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Summary.—The effects of shock-induced separation of turbulent boundary layers on two-dimensional aerofoils are introduced by considering the development of the surface-pressure distribution and flow pattern as the free-stream speed is increased in the transonic range (defined as that for which regions of supersonic flow exist on the aerofoils but are limited in chordwise extent). The progressive rearward extension of the supersonic flow, as the terminating shocks move rearwards over the surfaces, is an essential feature of this development.

Unless the incidence and thickness of an aerofoil at lift are both very small, the upper-surface shock, at some stage in its movement, induces a boundary-layer separation which tends to reduce the pressure rise through the shock. The consequences of this are usually not serious until the shock fails to re-establish subsonic flow immediately downstream. At that stage, however, the 'bubble' of separated flow begins to expand rapidly towards the trailing edge and beyond, and in so doing to exert a dominating influence on the development of the overall flow, *i.e.*, on the actual and relative rates of shock movement, or flow development, on the two surfaces. This influence wanes as soon as either the lower surface shock reaches the trailing edge or a centred supersonic expansion occurs there; the bubble finally collapses when the upper-surface shock moves on to the trailing edge.

The physical nature of the overall flow and the mechanism by which separation affects its development so strongly are described qualitatively. The picture presented has been made as complete as possible, even though this involves some ideas which must be regarded as speculative, in the hope that it might form a tentative basis for more rigorous treatments or for extension of the work to swept-back and finite wings.

Considerations of the flow at the trailing edge of the aerofoil and downstream along the wake figure prominently in the description, and the pressure at the trailing-edge position is used extensively.

Separation at the shock on the upper surface is first shown to cause a slowing up of the rearward movement of this shock with respect to the variation of trailing-edge pressure. The concept is then introduced that the steady flow for each free-stream Mach number has to be such that the trailing-edge pressure can satisfy two conditions, namely, (i) an approximate equality between the pressures on the two sides of the wake, and (ii) a compatibility with the free-stream static pressure, in the sense that the difference between the two pressures has to be the change that can be accommodated along the viscous flow downstream of the trailing edge. It is suggested that the former condition largely determines the relative rates of shock movement (or flow development) on the two surfaces, and the latter the actual rates with respect to free-stream Mach number. The effects of separation are then seen as (i) a slowing up of the flow development on the upper surface relative to that on the lower, and (ii) a rapid fall in trailing-edge pressure which leads to an acceleration of the actual development on the lower surface and compensates partially for the slowing-up on the upper surface that is noted when trailing-edge pressure is taken as variable.

The repercussions which these effects have on the variation of forces and moments are discussed briefly. The development and effects of separation for increasing incidence at constant Mach number are shown to be similar to those described for increasing Mach number at fixed incidence, and to include a reversal of the movement of the upper-surface shock from rearwards to forwards. Certain practical applications of the work and possible extensions of them are mentioned.

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1. *Introduction*.—Experiments on two-dimensional aerofoils at transonic speeds* produced some of the earliest evidence[1,2] of interaction between shock waves and boundary layers, and the induced boundary-layer separations have for long been known to be significant for such cases. The basic understanding of the effect of these phenomena on the overall flow has, however, developed continuously, a process which has been stimulated recently by the knowledge gained from systematic investigations such as those described in Ref. 3 of the interaction between shock waves and the boundary layers on flat plates. This work has demonstrated that the separation is of overriding importance and has provided a physical explanation of its effects for a flat plate. In addition to other results, it has established the conditions of shock strength, Mach number and Reynolds number under which separation can be expected to occur, for both laminar and turbulent layers, and has illustrated the factors affecting the rise of pressure along the separated layer.

Considerable differences arise, depending on whether the boundary layer is laminar or turbulent, and, especially when the layer is laminar, on the value of the Reynolds number. In order, therefore, to obtain results as nearly applicable to full-scale conditions as possible, it has been the practice in recent tests on aerofoils to make the boundary layer turbulent upstream of the region where it interacts with the shock wave.

As results are accumulated for aerofoils with turbulent boundary layers, our understanding of the mechanism by which separation on one surface develops and affects the flow about the whole aerofoil is gradually improving. The effects of the interaction are more straightforward and can be analysed more readily than those which occur when the boundary layer is laminar at the separation point, largely because the interaction itself is not beset by the extra complications which arise when transition occurs in the separated layer.

The present paper describes the mechanism qualitatively in an attempt to demonstrate its essential features and so to provide a tentative basis for further, more rigorous investigations of a theoretical or experimental nature, or for extensions of the work to swept-back and finite wings. The elucidation of certain details has permitted a more complete picture (albeit still highly speculative in many respects), to be presented now than was possible at the time of writing Ref. 4.

Each steady-state flow seems to be an equilibrium established by a fine balance of several interrelated features. The development through a succession of steady-flow equilibria, as the free-stream Mach number or the incidence is raised, is punctuated by critical stages in which significant changes in one or more of the individual features disturb this balance and react on the overall flow.

The discussion of these several features, the important changes in them and the manner in which they affect the overall flow extends to some length, so that a few remarks on the lay-out of the paper might prove helpful at this stage. A preliminary description of the overall flow is first given and is followed by an outline of the scheme to be adopted for the subsequent more detailed considerations. These considerations are summarised in the 'Concluding Remarks' which indicate those features which have been treated more fully now than in Ref. 4 and those which are new. The concluding remarks also mention very briefly some practical applications of the existing knowledge and possible extensions of these by further work of a similar nature. In this connection the detailed investigation of the flow over swept wings is of great importance but beyond the scope of the present paper.

Certain aspects of the approach used here in considering and describing shock movements over the surfaces are believed to be new, and potentially of wider use in a similar qualitative manner in more general considerations of the transonic flow past aerofoils.

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* 'Transonic' is used to describe the range of speeds for which regions of supersonic flow exist on the aerofoil but are limited in chordwise extent.

The all-important mixing process along the shear layer formed downstream of the separation point is not considered in any detail, and some of the effects of the transverse pressure gradients associated with the curved flow are ignored; in particular, the effects on the separated layers.

Although in considering the effects of separation much attention is paid to the relation between the flow on the two surfaces of an aerofoil, no entirely satisfactory substitute is used for the precise concept of circulation in shock-free and unseparated flow.

2. *Experimental Results.*—Results are drawn from a number of experiments made in the N.P.L. High-Speed Wind Tunnels, either with a two-dimensional aerofoil spanning the tunnel[5] or with a half-aerofoil, or 'bump', secured to one wall. The Reynolds number for these experiments was usually between $1\cdot5$ and $2\cdot0 \times 10^6$, based on aerofoil chord, although some results have been obtained for values outside this range. For the aerofoils, transition was fixed by one of various methods[6]; for the bump tests, the turbulent layer from the tunnel wall was allowed to pass over the bump, its thickness being reduced and controlled by suction upstream of the bump. The scale effects that may arise in extrapolating the results to full-scale conditions are discussed in Ref. 6, but should be too small to affect the present qualitative discussions.

The ideas developed and discussed are based on an analysis of surface pressures and flow patterns with the object of illustrating the effects of separation on the development of the flow: firstly, with changes of conditions on one of the actual surfaces, and then, more qualitatively, with increasing free-stream Mach number or incidence. Finally these effects are correlated with the variations in forces and moments derived from the surface pressures. Where comparisons are made to illustrate how the effects of separation depend on section shape and other parameters the aim is to further the understanding of the fundamental flow changes rather than to provide section data for prescribed Mach numbers or incidences, which in any case would have very limited application for contemporary aircraft designs with low aspect ratio and swept wings. We feel, therefore, that the validity of the conclusions is not seriously affected by certain limitations in experimental accuracy. These include uncertainties in blockage and other tunnel-interference corrections for conditions under which the regions of local supersonic flow extend to the tunnel walls and for conditions under which the 'bubbles' of separated flow extend beyond the trailing edge of the aerofoil into the wake[7,8] (*see* Section 6.7)*. The results for the aerofoils at incidence are also affected to some extent by the finite span/chord ratio of the models, in a manner similar to that which would apply for a finite, but fairly large, aspect ratio.

The schlieren flow photographs included in the paper were taken using a North graded-filter schlieren apparatus[9], with the density gradient in the neutral filter set perpendicular to the free-stream direction and, in most cases, to give a black boundary layer on the upper surface of the aerofoil. A spark light source was employed so that the exposure time was of the order of 1 microsecond. The first occurrence of separation can be detected from the photographs by the appearance of a small oblique limb, or 'toe', at the foot of the shock[4].

Static pressures are presented, with a few exceptions, in the form of the ratio, p/H_0, which for the flow upstream of the shocks and outside of the boundary layers is related explicitly to the local Mach number by the equations for isentropic flow. The changes in entropy through shocks of the strengths considered in this paper are very small and have been ignored in calculating the pressure corresponding to sonic flow at points downstream of the shocks, i.e., p_{sonic} has been assumed to be $0\cdot528H_0$ throughout the external flow.

3. *A Preliminary Description of the Overall Flow.*—3.1. *The Development of the Flow.*—Consider an aerofoil at lift, conveniently a symmetrical aerofoil at small positive incidence. With increasing free-stream Mach number, a region of supersonic flow forms first on the upper surface and then on the lower (Fig. 1). In each case a compression shock soon develops to terminate the

* It is suspected that the uncertainties in the presence of separated flow in transonic liners differ appreciably from those occurring in unventilated walls. The observations reproduced in Fig. 19b and Figs. 35 to 44 were obtained using transonic walls.

supersonic flow. The shape of the pressure distribution consequently changes in character from that which applies at low speeds to the 'sonic-range' distribution*, with the shock moved to the trailing edge and the flow supersonic over the whole of the surface except near the nose, which is similar to the distribution that would apply for supersonic free-stream speeds.

The change from the low-speed to the sonic-range distribution, with its associated changes in flow pattern, is referred to throughout this paper as the development of the flow, and figures prominently in the discussions. At first, before the formation of a shock, the rate of development on either surface can be represented by the variation of pressure at a single fixed point because the distributions remain similar. Once the shock has formed, however, the rate of fall of pressure at fixed points upstream slows up appreciably and finally ceases when the sonic-range distribution is established; the rate of development of the flow can then best be represented by the rate of movement of the shock because this determines the rate of extension of the area of supersonic flow.

3.2. *The Occurrence of Separation.*—As the shock moves back over the surface the static pressure, p_1, immediately upstream, falls both before and, provided the surface is not concave, after the sonic-range values are reached (*see* Fig. 1). The shock will remain normal to the surface so long as the flow remains attached, and the pressure, p_2, immediately downstream must therefore correspondingly rise. It can be argued that in these circumstances the shock must, in fact, move continuously rearwards both for increasing free-stream Mach number and for increasing incidence. It then follows that the shock strength, as defined by p_2/p_1, in general increases with increasing free-stream Mach number or increasing incidence, and, except for sections with very small trailing-edge angles and for small angles of incidence†, reach the maximum value that the boundary layer can withstand without separating. Further increase in either variable would entail a further fall in upstream pressure, which in inviscid flow would give a further increase in shock strength but in practice leads to a separation of the layer (and so to a change in shock pattern), which prevents or restricts this further increase in strength.

3.3. *Some of the Consequences of Separation; Comparison with the Low-Speed Stall.*—The pressure rise through the shock, which is thus prevented by separation from increasing in its normal manner, and the rise from the foot of the shock to the trailing edge, also reduced by separation, are the components of the recovery from the low-pressure supersonic flow to the high pressures at the trailing-edge position, and hence the major part of the recovery to the free-stream static pressure and to the pressures on the opposite surface. Clearly, therefore, the separation will have profound effects on the development of the flow, both with respect to the free-stream conditions and relative to the flow on the opposite surface.

The sequence so far has borne marked similarities to the occurrence of separation or stalling for increasing incidence at low speeds, with the shocks now taking the place of the adverse gradients which occur downstream of the nose for that case. There are, however, certain important differences. For example, the stall can now occur for increasing Mach number as well as for increasing incidence. Further, the losses in pressure recovery can often be accommodated only by shock movements and hence local but very severe changes in loading. The consequences of these are often quite different from the changes which occur over the whole chord in the low-speed stall. It will be argued below that changes in the flow along the wake play a vital part in the development of the effects of shock-induced separation, as is also the case for the development of the low-speed stall[8].

3.4. *The Flow in the Presence of Separation.*—Fig. 2 is a schlieren photograph of the flow about a two-dimensional aerofoil at 2-deg incidence and $0 \cdot 88$ free-stream Mach number, a value well above that at which separation would first occur for this case. The boundary layer separates

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* The chordwise distribution of pressure ratio, p/H_0, or local Mach number M, obtained from the stationary values which occur near $M_0 = 1 \cdot 0$ for fixed chordwise positions is described as the 'sonic-range' distribution.

† Holder[10] has shown that separation would be expected to occur for some shock position upstream of the trailing edge whenever the sum of the half trailing-edge angle and the angle of incidence ($\tau/2 + \alpha$), exceeds about $2 \cdot 4$ deg.

abruptly at the foot of a bifurcated shock[11] on the upper surface and then forms a vortex-, shear- or mixing-layer between the 'dead-air' region (or so-called internal dissipative flow[12]) and the external flow. The shear layer, of course, spreads as it moves downstream, and in addition its centre-line curves back towards the surface from a certain point onwards (*see* Section **5.5.**). It thus encloses a 'bubble', which for the example shown extends beyond the trailing edge and is closed in the wake when the separated shear layer joins the layer from the lower surface.

The flow is sketched diagrammatically in Fig. 3 with the corresponding static-pressure variations. The upper curve, with pressure decreasing upwards from the stagnation value on the axis, represents the variation from the subsonic free-stream pressure, p_0, up to the leading edge, then over the upper surface to the trailing edge and, finally, along the wake back to the free-stream value downstream. The lower curve, with decreasing pressure downwards, shows the corresponding changes for the lower surface, the parts upstream and downstream of the aerofoil being mirror images of the corresponding parts of the upper curve.

Most of the shock rise $(p_2 - p_1)$ on the upper surface occurs through the oblique 'toe' of the wave, and this determines the initial deflexion of the shear layer. The rate of pressure rise along the surface under the shear layer, which to the first order is the same as that in the external flow along the outer edge of the layer, depends on the mixing rate and on the depth of the dead-air region. A complex interplay occurs between the rate of pressure rise itself, the growth and curvature of the shear layer, and the depth of the dead-air region.

Downstream of the trailing edge the 'bubble' is bounded by two shear layers converging on one another. The pressure continues to rise along these layers, often at an increased rate. Unfortunately, very little is known about the variation of pressure along the wake, but the initial rise is likely to be from a value $p_{\mathrm{T.E.}}$ at the trailing-edge position to a value p_R at the point R at which the shear layers unite, such that $p_R > p_0 > p_{\mathrm{T.E.}}$. Provided the flow remains subsonic along the wake, at least from the point R onwards, the pressure should then fall again from p_R to p_0, in a manner similar to the change along a wake closed right from the trailing edge (in that case from the value $p_{\mathrm{T.E.}}$, at the trailing-edge position itself, to p_0 (*see* Section 6.3)). The magnitude of the rise would depend on certain characteristics of the wake at R, probably the relation between δ^* and θ, and on the subsequent changes between that point and a point far downstream[13, 14].

The effects of separation on the pressure variations can perhaps best be illustrated by comparison with the variations that would be expected for the same free-stream pressure and aerofoil incidence in the absence of separation (*see* Fig. 4). The full line is the same curve as sketched in Fig. 3, representing the distribution in the presence of separation, and the line of short dashes represents that expected in the absence of separation. The basic changes occur over the upper surface, where the pressure rise through the shock is greater in the absence of separation because the shock remains normal to the surface; the rate of rise downstream is also greater. The pressure now falls continuously along the wake to the free-stream value. Both the shock position and the trailing-edge pressure are quite different on the two curves, the separation causing a forward displacement of the shock and a fall in trailing-edge pressure. Each curve is, however, unique in these and indeed in all respects, in that it gives the only possible mode of pressure recovery from the supersonic distribution upstream of the shock to the free-stream pressure far downstream of the aerofoil for the particular conditions specified. This must remain true even when the interrelation with the flow on the lower surface is considered. It should be noted that the magnitude of the rise in pressure from just upstream to just downstream of the separated flow is not fixed irrespective of the manner in which it occurs for a given case such as sketched, as it is for a given shock and Mach number in the fundamental types of shock-wave boundary-layer interaction on flat plates[3]. Even the overall rise $p_0 - p_1$ depends on the details of the interaction in so far as they affect the shock position and hence p_1.

The interrelation between the two surfaces is, as shown in Ref. 4, governed by a further important factor. Since the wake cannot support any appreciable pressure difference between its two edges, the pressures at corresponding points on the upper and lower edges must be equal or nearly so. In particular, the pressures at the two edges of the wake at the trailing-edge position must be approximately equal. This means that the flow on the lower surface must be such as to give the same pressure at the trailing-edge position as that on the upper surface. Thus the basic changes on the upper surface leading to the reduced trailing-edge pressure will be accompanied by changes on the lower surface to give the same reduction. The changes on the lower surface will in turn affect the wake to some extent, and therefore reflect back on the flow on the upper surface until some equilibrium is established in the development of the flows on the two surfaces relative to one another. On the supposition that the flow remains attached to the lower surface, this reduction in pressure at the trailing edge can be achieved only by an acceleration of the development of the flow on that surface, *i.e.*, by a general increase in velocity and/or a rearward movement of the shock. This shock displacement is opposite to the one which separation produces on the upper surface.

4. *The Scheme Adopted for the more Detailed Consideration of the Separated Flow and its Effects.*—The features and effects of the separated flow are introduced by discussing their development for an aerofoil at fixed incidence and increasing free-stream Mach number. The corresponding development for increasing incidence and fixed Mach number is later briefly described.

The flow on the upper surface is first treated intrinsically, the variation of certain quantities being considered with respect to an arbitrary variation of p_1 within the range appropriate to a particular aerofoil and incidence. This reveals details of the development of the separation bubble and its effects, and the origins of many of the effects of section shape, incidence, control deflection, boundary-layer thickness, etc., consideration of which is deferred to a later paper. It also establishes the relationship between the shock position and trailing-edge pressure, which is of practical importance because it gives the rate of development of the supersonic flow for variation of trailing-edge pressure.

In considering next what changes must occur on the lower surface concurrently with those on the upper surface, and how the development on both surfaces is linked to the free-stream Mach number, we argue that the governing factors are, respectively, the condition of equality of pressure between the two surfaces at the trailing-edge position, and the relationship between the trailing-edge pressure and free-stream Mach number as determined by the flow along the wake.

The trailing-edge pressure is thus used as an important variable or parameter in three respects, namely:

(*a*) in its relationship to shock position on an individual surface

(*b*) in its effect on the interrelation between the two surfaces, *i.e.*, on the development of the flow on one surface relative to that on the other

(*c*) in its relationship to the free-stream Mach number, M_0, or pressure, p_0.

Its significance in all three respects will be illustrated more fully in a subsequent paper and justification derived for its wide use in this manner by reference to some experiments in which it was found possible to vary the trailing-edge pressure independently of the free-stream Mach number. Small strips were attached to one surface of an aerofoil, leaving the other surface clean, and the trailing-edge pressure varied by altering the height or position of the strip. The observations of the shock position on the clean surface, for example, give a unique curve over much of the range when plotted against $p_{\text{T.E.}}$ (Fig. 5), but show little or no correlation with free-stream Mach number.

5. *The Development of the Flow over the Upper Surface for fixed Incidence.*—5.1. *The Flow Pattern; Definitions.*—The surface-pressure distribution divides conveniently into the following components (*see* Figs. 1 and 6):

(*a*) The supersonic distribution upstream of the shock, leading to the value p_1 immediately upstream (p_1 is referred to as the shock upstream pressure)

(*b*) The abrupt pressure rise through the shock to the pressure p_2 (p_2 is referred to as the shock downstream pressure)

(*c*) The slower downstream pressure recovery between the foot of the shock and the trailing edge, leading to the pressure $p_{\text{T.E.}}$ at the trailing edge.

The shock can be considered to trace out a locus of p_1 as it moves over the surface. The variations of p_2 and $p_{T.E.}$ then follow as a result of changes in the shock configuration and flow deflexion at separation, and changes in the separated flow downstream.

The shock is normal at the surface in the absence of separation because there is no flow deflexion. When the flow separates, a fairly abrupt deflexion occurs and the corresponding compression takes the form of a straight oblique shock running from close to the edge of the layer. Fig. 6 shows a schlieren photograph of the flow pattern in the neighbourhood of the resulting interaction and a sketch of the pressure distribution along the surface. The pressure curves cannot be defined in precise detail from the limited number of observations available in the average aerofoil tests, and so the details in the sketch are inferred from those obtained in the tests on flat plates[3, 15]*. For the example illustrated, the value of p_1 is somewhat below that for which separation would first occur, *i.e.*, its value is such that, if separation could be prevented by some means, the shock pressure rise would be considerably greater than that which is normally just sufficient to cause separation. The separation occurs at a point S along the steep rise when the pressure has risen to a value p_s, the separation pressure. The considerations which determine the position of separation relative to the beginning of the compression must be similar to those discussed in Refs. 3 and 16 and, briefly, that the rise $p_s - p_1$ corresponds to the maximum that the boundary layer would withstand without separating for the same upstream conditions. The pressure continues to rise and the layer to be deflected after separation, due to the mixing in the shear layer and the consequent momentum transfer to the 'dead-air' region. The rate of pressure rise remains fairly large while the dead-air region is still shallow, but falls off progressively to a low value as the region thickens. Some of this downstream rise is included in the rise overall $(p_2 - p_1)$, determined from the surface pressures as sketched in the diagram. The pressure p_2 is analogous to the 'kink' pressure used in many experiments on flat plates and described for example in Refs. 3 and 15. We shall continue in the present context to use the term 'shock rise' to describe the quantity $(p_2 - p_1)$.

For cases such as that shown in Fig. 6, where p_2 is still less than the sonic pressure, the slow pressure rise continues to sonic pressure and above. The small normal shock visible in the photograph is not felt on the surface, and indeed the thick dead-air region which now exists could not support any appreciable abrupt rise. Presumably, therefore, this small, so-called normal, wave is either delayed until the pressure has risen almost to the sonic value or is in fact so inclined that it can be reflected from the shear layer to prevent a sudden change of pressure at its foot.

Results for a typical example have been analysed in Fig. 7 to show the variations in p_2, and downstream pressure-recovery, as p_1 decreases, *i.e.*, as the shock moves aft.

5.2. *The Variation of p_2 with p_1.*—The value of p_2 (Fig. 7) increases in the manner expected for a normal shock† until separation occurs at the foot of the shock (*see* Section 3.2.), whereupon it starts to fall. The value of p_2/p_1 is then approximately $1 \cdot 40$, which is therefore interpreted as the strength of the shock just sufficient to cause separation. The value of p_2/p_1 continues to

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* Pressure curves are drawn with pressure increasing downwards throughout this paper to conform to the usual practice for aerofoil pressure distributions. They are therefore mirror images about the horizontal axis of the pressure curves presented in the flat-plate investigations[3,15].

† The pressure rise measured on the surface under the shock is considerably less than the theoretical value for a normal shock in uniform plane flow at the same upstream pressure. This has been noted by many workers, including Ackeret *et al*[2] who drew attention to the existence of the rapid expansion which occurs immediately downstream of the shock, and near the surface, as a result of the rapid transverse changes necessary to restore the sign of the pressure gradient normal to the surface to the sense (positive outwards), which is compatible with the curvature of the flow. On the surface under the boundary layer, the pressure variation through the shock and expansion is not measured as a discrete rise followed by an expansion but as a single rise attenuated by the expansion. Rapid changes near the foot of the shock taking the form of a continuous compression upstream of the shock and an expansion downstream have been calculated theoretically[17,18] by considering the effect of the pressure gradient along the shock just upstream and hence of the variation in shock strength along its length. The expansions are often observed in schlieren photographs if the knife-edge is parallel to the shock direction. An abrupt increase in displacement thickness at the foot of the shock could also have some effect on the measured pressure rise[4].

rise for a further small decrease in p_1, but beyond this remains at a nearly constant value (about $1 \cdot 5$ for the present example). For a given value of p_1, the difference between the separation pressure, p_s (derived from $p_s = 1 \cdot 4 p_1$), and the pressure p_2 represents the rise which occurs along the separation bubble before the dead-air region has deepened sufficiently to reduce the rate of rise to a low value.

5.3. *Separation and 'kink' Pressure Ratios.*—The value $1 \cdot 40$ for the separation pressure ratio, p_s/p_1, was deduced in an analysis of a large number of examples in Ref. 4 and is consistent with more recent observations on aerofoils. It is also consistent with the values observed in other types of shock-wave boundary-layer interaction[3]. Such variation with upstream pressure as might have been expected from this other work is sufficiently small in the range encountered on aerofoils to be unnoticeable within the accuracy to which the ratio itself can be determined. The constant value of $1 \cdot 4$ has therefore been assumed in the qualitative discussions that follow.

The near constant values reached for p_2/p_1 also agree tolerably well with the 'kink' pressures observed in other experiments[3], but are affected to some extent by surface curvature, aerofoil incidence, etc.

The actual value of p_1 (and hence that of the corresponding local Mach number, M_1), for which separation first occurs, varies from example to example because, for an aerofoil shock, the value of p_1 required to give the pressure ratio $1 \cdot 4$ depends on a number of factors, including aerofoil thickness, incidence, etc. It was found in Ref. 4 that this dependence could be represented approximately by a linear variation of the local Mach number for separation with free-stream Mach number, between the points ($M_0 = 0 \cdot 7$, $M_1 = 1 \cdot 26$), ($M_0 = 0 \cdot 9$, $M_1 = 1 \cdot 22$).

5.4. *The Variation of the Downstream Pressure Recovery with p_1.*—The rise in pressure from p_2, at a point P_2, to the value $p_{\text{T.E.}}$ at the trailing edge is plotted in the form of the factor

$$\Pi = \frac{p_{\text{T.E.}} - p_2}{(p_{\text{T.E.}} - p_2)_{\text{low speed}}} \times \frac{(1 - p_{\text{T.E.}}/H_0)_{\text{low speed}}}{(1 - p_{\text{T.E.}}/H_0)}$$

where the 'low-speed' values are obtained from the pressure distribution for a speed below that at which a shock is first formed on the surface, with p_2 always taken at the chordwise position, P_2, used in the corresponding 'high-speed' case. The second ratio of the factor is included to allow approximately for the normal compressibility effects on the pressure gradients.

The typical variation of this factor (Fig. 7), seems at first sight rather surprising. The gradual fall present before separation is reversed after separation, and the factor then actually increases for a certain range of p_1 before it eventually falls very rapidly to a low value. In this intermediate range of p_1, where p_2 is falling but the downstream recovery factor is increasing, little change occurs in the trailing-edge pressure; it does, however, fall rapidly once the recovery factor starts to fall.*

The fact† that the rapid fall in pressure-recovery factor is delayed until p_2 has fallen to or below the sonic pressure, p_{sonic}, provides the clue to a tentative explanation of this delay (*see* Section 5.5), and also of the actual increase which occurs in the intermediate range of p_1.

5.5. *The Development of the Separation 'Bubble'; the Significance of the Stage at which $p_2 = p_{\text{sonic}}$.*—Consider the flow near and downstream of separation on a curved surface as sketched in Fig. 8, the curvature being greatly exaggerated in the sketch, to show a probable approximate pattern of streamlines, shocks and shear layer. Supersonic flow is postulated immediately downstream of the oblique shock because this is the case for which the pattern can be deduced with reasonable certainty from photographs and observed pressures.

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* It should be noted that we are considering here the variation of trailing-edge pressure with upstream conditions, *i.e.*, with p_1, which is not necessarily similar to its variation with downstream conditions, *i.e.*, with p_0 (*see* Section 6.4).

† First pointed out by Mr. L. H. Tanner.

Upstream of the shock, the supersonic flow expands around the convex surface with divergent stream tubes. A pressure rise is maintained along the surface downstream of the separation point by the mixing produced by the shear layer. In the usual way for supersonic external flow*, the rate of rise is very strongly influenced if not determined by the conditions for equilibrium between, on the one hand, the pressure gradient that the viscous momentum transfer can support in the dead-air region and, on the other, the deflexion that must occur in the streamlines to give this pressure gradient in the external flow. The streamlines are deflected outwards and are convergent. The consequent outward, or concave, deflexion of the shear layer and the convex curvature in the surface both tend to increase the depth of the dead-air region and, unless the spread of the shear layer more than compensates for their combined effects, the rate of compression will decrease with distance along the surface.

Once the pressure has risen to the sonic value, however, at some point T, say, a significant change occurs. The mixing process is continuous and the pressure continues to rise slowly provided the dead-air region is not excessively deep. But the rising pressure can now, in the subsonic flow, be associated only with diverging streamlines. Hence there must be a change in the sense of their deflexion, from outwards away from the surface to inwards towards the surface. If the pressure gradient has not fallen too low and if the surface curvature is not too large, this inwards deflexion, together with the spread of the shear layer, tends to reduce the height of the dead-air region and so to increase the rate of pressure rise. This in turn tends to increase the deflexion towards the surface and so on, a process which often leads to a closing of the 'bubble', with a comparatively rapid rate of rise near the reattachment.

This type of flow can occur as soon as the pressure at separation p_s (*see* Fig. 6), has fallen to the sonic value; on the assumption that p_s/p_1 remains at $1 \cdot 4$, this will be when p_1/H_0 has fallen to $0 \cdot 528/1 \cdot 4$, *i.e.*, $0 \cdot 377$†.

The development of the bubble and the distribution of pressure over it can then be traced by considering successive stages for a prescribed further reduction of p_1/H_0. The pressure distribution and a representative streamline near the edge of the bubble are sketched in Fig. 9 for each of four such stages, I to IV. The difference between the separation pressure and sonic pressure increases progressively and hence also the outward deflexion of the streamline which occurs downstream of separation. The deflection up to the separation point also increases very slightly (deduced from equations for oblique shocks[19]). For stage I, the sonic pressure would be reached almost immediately after separation. The dead-air region would still be shallow there and the rate of rise of pressure consequently still high, so that reattachment would occur relatively quickly under the influence of this gradient in the now subsonic external flow. The inflexions in the pressure curve would no doubt be so small as to be unnoticeable with the number of pressure holes usually available; and the rise $(p_2 - p_1)$ would thus include most of the total rise up to the reattachment point. The pressure recovery along the attached flow, between the point (P_2) at which p_2 was measured and the trailing edge, would be expected to show little or no effect of the separation.

In an even earlier stage, represented by the broken curve in Fig. 9, when p_1 is between the value just sufficient to give separation and that for which the separation pressure (approximately equal to $1 \cdot 4/p_1$) has fallen to p_{sonic}, the shock and flow patterns must be intermediate between the normal wave with zero flow deflexion and the pattern just described. The details of this intermediate pattern are not known, but since the flow must be subsonic immediately downstream of the shock, reattachment should occur even more readily than for stage I and the pressure recovery be affected even less.

* In the description of this flow, it has been tacitly assumed that the pressure changes other than the rise through the shock occur in simple waves. Certain effects arising from the finite extent of the supersonic flow will, however, be transmitted along the incoming family of characteristics.

† It will be seen later (*see* Section 6.7), that the stage for which $p_s = p_{\text{sonic}}$ is closer to that at which the serious effects of separation are first felt than is the stage at which separation first occurs. From the point of view of predicting the onset of the effects of separation, the stage for which $p_s = p_{\text{sonic}}$ has the additional practical advantage of being determined by the approximately constant value of p_1/H_0, namely, $0 \cdot 377$. On this basis the effects of separation on a two-dimensional aerofoil would not be expected to be serious unless $(\tau/2 + \alpha)$ exceeds about $4\frac{1}{2}$ deg (*see* footnote to Section 3.2).

For stage II, with a slightly lower value of p_1 than stage I, p_{sonic} would not be reached until slightly further along the separated flow. The dead-air region would by then be deeper and the pressure gradient smaller, but these could still be such as to permit reattachment fairly close to the foot of the shock. The pressure p_2 would be greater than p_{sonic} but would be reached before reattachment. The extra pressure rise associated with the reattachment would thus occur between the point (P_2) at which p_2 was measured and the trailing edge, and so increase the total recovery between p_2 and the trailing edge.

For a still lower value of p_1, stage III, the depth of the dead-air region would have become appreciable and the pressure gradient therefore fallen to a low value before the pressure had risen to p_{sonic}, i.e., p_2 would be less than p_{sonic}. The pressure would then continue to rise only slowly along the bubble. The point, T, say, at which sonic pressure was reached, and therefore the point at which the outward deflexion of the streamlines could cease and begin to reverse, would thus be delayed until considerably further along the bubble than before. Moreover, the dead-air region might well by then be very deep and the pressure gradient very small. The inward deflexion of the streamlines would as a result occur less rapidly and the increase in pressure gradient be correspondingly slowed down, and so on, with this interrelation between pressure gradient and deflexion leading to a marked delay in reattachment and a considerable expansion of the bubble. Further, the delay in reattachment and the removal downstream of the associated fairly abrupt increase in pressure might of itself tend to reduce the pressure gradients near the separation point.

A further small decrease in p_1, stage IV, and corresponding decrease in p_2, would lead to a further large relative expansion of the bubble.

The failure to reach sonic pressure whilst the dead-air region is still shallow, and before the pressure has ceased to rise fairly rapidly, can thus start an unstable divergence causing the bubble to expand very suddenly, often to a point beyond the trailing edge. The sudden expansion would be expected to lead to just such a rapid fall in downstream pressure recovery as is observed to occur when p_2 falls below p_{sonic} (Fig. 7). The correlation is good for all cases examined so far.

As will be seen in the next Section, the loss in downstream pressure recovery contributes to the adverse effects which separation has on the rearward movement of the upper-surface shock for a given variation in trailing-edge pressure. It will be shown later (*see* Section 6) that the divergence which occurs in the variation of trailing-edge pressure with free-stream conditions, and hence the onset of other adverse effects of separation on the overall flow, are also closely associated with the rapid expansion of the separation bubble. The stage at which $p_2 = p_{sonic}$ therefore usually marks the onset of the effects of separation.

The shape of the pressure curves for the upper surface in Fig. 1, near the foot of the shock and downstream, reveal the presence of the developments in the separation bubble described above. They are also well illustrated by photographs and pressure distributions obtained on half aerofoils; a sequence of four reproduced as Figs. 10 and 11 show the following stages:

(A) $p_1 = 0 \cdot 42H_0$; $p_2/p_1 < 1 \cdot 40$.

No separation occurs at the foot of the shock which is therefore normal at the surface.

(B) $p_1 = 0 \cdot 355H_0$; $p_2/p_1 > 1 \cdot 40$; $p_s < p_{sonic}$; $p_2 = p_{sonic}$.

Separation is present at the foot of the shock, giving an abrupt deflexion at the edge of the boundary layer and a very small oblique 'toe'. The direction of flow at the edge of the boundary layer is, however, almost immediately restored roughly to that of the surface. The downstream pressure-recovery factor is about the same as for A, having risen from a slightly lower minimum value between A and B.

(C) $p_1 = 0 \cdot 335H_0$; $p_2/p_1 > 1 \cdot 40$; $p_2 < p_{\text{sonic}}$.

The separation bubble has extended appreciably, but the deflexion of the shear layer back towards the surface still shows quite clearly when the pressure has risen above p_{sonic}. The downstream pressures also show signs of a local steeper rise of the type usually associated with reattachment. The value of the pressure-recovery factor has fallen, however, which suggests that even if reattachment does occur the bubble is sufficiently large to disturb the boundary-layer characteristics appreciably. This would certainly appear to be so from the photograph.

(D) $p_1 = 0 \cdot 32H_0$; $p_2/p_1 > 1 \cdot 40$; p_2 still further below p_{sonic}.

The bubble has now opened up to beyond the trailing edge and the pressure recovery between the shock and the trailing edge fallen to a very low value.

5.6. *The Movement of the Shock with Varying Trailing-Edge Pressure.*—The shock position* for a given trailing-edge pressure, or *vice versa*, the trailing-edge pressure associated with a given shock position, can be deduced from the locus of p_1 and the quantities which vary intrinsically with p_1 (*see* Fig. 7); the rate of shock movement for a given variation of trailing-edge pressure then follows.

The effects of separation on the rate of movement are illustrated in Fig. 12 for a typical example. (A striking similarity will be noticed between the diagrams of this Figure and those for the movement of a shock through a convergent-divergent nozzle with reduction of exit pressure. This latter movement is, in fact, analogous to the movement of the shock over an aerofoil with reduction of trailing-edge pressure[4].)

The chordwise pressure-distributions in the right-hand diagram are a series of actual observations with separation occurring at a certain stage in the shock movement; those on the left for no separation are the same up to that stage and are thereafter estimated for the same sonic-range distribution upstream of the shock, *i.e.*, the same locus of p_1, and for extrapolations of the pre-separation variation of p_2 and downstream pressure-recovery factor, with respect to p_1.

The variation of p_2 with p_1 can be superimposed on the chordwise distributions as a locus of shock downstream pressures. The downstream end of the shock is then located by the intersection of this locus with the particular curve which has the correct rate of downstream pressure recovery and passes through the prescribed trailing-edge pressure. The rest of the distribution can then be deduced by joining the downstream end of the shock thus found to the upstream end in a suitable way.

Distributions are drawn for regular intervals of trailing-edge pressure. In the absence of separation the shock moves back continuously for this imposed variation and reaches a point near to the trailing edge for the value h.

When separation occurs, the loss in pressure rise through the shock due to the break in the variation of p_2 with p_1 (Fig. 7), leads to a very different p_2 locus. The rate of rise of pressure between the downstream end of the shock and the trailing edge also falls once p_2 falls below p_{sonic} (*see* Fig. 7). Both these changes tend to displace the point of intersection of the locus and and downstream pressure curve in a forward direction, away from the trailing edge. In other words the effect of separation is to locate the shock further forwards for a given trailing-edge pressure, and to slow down its rearward movement for a given rate of decrease in trailing-edge pressure. For example, for trailing-edge pressure h the shock has reached only $0 \cdot 58c$ compared with approximately $0 \cdot 85c$ in the absence of separation, and the shock can move to this latter position only if the trailing-edge pressure is further reduced to the value i.

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* Determined by the onset of the rapid compression as sketched in Fig. 6; in the presence of turbulent separation this position coincides with that of the 'toe' of the oblique limb.

The variation of shock position with trailing-edge pressure is plotted out in the lower diagram of Fig. 13. The corresponding variation of p_1 is indicated in the upper diagram. The variation of p_2 is also included for the case in which separation occurs, so that the correlation can be examined between the stage at which $p_2 = p_{sonic}$ and that at which the rate of rearward shock movement suddenly falls. In this particular example the correlation is excellent. This implies that the slight increase in downstream pressure-recovery factor, which occurs between the stages at which separation first occurs and that at which $p_2 = p_{sonic}$, exactly compensates for the distortion in the p_2 locus to give the same shock position as would apply for the given trailing-edge pressure in the absence of separation. This may be partly fortuitous, however, because in two examples given below (Figs. 17 and 33*), the shock movement begins to slow up slightly before $p_2 = p_{sonic}$. These are for fairly low incidence with the shock and the separation occurring near the trailing edge. For several other examples at higher incidences, with the shocks nearer to the leading edge, the correlation is again very good (*see*, for example, Figs. 43 and 46). The shock position would not be expected to remain unaffected by separation once p_2 had fallen below p_{sonic} because the distortion of the p_2 locus and the loss in downstream pressure recovery would then have additive effects.

For the case represented in Fig. 13, and for many others, the movement of the shock speeds up again, with respect to the variation of trailing-edge pressure, once it has reached a point on the surface beyond which there is little or no further variation in p_1, and hence in p_2 (*see* Figs. 17 and 33 for other examples). This is typical for all sections having zero or small surface curvature for a substantial distance upstream of the trailing edge, *e.g.*, the RAE series of sections. The shock position plotted is the commencement of the compression, so that when this has reached about 0·90 to 0·95 chord the trailing edge itself comes within the shock region. The front of the shock can then move further back only comparatively slowly for a given rate of change of trailing-edge pressure (Figs. 17, 30 and 33).

6. *The Development of the Overall Flow Concurrent with that of the Flow on the Upper Surface for Fixed Incidence.*—6.1. *The Flow along the Wake; Two Necessary Conditions to be Fulfilled by the Trailing-Edge Pressure: Equality and Compatibility.*—As intimated in the introductory remarks (Section 3.4), the links connecting the development of the flow on the upper surface with the concurrent changes on the lower surface and with the free-stream Mach number (the chosen independent variable in the present discussion), lie in the flow at the trailing-edge position and along the wake. Thus, firstly, an equality must be maintained between the pressures at the two sides of the wake at the trailing-edge position. Secondly, the pressure, $p_{T.E.}$, at the trailing edge must be compatible with that, p_0, in the given free stream far downstream, that is, the difference $(p_{T.E.} - p_0)$ must be the change that can occur as a result of the static-pressure variation along the viscous flow in the wake.

The trailing-edge pressure can therefore be regarded as having to fulfil two necessary conditions which will be referred to, respectively, as the equality of pressure at the trailing-edge position and the compatibility between trailing-edge pressure and free-stream static pressure.

6.2. *The Relationship between Shock Position and Pressure far Downstream in the Presence of Separation; Similarity to Other Problems.*—When the separation bubble extends beyond the trailing edge, the shear layer is continuous from the foot of the shock well into the wake. The relationship that was shown in Section 5.6 to exist between the trailing-edge pressure and the position of the shock along the surface, and that connecting the trailing-edge pressure to the pressure far downstream, become in reality incidental to the more fundamental connexion between the pressure far downstream and the shock position. The trailing-edge pressure is kept as a definite link in the ensuing discussion and analysis, for the sake of continuity in presentation and, more important, because of its influence on the flow on the lower surface. No alternative exists at present, in fact, because detailed observations have so far not been extended beyond the trailing edge.

* The significance of the other curves in these diagrams and of the lower part of the Figures will be explained later.

Consider for the moment, however, the more fundamental connexion. The position of the shock and separation must be such as to give, from the pressure p_2 immediately downstream, the correct overall recovery along the shear layer and closed part of the wake to the static pressure p_0 far downstream (*see* Figs. 3 and 4); a recovery which, moreover, must occur in a certain particular manner for each shock position, depending on interwoven considerations of the mixing rates and of the path of the shear layer in relation to the surface and wake.

The situation is akin to that treated by Woods[20] for a circular cylinder; he calculated the separation position by assuming that the flow along the free streamline for the separated flow must give the correct magnitude and mode of pressure recovery, from the value at separation to the free-stream pressure far downstream. There is an even closer similarity to the flow pattern which controls the base pressure on bluff bodies in supersonic flow as described by Crocco and Lees[12] and by Holder and Gadd[3]. Crocco and Lees drew attention to the existence of a critical point in the wake, near to the coalescence of the two shear layers, where the properties of the wake and the velocity of the external flow (as determined by the history of the growth of the boundary layers and the growth and paths of the shear layers), have to be those for which the further variation in velocity along the edge of the wake will restore the velocity in the external flow to the free-stream value far downstream. They demonstrated that for each case the flow deflexion adjusts itself, such that the velocity in the external flow (and hence the base pressure) and the subsequent path and growth of the shear layer are those for which the compatibility condition can be met.

Similar considerations might reasonably be expected to apply in the present problem for the point R (Fig. 3), at which the shear layers unite; the position of the shock along the surface (or along the p_1 locus), would take the place of the flow deflexion at the base and adjust itself such that the shock downstream pressure and the subsequent path and growth of the shear layer were those for which the compatibility condition could be met.

6.3. *The Compatibility Condition, up to the Onset of the Effects of Separation.*—Consider the pressure distributions from a point just upstream of the shock on the upper surface for successive increases in free-stream Mach number, *i.e.*, decreases in p_0, such as are sketched in an idealized form in Fig. 14. The corresponding part of a low-speed distribution is also shown.

It is known from solutions of the wake momentum equation that, in the absence of shock waves and separated flow, the distribution of velocity along the wake is related to the variation of the form parameter[13, 14, 21] from its value $\mathscr{H}_{\text{T.E.}}$ at the trailing-edge position to unity far downstream. Young and Winterbottom[14] obtained the following approximate relation:

$$\left(\frac{U_{\text{T.E.}}}{U_0}\right)^{(\mathscr{H}_{\text{T.E.}}+5)/2} = \frac{\rho_0\theta_0}{\rho_{\text{T.E.}}\theta_{\text{T.E.}}} = \frac{\rho_0\delta_0^*}{\rho_{\text{T.E.}}\delta_{\text{T.E.}}^*}\,\mathscr{H}_{\text{T.E.}}$$

connecting the change in velocity along the wake to the properties of the boundary layer at the trailing-edge position and the subsequent change either in the momentum thickness of the wake or in the displacement thickness.

The velocity, and hence pressure, at the trailing-edge position have to be such as to satisfy this relation for each free-stream velocity, and hence pressure. At low speeds[22], the only appreciable departures from the potential-flow velocity distribution on the aerofoil which arise from the need to meet this compatibility condition are confined, for a given circulation, to the region near the trailing edge (the value of the circulation is affected, however, by the need to satisfy the equality of pressure on the two sides of the wake (*see* Section 6.6)).

The necessary conditions remain essentially unchanged as the speed is increased, but the repercussions are different when regions of supersonic flow exist. For an appreciable range of speeds, for example, any changes in trailing-edge pressure which result from changes in the flow along the wake can be accommodated on the aerofoil only by shock movements, involving variations in the extent of the supersonic flow. Moreover, as shown in Section 4, and Fig. 5 and other Figures, relatively small changes in trailing-edge pressure often have considerable effects on the position of the shock on both surfaces. The discrepancies between the pressure distributions for real flows and inviscid solutions are thus of a different nature and of a different order from those which occur at low speeds (the concept of circulation is also quite different).

For decreasing free-stream static pressure (Fig. 14), the distributions along the wake at first form a family of similar curves resulting in a linear relationship between $p_{T.E.}$ and p_0 (Fig. 15b). To the first order the advent of shock waves does not disturb this similarity of distributions, presumably because the relevant properties of the boundary layer at the trailing-edge position, e.g., $\mathscr{H}_{T.E.}$, are not appreciably disturbed. This remains true even after separation occurs at the foot of the shock so long as the bubble remains small, *e.g.*, curve 3, Fig. 14.

The rapid expansion of the bubble which occurs when $p_2 = p_{sonic}$ leads to a divergence from the linear variation of $p_{T.E.}$ with p_0 (Fig. 15), and this is because the bubble now disturbs the characteristics of the boundary layer at the trailing-edge position to give a smaller change in velocity along the wake (curve 4 of Fig. 14).

6.4. *The Cause of the Divergence in the Variation of Trailing-Edge Pressure with Free-Stream Static Pressure.*—It is correct to refer to the divergence in the variation of $p_{T.E.}$ with p_0 as being caused by the change in the variation along the wake and not directly by the decrease in the pressure recovery $(p_{T.E.} - p_1)$* upstream of the trailing edge for a fixed shock position, for the following reason. If this loss in pressure recovery could occur without any disturbance to the properties of the boundary layer at the trailing-edge position, then the pressure variation along the wake would retain the similar form (Fig. 14) and the relationship between $p_{T.E.}$ and p_0 remain undisturbed. The loss in pressure recovery would in such circumstances affect only the shock position (*see*, for example, Fig. 13). The significance of the fact that the loss in pressure recovery along the aerofoil and the divergence in the variation of $p_{T.E.}$ with p_0 appear to occur simultaneously is that the change in pressure variation along the aerofoil and the change in the variation along the wake are two effects of the same root cause, and not that one is due to the other. In some cases the movement of the upper-surface shock seems to be affected very slightly before the variation of trailing-edge pressure (*see*, for example, Fig. 33), which suggests that the changes may not in fact be exactly simultaneous. These are usually the cases for which the shock movement is affected before $p_2 = p_{sonic}$ (*see* Section 5.6); the correlation between $p_2 = p_{sonic}$ and divergence of $p_{T.E.}$ is nearly always very close.

6.5. *The Conditions after the Onset of the Effects of Separation; the Possible Existence of Two Critical Points.*—Once the bubble extends beyond the trailing edge a further important change occurs (*see* curve 5 of Fig. 14), namely, that the pressure continues to fall along the shear layers until the two unite at the point R (*see* Fig. 3). From that point onwards, downstream, the variation in pressure is probably determined in a manner similar to that for closed wakes from the trailing edge onwards, *i.e.*, according to some definite relationship between the pressure and properties of the wake at this point and the corresponding quantities far downstream. In other words, the point R could now take the place of the trailing edge as the critical point at which the compatibility condition has to be fulfilled. The equality of pressure at the two sides of the wake would remain a necessary condition at the trailing-edge position. In these circumstances two critical points would exist for the equilibrium steady state of the flow (just such a situation as might be expected to lead to the unsteady flow phenomena so often associated with shock-induced separation). It is perhaps significant that schlieren photographs of the flow (*e.g.*, Fig. 2), often show eddies springing from the edge of the wake near the point at which the two shear layers first meet.

The following example illustrates possible stages in the re-establishment of steady-flow equilibrium once some change has occurred to disturb it. Suppose the pressure distributions sketched in Fig. 16 are those over the rear of the two surfaces and along the wake of an aerofoil

* The term 'pressure recovery' is here used to include both the 'shock pressure rise' and the 'downstream pressure recovery' (*see* Notation, p. 29).

for a fixed free-stream Mach number at which separation is present on the upper surface. Suppose further, that some change occurs near the foot of the shock on the upper surface, a sudden local deformation of the surface, say, to disturb the pressure recovery downstream of the shock. This would lead to an instantaneous change in trailing-edge pressure and react on the flow on the opposite surface, causing the shock there to move rearwards. The hypothetical instantaneous position sketched is the one that it would adopt if the upper-surface shock remained in a fixed position. The static-pressure variation along the wake would also be disturbed and the pressure would not be able to fall from the new value at the new position of R to the correct free-stream value if the upper-surface shock position did in fact remain fixed. This would react on the flow in such a way as to increase the new pressure at R, *i.e.*, the shock and separation point on the upper surface would move forwards. The trailing-edge pressure for the new steady-flow equilibrium would lie between the original value and the hypothetical value for fixed upper-surface shock position. The equilibrium position for the lower-surface shock would also lie between the two extreme positions. In view of the finite time taken for disturbances to travel from one point to another, oscillations of both shocks are likely to occur before the new steady-flow equilibrium is established. For example, the lapse of time during which the disturbance is travelling from the trailing edge to the point R could cause the lower-surface shock to overshoot its equilibrium position.

6.6. *The Respective Roles of the Two Conditions for Trailing-Edge Pressure in Determining the Steady-Flow Equilibrium.*—The sketches in Fig. 16 illustrate a further point. The disturbance to the value of the trailing-edge pressure for a given shock position on the upper surface led to a change in the *relative* shock positions for the two surfaces, in order that the equality of pressure at the trailing-edge position might be maintained. The readjustment that was necessary to enable the pressure to return to the free-stream value along the wake, *i.e.*, to meet the compatibility condition for the trailing-edge pressure, led to a change in the *actual* shock positions, corresponding to the change in trailing-edge pressure from its hypothetical instantaneous value back to the new steady-flow value, intermediate between the hypothetical one and the undisturbed one.

It can be stated as a general principle that the condition for equality of pressure at the trailing-edge position largely determines the relative shock positions or, more generally, the relative stages in the flow development on the two surfaces; whilst the compatibility condition between the trailing-edge pressure, say, and the free-stream pressure largely determines the actual shock positions or actual stages in the flow development. This is somewhat analagous to that used by Spence[22] to allow for the effects of the boundary layer in lift calculations at low speeds, namely, that the circulation is determined by the condition of equality of pressures at the trailing-edge position, whilst the velocities on the aerofoil for a given circulation are determined by representing the boundary layer and wake by a suitable continuous distribution of sources.

This principle can be further illustrated by reference to the composite diagram in Fig. 17 prepared from the results for a typical aerofoil (10 per cent thick RAE 102 at $\alpha = 2$ deg). The upper part presents the variation with trailing-edge pressure of a representative parameter of the flow on each surface to show the stage of flow development on that surface. The shock position is used where possible and indicates the extent of the supersonic region, but for the lower surface before a shock forms there, the pressure at a typical point, mid-chord, is plotted to show the general level of velocity. The pair of values from the two parameters at any given value of the trailing-edge pressure represents the particular steady flow over the aerofoil which satisfies the equality of trailing-edge pressure at that given value. Each particular steady flow involves a certain relative development of the flow on the two surfaces which, it is suggested, can be regarded loosely as taking the place of the circulation in potential flow.

The comparative variation of the parameters illustrates how the relative flow development for the two surfaces varies with trailing-edge pressure. It will be seen that, whereas the development on the upper surface is slowed up abruptly when the separation bubble on that surface expands rapidly, *i.e.*, when p_2 falls below p_{sonic}, the rate of development on the lower surface is fairly constant with respect to this variable. The relative change between the two surfaces is thus quite marked.

The abscissa of the lower diagram is the free-stream pressure, p_0/H_0. There is a one-one correspondence between points on the two abscissa scales, defined by the relationship between trailing-edge pressure and free-stream pressure. Corresponding points are linked by straight lines. The curves of the two flow parameters can then be transferred from the upper diagram to the lower by taking the pair of ordinate values for each point on the upper abscissa scale and plotting them at the corresponding point on the lower abscissa scale. The pair of ordinate values at any given value of the free-stream pressure then represents the particular steady-flow past the aerofoil which satisfies the compatibility condition between the trailing-edge pressure and the given free-stream pressure as well as the condition of equality of pressure at the trailing edge.

The reversal in the change in slope of the linking straight lines as p_0/H_0 is decreased (moving to the right), and their subsequent rapid divergence, correspond to the divergence in the variation of $p_{\text{T.E.}}$ with p_0. We have seen above that this divergence is due to the rapid expansion of the separation bubble and that it therefore occurs at approximately the same value of $p_{\text{T.E.}}/H_0$ as the slowing up of the upper-surface shock. It is clear from comparison of the two parts of Fig. 17 that this divergence leads to a more rapid development of the flow on both surfaces with respect to the free-stream pressure than to the trailing-edge pressure, but that the relative development is the same. The development on the upper surface still shows the characteristic slowing-up due to the separation there, but that on the lower surface now reveals the abrupt acceleration due to the upper-surface separation that is so evident when results are analysed on the basis of free-stream Mach number only[4].

The situation is, of course, more complicated than just described. For example, the flow from the lower surface will affect the properties of the wake and the distribution of pressure along it, which will react on the flow development on both surfaces. Again, as will be discussed later, the separated flow on the upper surface sometimes has an appreciable effect on the distribution of pressure along the lower surface just upstream of the trailing edge. This applies especially to high aerofoil incidences (*see* Section 6.8); its effect is to disturb somewhat the continuity of flow development on the lower surface with variation of trailing-edge pressure, which in turn reflects on the relative flow developments for the two surfaces.

6.7. *Further Remarks on the Relationship between Trailing-Edge Pressure and Free-Stream Static Pressure.*—The linear variation of trailing-edge pressure, $p_{\text{T.E.}}$, with free-stream static pressure, p_0, up to a certain stage, as described above (Section 6.3) and illustrated in Fig. 15, is typical of all cases examined, with the qualification that very slight regular departures from the straight-line relation sometimes occur. The rapid expansion of the separation bubble towards the trailing edge and beyond always causes an abrupt divergence from the linear variation, and ultimately a change in sign of $(p_0 - p_{\text{T.E.}})$ from negative to positive.

In Ref. 4 it was suggested that the divergence (or rapid fall) in trailing-edge pressure occurs immediately after the first occurrence of separation, which was correct for the examples considered at that time for fairly low incidences (consideration then was limited to low incidences because no data were available for high incidences with transition fixed far enough forwards to be upstream of separation when it first occurred). It is now clear that appreciable delays often arise. For a given aerofoil at a given incidence, the rate of development of the separation bubble is governed by the rate of variation of p_1, so that the delay tends to be large if this pressure changes only slowly with p_0. In comparisons between different section shapes, incidences, etc., other factors also are involved to some extent, including the distance of the separation point from the trailing edge and an effect of the distribution of surface curvature on the depth of the dead-air region. The divergence of the trailing-edge pressure and the onset of the effects of separation on the overall flow are always closely associated, however, and the contention remains valid that observation of the divergence of the trailing-edge pressure provides a valuable indication of the onset of these effects.

In the past it has been usual to note the divergence of trailing-edge pressure from its regular variation with free-stream Mach number, M_0, by plotting either $C_{p\,\mathrm{T.E.}}$ or $p_{\mathrm{T.E.}}/H_0$ against M_0, and it is obviously convenient for many practical applications to obtain directly, in this way, the Mach number for divergence. The plot of $p_{\mathrm{T.E.}}/H_0$ against p_0/H_0 has the advantage, however, that the divergence from a straight line is often more precise; the plot of the difference between trailing-edge pressure and free-stream static pressure against free-stream pressure is yet another alternative. The various ways of plotting the observations are compared for one case in Fig. 15 which suggests that the definition of the divergence depends at least as much on the choice of suitable scales as on the mode of plotting.

The trailing-edge pressure falls rapidly after its initial divergence, in response to the changes in the static-pressure variations along the wake caused by the development of the separated flow. The rate of fall, and hence, as seen above, the rate of development of the flow on both surfaces including that of the separated flow itself, must be just that which will allow its value to remain compatible with the free-stream static pressure. Although this rate of fall is a very important parameter in the development of the overall flow in the presence of separation, little further can be added at present, partly because practically nothing is known about the variation along the wake of either the static pressure or the properties of the wake, and partly because of uncertainties regarding blockage effects in wind tunnels. The blockage effects in question are those which might, in the presence of separated flow, lead to errors in the static pressure far downstream[8]. The validity of this pressure must remain open to doubt unless observations are available to confirm that in the experiment it returned to the value for the free stream ahead of the model, or more strictly, if the changes in entropy through the shocks become important, to the appropriate slightly different value.

Values of $(p_{\mathrm{T.E.}} - p_0)/H_0$ are plotted in Fig. 18 for a number of examples to show the effect of section shape and incidence on the rate of fall of trailing-edge pressure after divergence. The full lines show the variation if the abscissa is taken to be p_0/H_0, and the broken line the variation if the abscissa is p_1/H_0, p_1 being the pressure immediately upstream of the shock. The rate of variation with respect to p_1 is not greatly affected by section shape and incidence, which suggests that the differences in the variation with respect to p_0 are largely due to the different rates of variation of p_1 with p_0, and therefore to the different rates of the development of the separation bubble, which depends strongly on the variation of p_1. Other factors would, however, be expected to have some effect on $p_{\mathrm{T.E.}} - p_0$, including the relative spacing of the shear layers which would be affected by incidence and by the occurrence of separation on the lower surface.

Comparison of the full and broken curves in Fig. 18 suggests that the delay between the first occurrence of separation, indicated by the change from open to closed symbols, and the onset of its effects, indicated by the divergence of $p_{\mathrm{T.E.}}$, is also more dependent on the variation of p_1 than on that of p_0.

6.8. *Further Remarks on the Interrelation between the Two Surfaces; Relative Rates of Flow Development.*—For the example shown in Fig. 17 the rate of flow development on the lower surface is reasonably constant with respect to the variation of trailing-edge pressure, which implies, when there is no shock on the lower surface, that the chordwise distributions of static pressure form a family of similar curves extending right to the trailing edge. This is found to be approximately true for all aerofoils up to moderate incidences, and to apply also over much of the chord when a shock is present, *e.g.*, in the region downstream of the shock (*see*, for example, Fig. 1). It follows that any divergence in the rate of variation of $p_{\mathrm{T.E.}}$ with free-stream static pressure, p_0 due to changes on the upper surface, affects the whole of the lower surface roughly proportionally when there is no shock there. In the presence of a shock, the pressures between the shock and the trailing edge are all affected roughly proportionally, and the shock moves in response to the changes in $p_{\mathrm{T.E.}}$ in precisely the same manner as the upper-surface one has been shown to do (*see* the left-hand diagram of Fig. 12, and Section 5.6).

The pressures at fixed points, including the trailing edge, on the lower surface of an aerofoil at fairly low incidence (10 per cent RAE 102, $\alpha = 2$ deg) are given in Fig. 19a to show how, in the absence of a shock there, the pressures over the whole surface diverge from their normal rate of variation with p_0 and fall rapidly in sympathy with the divergence and rapid fall at the trailing edge. For such cases the presence of separated flow on the upper surface clearly cannot be exerting any very strong influence on the distribution of pressure on the lower surface as distinct from the general level, or stage in the flow development. This no longer applies, however, when the aerofoil is at high incidence. An acceleration, or fall in pressure, then occurs locally along the rear of the lower surface due to the separated flow on the upper surface. Very crudely, this fall in pressure can be considered in terms of the angle through which the flow leaving the lower surface is deflected. For small angles of incidence, Fig. 20a, the direction ot the flow adjacent to the lower surface just upstream of the trailing edge is upwards relative to the free stream, and the streamlines tend to diverge as they turn parallel to the wake, aligned roughly along the free stream. The pressure rises continuously through the trailing-edge value. At high incidences (Fig. 20b), the direction of the flow on the upper surface is still not very different from that of the free stream, but the flow on the lower surface just upstream of the trailing edge is now downwards relative to the free stream. The streamlines there tend to deflect upwards towards the wake, leading to a convergence of the streamlines and fall in pressure. Downstream of the trailing edge, the pressure must rise again, and the streamlines diverge before finally becoming parallel.

This speculative explanation is supported by the results reproduced in Fig. 21 (from Ref. 23), to show that, in the presence of severe separation on the upper surface, the magnitude of the pressure fall just upstream of the trailing edge on the lower surface, for a given Mach number and incidence* and for related sections, depends on the angle between the lower surface and the chord-line at the trailing edge, and increases as this angle decreases. For symmetrical sections this angle is, of course, $\tau/2$ and the flow on the lower surface is along the stream direction when $\alpha = \tau/2$; thereafter the downwards deflexion relative to the stream is $(\alpha - \tau/2)$. For cambered sections the corresponding quantity is $(\alpha + \lambda - \tau/2)$, where λ is the angle between the centre-line and the chord at the trailing edge.

In general, as $(\alpha + \lambda - \tau/2)$ becomes positive and increases in magnitude, the divergence in trailing-edge pressure, due to the separation on the upper surface, ceases to affect the whole of the lower surface uniformly. The occurrence of the local fall in pressure near the trailing edge enables the equality of pressure at the trailing edge itself to be restored for a smaller fall over the rest of the surface. In other words, the local fall in pressure compensates partially for the effect of the upper-surface separation and thereby reduces the effect on the relative development of flow on the two surfaces.

Fig. 19b shows the variation of the pressures at fixed points on the lower suface of a 6 per cent thick RAE 104 section at 7·7 deg and, in contrast to the results in Fig. 19a, the effects of the divergence of the trailing-edge pressure are severe over the rear part of the lower surface only. The differences in α and $\tau/2$ both contribute to the change in $(\alpha - \tau/2)$ from $-3{\cdot}5$ deg for Fig. 19a to $+3{\cdot}6$ deg for Fig. 19b.

Associated with the loss in downwash behind the aerofoil (*i.e.*, in the downwards deflexion of the dividing streamline) due to the upper-surface separation, one would expect a movement of the stagnation point forwards along the lower surface. This would be consistent with the changes in velocity on the lower surface, and there may well be a much closer connection between the loss in downwash and the changes in the overall flow than suggested in the speculative approach used here.

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* The example shown is for fairly low Mach number and very high incidence, with the upper-surface flow stalled completely. This was selected from observations covering a wide range of both variables because the pressure distribution on the upper surface was the same for all four sections. Similar qualitative differences in the lower-surface pressures applied fairly generally.

A further point of interest concerning the interrelation between the two surfaces arises when there are shocks on both surfaces, *i.e.*, for fairly low incidences and for Mach numbers just below unity. The relative development of the flow is then determined solely by the relative shock positions.

Suppose the loci of shock upstream pressures, p_1, are represented by the broken lines in the sketches of Fig. 22 (corresponding very closely to the 'sonic-range' pressure distributions), and the loci of shock downstream pressures, p_2, by the chain-dotted lines. In the absence of separation (Fig. 22a), the values of p_2 will be lower for the lower surface, *i.e.*, the locus will lie above that for the upper surface in the sketch. For any given trailing-edge pressure the upper-surface shock will therefore be further back than the lower-surface one, as sketched (the rate of downstream pressure recovery has been assumed to be the same on the two surfaces; any difference in practice would be such as to increase the difference between the shock positions).

In the presence of separation on the upper surface only, the p_2 locus for this surface curves upwards towards that for the lower surface (*see* right-hand diagram of Fig. 12) and often crosses it as shown in Fig. 23b. This leads to quite different relative shock positions (those sketched in Fig. 23b are for unchanged trailing-edge pressure and rate of downstream pressure recovery; the differences that would occur in these quantities in practice due to separation would increase the effect of separation on shock positions).

As the shocks approach the trailing edge, the overall loading on the aerofoil consists simply of the pressure difference between the sonic-range distributions integrated back to the shocks, together with the extra loading which occurs between the two shocks. The nature of the p_2 loci can then be used to deduce that in the absence of separation the upper-surface shock must reach the trailing edge first, and therefore that the loading must approach the sonic value, *i.e.*, that attained when both shocks are at the trailing edge, from above. If in the presence of separation the p_2 loci cross, then the lower-surface shock will reach the trailing edge before the upper-surface one; the loading would then approach the sonic value from below, or in other words a trough would occur in the loading curve (*see* Section 8.1).

7. *The Decay of the Effects of Separation for Fixed Incidence; The Significance of Supersonic Flow at the Trailing Edge and Along the Wake.*—7.1. *The Stage at Which the Decay Begins.*—It is fairly well known that the effects of separation begin to decrease once the pressure at the trailing edge, i.e., $p_{\mathrm{T.E.}}$, falls to the sonic value, p_{sonic}, and below[24, 4]. This process can start, and the sequence of changes occur, in different ways ; these will be examined by considering first the change in the interrelation between the two surfaces, or relative flow development, and then the change in static-pressure variation along the wake.

7.2. *The Change in the Interrelation between the Two Surfaces, or in the Relative Rates of Flow Development.*—In general, the existence of supersonic phenomena at the trailing-edge effectively isolates one surface from the point at which the equality of pressure at the two sides of the wake must be satisfied, and in so doing isolates the two surfaces from one another.

Consider pressures at stations a and w (Fig. 23), respectively just upstream of the trailing edge on the aerofoil and just downstream on the wake.

When the flow is subsonic over the trailing edge (Fig. 23 (i)),

$$p_{a\,u} \simeq p_{w\,u} \simeq p_{w\,l} \simeq p_{a\,l}\,.$$

If supersonic flow exists at the trailing edge, with lower pressure on the lower surface, say, then a shock can occur at the trailing edge on this surface Fig. 23 (ii) and

$$p_{a\,u} \simeq p_{w\,u} \simeq p_{w\,l} > p_{a\,l}\,.$$

The shock introduces an inequality between $p_{a\,u}$ and $p_{a\,l}$, and the condition for equality of pressure at the two sides of the wake no longer controls the relative flow on the two surfaces.

Similarly, if the lower-pressure supersonic flow is on the upper surface, an expansion can occur at the trailing edge on the lower surface (Fig. 23 (iii)), and

$$p_{a\,u} \simeq p_{w\,u} \simeq p_{w\,l} < p_{a\,l}\,.$$

The presence of a shock or an expansion at the trailing edge no doubt affects the validity of the assumption that there is no appreciable static-pressure difference across the wake, but the arguments used here should remain qualitatively correct.

The situation with a shock at the trailing edge arises for small positive incidences. Suppose the free-stream Mach number increases, *i.e.*, $p_{\mathrm{T.E.}}$ decreases, through the stages represented by the sketches in Fig. 24. The sonic-range pressure distributions are as usual traced out by the rearward-moving shocks, and coincide very nearly with the p_1 loci; the corresponding p_2 loci are as sketched.

For stage (*a*) both shocks are upstream of the trailing edge as described in Section 6.8.

The lower-surface shock reaches the trailing edge, or at least its downstream end does*, at stage (*b*) when $p_{\mathrm{T.E.}}$ reaches the p_2 locus for this shock.

$p_{\mathrm{T.E.}}$ continues to fall and the shock on the upper surface to move rearwards (*see* stage (*c*)), but practically no change can occur on the lower surface where the full sonic-range pressure distribution was already established at stage (*b*). The decrease in magnitude in the shock pressure rise from just upstream of the lower-surface shock to the trailing edge is probably associated with a slight movement of the shock over the trailing edge on to the wake.

Note that $p_{\mathrm{T.E.}}$ falls below p_{sonic} between stages (*b*) and (*c*). The lower surface actually becomes effectively isolated from the upper surface at the trailing edge just before $p_{\mathrm{T.E.}} = p_{\mathrm{sonic}}$, when $p_{\mathrm{T.E.}} = (p_2)_{\mathrm{lower\ surface}}$ at stage (*b*). The difference between the value of M_0 for $p_{\mathrm{T.E.}} = (p_2)_{\mathrm{lower\ surface}}$ and the value for $p_{\mathrm{T.E.}} = p_{\mathrm{sonic}}$ is usually very small (*see*, for example, Fig. 15b), and for a given aerofoil decreases as the incidence increases because $(p_2)_{\mathrm{lower\ surface}} \to p_{\mathrm{sonic}}$.

When $p_{\mathrm{T.E.}}$ has fallen to the p_2 locus for the upper surface, the upper-surface shock finally moves right on to the trailing edge (Fig. 24d). The pattern then assumed by the flow, and the subsequent changes, follow very closely those described in Ref. 10.

Fig. 25 shows observations of shocks moving on to the trailing-edge of an aerofoil, for comparison with the sketches of Fig. 24. Stage (i) corresponds to stage (*a*) of Fig. 24, stage (ii) falls between (*b*) and (*c*) and stage (iii) corresponds to (*d*).

The trailing-edge expansion, between the lower surface and the wake, occurs at high incidence, and it was in this context that the interrelation between the two surfaces of an aerofoil was first shown to be affected significantly when $p_{\mathrm{T.E.}}$ became equal to p_{sonic}[24].

In the presence of separation on the upper surface, the flow near the trailing edge when $p_{\mathrm{T.E.}}$ is just slightly greater than p_{sonic} is as sketched in Fig. 26a, the static pressure on the lower surface falling to a minimum value at the trailing edge as discussed in Section 6.8.

As for Fig. 24, successive stages are considered in which successive falls in trailing-edge pressure are produced by increase in free-stream Mach number.

Fig. 26b shows the flow when $p_{\mathrm{T.E.}}$ has fallen to p_{sonic}.

The expansion develops for further decrease in $p_{\mathrm{T.E.}}$ as shown in Fig. 26c, but the pressures over the lower surface fall only very slightly, if at all, from those at stage (*b*). Thus, unlike the change from (*a*) to (*b*), the upper surface flow can develop between (*b*) and (*c*) without appreciably affecting the flow over the lower surface. This is illustrated by the observed pressure distributions in Fig. 27, which were obtained for an aerofoil at 8-deg incidence and successive Mach numbers, 0·7, 0·75 and 0·8. The first two are similar to stage (*a*) of Fig. 26. Stage (*b*) of Fig. 26 would occur between the second and third curves of Fig. 27, stage (*c*) being similar to the third curve.

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* Because of the finite rate of pressure rise in the shock, the trailing edge falls within the shock region, *i.e.*, the downstream end of the shock reaches the trailing edge, when the shock position, x_{sh}/c as defined in Fig. 6, is still appreciably upstream of the trailing edge (about 0·90 to 0·95c).

It is of interest to note that in the flow represented by the final sketch of Fig. 26, as also in the final stages shown in Fig. 24, the velocity along the edge of the wake is at first supersonic and that, if this supersonic flow persists to a point beyond that at which the bubble closes, shocks whose strengths are not negligible can impinge on the closed part of the wake.

7.3. *The Change in the Static-Pressure Variation along the Wake; The Collapse of the Bubble of Separated Flow.*—In addition to the changes in the relative development of the flow on the two surfaces as just described, an acceleration in the actual development with increasing free-stream Mach number, M_0, is often observed in the final stages as the shocks move towards the trailing edge or as $p_{\text{T.E.}}$ falls below p_{sonic}. Several factors seem to contribute to this.

Firstly, there is an increase in the rate at which $p_{\text{T.E.}}$ falls with increasing M_0, i.e., an increase in the total change $(p_0 - p_{\text{T.E.}})$ in static pressure along the wake. The acceleration shows very clearly in the rearward movement of the upper-surface shock, as illustrated in Fig. 17, lower diagram. This Figure also gives the variation of shock position with trailing-edge pressure; the corresponding relation between trailing-edge pressure and free-stream static pressure is shown in Fig. 15b. The increased rate of fall of $p_{\text{T.E.}}$ commences when the lower-surface shock reaches the trailing edge, *i.e.*, when $p_{\text{T.E.}} = (p_2)_{\text{lower surface}}$ and just before $p_{\text{T.E.}} = p_{\text{sonic}}$, and it is therefore probably associated with the presence of supersonic flow along the wake.

Secondly, for the RAE series of aerofoils, 100–104, and many others which have flat or nearly flat surfaces immediately upstream of the trailing edge, the upper-surface shock, once it has reached this flat part of the surface, begins to move more rapidly for a given rate of fall of $p_{\text{T.E.}}$, irrespective of the increased rate of fall of $p_{\text{T.E.}}$ (*see* Section 5.6).

Finally, when the upper-surface shock has eventually moved on to the trailing edge and become inclined in its final sonic position[10], the separation bubble must be very small indeed. There will, in fact, be no separation at all unless $(\tau/2 + \alpha)$ exceeds about $12\frac{1}{2}$ deg[3, 25]. The movement of the shock on to the trailing edge must therefore be accompanied by a collapse of the separation bubble, a process which itself would be expected to have a marked effect on the flow along the wake, irrespective of whether any part of that flow was supersonic.

Consider successive shock and separation positions over the rear of the upper surface as represented in Fig. 28 by sketches of a streamline near the edge of the bubble for each stage (the lower-surface boundary layer, and the lower shear layer downstream of the trailing edge, are assumed to be the same at all stages; in practice they most probably would change slightly). The corresponding distributions of static pressure over the surface are also shown. The sketches are for equal intervals of shock position and not necessarily equal intervals of $p_{\text{T.E.}}$ or p_0. When the shock is moving on the part of the surface which has appreciable curvature, the outward deflexion at the front of the bubble can increase and the subsequent inward deflexion be delayed*, so that the distance between the shear layers at the trailing-edge position, and also the length of the bubble downstream, can increase even though the height of the separation point above the trailing edge decreases, *e.g.*, the change from stage 1 to stage 2 in Fig. 28.

As the shock moves on to the flatter part of the surface, however, as in stage 3, the outward deflexion ceases to increase and the distance between the shear layers at the trailing edge begins to decrease. The edge of the bubble then moves down the surface for succeeding stages roughly as sketched, the bubble becoming shallower and shorter as the shock moves aft. The rate of static pressure rise along the wake would be expected to increase as the bubble became smaller. The rate of recovery over the rear of the aerofoil would probably also increase slightly†, which would tend to increase the value of $p_{\text{T.E.}}$ for a given shock position or, alternatively, to speed up the movement of the shock for a given rate of fall of $p_{\text{T.E.}}$ (this movement is already accelerating due to the flattening of the p_2 locus; *see* Section 5.6).

* This follows from the fact that p_1 is decreasing, the reasoning being similar to that used in Section 5.5 to describe the rapid expansion of the bubble.

† Some evidence of this has been observed (*see*, for example, the curve of pressure-recovery factor reproduced here in Fig. 10).

An interesting situation arises if the increased rate of fall in $p_{T.E.}$ with respect to p_0, discussed as the first factor contributing to the accelerating flow development and illustrated in Fig. 15, is connected with this collapsing of the bubble. Such a tendency, for $p_0 - p_{T.E.}$ to increase as the bubble collapsed, would tend to give rise to a process which was unstable with decrease in p_0, similar to, but the reverse of, the rapid expansion of the bubble with decrease in p_1 described in Section 5.6. Successive small changes in p_0 would lead to diverging changes in $p_{T.E.}$ and therefore to an even greater acceleration of the shock wave; this in turn would speed up the collapse of the bubble, the very feature which, as postulated, initiated the acceleration and led to the increased rate of fall of $p_{T.E.}$.

The connection between the increased rate of fall in trailing-edge pressure and the changes associated with the occurrence of some supersonic flow along the wake is not very well undertsood.

Take first the case for which the supersonic flow is introduced when the lower-surface shock moves on to the trailing edge and becomes inclined. It is possible that, if the bubble closes at a point fairly near to the trailing edge and where the flow is still supersonic, a small breakdown shock will occur along the closed part of the wake and so augment the total pressure recovery along the wake. The fact that static pressure rises along the closed part of a wake when the velocity is supersonic[18], instead of falling as in subsonic flow, would also tend to increase the total pressure recovery. These changes would introduce progressively the type of flow which exists when the upper-surface shock has moved right on to the trailing edge and assumed the inclination which will deflect the flow along the stream direction[10] (*see* Fig. 29), *i.e.*, assumed its 'sonic' position. The bubble has then disappeared (assuming $(\tau/2 + \alpha)$ to be less than about $12\frac{1}{2}$ deg), and the pressure rise at the shock fallen. The recovery to free-stream pressure is assisted by the small normal shock on the wake which moves off downstream as the free-stream static pressure, p_0, is further reduced, and presumably disappears as this approaches the sonic value.

In the case for which the supersonic flow along the wake is introduced when an expansion occurs at the trailing edge, it is clear from the sketch in Fig. 26 that the expansion will help to close the bubble downstream of the trailing edge and consequently to increase the rate of static-pressure rise along the wake. Considerations of how this might increase the total recovery, $p_0 - p_{T.E.}$, are then very similar to those just described for a shock at the trailing edge. The presence of fairly strong breakdown shocks on the closed wake has been observed for such cases[24]. In the examples shown in Fig. 27, the expansion leads to a rapid fall in $p_{T.E.}$ and, because of this, to a considerable rearwards movement of the upper-surface shock.

Just as before, the bubble must eventually close completely provided $(\tau/2 + \alpha)$ is less than about $12\frac{1}{2}$ deg, and the flow at the trailing edge approach the inviscid type of flow, now either with a pair of oblique shocks or with an oblique shock on the upper surface and an expansion on the lower surface, depending on whether $(\tau/2 - \alpha)$ is greater or less than zero.

8. *The Effects of Separation on the Forces and Moments for an Aerofoil at Fixed Incidence.*—The effects of separation on the variation with free-stream Mach number of the forces and moments for an aerofoil at fixed incidence were fairly fully described in Ref. 4. Brief consideration again here by reference to one or two specific cases will serve to recapitulate the various flow developments described above and help to illustrate the practical importance of the features which have emerged since Ref. 4 was written.

8.1. *The Development of the Flow over a 6 per cent. thick RAE 104 Aerofoil at 2 deg Incidence; The Effects on C_L.*—Results for a range of free-stream Mach numbers are shown in:

Fig. 30	Surface-pressure distributions
Fig. 31	Flow photographs
Fig. 32	Quantities derived from the upper-surface pressures
Fig. 33	Variation of a flow parameter (shock position or pressure at a fixed point) for each surface, and of $p_{T.E.}$
Fig. 34	Variation of total C_L and of the contributions from the separate surfaces.

Photographs (Fig. 31), are shown for eight stages in the development of the flow, defined by specified free-stream Mach numbers and labelled (*a*) to (*h*). These letters are used on the other Figures to identify the various observations.

For stage (*a*), $M_0 = 0 \cdot 84$, the value of p_2/p_1 on the upper surface (Fig. 32) is less than $1 \cdot 4$ and separation does not occur at the foot of the shock (Fig. 31).

At stage (*b*), $M_0 = 0 \cdot 86$. p_2/p_1 is greater than $1 \cdot 4$ and the flow separates at the foot of the shock, but p_2 is still greater than p_{sonic} (Fig. 32); the pressure recovery over the rear of the aerofoil increases slightly (Fig. 32), but the trailing-edge pressure (Fig. 33), the overall flow (Fig. 33) and C_L (Fig. 34) are not affected.

For stage (*c*), $M_0 = 0 \cdot 88$, p_2 is equal to p_{sonic} and the trailing-edge pressure has started to fall; the movement of the upper-surface shock has started to slow up with variation of $p_{\text{T.E.}}$ (Fig. 33a) and so to affect the relative rate of development of the flow on the two surfaces. The plot of shock positions, or pressure at a fixed point, against p_0 (Fig. 33b) shows how, in addition to the slowing up on the upper surface, the development on the lower surface is accelerated with respect to free-stream conditions.

This process continues throughout stages (*d*), (*e*) and (*f*), i.e., up to $M_0 = 0 \cdot 93$, a supersonic region and shock having formed on the lower surface at stage (*d*), $M_0 = 0 \cdot 90$.

The effects on loading of the changes in the rate of development on the two surfaces with respect to free-stream Mach number are best demonstrated by reference to the curves in Fig. 34b showing the integrated contributions from the separate surfaces. The difference in ordinate between these curves at any given value of the Mach number represents the total C_L at that Mach number. Both curves show a definite divergence in the mode of variation after stage (*b*), the change in both cases being such as to contribute to an abrupt fall in total C_L.

The C_L (Fig. 34a), reaches a minimum value at stage (*f*), $M_0 = 0 \cdot 93$, just as the lower-surface shock is about to move on to the trailing edge; the value of $p_{\text{T.E.}}$ at this stage is just approaching the value given by the p_2 locus for the lower surface (Fig. 30).

The effect of separation on the relative flow (Fig. 33a) then begins to dimimish (*see* stage (*g*), for example); the upper shock accelerates on to the trailing edge, thereby increasing the contribution to the lift from the upper surface. The total C_L rises from the trough to a value corresponding to a flow pattern in which both shocks have reached the trailing edge.

The sonic-range pressure distribution then exists over the whole of both surfaces and little change in flow pattern occurs with further increase in free-stream Mach number. Thus the only change from stage (*g*) ($M_0 = 0 \cdot 94$) to stage (*h*) ($M_0 = 0 \cdot 96$) is in the inclination of the trailing-edge shocks as the bubble finally collapses completely (Figs. 30 and 31). C_L falls gradually, however, because of the increase in the quantity $\frac{1}{2}\rho U_0^2$, and this rate of fall would be expected to persist as the free-stream Mach number was increased through unity up to a value at which the sonic-range distribution ceases to apply.

The trough in the curve of C_L versus M_0, of which that in Fig. 34a is typical, has become perhaps the most widely recognised effect of separation on steady-flow characteristics, and is of course far more severe on thicker sections; the C_L often falls almost to zero, for example. From the deduction made in Section 6.8, that in the absence of separation the loading must approach the sonic-range value from above, it follows that the trough would not occur in the absence of separation and that C_L would therefore fall abruptly but monotonically to the sonic and supersonic values.

8.2. *The Effects on Other Quantities.*—By analyses similar to those just illustrated it is possible to show how the changes in chordwise distribution of the loading on the separate surfaces affects the section pitching moments[4]. The most serious effect is a halt in the normal compressibility nose-down changes and a reversal to a violent nose-up change, due to the rapid rearward movement of the lower-surface shock at a stage when the upper-surface one hardly moves at all.

This is followed by a further reversal to a nose-down change, the upper-surface shock moving back at a stage when the lower-surface one is fixed at the trailing edge, before the variation finally settles down to the gradual sonic-range variation. The corresponding changes in the centre-of-pressure position are a halt in the normal rearward movement, followed by a violent forward 'kick' before the final rearward movement to the sonic-range position.

The slowing up of the rearward movement of the upper-surface shock, *i.e.*, of the rearward development of the low-pressure supersonic flow, slows up the rate of rise of drag coefficient for the fixed incidence[4]. The rate of rise of C_D for a given C_L might be expected to increase, however, due to the loss of lift coefficient at fixed incidence.

The loss in effectiveness of flap-type controls, and in particular the troughs in the variation of control effectiveness with free-stream Mach number, are known[26,4] to be associated with separation effects similar to those just described. The deflected control aggravates the separation because it leads to higher local Mach numbers upstream of the shock and deeper dead-air regions downstream.

The correlation between the onset of buffeting and the occurrence of separation is fairly well established[4].

8.3. *The Development of the Flow over a* 6 *per cent thick RAE* 104 *Aerofoil at* 3·7 *deg Incidence.*—Results for a range of free-stream Mach numbers are shown in:

Fig. 35	Surface-pressure distributions
Fig. 36	Flow photographs
Fig. 37	Quantities derived from the upper-surface pressures
Fig. 38	Variation of a flow parameter for each surface
Fig. 39	Variation of C_L and $p_{\mathrm{T.E.}}$ with M_0.

Photographs (Fig. 36), are shown for six stages, (*a*) to (*f*), defined by specified free-stream Mach numbers from 0·7 to 0·95.

The main difference between these results and those for 2 deg is the much greater delay between the first occurrence of separation, immediately after stage (*a*), $M_0 = 0 \cdot 7$, and the onset of its effects on the overall flow. This is indicated by the divergence in the variation of $p_{\mathrm{T.E.}}$ and occurs as the bubble expands rapidly between stages (*c*) and (*d*), i.e., between $M_0 = 0 \cdot 80$ and $0 \cdot 85$. Separation first occurs and the pressure p_2 becomes equal to p_{sonic} at almost the same values of p_1 as for 2 deg, and the greater delay is therefore due entirely to the slower variation of p_1 with shock position, and hence with free-stream Mach number, M_0, during the stage in which the bubble is first developing or, in other words, to the flatness of the p_1 locus in the region where separation first occurs. For this same reason, the movement of the upper-surface shock is not slowed up so abruptly as at the lower incidence. There is good correlation between: the lift divergence, *i.e.*, the fall in the rate of increase of C_L with M_0 followed by a rapid decrease (Fig. 39); the divergence in the variation of $p_{\mathrm{T.E.}}$; and the stage at which $p_2 = p_{\mathrm{sonic}}$, i.e., at which the separation bubble expands rapidly. The observations were not continued far enough nor spaced at close enough intervals of Mach number to define the minimum in the curve of C_L versus Mach number.

9. *The Effects of Separation on the Development of the Flow with Increasing Incidence.*—The developments in the flow which lead to shock-induced separation are much the same for increasing incidence as for increasing free-stream Mach number, namely, that the upper-surface shock moves progressively rearwards and increases in strength up to a value which the boundary layer can no longer withstand without separating. The developments after separation are also very similar, including the rapid expansion of the bubble of separated flow when the pressure downstream of the shock falls to the sonic pressure, and the effects of this on the pressure recovery

over the rear of the aerofoil and on the static-pressure variation along the wake. Just as was found for fixed incidence, these changes affect the variation of trailing-edge pressure, the movement of the shocks and hence the value and distribution of the chordwise loading.

These developments are illustrated for a typical example, 6 per cent RAE 104 aerofoil at $M_0 = 0 \cdot 75$, in the following series of Figures:

Fig. 40	Surface-pressure distributions
Fig. 41	Flow photographs
Fig. 42	Quantities derived from the upper-surface pressures
Fig. 43	Variation with incidence of $p_{\text{T.E.}}$, upper-surface shock position and C_L.

Separation first occurs at the foot of the upper-surface shock when the incidence is between $2 \cdot 7$ and $3 \cdot 7$ deg (Fig. 41, observations (*a*) and (*b*)), *i.e.*, when p_2/p_1 exceeds $1 \cdot 4$ (Fig. 42). The gradual growth of the bubble whilst p_2 is still greater than p_{sonic} is shown clearly by the upper-surface pressure distributions (Fig. 40), and also by the photographs (Fig. 41). p_2 falls below p_{sonic} when the incidence is between $4 \cdot 7$ and $5 \cdot 7$ deg (observations (*c*) and (*d*)), and the bubble then expands rapidly, causing the trailing-edge pressure to diverge from its nearly constant value (Fig. 43), and the shock to halt and move forwards.

The fact that the value of $p_{\text{T.E.}}/H_0$ remains nearly constant for increasing incidence at constant free-stream Mach numbers up to the incidence at which the bubble of separated flow extends rapidly towards the trailing edge seems to be typical of many examples, at least when transition is fixed near the nose on both surfaces, and constitutes a useful empirical result.

For $p_{\text{T.E.}}$ to be constant, the total change in static pressure along the wake $(p_0 - p_{\text{T.E.}})$, must also be constant, since p_0 is fixed for each constant Mach number. The implies that those properties of the wake at the trailing-edge position which determine the static-pressure changes along the wake are, to the first order, unaffected by increase in incidence until the bubble of separated flow has become relatively large.

The separation has a more pronounced effect on the movement of the upper-surface shock than it does for fixed incidence. This difference arises mainly because of the different ways in which the shock movement has to respond to a change in the variable, either incidence or free-stream Mach number, in order to control the development of the separated flow just sufficiently to allow the trailing-edge pressure to remain compatible with the free-stream static pressure. The development of the separated flow is largely determined, for a given aerofoil section, by the variation of shock upstream pressure, p_1. For increasing free-stream Mach number, the value of p_1 often decreases by virtue only of the rearward shock movement, and a retardation of this movement can then give the necessary control on the rate of development of the separated flow*. For increasing incidence at fixed free-stream Mach number, on the other hand, the increasing incidence and the rearward moving shock often both contribute to the fall in p_1 simultaneously, and a halt of the rearward shock movement is then insufficient by itself to give the necessary control on the rate of development of the separated flow, so that the shock actually moves forwards. The fact that the free-stream static pressure is now fixed also tends to increase the change in shock movement required to maintain compatibility between trailing-edge pressure and free-stream static pressure.

For the example under consideration, the point at which separation occurs on the upper surface moves forward with the shock until it reaches the leading edge (Figs. 41 and 43). The peak suctions then begin to show signs of a collapse similar to that which occurs at the low-speed stall. The forward movement of the separation point seems in fact to be analagous to that which occurs at low speeds when the stall occurs as a result of the development of a rear separation.

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* Some examples have, however, been observed in which the increase in free-stream Mach number contributed directly to the fall in p_1 and thereby induced a slight forward movement of the shock.

The divergence of trailing-edge pressure affects the flow on the lower surface (Fig. 40) in such a way as to retard, in come cases even to halt, the rate at which the pressures at fixed points increase with increasing incidence. For other cases in which the separation occurs at a sufficiently low positive incidence for a shock to be present on the lower surface, *i.e.*, for free-stream Mach numbers much nearer to unity, the fall in trailing-edge pressure leads to a reversal of the movement in this shock also, from forwards with increasing incidence to backwards.

The changes described for the shock movements and pressures on the two surfaces must affect the chordwise loading on the aerofoil and be reflected in the variation of force and moment coefficients and their derivatives. The sudden fall in $\partial C_L/\partial\alpha$ shown in Fig. 43 is a typical example.

The position of the upper-surface shock on the 6 per cent RAE 104 aerofoil is plotted against incidence for several different constant Mach numbers in Fig. 44. The presence of separation is denoted by filled symbols; its position, of course, coincides with that of the shock. The divergence in the variation of $p_{\text{T.E.}}$ is delayed, until the rapid expansion of the bubble occurs, as shown by the broken line which is the locus of points (α, M), for the divergence of $p_{\text{T.E.}}$. This locus forms a boundary beyond which the effects of separation are felt on the overall flow. When plotted with α as ordinate and M as abscissa, the boundary bears a marked resemblance to flight determined boundaries for the onset of certain adverse effects of separation.

It is of interest to note from Fig. 44 that separation and the onset of its effects occur at progressively lower Mach numbers and progressively nearer to the leading edge as the incidence is increased. It becomes difficult to judge at what stage the separation ceases to be induced by a shock and is induced instead by the adverse pressure gradient immediately downstream of the peak suction, as at low speeds.

The boundary in Fig. 44, giving the position of the shock, and hence of separation, for the onset of the effects of separation, meets the leading edge for a certain incidence and certain Mach number. The curves giving shock or separation positions for fixed Mach numbers greater than this value approach the leading edge at certain higher incidences. The flow patterns are then almost identical to those which apply for normal leading-edge separations; if separation were to be suppressed, however, a region of attached supersonic would be expected to develop around the leading edge and downstream, with its terminating shock moving back along the chord with increasing incidence or Mach number.

The pressure distributions in Fig. 45 and other observations in Fig. 46 have been included to show how, in a manner similar to that which occurs when the free-stream Mach number is the variable, the effects of separation on the relative flow development on the two surfaces diminish once the pressure at the tailing edge has fallen to the sonic value and so allowed a supersonic expansion to occur there, on the lower surface.

For this example, the effects of the separation, occurring at the foot of the shock on the upper surface, are first felt between 2- and 4-deg incidence (observations (*a*) and (*b*)). They then halt and reverse the rearward movement of the upper-surface shock and the normal increase in pressure at fixed points over the forward part of the lower surface, $\alpha = 5$ deg (observation (*c*)). The trailing-edge pressure falls and reaches the sonic value when the incidence is about 6-deg incidence (not reproduced). As the incidence is increased beyond this, the trailing-edge expansion develops and leads to a further fall in trailing-edge pressure. The upper-surface shock can then resume its rearward movement, $\alpha = 8$ deg (observation (*d*)). Moreover, the fall in trailing-edge pressure can now occur without inducing a fall over the forward part of the lower surface and the pressures at fixed points there now begin to rise again.

As would be expected from the changes in pressure distributions, $\partial C_L/\partial\alpha$ increases again once the trailing-edge pressure has fallen to the sonic value (Fig. 46). At fairly high Mach numbers the occurrence of sonic pressure at the trailing edge often leads to the absence of any definite maximum in the curve of C_L versus α[24].

10. *Concluding Remarks.*—The value of the local Mach number upstream of the shock, and that of the corresponding shock strength, for which separation can be expected to occur on a two-dimensional aerofoil were already fairly well established[4]; the present analysis has revealed that, for increasing upstream Mach number beyond this value, the bubble does not extend far downstream or have any serious consequences until the pressure rise through the shock has failed to restore subsonic flow immediately downstream, *i.e.*, until the shock downstream pressure has fallen to the sonic value or below. The bubble thereupon expands rapidly and begins to affect the overall flow. The delay, in terms of free-stream Mach number or incidence, between the first occurrence of separation and the onset of its most serious effects depends on the rate at which the local conditions change with these variables, and in general is greater for high incidences than for low and for thin sections than for thick ones.

The rapid expansion of the separation bubble on the upper surface causes a loss in downstream pressure recovery which combines with the loss in shock pressure rise (below that expected in the absence of separation), to give as shown previously in Ref. 4, a more forward position of this shock for a given trailing-edge pressure. Since the trailing-edge pressure has to be the same on both sides of the wake (to fulfil the condition of equality), this implies a more forward upper-surface shock position for a given flow on the lower surface and has a marked adverse effect on the relative rates of flow development on the two surfaces, including the relative shock movements.

It is well known that the trailing-edge pressure falls rapidly with increase of either free-stream Mach number or incidence beyond the value for which the effects of separation are first felt. It is now suggested that this fall occurs because the development of the separated flow affects the variation of static pressure along the wake, which often contains the rear part of the 'dead-air' region between the shear layers from the two surfaces. The trailing-edge pressure falls in response to the changes along the wake, and the flow on the aerofoil develops, at just the rate that will allow its value to remain compatible with the free-stream static pressure, *i.e.*, to fulfil the condition of compatibility. It follows that the flow along the wake controls the actual rates of shock movement or flow development on the two surfaces of the aerofoil, as distinct from the relative rates.

The distinction between relative and actual rates of flow development helps, we believe, to explain the significance of the stage at which the shock on one of the two surfaces reaches the trailing edge, or alternatively, for higher incidences, the stage at which a supersonic expansion occurs there. The relative and actual rates of flow development are both affected in such a way as to encourage a fairly rapid decay of the effects of separation. This decay is associated also with the collapse of the bubble of separated flow which occurs as the flow pattern changes to that which applies for the sonic range of speeds.

Clearly, further and more rigorous work of a fundamental nature would be required to check the validity and usefulness of the speculative ideas which have been presented on these matters, and will be essential if the problem is to be understood fully. Meanwhile attention is likely to be attracted to the practical applications of what is already known with some certainty and to the possibilities of extending them by broadening the scope of the investigations. The most important applications for aircraft at present fall broadly under the headings:

(i) Prediction of the occurrence of separation and of the onset of its effects

(ii) Detection of the onset of its effects

(iii) Methods of prevention and alleviation.

(i) *Prediction.*—The knowledge of the value of the local static pressure p_1, or Mach number M_1, immediately upstream of the shock for which separation will first occur on an aerofoil[4] can be used in conjunction with the appropriate sonic-range pressure distribution to determine whether or not separation will occur for a given aerofoil and incidence at some free-stream Mach number below $1 \cdot 0$ (*see* footnote to Section 3.1).

Although the various effects of separation are not always first felt exactly simultaneously, the stage at which $p_2 = p_{\text{sonic}}$ often marks quite well the onset of serious overall effects. The local conditions, upstream of the shock, for which this stage is reached have not been investigated systematically and are probably affected more by surface curvature than are those for the first occurrence of separation; further work is needed here and analysis of existing observations might prove helpful. Meanwhile, the stage at which the separation pressure p_s equals p_{sonic} approximates more closely to the onset of the effects of separation than does the stage at which separation first occurs (it is, in fact, sometimes closer than the stage at which $p_2 = p_{\text{sonic}}$ at low incidences). This has the very real advantage of being defined approximately by a single value of shock upstream pressure, p_1, namely,

$$p_1 = p_s/1\cdot 4 = 0\cdot 528H_0/1\cdot 4 = 0\cdot 377H_0$$

or by

$$M_1 = 1\cdot 27\ ,$$

and therefore of lending itself more readily to prediction for any given section (*see* footnote to Section 5.5, para. 4).

More data on aerofoil pressure distributions or, alternatively, satisfactory theoretical or empirical methods of calculating them, are required before predictions can be made of the free-stream Mach numbers for which the critical condition will be reached for any aerofoil and incidence.

Very little is known about the conditions for the occurrence of turbulent separation on swept wings at transonic speeds and some preliminary work is planned in this important field. Attention will also be paid to the manner in which the effects of shock-induced separation first develop and, in particular, to whether the boundary-layer outflow downstream of separation modifies the conditions for reattachment and so reduces the delay between the occurrence of separation and the onset of its effects.

(ii) *Detection.*—The onset of the effects of separation on the overall flow for a two-dimensional aerofoil, or for a given spanwise station of a three-dimensional wing, can be detected by observations of the divergence of the trailing-edge pressure from its normal variation, or of that of the static pressure at a point just upstream of the trailing edge on the suction surface.

(iii) *Prevention and Alleviation.*—Much can be done to avoid, to delay or to minimise the serious effects of separation by suitable choice of section, plan-form and control system, but many more data are still required for design purposes, even on the relative merits of section shapes. As an example, it is known that thin sections have many advantages under certain conditions, including the delay in the occurrence of separation (and possible elimination) due to the reduced local Mach numbers, and the reduction in the severity of its effects due to the reduced surface curvatures and trailing-edge angles (*see* Section 6.8). Some doubt remains, however, as to the extent to which these advantages persist at high incidences for conditions under which separation occurs near the nose.

The possibilities of boundary-layer control to suppress the separation or to reduce its severity are being actively considered as described in Ref. 27. In addition to developing methods for realizing the obvious practical gains which can be anticipated from the effects of separation on section lift and associated quantities (*see* Sections 8 and 9), this work should provide data in the absence of separation under conditions for which it would otherwise occur; data which will therefore be of great value in assessing the accuracy of the deductions and speculations which have been made about the effects of separation (such few relevant data as are so far available are reassuring in this respect; *see*, for example, Refs. 6 and 27).

11. *Acknowledgments.*—Many of the ideas put forward in this paper have developed from discussions with colleagues in the High-Speed Group of the Division. Several members of the Group have been involved from time to time in obtaining the experimental results used. The help received in the preparation of the paper is also gratefully acknowledged.

NOTATION

Symbols

p	Static pressure
H	Stagnation pressure
M	Mach number
U	Velocity just outside of the boundary layer
θ	Density
Π	Pressure-recovery factor, downstream of a shock (defined in Section 5.4)
δ^*	Wake displacement thickness
θ	Wake momentum thickness
$\mathscr{H}$	$\dfrac{\delta^*}{\theta}$
C_L	Lift coefficient
C_D	Drag coefficient
c	Aerofoil chord
x	Distance along the chord from the leading edge
α	Aerofoil incidence
τ	Trailing-edge angle
λ	Angle between centre-line and chord-line at the trailing edge

Suffixes

$_0$	Value of the quantity in the free stream
$_{\mathrm{T.E.}}$	Value at the trailing edge
$_1$	Value just upstream of the shock
$_2$	Value just downstream of the shock
$_s$	Value at the separation point
$_{\mathrm{sonic}}$	Value at the point at which the velocity is sonic
$_{\mathrm{sh}}$	Value at the shock (e.g., x_{sh} = chordwise position of shock)

Definitions

p_1	Shock upstream pressure
p_2	Shock downstream pressure
p_2/p_1	Shock strength
$p_2 - p_1$	Shock pressure rise
$p_{\mathrm{T.E.}} - p_2$	Downstream pressure recovery

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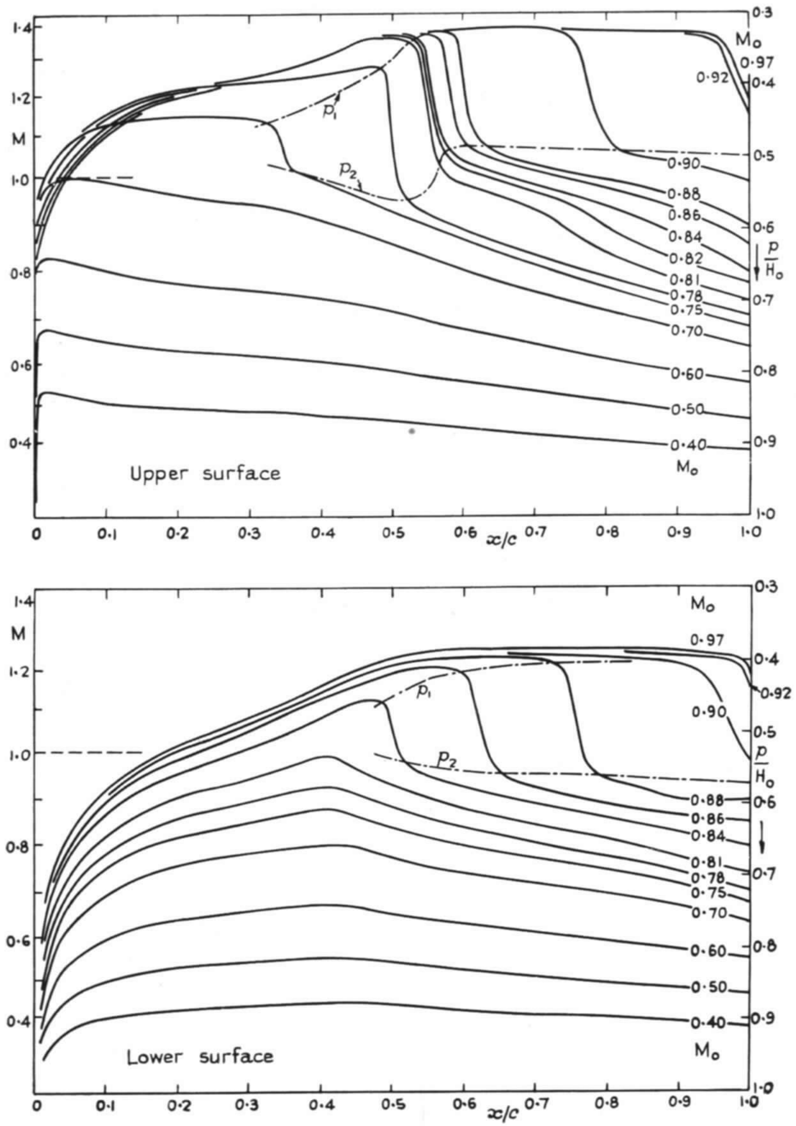


FIG. 1. Surface pressure distributions for a symmetrical aerofoil at a fixed positive incidence (10 per cent thick RAE 102 section at 2-deg incidence).

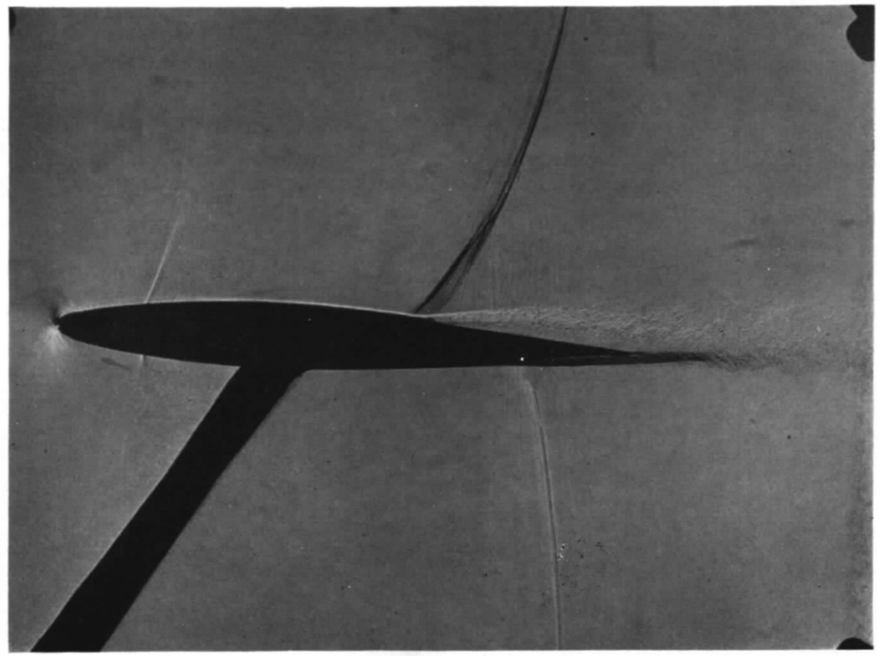


FIG. 2. Schlieren photograph of the flow past an aerofoil with severe separation at the foot of the upper-surface shock; 10 per cent RAE 102 section at 2-deg incidence ($M_0 = 0 \cdot 88$).

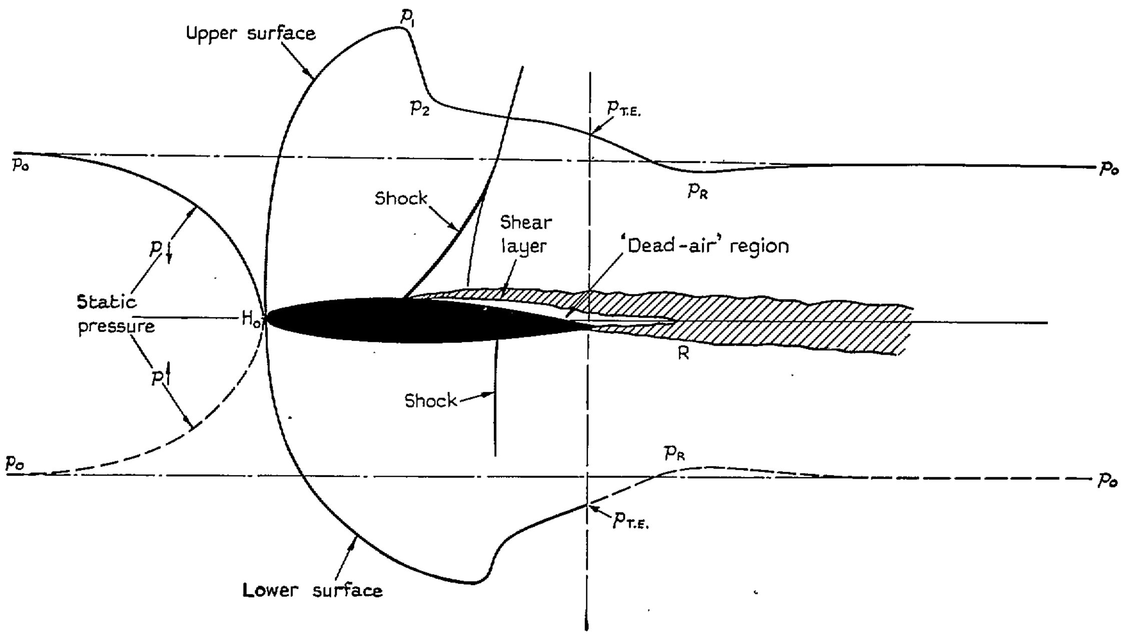


FIG. 3. Sketch of certain features of the flow about an aerofoil and along its wake in the presence of shock-induced separation on the upper surface.

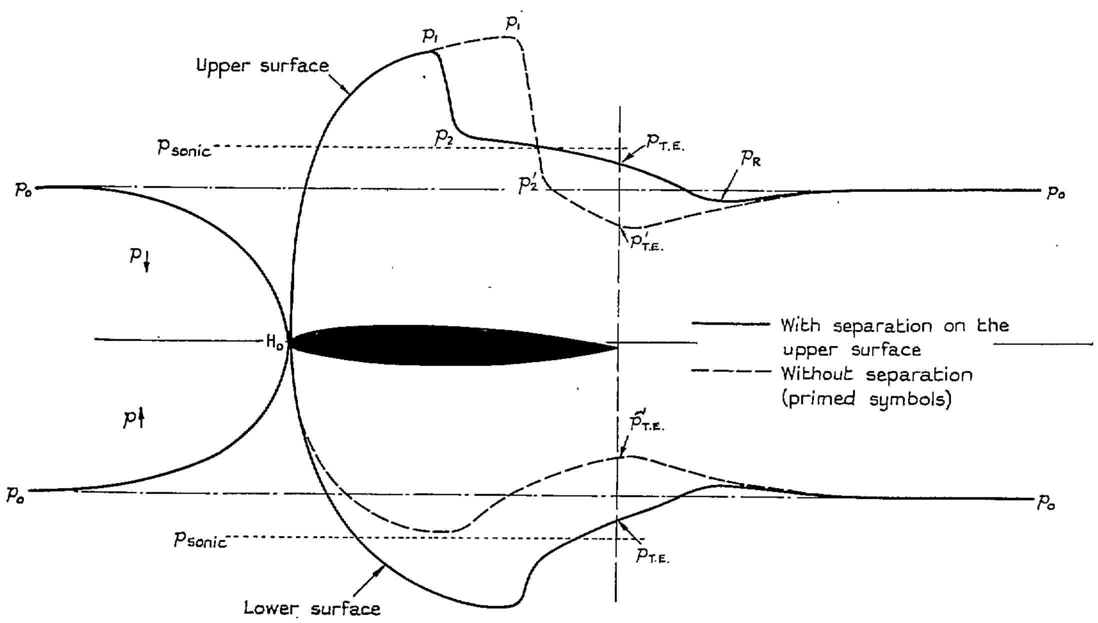


FIG. 4. Sketch of pressure distributions over the surfaces of an aerofoil and along the wake, with and without separation on the upper surface.

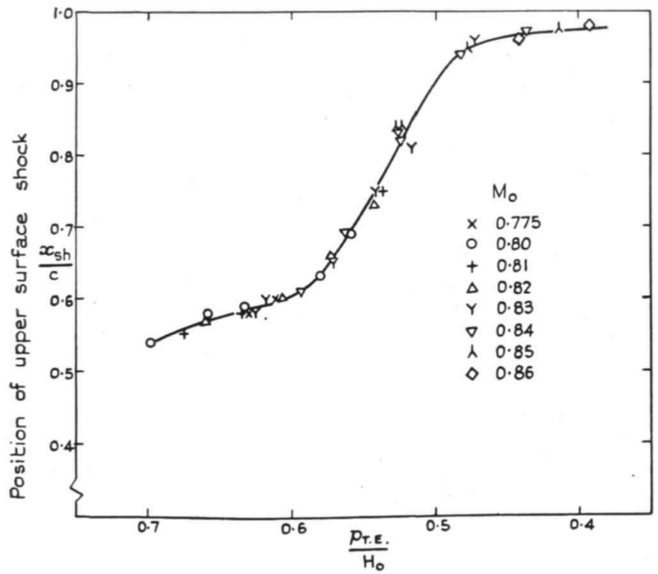


FIG. 5. Variation of upper-surface shock position with trailing-edge pressure, as adjusted by changing the height and position of a small strip on the lower surface.

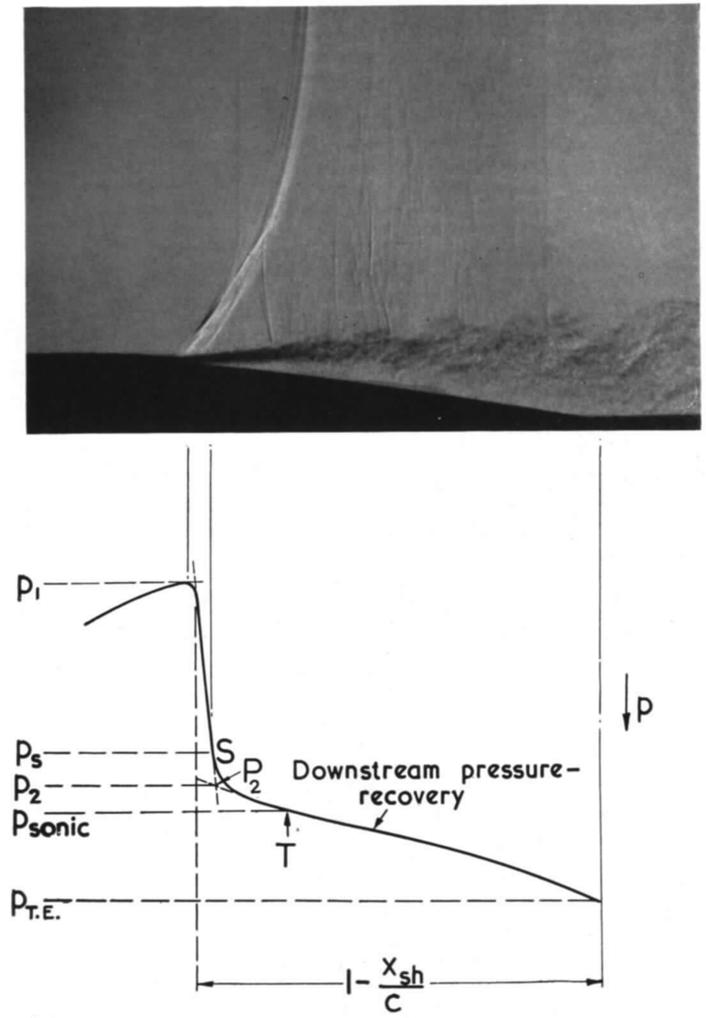


FIG. 6. Details of the flow and surface-pressure distribution near and downstream of separation; definition of certain pressures.

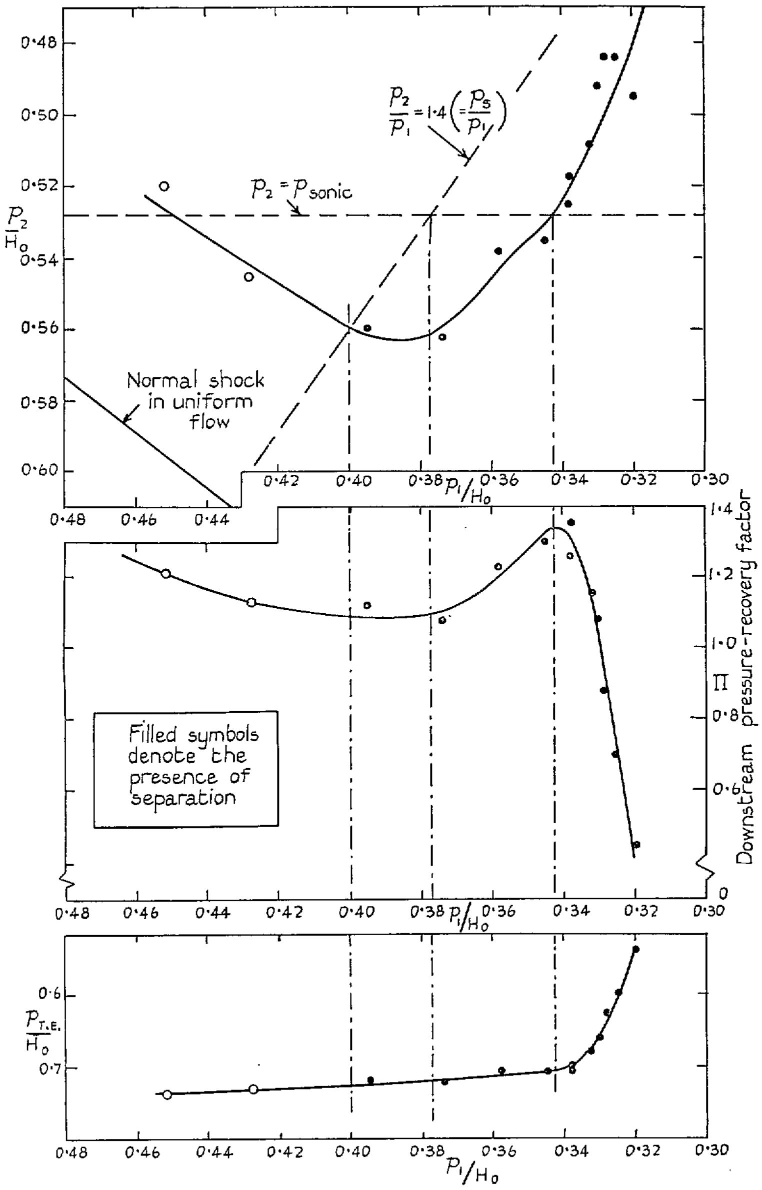


FIG. 7. Variation with shock upstream pressure p_1 of certain quantities derived from the upper-surface pressure distribution (for definitions *see* Fig. 6) (10 per cent thick RAE 102 aerofoil at 2-deg incidence).

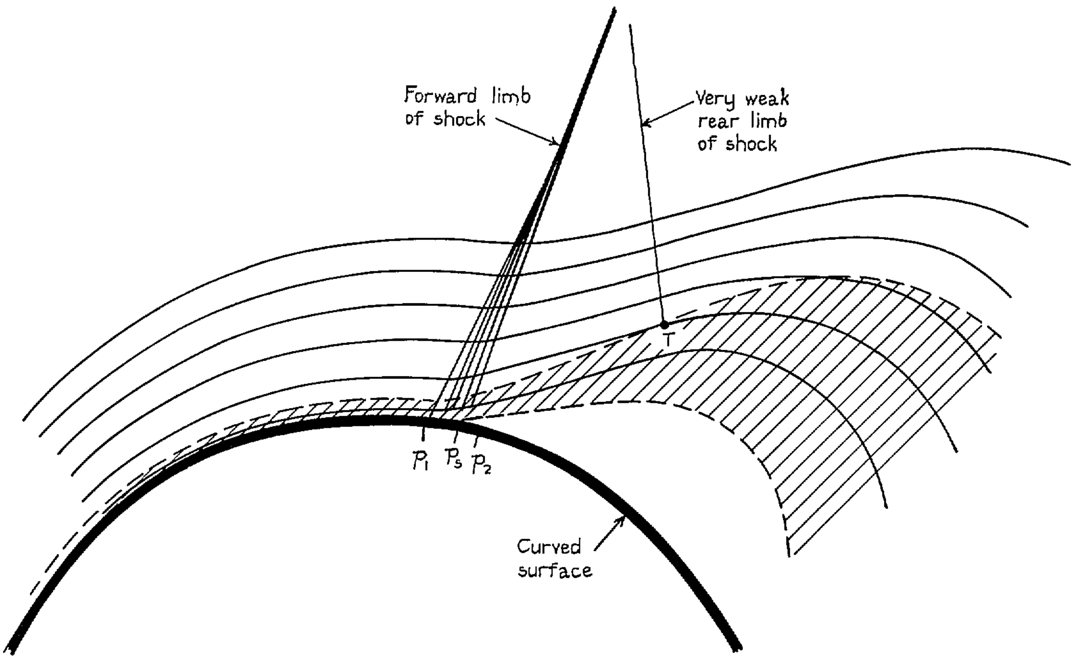


FIG. 8. Diagrammatic sketch of streamlines and boundary flow near separation on a curved surface (curvature greatly exaggerated). T is the approximate position at which the velocity along the streamline has fallen to the sonic value.

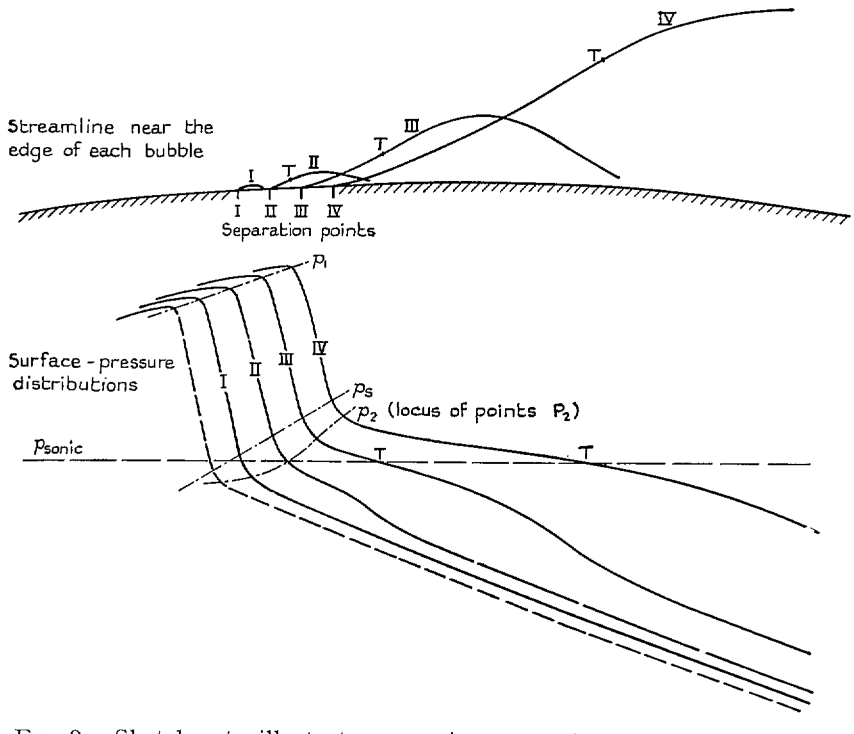


FIG. 9. Sketches to illustrate successive stages in the development of the separation 'bubble' for a prescribed variation of p_1.

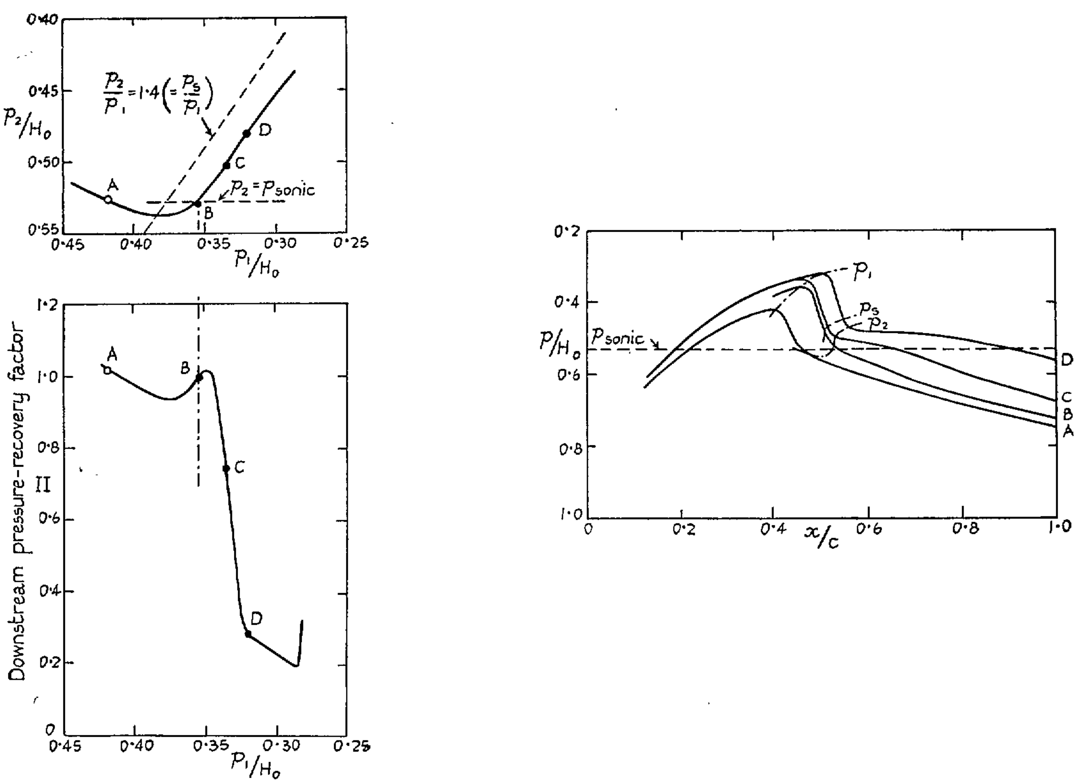
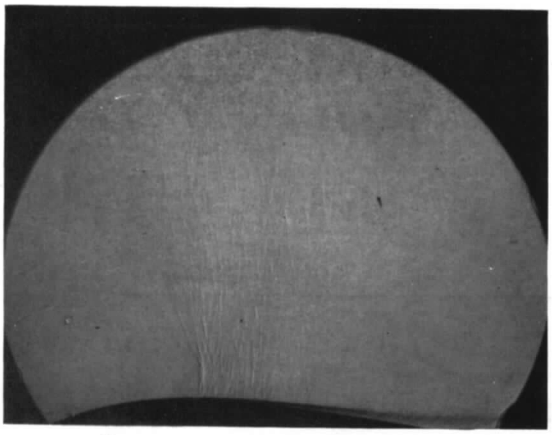
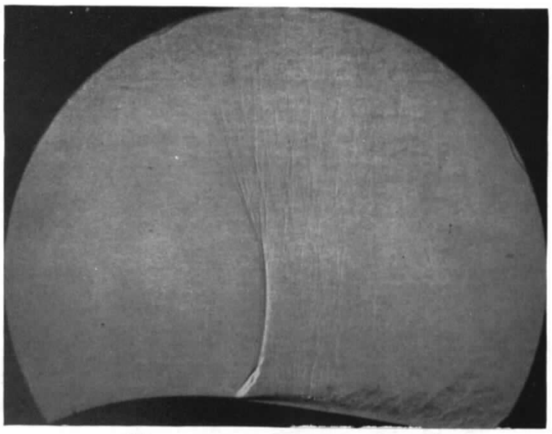


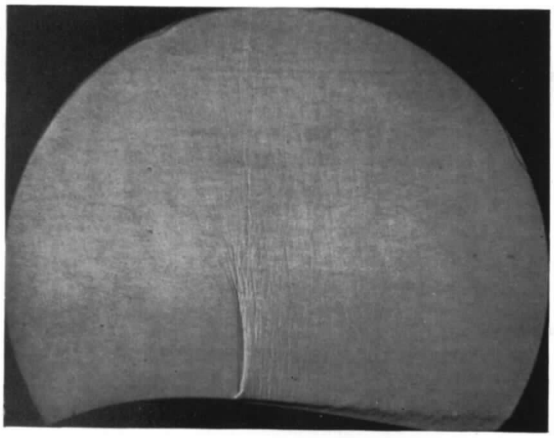
FIG. 10. Surface-pressure distributions, and quantities derived therefrom, at stages in the development of the separated flow (8 per cent thick half-aerofoil) (*See* Fig. 11 for corresponding flow photographs).



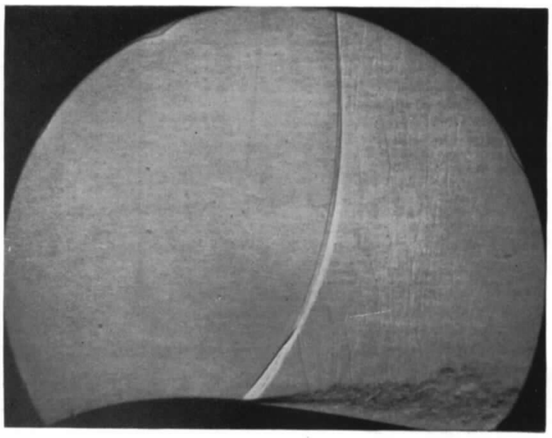
A; $p_1 = 0{\cdot}42\ H_0$



C; $p_1 = 0{\cdot}335\ H_0$



B; $p_1 = 0{\cdot}355\ H_0$



D; $p_1 = 0{\cdot}32\ H_0$

FIG. 11. Photographs showing the development of the separated flow (8 per cent thick half-aerofoil) (*See* also Fig. 10).

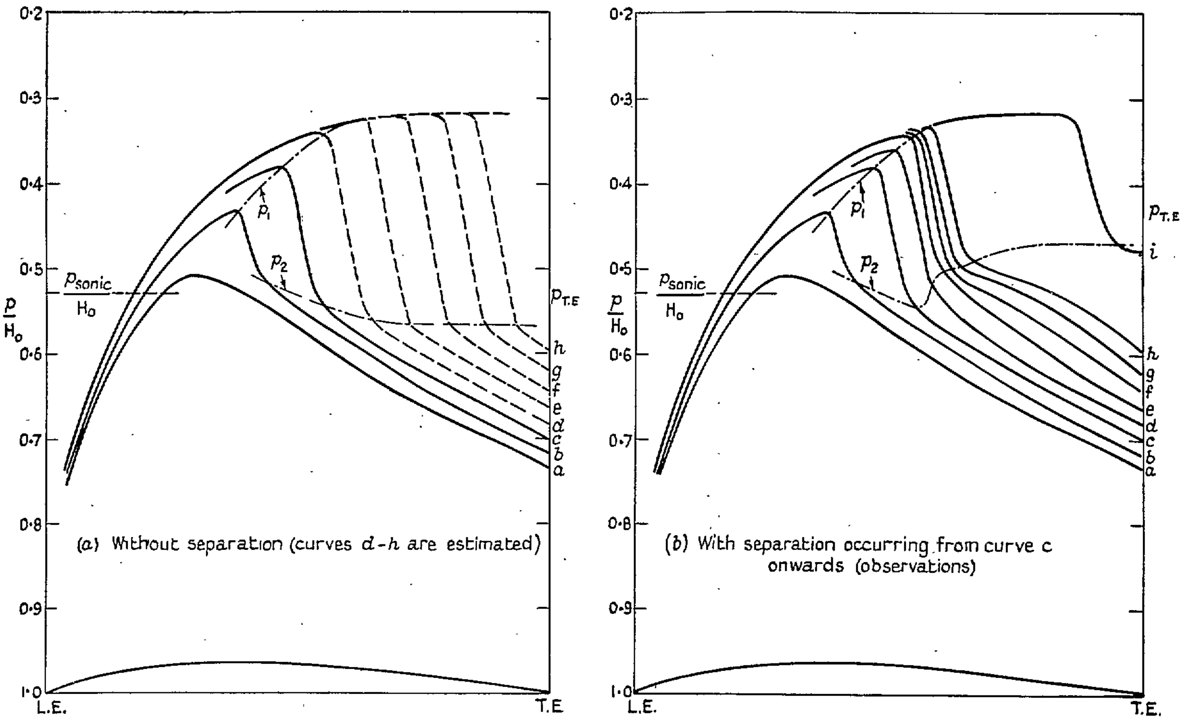


FIG. 12. Surface-pressure distributions showing the movement of the shock for a prescribed variation in trailing-edge pressure (*See* Fig. 13 for shock position plotted against $p_{T.E.}/H_0$).

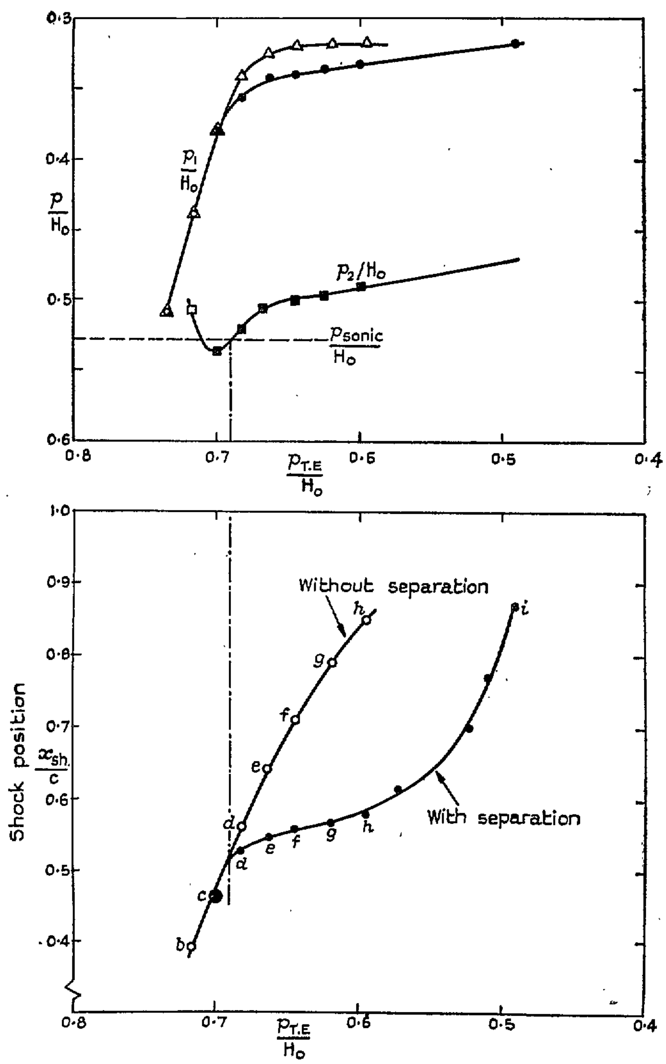


FIG. 13. Variation of shock position with trailing-edge pressure (*See* Fig. 12 for corresponding pressure distributions).

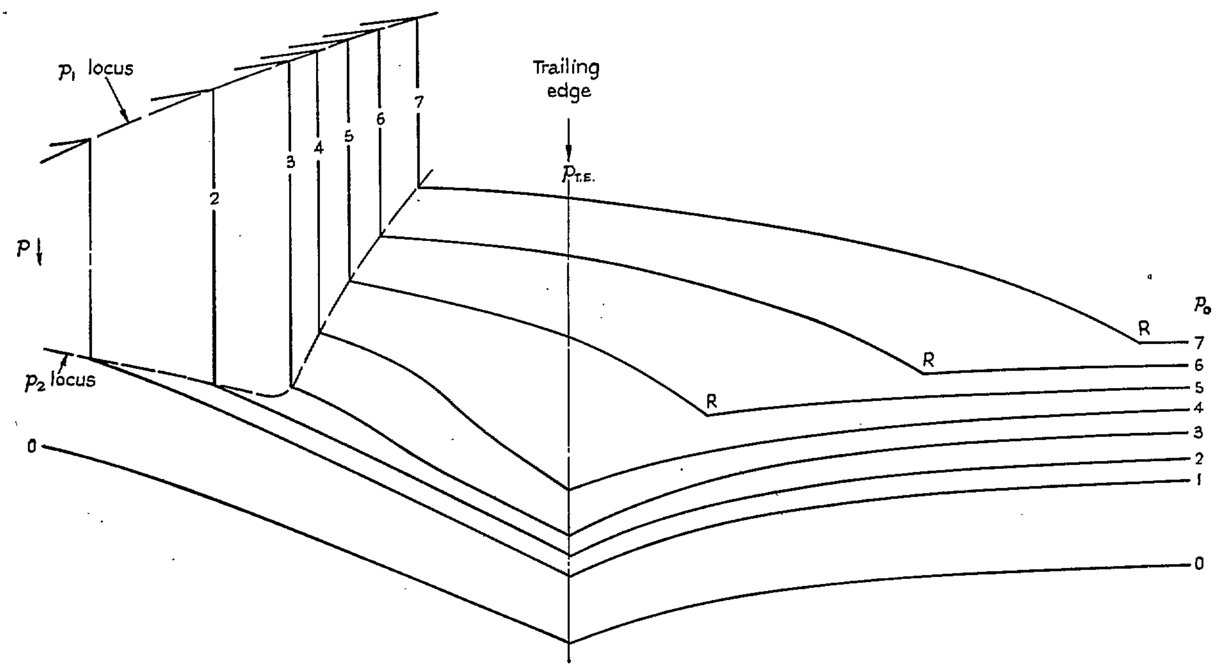


FIG. 14. Idealized sketches of the distributions of pressure from a point just upstream of the upper-surface shock, along the surface and wake to successive values of the free-stream static pressure.

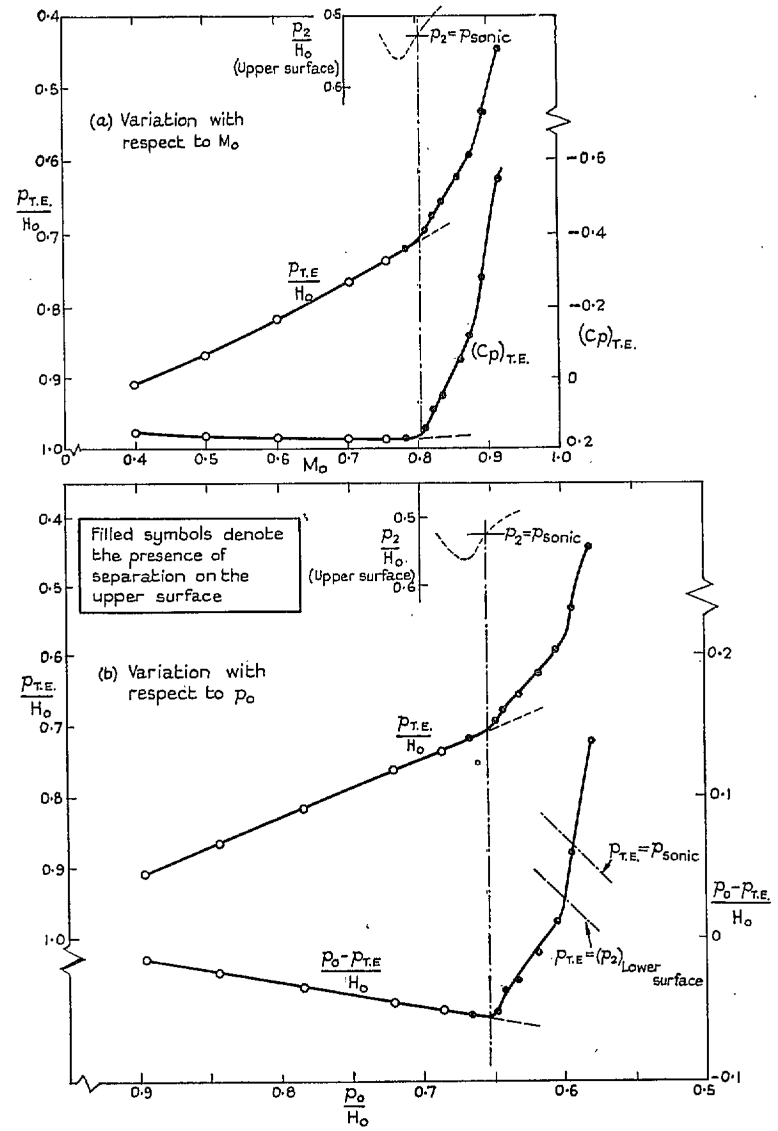


FIG. 15. Variation of trailing-edge pressure $p_{T.E.}$ for fixed incidence; different methods of plotting to show the divergence from the normal (pre-separation) variation (10 per cent thick RAE section at 2-deg incidence).

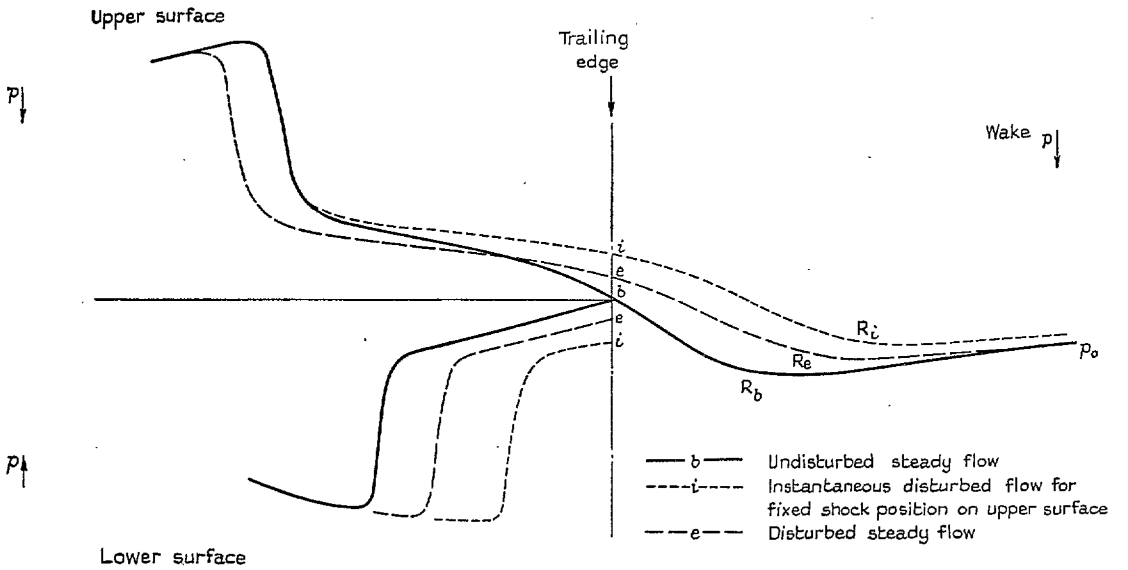


FIG. 16. Sketch of pressure distributions over the rear of an aerofoil and along the wake to illustrate the effect of a hypothetical disturbance to the separated flow near the foot of the shock on the upper surface.

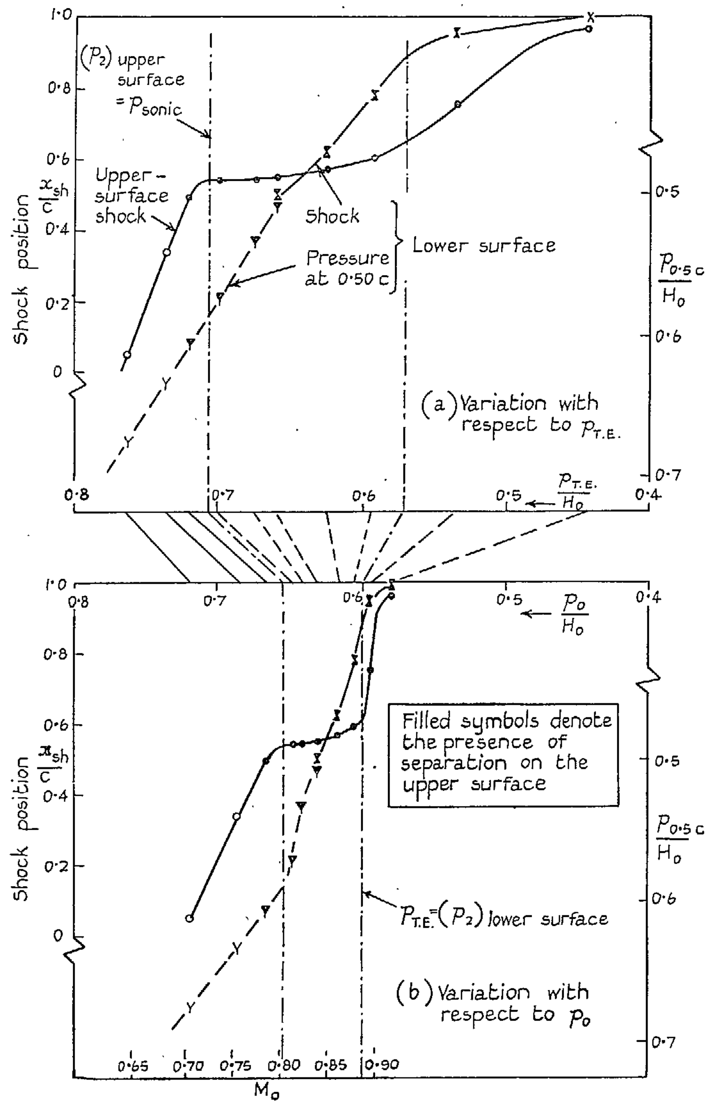


FIG. 17. Variation of a flow parameter for each surface of an aerofoil to show the rate of development of the flow on the two surfaces relative to one another (both diagrams) and with respect to the free-stream static pressure (lower diagram only) (10 per cent thick RAE 102 section at 2-deg incidence).

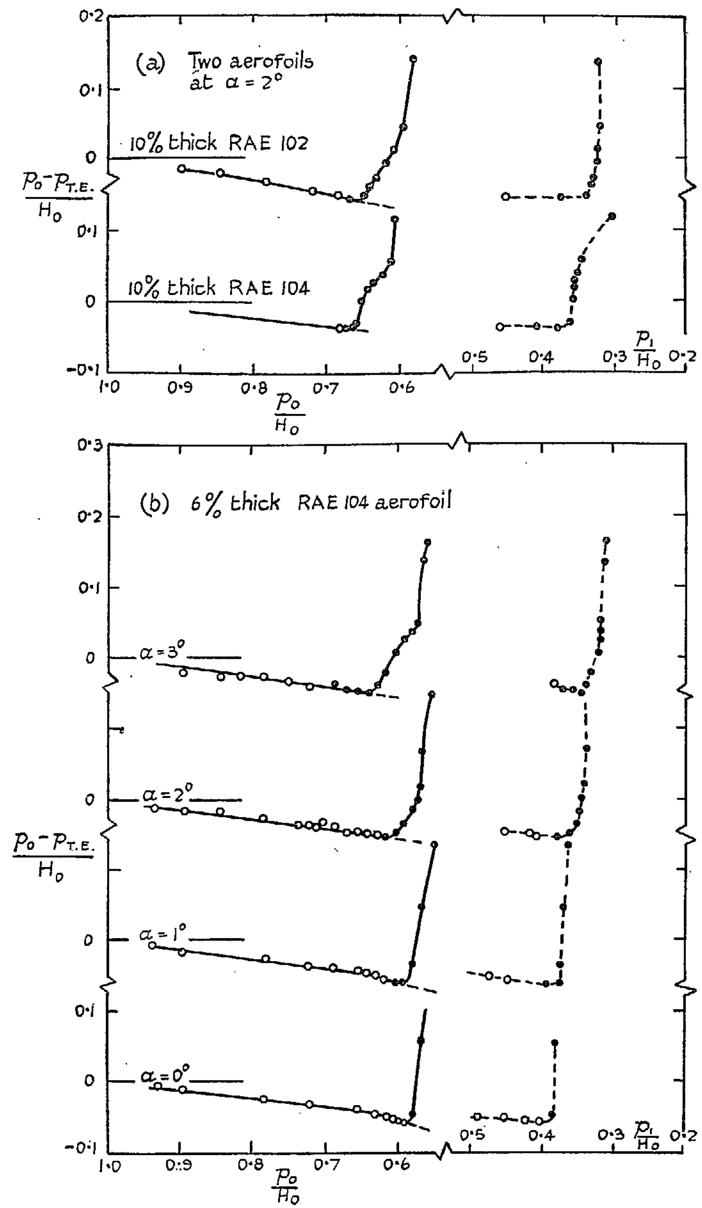


FIG. 18. Effect of section shape and aerofoil incidence on the variation of trailing-edge pressure (Filled symbols denote the presence of separation on the upper surface).

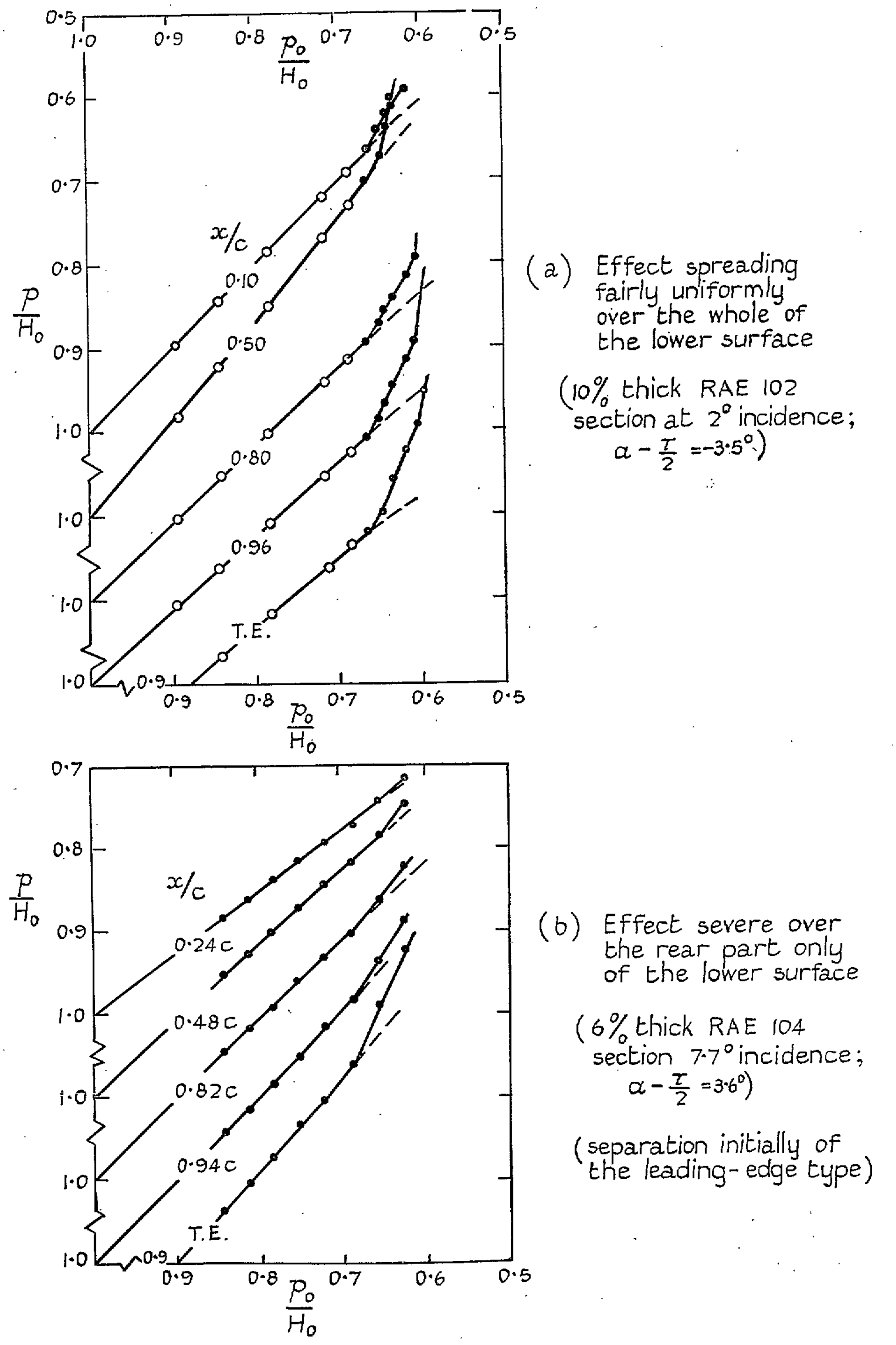


FIG. 19. Variation of pressures at fixed positions on the lower surface showing the effect of the divergence of the trailing-edge pressure (Filled symbols denote the presence of separation on the upper surface).

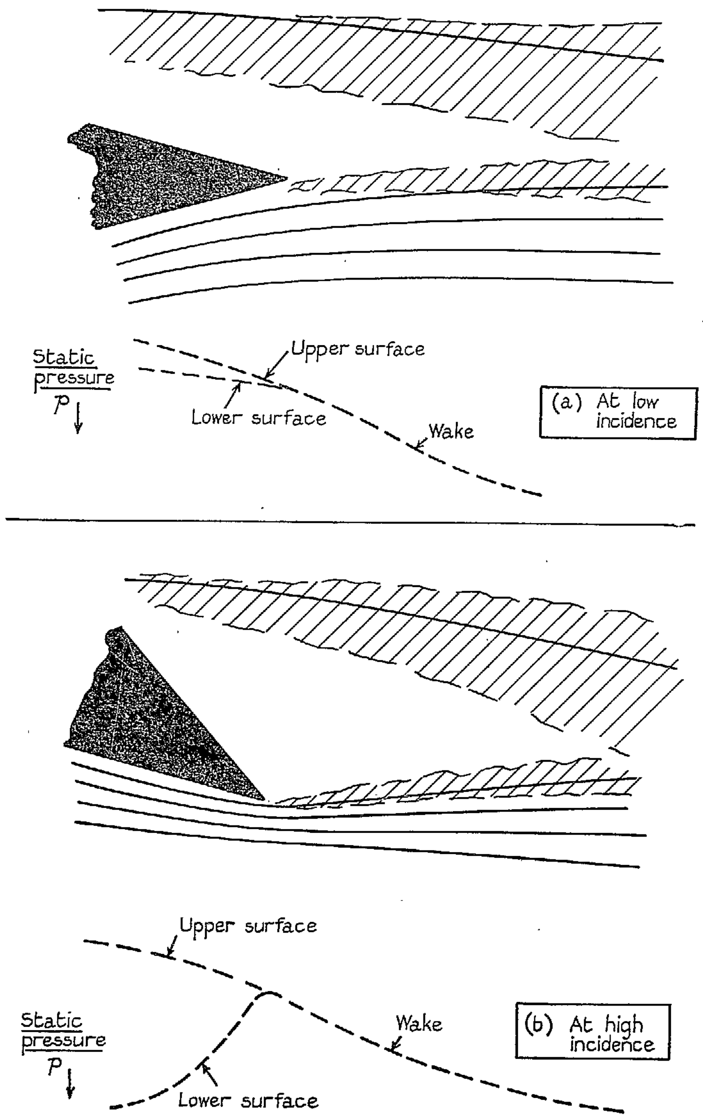
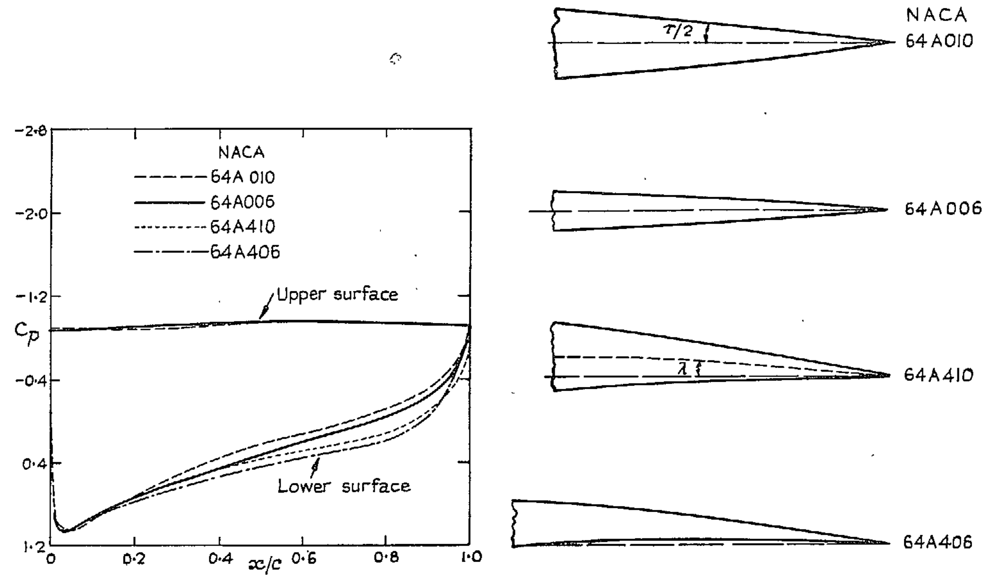


FIG. 20. Sketches to illustrate the influence of the upper-surface separation on the flow near the trailing edge on the lower surface; low and high incidence.



(a) Surface-pressure distributions for $\alpha = 28°$, $M_0 = 0{\cdot}5$.

(b) The trailing-edges of the four related sections.

FIG. 21. The influence of trailing-edge angle and camber on the pressures over the lower surface in the presence of severe separation on the upper surface (results for four related NACA sections; Ref. 23).

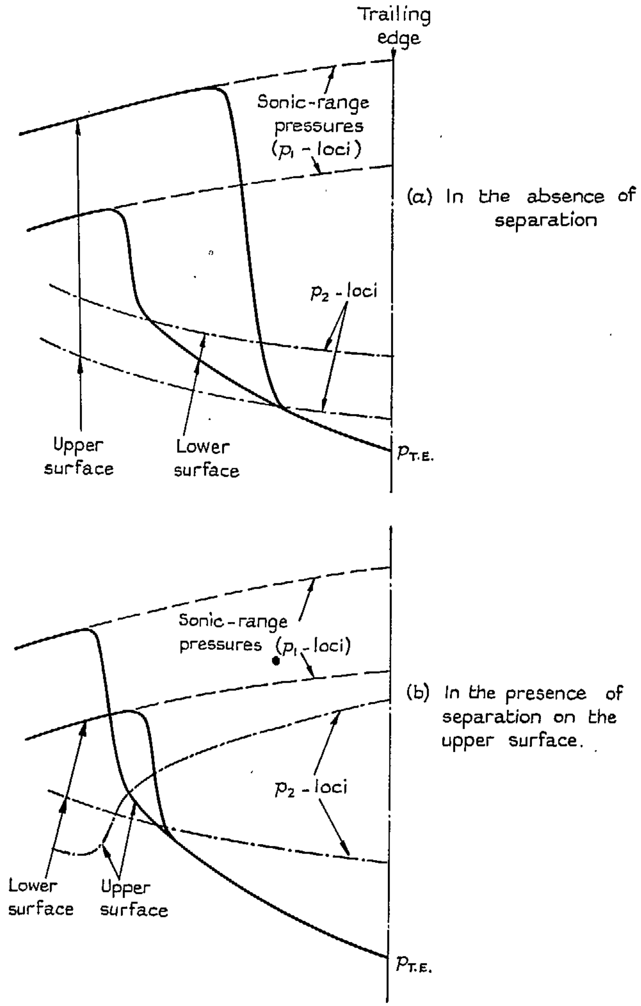


FIG. 22. Sketches of surface-pressure distributions to show relative shock positions for an aerofoil at a fairly low incidence and a Mach number close to unity.

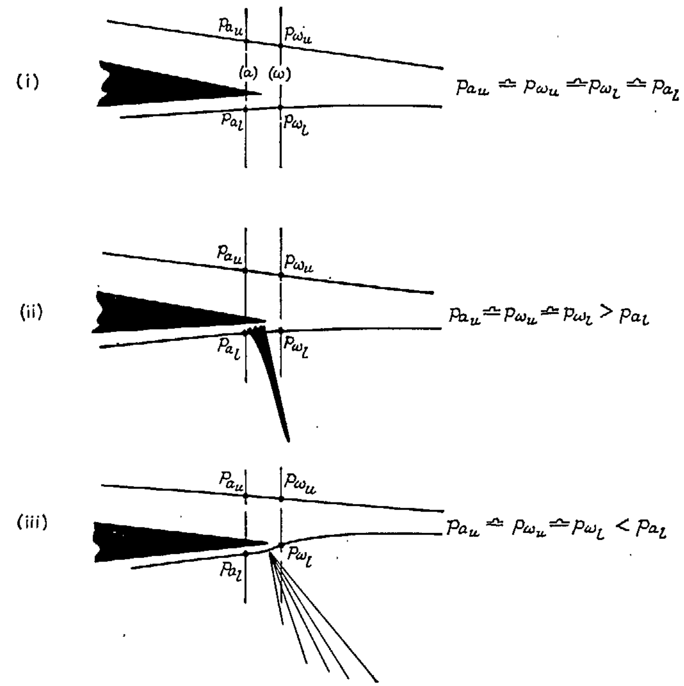


FIG. 23. Pressures at points immediately upstream and immediately downstream of the trailing edge.

(i) With subsonic flow over the trailing edge
(ii) With a shock at the trailing edge on the lower surface
(iii) With a centred expansion at the trailing edge on the lower surface.

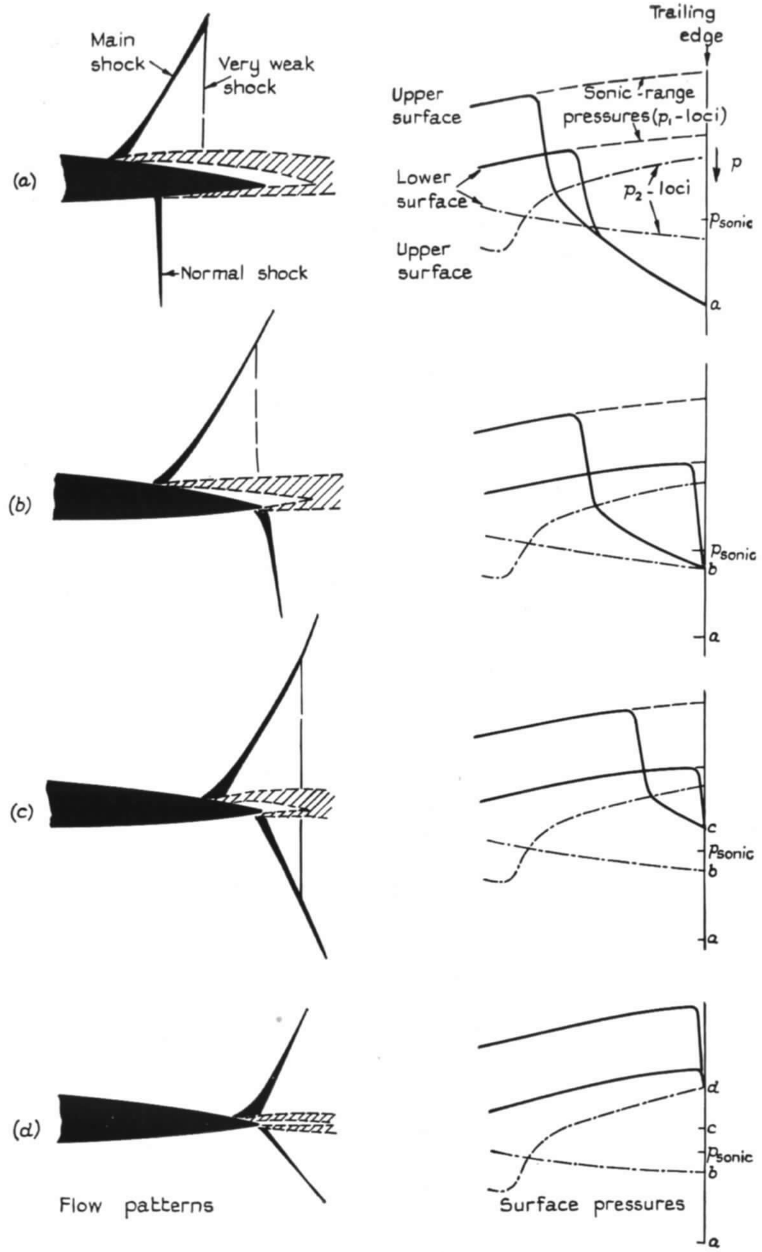


FIG. 24. Sketches to illustrate the changes in the flow on the two surfaces of an aerofoil relative to one another once the lower-surface shock has reached the trailing edge.

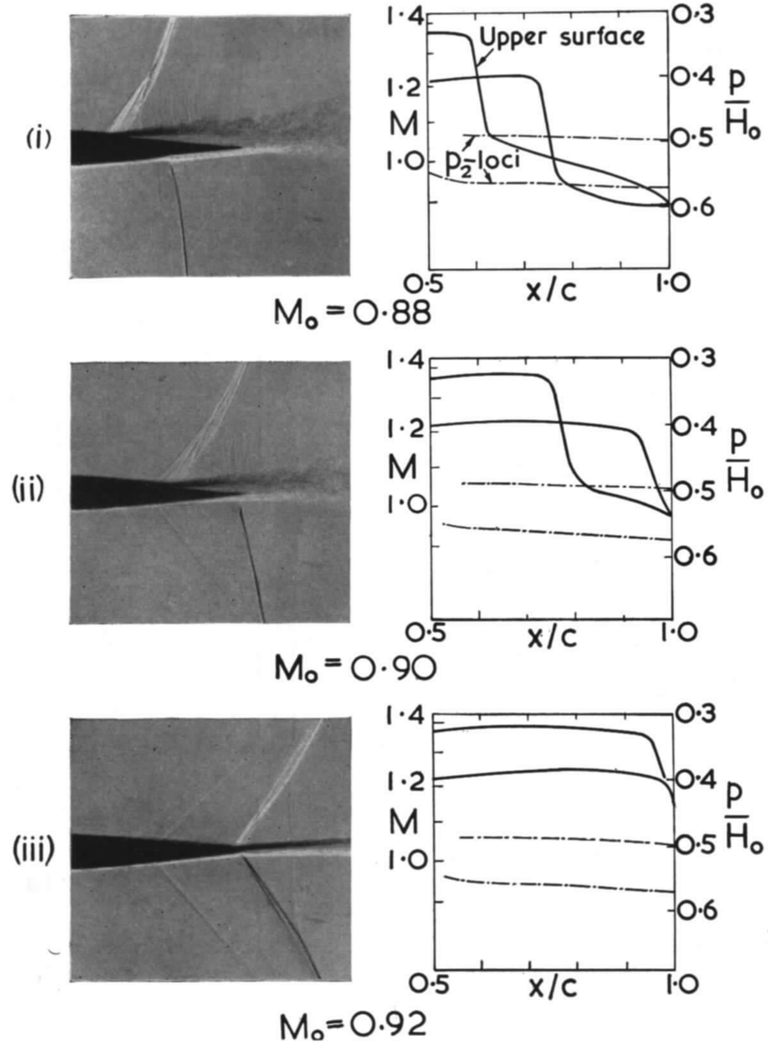


FIG. 25. Observations showing the changes in the flow on the two surfaces of an aerofoil relative to one another once the lower-surface shock has reached the trailing edge (10 per cent RAE 102 section at 2-deg incidence) (*See* sketches in Fig. 24).

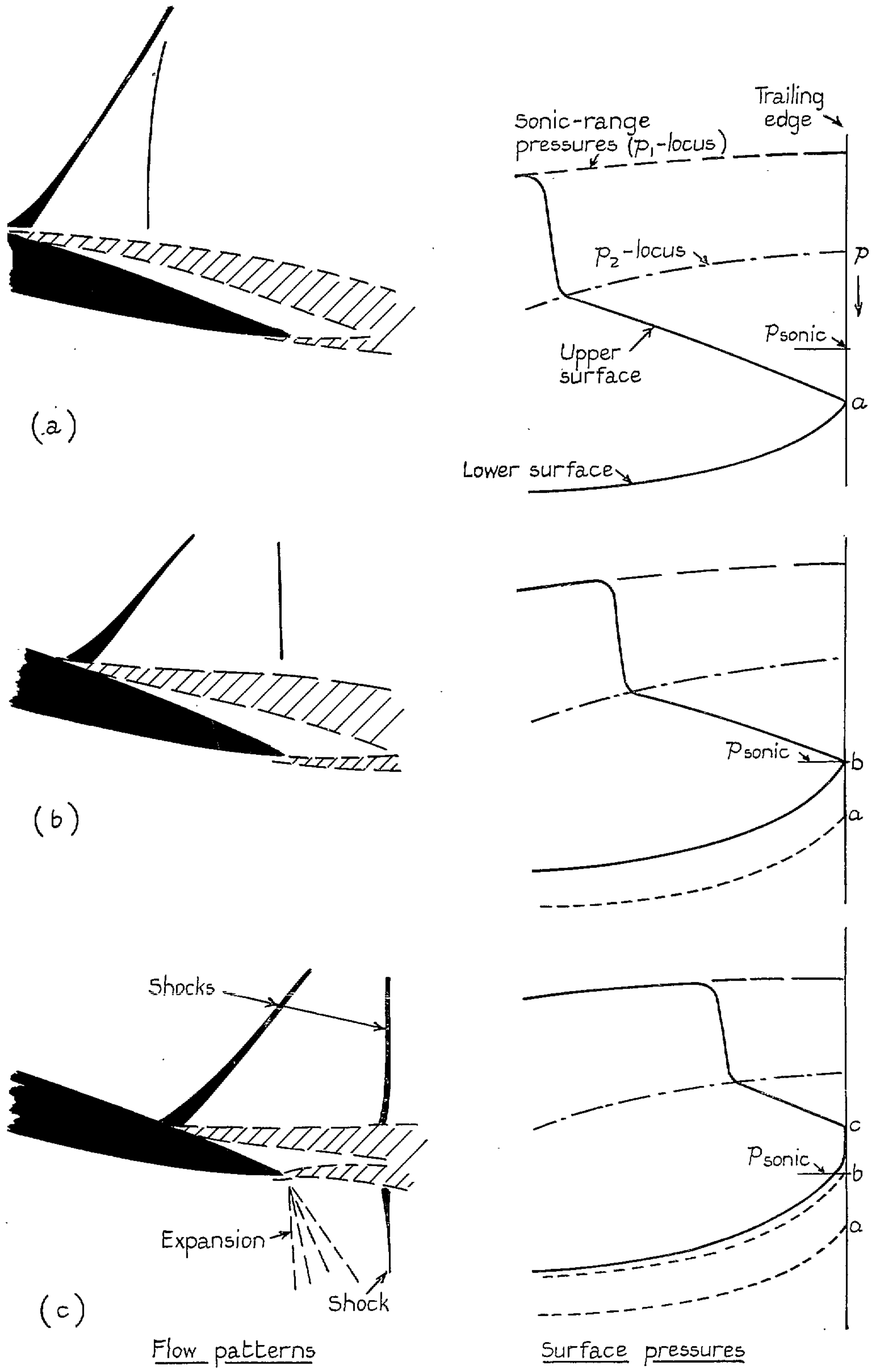


FIG. 26. Sketches to illustrate the changes in the flow on the two surfaces of an aerofoil relative to one another once a supersonic expansion can occur at the trailing edge on the lower surface.

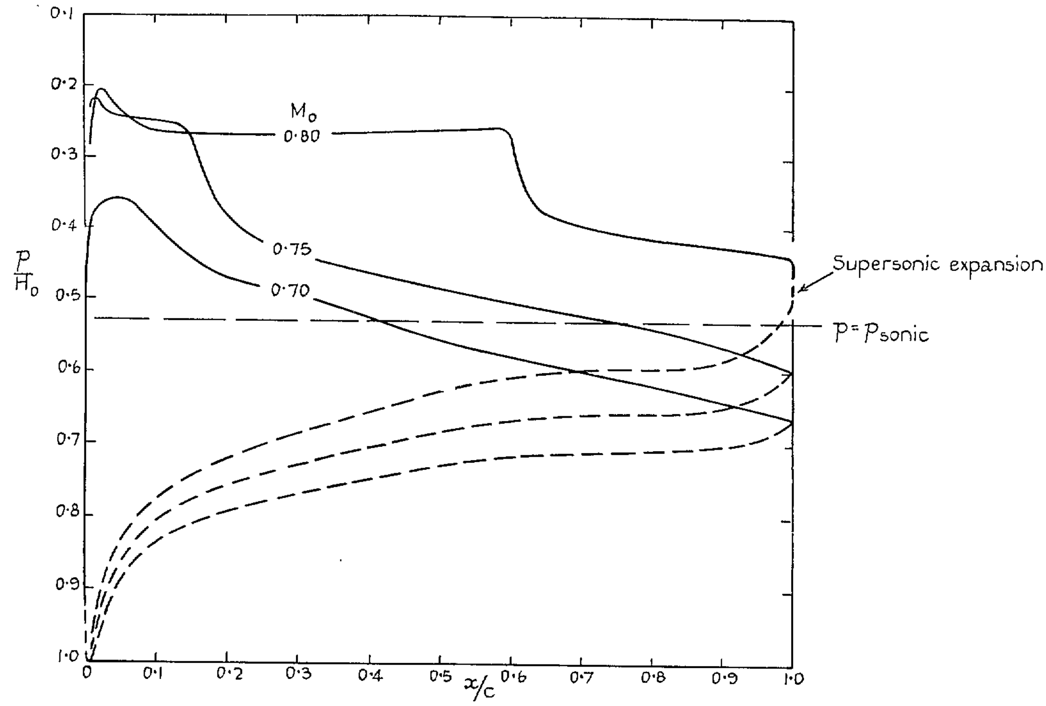


FIG. 27. Observed pressure distributions showing the changes in the flow on the two surfaces of an aerofoil relative to one another when a supersonic expansion occurs at the trailing edge on the lower surface (6 per cent thick aerofoil at 8-deg incidence) (*See* sketches in Fig. 26).

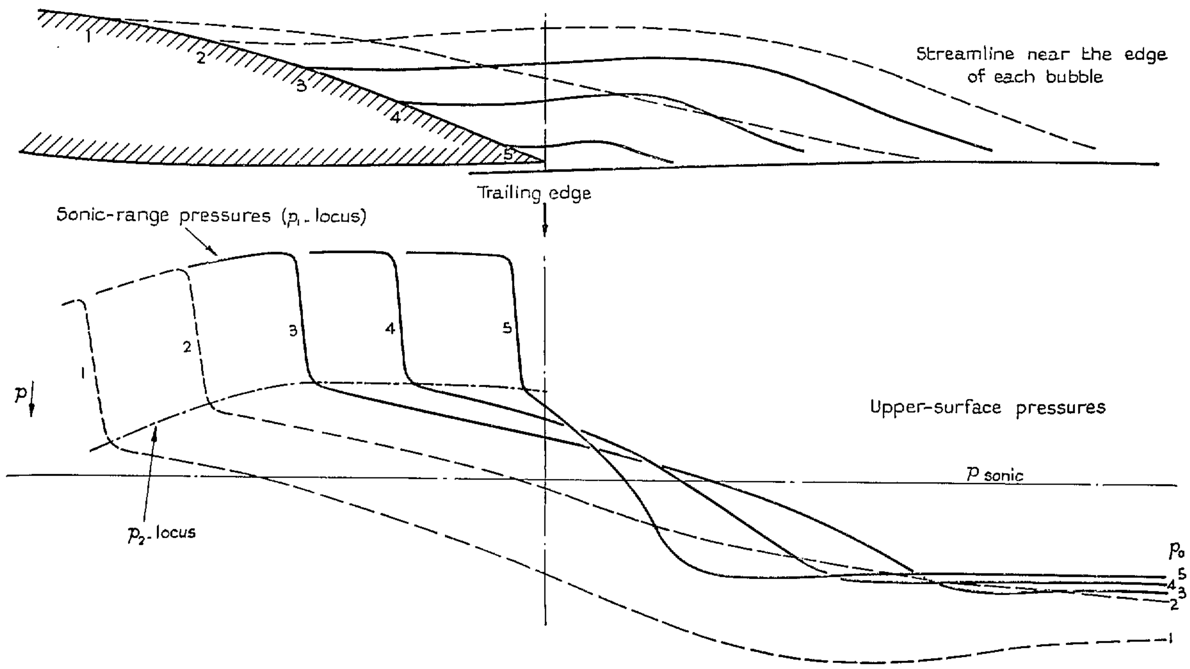


FIG. 28. Sketches to illustrate probable successive stages in the collapse of the separation 'bubble' as the upper-surface shock approaches the trailing edge.

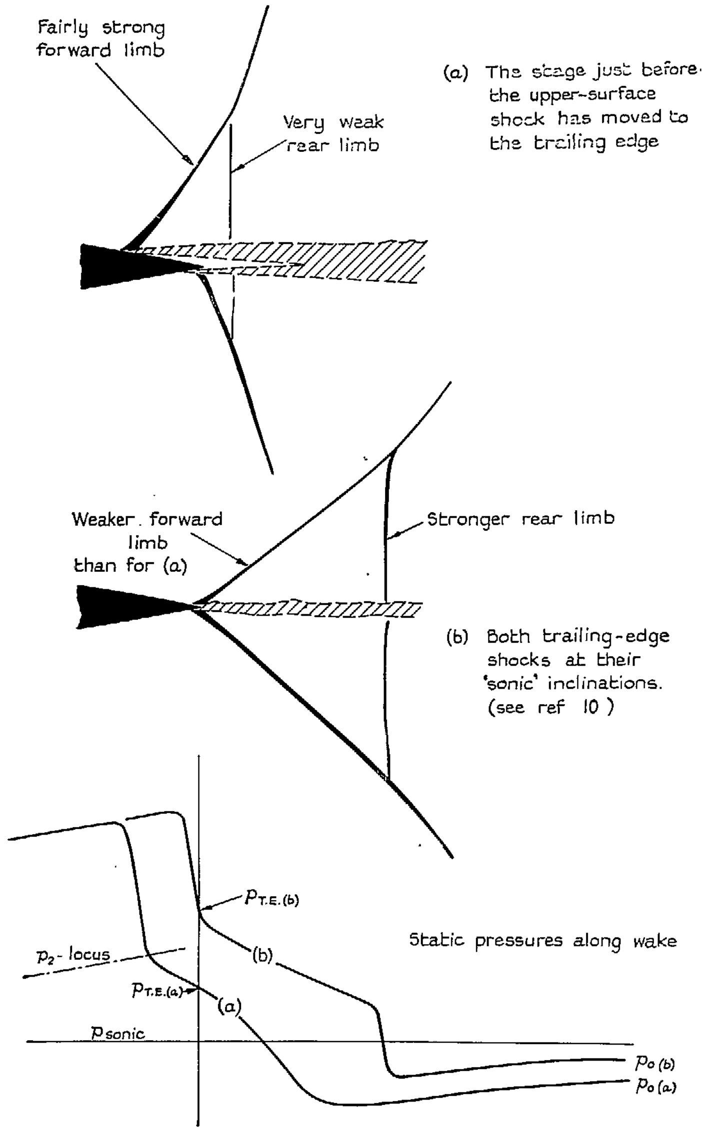


FIG. 29. Movement of the shocks into their 'sonic' positions and inclinations.

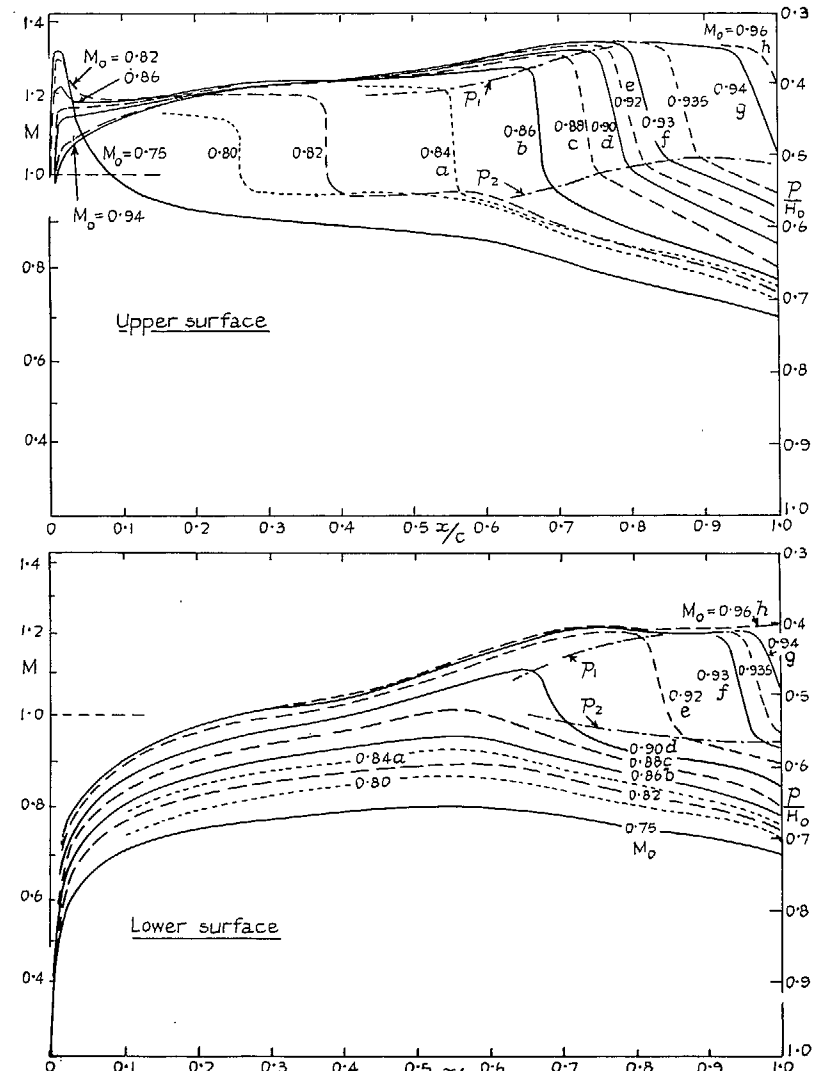


FIG. 30. Observations for a 6 per cent thick RAE 104 areofoil at 2-deg incidence; surface-pressure distributions (*See* also Figs. 31 to 34).

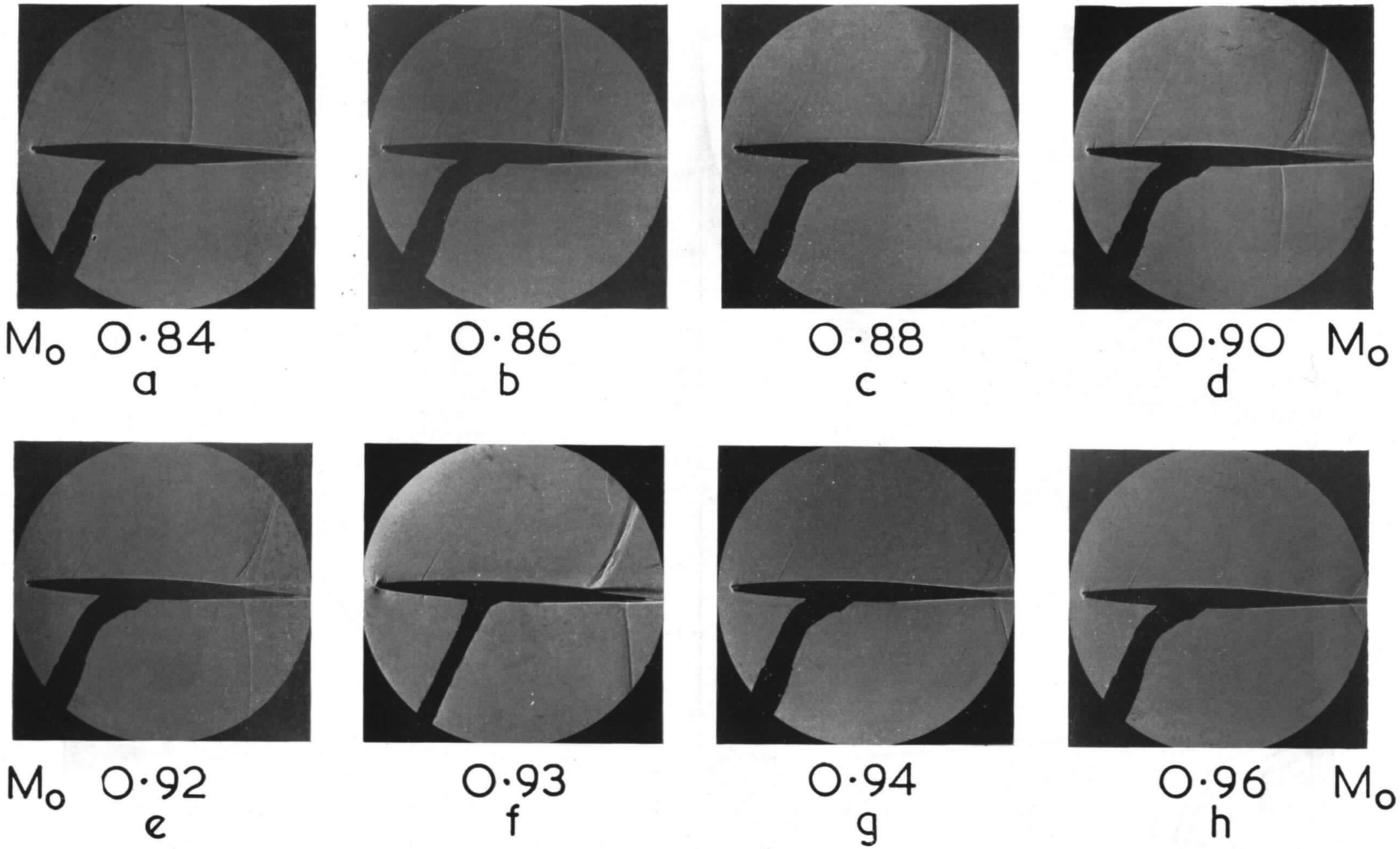


FIG. 31. Observations for a 6 per cent thick RAE 104 aerofoil at 2-deg incidence; flow photographs (*See* also Figs. 30 and 32 to 34).

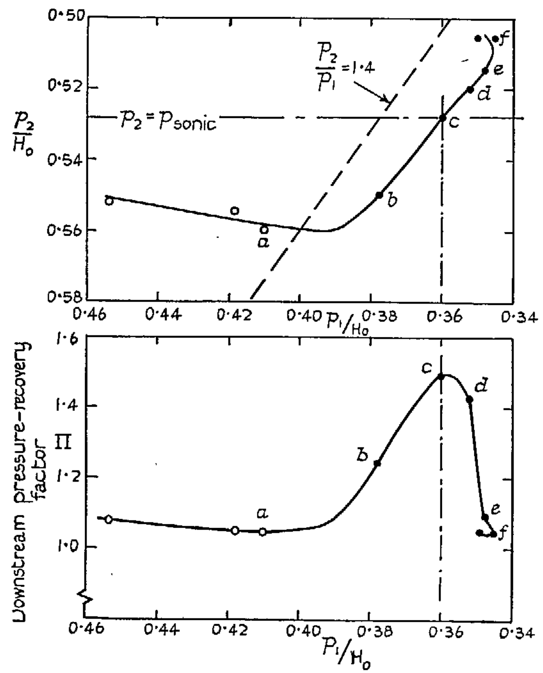


FIG. 32. Observations for a 6 per cent thick RAE 104 aerofoil at 2-deg incidence; quantities derived from the upper-surface pressures (*See* also Figs. 30, 31, 33 and 34). (Filled symbols denote the presence of separation.)

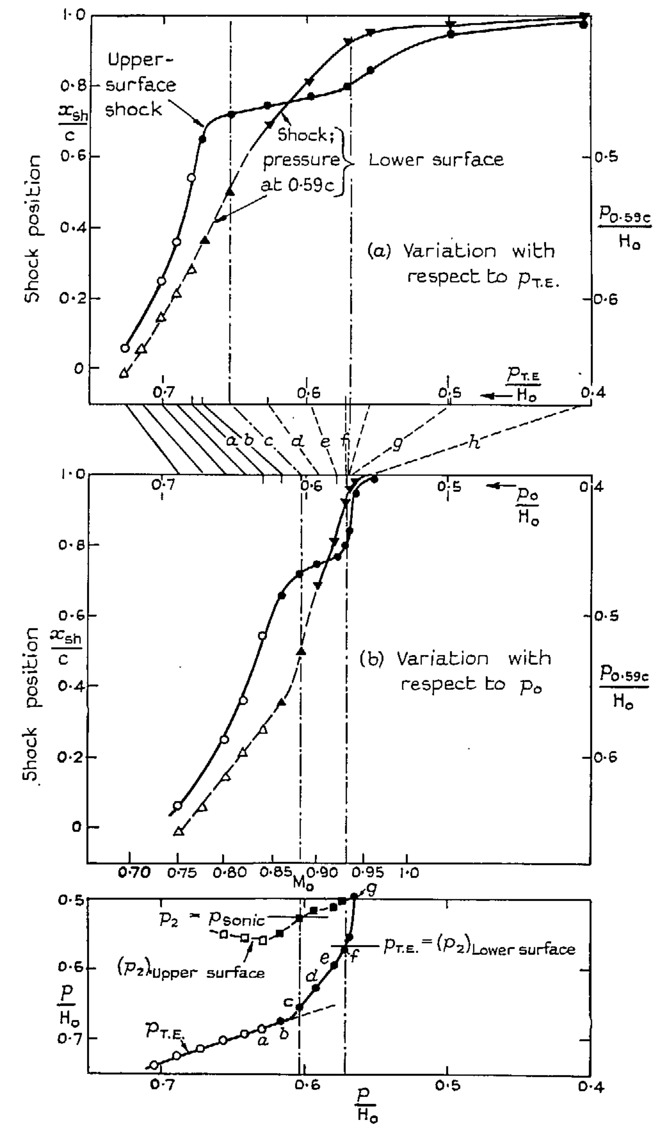


FIG. 33. Observations for a 6 per cent thick RAE 104 aerofoil at 2-deg incidence; variation of a flow parameter (shock position or pressure at a fixed point) for each surface (*See* also Figs. 30, 31, 32 and 34).

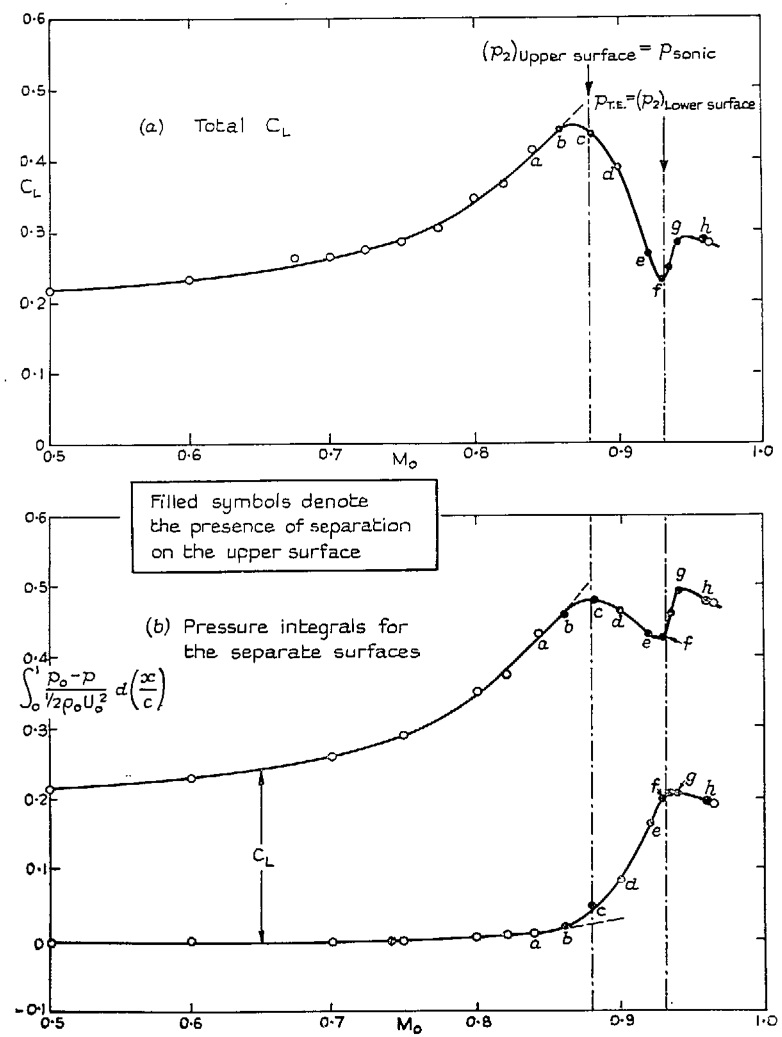


FIG. 34. Observations for a 6 per cent thick RAE 104 aerofoil at 2-deg incidence; variation with free-stream Mach number of the total C_L and of the contributions from the separate surfaces (*See* also Figs. 30 to 33).

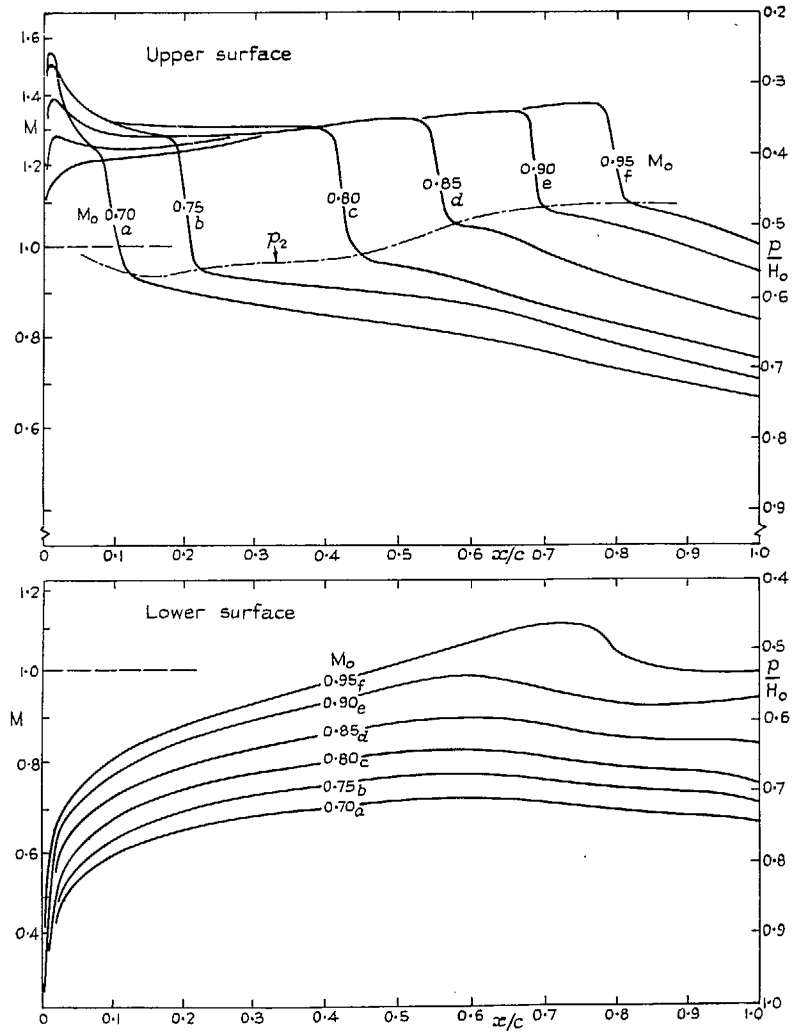


FIG. 35. Observations for a 6 per cent thick RAE 104 aerofoil at 3·7-deg incidence; surface-pressure distributions (*See* also Figs. 36 to 39).

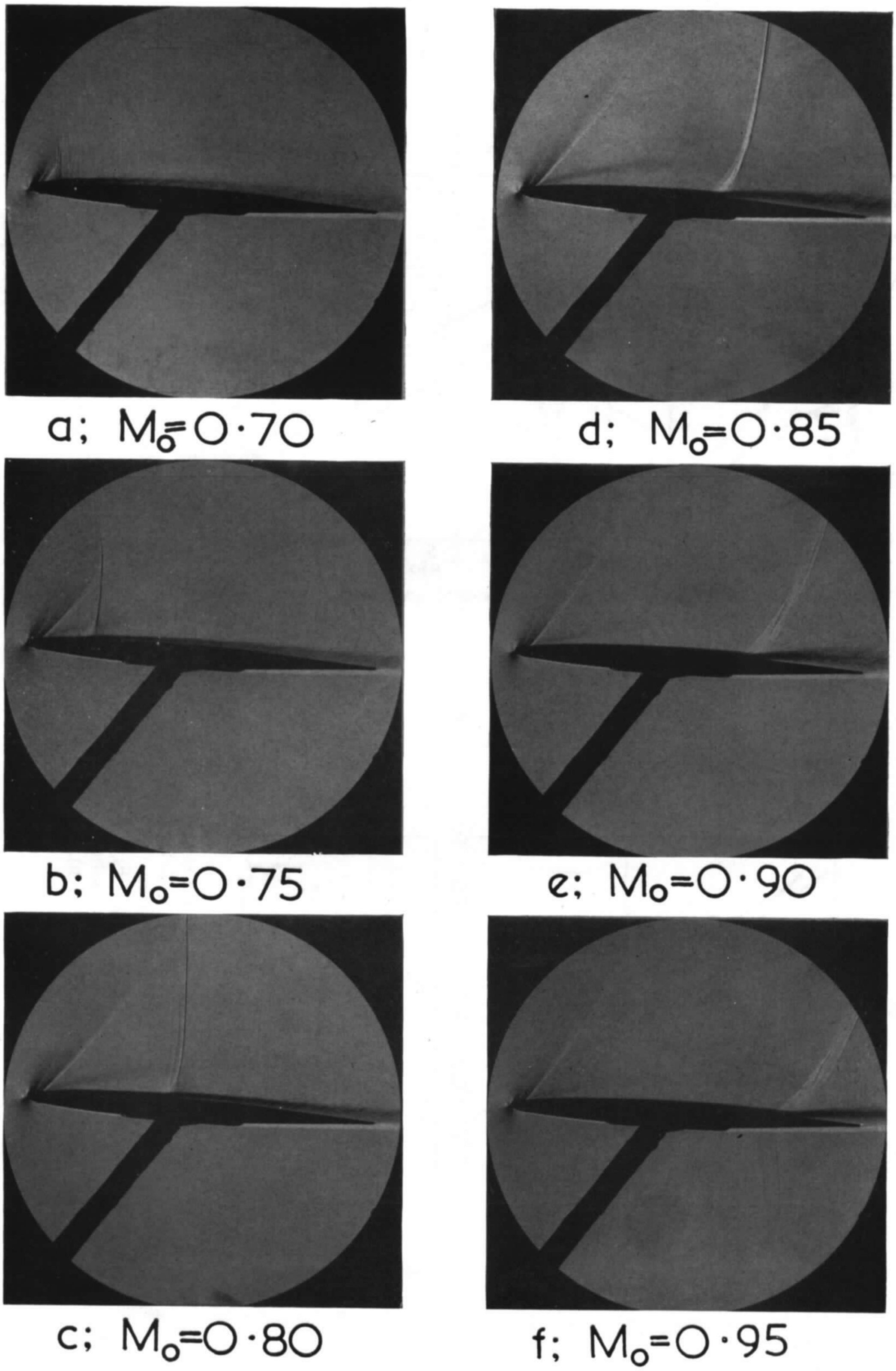


FIG. 36. Observations for a 6 per cent thick RAE 104 aerofoil; at 3·7-deg incidence; flow photographs (*See* also Figs. 35, 37, 38 and 39).

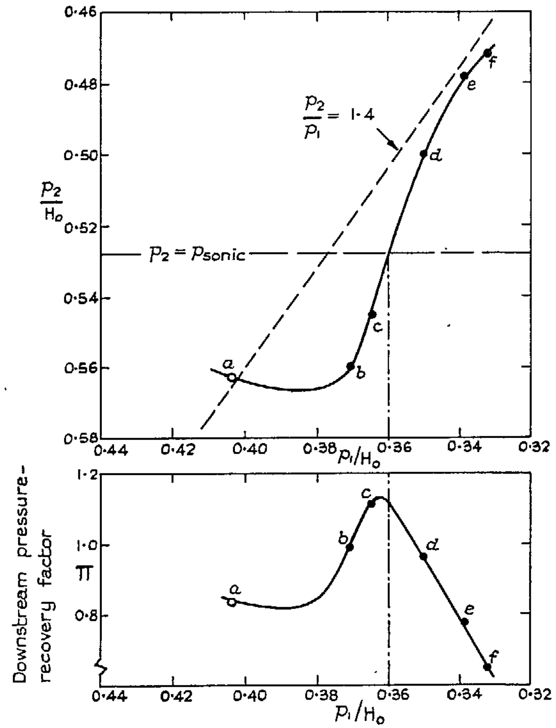


FIG. 37. Observations for a 6 per cent thick RAE 104 aerofoil at 3·7-deg incidence; quantities derived from the upper-surface pressures (*See* also Figs. 35, 36, 38 and 39) (Filled symbols denote the presence of separation).

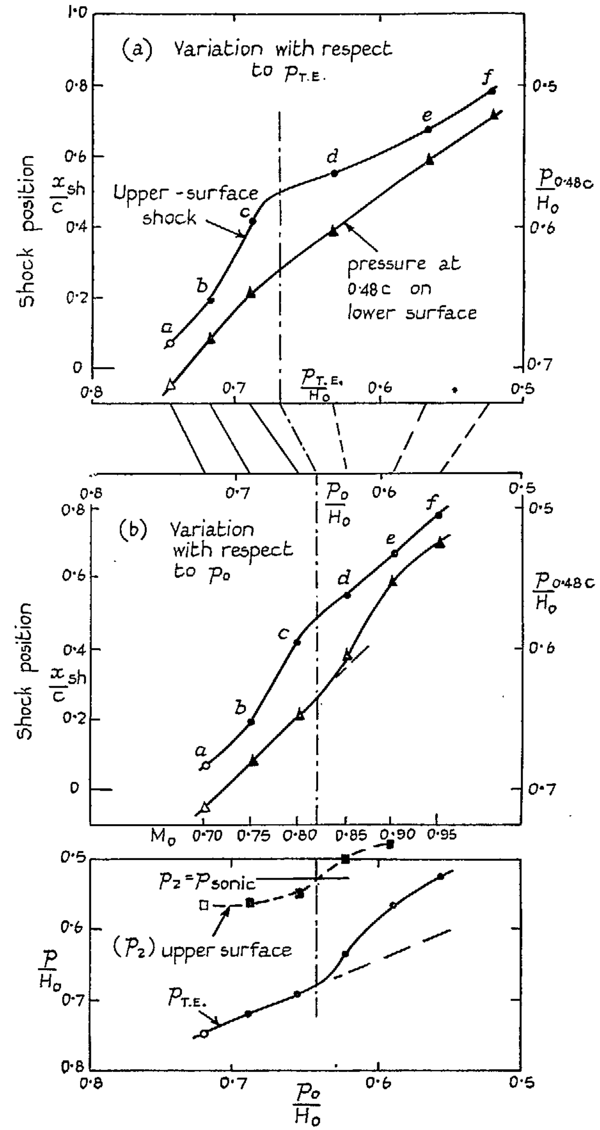


FIG. 38. Observations for a 6 per cent thick RAE 104 aerofoil at 3·7-deg incidence; variation of a flow parameter (shock position and pressure at a fixed point) for each surface (*See* also Figs. 35, 36, 37 and 40).

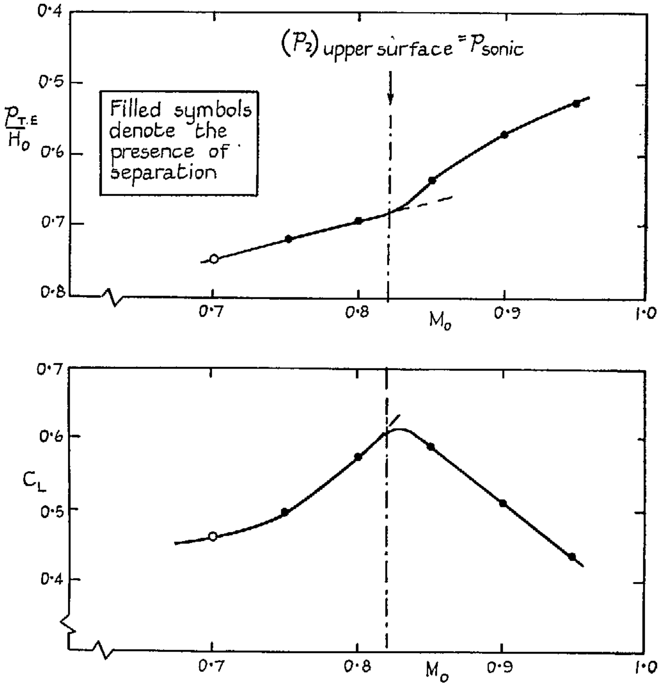


FIG. 39. Observations for a 6 per cent thick RAE 104 aerofoil at 3·7-deg incidence; variation with free-stream Mach number of C_L and $p_{T.E}$ (*See* also Figs. 35 to 38).

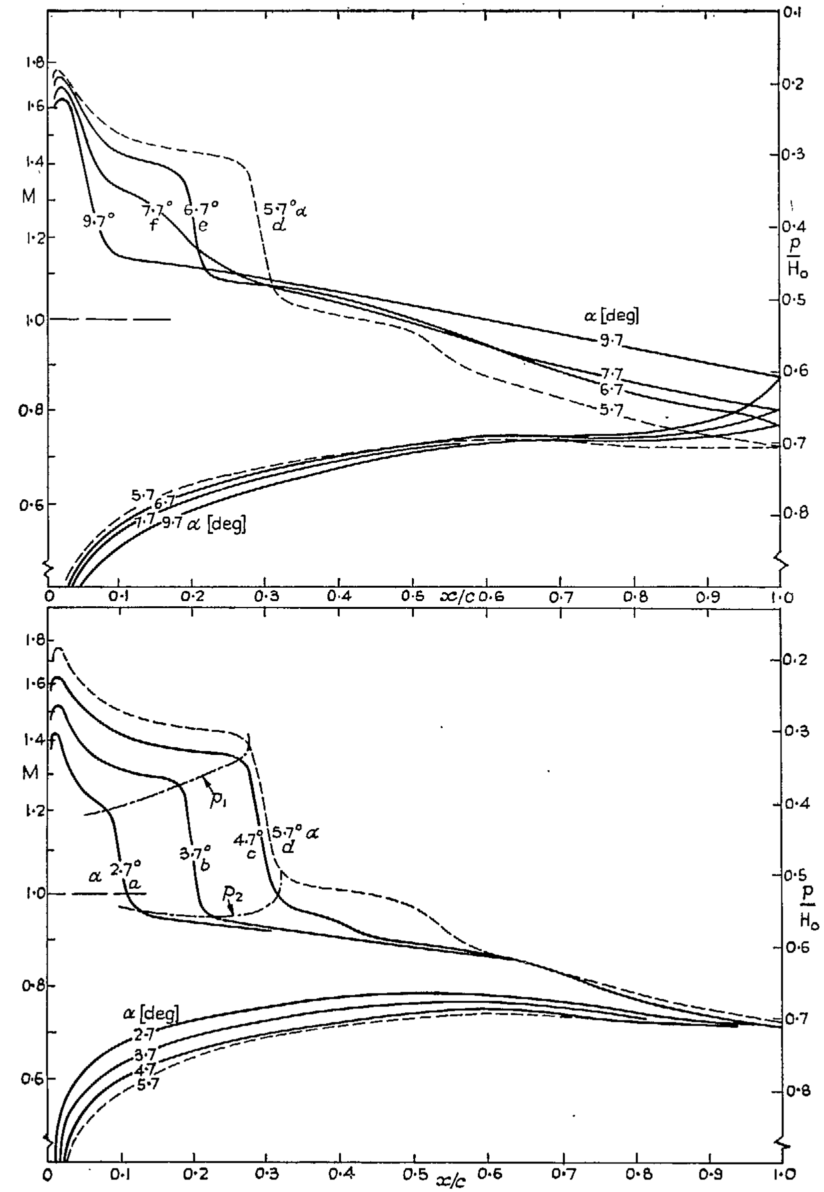


FIG. 40. Observations for a 6 per cent thick RAE 104 aerofoil through a range of incidence; $M_0 = 0 \cdot 75$; surface-pressure distributions (*See* also Figs. 41 to 43).

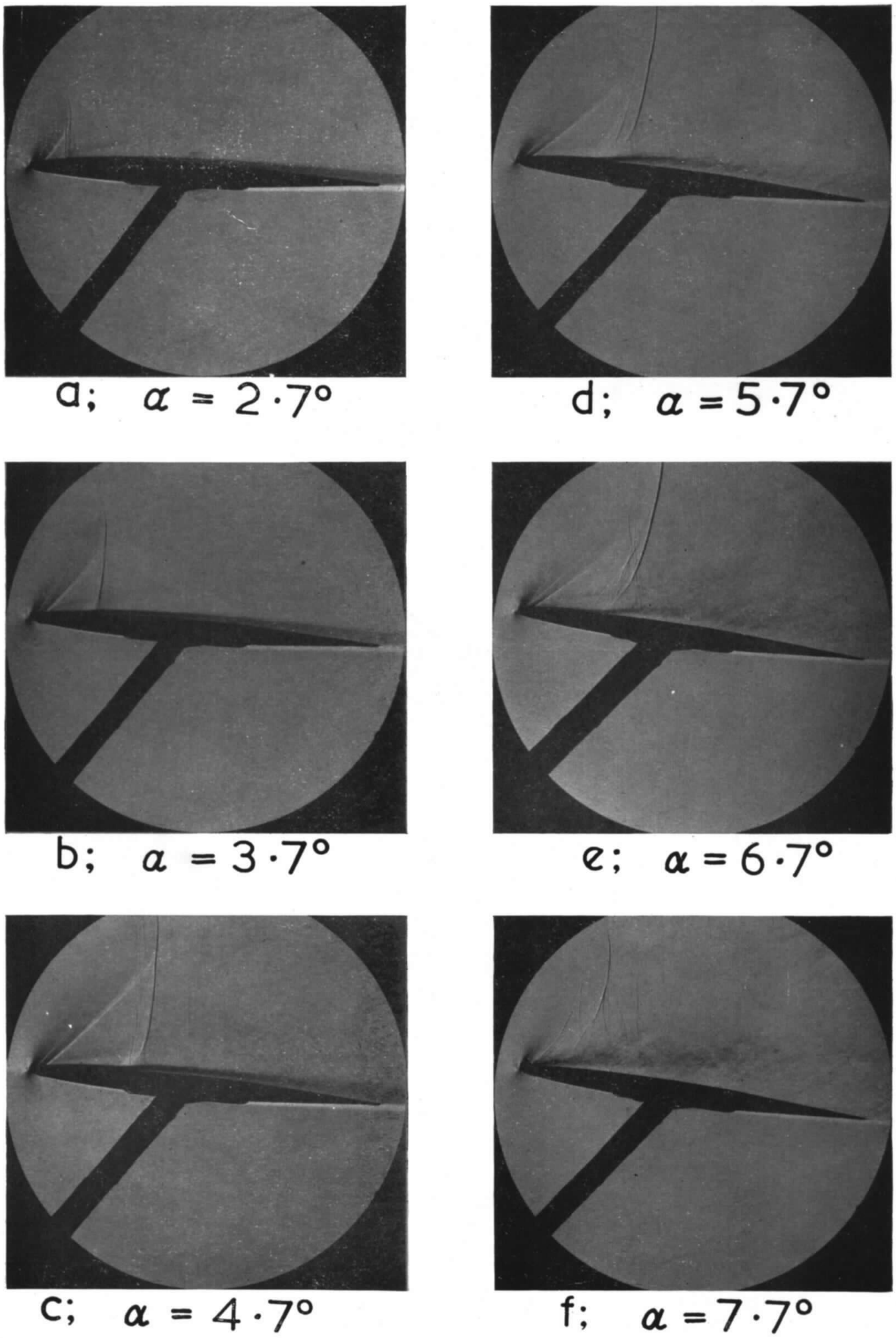


FIG. 41. Observations for a 6 per cent thick RAE 104 aerofoil through a range of incidence; $M_0 = 0{\cdot}75$; flow photographs (*See* also Figs. 40, 42 and 43).

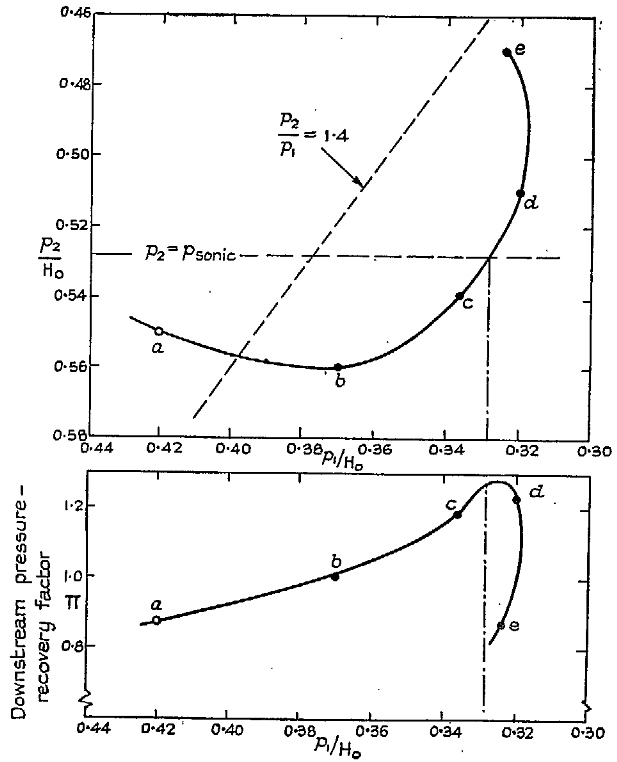


FIG. 42. Observations for a 6 per cent thick RAE 104 aerofoil through a range of incidence; $M_0 = 0·75$; quantities derived from the upper-surface pressures (*See* also Figs. 40, 41 and 43) (Filled symbols denote the presence of separation).

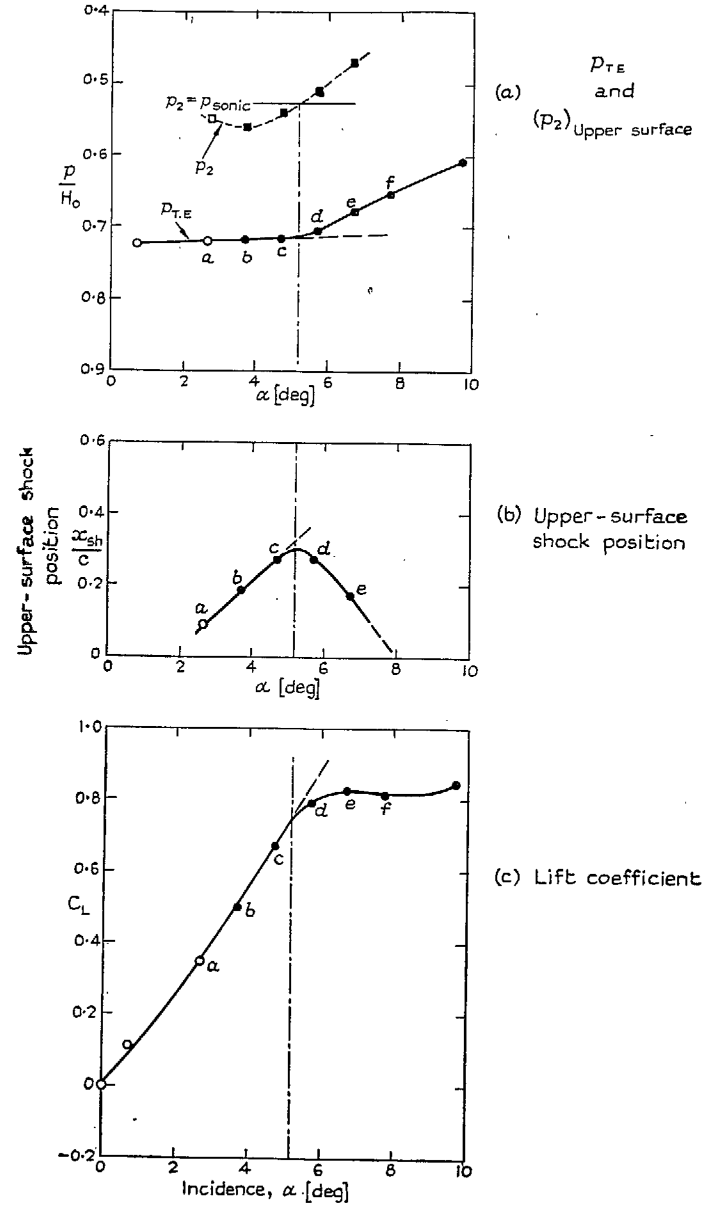


FIG. 43. Observations for a 6 per cent thick RAE 104 aerofoil through a range of incidence; $M_0 = 0·75$ (*See* also Figs. 40, 41 and 42) (Filled symbols denote the presence of separation).

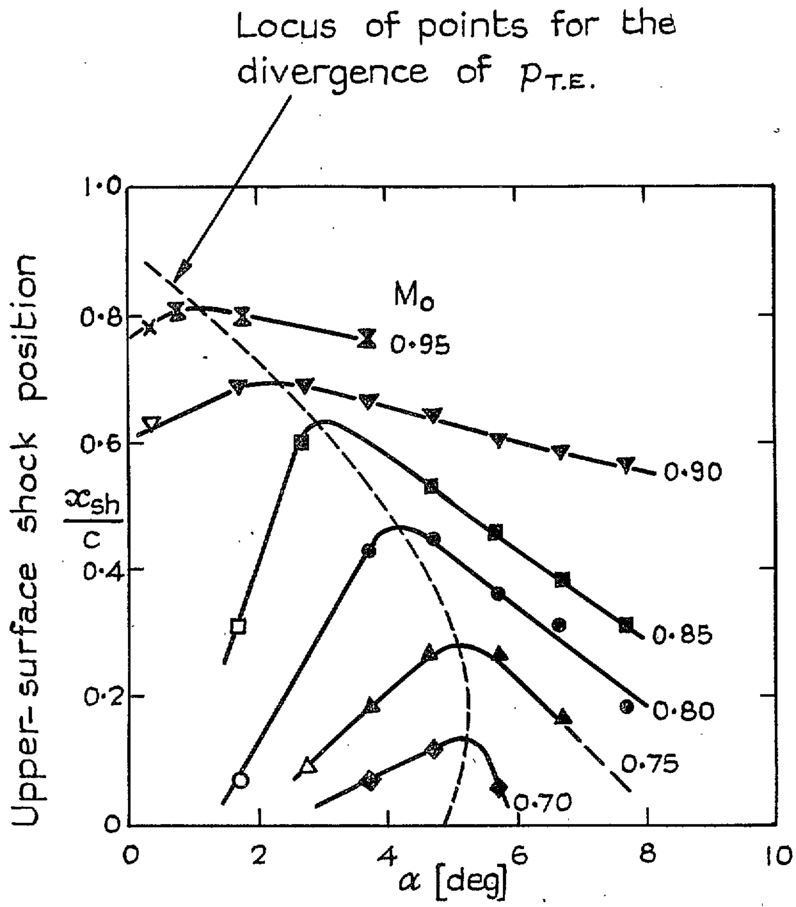


FIG. 44. Observations for a 6 per cent thick RAE 104 aerofoil through a range of incidence; upper-surface shock positions for several different constant Mach numbers (Filled symbols denote the presence of separation).

(75401) Wt. 53/2293 K.7 1/60 Hw.

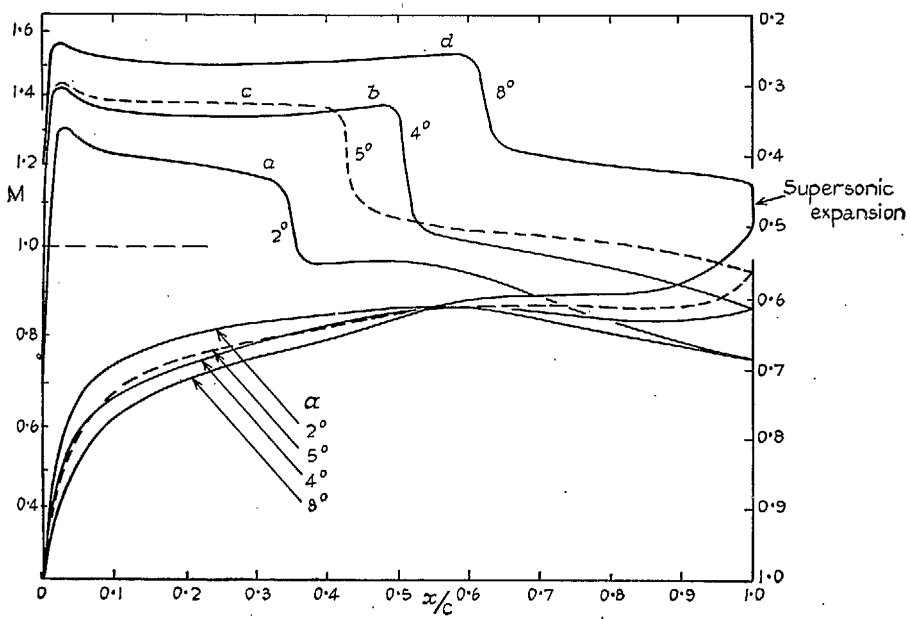


FIG. 45. Observations for a 6 per cent thick aerofoil through a range of incidence at $M_0 = 0\cdot8$ (*See* also Fig. 46).

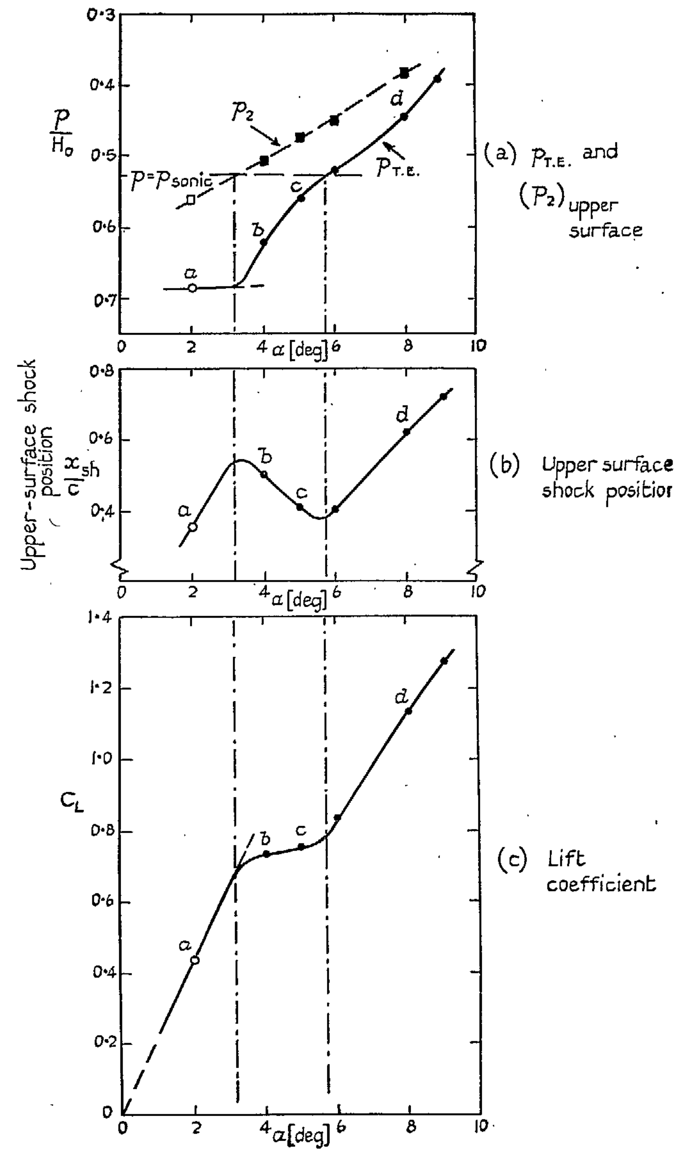


FIG. 46. Observations for a 6 per cent thick aerofoil through a range of incidence at $M = 0\cdot8$ (*See* also Fig. 45) (Filled symbols denote the presence of separation).

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