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SUMMARY

An investigation of the effects of elevator circuit stiffness on the loading conditions of an aircraft in a rapid pilot induced pull-out manoeuvre is presented with a particular reference to the airworthiness aspects.

The solution to the problem is given in general terms; it includes expressions for the incremental elevator angle, stick force, normal acceleration at the centre of gravity and the tailplane load in response to a chosen sequence of pilot induced stick movements to initiate a rapid pull-out manoeuvre. The solution is a complex one and the influence of significant parameters such as the degree of mass balance of the elevator circuit, the forward speed of the aircraft and the rate of application of the stick movement is discussed in relation to a numerical example.

Brief mention is made of further implications of this investigation.

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1 Introduction

In the course of a recent accident investigation it was found that the stiffness of the elevator circuit of the aircraft concerned was somewhat lower than that normally obtained in practice, and it was required to determine what was the effect of this comparatively low stiffness on the loading condition in rapid, pilot induced pull-out manoeuvres.

It is usual in the consideration of the loading conditions in pilot induced manoeuvres to assume that the linkages between the pilot's control and the associated aerodynamic control are rigid, so that in the case of a pull-out manoeuvre for example, the movements of the elevator may be expressed directly in terms of movements of the stick. With this assumption, and others relevant to the short period mode of longitudinal oscillation of the aircraft, e.g. constant forward speed, the aircraft may be regarded as a dynamic system with two degrees of freedom. A number of solutions of this case for different types of elevator motion are available[2,3]. If, however, the elevator circuit is not rigid, a further degree of freedom is introduced into the system; this adds considerably to the complexity of the problem, and the effects of the addition on the overall response of the aircraft are not readily apparent.

The purpose of the present report is to consider qualitatively some of the effects of elevator circuit stiffness in relation to the loading conditions at the wing and at the tail in rapid pilot induced pull-out manoeuvres. A general solution to the problem is given, but in view of its complexity, it has been found convenient to discuss the important effects with the aid of an example.

2 The representation of the elevator circuit

In practice the overall stiffness of a control circuit is a sum of the stiffnesses of the mountings which carry the various linkages as well as the stiffnesses of the linkages themselves. For the purposes of the present investigation all these effects are assumed to be of a similar nature so that the overall circuit may be replaced by a single linear spring of appropriate stiffness in an otherwise rigid system; for convenience it is assumed that this spring is located at the top of the stick. (See Fig.1.) In addition it is assumed that the moment of inertia of the elevator circuit is incorporated in that of the elevator itself and that the circuit is frictionless.

3 Details of the analytical investigation

With the above representation of the elevator circuit, the equation of motion of the elevator about its hinge, has been developed, and, together with the equations governing the pitching and heaving motions of the aircraft, has been solved analytically for a specified sequence of movements of the stick. See Appendix. Expressions are derived for the incremental elevator angle, the incremental normal acceleration at the centre of gravity of the aircraft and at the tail, the incremental stick force and the incremental tailplane load. The assumptions used in setting up the equations of motion are those normally made in connection with the analysis of the short period mode of response of the aircraft and are enumerated, for example, in Ref.2.

4 The pilot's action

The airworthiness aspects of pilot induced pull-out manoeuvres have already been considered[2] for the case in which the elevator circuit is rigid, the assumed elevator (and stick) movement being of an exponential form. A similar type of stick movement is used for the present investigation. The sequence of movements is defined by equation (17) of the Appendix. The parameter k in this equation governs the initial rate at which the stick is moved to its final deflected position, and the significance of this parameter is the same as in the "rigid" case. With this choice of stick movements, the effects of stiffness of the elevator circuit may be assessed against the "rigid" case.

The pull-out manoeuvre considered is a rapid one, and the numerical value ascribed to the parameter k is that suggested in the design requirement relating to the pull-out manoeuvre[2,4] namely 4J+R, where J is the non-dimensional frequency of the pitching oscillations of the aircraft alone, and R is the non-dimensional damping coefficient of these oscillations.

5 Details of the example, and of the calculations

In view of the complexity of the general solutions for the relevant response quantities presented in the Appendix, a general discussion of the effects of elevator circuit stiffness on the loading conditions of the aircraft is not practicable, and recourse has been made to an example to illustrate the salient features of the case. The chosen example is based on the aircraft which prompted the initial investigation, having a conventional cable type elevator circuit. The basic numerical data for the example are given in Table I. It will be noted that in the basic configuration the elevator is not mass balanced.

The calculations have been made to illustrate the direct effects of elevator circuit stiffness on the loading conditions of the wing and tailplane under the chosen set of conditions Table I 1(i), but additional calculations have been made to provide an idea of the significance of the degree of mass balance of the elevator, the forward speed of the aircraft and the rate of stick movement on the overall problem.

The preliminary study of the various effects was made with the aid of an analogue computer, but all the results discussed below have been obtained by direct calculation. The results are presented in the form of time histories of the manoeuvre.

6 Discussion

(i) The basic effect of elevator circuit stiffness

The responses in elevator angle, stick force, normal acceleration at the centre of gravity and net tailplane load (i.e. allowing for inertia effects) to the assumed sequence of stick movements for a number of circuit stiffnesses are illustrated in Fig. 2(a-d). The first observation to be made is that the damping of the elevator mode is below its "critical" value, and that for finite values of circuit stiffness, the response in elevator angle to the stick movement exhibits a transient oscillation. The numerical value of the elevator damping coefficient ν_e, (see equations (10)-(13)) used in the calculations has been based on a simple formula derived by Adamson and Lyons[1] from a few experimental data, but it is thought that it represents a realistic value.

The response of the aircraft is directly dependent on the sequence of elevator movements and it is to be expected that all the other quantities illustrated in Fig.2 should exhibit similar transient oscillations. In the case of the stick force and tailplane load this is most marked, mainly because both these quantities depend on the elevator angle as well as the response of the aircraft itself. In the case of the normal acceleration at the centre of gravity, the effect is not readily discernable since the growth of acceleration is relatively insensitive to the rapid fluctuations of the elevator.

In the early stages of the manoeuvre, the transient oscillations in the stick force and tailplane load time histories are of comparatively large initial amplitude and it is evident that still higher amplitudes could be obtained at higher finite stiffnesses. However, as often happens in dynamic phenomena, a stiffness would eventually be reached beyond which the amplitude would begin to decrease; it will be noticed that in the case of the tailplane load, the rate of increase of maximum down load with increase in stiffness falls quite rapidly even with the values of stiffness considered here. In addition it should be pointed out that if the oscillations of the elevator were much more rapid than those shown in Fig. 2(a) "unsteady flow" conditions around the tailplane would tend to modify the overall picture obtained, and the predicted maximum loads and forces would probably not be realized in practice.

However, assuming that the chosen elevator damping is of the correct order, the chief effect of reducing the stiffness of the elevator circuit is to reduce the loading conditions at the wing and at the tail as well as the level of stick forces for a given stick movement.

(ii) The effect of mass balance

In the early stages of a rapid pull-out manoeuvre, the motion of the aircraft is continuously disturbed, and there are continuous changes in the normal acceleration at the centre of gravity and in the angular pitching velocity. It follows that the normal acceleration at the tail differs from that at the centre of gravity until steady conditions are reached. Since the tailplane and elevator are subjected to acceleration during the pull-out manoeuvre it is evident that if the elevator is not mass-balanced and its circuit is not rigid, this acceleration will affect the total hinge moment of the elevator and also its displacement, which in turn will affect the motion of the aircraft.

Some evidence of these effects may be obtained from Fig.3. Here mass balancing has been achieved by the addition of weights forward of the elevator hinge line, and this has increased the overall moment of inertia of the elevator system, as indicated in the Table I 1(ii). The elevator response lags further behind the sequence of stick movements than in the unbalanced state, and in addition the transient oscillation is more vigorous; further, the removal of the elevator hinge moments due to acceleration effects results in a larger asymptotic elevator angle. The stick forces in the early stages of the manoeuvre are increased because the relief, due to the effects of the negative acceleration at the tail in this period, is removed, but eventually are decreased as a result of the positive acceleration effects which arise. The main effect on the normal acceleration at the centre of gravity is to increase its maximum value, because of the larger elevator angle attainable. The effect on the critical loading condition at the tail, i.e. the maximum download in the present case, is negligible.

(iii) Effect of forward speed of the aircraft

The analytical work in the Appendix is presented in terms of non-dimensional quantities. Such a presentation allows a consistent comparison of results of different cases and also gives a true indication of the influence of the various pertinent parameters. .The non-dimensional damping coefficient of the elevator ν_e, see equation (13), is a function of the forward speed of the aircraft, and thus, as illustrated in Fig.4, a change in this speed results in a change in both the amplitude and "duration" of the transient oscillation in the elevator response, and in its asymptotic value, leading to changes in the stick force, normal acceleration and tailplane load in such a manner that the stick force/g and steady tailplane load/g remain constant.

(iv) Effect of rate of stick movement

The primary effect of a change in the rate of stick movement (see Fig.5) is to change the amplitude of the transient oscillations of the elevator in the early stages of the manoeuvre, and thus to change the stick force and tailplane load in this period. The changes are quite marked, and it is clear that the precise rate of stick movement has a profound effect on the magnitude of the forces and loads in the initial stages of the manoeuvre whether the elevator circuit is rigid or not. The steady state conditions are not affected by the rate of movement of the stick.

(v) Airworthiness aspects

So far the discussion has been confined to the consideration of the effects of circuit stiffness on the response of an aircraft to a specified sequence of stick movements. However, such considerations do not fully illustrate the airworthiness aspects of the case, since, in practice, these aspects are closely linked to a flight envelope of normal accelerations at the centre of gravity of the aircraft. As indicated in (i) above the primary effect of a reduction in control circuit stiffness is a reduction in the overall level of response of the aircraft, and in particular the normal acceleration at the centre of gravity, if the sequence of stick movements is unchanged. For this reason it is desirable that the case should be re-examined with reference to a specified value of normal acceleration to be reached in the manoeuvre.

An appreciation of the effects of circuit stiffness on the airworthiness aspects of the case may be obtained by a simple re-evaluation of the results previously obtained since the magnitudes of all the response quantities involved are directly proportional to the magnitude of the sequence of stick movements. In Fig.6 the stick displacement at each value of circuit stiffness has been adjusted in such a manner that the asymptotic normal acceleration at the centre of gravity (not illustrated), obtained in the ensuing manoeuvre is always the same. Adjustment has been made only to the asymptotic stick displacement; the parameter k has not been changed. It follows that the maximum (initial) rate of stick movement is different for each value of circuit stiffness. This fact must be borne in mind when considering the various responses in Fig.6, since the effects illustrated by the curves are not now due solely to the effect of circuit stiffness, but include the effects of changes in the initial rate of stick movement, which, as noted in (iv) above can be quite marked. However, it is believed that if flight tests were made to investigate the airworthiness aspects of circuit stiffness in rapid pull-out manoeuvres, the results obtained would indicate a tendency on the part of the pilot to increase both the initial rate and amplitude of stick movement to produce the required steady acceleration as the stiffness of the elevator circuit was reduced rather than increase the amplitude of stick movement alone. It follows that with this plausible assumption the method of presentation of results in Fig.6 may be considered to be a realistic one.

The important feature to be noted is the marked increase in the maximum tailplane load at the beginning of the manoeuvre when the elevator circuit stiffness is reduced. In the present example, this maximum load is also the critical load in the manoeuvre, and since it is evident that the values of stiffness covered in Fig.6 do not lead to the most critical conditions, it is concluded that the airworthiness aspects of the rapid pull-out manoeuvre may be significantly affected by the actual stiffness of the elevator circuit. However, the results presented in Fig.7 indicate that the effects will not be so important if the elevator circuit is mass balanced, although the steady stick force/g will be reduced.

7 Implications

The general analysis in the Appendix applies directly to the case in which the stick and elevator are connected by non-rigid linkages. However, it is suggested that this analysis could easily be recast to solve other problems in which there exists an interaction between dynamic structural distortions and the elevator hinge moments e.g. cases in which the elevator torsional stiffness is low or in which a servo or power control of definite stiffness is present in the elevator circuit. It is also suggested that the present analysis demonstrates that problems may arise with regard to airworthiness conditions when the assumption of a rigid structure is not justifiable.

8 Conclusions

The effects of elevator circuit stiffness on the loading conditions of an aircraft in rapid pilot induced pull-out manoeuvres are considered. A solution to the problem in general terms is given, but in view of its complexity, the salient features of the problem are discussed with reference to a numerical example.

In particular, it is concluded that the loading conditions at the tail in a rapid pull-out manoeuvre may be significantly dependent on the degree of stiffness present in the elevator circuit, and that acceptance of the assumption that the circuit is rigid in the analysis of such manoeuvres when it is not justifiable may lead to an underestimate of the critical loading conditions.

In addition it is suggested that the investigation may serve to illustrate a method of approach to other problems in which there exists an interaction between dynamic structural distortions and elevator hinge moments.

LIST OF SYMBOLS

A, B, C		-	coefficients in equation (26)
$A_1, B_1, C_1, D_1, E_1, F_1$		-	" " " (12)
A_2, B_2, C_2, D_2, E_2		-	" " " (20)
$a = \frac{\partial C_L}{\partial \alpha}$		-	for the whole aeroplane
$a_1 = \frac{\partial C_L'}{\partial \alpha'}$		-	
$a_2 = \frac{\partial C_L'}{\partial \eta}$		-	including the effects of tabs if used
a_t	(ft/sec^2)		vertical acceleration at the tail (positive down)
$b_1 = \frac{\partial C_h}{\partial \alpha'}$		-	

$b_2 = \frac{\partial C_h}{\partial \eta}$	-	including the effects of tabs if used
C_h	-	hinge moment coefficient of elevator
C_L	-	lift coefficient of the aeroplane
C_L'	-	lift coefficient of the tailplane
C_m	-	pitching moment coefficient of the aeroplane about its cg
c	(ft)	standard mean chord of wing
c_e	(ft)	standard mean chord of elevator aft of hinge
D	-	coefficient in equation (24)
F	(lb)	stick force
g	(ft/sec^2)	acceleration due to gravity
H_s	(lb/ft)	hinge moment due to stick force, see equation (8)
$I_e = \frac{W_e}{g} k_e^2$	slug ft^2	moment of inertia of the elevator circuit and elevator relative to the elevator hinge
J	-	non-dimensional frequency of pitching oscillations of the aircraft - elevator circuit rigid
k	-	parameter in the definition of the stick movement, see equation (17)
k_B	(ft)	radius of gyration of the aeroplane about its lateral axis
k_e	(ft)	radius of gyration of the elevator about its hinge line (including effects of elevator circuit)
k_s	(lb/ft)	total stiffness of the elevator circuit
ℓ	(ft)	distance from the cg of the aeroplane to the quarter chord point of the tailplane
m_e	(rad/ft)	stick gearing
m_q less tail	-	damping derivative in pitch of the aeroplane less tail
n	-	coefficient of incremental normal acceleration of the cg of the aeroplane (positive up)
n_t	-	coefficient of total normal acceleration at the tailplane (positive up)
P	(lb)	incremental aerodynamic tailplane load, see equation (26)
q	(rad/sec)	angular velocity of the aeroplane in pitch

$\hat{q} = \hat{t}q$	-	non-dimensional form of q
R	-	non-dimensional damping factor of the pitching oscillations of the aeroplane system - elevator circuit rigid
R_1, J_1, R_e, J_e	-	parameters, see equation (16)
S	(ft^2)	wing area
S'	(ft^2)	tailplane area
S_e	(ft^2)	elevator area aft of hinge line
s	(ft)	stick movement
s_m	(ft)	maximum stick movement
$\hat{s} = \frac{s}{s_m}$	-	
t	(sec)	time
$\hat{t} = \frac{W}{g\rho SV}$	(sec)	unit of aerodynamic time
V	(ft/sec)	true airspeed of the aeroplane
W	(lb)	weight of the aeroplane
W_e	(lb)	weight of the elevator
w	(ft/sec)	incremental velocity component along z axis (positive down)
$\hat{w} = \frac{w}{V}$	-	
x_e	(ft)	distance of the cg of the elevator from its hinge line (positive aft)
α	(rad)	incremental wing incidence
$\dot{\alpha} = \frac{d\alpha}{dt}$	(rad/sec)	
α'	(rad)	incremental tailplane incidence
$\alpha_1, \alpha_e, \beta_1, \beta_e, \gamma_1, \gamma_e, \delta_1, \delta_e$	-	coefficients in equation (19)
$\bar{\beta}_1$ $\bar{\bar{\beta}}_1$ $\bar{\beta}_e$ $\bar{\bar{\beta}}_e$ $\bar{\gamma}_1$ $\bar{\bar{\gamma}}_1$ $\bar{\gamma}_e$ $\bar{\bar{\gamma}}_e$	-	coefficients in equation (23)
$\gamma = \frac{1}{2}\rho V^2 S_e c_e$	(lb/ft)	
$\Delta = \frac{\gamma \hat{t}^2}{I_e}$	-	
$\frac{\partial C_m}{\partial \alpha}$	-	static stability derivative

LIST OF SYMBOLS (CONTD)

δ	-	elevator effectiveness
ε	(rad)	downwash angle at the tail
η	(rad)	elevator angle
$\dot{\eta} = \frac{d\eta}{dt}$	(rad/sec)	
$\mu = \frac{W}{g\rho S\ell}$	-	relative density of the aeroplane
$\nu = \nu_{tail} + \nu_{less\ tail}$	-	rotary damping coefficient
$\nu_{tail} = \frac{1}{2}\frac{S'\ell^2}{S{k_B}^2}$	-	
$\nu_{less\ tail} = -\frac{\ell^2}{{k_B}^2}(m_q)_{less\ tail}$	-	
$\bar{\nu}$	(sec)	damping coefficient of the elevator, see equation (10)
ν_e	-	non-dimensional damping derivative of the elevator
ρ	(slug/ft^3)	true air density
$\Sigma = \frac{x_e\ell}{{k_e}^2}$	-	
τ	-	non-dimensional time
$\varphi = -\delta F_1$	-	
$\chi = \frac{d\varepsilon}{d\alpha}\nu_{tail}$	-	downwash damping derivative
$\omega = \frac{Wc}{2g\rho S{k_B}^2}\frac{\partial C_m}{\partial\alpha}$	-	static stability coefficient

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APPENDIX I

The equations of motion and their solution

A1 Equations of motion

The equations which describe the pitching and heaving motions of an aircraft in a pilot induced pull-out manoeuvre can be written thus[2]

$$\frac{W}{g} k_B^{\,2} \frac{dq}{dt} - \tfrac{1}{2}\rho V^2 Sc \frac{\partial C_m}{\partial \alpha} \alpha - \tfrac{1}{2}\rho V^2 Sc \frac{\partial C_m}{\partial q} q + \tfrac{1}{2}\rho V^2 S' \ell \frac{\partial C_L'}{\partial \alpha'} \frac{\ell}{V} \frac{d\varepsilon}{d\alpha} \frac{d\alpha}{dt} = -\tfrac{1}{2}\rho V^2 S' \ell \frac{\partial C_L'}{\partial \eta} \eta \tag{1}$$

$$\frac{W}{g} \left(\frac{dw}{dt} - qV\right) + \tfrac{1}{2}\rho V^2 S \frac{\partial C_L}{\partial \alpha} \alpha = 0 \tag{2}$$

or in the usual non-dimensional form

$$\chi \frac{d\hat{w}}{d\tau} + \omega\hat{w} + \frac{d\hat{q}}{d\tau} + \nu\hat{q} = -\delta\eta \tag{3}$$

$$\frac{d\hat{w}}{d\tau} + \tfrac{1}{2}a\hat{w} - \hat{q} = 0. \tag{4}$$

If, however, the elevator circuit is not rigid, a further equation, an equation of elevator hinge moments, is needed to describe completely the motion of the aircraft. For the present investigation the flexibility of the elevator circuit has been represented by a single linear spring in an otherwise rigid circuit, as illustrated in fig.(1). It has been further assumed that the moment of inertia of the elevator circuit can be added to that of the elevator itself, and that the circuit is frictionless. With this representation of the elevator circuit, the equation of hinge moments may be written thus:

$$\frac{W_e}{g} k_e^{\,2} \frac{d^2\eta}{dt^2} - \tfrac{1}{2}\rho V^2 S_e c_e \frac{\partial C_h}{\partial \alpha} \alpha - \tfrac{1}{2}\rho V^2 S_e c_e \frac{\partial C_h}{\partial \dot{\alpha}} \dot{\alpha} - \tfrac{1}{2}\rho V^2 S_e c_e \frac{\partial C_h}{\partial q} q - \tfrac{1}{2}\rho V^2 S_e c_e \frac{\partial C_h}{\partial \eta} \eta$$

$$-\tfrac{1}{2}\rho V^2 S_e c_e \frac{\partial C_h}{\partial \dot{\eta}} \dot{\eta} + \left(\frac{W_e x_e}{g} \cdot a_t + \frac{W_e}{g} k_e^{\,2} \frac{dq}{dt}\right) - \left(\frac{\partial H_s}{\partial s} s + \frac{\partial H_s}{\partial \eta} \eta\right) = 0 \tag{5}$$

where the first bracket relates to the hinge moment due to acceleration effects at the tail and the second bracket relates to the hinge moment due to stick forces.

The vertical acceleration at the tail is[2]

$$a_t = \frac{dw}{dt} - qV + \frac{\ell dq}{dt} \tag{6}$$

The stick force is, see fig.(1)

$$F = k_s \left(s + \frac{\eta}{m_e}\right) \tag{7}$$

so that the hinge moment due to stick forces is

$$\left.\begin{aligned} H_s &= -\frac{F}{m_e} = -\frac{k_s}{m_e}\left(s + \frac{\eta}{m_e}\right) \\ \frac{\partial H_s}{\partial s} &= -\frac{k_s}{m_e} \\ \text{and} \qquad \frac{\partial H_s}{\partial \eta} &= -\frac{k_s}{m_e^2} \end{aligned}\right\} \tag{8}$$

In addition, the effective incidence at the tail is[2]

$$\alpha' = \left(1 - \frac{d\varepsilon}{d\alpha}\right)\alpha + \frac{\ell}{V} q + \frac{\ell}{V}\frac{d\varepsilon}{d\alpha} \cdot \frac{d\alpha}{dt} \tag{9}$$

so that the derivatives in equation (5) take the form

$$\left.\begin{aligned} \frac{\partial C_h}{\partial \alpha} &= \left(1 - \frac{d\varepsilon}{d\alpha}\right) b_1 \\ \frac{\partial C_h}{\partial \dot{\alpha}} &= \frac{\ell}{V}\frac{d\varepsilon}{d\alpha} b_1 \\ \frac{\partial C_h}{\partial q} &= \frac{\ell}{V} b_1 \\ \frac{\partial C_h}{\partial \eta} &= b_2 \\ \frac{\partial C_L}{\partial \eta} &= \bar{\nu} \end{aligned}\right\} \tag{10}$$

Thus, introducing the portmanteau symbols $\gamma = \frac{1}{2}\rho V^2 S_e c_e$ and $I_e = \frac{W_e}{g} k_e^2$, and using equations (6), (8) and (10), equation (5) may be rewritten in the following manner.

$$\left(\frac{d^2\eta}{dt^2}+\frac{dq}{dt}\right)-\frac{\Upsilon b_1}{I_e}\left[\left(1-\frac{d\varepsilon}{d\alpha}\right)\alpha+\frac{\ell}{V}\,q+\frac{\ell}{V}\frac{d\varepsilon}{d\alpha}\frac{d\alpha}{dt}\right]-\frac{\bar{\nu}\Upsilon}{I_e}\frac{d\eta}{dt}+\frac{1}{I_e}\left[\frac{-W_e x_e}{g}\left(\frac{dw}{dt}-qV+\frac{\ell dq}{dt}\right)\right]$$

$$-\frac{\Upsilon}{I_e}\left(b_2-\frac{k_s}{\Upsilon m_e^2}\right)\eta \;=\; -\frac{k_s}{I_e m_e}\,s \tag{11}$$

or in non-dimensional form

$$A_1\frac{d^2\hat{w}}{d\tau^2}+B_1\frac{d\hat{w}}{d\tau}+C_1\hat{w}+\frac{d^2\eta}{d\tau^2}+D_1\frac{d\eta}{d\tau}+E_1\eta \;=\; F_1\hat{s} \tag{12}$$

where

$$\left.\begin{aligned}
A &= 1+\Sigma & D_1 &= -\Delta\nu_e\\
B_1 &= \frac{A_1 a}{2}-\Delta\left(1+\frac{d\varepsilon}{d\alpha}\right)\frac{b_1}{\mu} & E_1 &= -\Delta\left(b_2-\frac{k_s}{m_e^2}\right)\\
C_1 &= -\Delta\left(1-\frac{d\varepsilon}{d\alpha}+\frac{a}{2\mu}\right)b_1-\frac{a}{2}\,\mu\Sigma & F_1 &= -\frac{k_s\Delta s_m}{\Upsilon m_e}\\
&\text{where}\\
\Sigma &= \frac{x_e\ell}{k_e^2} & \Delta &= \frac{\Upsilon \hat{t}^2}{I_e}\\
\nu_e &= \frac{\bar{\nu}}{\hat{t}} & \hat{s} &= \frac{s}{s_m}
\end{aligned}\right\} \tag{13}$$

and where s_m is a characteristic value of the stick movement.

Now equation (3) and (4) may be combined to give a second order equation of the form

$$\frac{d^2\hat{w}}{d\tau^2}+2R\frac{d\hat{w}}{d\tau}+(R^2+J^2)\hat{w} \;=\; -\delta\eta \tag{14}$$

where $R=\frac{1}{2}(\nu+\chi+\frac{1}{2}a)$ and $J=\sqrt{\omega+\frac{1}{2}\,a\nu-R^2}$ are identically equal to the non-dimensional damping factor and frequency respectively of the pitching oscillations of the aircraft system - elevator circuit rigid. It is assumed here that the characteristic two degree of freedom stability equation associated with equation (14) does in fact lead to a pair of complex roots and therefore an oscillatory response. Further it will be assumed that in the formal presentation of the following analysis of the coupled system of aircraft and elevator, the motions in the aircraft and elevator modes are oscillatory in nature.

Combining equation (12) and (14) we obtain a quartic equation

$$\frac{d^4\hat{w}}{d\tau^4} + (2R + D_1)\frac{d^3\hat{w}}{d\tau^3} + (R^2 + J^2 + 2RD_1 + E_1 - A_1\delta)\frac{d^2\hat{w}}{d\tau^2}$$

$$+ \left[(R^2+J^2)D_1^{\circ} + 2RE_1 - B_1\delta\right]\frac{d\hat{w}}{d\tau} + \left[(R^2+J^2)E_1 - C_1\delta\right]\hat{w} = -\delta F_1\hat{s} \tag{15}$$

which, for the purposes of solution, is conveniently factorized in the form

$$\left[\frac{d^2}{d\tau^2} + 2R_1\frac{d}{d\tau} + (R_1^{\ 2}+J_1^{\ 2})\right]\left[\frac{d^2}{d\tau^2} + 2R_e\frac{d}{d\tau} + (R_e^{\ 2}+J_e^{\ 2})\right]\hat{w} = \varphi\hat{s} \tag{16}$$

where $\varphi = -\delta F_1$. For all practical purposes the values of R_1 and J_1 and R_e and J_e will be, for the present problem, approximately equal to the damping factors and frequencies of the oscillations of the separate, uncoupled aircraft and elevator systems respectively.

A2 General solutions

The stick movement to initiate the rapid pull-out is assumed to be

$$\hat{s} = (1 - e^{-k\tau}) \tag{17}$$

in which case s_m is equal to the maximum displacement of the stick. The solution of equation (16) for $\hat{w}$ with this disturbing function is, with initial conditions $\tau = \hat{w} = \hat{q} = 0$,

$$\frac{1}{\varphi}\hat{w} = (\delta_1+\delta_e) + (\alpha_1+\alpha_e)e^{-k_1\tau} + \beta_1 e^{-R_1\tau}\sin J_1\tau + \gamma_1 e^{-R_1\tau}\cos J_1\tau +$$

$$+ \beta_e e^{-R_e\tau}\sin J_e\tau + \gamma_e e^{-R_e\tau}\cos J_e\tau \tag{18}$$

where

$$\left.\begin{aligned} \alpha_1 &= -\frac{B_2 - A_2k}{(R_1-k)^2 + J_1^{\ 2}} & \gamma_1 &= -\alpha_1 - \delta_1 \\ \delta_1 &= \frac{B_2}{R_1^{\ 2} + J_1^{\ 2}} & \beta_1 &= -\frac{R_1}{J_1}\delta_1 - \frac{R-k}{J_1}\alpha_1 \end{aligned}\right\} \tag{19}$$

$$\left.\begin{aligned}
\alpha_e &= \frac{D_2 - C_2 k}{(R_e - k)^2 + J_e^2} & \qquad \gamma_e &= -\alpha_e - \delta_e \\
\delta_e &= \frac{D_2}{R_e^2 + J_e^2} & \qquad \beta_e &= -\frac{R_e}{J_e}\delta_e - \frac{R_e - k}{J_e}\alpha_e
\end{aligned}\right\} \quad (19)$$

and where

$$\left.\begin{aligned}
A_2 &= -\frac{2(R_1 - R_e)}{E_2} \\
B_2 &= \frac{(R_1^2 + J_1^2) - (R_e^2 + J_e^2) + 4 R_1 (R_e - R_1)}{E_2} \\
C_2 &= -A_2 \\
D_2 &= \frac{(R_1^2 + J_1^2) - (R_e^2 + J_e^2) + 4 R_e (R_e - R_1)}{E_2} \\
E_2 &= 2(R_1^2 + J_1^2)(R_e^2 + J_e^2) + (R_1^2 + J_1^2)\left[4R_1R_e - 4R_e^2 - (R_1^2 + J_1^2)\right] \\
&\quad + (R_e^2 + J_e^2)\left[4R_1R_e - 4R_1^2 - (R_e^2 + J_e^2)\right]
\end{aligned}\right\} \quad (20)$$

Further

$$\frac{1}{\varphi}\frac{d\hat{w}}{dt} = -k(\alpha_1 + \alpha_e)e^{-k\tau} + \bar{\beta}_1 e^{-R_1\tau}\sin J_1\tau + \bar{\gamma}_1 e^{-R_1\tau}\cos J_1\tau + \bar{\beta}_e e^{-R_e\tau}\sin J_e\tau + \bar{\gamma}_e e^{-R_e\tau}\cos J_e\tau \quad (21)$$

and

$$\frac{1}{\varphi}\frac{d^2\hat{w}}{d\tau^2} = k^2(\alpha_1 + \alpha_e)e^{-k\tau} + \bar{\bar{\beta}}_1 e^{-R_1\tau}\sin J_1\tau + \bar{\bar{\gamma}}_1 e^{-R_1\tau}\cos J_1\tau + \bar{\bar{\beta}}_e e^{-R_e\tau}\sin J_e\tau + \bar{\bar{\gamma}}_e e^{-R_e\tau}\cos J_e\tau \quad (22)$$

$$\left.\begin{aligned} \text{where}\quad \bar{\beta}_1 &= -R_1\beta_1 - J_1\gamma_1 & \bar{\beta}_e &= -R_e\beta_e - J_e\gamma_e \\ \bar{\gamma}_1 &= -R_1\gamma_1 + J_1\beta_1 & \bar{\gamma}_e &= -R_e\gamma_e + J_e\beta_e \\ \bar{\bar{\beta}}_1 &= -R_1\bar{\beta}_1 - J_1\bar{\gamma}_1 & \bar{\bar{\beta}}_e &= -R_e\bar{\beta}_e - J_e\bar{\gamma}_e \\ \bar{\bar{\gamma}}_1 &= -R_1\bar{\gamma}_1 + J_1\bar{\beta}_1 & \bar{\bar{\gamma}}_e &= -R_e\bar{\gamma}_e + J_e\bar{\beta}_e \end{aligned}\right\} \qquad (23)$$

A3 <u>Derived response quantities</u>

(i) Normal acceleration[2]:

at the cg: $$n = D\hat{w} \qquad (24)$$

where $$D = \frac{\frac{1}{2}\rho V^2}{W/S}\,a$$

at the tail: $$n_t = D\left[\hat{w} - \frac{1}{\mu}\left(\frac{2}{a}\frac{d^2\hat{w}}{d\tau^2} + \frac{d\hat{w}}{d\tau}\right)\right]. \qquad (25)$$

(ii) Tailplane load[2]:

$$P = A\left(B\hat{w} + C\frac{d\hat{w}}{d\tau} + a_2\eta\right) \qquad (26)$$

where $$A = \tfrac{1}{2}\rho V^2 S'$$

$$B = \left(1 - \frac{d\varepsilon}{d\alpha} + \frac{a}{2\mu}\right)a_1$$

$$C = \left(1 + \frac{d\varepsilon}{d\alpha}\right)\frac{a_1}{\mu}\ .$$

(iii) Elevator angle:

$$\eta = -\frac{1}{\delta}\left(\frac{d^2\hat{w}}{d\tau^2} + 2R\frac{d\hat{w}}{d\tau} + (R^2 + J^2)\,\hat{w}\right). \qquad (27)$$

(iv) Stick force:

$$F = k_s\left(s + \frac{\eta}{m_e}\right). \qquad (7)$$

TABLE I

Numerical data for the example

1 **Basic effects**

(i) Basic data - Fig.(2)

$R = 3.17$	$\mu = 10.7$	$b_1 = -0.29$	$m_e = 0.9$ rad/ft
$J = 3.12$	$\hat{t} = 0.57$ sec	$b_2 = -0.675$	$I_e = 0.15$ slug ft^2
$\delta = 24.4$	$\ell = 12.6$ ft	$S_e = 11.4\ ft^2$	$\nu_e = -0.0105$
$a = 4.16$	$A = 10,500$ lb	$c_e = 1.1$ ft	$\gamma = 830$ lb ft
$a_1 = 3.31$	$D = 26.8$	$k_e^2 = 0.40\ ft^2$	$s_m = 0.0833$ ft
$a_2 = 2.54$	$V = 140$ kts TAS	$x_e = 0.35$ ft	

$\frac{d\varepsilon}{d\alpha} = 0.33 \quad k = 4J+R = 15.65$

(ii) Effect of mass balance - Fig.(3)

As in 1(i) but with

(a) $k_s = 500$ lb/ft $k_e^2 = 0.34\ ft^2$ and (b) $k_s = 500$ lb/ft $k_e^2 = 0.40\ ft^2$
$x_e = 0$ $x_e = 0.35$ ft
$I_e = 0.216$ slug ft^2 $I_e = 0.15$ slug ft^2

(iii) Effect of forward speed - Fig.(4)

As in 1(i) but with

(a) $V = 120$ kts TAS $\hat{t} = 0.67$ sec and (b) $V = 140$ kts TAS $\hat{t} = 0.57$ secs
$D = 19.6$ $\gamma = 610$ lb/ft $D = 26.8$ $\gamma = 830$ lb ft
$A = 7700$ lb $k_s = 500$ lb/ft $A = 10,500$ lb $k_s = 500$ lb/ft

(iv) Effect of rate of stick movement - Fig.(5)

As in 1(i) but with

(a) $V = 120$ kts TAS $\hat{t} = 0.67$ sec and (b) $V = 120$ kts TAS $\hat{t} = 0.67$ sec
$D = 19.6$ $k = 18.26$ $D = 19.6$ $k = 15.65$
$H = 7700$ lb $k_s = 500$ lb/ft $A = 7700$ lb $k_s = 500$ lb/ft

2 **Airworthiness aspects - Figs.(6) and (7)**

(i) As in 1(i) but with
$n_{t=\infty} = 0.58$'g' and with s_m to suit.

(ii) As in 1(i) but with

(a) $n_{t=\infty} = 0.58$'g' $k_s = 500$ lb/ft and (b) $n_{t=\infty} = 0.58$'g' $k_s = 500$ lb/ft
$I_e = 0.216$ slug ft^2 $s_m = 0.0333$ ft $I_e = 0.15$ slug ft^2 $s_m = 0.0333$ ft
$x_e = 0$ $x_e = 0.35$ ft
$k_e^2 = 0.34\ ft^2$ $k_e^2 = 0.40\ ft^2$

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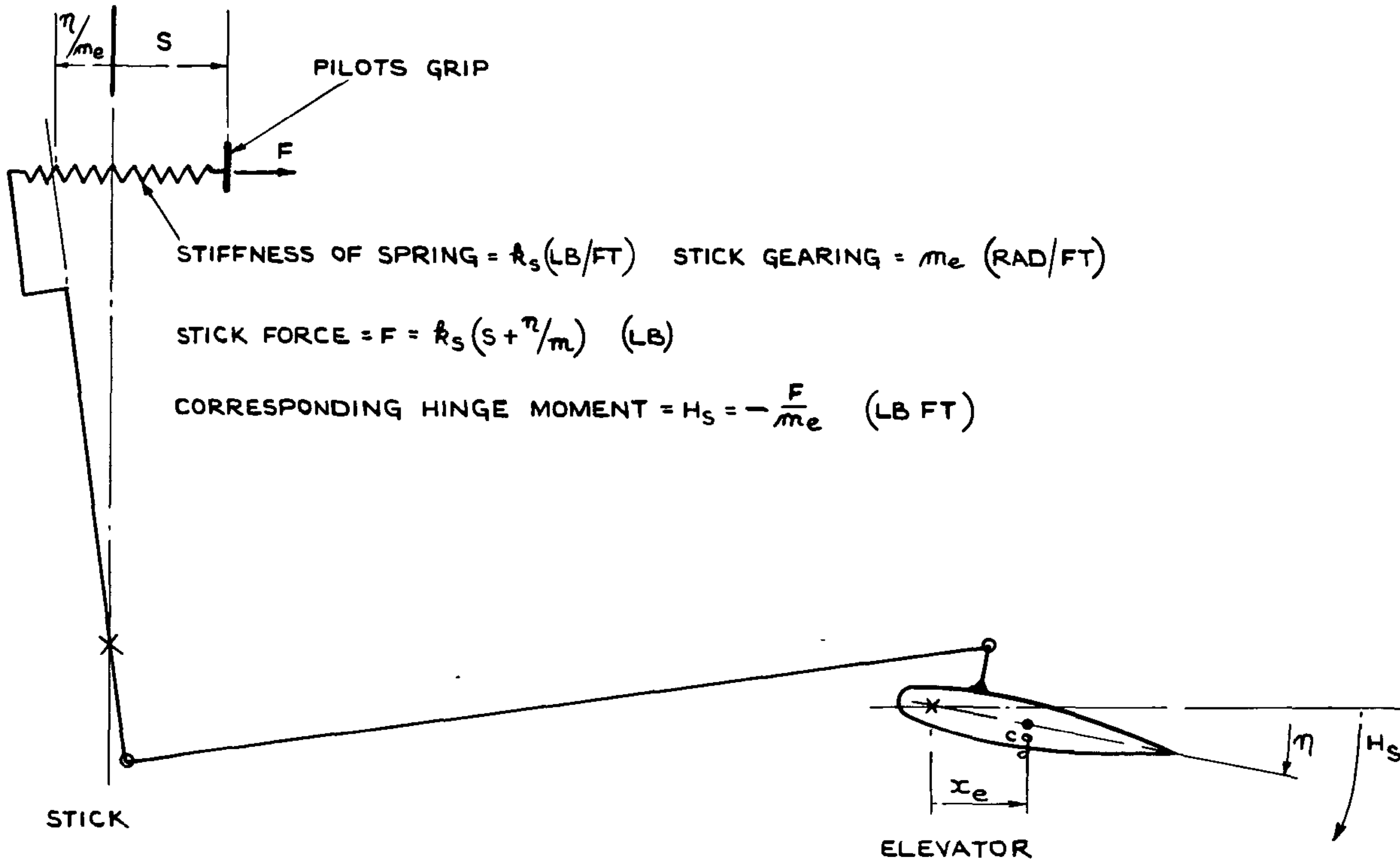


FIG 1. DIAGRAMMATIC REPRESENTATION OF THE ELEVATOR CIRCUIT.

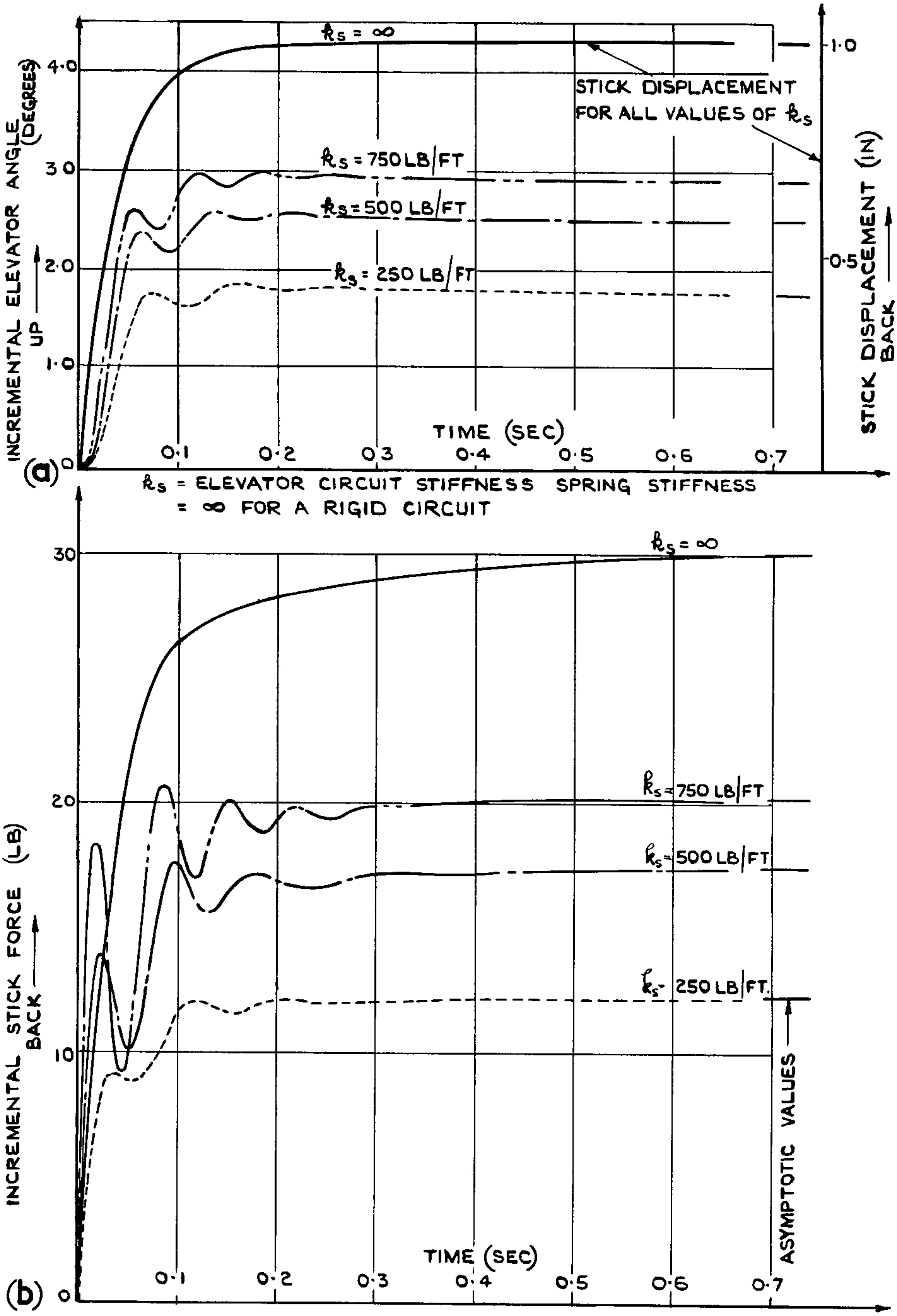


FIG 2 (a & b) EFFECT OF CIRCUIT STIFFNESS ON THE LOADING CONDITIONS IN A RAPID PULL-OUT MANOEUVRE

($V = 140$ KTS $x_e = 0{\cdot}35$ FT, $k = 15{\cdot}65$)

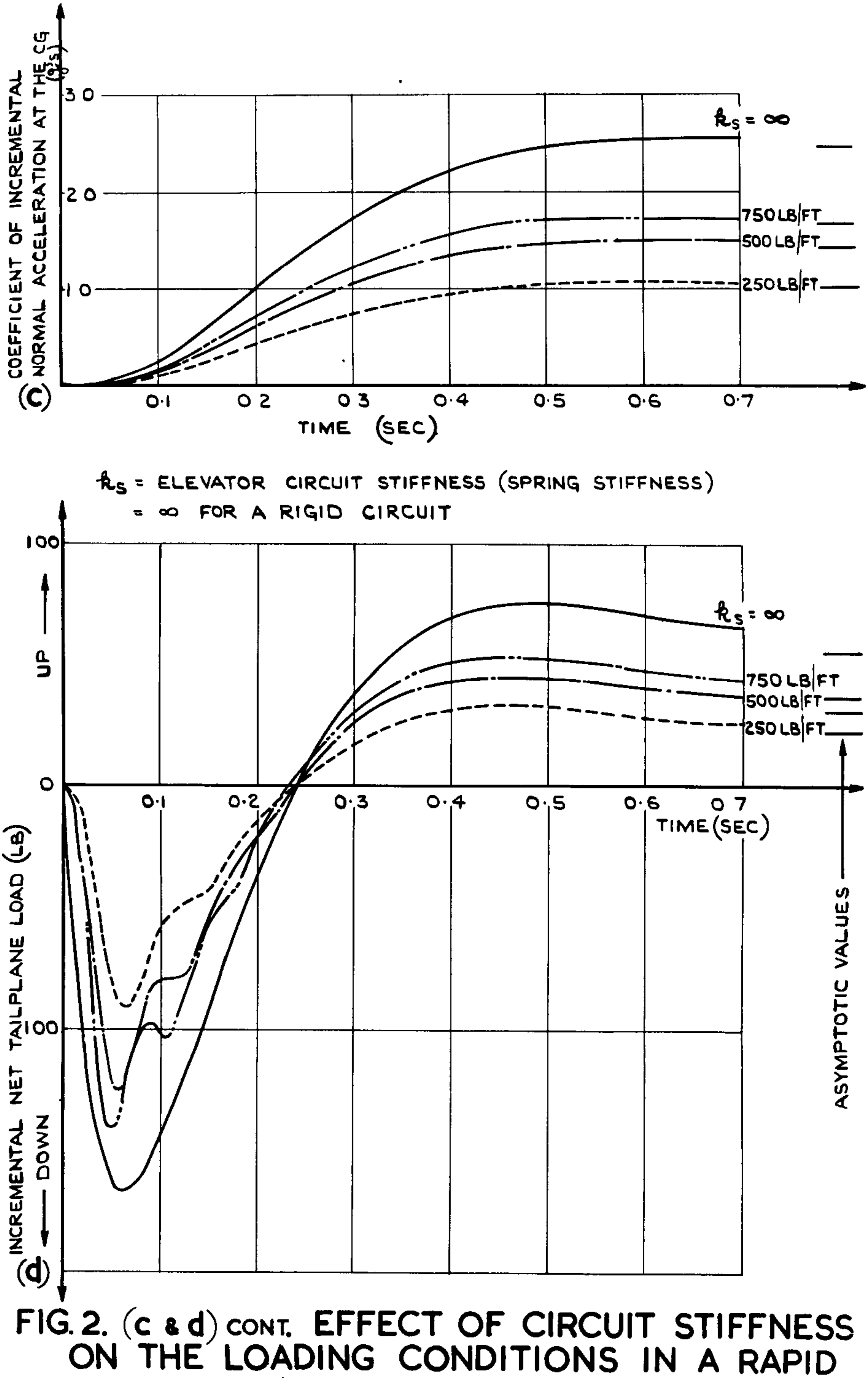
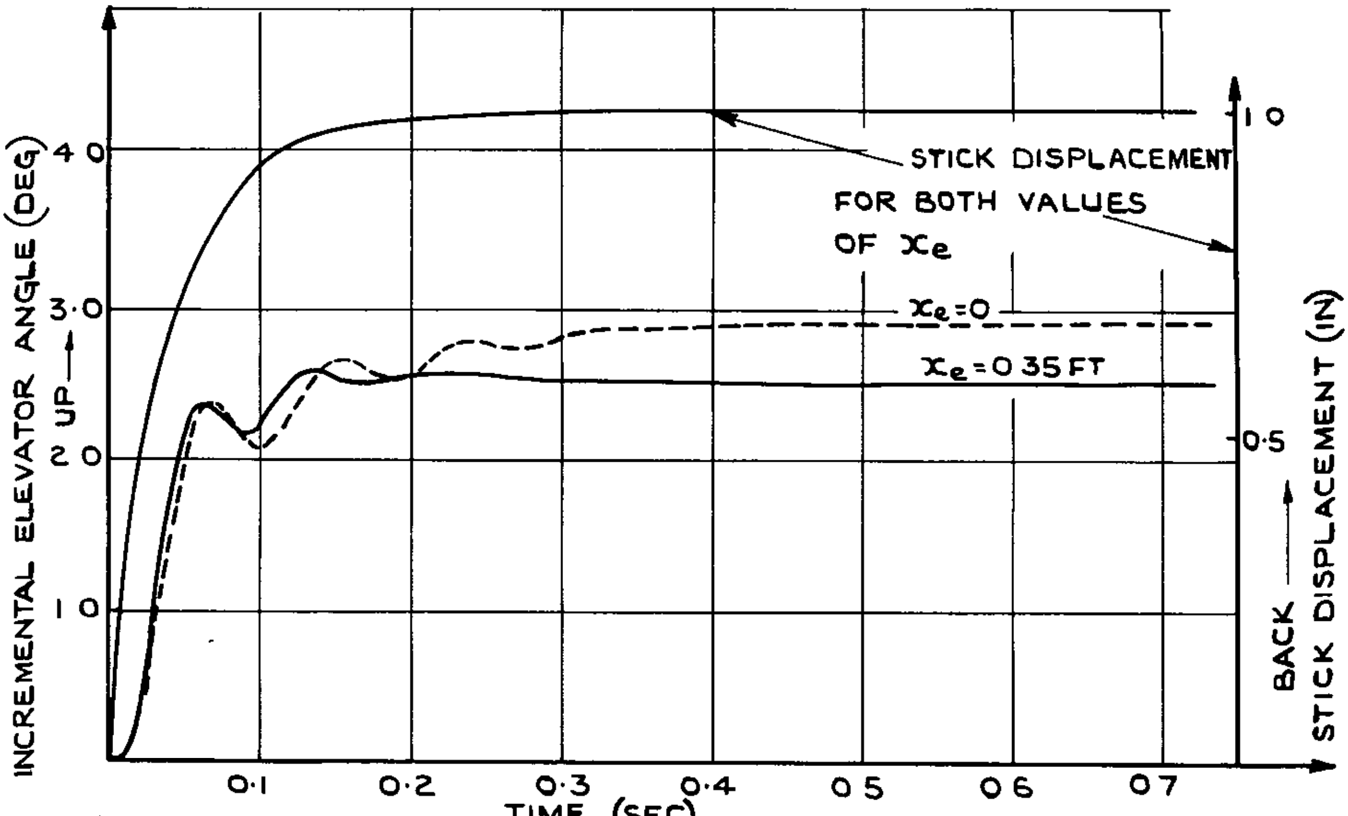


FIG. 2. (c & d) CONT. EFFECT OF CIRCUIT STIFFNESS ON THE LOADING CONDITIONS IN A RAPID PULL-OUT MANOEUVRE

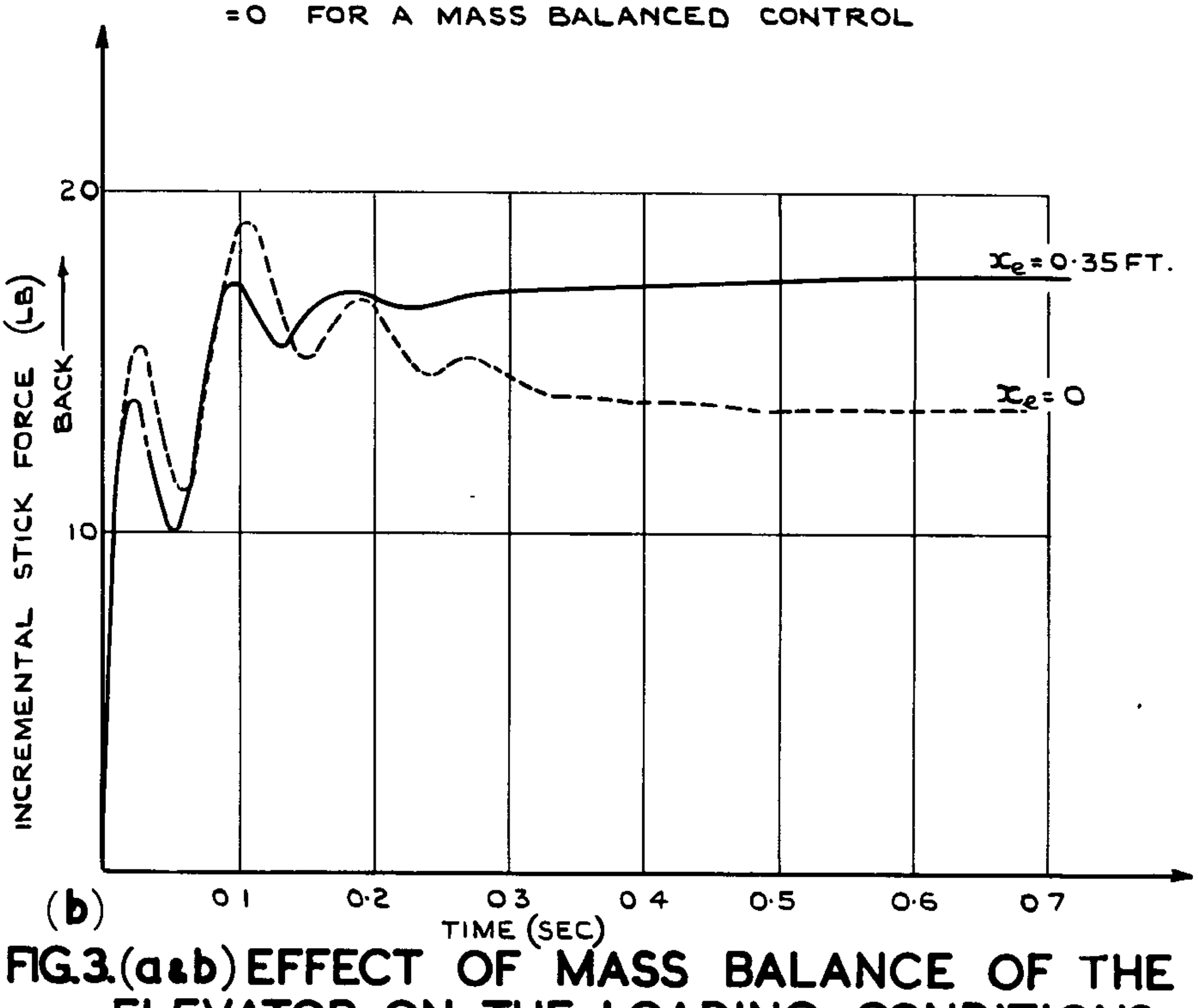
($V = 140$ KTS, $x_e = 0{\cdot}35$ FT., $k = 15{\cdot}65$)



(a)

x_e = DISTANCE OF CG OF ELEVATOR AFT OF ITS HINGE LINE

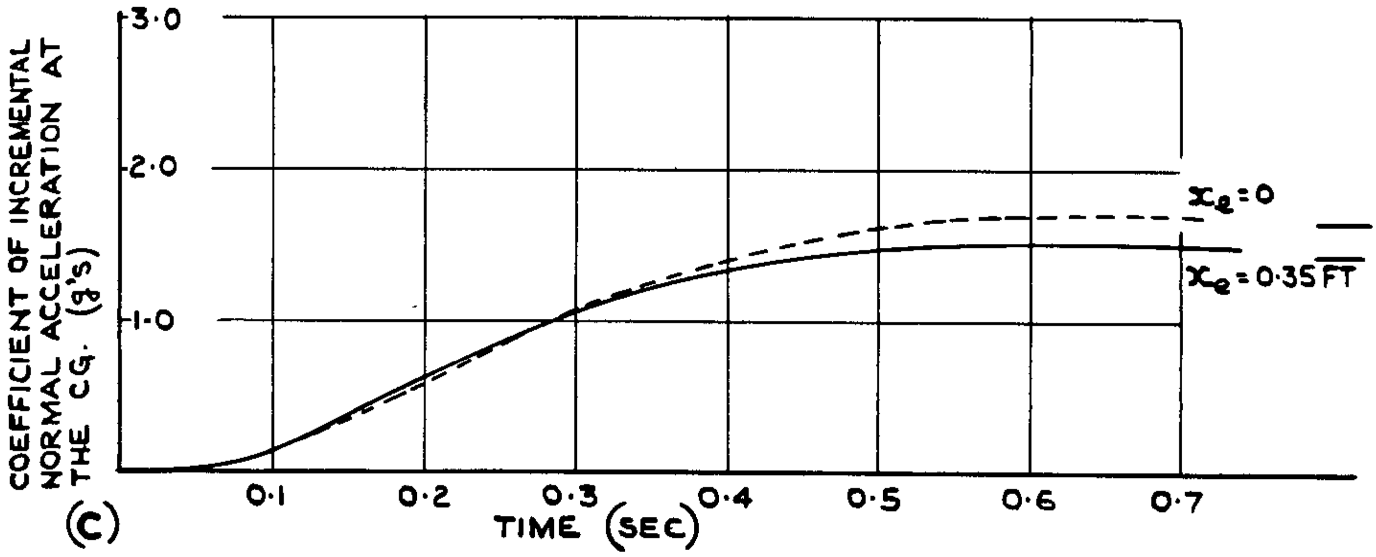
= 0 FOR A MASS BALANCED CONTROL



(b)

FIG.3.(a&b) EFFECT OF MASS BALANCE OF THE ELEVATOR ON THE LOADING CONDITIONS IN A RAPID PULL-OUT MANOEUVRE WHEN THE ELEVATOR CIRCUIT IS NOT RIGID.

($V = 140$ KTS, $k = 15.65$, $k_s = 500$ LB/FT)



x_e = DISTANCE OF CG OF ELEVATOR AFT OF ITS HINGE LINE
= 0 FOR A MASS BALANCED CONTROL

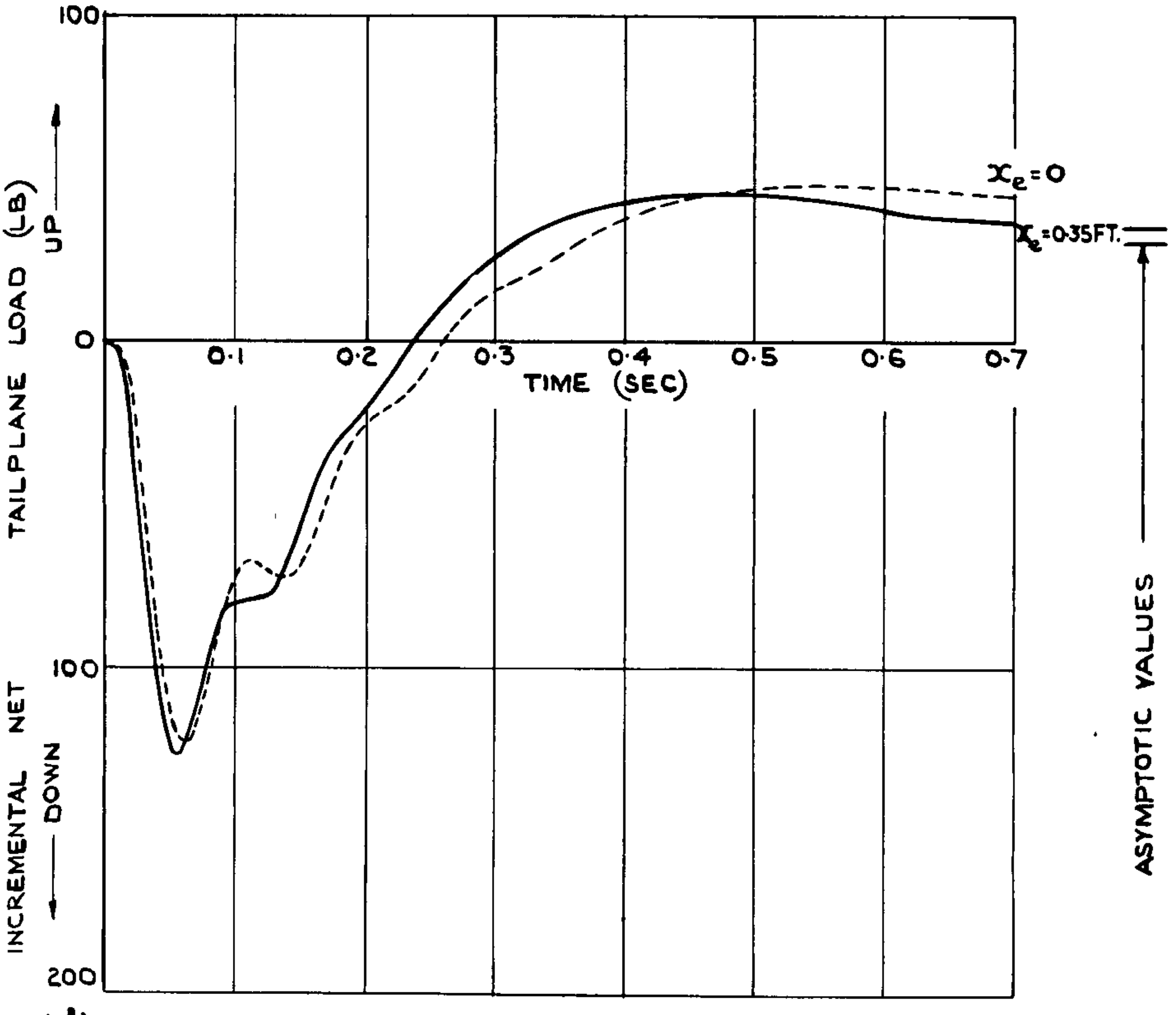


FIG 3 (c & d) CONT. EFFECT OF MASS BALANCE OF THE ELEVATOR ON THE LOADING CONDITIONS IN A RAPID PULL-OUT MANOEUVRE WHEN THE ELEVATOR CIRCUIT IS NOT RIGID.
($V = 140$ KTS, $k = 15{\cdot}65$, $k_s = 500$ LB/FT.)

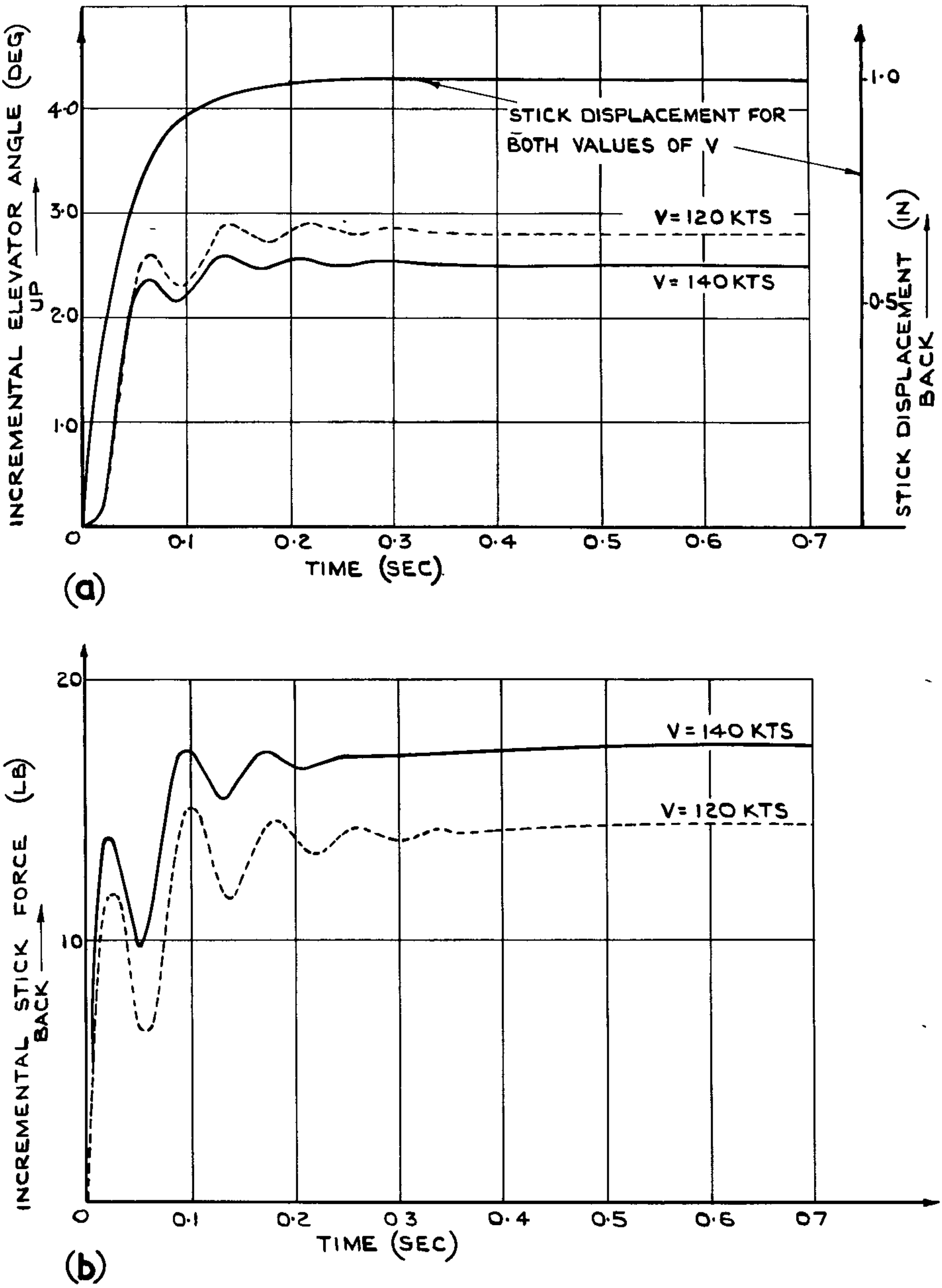


FIG 4 (a & b) EFFECT OF FORWARD SPEED ON THE LOADING CONDITIONS IN A RAPID PULL-OUT MANOEUVRE WHEN THE ELEVATOR CIRCUIT IS NOT RIGID.

(x_e = 0·35 FT, k = 15·65, k_s = 500 LB/FT.)

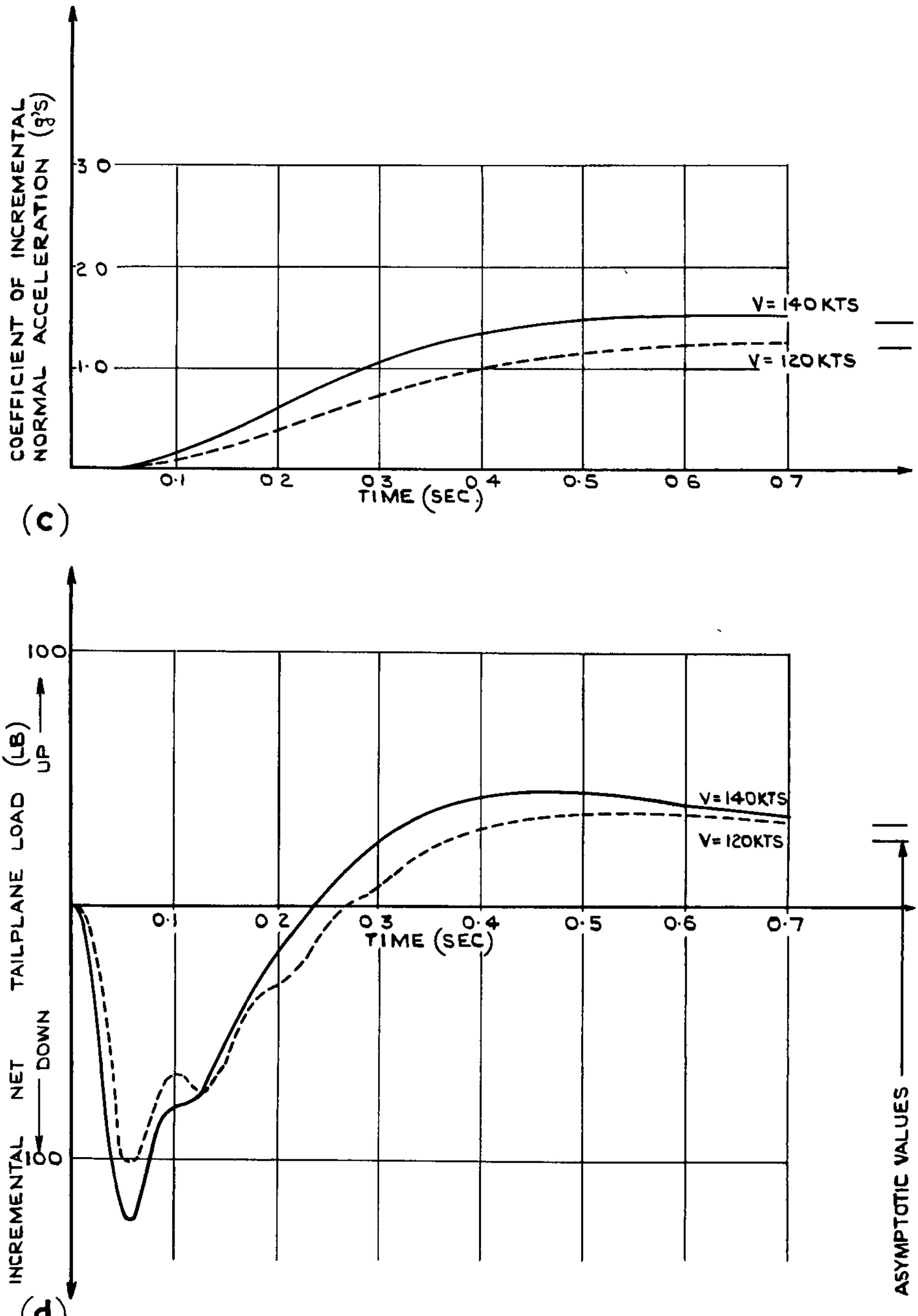
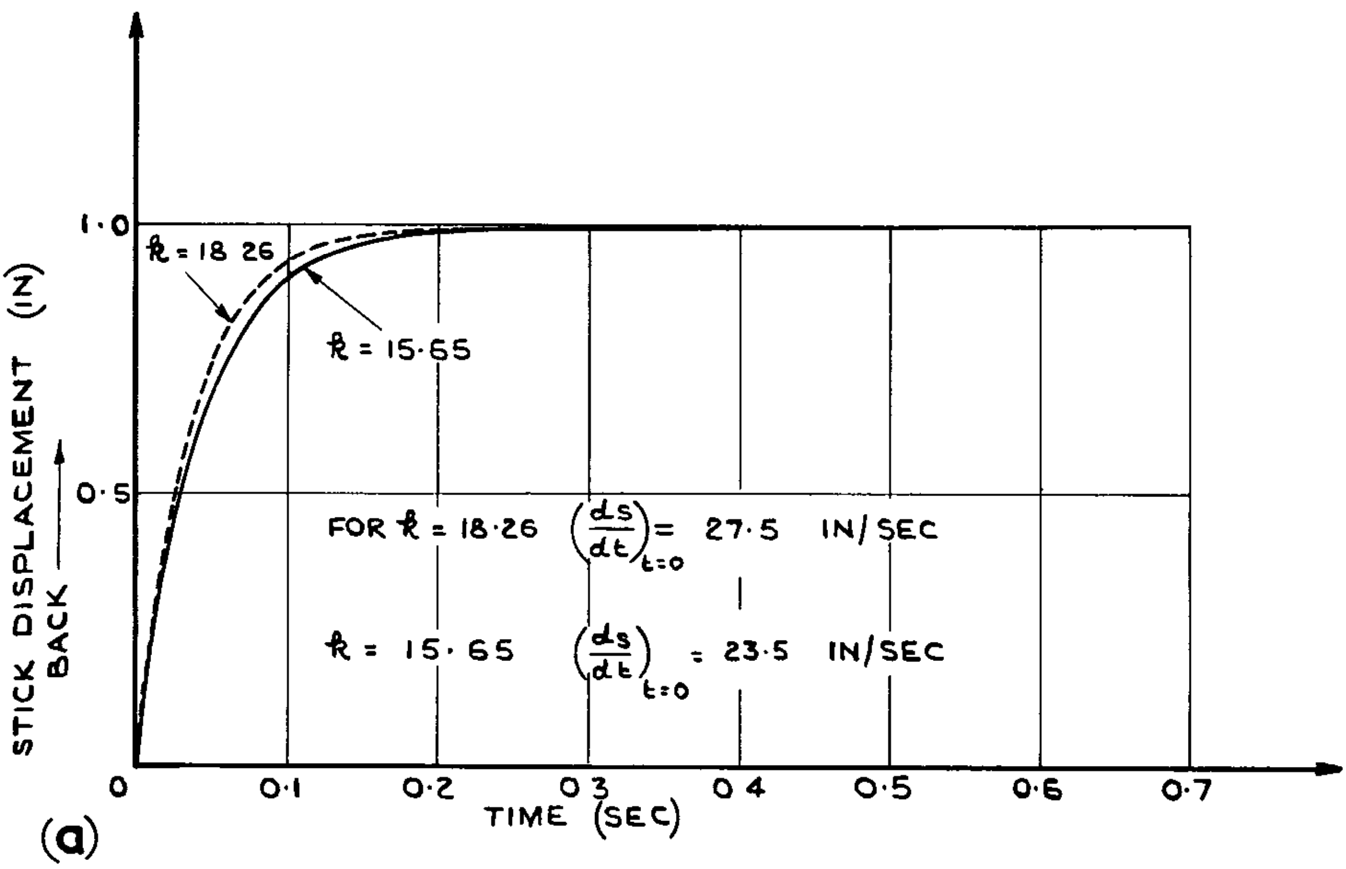


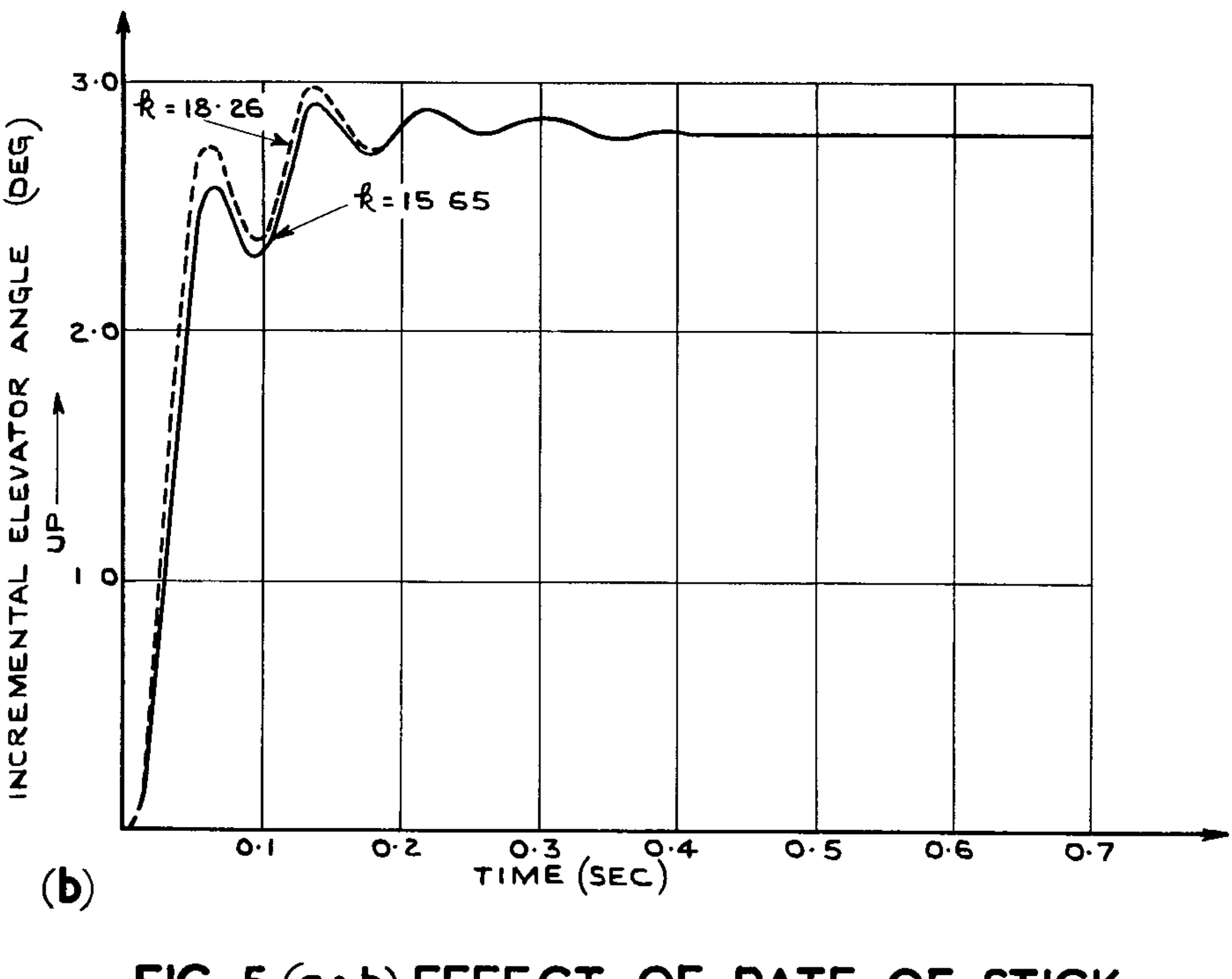
FIG. 4. (c & d) CONT. EFFECT OF FORWARD SPEED ON THE LOADING CONDITIONS IN A RAPID PULL-OUT MANOEUVRE WHEN THE ELEVATOR CIRCUIT IS NOT RIGID

($x_e = 0{\cdot}35$ FT $k = 15{\cdot}65$, $k_s = 500$ LB/FT)



(a)

INCREMENTAL STEADY NORMAL ACCELERATION = 1·18 g IN BOTH CASES.



(b)

FIG. 5.(a & b) EFFECT OF RATE OF STICK MOVEMENT ON THE LOADING CONDITIONS IN A RAPID PULL-OUT MANOEUVRE WHEN THE ELEVATOR CIRCUIT IS NOT RIGID

($V = 120$ KTS, $x_e = 0{\cdot}35$ FT, $k_s = 500$ LB/FT)

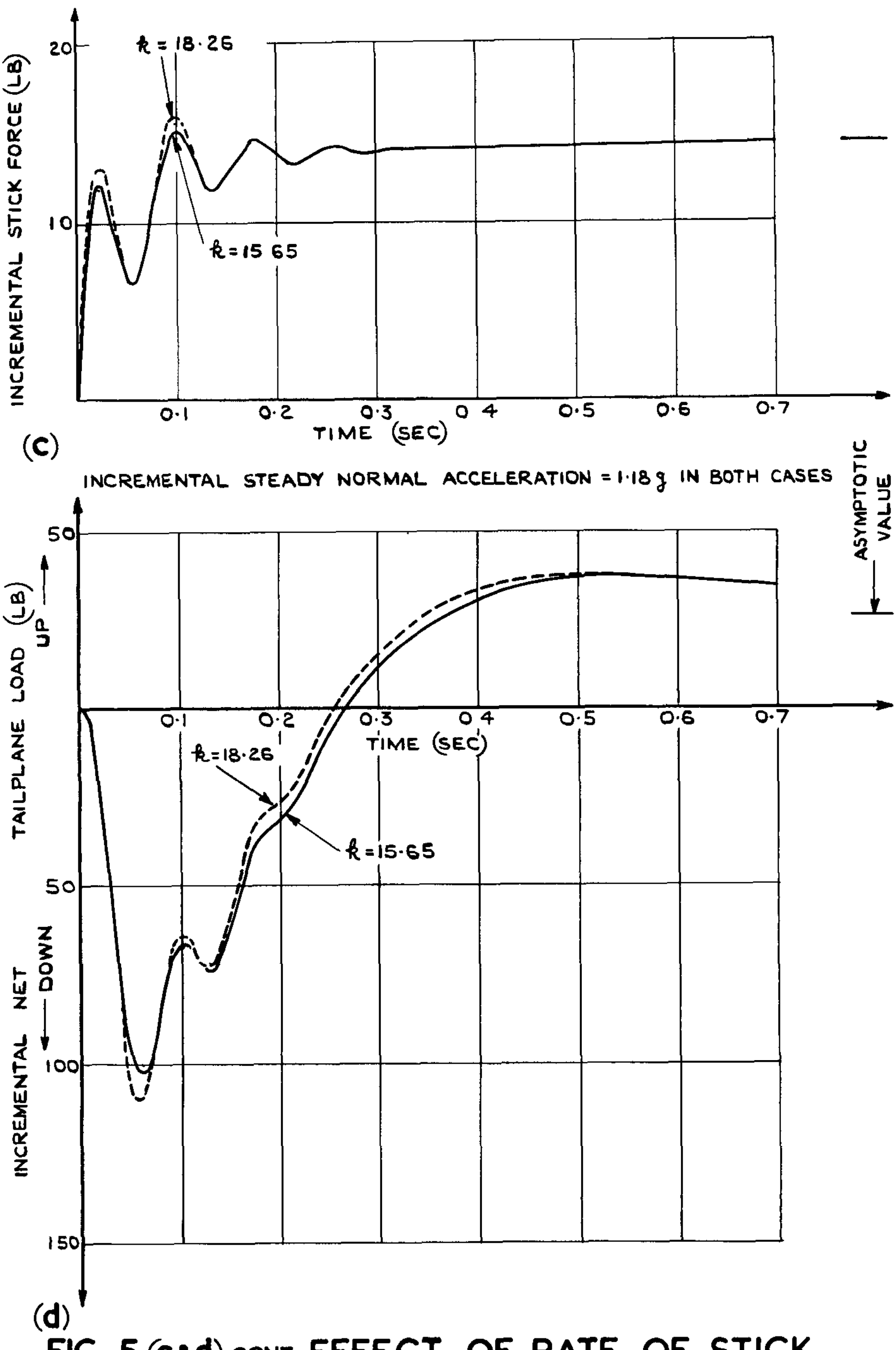


FIG. 5.(c&d) CONT. EFFECT OF RATE OF STICK MOVEMENT ON THE LOADING CONDITIONS IN A RAPID PULL-OUT MANOEUVRE WHEN THE ELEVATOR CIRCUIT IS NOT RIGID

(V=120KTS, x_e=0·35FT, k_s=500LB/FT)

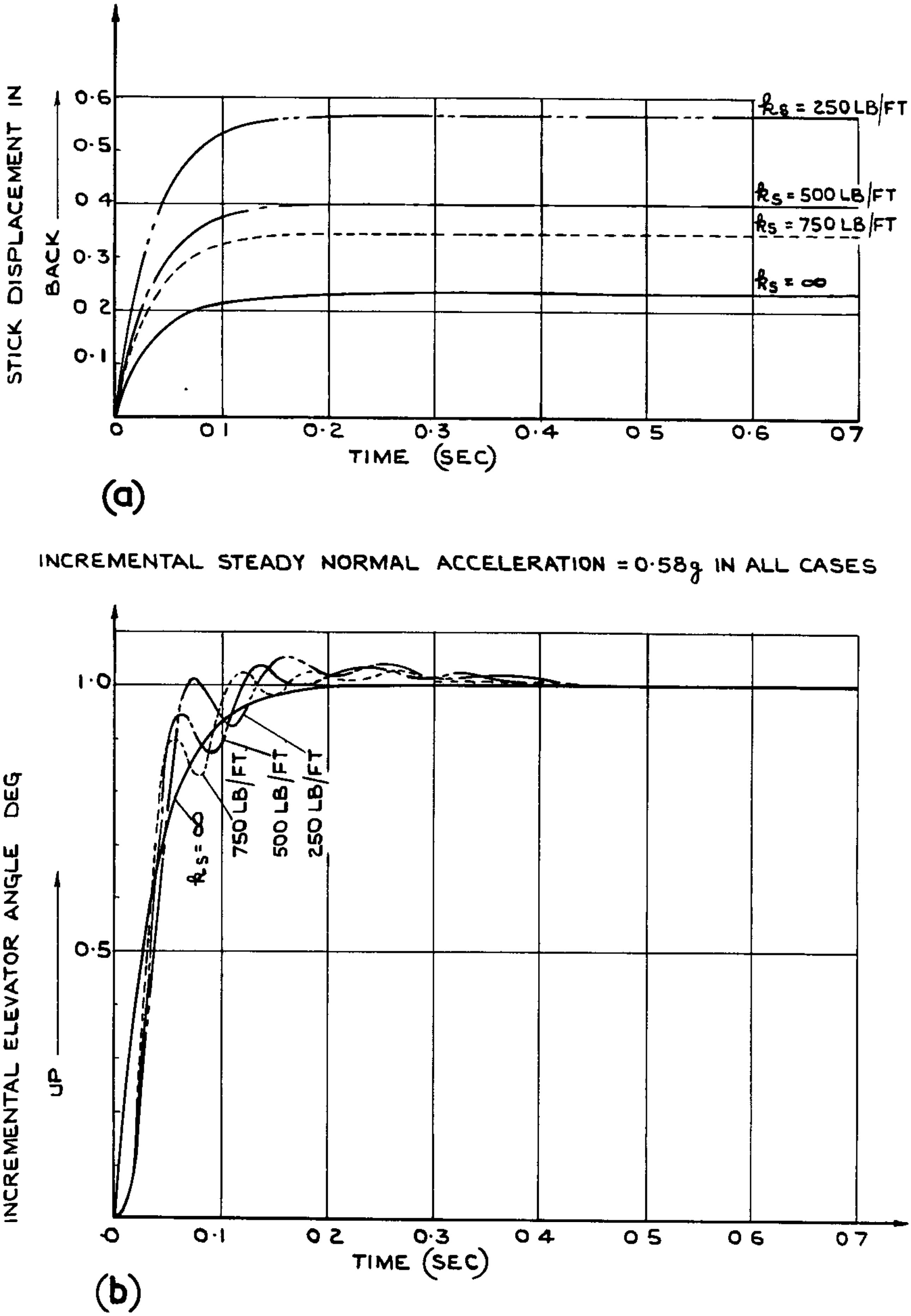


FIG 6 (a & b). EFFECT OF ELEVATOR CIRCUIT STIFFNESS ON THE LOADING CONDITIONS IN A RAPID MANOEUVRE TO A GIVEN NORMAL ACCELERATION.

(V = 140 KTS, x_e = 0·35 FT, k = 15-65)

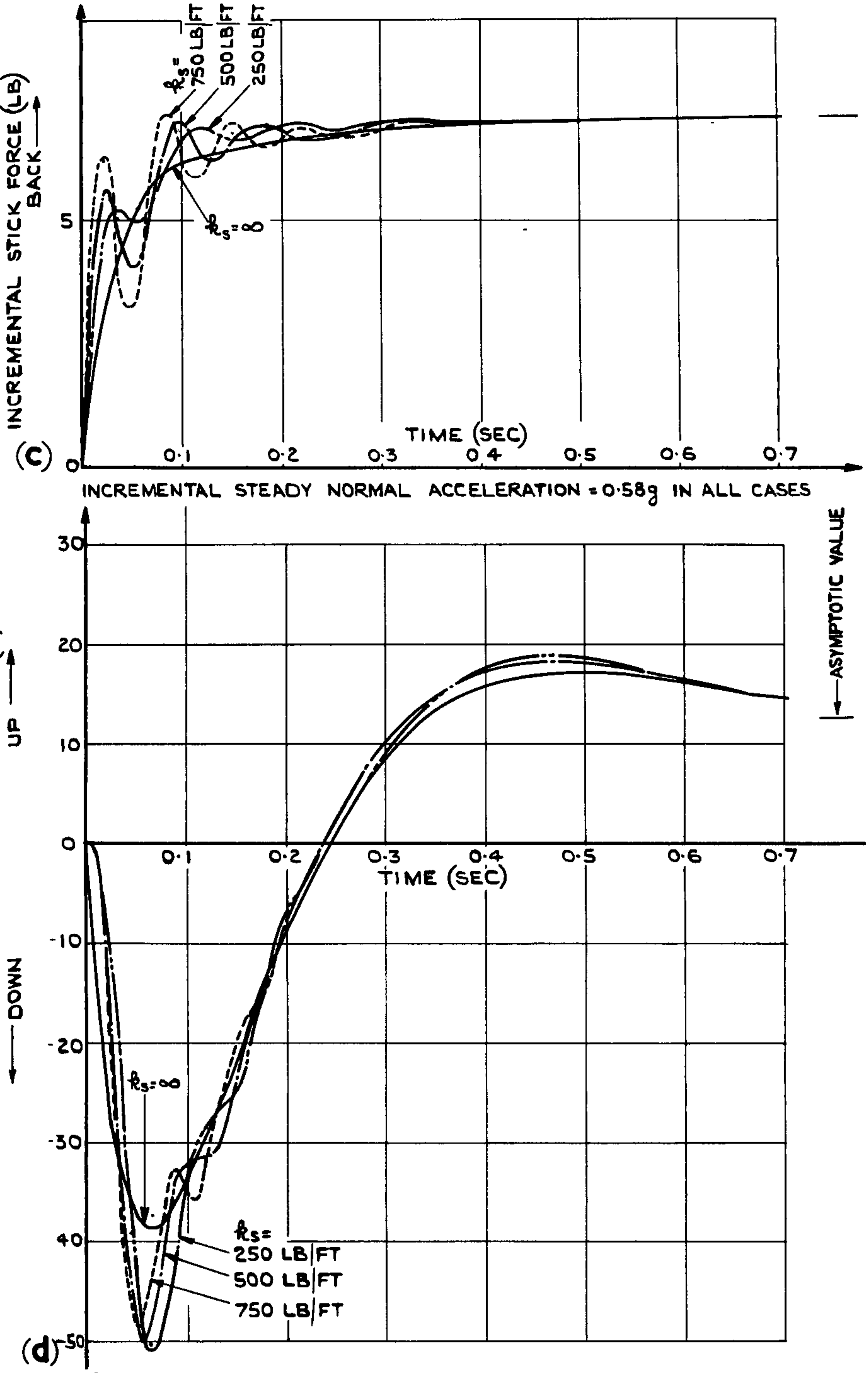
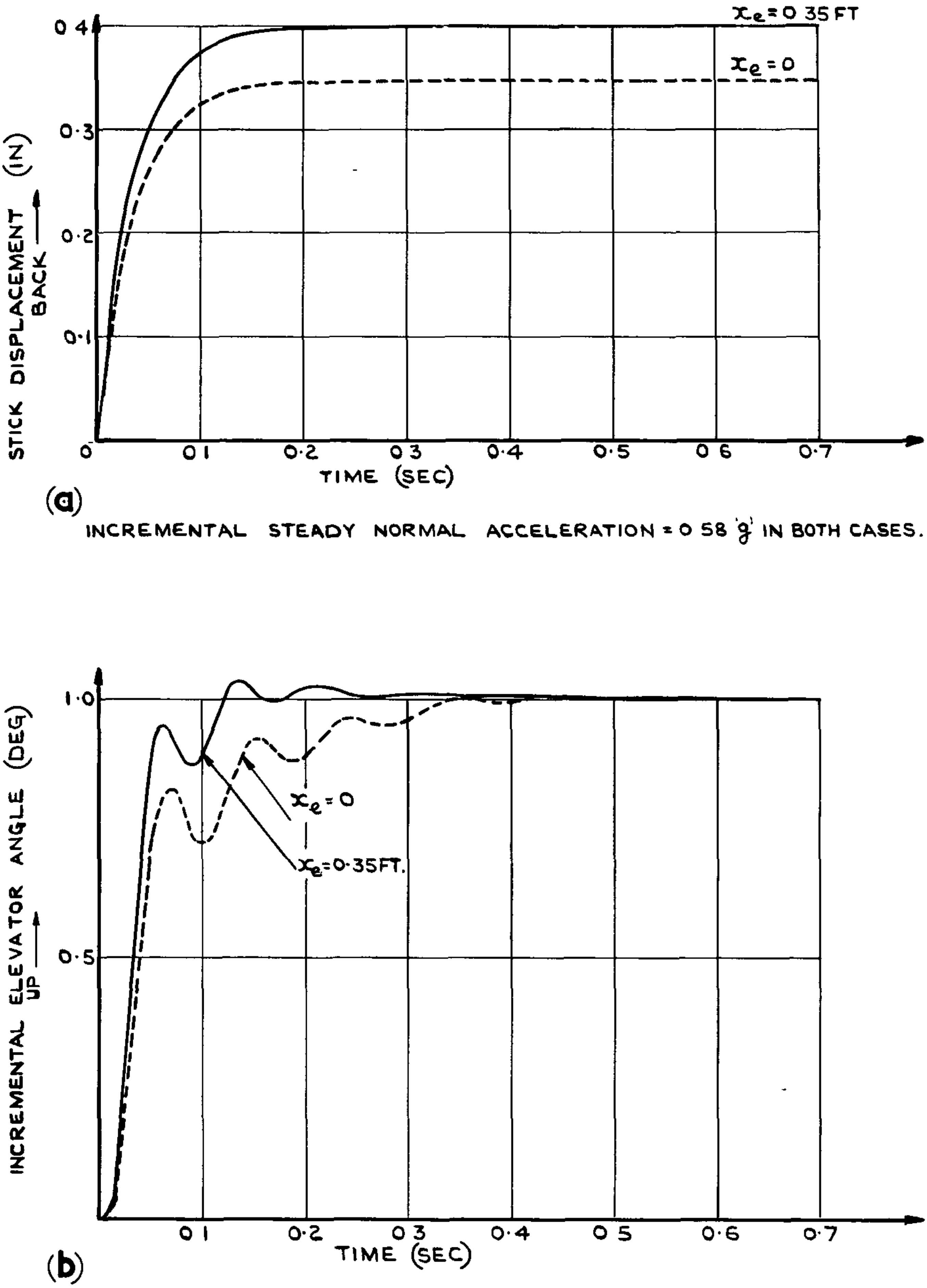


FIG. 6.(c & d) CONT. EFFECT OF ELEVATOR CIRCUIT STIFFNESS ON THE LOADING CONDITIONS IN A RAPID MANOEUVRE TO A GIVEN NORMAL ACCELERATION

($V = 140$ KTS, $x_l = 0{\cdot}35$ FT, $k = 15{\cdot}65$)



(a)

INCREMENTAL STEADY NORMAL ACCELERATION = 0.58 'g' IN BOTH CASES.

(b)

FIG. 7. (a & b) EFFECT OF ELEVATOR MASS BALANCE ON THE LOADING CONDITIONS IN A RAPID PULL-OUT MANOEUVRE TO A GIVEN NORMAL ACCELERATION

($V = 140$ KTS, $k = 15.65$, $k = 500$ LB/FT)

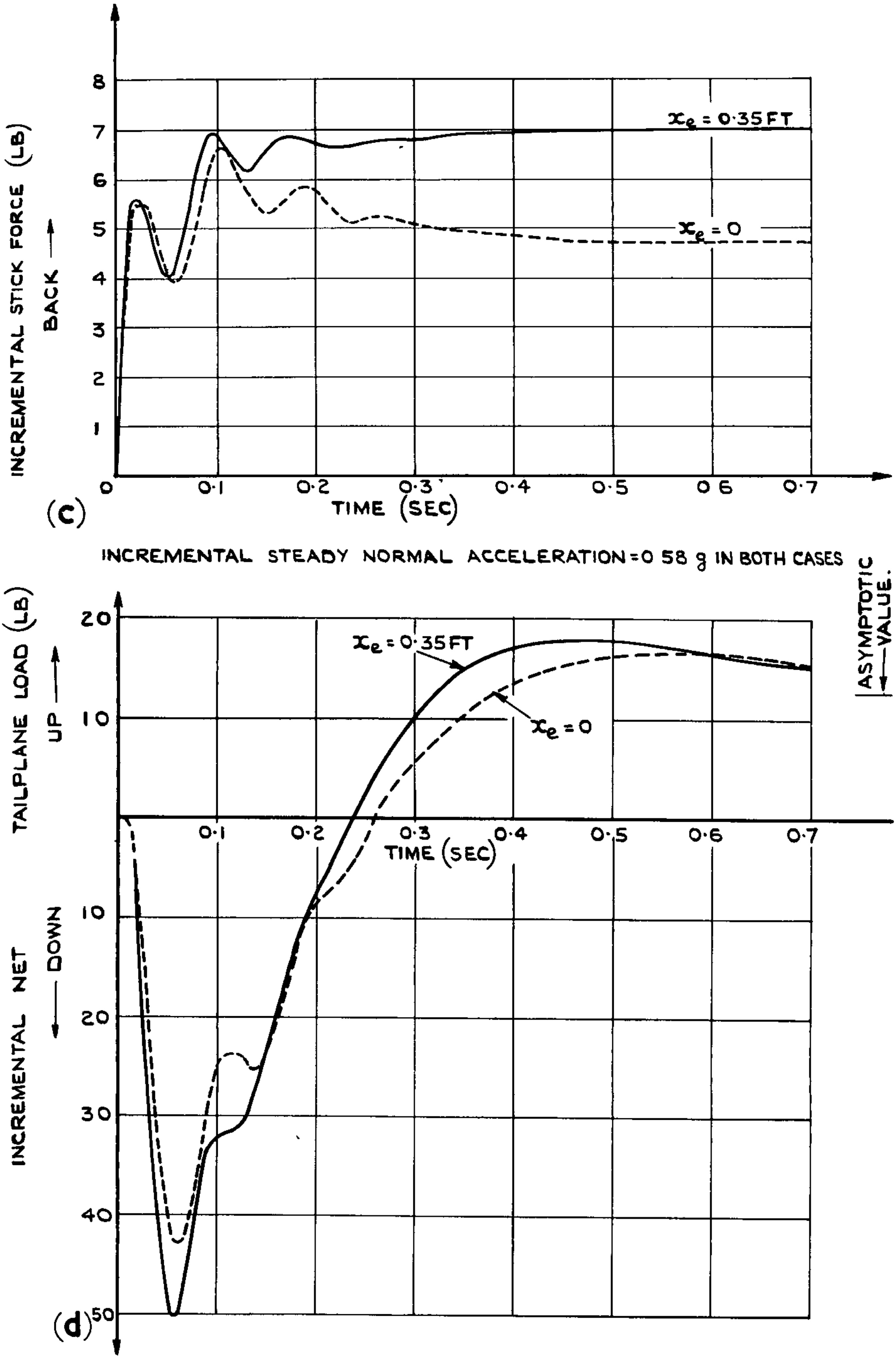


FIG. 7. (c & d) CONT. EFFECT OF ELEVATOR MASS BALANCE ON THE LOADING CONDITIONS IN A RAPID PULL-OUT MANOEUVRE TO A GIVEN NORMAL ACCELERATION

($V = 140$ KTS, $k = 15{\cdot}65$, $k_s = 500$ LB/FT)

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