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Summary.—The initial lift for wings of finite aspect ratio due to a sudden unit change of incidence is considered and, by the use of the method developed in R. & M. 2117[1], the ratio of the initial to the final values of the lift is determined for rectangular wings and cropped delta wings of taper ratio 1/7. These values indicate that the initial lift may be greater than the final lift for aspect ratios $A \leqslant 2$. This result appears to be supported by the values of instantaneous lift which were measured* for the delta wing of aspect ratio $1 \cdot 2$ as its incidence was rapidly changed.

1. *Introduction.*—The determination of the aerodynamic forces acting on wings of finite span in non-uniform motion in an incompressible, inviscid fluid has been considered by W. P. Jones[1] (1945). In particular, he showed that the growth of lift function $k_1(s)$ corresponding to a sudden unit change of incidence at $s = 0$, can be derived by operational methods from the lift function corresponding to simple harmonic translational motion of the wing. In the present note, this method is used to calculate the function $k_1(s)$ for rectangular and cropped delta wings of aspect ratios ranging from $1 \cdot 2$ to 4. The values of the lift function corresponding to simple harmonic translational motion which are used in the calculations, were computed by vortex lattice methods[2,3,4]. Values of the ratio of the initial to the final values of $k_1(s)$ are tabulated in Table 1 and it can be seen from Fig. 1 that the present results correlate quite well with values previously obtained for rectangular wings[1] and elliptic wings[5] of larger aspect ratio.

The ratio $k_1(s = 0)/k_1(s = \infty)$ obtained for the wings $A \leqslant 2$, indicated that the initial lift might be greater than the final value. To check this result it therefore seemed advisable to obtain experimental information for a wing of low aspect ratio, and the delta wing, $A = 1 \cdot 2$, was chosen for further investigation. Values of instantaneous lift and incidence were measured by Scruton and Woodgate, during a fairly rapid change of incidence. It was assumed that the experimental incidence curve, which is plotted in Fig. 2, could be represented approximately by a series of sudden changes of incidence, and for comparison with experiment, values of the instantaneous lift were then calculated by the use of the appropriate growth of lift function $k_1(s)$ (*see* equation (8)).

* The wind-tunnel tests were made by Scruton and Woodgate at the N.P.L. No report describing these tests is at present available.

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2. *Theory*.—When the effect of aerodynamic inertia is omitted, the lift force L on a finite wing describing simple harmonic translational oscillations may be expressed in terms of the aerodynamic derivatives l_z and $l_{\dot{z}}$ by:

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = 2(l_z + i\omega_m l_{\dot{z}})z\, e^{ipt} \quad \ldots \quad (1)$$

where l_z does not include the usual aerodynamic inertia term $l_{\ddot{z}}$. In equation (1), $z' = c_m z\, e^{ipt}$ is the normal downward displacement of the wing, and $\omega_m = pc_m/V$ is the frequency parameter in terms of the mean chord c_m. If the instantaneous incidence of the wing is α, then in the usual notation:

$$\alpha = \frac{w}{V} = i\omega_m z\, e^{ipt}\,, \quad \ldots \quad (2)$$

where w denotes the downwash. It follows from (1) that $C_{L\alpha}$, which corresponds to the lift slope coefficient, is given by:

$$C_{L\alpha} = \frac{2(l_z + i\omega_m l_{\dot{z}})}{i\omega_m}\,. \quad \ldots \quad (3)$$

It is shown in Ref. 1 that $C_{L\alpha}$ corresponding to simple harmonic translational motion of a wing, is the operational image of the lift function $k_1(s)$ due to a sudden unit change of incidence when the wing is at the position $s = 0$. Hence, if $C_{L\alpha}$ is a known function of the frequency the function $k_1(s)$ can be determined. It is assumed that $C_{L\alpha}$, as defined by equation (3), can be represented to sufficient accuracy by the expression:

$$C_{L\alpha} = A_0 - A_1\left(\frac{i\omega_m}{2a_1 + i\omega_m}\right). \quad \ldots \quad (4)$$

The arbitrary coefficients A_0 and A_1 are determined by the values of $C_{L\alpha}$ for $\omega_m = 0$ and $\omega_m = \infty$, and the arbitrary coefficient a_1 is determined by trial and error to give the best approximation to (3). Then by Ref. 1, the growth of lift function $k_1(s)$ corresponding to $C_{L\alpha}$ as defined by (4), is of the form:

$$k_1(s) = A_0 - A_1\, e^{-a_1 s}\,, \quad \ldots \quad (5)$$

where s is the distance travelled/half mean chord. In particular it follows from (3), (4) and (5) that:

$$\left.\begin{aligned} k_1(s = 0) &= C_{L\alpha}(\omega_m = \infty) = 2l_{\dot{z}}(\omega_m = \infty) \\ k_1(s = \infty) &= C_{L\alpha}(\omega_m = 0) = 2l_{\dot{z}}(\omega_m = 0) \end{aligned}\right\} \quad \ldots \quad (6)$$

since the derivative $l_z = O(\omega_m^2)$ for finite wing when $\omega_m \to 0$, and $l_z = 0$ when $\omega_m \to \infty$. Hence the ratio of the initial growth of lift to the final value is the same as the ratio of the damping derivative $l_{\dot{z}}$ at $\omega_m = \infty$ to that at $\omega_m = 0$.

Thus

$$\frac{k_1(s = 0)}{k_1(s = \infty)} = \frac{l_{\dot{z}}(\omega_m = \infty)}{l_{\dot{z}}(\omega_m = 0)} \equiv k_1(0/\infty) \text{ say}\,. \quad \ldots \quad (7)$$

If the growth of lift function $k_1(s)$ is known, then, with effects of aerodynamic inertia omitted, the lift coefficient $C_L(s)$ corresponding to a wing with incidence varying as $\alpha(s)$ is, by equation (42) of Ref. 1:

$$C_L(s) = \alpha(0)\, k_1(s) + \int_0^s k_1(s - s_1)\frac{\partial\alpha}{\partial s_1}\, ds_1\,. \quad \ldots \quad (8)$$

3. *Calculation of $k_1(0/\infty)$*.—The ratio $k_1(0/\infty)$ of the initial value to the final value of the function $k_1(s)$ was calculated for wings of various plan-form; the values are plotted against aspect ratio in Fig. 1.

The results for the cropped delta wings (taper ratio 1/7) of aspect ratio $A = 1 \cdot 2$, 2 and 3, and the rectangular wings $A = 2$, 8/3 and 4, are tabulated in Table 1. The values of $k_1(0/\infty)$ were evaluated by using equation (7) with values of the derivative $l_{\dot{z}}$ which were calculated by vortex lattice methods*. The derivatives $l_{\dot{z}}$ for $\omega_m \to 0$ were calculated by the vortex lattice method for stability derivatives[3]. The method suggested by W. P. Jones[2] (1946) for the calculation of flutter derivatives, was used to compute $l_{\dot{z}}$ for $\omega_m \to \infty$.

Also shown in Fig. 1 are the ratios of the values $k_1(s = 0)$ and $k_1(s = \infty)$ given in Ref. 1 for rectangular wings of aspect ratio 4 and 6. The values $k_1(0/\infty)$ for elliptic wings of aspect ratio 3 and 6 were derived from the formulae given by R. T. Jones[5] (1940). Garrick's low aspect-ratio theory (Ref. 6 (1951)) gives the value $k_1(0/\infty) = 1 \cdot 0$ for triangular and rectangular wings when the aspect ratio tends to zero.

4. *Experimental and Calculated values of Lift for the Delta Wing $A = 1 \cdot 2$*.—The values of $k_1(0/\infty)$ for $A \leqslant 2$ indicate that the initial lift may be greater than the final lift. To check this theoretical result, instantaneous values of lift and incidence were measured on the delta wing $A = 1 \cdot 2$ during a fairly rapid rotation about an axis positioned $0 \cdot 973 c_m$ back from the apex. These tests were made by Scruton and Woodgate in the N.P.L. Low Turbulence Tunnel at a wind speed of 80 ft/sec. Apparatus previously used by Scruton, Woodgate and Alexander[7] (1952) for derivative measurements was suitably modified for this purpose. The experimental results were expressed as lift L/final lift L_F and incidence α/final incidence α_F and, in Fig. 2, these ratios are plotted against s, the distance travelled in half mean chords.

4.1. Calculated values of instantaneous lift corresponding to the motion α/α_F described by the wing in the wind-tunnel tests, were obtained as follows:

It was assumed that the incidence curve α/α_F could be represented approximately by a series of sudden changes in incidence. The ratio of the lift/final lift was then evaluated from equation (8) in the form $C_L(s)/C_L(s = \infty) \equiv C_L(s/\infty)$ say, by using the growth of lift function $k_1(s)$ as defined in section 2. For the delta wing $A = 1 \cdot 2$ $k_1(s)$ was derived from equations (3), (4) and (5) as:

$$k_1(s) = 1 \cdot 629 + 0 \cdot 417\, e^{-0 \cdot 5 s} . \qquad (9)$$

The value of $0 \cdot 5$ for the coefficient a_1 in equation (5) is doubtful since there is not sufficient information available to determine it accurately. Arbitrary values $a_1 = 0 \cdot 25$, $a_1 = 1 \cdot 0$ and $a_1 = \infty$ were therefore assigned in equation (9) and the corresponding values of $C_L(s/\infty)$ were evaluated. It can be seen from the results given in Table 2 that the variation in the value a_1 has only a small effect on the values of $C_L(s/\infty)$. The values of $C_L(s/\infty)$ for $a_1 = 0 \cdot 25$ and $a_1 = 0 \cdot 5$ are plotted in Fig. 2.

Concluding Remarks.—The indication that the initial lift may be greater than the final lift for wings of low aspect ratio appears to be supported by the values of instantaneous lift which were measured on the delta wing $A = 1 \cdot 2$. It should be noted in comparing the calculated and the experimental values of the instantaneous lift/final lift that the measured lift includes a contribution due to aerodynamic inertia, and also an effect due to the rotation of the wing about its axis. Allowances for these effects could be made theoretically but, since they were expected to be unimportant, they were not taken into account in the analysis presented.

* These solutions were based on the 21 × 6 lattice with six collocation points as applied in Refs. 3 and 4.

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TABLE 1

Growth of Lift Function $k_1(0/\infty)$ (see equation (7))

Plan-form	Aspect ratio A	$l_{\dot{z}}$ ($\omega_m = 0$)	$l_{\dot{z}}$ ($\omega_m = \infty$)	k_1 $(0/\infty)$	Remarks
Delta, taper 1/7	1·2	0·815	1·023	1·26	Values of $l_{\dot{z}}$ calculated by vortex-lattice methods
	2	1·191	1·214	1·02	
	3	1·539	1·334	0·87	
Rectangle	2	1·223	1·206	0·99	
	2·67	1·462	1·289	0·88	
	4	1·791	1·376	0·77	

TABLE 2

Delta Wing $A = 1 \cdot 2$. Calculated lift/final lift corresponding to the experimental wing motion α/α_F

s	C_L (s/∞)			
	$a_1 = 0 \cdot 25$	$a_1 = 0 \cdot 5$	$a_1 = 1 \cdot 0$	$a_1 = \infty$ *
0	0	0	0	0
2	0·010	0·010	0·010	0·008
4	0·029	0·028	0·027	0·024
6	0·065	0·063	0·061	0·055
8	0·139	0·134	0·130	0·118
10	0·271	0·262	0·254	0·232
12	0·424	0·408	0·397	0·369
14	0·604	0·582	0·566	0·533
16	0·782	0·753	0·735	0·701
18	0·945	0·912	0·892	0·860
20	1·084	1·048	1·028	1·000
22	1·135	1·100	1·084	1·069
24	1·112	1·084	1·074	1·072
26	1·057	1·038	1·034	1·040
28	1·015	1·004	1·006	1·011
30	0·995	0·991	0·993	0·996
:	:	:	:	:
∞	1·000	1·000	1·000	1·000

* a_1 is the parameter in equation (5).

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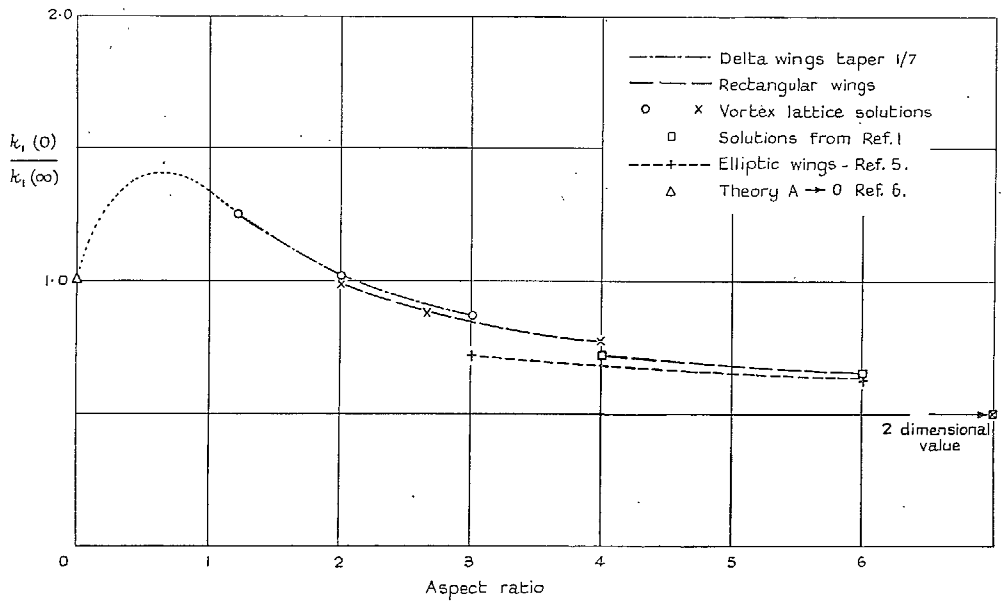

FIG. 1. Ratio of initial lift to final lift. $k_1(s)$ is growth of lift due to a sudden unit change of incidence at position $s = 0$.

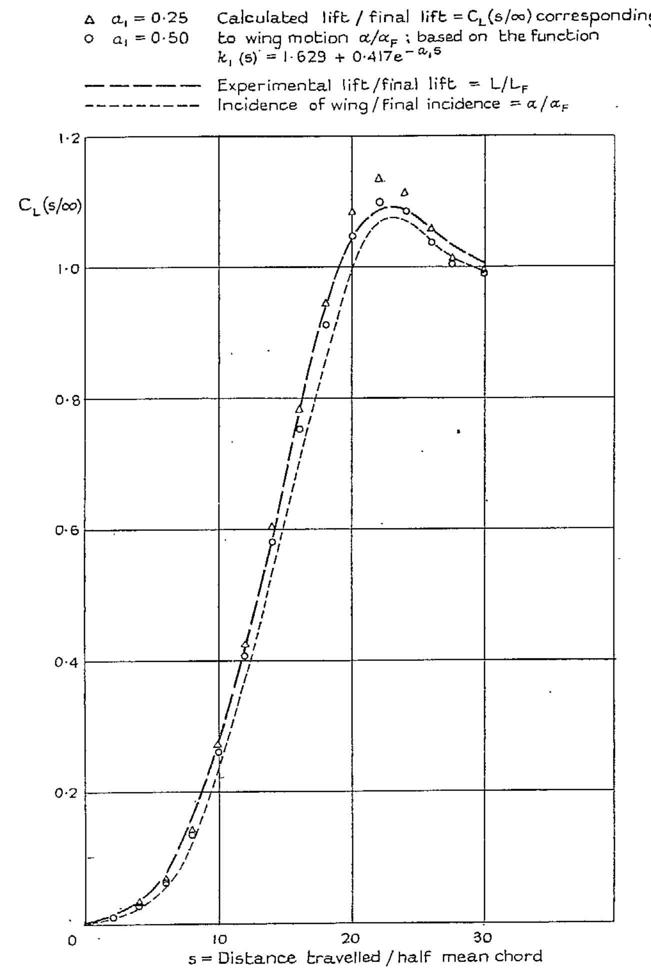

FIG. 2. Growth of lift: Delta wing, $A = 1{\cdot}2$ (taper ratio 1/7).

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