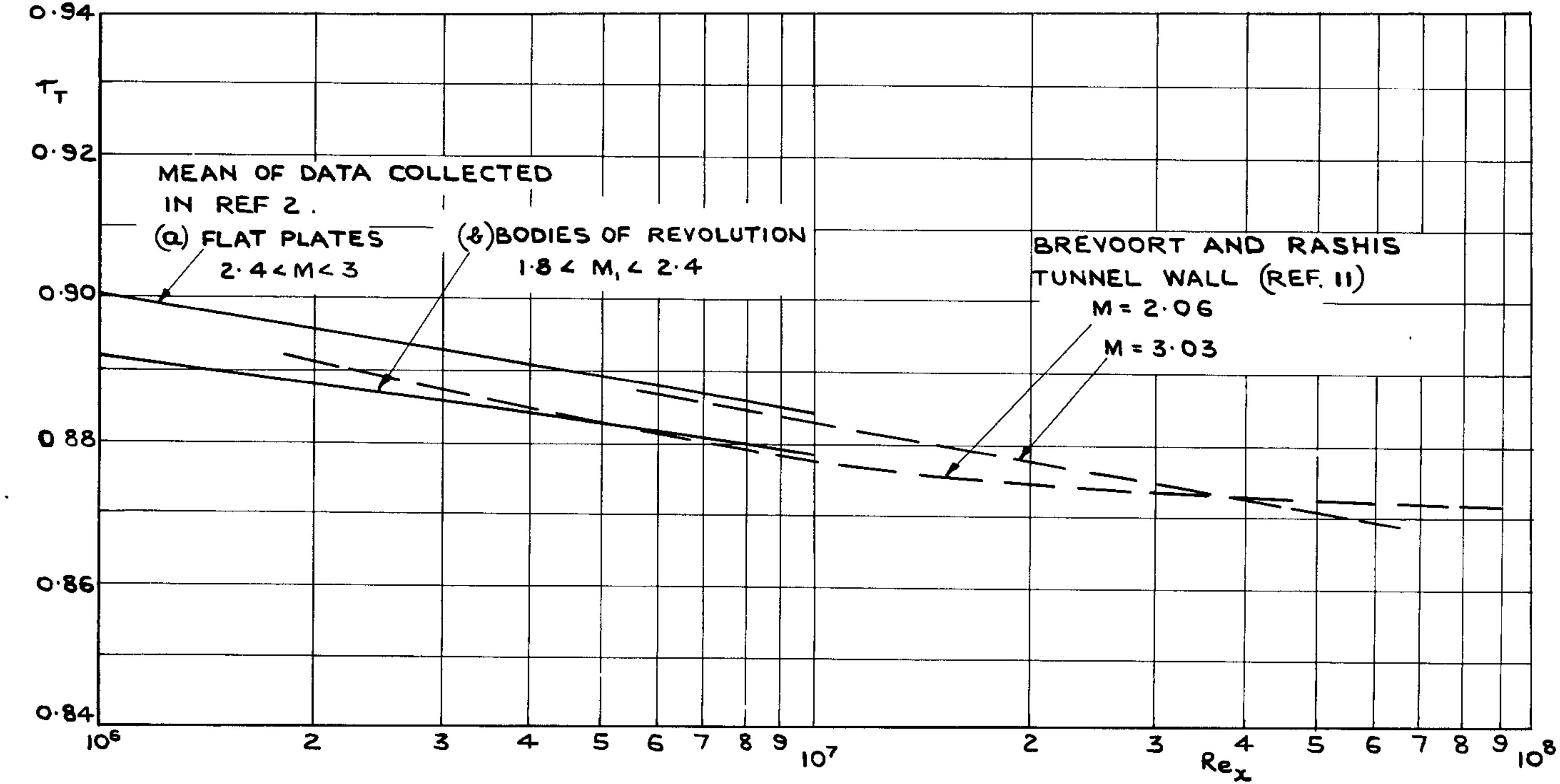


FIG. 4. MEAN VALUES OF TEMPERATURE RECOVERY FACTORS FOR TURBULENT BOUNDARY LAYERS. (WIND TUNNEL TEST RESULTS)

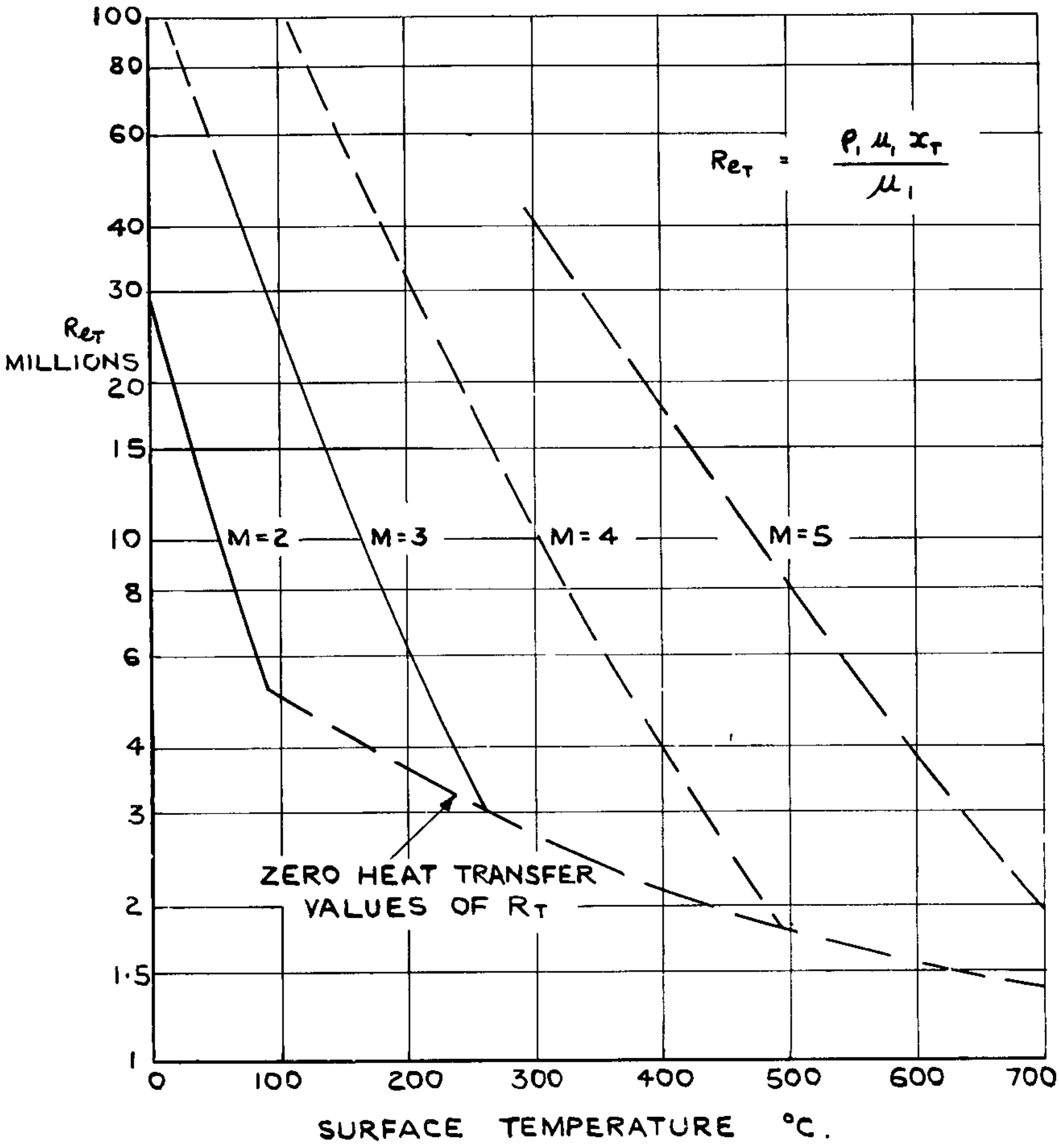


FIG 5. POSSIBLE TRANSITION REYNOLDS NUMBERS ON A SMOOTH SURFACE ZERO PRESSURE GRADIENT. (FLIGHT IN STRATOSPHERE.)

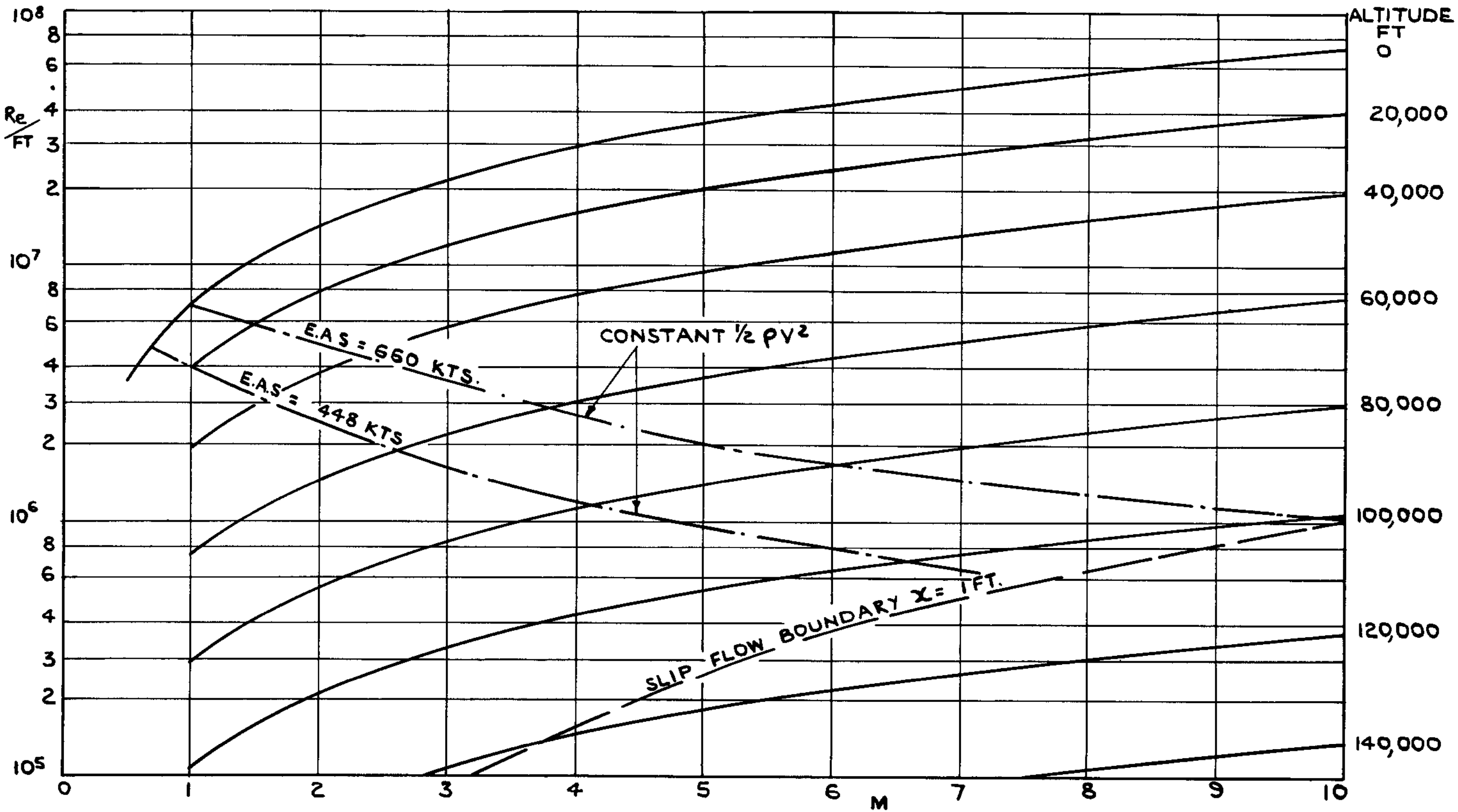
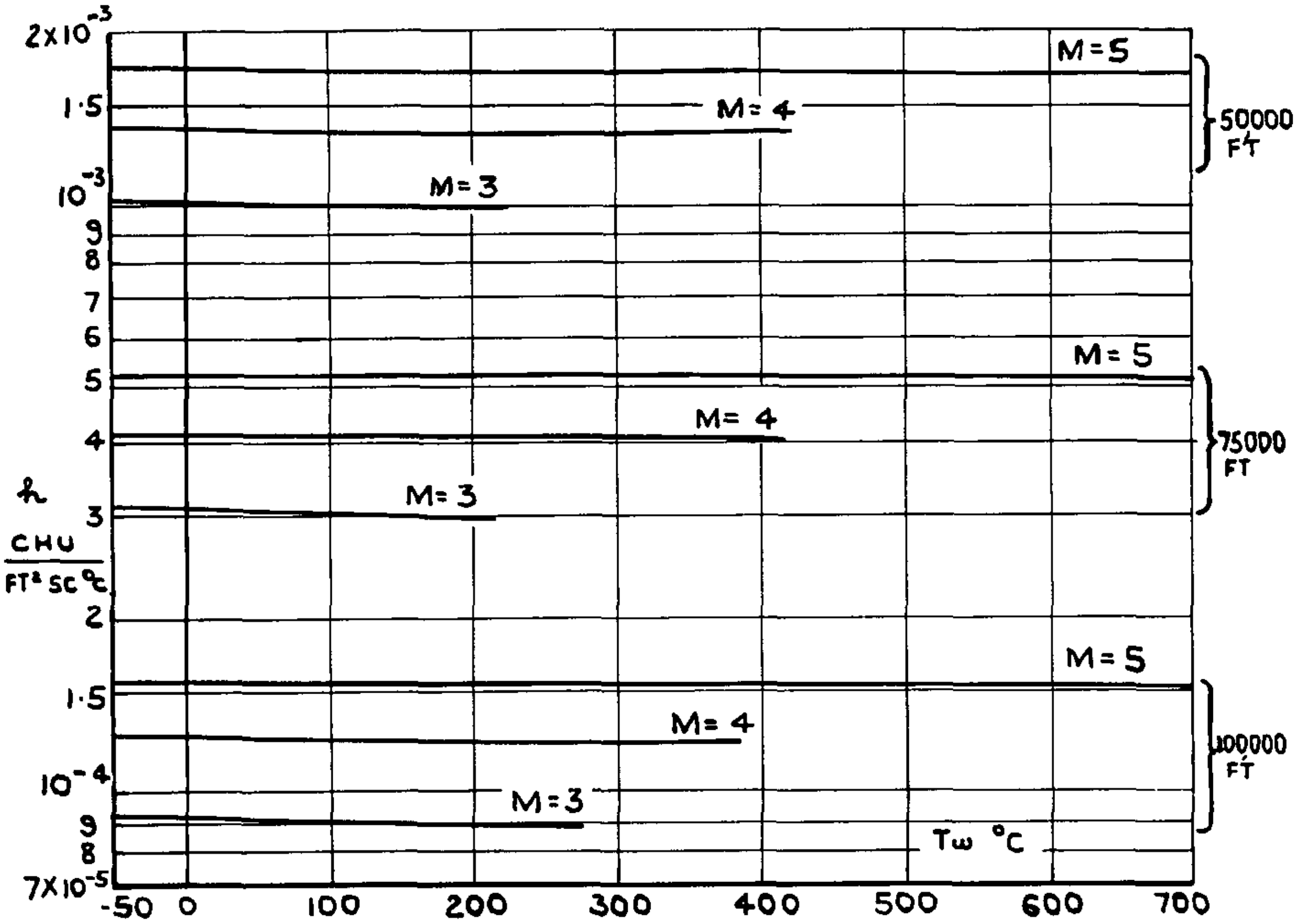
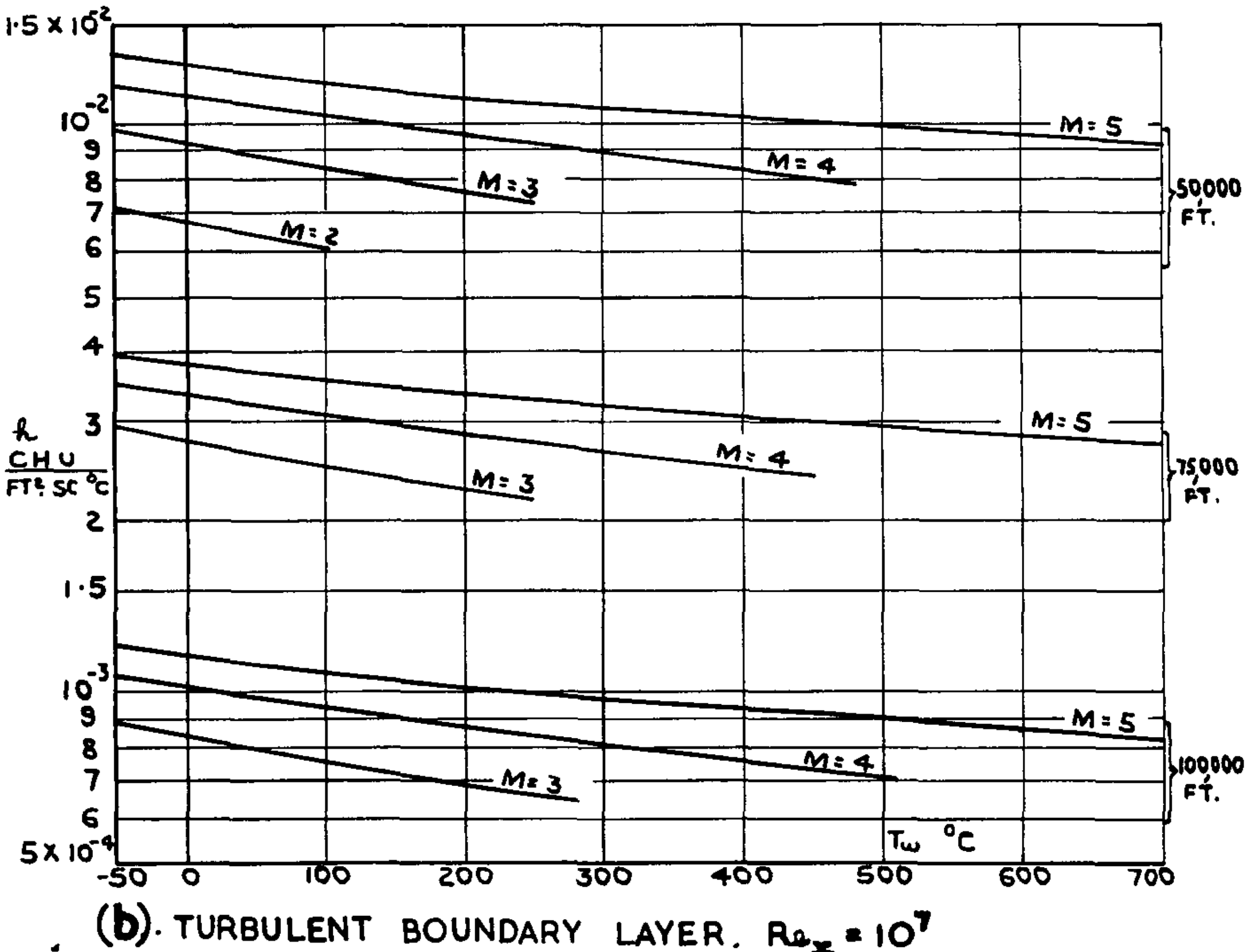


FIG 6. REYNOLDS NUMBERS PER FOOT OBTAINED IN FLIGHT AT VARIOUS ALTITUDES.



(a). LAMINAR BOUNDARY LAYER. $Re_x = 10^7$



(b). TURBULENT BOUNDARY LAYER. $Re_x = 10^7$

FIG 7(a & b) EFFECTS OF MACH. NUMBER, ALTITUDE & SURFACE TEMPERATURE ON LOCAL HEAT TRANSFER FACTOR h. (AT CONSTANT REYNOLDS NUMBER.)

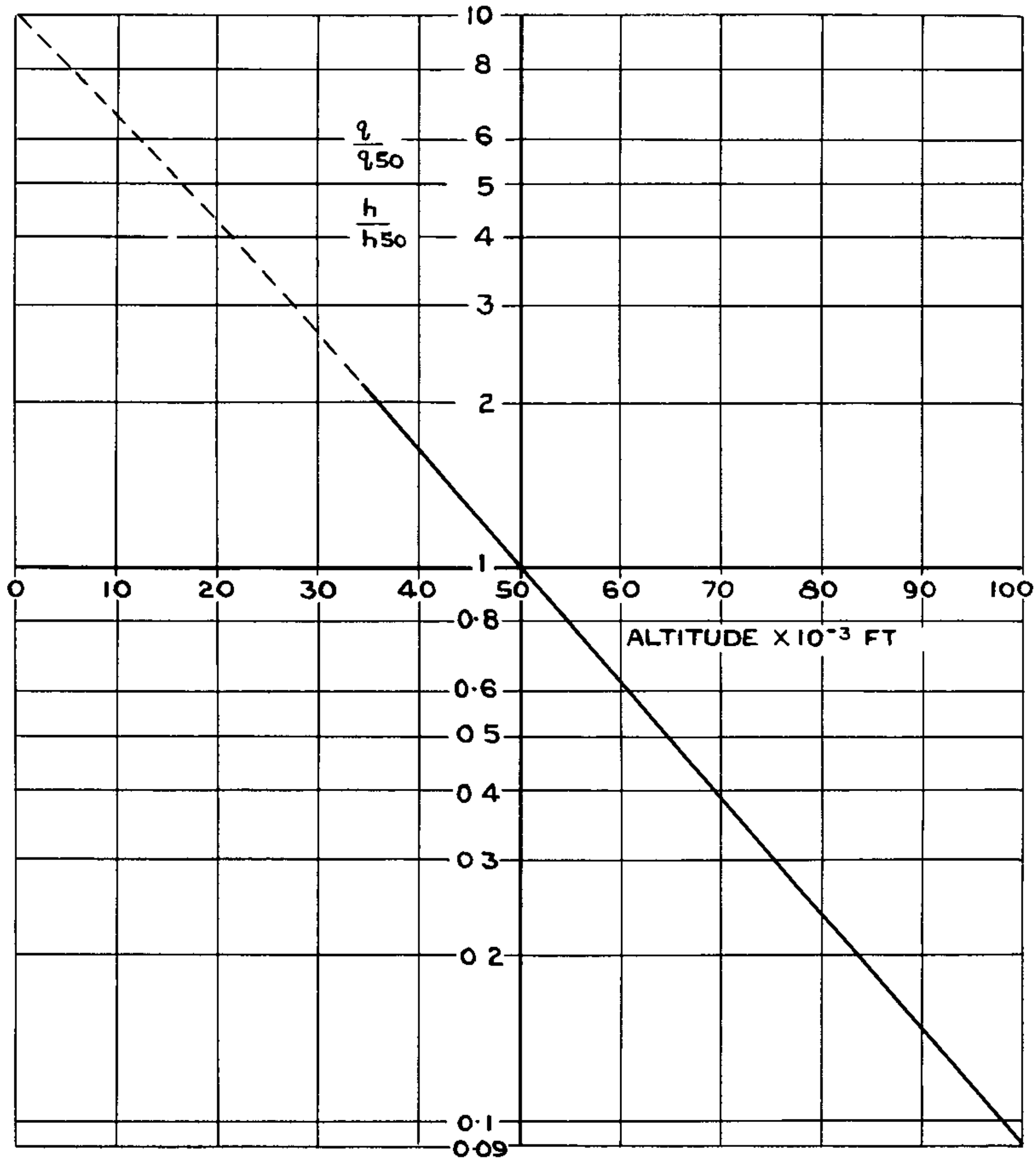


FIG. 8. EFFECT OF ALTITUDE ON AERODYNAMIC HEATING RATE FOR CONSTANT Re_x, M & T_w/T_1.

(RELATIVE TO VALUES AT 50,000 ft.)

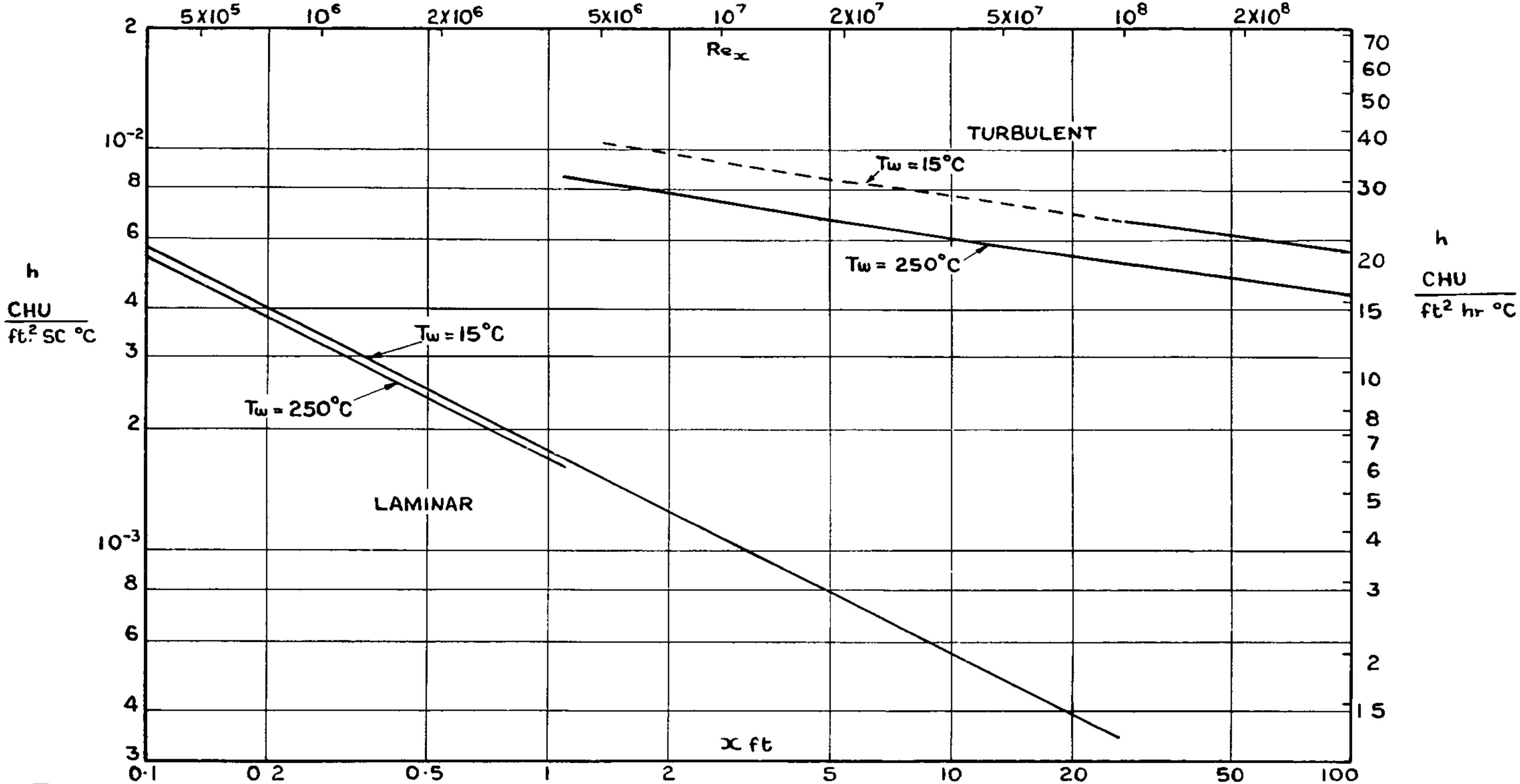


FIG. 9. VARIATION OF LOCAL HEAT TRANSFER FACTOR h WITH DISTANCE FROM NOSE OR LEADING EDGE, FOR M=3·1, ALTITUDE 50,000 ft. & UNIFORM SURFACE TEMPERATURE.
(HOFF'S CASE)

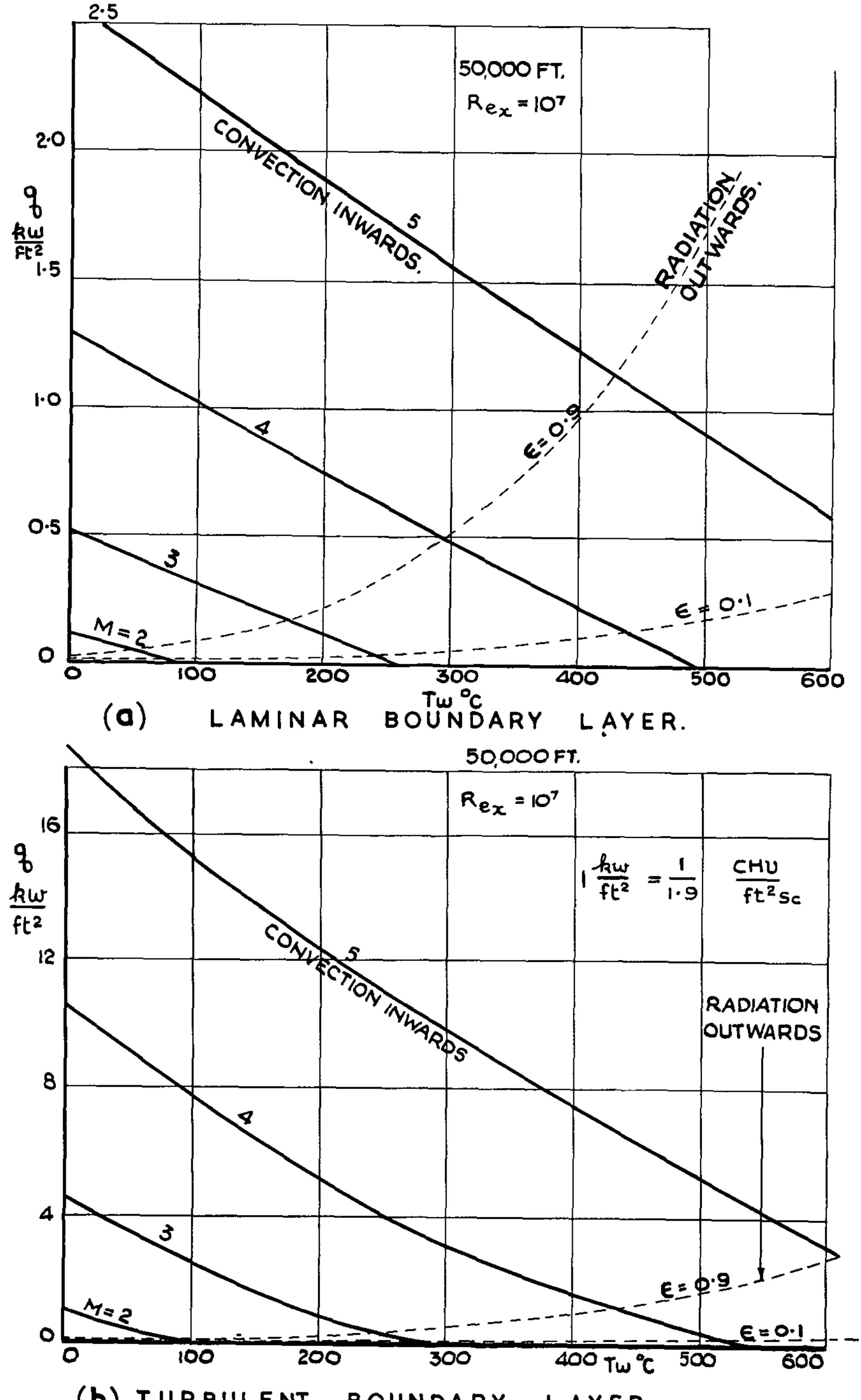


FIG.10(a & b) LOCAL HEAT TRANSFER RATES TO & FROM A FLAT PLATE UNDER FLIGHT CONDITIONS.

$R_{ex} = 10^7$ ALTITUDE 50,000 FT.

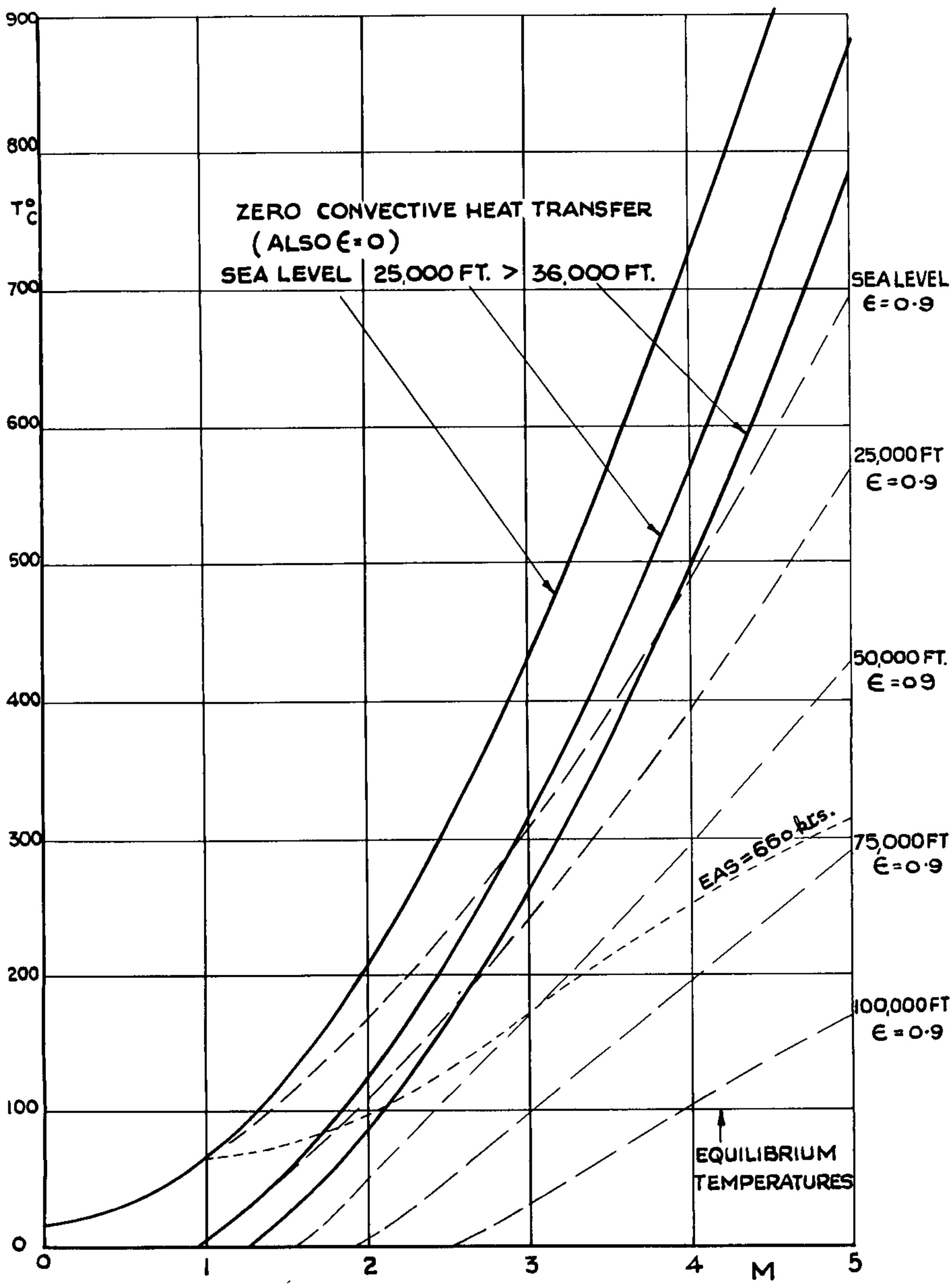


FIG 11(a) EQUILIBRIUM TEMPERATURES WITH A LAMINAR BOUNDARY LAYER.

LOCAL VALUES AT $R_{e_x} = 10^7$

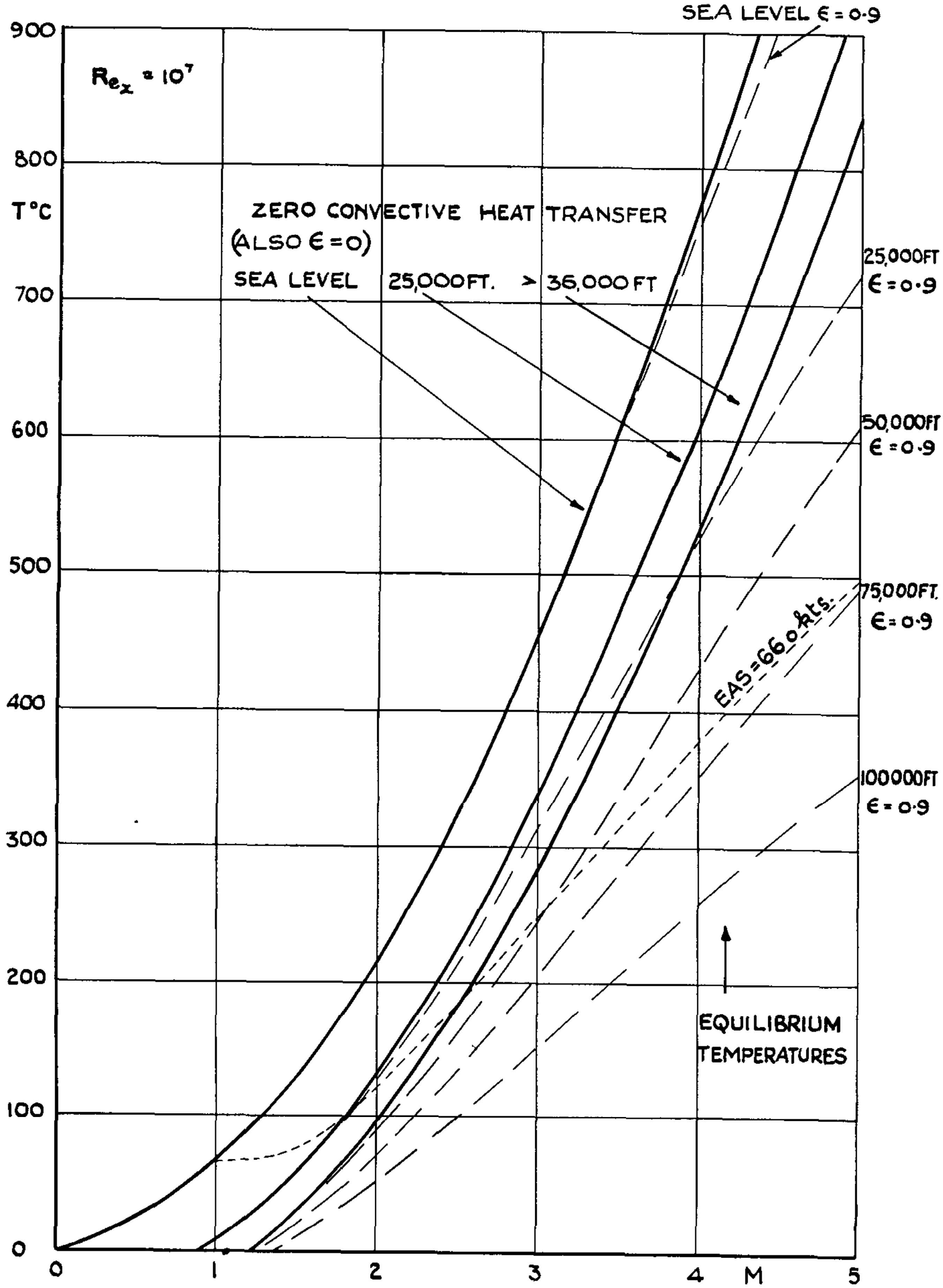
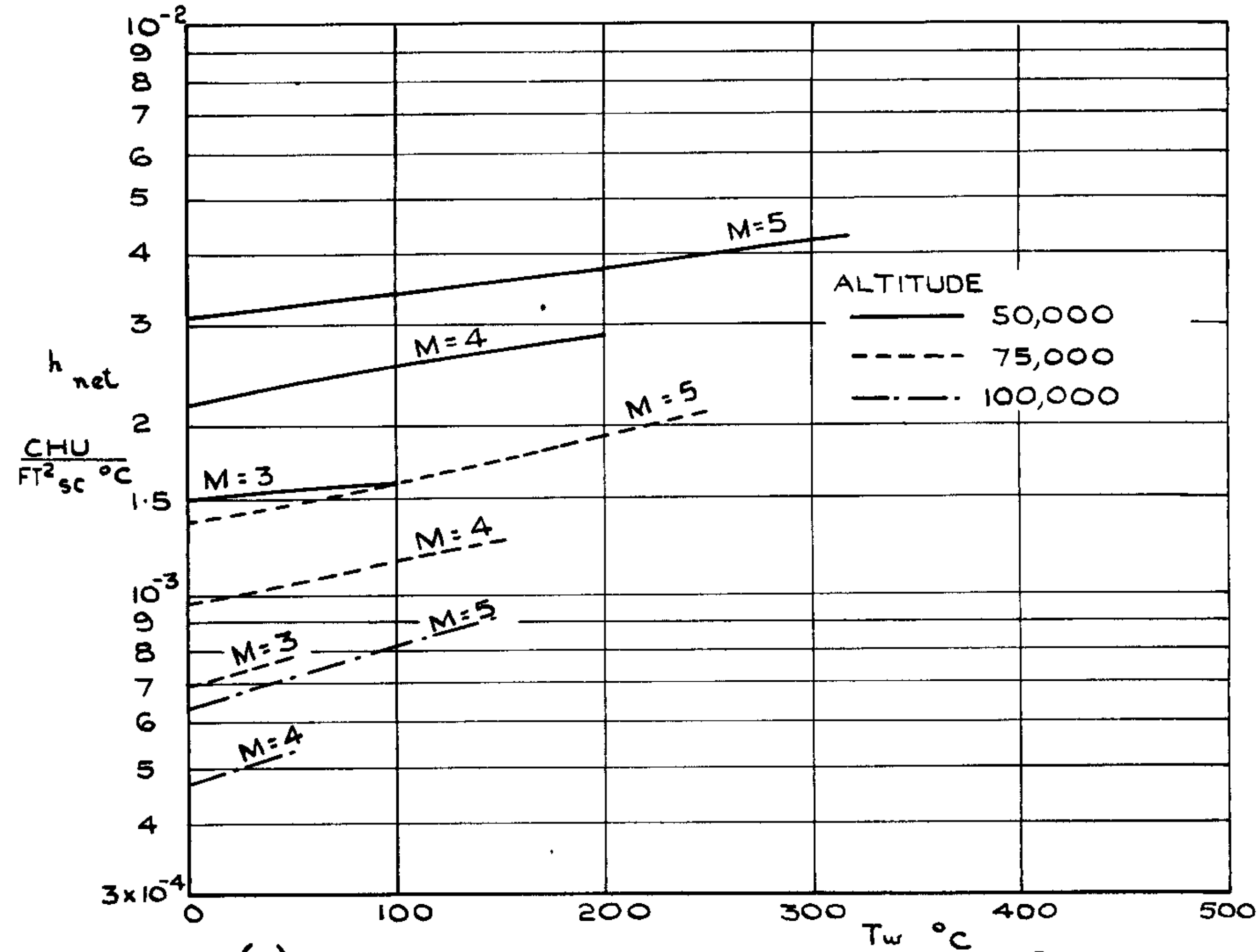
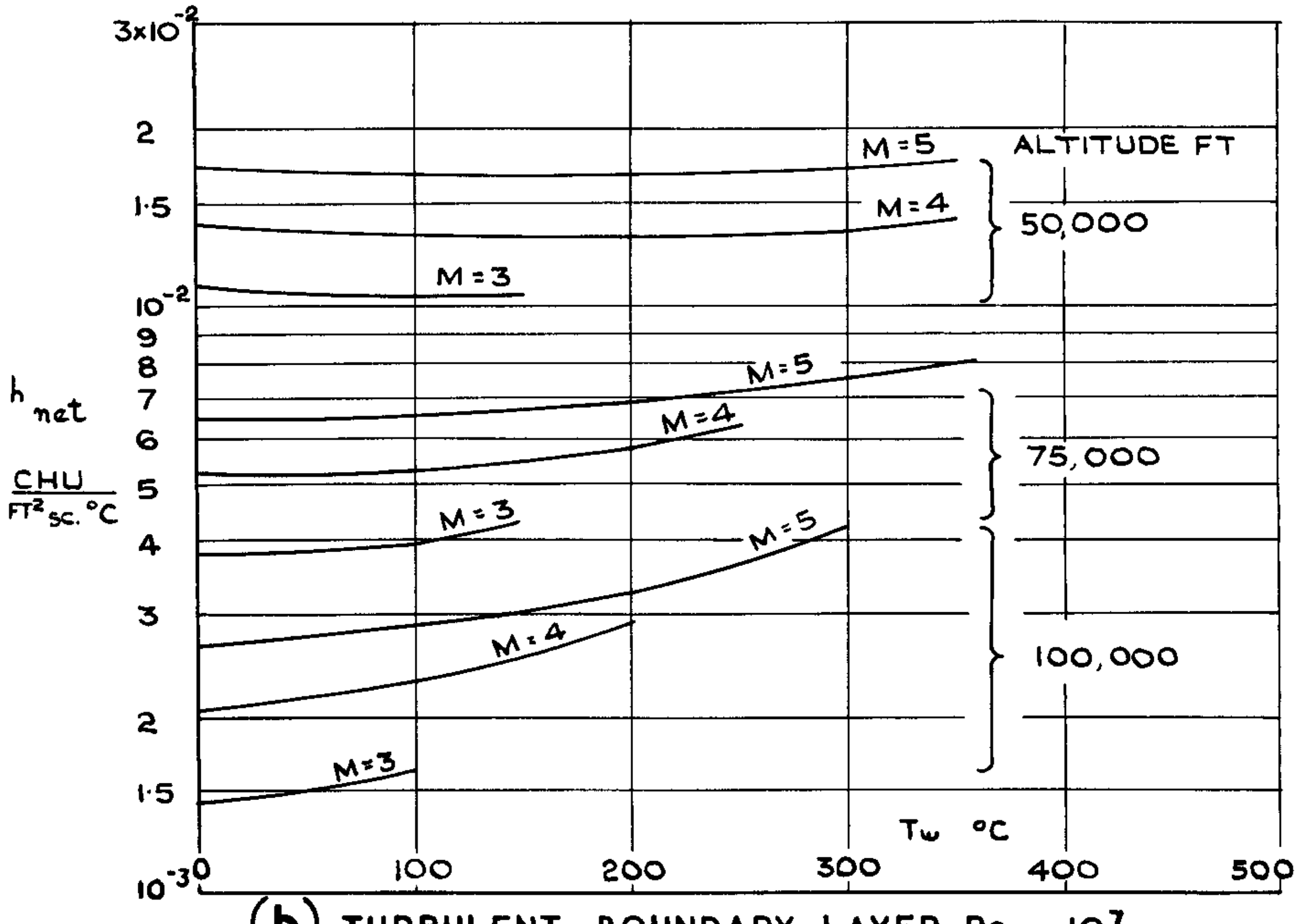


FIG.11(b). EQUILIBRIUM TEMPERATURES WITH A TURBULENT BOUNDARY LAYER.
LOCAL VALUES AT $R_{e_x} = 10^7$



(a) LAMINAR BOUNDARY LAYER $Re_x = 10^7$



(b) TURBULENT BOUNDARY LAYER $Re_x = 10^7$

FIG. 12. (a & b) VALUES OF NET HEAT TRANSFER FACTOR h_{net}

(CONVECTION – RADIATION) FOR $\varepsilon = 0{\cdot}9$

$$h_{net} = \frac{q_{net}}{T_{we} - T_w}$$

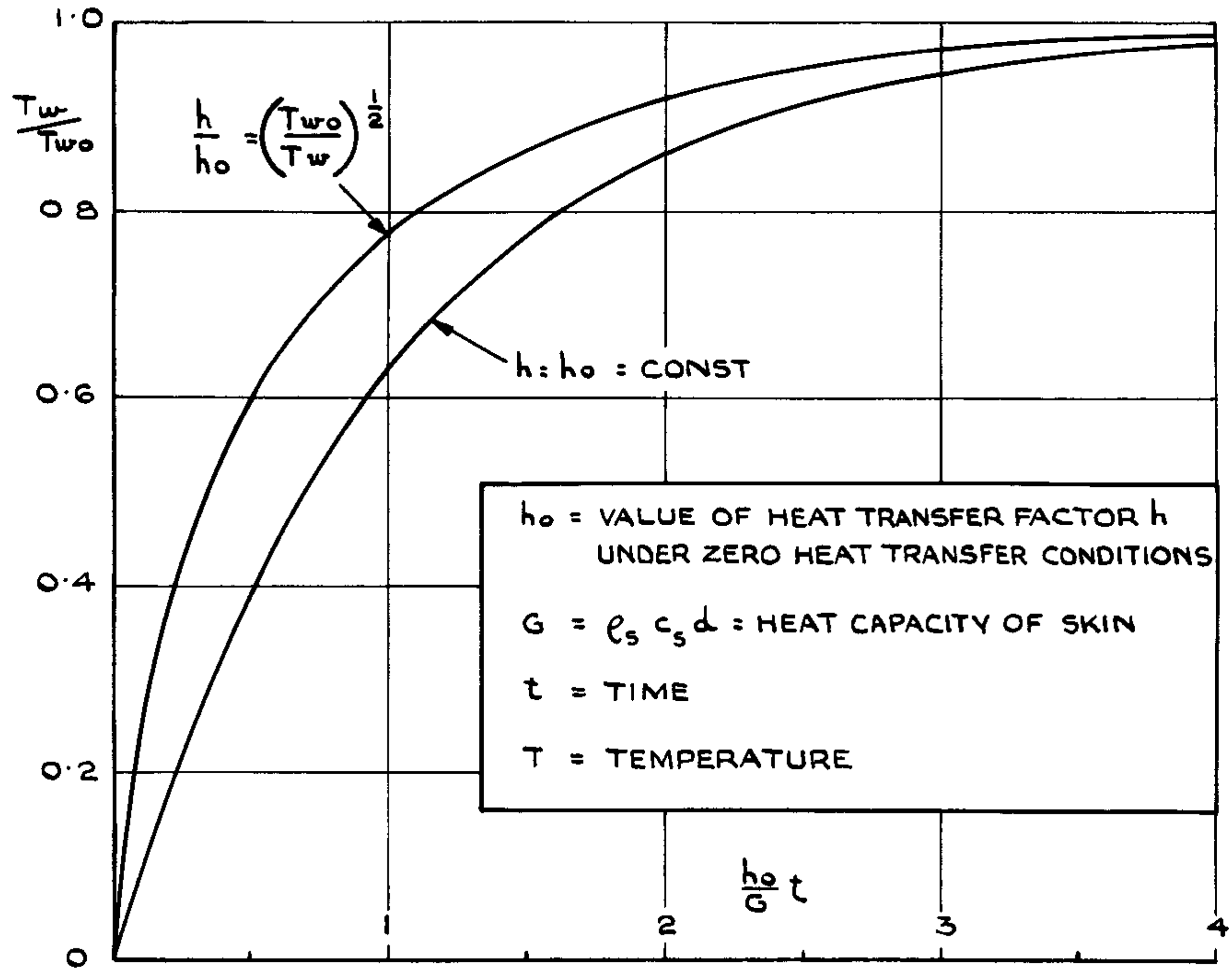


FIG. 13. AERODYNAMIC HEATING OF THIN SKINS.
EFFECT OF VARIATION OF HEAT TRANSFER COEFFICIENT.

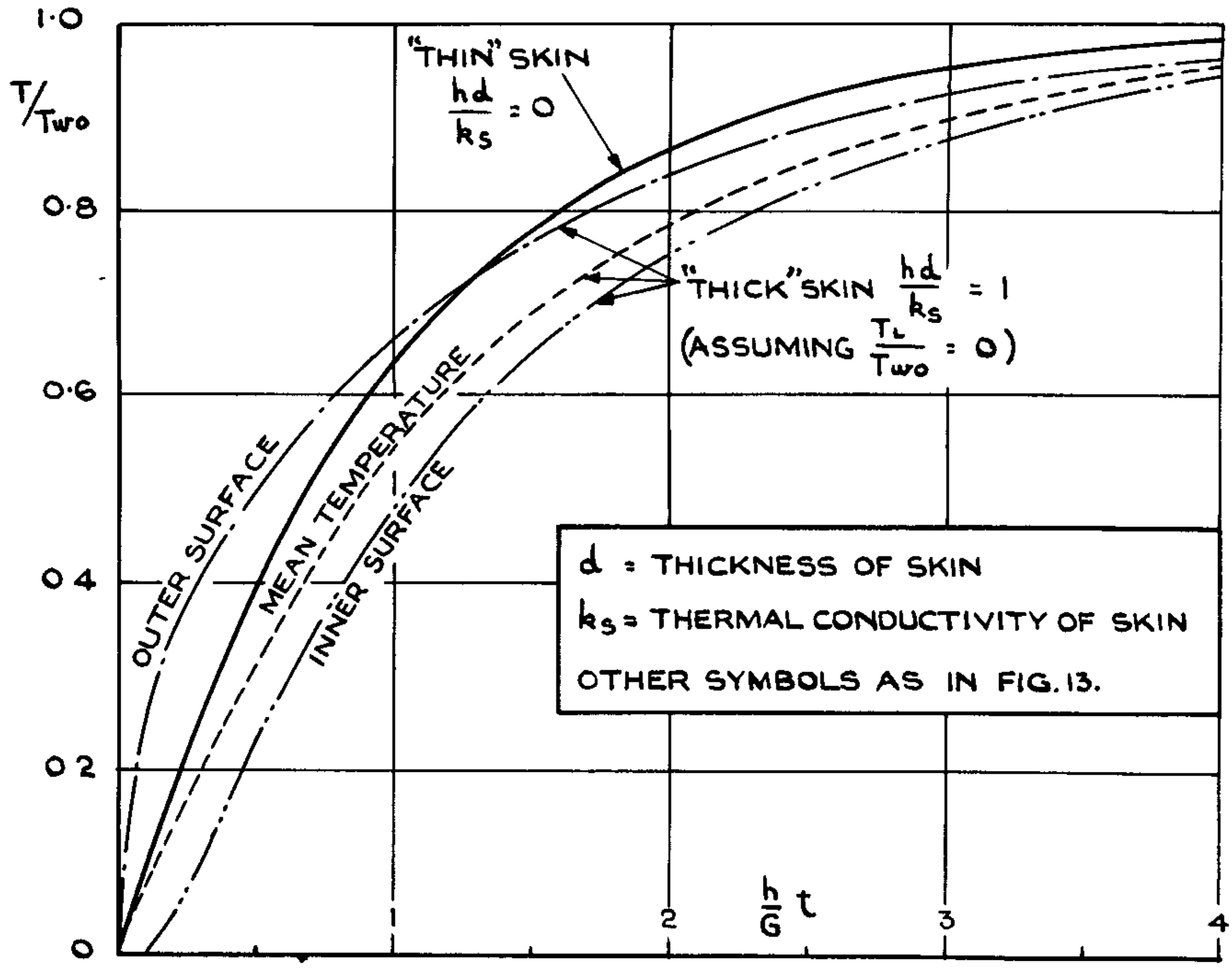


FIG. 14. COMPARISON OF HEATING RATES OF "THIN" AND OF "THICK" SKINS.
(ASSUMING h = CONSTANT)

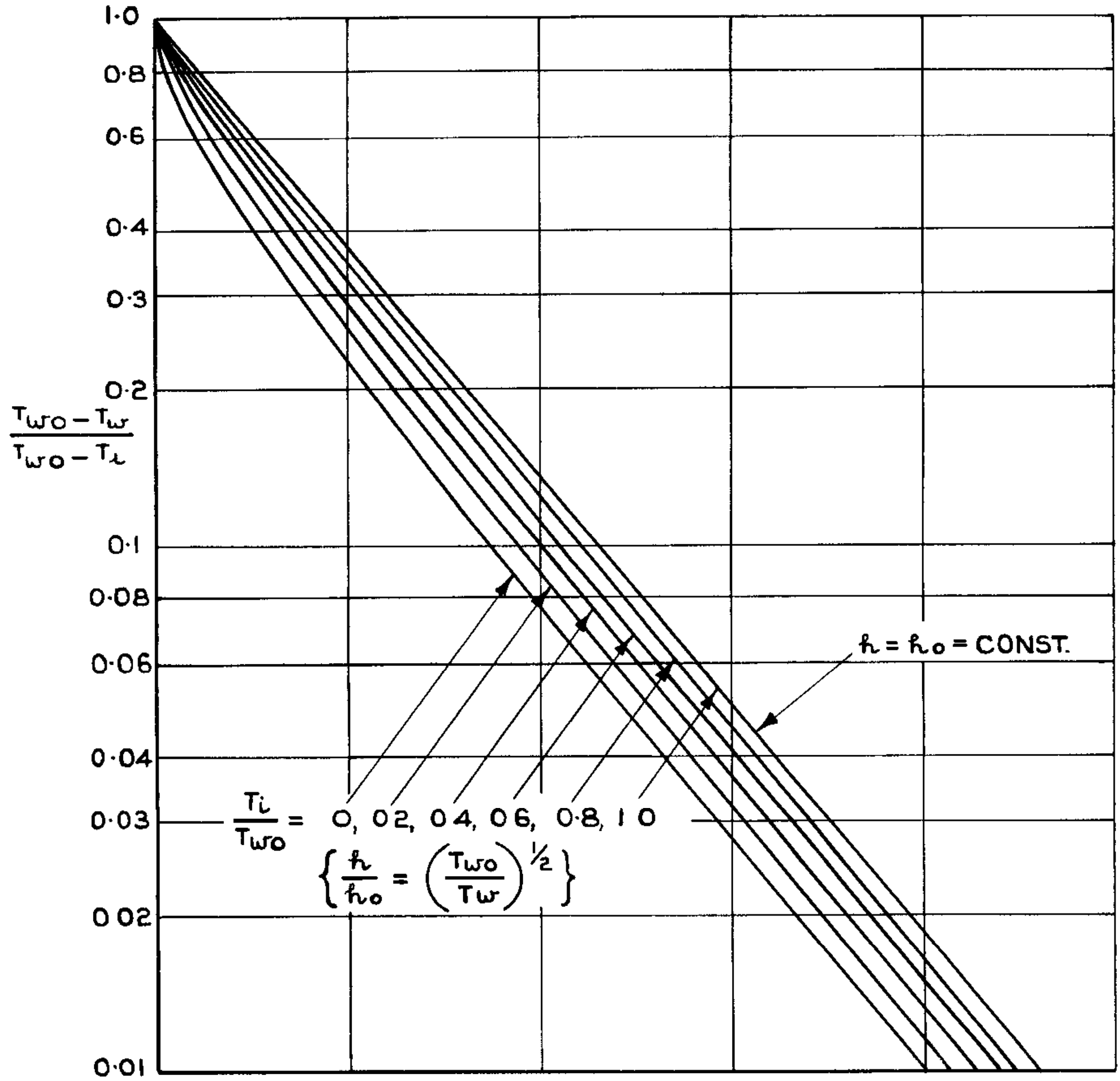
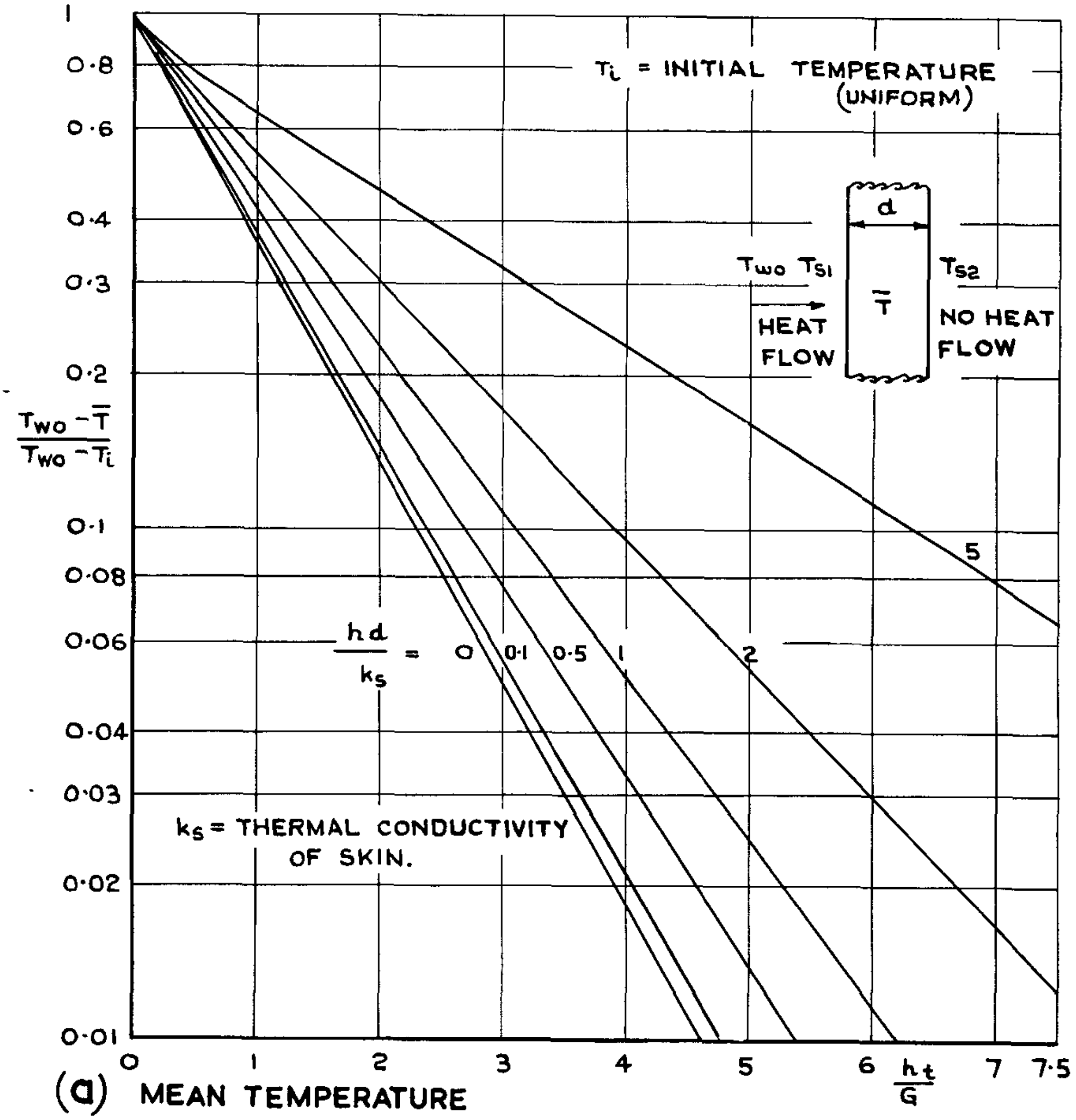


FIG. 15. AERODYNAMIC HEATING OF THIN SKINS. EFFECT OF VARIATION OF HEAT TRANSFER COEFFICIENT.

(ALTERNATIVE PLOT TO FIG. 13.)



(a) MEAN TEMPERATURE

FIG. 16 (a) DETAILED COMPARISON OF HEATING RATES THROUGH A SKIN FOR h = CONST. AND INNER SURFACE INSULATED.

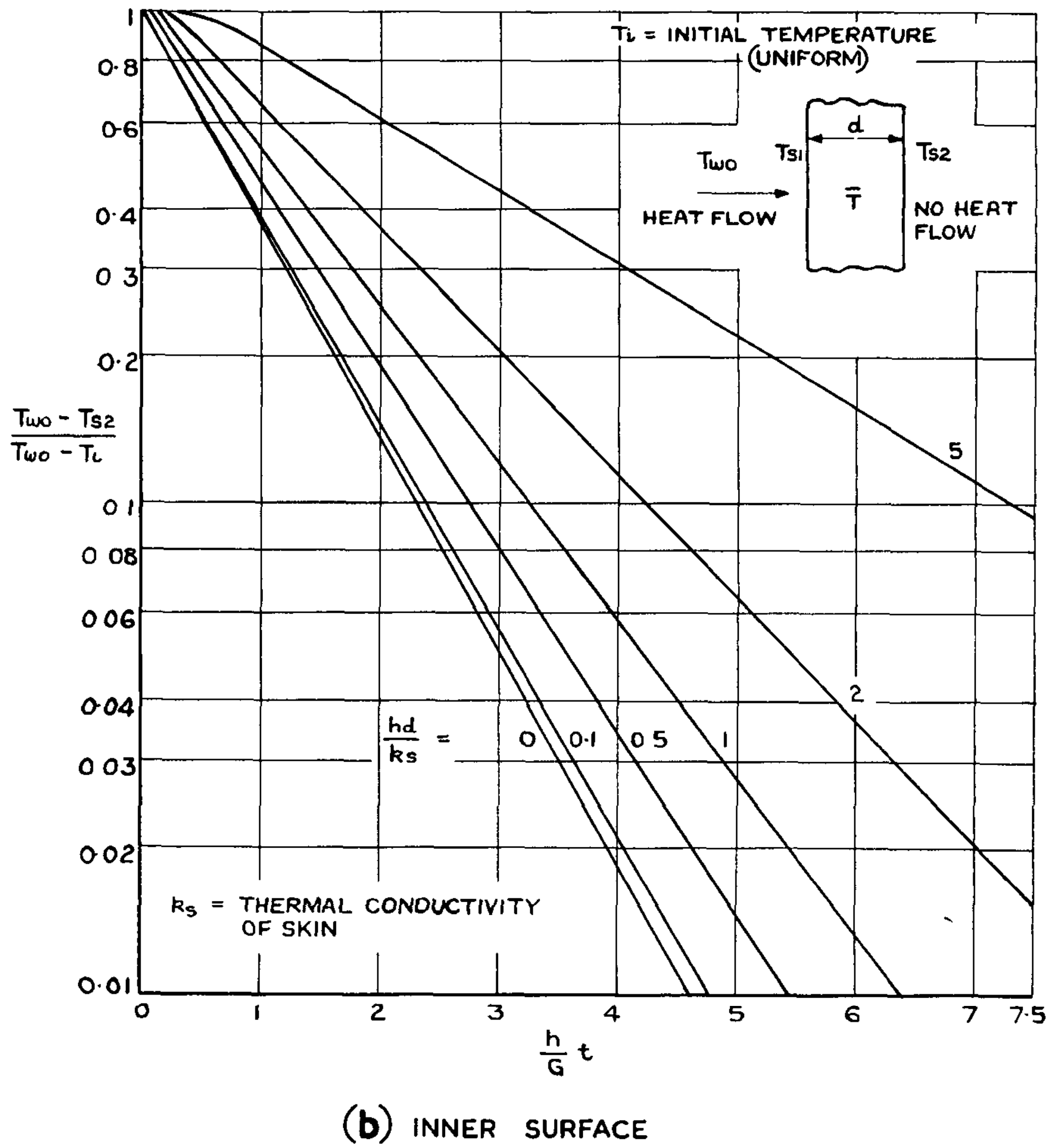


FIG. 16 (b) DETAILED COMPARISON OF HEATING RATIO THROUGH A SKIN FOR h = CONST. & INNER SURFACE INSULATED.

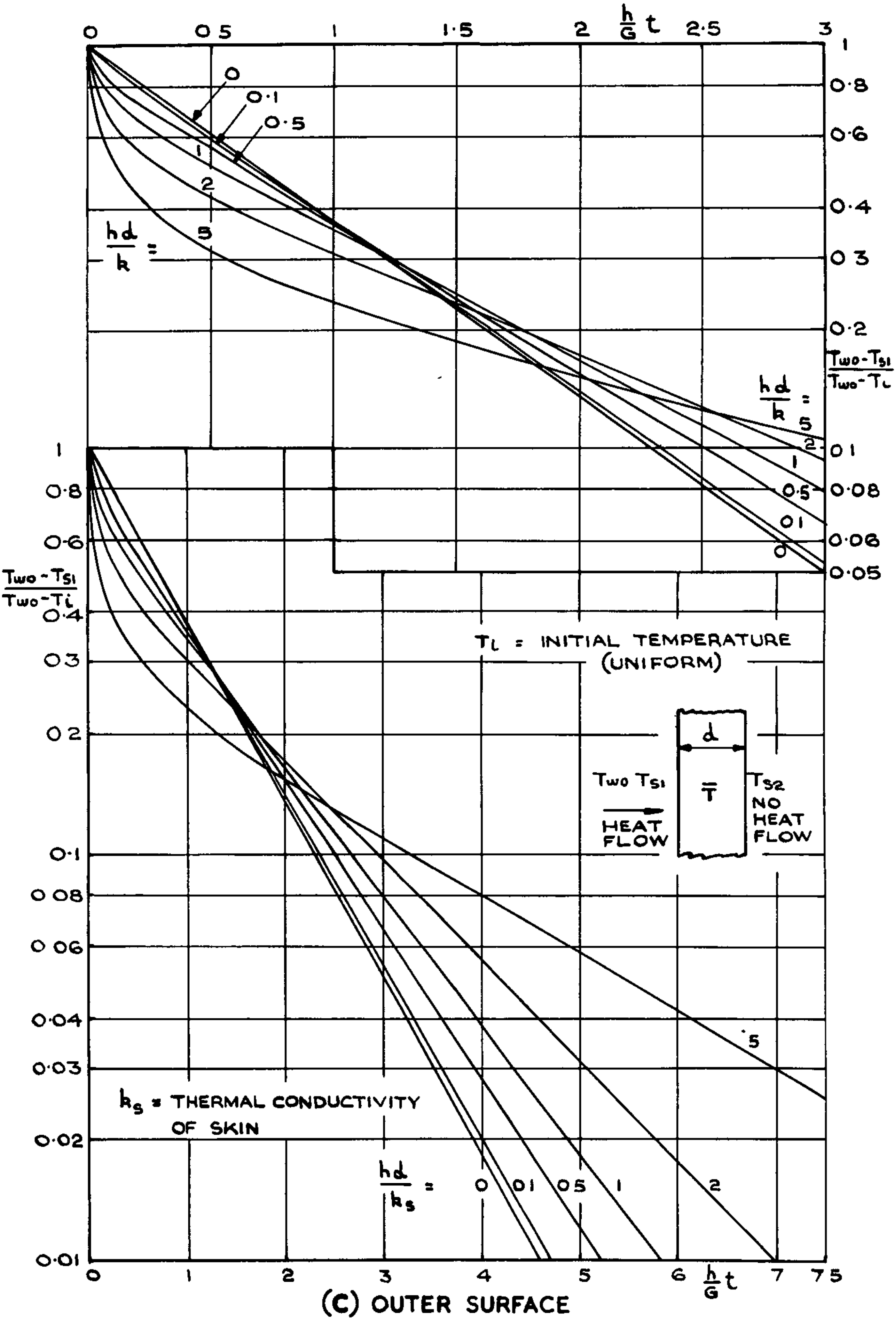
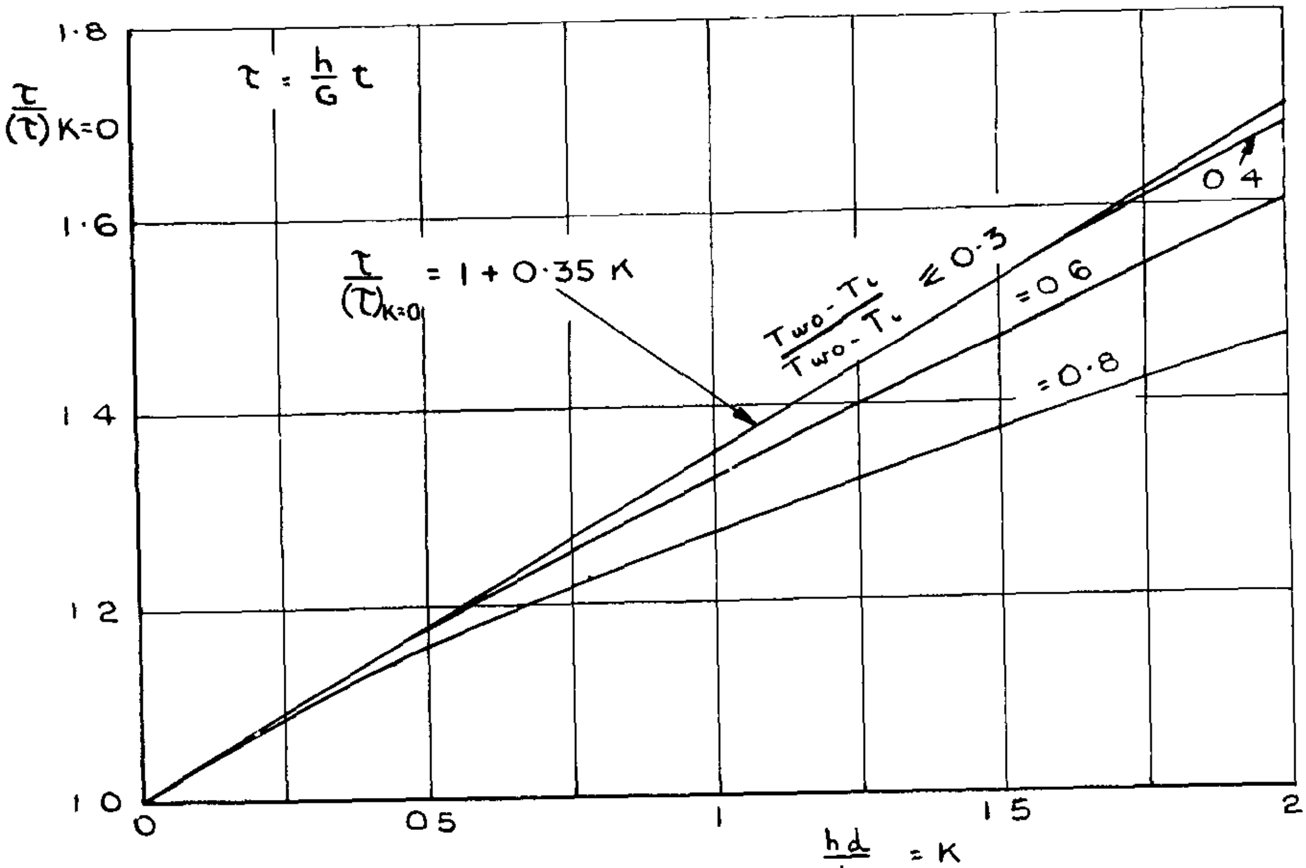
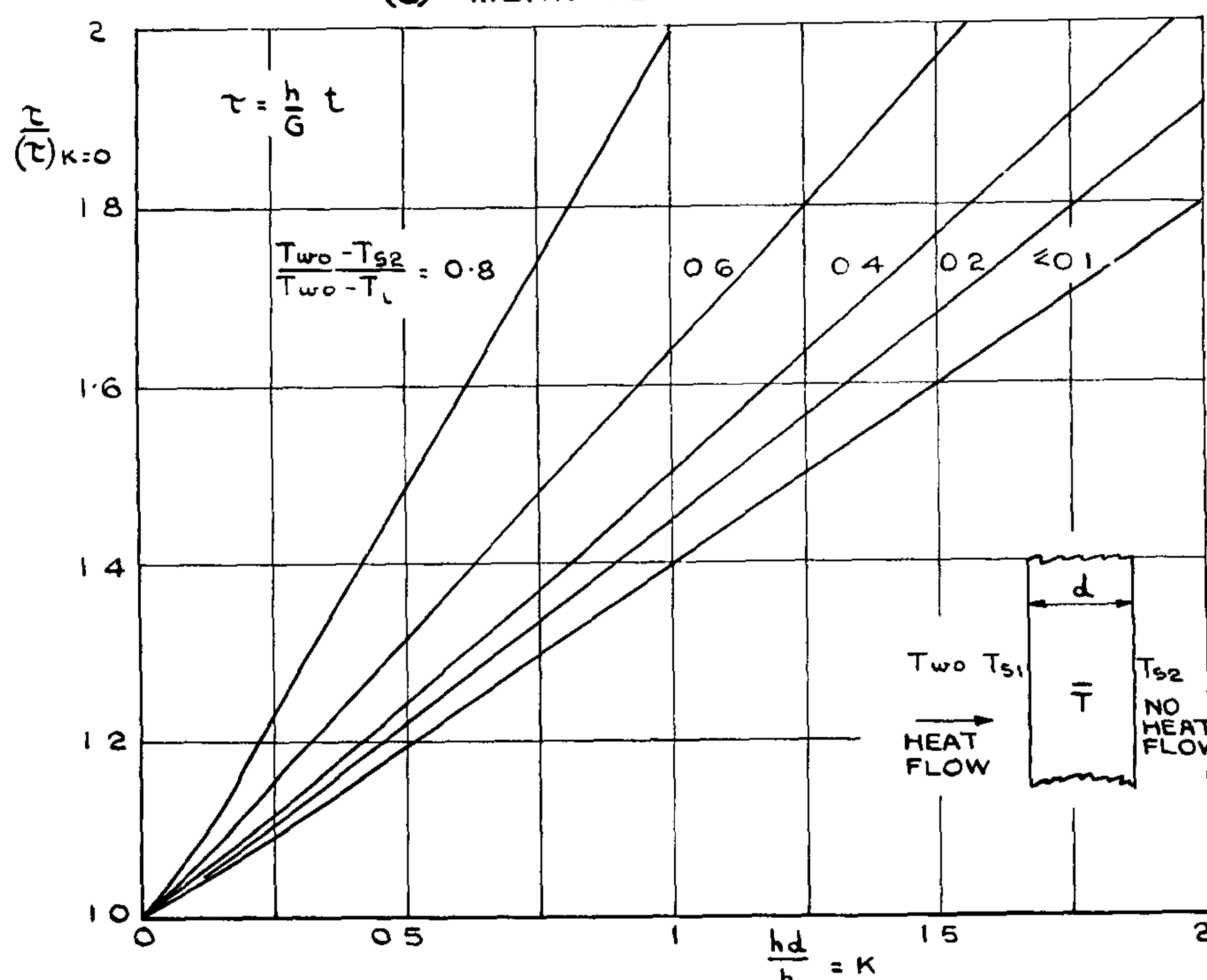


FIG. 16.(c) DETAILED COMPARISON OF HEATING RATES OF "THIN" & "THICK" SKINS.

(h = CONSTANT INNER SURFACE INSULATED)



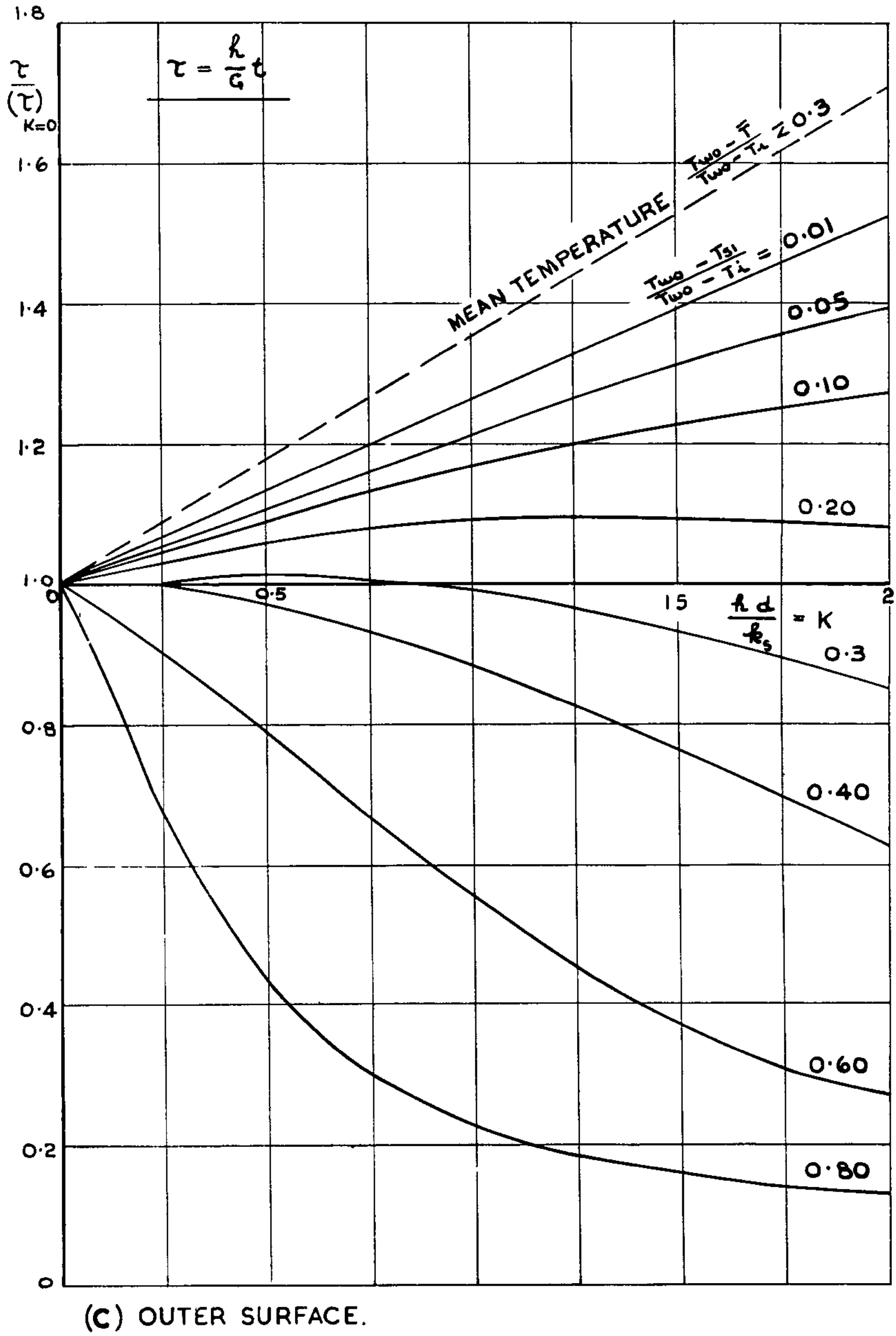
(a) MEAN TEMPERATURE



(b) INNER SURFACE.

FIG. 17. (a & b) EFFECT OF SKIN THICKNESS AND CONDUCTIVITY ON TIME TO REACH GIVEN TEMPERATURES.

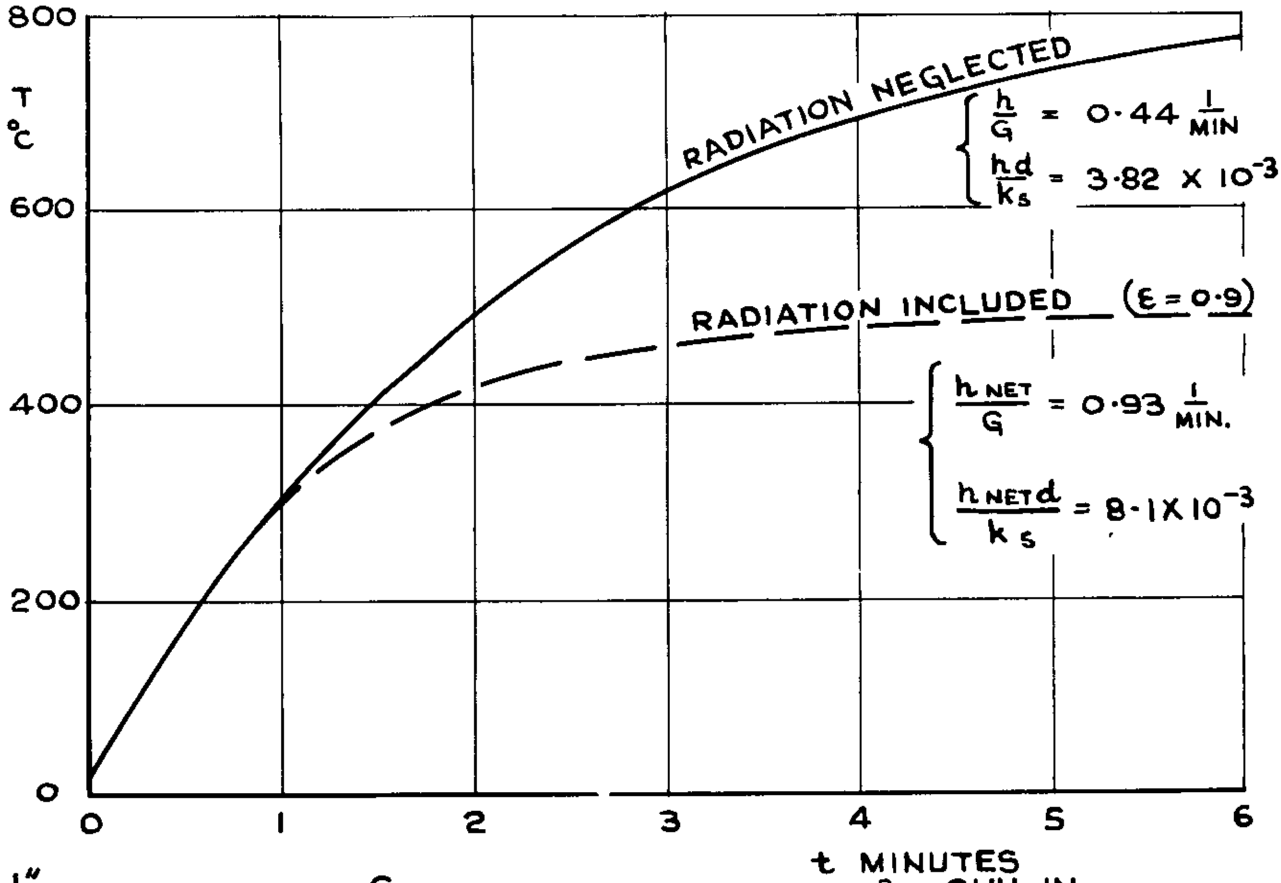
(h=CONST INNER SURFACE INSULATED FROM INTERIOR)



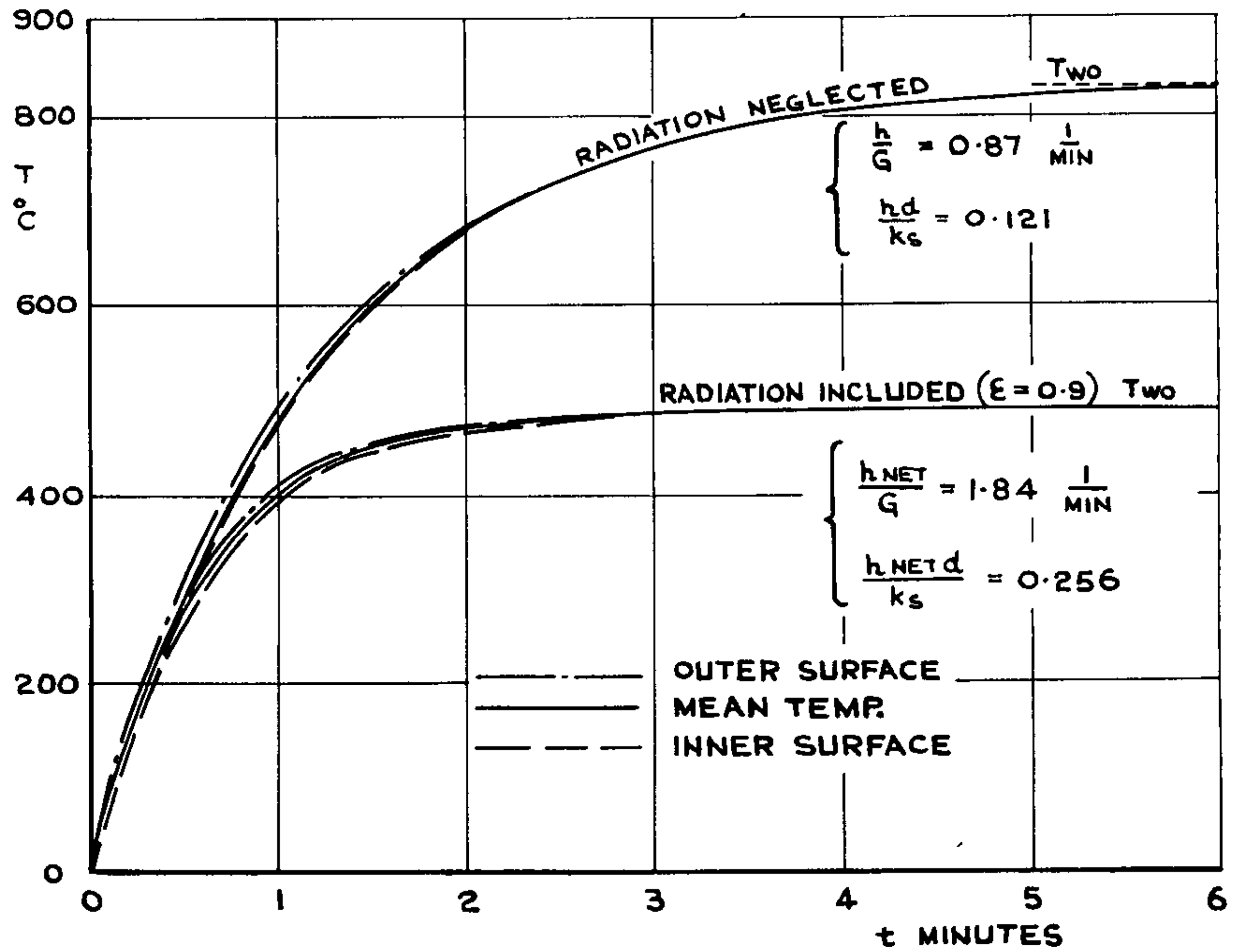
(C) OUTER SURFACE.

FIG: 17(c). EFFECT OF SKIN THICKNESS AND CONDUCTIVITY ON TIME TO REACH GIVEN TEMPERATURES.

(h = CONST. INNER SURFACE INSULATED FROM INTERIOR)



(a) $\frac{1''}{10}$ MILD STEEL. $\frac{G}{IN} = 4{\cdot}5$ $k_s = 8{\cdot}65 \times 10^{-2} \frac{\text{CHU IN}}{\text{FT}^2\ \text{SC.}\ ^\circ\text{C}}$



(b) $\frac{1''}{10}$ FUSED SILICA $\frac{G}{IN} = 2{\cdot}28$ $k_s = 2{\cdot}74 \times 10^{-3} \frac{\text{CHU IN}}{\text{FT}^2\ \text{SC.}\ ^\circ\text{C}}$

FIG.18 (a & b) EFFECTS OF HEAT CAPACITY AND RADIATION ON LOCAL HEATING RATES.

$M = 5$, ALTITUDE = 75,000 FT., $Re_x = 10^7$

TURBULENT BOUNDARY LAYER.

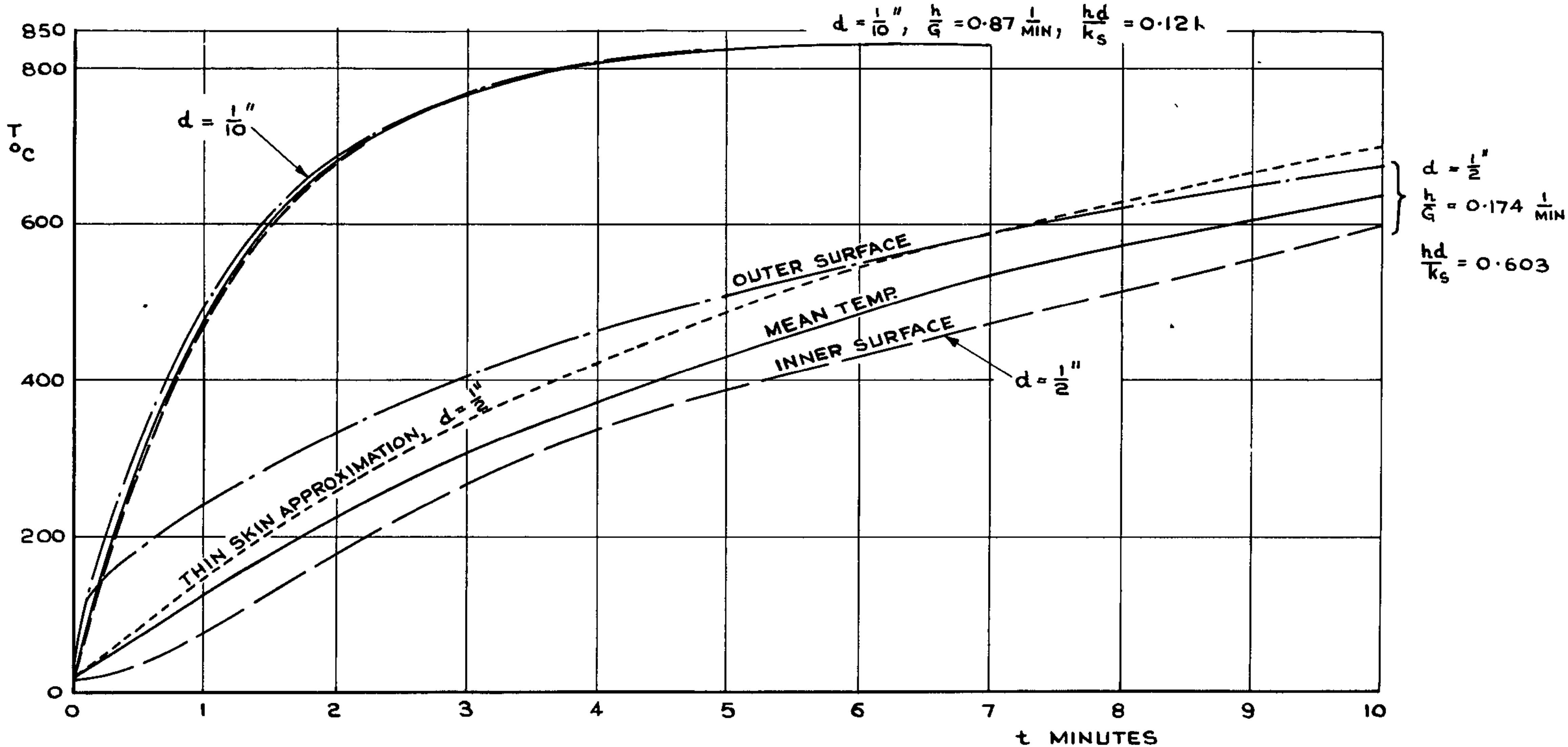


FIG. 19. EFFECTS OF SKIN THICKNESS ON LOCAL HEATING RATES OF FUSED SILICA.

M = 5, ALTITUDE = 75,000 FT., $Re_x = 10^7$

TURBULENT BOUNDARY LAYER.

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