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Summary.—The requirements for simulating in a wind tunnel flutter conditions appropriate to high-speed flight are discussed, and an assessment is made of the desirable features of a wind tunnel suitable for flutter testing at transonic and supersonic speeds.

It is concluded that such a tunnel should have either the Mach number or the stagnation pressure variable during the tunnel run, and that it is of considerable advantage, and for some purposes essential, for high stagnation pressures to be available. The stagnation pressure required to allow flight conditions to be simulated with a flutter model is considered to range from at least 2 atmospheres for transonic speeds to about 15 atmospheres for $M = 4$. No attempt to simulate kinetic heating is envisaged, although its effect on stiffness should be allowed for in the design of the model. To minimise uncertainties due to the variation of the model stiffness with temperature it is desirable that means for controlling the stagnation temperature should be incorporated in the tunnel.

1. *Introduction.*—Flutter experiments on models in low-speed wind tunnels have been used extensively in the past, both for general research purposes and for investigation of the flutter characteristics of specific aircraft. Similar investigations covering the conditions of high-speed flight have been carried out in the U.S.A.[1,2,3], but few have as yet been attempted in this country[4]. The increased difficulties of experimentation arise mainly from the smallness of the available high-speed wind tunnels and from the more exacting airflow conditions required. The present report is based on considerations recently given at the National Physical Laboratory to the practicability of carrying out flutter experiments in the Laboratory's existing high-speed wind tunnels, and also to the design features of a proposed 15-in. by 10-in. blow-down tunnel desirable for such experiments. Since these wind tunnels are designed to use air as the working medium no attention has been given to the possible advantages of using other gases such as Freon.

The main text is devoted to the requirements for simulating flight conditions relevant to flutter with models in high-speed wind tunnels, and to some experimental techniques which might be adopted for general research and for flutter tests of a particular aircraft. The suitability of some existing wind tunnels for flutter experiments, and some recommended design features for the proposed 15-in. × 10-in. blow-down tunnel, are discussed in the Appendix.

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2. *Simulation of Flight Conditions.*—The conditions leading to an aeroelastic instability in flight can be simulated with a model in a wind tunnel provided the model has similar aerodynamic (*i.e.*, external) shape, similar stiffness and mass distributions as its full-scale counterpart and also provided a number of non-dimensional parameters are the same as for full scale. If we assume that the structural damping of the material of the model is approximately the same as that of full scale, that gravitational effects are negligible and that the Reynolds number for the model is not excessively low, the independent non-dimensional parameters which need to achieve full-scale values can be limited to the following three :

(*a*) Mach number M

(*b*) Density parameter σ/ρ

(*c*) Stiffness parameter $\varepsilon/\rho L^3 V^2 \equiv S/\rho V^2$
or alternatively $\omega L/V$.

Ideally the requirement for general research investigations is that the ranges of values of these parameters met with in practice shall be covered in the wind tunnel unless knowledge is already available concerning their separate influences. For prototype or 'clearance' testing it would be necessary to simulate one or more particular conditions of flight. It is worth while examining the required test conditions for this in more detail since the results are also applicable to the requirements of the more general type of investigation.

For the present purpose a flight condition is completely specified by the Mach number M and the altitude h, since on the basis of a standard atmosphere these two quantities determine the values of V_a, p_a, ρ_a, and T_a. On the other hand the flow conditions in a tunnel are completely specified by three independent quantities conveniently chosen from the standpoint of tunnel operation as Mach number, stagnation temperature and stagnation pressure. From these quantities the values of V_m, p_m, ρ_m, and T_m, can be derived by means of the usual equations of isentropic flow. Fig. 1 shows the variations of pressure, density and temperature over a range of Mach number. For a tunnel in which stagnation conditions remain constant and atmospheric (i.e., $p_0 = 1$ Atm and $T_0 = 288$ deg K), the right-hand scales in Fig. 1 give the actual values of temperature, pressure and density which are compared with the values for flight at four altitudes. It will be noted that in a tunnel having atmospheric stagnation conditions, the values of all three quantities at the higher Mach numbers are considerably less than those of flight at all but the higher altitudes. Since V is proportional to $\sqrt{T}$, for any particular high Mach number the velocities in a wind tunnel having atmospheric stagnation temperature are also less than those in flight as shown in Fig. 2.

The wind-tunnel flow and model structural requirements for similarity can be restated as :

$$M_m = M_a \qquad \text{(1)}$$

$$\frac{S_m}{\rho_m {V_m}^2} = \frac{S_a}{\rho_a {V_a}^2} \qquad \text{(2)}$$

$$\frac{\sigma_m}{\rho_m} = \frac{\sigma_a}{\rho_a}. \qquad \text{(3)}$$

It should be noted that the values of S_a at high Mach numbers may differ from those measured on the ground because of temperature and thermal effects on the structure[5].

By rearrangement of the above equations we obtain :

$$\frac{S_m}{\sigma_m} \Big/ \frac{S_a}{\sigma_a} = \left(\frac{V_m}{V_a}\right)^2 = \left(\frac{T_m}{T_a}\right) = \frac{T_0 F_T(M)}{T_a}. \qquad \text{(4)}$$

Thus the structural efficiency (S/σ) scale of the model is dependent on Mach number, tunnel stagnation temperature and altitude (i.e., T_a) and is independent of stagnation pressure. Also since,

$$\frac{S_m}{\sigma_m} \Big/ \frac{S_a}{\sigma_a} = \left(\frac{\omega_m L_m}{\omega_a L_a}\right)^2 \quad \ldots \quad (5)$$

the frequency scale is additionally dependent on the linear scale.

From equation (3) we obtain

$$\frac{\sigma_m}{\sigma_a} = \frac{\rho_m}{\rho_a} = \frac{\rho_0 F_\rho(M)}{\rho_a} = \frac{p_0}{p_a}\frac{T_a}{T_0} F_\rho(M) . \quad \ldots \quad (6)$$

Thus the density scale is dependent on Mach number, tunnel stagnation pressure and temperature, and altitude (i.e., p_a and T_a).

From equations (3) and (4) we have

$$\frac{S_m}{S_a} = \frac{\rho_m T_m}{\rho_a T_a} = \frac{p_m}{p_a} = \frac{p_0}{p_a} F_p(M) . \quad \ldots \quad (7)$$

That is, the scale for the structural elastic moduli is dependent on Mach number, tunnel stagnation pressure and altitude (i.e., p_a) and is independent of stagnation temperature.

Without regard to their practical fulfilment, the theoretical conditions for similarity in tests to cover the effects of both Mach number and altitude are now considered under three separate headings.

2.1. *The Flow Characteristics in the Tunnel Required to Simulate Flight over a Range of Mach Number and Altitude with a Single Model.*—The fixed quantities are S_m and σ_m and the disposable* quantities in the tests are M, p_0 and T_0.

For each Mach number the required stagnation temperature is obtained from equation (4) :

$$T_0 = \frac{T_a}{F_T(M)} \frac{S_m}{\sigma_m} \frac{\sigma_a}{S_a} , \quad \ldots \quad (8)$$

whilst the required stagnation pressure is obtained from equation (7) :

$$p_0 = \frac{p_a}{F_p(M)} \frac{S_m}{S_a} . \quad \ldots \quad (9)$$

In Figs. 3 and 4 the required values of T_0 and $p_0/(S_m/S_a)$ are plotted as functions of M for a range of altitude, the temperatures being shown for values of $(S_m/\sigma_m)/S_a/\sigma_a) = 1{\cdot}0, 0{\cdot}5$ and $0{\cdot}25$. These two diagrams show that

(*a*) with a single model, complete flutter simulation for a particular altitude and over a range of Mach number can only be obtained provided both stagnation pressure and stagnation temperature are disposable ; for a wide range of Mach number the necessary variation of these quantities is very large

(*b*) with a single model, complete flutter simulation at a particular Mach number over a range of altitudes also requires a variation of both stagnation temperature and pressure

(*c*) unless $(S_m/\sigma_m)/(S_a/\sigma_a)$ is less than unity very high stagnation temperatures are required at high Mach numbers. However, in the transonic region the required stagnation temperatures are about normal provided $(S_m/\sigma_m)/(S_a/\sigma_a)$ is near unity.

* For the present purpose a disposable quantity signifies one which can either be continuously variable during a tunnel run or can be pre-set at a chosen value before the run.

It will be noted that when $S_m = S_a$ and $\sigma_m = \sigma_a$, as for instance with an exact structural replica, then at high Mach numbers both the stagnation pressure and temperature need to be high.

2.2. *The Model Characteristics Required to Simulate Flight over a Range of Mach Number and Altitude in a Tunnel with Fixed Stagnation Conditions.*—The fixed quantities are p_0 and T_0 (taken as 1 Atm and 288 deg K respectively) and the disposable quantities in the tests are M, S_m and σ_m.

The ratios (σ_m/σ_a), (S_m/S_a) are given in equations (6) and (7) respectively and are shown plotted against Mach number for four altitudes in Figs. 5 and 6. Corresponding values of $(S_m/\sigma_m)/(S_a/\sigma_a)$ are plotted in Fig. 7. If we are concerned only with the transonic region, the required values of (S_m/S_a), (σ_m/σ_a), and $(S_m/\sigma_m)/(S_a/\sigma_a)$ range both below and above unity.

2.3. *The Stagnation Pressure and Model Density Required to Simulate Flight over a Range of Mach Number and Altitude for a Model of Fixed Stiffness in a Tunnel with Fixed Stagnation Temperature.*—The fixed quantities are S_m and T_0 (taken as 288 deg K), and the disposable quantities are M, σ_m and p_0.

The stagnation pressure is obtained directly from equation (7) and the required values of $p_0/(S_m/S_a)$ are shown in Fig. 4.

The density scale is given by equation (4)

$$\frac{\sigma_m}{\sigma_a} = \frac{S_m}{S_a}\frac{T_a}{T_0 F_T(M)}, \quad \ldots \quad (10)$$

and may be obtained in relation to S_m/S_a from Fig. 8.

It will be seen that a range of Mach number and altitude can be simulated with a single elastic structure provided it is possible to vary the tunnel stagnation pressure and the model density. However, the required variation of stagnation pressure would be quite large; for simulation of flight at constant altitude it would be necessary to increase the stagnation pressure by a factor of about 80 between $M = 1$ and $M = 4$. The model density would need to be increased by a factor of about 3·5.

It is of interest to note from equation (10) that, for a fixed model stiffness and Mach number, the equivalent flight temperature of a test is proportional to the model density.

2.4. It has been shown that with a model with fixed values of σ_m and S_m the simulation in the tunnel of flight conditions over a range of Mach number and altitude requires the variation of both p_0 and T_0. Quite apart from the practical difficulties of providing a disposable stagnation temperature, considerations regarding the models themselves lead to the conclusion that the stagnation temperature should not be much different from that of the room. An alternative and more acceptable way of covering a range of Mach number and altitude using a single model of fixed stiffness would be to vary stagnation pressure and to alter the model density by a distributed mass loading. However, the range of stagnation pressure needed to cover a wide range of Mach number is very large and in practice several models of different stiffnesses would almost certainly be necessary.

3. *Considerations Regarding the Models.*—If acceptable from the aerodynamic standpoint, the most convenient arrangement would appear to be a component model (*e.g.*, a semi-span wing or a tail unit) supported directly at the wall of the tunnel. Body freedoms if required could be provided outside the working-section. Where more than one lifting surface is involved it may be necessary to mount the model on a central body situated along the tunnel axis. With either of these methods, it would be possible by suitable design of working-section to insert the model into the stream after the flow is established if this is necessary to avoid starting loads at supersonic speeds.

The maximum permissible size of model depends on the size of the tunnel and on the Mach number. The limitations are well known for supersonic speeds but at present there is insufficient knowledge of this aspect of transonic tunnel testing to enable model sizes to be stated with confidence. It is necessary to avoid large area blockage of the working-section and also to ensure that shocks of significant strength do not strike the model after reflection from the tunnel walls. For Mach numbers greater than unity, with a fixed size of working-section, the model can be larger the higher the Mach number. A tunnel with a working-section 15 in. × 10 in. might permit the use of model spans as much as 12 in. at $M = 4 \cdot 0$ but perhaps no more than 5 in. at transonic speeds. It may be noted here that for a fixed stagnation temperature, the model frequencies are inversely proportional to the model scale and hence for small models they will be high. Thus the smaller the model the greater the difficulty of providing an effective support at the tunnel even with the use of a 'seismic' support. This particular difficulty is absent with complete models.

Both of the above model sizes are regarded as small from constructional considerations and the structures would need to be very simple, perhaps either machined from the solid or consisting of a single elastic spar carrying the aerodynamic shape. The need for simplicity, particularly in the building of research models, also follows from the fact that it may not be possible to prevent models breaking up when they flutter. If the determination of each critical condition requires a new model, the method of construction must be as rapid and inexpensive as possible. In this event and where a number of models of the same or slightly different stiffnesses is required for a series of tests, a technique of model making using a plastic material and a master mould might prove to be the most suitable. Other factors which influence the choice of constructional material and method are the temperature which will be encountered in the tunnel, the stiffness and density requirements to match the available flow parameters, and the strength.

Since no attempt would be made to reproduce kinetic heating in the tunnels under discussion (*see* Appendix), no high temperatures are envisaged. However, the temperature changes that are commonly encountered in high-speed tunnels could be enough with some structural materials to cause appreciable changes in the stiffnesses of the model. The elastic material of models suitable for use with large temperature changes would need to be restricted to those substances having a low temperature coefficient of elastic modulus. From this standpoint metals would be most suitable. For example, a 1 per cent change in Young's modulus requires temperature changes of approximately 40 deg C and 17 deg C in steel and Duralumin respectively, while experience has shown that the stiffness of a wooden spar can change by as much as $0 \cdot 5$ per cent deg C. This large variation would prohibit the use of wood for elastic members when the temperature changes are more than a few degrees. The use of plastic materials is probably also limited to experiments in which the temperature remains reasonably constant.

To maintain the model at a temperature close to that of the tunnel room would require an elevated stagnation temperature, particularly at the higher Mach numbers. The equilibrium temperature of the model T_w can be related to the stagnation temperature and Mach number by the relation

$$T_w = T_0[C_\theta + F_T(M)(1 - C_\theta)] , \quad \ldots \quad (11)$$

where C_θ is the temperature recovery factor. For $C_\theta = 0 \cdot 9$ equation (11) gives

$$T_0 = \frac{T_w}{0 \cdot 9 + 0 \cdot 1 \, F_T(M)} . \quad \ldots \quad (12)$$

This function is plotted in Fig. 9 which shows the required stagnation temperature to maintain the model temperature at specified values. For a model to be maintained at a temperature of about 15 deg C, values of T_0 are required which range from 20 deg C at $M = 1$ to nearly 40 deg C at $M = 4$.

It has been shown in a previous section that the values of S_m and σ_m must be related to the flow parameters of the experiment. It follows therefore that the method of model construction and the choice of materials are inter-related with the range of flow parameters available in the tunnel. Unless the flow parameters fall within certain ranges, the construction of suitable models may prove difficult. For instance, since a high value of S/σ is a principal aim of full-scale design, it can be assumed that it will be difficult to design models for which the value of S/σ is much greater than full-scale values, although no difficulty would be encountered in designing structures of lower efficiency. The quantity S/σ has to be related to the stagnation temperature of the tunnel and the specification that this temperature should not differ greatly from that of the room, leads to a number of consequences concerning models required to simulate flight conditions. Fig. 7 shows that when $T_0 = 288$ deg K, for Mach numbers about unity, the value of $(S_m/\sigma_m)/(S_a/\sigma_a)$ required for flight simulation is also about unity, values higher than unity being required for high-altitude simulation. Thus at transonic speeds the model must have a structure which has about the same efficiency as the full-scale structure. For full-scale wings which are highly efficient stressed-skin structures, the model must also be a stressed-skin structure using a material for which the ratio E/σ_m' is the same as for the full-scale material. The construction of such models to small linear scales would be both costly and troublesome. The difficulties associated with small skin thicknesses could be mitigated to some extent by the use of relatively thicker sheets since this would increase both S and σ in the same ratio. The resultant increase in the stiffness could be offset by using a material having a low value of E, but in view of the need for maintaining the full-scale value of E/σ_m', the choice is limited (*see* Table 1). The preceding considerations lead to the general conclusion that for models in which both the density and the stiffness are correctly simulated the values of S_m/S_a for transonic speeds will be about unity or even greater.

If, as is probably most convenient for clearance tests, the model is built to represent a reduced full-scale stiffness (reduced say by the factor $0 \cdot 7$ to allow the establishment of a stiffness margin) the difficulties of model construction are relieved to some extent since the required structural efficiency is similarly reduced.

The difficulties of making true simulation models for use at the higher Mach numbers would not appear to be so severe. Fig. 7 shows that for $T_0 = 288$ deg K the required value of $(S_m/\sigma_m)/(S_a/\sigma_a)$ is only about $0 \cdot 3$ at $M = 4$ and this would be lowered if the model is required to represent a reduced stiffness. It is thus possible for a stressed-skin structure to be represented by a more solid model.

Should the full-scale component itself be solid or nearly so, the construction of representative models is considerably easier. At transonic Mach numbers the requirement is for a solid model of a material for which the ratio E/σ_m' is about the same as full scale, whilst at higher Mach numbers materials of a lower E/σ_m' can be used (*see* Table 1).

In addition to relating the value of S/σ to the stagnation temperature, the model stiffness must be related to the range of stagnation pressure ; if only low values of the latter quantity are available it is conceivable that difficulties may be encountered in constructing a model flexible enough to achieve the required values of $\rho V^2/S$. Alternatively, if the available stagnation pressure can only be high, it may be difficult to construct models stiff enough to reach the lowest values of $\rho V^2/S$ which are required. Thus the limitations which model design and construction impose on the stiffness may be regarded as resulting in requirements for the tunnel stagnation pressure. It has already been concluded that in the transonic region the models required for correct simulation will have values of S_m/S_a about unity and thus it follows from Fig. 4 that the available stagnation pressure should be at least 2 Atm. At $M = 4 \cdot 0$, where a value of $(S_m/\sigma_m)/(S_a/\sigma_a)$ about $0 \cdot 3$ is required, a value of S_m/S_a as little as $0 \cdot 1$ can probably be achieved without undue difficulty. Then from Fig. 4 the required stagnation pressure will be about 15 Atm. For correct simulation of high altitude flight at transonic and low supersonic speeds some difficulties might be met in obtaining the required model stiffness unless the stagnation pressure can be reduced below 1 Atm.

The above considerations are mainly directed towards achieving the full-scale density and stiffness parameters with the possibility of a specified stiffness reduction factor. The necessary conditions impose severe restrictions on the models. A solid model or one consisting of a central spar with balsa-wood boxes to provide the shape could easily be made but such models would have low values of S/σ. For many research investigations it may not be important to achieve representative values of the density parameter, and thus these types of model would be adequate. When the density parameter is ignored the only relation to be considered is that between the stagnation pressure and the model stiffness. It is expected that no particular difficulties would be encountered in reaching values of S_m/S_a as low as 1/100. Then to achieve in the experiments values of $\rho V^2/S$ not less than three times those attained in flight, the required stagnation pressure at Mach numbers even as high as 4 is only 4·5 Atm.

For externally similar wings at the same Mach number a typical aerodynamic load per unit area is proportional to ρV^2 and thus proportional to the stagnation pressure. On the basis that a certain value of $\rho V^2/S$ will be achieved in the tunnel tests, it follows that the loading will be proportional to S_m. When $S_m \simeq S_a$ the loading of the model under the conditions of flight simulation is as severe as that of the full-scale structure. Models having reduced values of S_m/S_a will be subjected to lower loadings and may be less prone to failure.

4. *Test Procedure.*—4.1. *For General Investigations.*—The object of the flutter tests is to locate the critical boundary corresponding to a state of stable oscillation and additionally to determine the frequency and mode of distortion. The test procedure thus requires an initially stable system and the modification of a suitable parameter until flutter occurs. If modification of a flow parameter is not possible whilst the tunnel is running, as for instance with some supersonic tunnels, the method of testing would entail observing whether or not flutter occurs during each of a number of runs, some modification being made either to the model or to the flow characteristics between each run. This procedure has been followed by Tuovila, Baker and Regier[2] with a tunnel running at fixed Mach number and fixed stagnation pressure. A cantilever semi-span model was inserted through the tunnel wall after the tunnel had been started. The models were designed not to flutter during the first run and were subsequently modified in gradual stages by either reducing a stiffness or altering an inertia until flutter occurred. In this way a critical value of a model parameter was obtained. In comparison with continuous variation, the successive modification of a parameter appears laborious since every tunnel run does not yield a critical condition. However, the method in which the flow parameters remain constant whilst the model is successively modified has the advantage that a critical condition can be reached whilst either the density or the stiffness parameter remains at a prescribed value. Both these parameters can be held constant if it is desired to reach a critical condition by redistribution of mass. However, a more expeditious way of conducting a test is for the flow in the tunnel to be started, the model remaining in a flutter-free condition, and then for a suitable flow parameter to be gradually changed until flutter occurs.

For a particular structure with specified elastic and inertial properties in a specified gas the occurrence of flutter depends on the speed and the state of the gas, that is, on three independent quantities (e.g., V, p and T). In dealing with tunnel tests the three quantities are conveniently chosen as stagnation pressure, stagnation temperature and Mach number. In principle at least, a flutter condition may be approached from an initial condition of stability by varying any one or combination of these quantities. It is therefore necessary to examine to what extent these variations are desirable or practicable.

It will be remembered that for any particular value of M, the quantity ρV^2 is directly proportional to p_0 and independent of the value of T_0, whilst V^2 is directly proportional to T_0 and independent of p_0. Thus

(*a*) If M and T_0 are held constant, V also remains constant and ρ is proportional to p_0. Both the density and the stiffness parameters then vary inversely with p_0.

(*b*) Alternatively if M and p_0 are held constant whilst T_0 is varied, ρV^2 remains constant whilst $1/\rho$(and V^2) is proportional to T_0. Now only the density parameter varies.

(*c*) If M is the variant each of the quantities ρ, V, and ρV^2 change and hence all three of the flutter parameters vary during the run. Increase of M leads to an increase in V, a decrease in ρ and an increase or a decrease in ρV^2 according to whether M is less or greater than $1 \cdot 4$ approx.

Controlled variation of temperature during a run would be difficult and costly to achieve in practice particularly with an intermittent tunnel. Quite apart from the difficulties of its achievement such a variation is undesirable because of temperature effects on the models. A further consideration is that a variation of T_0 at constant M and p_0 is equivalent to a variation of wing density, and there is some evidence to suggest that flutter, particularly of heavy wings, may be insensitive to variation of this quantity.

The variation of the critical value of ρV^2 with Mach number for flexure-torsion flutter of an unswept cantilever wing is of the type shown in Fig. 10 (*see* Ref. 1). The Mach number for which $(\rho V^2)_c$ rises rapidly is probably higher for swept wings. The same flutter boundary is shown on a p_0, M diagram in Fig. 11. It may be noted that whereas point Y_1 on the flutter boundary can be approached either by an increase of p_0 or by an increase of M, point Y_2 must be approached either by an increase of p_0 or by a *decrease* of M.

Continuous variation of Mach number is inherent in the operation of a transonic tunnel and it is possible by adjustment of the pressure ratio across the working-section to vary the Mach number from a subsonic value to a low supersonic value. For higher supersonic values, variation of Mach number entails altering the geometry of the working-section of the tunnel. This cannot usually be done without stopping the tunnel, although McCarthy and Halfman in Ref. 3 describe a tunnel specially designed for flutter tests in which a sliding block nozzle is used to vary Mach number continuously during the run.

4.2. *Investigations Concerning Specific Aircraft.*—For the purpose of determining the flutter safety of a particular design it would be necessary to explore the possibility of flutter within a flight range specified in terms of altitude and Mach number. Fig. 12 shows hypothetical curves for the level flight performance and flutter boundaries drawn on an h, M diagram. For the purpose of the present argument the flutter boundary has been drawn on the assumption that if, for any Mach number, a flutter region exists, it will extend from sea-level to a certain critical altitude. The available evidence suggests that this is so, except perhaps for wings of low density.

The aim of the tunnel tests would be

(*a*) to determine whether the flutter boundary intercepts the flight boundary or whether a certain safety margin exists

or (*b*) to find the position of the flutter boundary.

To establish whether the flutter boundary intercepts the flight boundary it would probably be sufficient to reproduce in the tunnel the conditions appropriate to the lower part, ABC, of the flight boundary. This could be done, in principle at least, by using a model of fixed stiffness and varying the density by mass loading to suit the Mach number and altitude. For each Mach number, the model density would be adjusted to the specified altitude by means of equation (10). The absence or otherwise of an intercept between the flight and flutter boundaries at this Mach number would be established when the stagnation pressure is raised up to the value determined by equation (9), in which the value of p_a is appropriate to the specified altitude.

In a practical application it may be preferable for the model stiffness to represent the full-scale stiffness reduced by a suitably chosen factor. The tests would then determine whether a stiffness margin of specified amount existed between the flight and flutter boundaries.

A procedure for finding the actual position of the flutter boundary using a tunnel having a fixed stagnation temperature might be to test at each of several Mach numbers a number of models of different but suitably chosen stiffnesses and densities (if the tests were not destructive these could be the same model modified in stages). The stagnation pressure would be increased until flutter occurred and thus from each test a critical value of p_0 would be determined from which the equivalent flight air pressure and density could be obtained from the following equations :

$$p_a = \frac{S_a}{S_m} F_p(M)\, p_{0\,\mathrm{crit}} \qquad (13)$$

$$\rho_a = \frac{\sigma_a}{\sigma_m} \frac{F_\rho(M)}{RT_0} p_{0\,\mathrm{crit}} \qquad (14)$$

The values of p_a and ρ_a thus obtained using the several models could then be plotted on a (p_a/S_a), (ρ_a/σ_a) diagram to give for each value of M a flutter boundary (*see* Fig. 13). The intercept of this curve with that of the atmosphere line (determined by the values of p and ρ with increasing altitude) would give the critical altitude for flutter. From the values of the critical altitude obtained at several Mach numbers the flight flutter boundary shown in Fig. 12 could then be drawn.

In an experiment in which the stagnation pressure is increased, the characteristic point P on a (p_a/S_a), (ρ_a/σ_a) diagram moves along a line through the origin the slope of which is $(\sigma_m/S_m)RT_0F_T(M)$.

The slope of the characteristic line is thus inversely proportional to S_m/σ_m and the most suitable models for the experiment would be those designed to have characteristic lines which would be close to the atmosphere line.

The above methods imply that it is practicable to construct models of the required density and stiffness to suit the available flow characteristics. It may be found, however, particularly at low supersonic speeds with only small models and a limited stagnation pressure, that the combinations of model stiffness and density necessary to obtain experimental points in the neighbourhood of the atmosphere line cannot be achieved due to practical limitations of model construction in relation to the available flow characteristics. The effects of such limitations are now considered in more detail by reference to Fig. 14 which shows a possible (p_a/S_a), (ρ_a/σ_a) diagram appropriate to one Mach number. It may be noted that the abscissa of any point is the inverse of the density parameter, whilst, since Mach number is specified, the ordinate is inversely proportional to the stiffness parameter. The equivalent full-scale condition of a particular model whilst the tunnel stagnation pressure is increased over a certain range is represented by AB, point B corresponding to the maximum available stagnation pressure. Decreasing the model stiffness alone would move B upwards along an ordinate ; decreasing the model density would move B horizontally to the right. A further increase of stagnation pressure, if it were possible, would move B farther in the direction AB. The region of the diagram that can be explored by experiments in the tunnel is limited by the following hypothetical boundaries :

(i) A line A_1B_1 of minimum slope which corresponds to models with the maximum available S/σ. To obtain a high structural efficiency the stiffness will be high and thus the extreme point B_1 will be fairly close to the horizontal axis corresponding to a low value of p_a/S_a

(ii) A line B_1B' which is the locus of the extreme point B when the value of S/σ for the model is varied. This line corresponds to the limit of model flexibility coupled with the maximum available stagnation pressure

(iii) A line A_1A' which is the locus of point A for variation of S/σ. This line corresponds to the upper limit of model stiffness coupled with the minimum available stagnation pressure.

Also shown on the diagram are an atmosphere line and FF′, a flutter boundary, both of which are associated with the particular structure specified by S_a and σ_a.

As drawn, the diagram shows that models having high structural efficiencies can simulate altitudes within the range corresponding to H_1H_2 but cannot reach the flutter boundary. A model having a lower structural efficiency and correspondingly lower stiffness for which the tunnel experiment is represented by AB encounters the flutter boundary at P, and as drawn the only part of the boundary that can be reproduced in the tunnel is FF_1. If only a single point P on the flutter boundary has been determined, certain deductions can be made if it is possible on the basis of experience to assume a boundary of a certain shape. For instance if the slope of the boundary can be assumed to be positive, the full-scale condition represented by Q would be free from flutter but flutter would occur if the stiffness S_a were reduced by the ratio UQ/UP. The full scale structure at an altitude represented by R would also be flutter free but would flutter if its density were increased in the ratio WR/WP. In other words stiffness and density margins can be determined for conditions Q and R respectively.

5. *Starting and Stopping Loads in a Supersonic Tunnel.*—Since flutter models are less robust than the models commonly used in tunnel tests they will be more vulnerable to the severe transient loading which can occur during the stopping and starting of a supersonic wind tunnel. High loading can occur just before the supersonic flow is fully established over the model and is probably associated with a fluctuating flow that precedes the passage of the tunnel shock. The magnitude of the starting load increases with Mach number and it would seem reasonable to assume that it also increases with the value of the stagnation pressure obtaining at the instant when supersonic flow becomes established at the model. The model is likely to be less affected by a rapid, rather than a slow passage of the tunnel shock along the working-section. In some types of supersonic tunnel it is possible to arrange for the establishment and breakdown of supersonic flow to occur at a low value of the instantaneous stagnation pressure. With a closed circuit tunnel, for instance, this may be done by partial evacuation before the flow is started or stopped. However, with a blow-down tunnel discharging to a fixed pressure the 'starting' stagnation pressure for any particular mach number is also fixed, but it would be possible to provide the necessary control if the diffuser is connected to an evacuated vessel for at least the stopping and starting periods. It may be noted that with a variable stagnation-pressure blow-down tunnel discharging to any particular pressure the importance of the starting loads will be less when the working stagnation pressure is higher because the models will be made stiffer and stronger.

If the required control of stagnation pressure is not practicable, then damage to the model could be avoided by inserting it into the working-section after supersonic conditions have been reached and withdrawing it again before shutting down[3]. Alternatively, the model might be clamped. It may be noted that one or other of these methods might prove to be effective in preventing damage to the model when flutter occurs.

6. *Summary of Desirable Characteristics of Tunnels to be used for Flutter Tests.*—Distinction is made between those disposable flow parameters which can be pre-set at specified values before the run and those which need to be varied during the run. Certain conclusions apply generally throughout the range of high-speed testing. These are :

(*a*) In addition to the requirement for covering a range of Mach number, it is an advantage for the stagnation pressure of the flow to be disposable

(*b*) It is considered almost essential for either Mach number or stagnation pressure to be capable of controlled variation whilst the tunnel is running

(*c*) In order to avoid significant changes of model stiffness due to temperature changes, the stagnation temperature of the flow should be maintained at a value which will keep the model at approximately room temperature

(*d*) The tunnel itself should not be vulnerable to model break-away.

It is more convenient to deal with further conclusions separately for tunnels operating in the transonic and supersonic ranges.

(i) *Transonic Range.*—The transonic type of tunnel covers a range of Mach number through $1 \cdot 0$ up to, possibly, $1 \cdot 4$. Since Mach number can be varied during the run it is not so important that stagnation pressure should also be variable. However, such a provision might be valuable in approaching a critical condition. A disposable stagnation pressure would provide some latitude in model design and a range up to 2 Atm or more would ease the problems of designing models for flight simulation. A running time of about 30 seconds is considered ample provided the range of Mach number can be covered in this time. Since in this speed range the model can only be small in relation to the working-section, it is important that the tunnel should be as large as possible.

(ii) *Supersonic Range.*—With a supersonic tunnel operating above a Mach number of about $1 \cdot 2$. Mach number is determined by the nozzle geometry and may either be varied in steps by means of interchangeable liners or continuously by the use of a flexible nozzle or a sliding block. At low supersonic speeds, theory and experiment indicate that flutter characteristics undergo rapid changes, and up to $M = 1 \cdot 6$ at least, provision is needed for tests at close intervals of Mach number. For the simulation of flight conditions, and for latitude in model construction, the available stagnation pressure should increase with Mach number, a value of about 15 atmospheres being considered necessary at $M = 4$. As stated previously, it would be an advantage for a range of stagnation pressures to be available at each Mach number.

Consideration of the alternative procedures for the determination of critical conditions suggests that there is little to choose between variation of Mach number and variation of stagnation pressure. Experimental evidence suggests that in the region of low supersonic speeds an approach to the flutter boundary may entail either an increase of Mach number upwards from $1 \cdot 2$ (say) or a decrease from some initially high value. Thus the safe approach may not be known beforehand. With the alternative procedure, provided the initial stagnation pressure is sufficiently low, the flutter boundary can always be approached from the safe side by an increase of pressure. When Mach number is varied with fixed stagnation pressure, the remaining similarity parameters (density and stiffness) also vary. In the procedure in which the stagnation pressure is varied, one of the similarity parameters (Mach number) remains constant, and this enables tests to be made more systematically. Because of these considerations there would seem to be an advantage in determining critical speeds by variation of stagnation pressure.

The rate of variation of the chosen parameter must be matched to the available running time of the tunnel. If this time is short, and if a disposable stagnation pressure is already available, the cost of providing a rapid variation is likely to be small in comparison with the cost of providing a facility for rapidly changing the Mach number.

NOTATION

(a) Flight and Tunnel Quantities

M		Mach number	
h		Altitude	
V		Air speed	
ρ		Air density	Suffix $_0$ is used to refer to wind-tunnel stagnation values
T		Air temperature	
p		Pressure	
R		Gas constant	
$F_\rho(M)$	=	ρ/ρ_0	The relations between stream and stagnation quantities for isentropic flow
$F_T(M)$	=	T/T_0	
$F_p(M)$	=	p/p_0	

(b) Structural Quantities

L		A characteristic external length (*e.g.*, chord or span)
σ		An overall density proportional to the total mass divided by L^3
σ'		Density of material
ε		An elastic stiffness expressed as a moment per unit angular displacement
S	=	ε/L^3, an elastic modulus of the structure
E		An elastic modulus of the structural material
ω		A characteristic frequency (For all the above symbols, suffices $_a$ and $_m$ are used to refer to flight and model conditions respectively)
T_w		Temperature of model during test in wind tunnel

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APPENDIX

N.P.L. Tunnels Suitable for Flutter Tests

Of the tunnels at N.P.L., those considered at all suitable for flutter tests over a portion or the whole of their range of operation are tabulated below.

Designation in terms of nominal working-section size	Mach number	Stagnation pressure (Atm.)	Mach number range over which tunnel is suitable for flutter tests
18 in. × 14 in.	0 to 1·8	1 to 3	0·7 to 1·1 and 1·25 to 1·8
36 in. × 14 in.	0 to 1·8	1	0·7 to 1·1
25 in. × 20 in.	0 to 1·8	1	0·7 to 1·1
13 in. × 11 in.	1·4 to 2·5	0·3 to 1	1·4 to 2·5
15 in. × 10 in. (proposed)	0 to 4	up to 15 (depending on Mach number)	0·7 to 4·0

The merits of each tunnel are discussed below.

1. 18-*in.* × 14-*in. Tunnel.*—This is an intermittent induced flow-tunnel with return circuit and pressurisation up to 3 Atm. The tunnel is not very vulnerable to damage from a model break-away, but is not considered suitable for flutter tests in which frequent model failure is expected. The models for this tunnel could be constructed without serious difficulty provided they were restricted to simple structures. The available stagnation pressure of 3 Atm would adequately fulfil the requirement of low-altitude flight simulation at transonic speeds and would probably be sufficient to cover similar requirements at $M = 1 \cdot 8$. If, however, flight is to be simulated at high altitudes, some difficulty may be experienced, especially at transonic speeds, in obtaining the high model stiffnesses which would be required because the stagnation pressure cannot be reduced below 1 Atm.

There is a fall of stagnation temperature during the run (about 5 deg C per minute), which would probably preclude the use of elastic materials other than metals.

(*a*) *Tests at Subsonic and Transonic Speeds.*—The tunnel can be fitted with a subsonic working-section or a transonic one in which the top and bottom walls are slotted. Semi-span models could be mounted at a side wall or complete-span models could be mounted centrally. Mach number is continuously variable during a run up to the value 1·1, and thus a flutter critical condition could be determined either by this variation at constant stagnation pressure or by varying the stagnation pressure at constant Mach number.

(*b*) *Tests at Supersonic Speeds.*—The working-section can be fitted with liners to give constant Mach numbers 1·25, 1·4 and 1·6 and for these values a flutter critical condition could be determined by variation of stagnation pressure. The stagnation pressure during starting and stopping can be no lower than 1 Atm and thus a model may need to be inserted and withdrawn whilst the flow is established.

2. 36-*in.* × 14-*in. and* 25-*in.* × 20-*in. Tunnels.*—Both these tunnels have characteristics similar to those of the 18-in. × 14-in. Tunnel except that the stagnation pressure is fixed at 1 Atm. The tunnels would thus only be suitable for flutter tests up to $M = 1 \cdot 1$, over which range the Mach number is continuously variable. Advantage is gained from the increased tunnel dimensions but the absence of an elevated stagnation pressure makes flight simulation more difficult.

3. 13-*in.* × 11-*in. Supersonic Tunnel.*—This tunnel is driven by a compressor in the return circuit and it is therefore vulnerable to model break-away. Although means to prevent damage could probably be devised, it is not expected that the tunnel could be made suitable for investigations where model break-away is a frequent occurrence. Simulation of flight, except at high altitude, would be very difficult due to the absence of pressurisation, although the tunnel might prove to be useful for research investigations in which it is unnecessary to obtain representative values of the density parameter. The stagnation temperature is higher than that of the room and is to some extent controllable.

4. 15-*in.* × 10-*in. Tunnel.*—This is a proposed blow-down tunnel for general aerodynamic investigations and will provide a Mach number range of up to 4. Its present design fulfils many of the requirements for flutter and, with suggested modifications, it would be in many respects ideal for flutter tests. However, the size of the working-section is considered to be rather small, especially for testing in the transonic range.

It is proposed to connect the exit of the working-section to either a low-pressure reservoir or to atmosphere, and hence starting loads can be reduced by starting under reduced pressure. If the design could be modified to enable the flow to be stopped at reduced pressure, means for inserting or withdrawing the model might not be required. The addition of a heater to ensure a reasonably constant temperature during the run is desirable. The available range of stagnation pressure is shown as a function of Mach number in Fig. 15. Curve (1) shows the upper limit to the stagnation pressure which is based on tunnel strength and noise considerations. Curve (2) shows the minimum pressure which can be used when the running time is restricted to 30 seconds or less, whilst curve (3) shows the minimum pressure when the time of run exceeds that necessary to fill the vacuum reservoir. Plotted on the same diagram is curve (4), which has been deduced on a somewhat arbitrary basis, to give upper limits to the stagnation pressures required for flight simulation. It will be noted that in its present design the tunnel will provide at transonic speeds an excess of stagnation pressure over the estimated requirement for flight simulation.

TABLE 1

Elastic Moduli and Specific Gravities of Some Full-Scale and Model Materials

I Material	II $E \times 10^{\times 6}$ (lb/in.^{2})	III Specific gravity ($\propto \sigma'$)	IV $\frac{\text{Col. II}}{\text{Col. III}} = k\frac{E}{\sigma'}$
METALS			
Steel	30	7·73	3·9
Stainless steel	26·3	7·73	3·4
Titanium	15·5	4·50	3·5
Duralumin	10	2·75	3·6
Magnesium	6	1·75	3·4
WOODS*			
Hickory	1·9	0·79	2·4
Ash	1·4	0·62	2·3
Mahogany	1·2	0·51	2·4
Spruce	1·3	0·40	3·2
Balsa	—	0·14	—
PLASTICS*			
Cellulose nitrate	0·3	1·38	0·22
Perspex	0·46	1·18	0·39
Polyester resin	0·3	1·3	0·23
Glass-reinforced polyester resin (cloth)	2 to 3	1·7 to 1·9	1·4
Glass-reinforced polyester resin (chopped strand mat)	1·2 to 1·7	1·5 to 1·6	0·94
Bakelite-base laminated board	3	1·4	2·1

* The values quoted must be regarded as subject to wide variation.

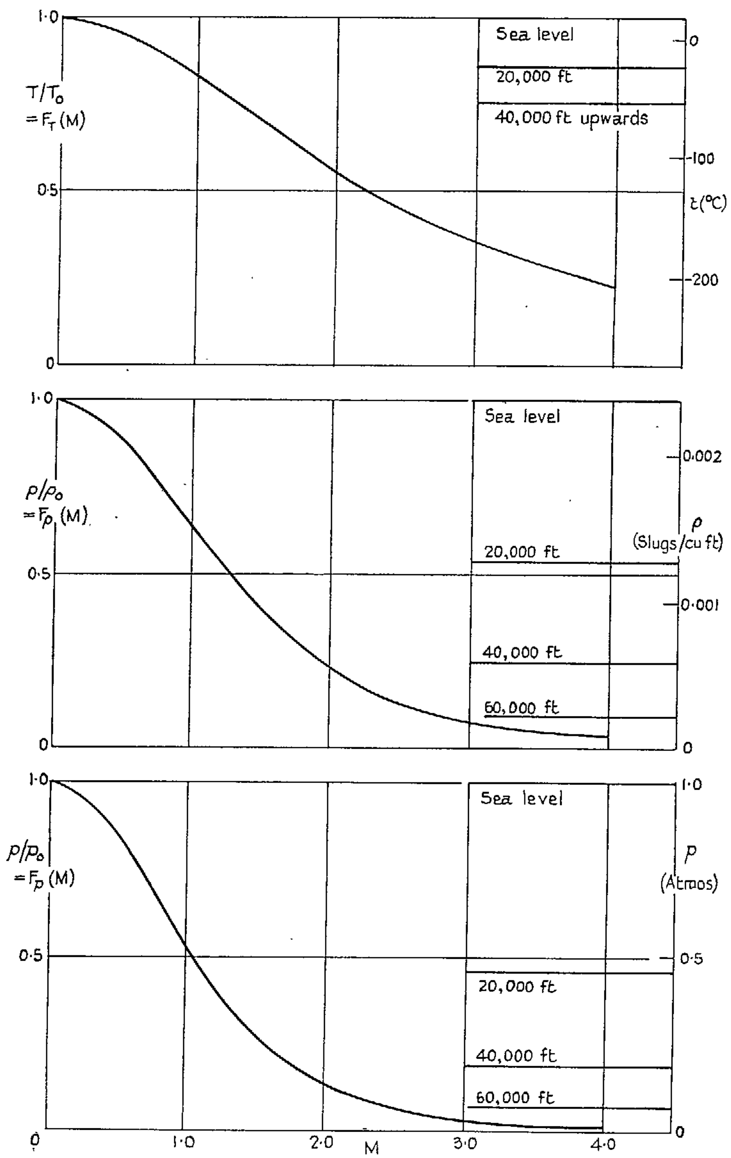

FIG. 1. Variation of T/T_0, ρ/ρ_0, and p/p_0 with M.

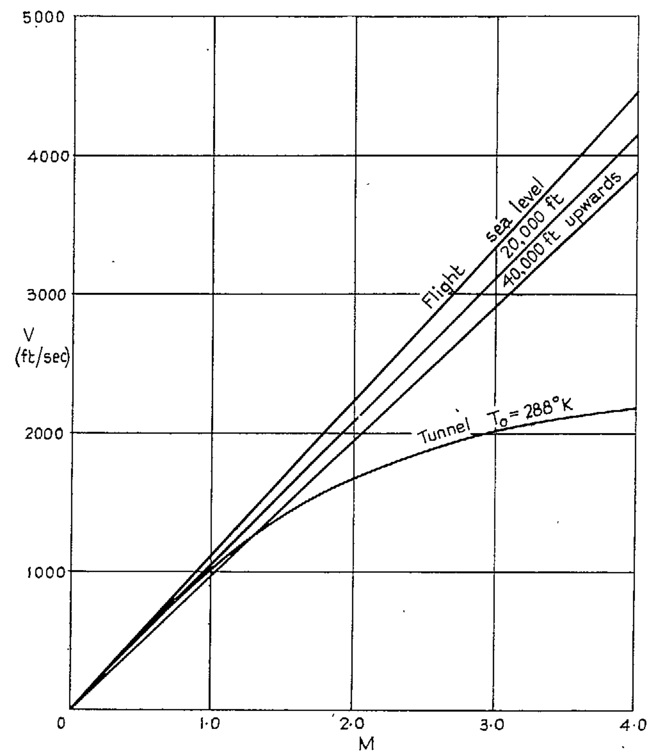

FIG. 2. The variation of V with M.

(*a*) In a tunnel with $T_0 = 288$ deg K

(*b*) In flight at $h = 0$; 20,000ft ; 40,000 ft and upwards.

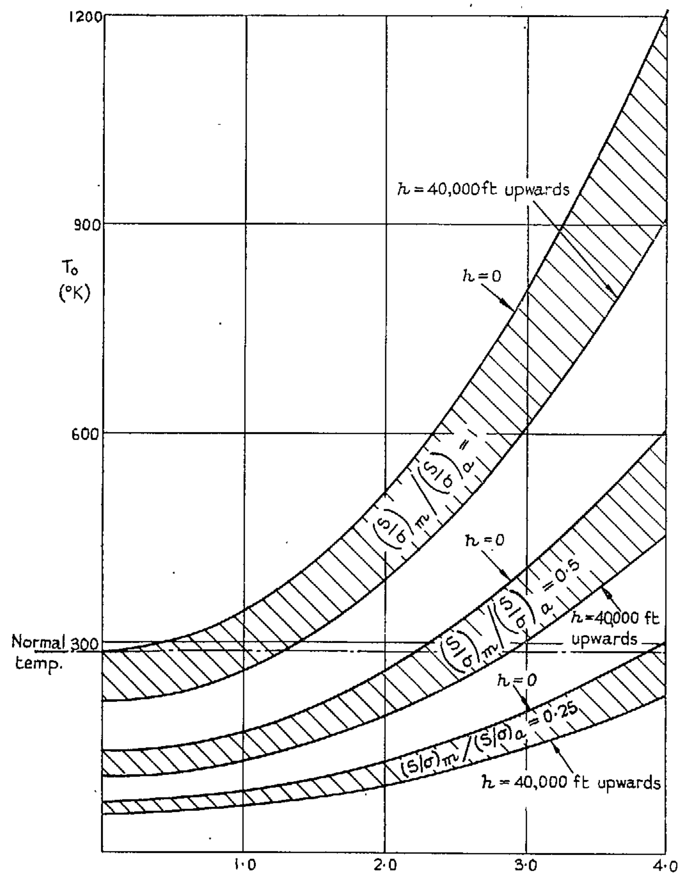

FIG. 3. Stagnation temperatures required for flight simulation for models with various values of the structural efficiency (S/σ).

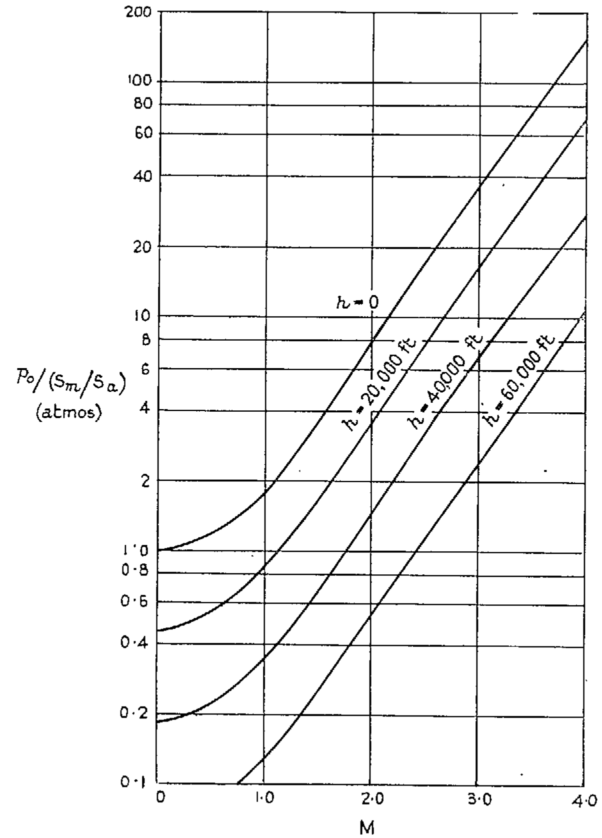

FIG. 4. Stagnation pressure required for flight simulation with models of stiffness scale (S_m/S_a).

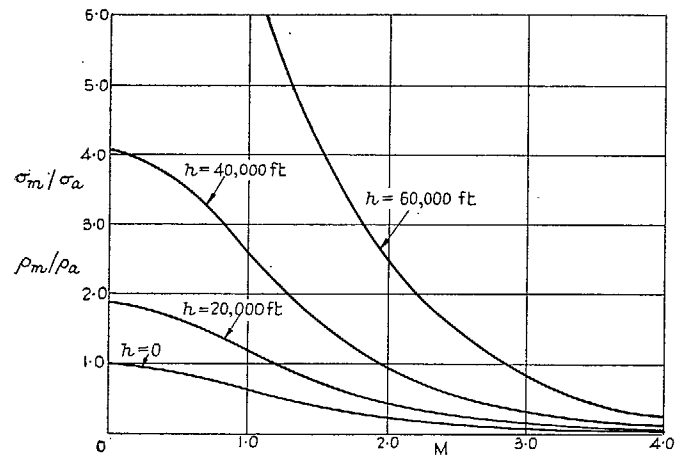

FIG. 5. Variation of density scale with M for flight simulation in tunnel with stagnation pressure = 1 Atm and stagnation temperature = 288 deg K.

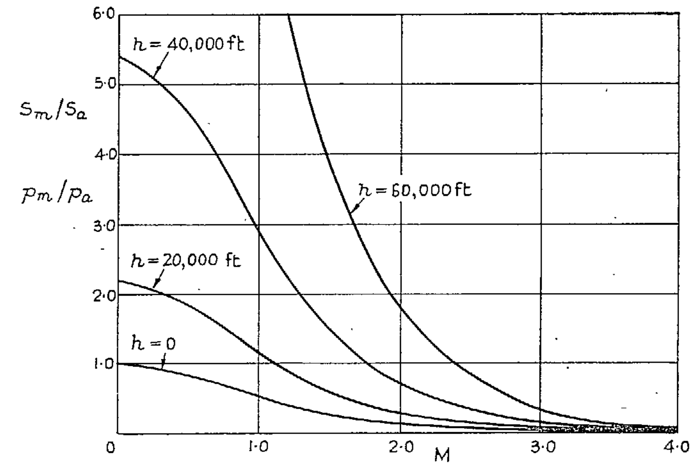

FIG. 6. Variation of stiffness scale with M for flight simulation in tunnel with stagnation pressure = 1 Atm.

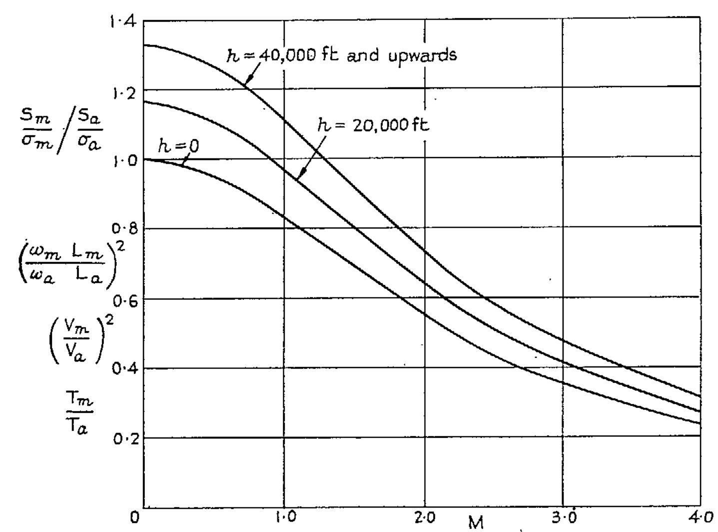

FIG. 7. Variation of S/σ scale with M for flight simulation in tunnel with stagnation temperature = 288 deg K.

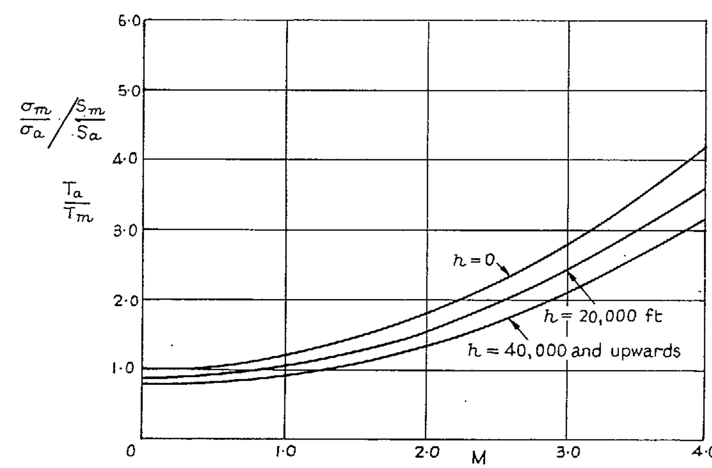

FIG. 8. Variation of density scale for flight simulation in tunnel with stagnation temperature = 288 deg K and with S_m/S_a fixed.

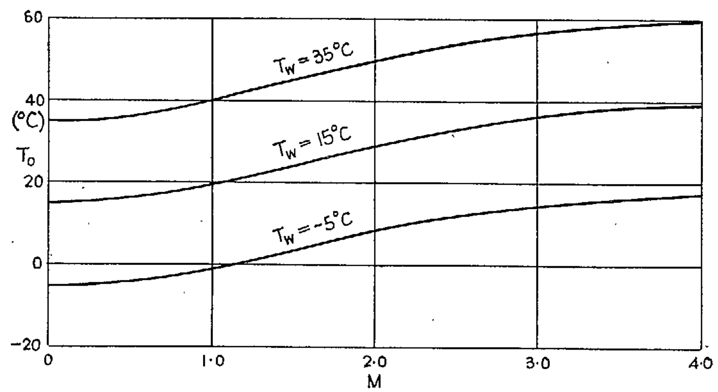

FIG. 9. Stagnation temperature required to give specified equilibrium values of model temperature T_w based on temperature recovery factor 0·9.

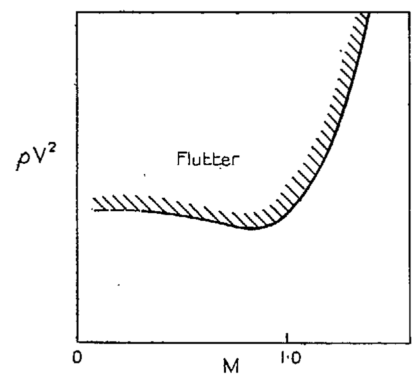

FIG. 10. Typical plot of $(\rho V^2)_c$ vs. M for an unswept cantilever wing.

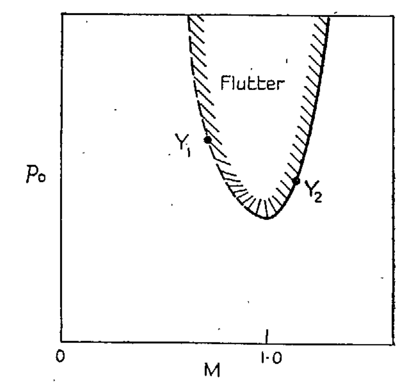

FIG. 11. Typical p_0 vs. M flutter boundary for an unswept cantilever wing. (derived from FIG. 10).

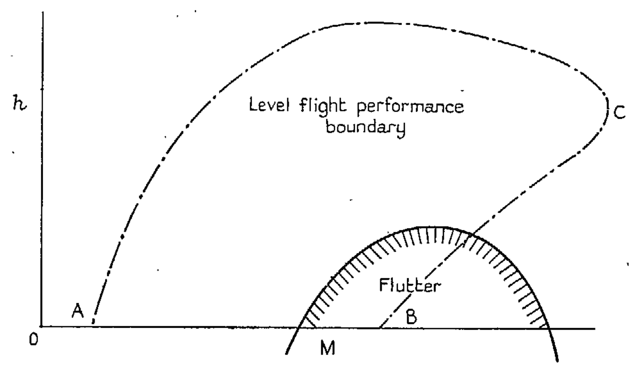

FIG. 12. Hypothetical boundaries for flight and flutter.

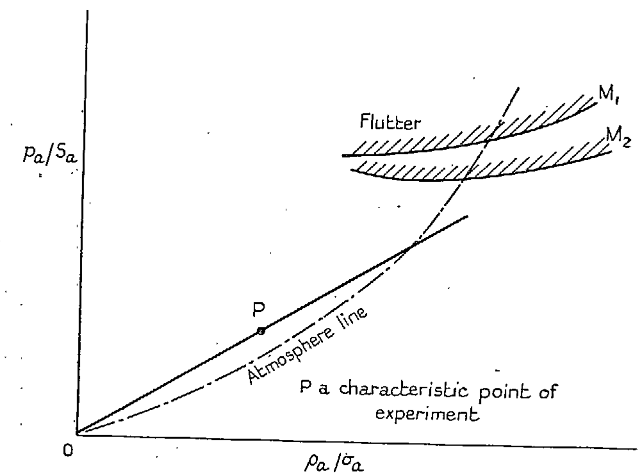

FIG. 13. Hypothetical (p_a/S_a), (ρ_a/σ_a) diagram showing flutter boundaries in relation to atmosphere line.

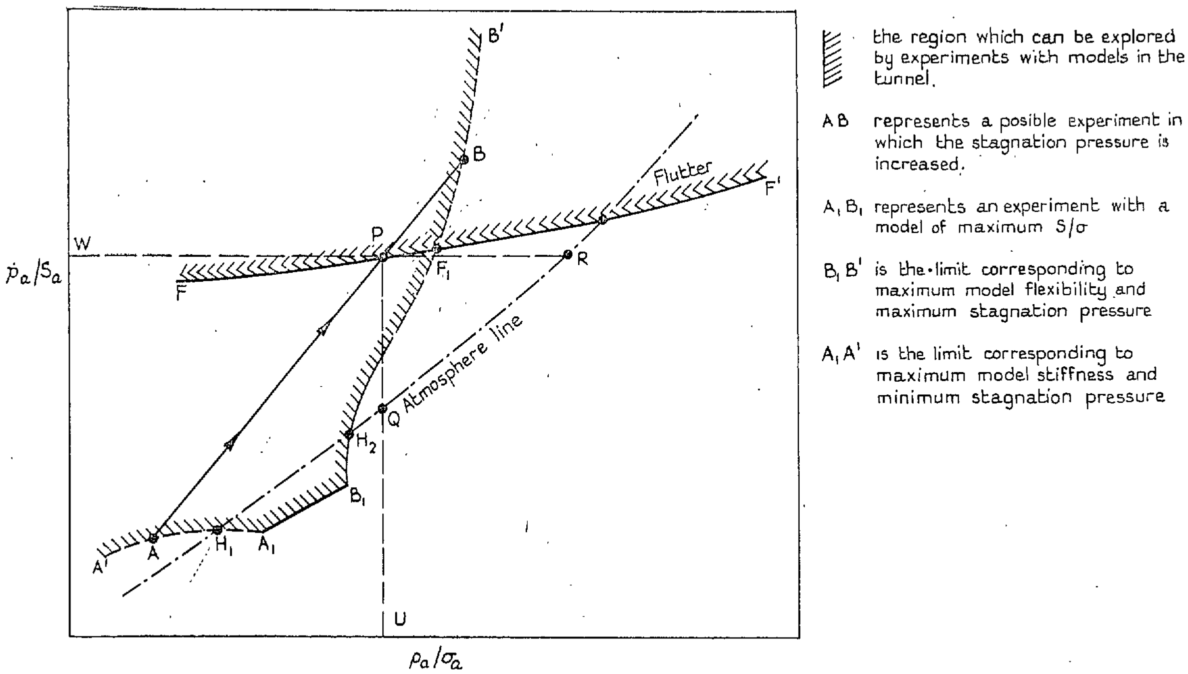

FIG. 14. Hypothetical (p_a/S_a), (ρ_a/σ_a) diagram showing limitations which may be encountered in an attempt to simulate flight conditions in a tunnel.

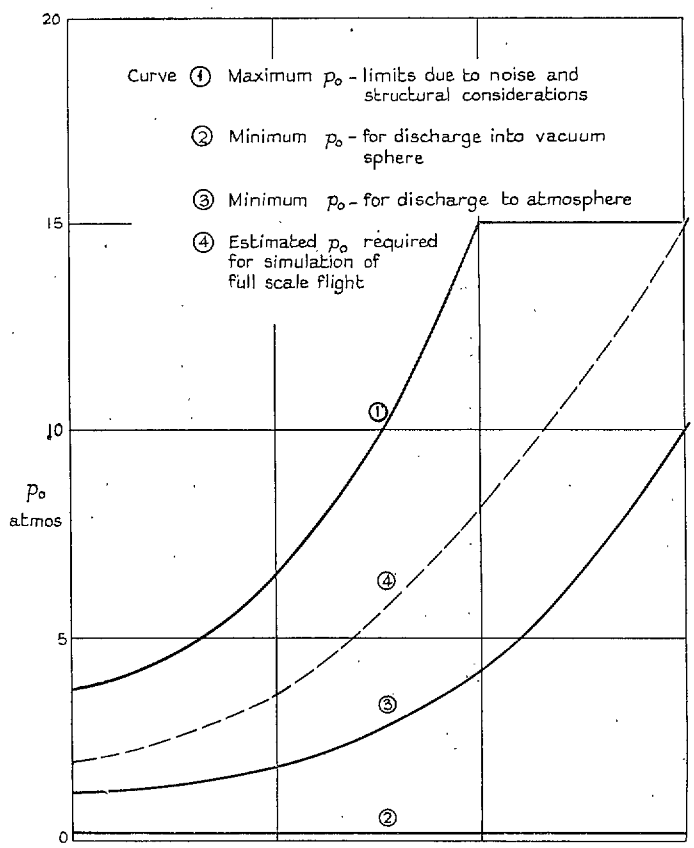

FIG. 15. Available range of stagnation pressure for the proposed 15 in. × 10 in. tunnel.

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