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# A Survey of the Alternating Pressures Exciting High Frequency Vibrations in Gas Turbines

*By*

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# A Survey of the Alternating Pressures Exciting High Frequency Vibrations in Gas Turbines

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*Summary*.—A survey of the predominant harmonic components of the alternating pressures is made for three centrifugal compressors, three axial-flow compressors and a single-stage turbine to ascertain the forces exciting vibration in a compressor or turbine stage. The predominant harmonic components of the alternating pressures in the casing are the first-order components of the impulse from the rotor blades or impeller vanes. Harmonics up to the seventh of these impulses can be detected in parts of the speed range. The alternating pressures in a compressor annulus have components similar to those observed in the casing for the stage but in addition have components of greater magnitude excited by dissimilar flows in individual blade passages. The largest orders are those which excite the fundamental flexural mode of vibration of the blades and the corresponding blade movements amplify the alternating pressures.

The amplitude of the alternating pressures is dependent on the work done in a blade or impeller vane passage, the incidence at entry to the stage, the exit conditions and the general flow conditions and there is usually a relative reduction when operating near the design point.

There is a reduction in the alternating pressures with distance from the source particularly across a blade row.

There are three other sources of excitation at part load :

(*a*) Stalling flutter at high positive incidence can occur in axial-flow compressors for blades of low stiffness over wide ranges of operation provided the incidence and mass flow are greater than the boundary conditions

(*b*) Alternating pressures can be excited near the surge by one or more stall cells rotating at some fraction of compressor speed

(*c*) At the stalling incidence of a stage, if a proportion of the stage stalls due to dissimilar geometry or flow and the remainder of the stage and compressor as a whole is operating at relatively high efficiency, large alternating pressures of all forcing orders can be excited. This effect is the more severe the lower the natural frequencies of the blades.

1. *Introduction*.—The investigation of alternating pressures in gas turbines is difficult. The static pressures change between certain limits over small distances and during small time intervals and the alternating pressures reduce to small amplitudes in a small distance from the source.

It was thought that the alternating pressures observed at one point would give the forces exciting vibration in a particular stage but the predominant harmonic components of the alternating pressures taken at the compressor casing are dissimilar from the harmonic components of the strain in the blades. The alternating pressures in the compressor annulus at mean diameter or nearer the tip of the blade give much better agreement, but for these readings a special approach has to be made and there is a loss in accuracy or frequency response of the pressure element.

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Comprehensive observations of the alternating pressures in a particular stage have not been obtained but a survey is made of a large number of pressure readings and general information is presented.

There have been a number of high alternating stresses in compressors and the majority have occurred at part load. Two of the curves of alternating pressures given in the report were taken during such an investigation but the alternating pressures taken at one point in the compressor casing does not give sufficient information on the excitation. The part-load operation of compressors is discussed together with stalling flutter and the excitation from rotating stall cells at or near the surge. Strain readings have indicated that there is excitation from dissimilar flows in individual blade passages at all conditions of operation and at the stalling incidence of the stage there is a critical increase in the excitation. The summation of the pressures acting on the blades around a circumference is analysed and a Fourier analysis is made for square waves covering various proportions of the circumference and the effect on this critical excitation is discussed.

2. *Static-Pressure Distribution in Axial-flow Compressors.*—It is difficult to give general trends in the pressure distribution in compressors and quite different values can be obtained in some stages for other operating conditions. A typical overall characteristic for a medium performance compressor is given in Fig. 1. The stage pressures for a high-performance compressor are plotted in Fig. 2 for maximum pressure ratio at various speeds. For compressors designed for lower values of work done per stage with greater number of stages the region of inlet pressure drop due to acceleration of the air stream extends over more blade rows. The static-pressure distribution along a blade of a test compressor (Ref. 1) is given in Fig. 3 and the static pressures at inner and outer diameter for stages 2 and 5 are plotted on the pressure characteristics. The pressure distribution is complex and has been plotted for favourable operating conditions. Different results can be obtained at part load particularly when some of the early stages are stalled.

The pressure distribution around a 11C2/33P40 aerofoil has been obtained from Ref. 2 and is plotted for incidences of $-2$, 2, $4 \cdot 5$ and 13 deg in Fig. 4. The maximum value of the suction peak is of the order of $2 \times$ velocity head and the maximum reduction in the static pressure around the aerofoil is not much greater. The pressure changes do not depend directly on the static pressure at entry to the stage. The change in pressures occurs over short distances and for short time intervals. This is also true of the alternating pressures which reduce to small amplitude in a short distance from the source.

3. *Alternating Pressures in Gas Turbines.*—3.1. *Type of Pressure Elements Used in Investigation.*—The balanced diaphragm pressure element with stretched diaphragm as described in Ref. 3 was used in most of the investigations. Some development was necessary on the balance air control system and time was taken to develop the technique of following the pressure on the face of the diaphragm when obtaining the engine or compressor operating condition.

As the investigations progressed the readings of alternating pressure and the strain readings taken from a blade in the same stage comprised different harmonic components some of which did not appear on the other records. The strain and pressure readings given in Ref. 3 gave much better agreement and although these techniques could not be applied to the particular compressors, it was imperative to obtain a pressure reading at mean diameter even with a loss in accuracy of reading. A nose was attached to the standard element as is shown in Fig. 5 which comprised a $\frac{3}{32}$-in. diameter passage $1\frac{3}{4}$-in. long which opened up to the diameter of the diaphragm in a further $\frac{1}{4}$ in. The compressor casing was drilled between two stator blades and the pick-up and nose was inserted in the blade passage with the connecting hole facing the direction of the flow. Another pick-up was placed in the compressor casing in the same position in the stage and relative readings were taken. The two elements were calibrated against each other in the end of the resonant tube described in Ref. 3. The readings obtained from such an element will have a limited frequency response and this should be taken into account in the interpretation of the results. The readings will be accurate for the lower frequencies when the corrections are applied.

3.2. *Alternating Pressures Recorded by Element.*—The pressure recorded by the element is proportional to the resultant force on the diaphragm. The alternating pressures on the diaphragm will comprise changes in the pressure of the air stream and also a component of the velocity head due to parts of the stream impinging on the diaphragm, the magnitude of this depending on the relative direction of stream and diaphragm. It will not be possible to obtain a strict comparison with the static and velocity heads measured in steady flow because a diaphragm of similar dimensions to a pitot head would be required or the measurements taken at the end of a connecting passage. The readings taken with the extended nose will comprise greater proportions of the velocity head whereas the readings taken in the casing will contain a small component of the velocity head due to the flow off the tips of the blades.

3.3. *Alternating Pressures in Centrifugal Compressors.*—The alternating pressures obtained in the casing at outlet of three centrifugal compressors are given in Figs. 6 and 7 for single-sided impellers and Fig. 8 for a double-sided impeller. The amplitude of the alternating pressures reduce approximately inversely as the number of passages on the impeller. The predominant harmonic component is the first order of the impulse from the impeller vanes and appears throughout the speed range for all impellers. Orders up to the 6th or 7th harmonics of the impulse appear in some parts of the speed range. The first and second order of the impeller speed appears at low operating speeds.

The amplitude of the alternating pressures are more dependent on the incidence to the rotor and the direction of the flow into the diffuser and the general conditions of the flow rather than the delivery pressure. There is usually a relative reduction in the alternating pressures when operating near the design point.

3.4. *Alternating Pressures in Axial-flow Compressors.*—The alternating pressures in the casing of an axial-flow compressor above the last-stage rotor blade is given in Fig. 9 and above the fifth-stage rotor blade of another compressor in Fig. 10. The corresponding alternating pressures at mean diameter to the latter is given in Fig. 11. The alternating pressures obtained in Ref. 4 are reproduced in Fig. 12 which are the alternating pressures at 0·85 length of the first-stage stator blade obtained by letting a pressure capsule 0·5 in. in diameter and 0·04 in. thick into the blade with the diaphragm approximately flush with the surface. Readings have been obtained on other compressors for limited ranges but one set of readings were obtained on a compressor with differing numbers of blades in all blade rows and on all records obtained the forcing impulses were confined to the particular stage.

For the readings taken in the compressor casing the predominant harmonic component is the first order of the impulse from each rotor blade and harmonics of this impulse up to the fifth appear on the records. First- and second-order harmonics of compressor speed appear on the records mainly at low operating speeds.

For the readings taken in the compressor annulus a different frequency spectrum is given. The pressure elements will give higher components of the velocity head. The alternating pressures from the blade impulses are similar to the readings obtained in the casing but the predominant harmonics are those due to dissimilar flows in individual blade passages the largest orders being those which excite the blades in their fundamental flexural mode of vibration. It appears that the blade vibration amplifies these components of the alternating pressures.

The alternating pressures in axial-flow compressors are much less than those in centrifugal compressors and the amplitude is not dependent on the stage. Again the amplitude of the alternating pressures are dependent on incidence, deviation and the general flow in the stage rather than the stage pressure and there is usually a relative reduction when operating near the design point.

3.5. *Changes in Alternating Pressure with Position on Characteristic.*—The alternating pressures were recorded for stages 1, 3, 4 and 5 of the axial-flow compressor for which the overall characteristics are given in Fig. 1. The predominant harmonic components have been plotted in

Figs. 13, 14, 15 and 16 respectively on stage characteristics obtained from the stage pressures and assuming the stage surges at the same time that the compressor as a whole surges. The amplitudes of the alternating pressures are similar to the other readings on axial-flow compressors, one difference being a greater number of harmonics from the stator blade impulses in the minor harmonics.

There is little change in the harmonic components of the rotor and stator blade impulses with position on the characteristic except for a slight reduction at the lower pressure ratios. In Ref. 5 the readings of strain in a blade excited by the harmonic components associated with dissimilar flows in individual blade passages also show small changes in amplitude with position on the characteristic except for radical changes in the flow which will be discussed in sections 4.4, 4.5 and 4.6.

3.6. *Alternating Pressures in a Single-Stage Turbine.*—A standard element with a brazed copper-gauze heat-dissipator pressed on to the nose was fitted in a cooling shroud and the assembly was fitted to the shroud ring of a single-stage turbine as stated in Ref. 3. The results are plotted in Fig. 19.

The predominant harmonic component is the first order of the impulse from a rotor blade and harmonics up to the seventh appear in parts of the speed range. First and second order of turbine speed appear on the records mainly at low operating speeds. The alternating pressures are similar to those obtained in the casing of an axial-flow compressor except that the amplitudes are much larger and are of the same order as those recorded at the outlet of a centrifugal compressor.

3.7. *Changes in Alternating Pressures with Position in Engine.*—The alternating pressures at outlet of a centrifugal compressor are plotted in Fig. 17 and the alternating pressures observed six inches downstream in the air casing are plotted in Fig. 18. The frequency spectrum is similar in both cases and the amplitudes fall to approximately 50 per cent between the two positions.

The same comparison for the turbine is given in Figs. 19 and 20. There is a greater reduction in the harmonics of the blade impulses but the harmonics from the turbine speed do not decrease to the same extent.

4. *Discussion of Results.*—4.1. *Summary of the Alternating Pressures.*—The predominant harmonic components of the alternating pressures in the casing of a compressor are the first-order components of the rotor blade or impeller vane impulses. Harmonics up to the seventh have been recorded in some parts of the speed-range.

Small amplitude harmonic components of the stator blade impulse can be detected and these components are larger if the same numbers of stator blades are used in adjacent stages.

The amplitude of the harmonic components is independent of pressure, position on the characteristic at which the compressor is operated, stage in an axial-flow compressor, but is dependent on the work done per blade or impeller vane passage in the stage.

The alternating pressures are dependent on the incidence at entry to the stage, the exit conditions and the general flow conditions in the stage. There is usually a relative reduction in the alternating pressures when operating near the design point.

First- and second-order harmonic components of rotor speed appear throughout on the records but have largest amplitude at the lower operating speeds.

If the number of rotor and stator blades differ and the numbers of blades in adjacent stages also differ there is little transfer of the alternating pressure between stages.

The alternating pressures in the compressor annulus have components similar to those obtained in the casing but in addition have components of greater magnitude which are excited by dissimilar flows in individual blade passages. The largest orders are those which excite the fundamental flexural mode of vibration of the blades and the corresponding blade movements amplify the harmonics.

The alternating pressures in single-stage turbines are similar to those in compressors except that the amplitude is greater due to the greater work done in a blade passage.

There is a sharp reduction in the alternating pressures with distance from the source particularly across adjacent blade rows.

4.2. *Analysis of Alternating Pressures.*

If $\Delta p_n'$ is the mean pressure over the section of blade $n'$ at radius $\delta r$

$\Delta P$ the pressure on width $\delta r$ of annulus

$$\text{Then } \Delta P = \frac{c}{s}\frac{1}{N}\sum_{n'=1}^{n'=N}\Delta p_n'$$

where $N$ is the number of blades

$c$ chord of blade

$s$ pitch of blade.

$$\text{If } \Delta p_n' = p_{0n}' + p_{1n}' \cos\theta + p_{2n}' \cos 2\theta \ldots + p_{mn}' \cos n\theta.$$

The axial component of pressure for order $m$ is :

$$\Delta P = \frac{c}{s}\sum_{n'=1}^{n'=N} p_{mn}' \cos\left(m\theta + \frac{2\pi n'}{N}\right) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (1)$$

If the force on each blade is the same

$$\Delta P = \frac{c}{s} p_m \sum_{n'=1}^{n'=N} \cos\left(m\theta + \frac{2\pi n'}{N}\right). \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (2)$$

The alternating pressures obtained in the compressor casings indicate that equal forces on each blade could be assumed and equation (2) would apply.

The alternating pressures obtained in the compressor annuli show differences in the forces on individual blades and equation (1) would apply.

Different numbers of rotor and stator blades produce a different frequency spectrum in each. If the same numbers of blades are used in adjacent stages the same harmonics are in phase in the stages and alternating pressures are transferred between them. If different numbers of blades are used in adjacent stages there is a tendency to restrict the alternating pressures to the source.

The transfer of energy for the same numbers of blades in adjacent stages is shown in Figs. 13, 14, 15 and 16 in which the impulses from the stator blades are more prominent and in Fig. 9 where an alternating pressure has been transferred across five stages.

4.3. *The Part Load Performance of Axial-flow Compressors.*—The estimation of the part load performance of a compressor is difficult and scatter appears in the results. However some general conclusions can be made.

At low operating speeds the decrease in mass flow is greater than the decrease in peripheral speed resulting in high positive incidence values in the early stages. The high incidences occur for a small number of blade rows but the drop in efficiency reduces the work done and the increase in density is not sufficient to maintain approximately constant axial velocity in the converging passage. There is an increase in the axial velocity with a tendency to reduce incidence. This effect continues through the compressor resulting in negative incidences in the later stages and may lead to choking of the outlet stages.

4.4. *Alternating Pressures at Part Load.*—The minimum excitation of vibration relative to the operating loads for compressors and turbines occurs at and around the design point. With reductions in speed the excitation of vibration or alternating pressures increase slightly.

In compressors with further reduction in speed the incidences in the early stages increase, and when this approaches the stalling incidence for the stage if the blades are sufficiently flexible there is a possibility of the excitation of stalling flutter over wide ranges of operation of the compressor.

With further reduction in speed or operation of compressor at higher pressure ratio there is a possibility near the surge or if some of the stages can be operated in surge of stall cells which rotate at irregular fractions of the rotor speed. More than one cell can occur so that higher frequency modes can be excited.

In addition at the stalling incidence of a stage there is a critical operating condition where high alternating pressures can occur. A stall occurs in part of the stage due to dissimilarities in the geometry of the stage and in the flow, and the remainder of the stage is operating at relatively high efficiency. For this condition there is excitation at all frequencies. As the speed is reduced from this critical value the flow in the stage deteriorates generally but is more uniform around the circumference and the excitation is reduced. The stalling incidence can be exceeded at part load for about a third of the compressor and will occur at higher speeds in the later stages. This critical excitation is the more severe the higher the efficiency of the remainder of the stage and the higher the efficiency of the compressor as a whole.

4.5. *Flutter of Blades in Compressors.*—Stalling flutter at high positive incidences can occur in axial-flow compressors for blades of low stiffness over wide ranges of operation provided the incidence and mass flow are greater than the lower boundary conditions.

Quantitative information on the flutter of blades in compressors is not available as yet. It would be indicated by a pressure element in the annulus probably by a sporadic signal at the natural frequency of the blades of indefinite amplitude but it would not resemble the multiple harmonic signal associated with forced vibration.

4.6. *Excitation of Alternating Pressures by Rotating Stall Cells.*—A number of investigators have observed that near the surge of a compressor stall cells are set up that rotate at irregular fractions of compressor speed. One instance is reported in Ref. 6. Large-amplitude alternating pressures can result and more than one cell can occur which would excite higher frequency modes of vibration. For some compressor designs the stall cells could occur prior to the surge and for others occur when some of the stages are operating in a surged condition.

It is not thought that excessive stresses will be excited in the blades because the compressor will be operating with low mass flow and at low efficiency.

4.7. *Alternating Pressures Due to Stall of Part of Stage.*—Strain-gauge investigations on compressors have shown there is excitation from dissimilar flow in individual blade passages at all operating speeds. In Ref. 5 it has been shown that at a critical operating condition at or near stalling incidence there is an abrupt increase in the excitation such as would occur for a stall of a part of the stage.

A Fourier analysis has been made of a series of square waves to represent stalls of various proportions of the circumference. The sine, cosine and sum terms are given for harmonics up to the twentieth in Tables 1, 2 and 3. The harmonics up to the eighth for the sum terms are plotted in Fig. 21 and the maximum value of the coefficients for any entry condition is given in Fig. 22. The proportion of the circumference to be covered by the square wave to produce the maximum excitation is plotted in Fig. 23.

If the excitation from the dissimilar flow in a stage could be assumed to be equivalent to a square wave extending for 2 deg of the circumference the harmonic coefficient for all orders is approximately 0·01. If part of the stage stalls the increase in the coefficient for the various orders for various proportions of the circumference can be obtained from the curves.

The advantage of a high natural fundamental flexural frequency of the blades is demonstrated. For compressors of medium flows and medium pressure ratios the fundamental flexural mode is excited by fifth, sixth, seventh or eighth orders and for compressors with high mass flow and high pressure ratios the fundamental flexural mode is excited by third, second and first orders. If a stall occurred of a sufficient proportion of the stage, the alternating pressures would be from three to six times more severe for the latter.

There is not sufficient information to define if the stall is fixed or rotates relative to the blades. There is a change in the excitation in the critical zone with a reduction in the frequency of the forcing harmonics which is greatest for the greatest amplitude. This indicates an increase in the stalled zone. The amplitudes of vibration of the blades decrease with increase in incidence to a value slightly higher than that before the critical zone.

5. *Conclusion.*—The predominant harmonic components of the alternating pressures in the casing of gas turbines is the fundamental of the impulse from the rotor or impeller vanes. Harmonics up to the seventh of this impulse can be detected in parts of the speed range.

The alternating pressures in a compressor annulus have components similar to those in the casing but in addition have components of greater magnitude excited by dissimilar flows in individual blade passages. The largest orders are those which excite the fundamental flexural mode of vibration of the blades and the corresponding blade movements amplify the harmonics.

The amplitude of the alternating pressures is dependent on the work done per blade or impeller vane passage. The pressures are dependent on the incidence at entry to the stage, the exit conditions and the general flow conditions and there is usually a relative reduction when operating near the design point.

The lower harmonics of the rotor speed appear at all speeds but have largest amplitude at the lower operating speeds.

There is a reduction in the alternating pressures with distance from the source particularly across a blade row.

In addition there are three other sources of alternating pressures when operating at part load.

(*a*) Stalling flutter at high positive incidences can occur in axial-flow compressors for blades of low stiffness over wide ranges of operation provided the incidence and mass flow are greater than the lower boundary conditions.

(*b*) Alternating pressures can be set up near the surge by stall cells which rotate at irregular fractions of compressor speed. More than one cell can occur which will excite higher frequency modes of vibration.

(*c*) At the stalling incidence of a stage, if a proportion of the stage stalls due to dissimilar geometry or flow and the remainder of the stage and compressor as a whole is operating at high efficiency, large alternating pressures of all forcing orders can be excited. This effect is the more severe the lower the natural frequency of the blades.

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## TABLE 1

*Cosine Coefficient in Fourier Analysis of Various Square Waves*

| Proportion of circumference | Angle subtended (deg) | Order | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $\frac{1}{180}$ | 2 | 0·0111 | 0·0111 | 0·01108 | 0·01107 | 0·01105 | 0·01103 | 0·0110 | 0·0109 | 0·0109 | 0·0109 |
| $\frac{1}{90}$ | 4 | 0·0222 | 0·0221 | 0·0220 | 0·0219 | 0·0217 | 0·0216 | 0·0213 | 0·0211 | 0·0208 | 0·0205 |
| $\frac{1}{60}$ | 6 | 0·033 | 0·033 | 0·033 | 0·032 | 0·032 | 0·031 | 0·030 | 0·029 | 0·029 | 0·027 |
| $\frac{1}{45}$ | 8 | 0·044 | 0·044 | 0·043 | 0·42 | 0·041 | 0·039 | 0·038 | 0·036 | 0·034 | 0·031 |
| $\frac{1}{20}$ | 18 | 0·098 | 0·094 | 0·086 | 0·076 | 0·064 | 0·051 | 0·037 | 0·023 | 0·011 | 0 |
| $\frac{1}{15}$ | 24 | 0·129 | 0·118 | 0·101 | 0·079 | 0·055 | 0·031 | 0·0095 | $0\cdot\bar{0}09$ | $0\cdot\bar{0}21$ | $0\cdot\bar{0}27$ |
| $\frac{1}{12}$ | 30 | 0·159 | 0·138 | 0·106 | 0·069 | 0·032 | 0 | $0\cdot\bar{0}23$ | $0\cdot\bar{0}35$ | $0\cdot\bar{0}32$ | $0\cdot\bar{0}28$ |
| $\frac{1}{10}$ | 36 | 0·187 | 0·151 | 0·101 | 0·047 | 0 | $0\cdot\bar{0}31$ | $0\cdot\bar{0}43$ | $0\cdot\bar{0}38$ | $0\cdot\bar{0}18$ | 0 |
| $\frac{1}{9}$ | 40 | 0·204 | 0·157 | 0·092 | 0·028 | $0\cdot\bar{0}22$ | $0\cdot\bar{0}46$ | $0\cdot\bar{0}45$ | $0\cdot\bar{0}26$ | 0 | 0·021 |
| $\frac{1}{8}$ | 45 | 0·224 | 0·159 | 0·075 | 0 | $0\cdot\bar{0}45$ | $0\cdot\bar{0}53$ | $0\cdot\bar{0}32$ | 0 | 0·025 | 0·032 |
| $\frac{1}{6}$ | 60 | 0·275 | 0·138 | 0 | $0\cdot\bar{0}69$ | $0\cdot\bar{0}55$ | 0 | 0·039 | 0·034 | 0 | $0\cdot\bar{0}27$ |
| $\frac{1}{5}$ | 72 | 0·302 | 0·093 | $0\cdot\bar{0}62$ | $0\cdot\bar{0}76$ | 0 | 0·051 | 0·027 | $0\cdot\bar{0}23$ | $0\cdot\bar{0}34$ | 0 |
| $\frac{1}{4}$ | 90 | 0·318 | 0 | $0\cdot\bar{1}06$ | 0 | 0·064 | 0 | $0\cdot\bar{0}45$ | 0 | 0·035 | 0 |
| $\frac{1}{3}$ | 120 | 0·276 | $0\cdot\bar{1}38$ | 0 | 0·069 | $0\cdot\bar{0}55$ | 0 | 0·039 | $0\cdot\bar{0}34$ | 0 | 0·028 |
| $\frac{1}{2}$ | 180 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| Proportion of circumference | Angle subtended (deg) | Order | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| $\frac{1}{180}$ | 2 | 0·0108 | 0·0108 | 0·0107 | 0·0107 | 0·0106 | 0·0105 | 0·0105 | 0·0104 | 0·0103 | 0·0102 |
| $\frac{1}{90}$ | 4 | 0·0201 | 0·0195 | 0·0192 | 0·0188 | 0·0184 | 0·0178 | 0·0173 | 0·0168 | 0·0161 | 0·0154 |
| $\frac{1}{60}$ | 6 | 0·026 | 0·025 | 0·024 | 0·023 | 0·021 | 0·019 | 0·018 | 0·017 | 0·015 | 0·014 |
| $\frac{1}{45}$ | 8 | 0·029 | 0·026 | 0·024 | 0·021 | 0·018 | 0·016 | 0·013 | 0·010 | 0·008 | 0·005 |
| $\frac{1}{20}$ | 18 | $0\cdot\bar{0}09$ | $0\cdot\bar{0}16$ | $0\cdot\bar{0}19$ | $0\cdot\bar{0}22$ | $0\cdot\bar{0}21$ | $0\cdot\bar{0}19$ | $0\cdot\bar{0}15$ | $0\cdot\bar{0}10$ | $0\cdot\bar{0}05$ | 0 |
| $\frac{1}{15}$ | 24 | $0\cdot\bar{0}29$ | $0\cdot\bar{0}25$ | $0\cdot\bar{0}18$ | $0\cdot\bar{0}09$ | 0 | 0·008 | 0·014 | 0·017 | 0·017 | 0·014 |
| $\frac{1}{12}$ | 30 | $0\cdot\bar{0}14$ | 0 | 0·012 | 0·019 | 0·021 | 0·017 | 0·009 | 0 | $0\cdot\bar{0}08$ | $0\cdot\bar{0}1$ |
| $\frac{1}{10}$ | 36 | 0·017 | 0·025 | 0·023 | 0·013 | 0 | $0\cdot\bar{0}11$ | $0\cdot\bar{0}18$ | $0\cdot\bar{0}17$ | $0\cdot\bar{0}09$ | 0 |
| $\frac{1}{9}$ | 40 | 0·028 | 0·023 | 0·008 | $0\cdot\bar{0}07$ | $0\cdot\bar{0}18$ | $0\cdot\bar{0}18$ | $0\cdot\bar{0}12$ | 0 | 0·011 | 0·015 |
| $\frac{1}{8}$ | 45 | 0·021 | 0 | $0\cdot\bar{0}17$ | $0\cdot\bar{0}21$ | $0\cdot\bar{0}15$ | 0 | 0·132 | 0·017 | 0·011 | 0 |
| $\frac{1}{6}$ | 60 | $0\cdot\bar{0}21$ | 0 | 0·021 | 0·019 | 0 | $0\cdot\bar{0}17$ | $0\cdot\bar{0}16$ | 0 | 0·014 | 0·014 |
| $\frac{1}{5}$ | 72 | 0·028 | 0·016 | $0\cdot\bar{0}14$ | $0\cdot\bar{0}22$ | 0 | 0·019 | 0·011 | $0\cdot\bar{0}10$ | $0\cdot\bar{0}16$ | 0 |
| $\frac{1}{4}$ | 90 | $0\cdot\bar{0}29$ | 0 | 0·024 | 0 | $0\cdot\bar{0}21$ | 0 | 0·018 | 0 | $0\cdot\bar{0}16$ | 0 |
| $\frac{1}{3}$ | 120 | $0\cdot\bar{0}25$ | 0 | 0·021 | $0\cdot\bar{0}19$ | 0 | 0·017 | $0\cdot\bar{0}16$ | 0 | 0·015 | $0\cdot\bar{0}14$ |
| $\frac{1}{2}$ | 180 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

## TABLE 2

*Sine Coefficient in Fourier Analysis of Various Square Waves*

| Proportion of circumference | Angle subtended (deg) | Order | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| $\frac{1}{180}$ | 2 | 0·0002 | 0·0004 | 0·0006 | 0·0008 | 0·0009 | 0·0012 | 0·0014 | 0·0015 | 0·0017 | 0·0019 | 0·0021 | 0·0023 | 0·0025 | 0·0027 | 0·0028 | 0·0030 | 0·0032 | 0·0034 | 0·0036 | 0·0037 |
| $\frac{1}{90}$ | 4 | 0·0008 | 0·0016 | 0·0023 | 0·0031 | 0·0038 | 0·0046 | 0·0053 | 0·0061 | 0·0068 | 0·0075 | 0·0081 | 0·0088 | 0·0094 | 0·0105 | 0·0105 | 0·0112 | 0·0117 | 0·0122 | 0·0127 | 0·0132 |
| $\frac{1}{60}$ | 6 | 0·002 | 0·003 | 0·005 | 0·007 | 0·008 | 0·010 | 0·012 | 0·013 | 0·015 | 0·016 | 0·017 | 0·018 | 0·019 | 0·020 | 0·021 | 0·022 | 0·023 | 0·023 | 0·024 | 0·024 |
| $\frac{1}{45}$ | 8 | 0·003 | 0·006 | 0·009 | 0·012 | 0·015 | 0·017 | 0·020 | 0·022 | 0·024 | 0·026 | 0·028 | 0·029 | 0·030 | 0·031 | 0·032 | 0·032 | 0·032 | 0·032 | 0·031 | 0·031 |
| $\frac{1}{20}$ | 18 | 0·016 | 0·031 | 0·044 | 0·055 | 0·064 | 0·069 | 0·072 | 0·072 | 0·069 | 0·064 | 0·057 | 0·048 | 0·043 | 0·029 | 0·021 | 0·014 | 0·008 | 0·003 | 0·001 | 0 |
| $\frac{1}{15}$ | 24 | 0·028 | 0·053 | 0·073 | 0·088 | 0·095 | 0·096 | 0·090 | 0·079 | 0·064 | 0·048 | 0·032 | 0·018 | 0·008 | 0·002 | 0 | 0·002 | 0·006 | 0·012 | 0·018 | 0·024 |
| $\frac{1}{12}$ | 30 | 0·043 | 0·079 | 0·106 | 0·119 | 0·119 | 0·106 | 0·085 | 0·059 | 0·035 | 0·016 | 0·004 | 0 | 0·003 | 0·011 | 0·021 | 0·029 | 0·035 | 0·035 | 0·031 | 0·024 |
| $\frac{1}{10}$ | 36 | 0·061 | 0·11 | 0·139 | 0·144 | 0·127 | 0·096 | 0·060 | 0·028 | 0·007 | 0 | 0·005 | 0·018 | 0·032 | 0·041 | 0·042 | 0·036 | 0·025 | 0·011 | 0·003 | 0 |
| $\frac{1}{9}$ | 40 | 0·075 | 0·132 | 0·159 | 0·155 | 0·124 | 0·080 | 0·038 | 0·009 | 0 | 0·007 | 0·002 | 0·039 | 0·048 | 0·044 | 0·032 | 0·016 | 0·004 | 0 | 0·004 | 0·013 |
| $\frac{1}{8}$ | 45 | 0·093 | 0·159 | 0·181 | 0·159 | 0·108 | 0·053 | 0·013 | 0 | 0·010 | 0·032 | 0·049 | 0·053 | 0·042 | 0·023 | 0·006 | 0 | 0·005 | 0·018 | 0·029 | 0·032 |
| $\frac{1}{6}$ | 60 | 0·159 | 0·238 | 0·212 | 0·119 | 0·032 | 0 | 0·023 | 0·060 | 0·071 | 0·048 | 0·015 | 0 | 0·012 | 0·034 | 0·042 | 0·030 | 0·009 | 0 | 0·008 | 0·024 |
| $\frac{1}{5}$ | 72 | 0·22 | 0·288 | 0·192 | 0·055 | 0 | 0·037 | 0·082 | 0·072 | 0·024 | 0 | 0·020 | 0·048 | 0·044 | 0·016 | 0 | 0·014 | 0·034 | 0·032 | 0·012 | 0 |
| $\frac{1}{4}$ | 90 | 0·318 | 0·318 | 0·106 | 0 | 0·064 | 0·106 | 0·046 | 0 | 0·035 | 0·064 | 0·026 | 0 | 0·025 | 0·045 | 0·021 | 0 | 0·019 | 0·035 | 0·017 | 0 |
| $\frac{1}{3}$ | 120 | 0·477 | 0·238 | 0 | 0·119 | 0·095 | 0 | 0·068 | 0·060 | 0 | 0·048 | 0·043 | 0 | 0·037 | 0·034 | 0 | 0·030 | 0·028 | 0 | 0·025 | 0·024 |
| $\frac{1}{2}$ | 180 | 0·636 | 0 | 0·212 | 0 | 0·127 | 0 | 0·091 | 0 | 0·071 | 0 | 0·052 | 0 | 0·049 | 0 | 0·043 | 0 | 0·038 | 0 | 0·034 | 0 |

## TABLE 3

*Cosine and Sine Coefficients Sum in Fourier Analysis of Various Square Waves*

| Proportion of circumference | | Angle subtended (deg) | Order | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1 | 2 | 3 | .4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 0·00556 | $\frac{1}{180}$ | 2 | 0·0111 | 0·0111 | 0·0111 | 0·0111 | 0·0111 | 0·0111 | 0·0111 | 0·0111 | 0·0110 | 0·0110 | 0·0110 | 0·0110 | 0·0110 | 0·0110 | 0·0109 | 0·0109 | 0·0109 | 0·0109 | 0·0109 | 0·0108 |
| 0·0112 | $\frac{1}{90}$ | 4 | 0·0222 | 0·0222 | 0·0222 | 0·0221 | 0·0220 | 0·0220 | 0·0220 | 0·0219 | 0·0219 | 0·0218 | 0·0216 | 0·0214 | 0·0214 | 0·0216 | 0·0211 | 0·0210 | 0·0209 | 0·0208 | 0·0205 | 0·0203 |
| 0·166 | $\frac{1}{60}$ | 6 | 0·033 | 0·033 | 0·033 | 0·033 | 0·033 | 0·033 | 0·032 | 0·032 | 0·032 | 0·032 | 0·031 | 0·031 | 0·031 | 0·030 | 0·030 | 0·029 | 0·029 | 0·029 | 0·028 | 0·027 |
| 0·222 | $\frac{1}{45}$ | 8 | 0·044 | 0·044 | 0·044 | 0·044 | 0·043 | 0·043 | 0·043 | 0·042 | 0·042 | 0·041 | 0·040 | 0·039 | 0·039 | 0·038 | 0·037 | 0·036 | 0·035 | 0·034 | 0·032 | 0·031 |
| 0·05 | $\frac{1}{20}$ | 18 | 0·099 | 0·098 | 0·096 | 0·094 | 0·090 | 0·086 | 0·081 | 0·076 | 0·070 | 0·064 | 0·057 | 0·051 | 0·047 | 0·037 | 0·029 | 0·023 | 0·017 | 0·011 | 0·005 | 0 |
| 0·066 | $\frac{1}{15}$ | 24 | 0·132 | 0·130 | 0·125 | 0·118 | 0·110 | 0·101 | 0·091 | 0·079 | 0·067 | 0·055 | 0·043 | 0·032 | 0·020 | 0·009 | 0 | 0·008 | 0·023 | 0·021 | 0·025 | 0·027 |
| 0·0834 | $\frac{1}{12}$ | 30 | 0·165 | 0·159 | 0·15 | 0·137 | 0·122 | 0·106 | 0·088 | 0·069 | 0·047 | 0·030 | 0·015 | 0 | 0·013 | 0·023 | 0·030 | 0·034 | 0·036 | 0·035 | 0·032 | 0·028 |
| 0·1 | $\frac{1}{10}$ | 36 | 0·197 | 0·187 | 0·172 | 0·151 | 0·127 | 0·101 | 0·074 | 0·047 | 0·020 | 0 | 0·018 | 0·031 | 0·040 | 0·043 | 0·042 | 0·038 | 0·030 | 0·021 | 0·010 | 0 |
| 0·112 | $\frac{1}{9}$ | 40 | 0·218 | 0·205 | 0·184 | 0·157 | 0·125 | 0·092 | 0·059 | 0·027 | 0 | 0·022 | 0·030 | 0·046 | 0·048 | 0·045 | 0·037 | 0·025 | 0·013 | 0 | 0·012 | 0·020 |
| 0·125 | $\frac{1}{8}$ | 45 | 0·243 | 0·227 | 0·196 | 0·159 | 0·118 | 0·075 | 0·035 | 0 | 0·027 | 0·045 | 0·054 | 0·053 | 0·045 | 0·034 | 0·016 | 0 | 0·014 | 0·025 | 0·031 | 0·032 |
| 0·166 | $\frac{1}{6}$ | 60 | 0·318 | 0·275 | 0·212 | 0·138 | 0·064 | 0 | 0·045 | 0·069 | 0·071 | 0·056 | 0·026 | 0 | 0·024 | 0·039 | 0·042 | 0·034 | 0·019 | 0 | 0·017 | 0·028 |
| 0·2 | $\frac{1}{5}$ | 72 | 0·374 | 0·303 | 0·202 | 0·094 | 0 | 0·062 | 0·087 | 0·076 | 0·042 | 0 | 0·034 | 0·051 | 0·047 | 0·027 | 0 | 0·023 | 0·036 | 0·034 | 0·020 | 0 |
| 0·25 | $\frac{1}{4}$ | 90 | 0·45 | 0·318 | 0·15 | 0 | 0·090 | 0·106 | 0·064 | 0 | 0·050 | 0·064 | 0·039 | 0 | 0·035 | 0·045 | 0·030 | 0 | 0·026 | 0·035 | 0·024 | 0 |
| 0·333 | $\frac{1}{3}$ | 120 | 0·551 | 0·275 | 0 | 0·138 | 0·110 | 0 | 0·079 | 0·069 | 0 | 0·055 | 0·051 | 0 | 0·042 | 0·039 | 0 | 0·035 | 0·032 | 0 | 0·029 | 0·028 |
| 0·5 | $\frac{1}{2}$ | 180 | 0·636 | 0 | 0·212 | 0 | 0·127 | 0 | 0·091 | 0 | 0·071 | 0 | 0·052 | 0 | 0·049 | 0 | 0·043 | 0 | 0·038 | 0 | 0·033 | 0 |

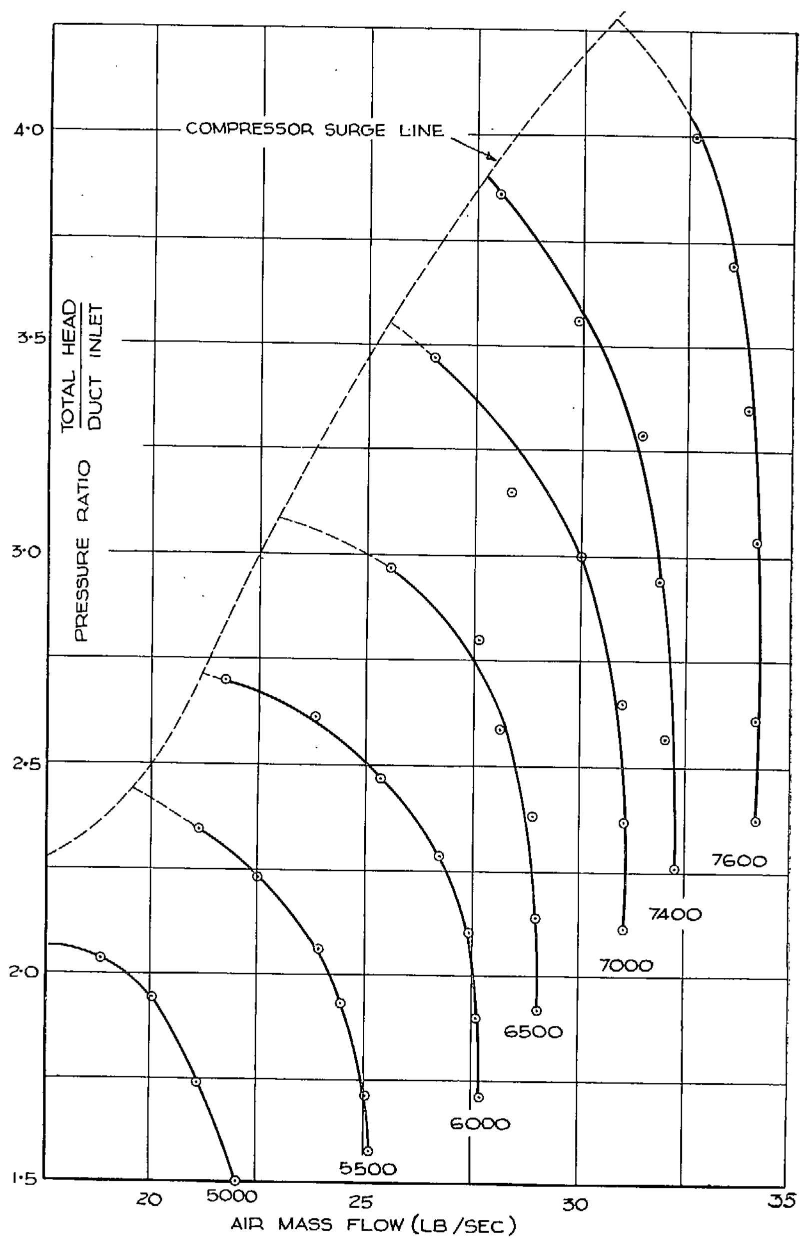



FIG. 1. Overall compressor characteristics.

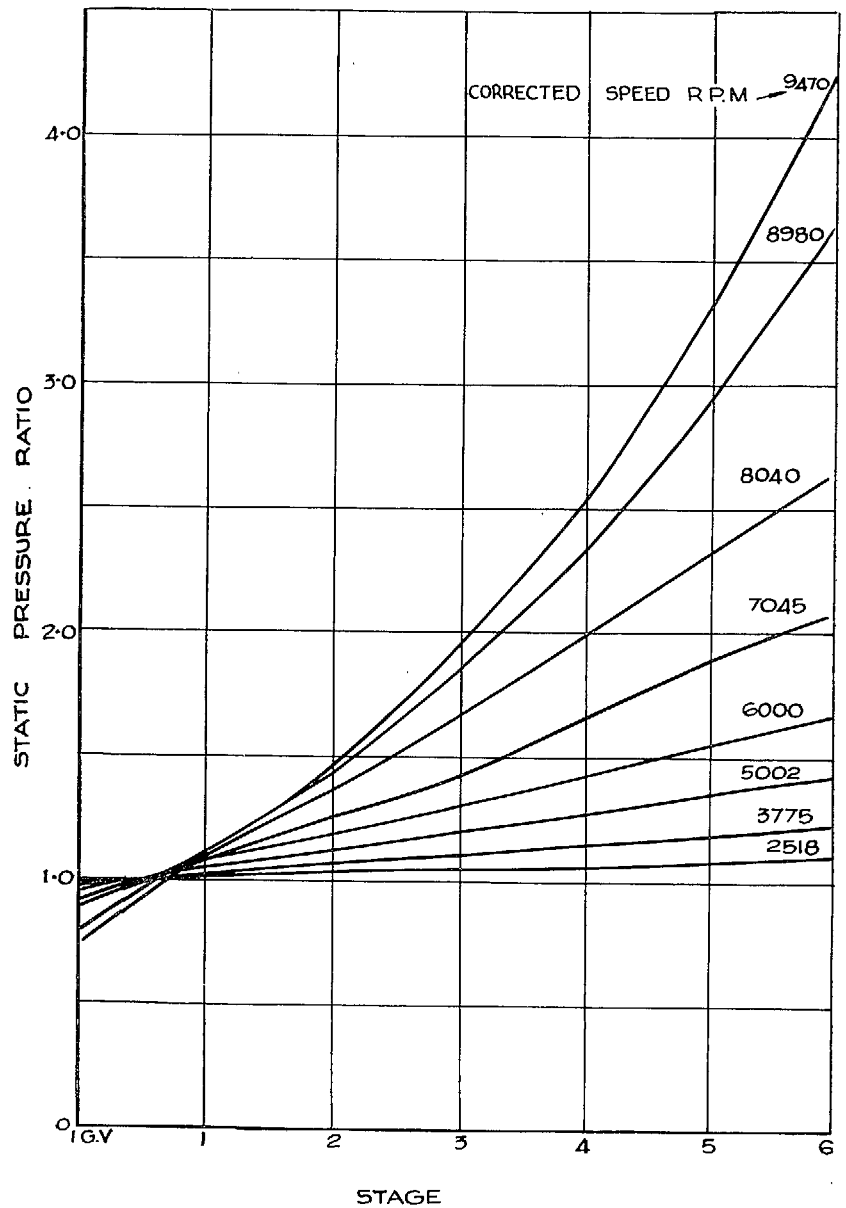



FIG. 2. Pressure distribution through compressor.

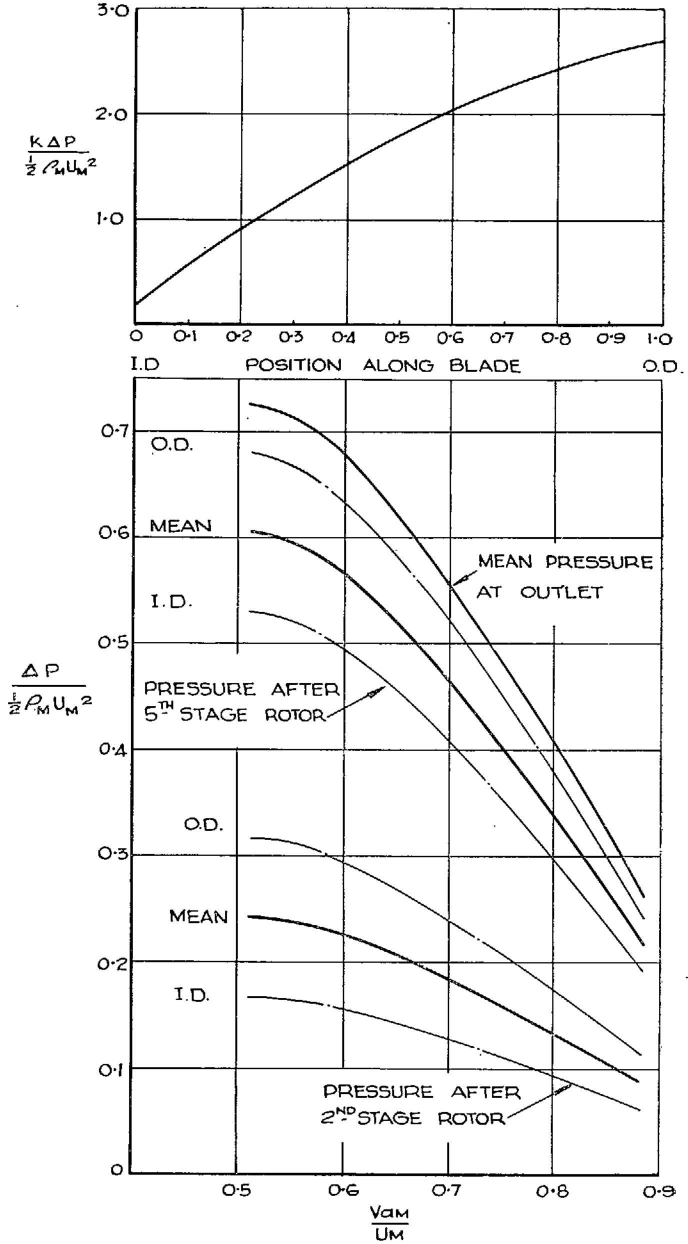
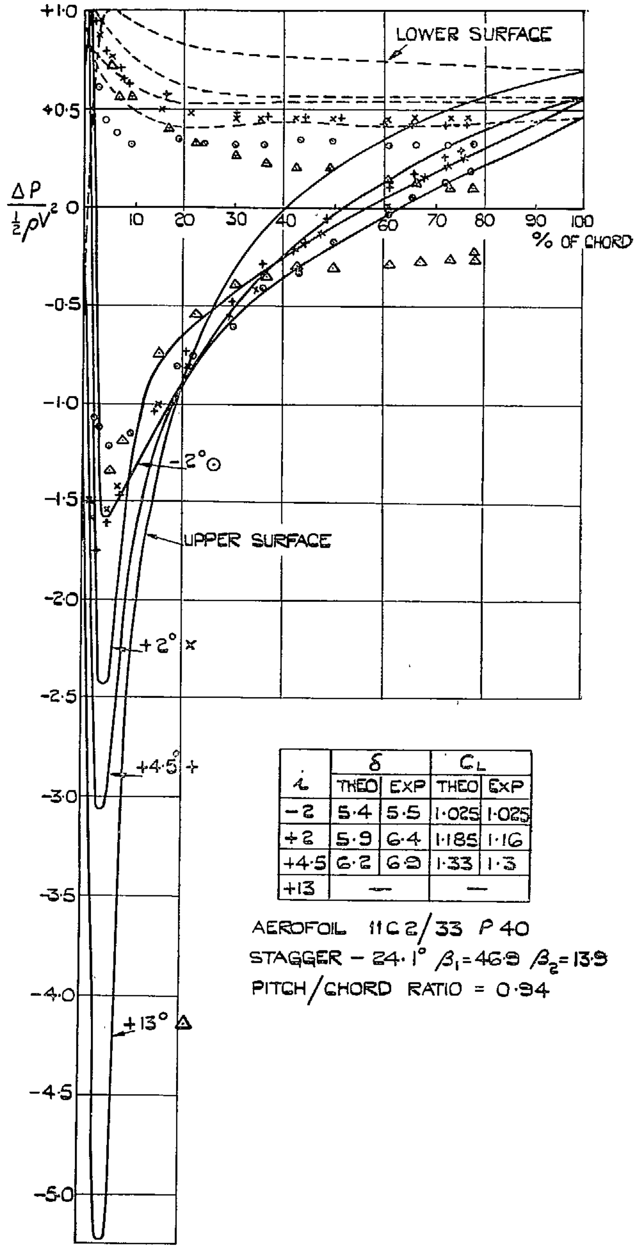



FIG. 3. Pressure distribution along blade.




| $i$ | $\delta$ THEO | $\delta$ EXP | $C_L$ THEO | $C_L$ EXP |
|---|---|---|---|---|
| −2 | 5·4 | 5·5 | 1·025 | 1·025 |
| +2 | 5·9 | 6·4 | 1·185 | 1·16 |
| +4·5 | 6·2 | 6·9 | 1·33 | 1·3 |
| +13 | — | | — | |

FIG. 4. Pressure distribution around aerofoil.

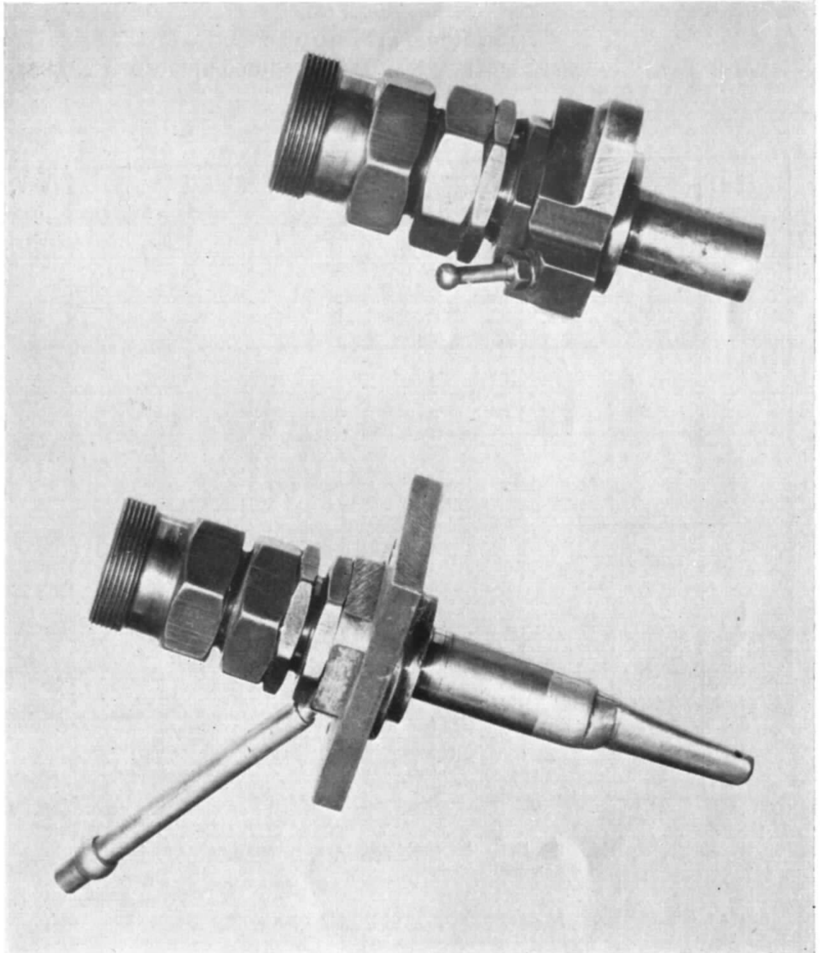


FIG. 5. Pressure elements showing extended nose.

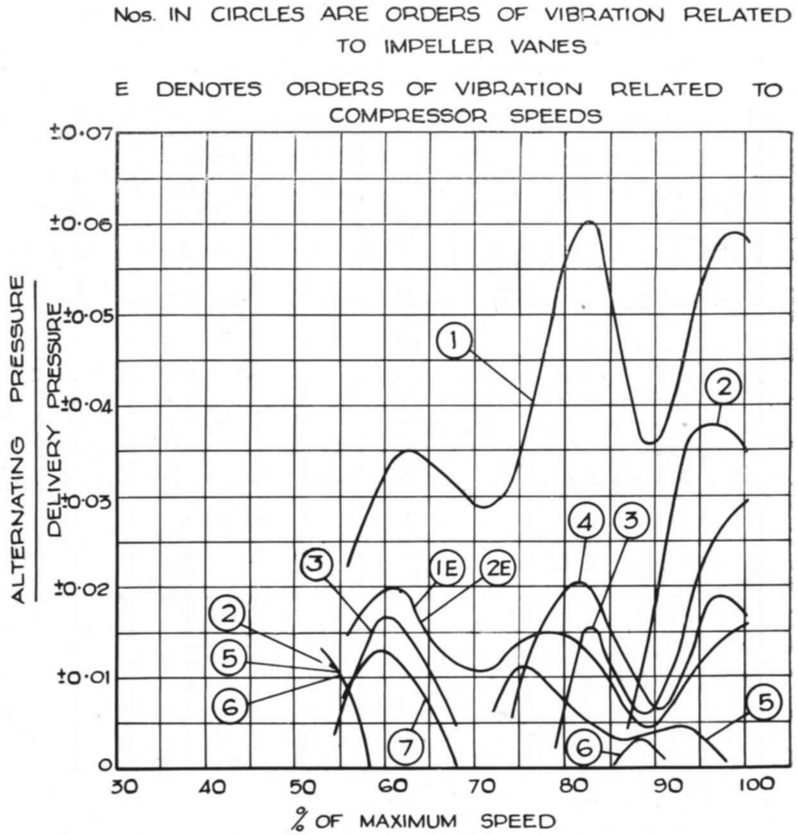



FIG. 6. Harmonic components of alternating pressure at outlet of centrifugal compressor.

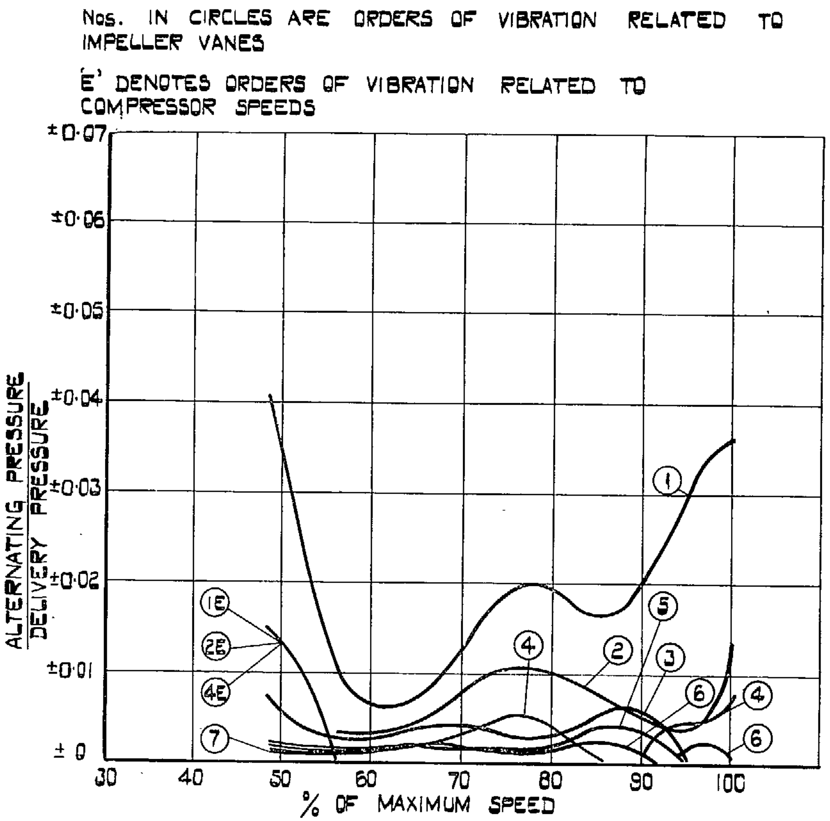



FIG. 7. Harmonic components of alternating pressure at outlet of centrifugal compressor.

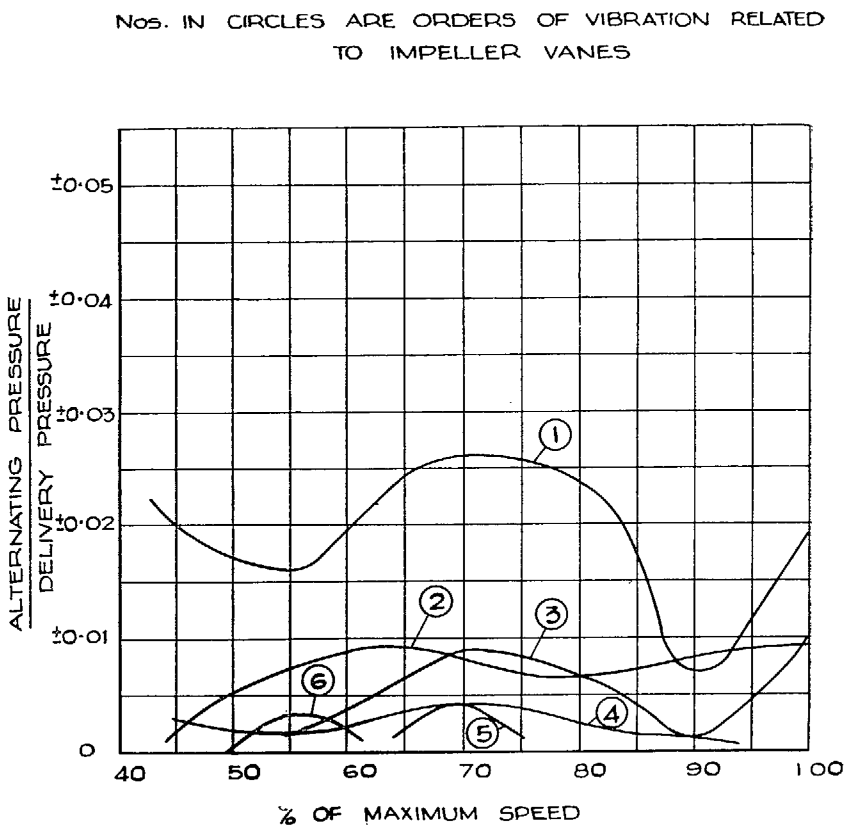



FIG. 8. Harmonic components of alternating pressure at outlet of centrifugal compressor.

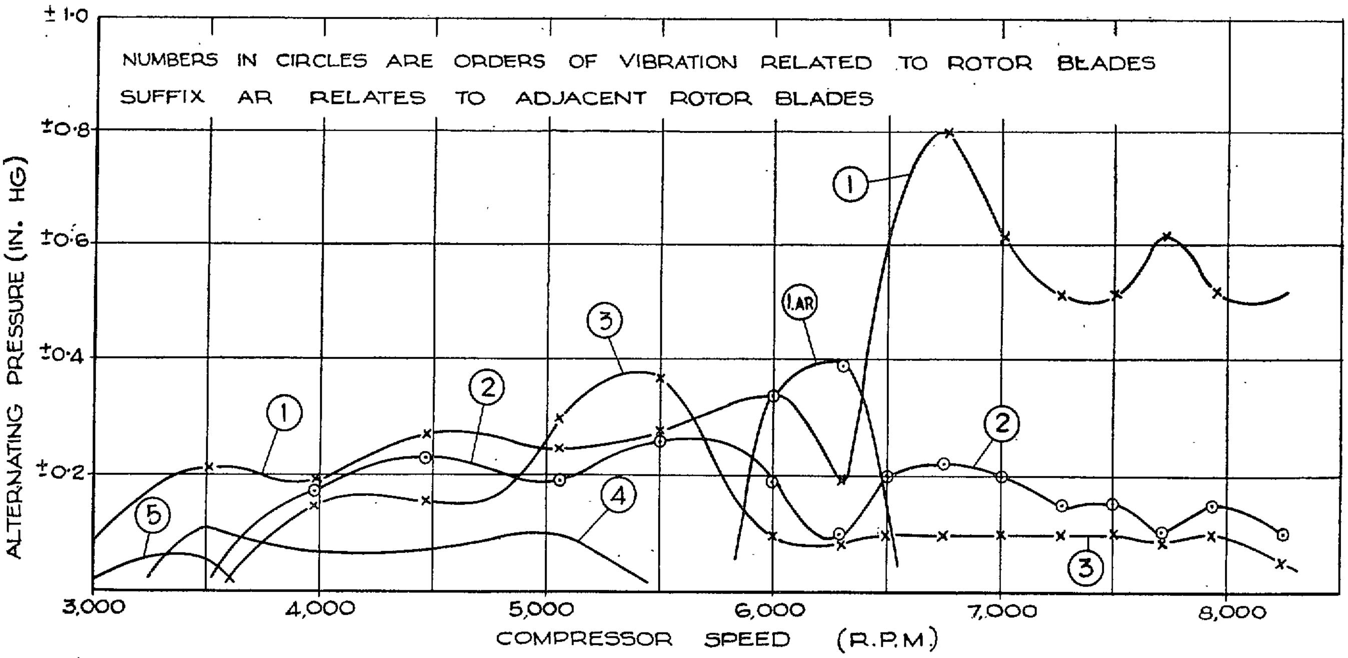



FIG. 9. Alternating pressure in casing of last stage of an axial-flow compressor.

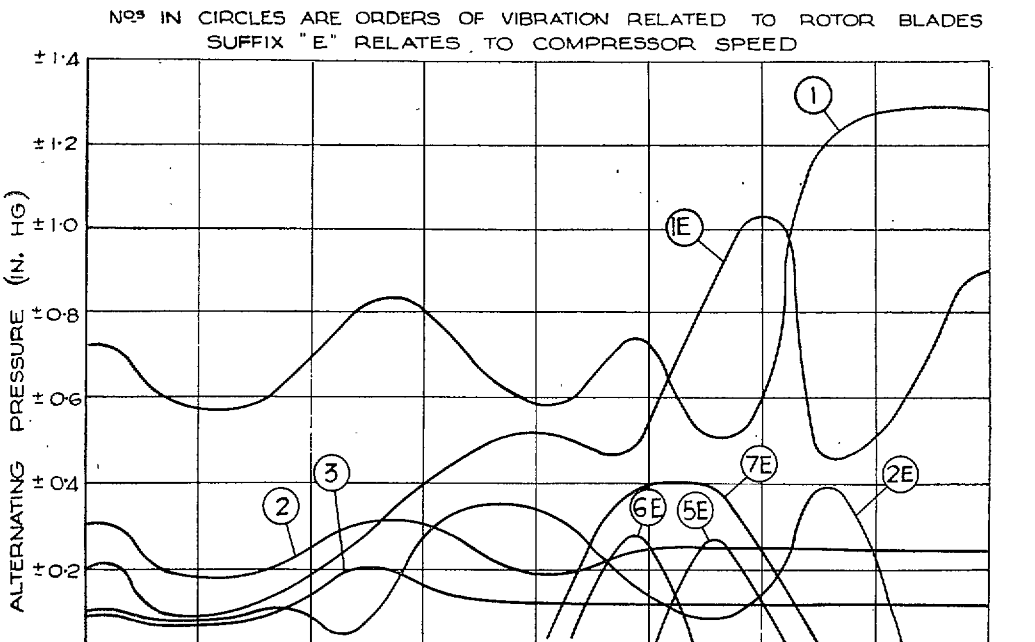



FIG. 10. Alternating pressures in casing of fifth stage of an axial-flow compressor.

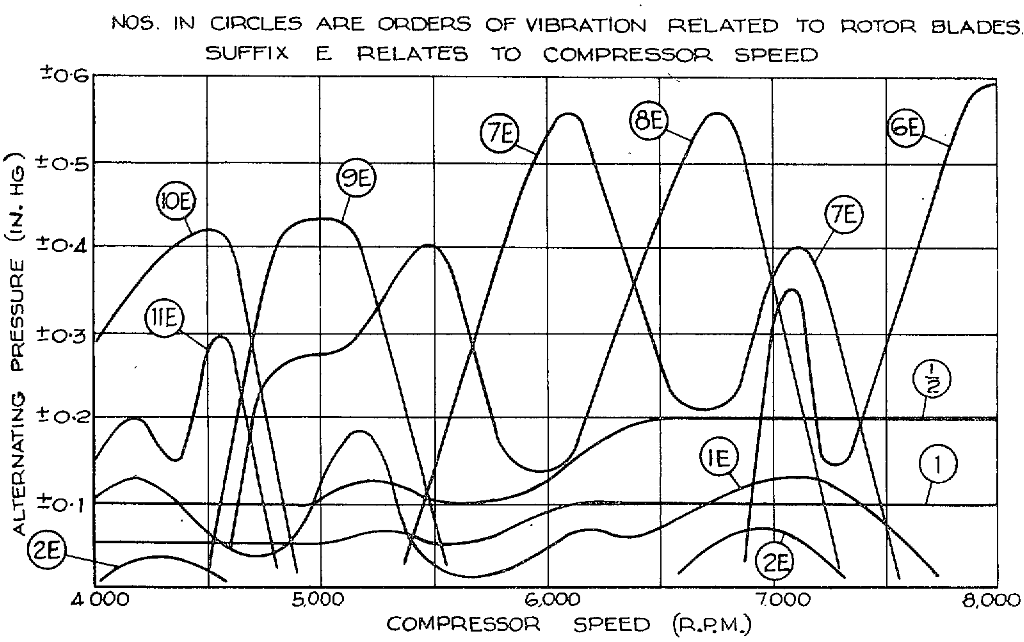



FIG. 11. Alternating pressures at mean diameter of fifth stage of an axial-flow compressor.

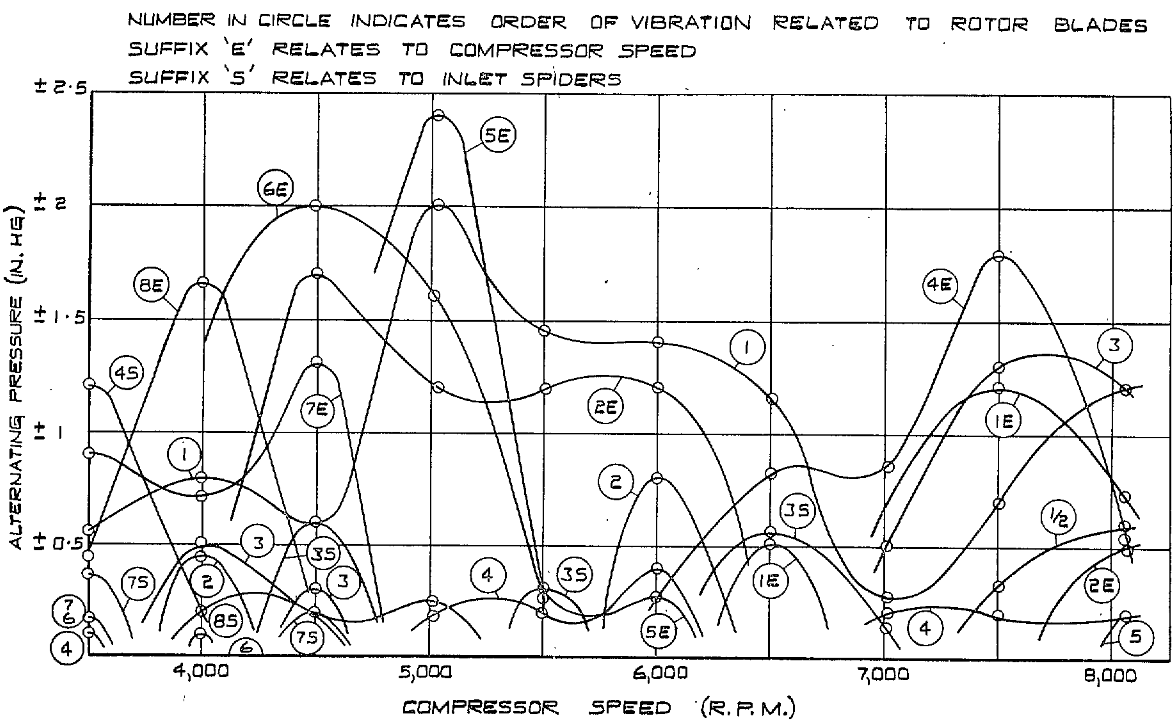



FIG. 12. Alternating pressures at mean diameter of first stage of an axial-flow compressor.

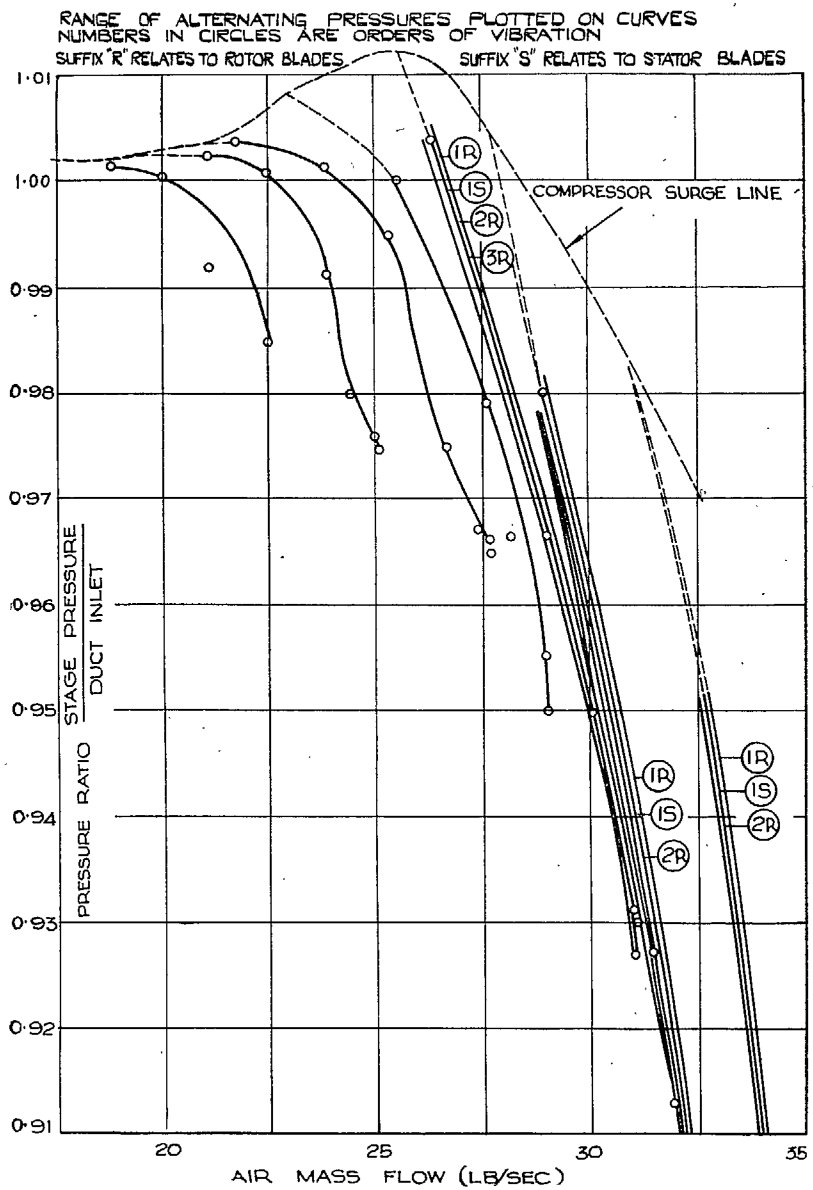



FIG. 13. Alternating pressures in stage 1 plotted on stage characteristics.

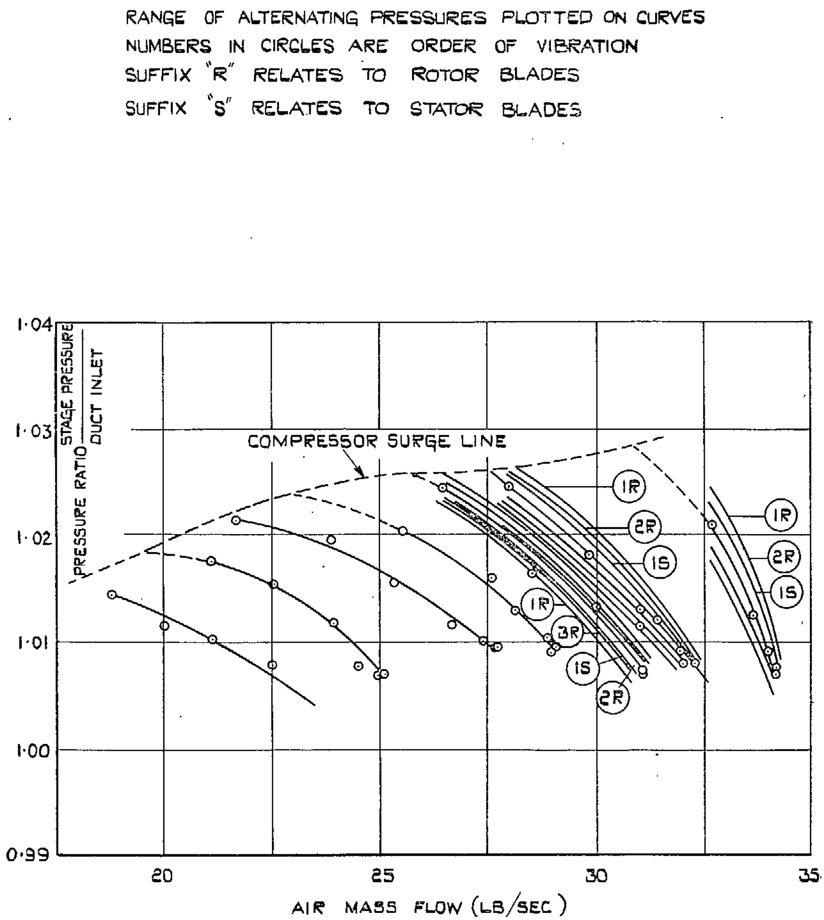



FIG. 14. Alternating pressures in stage 3 plotted on stage characteristics.

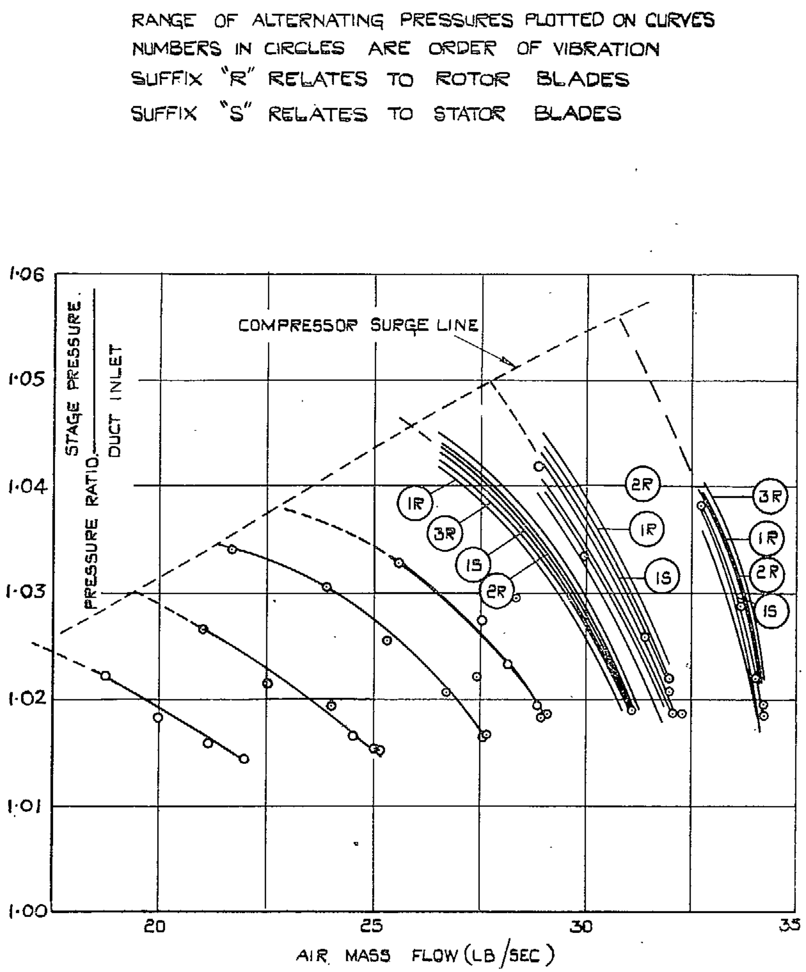



FIG. 15. Alternating pressures in stage 4 plotted on stage characteristics.

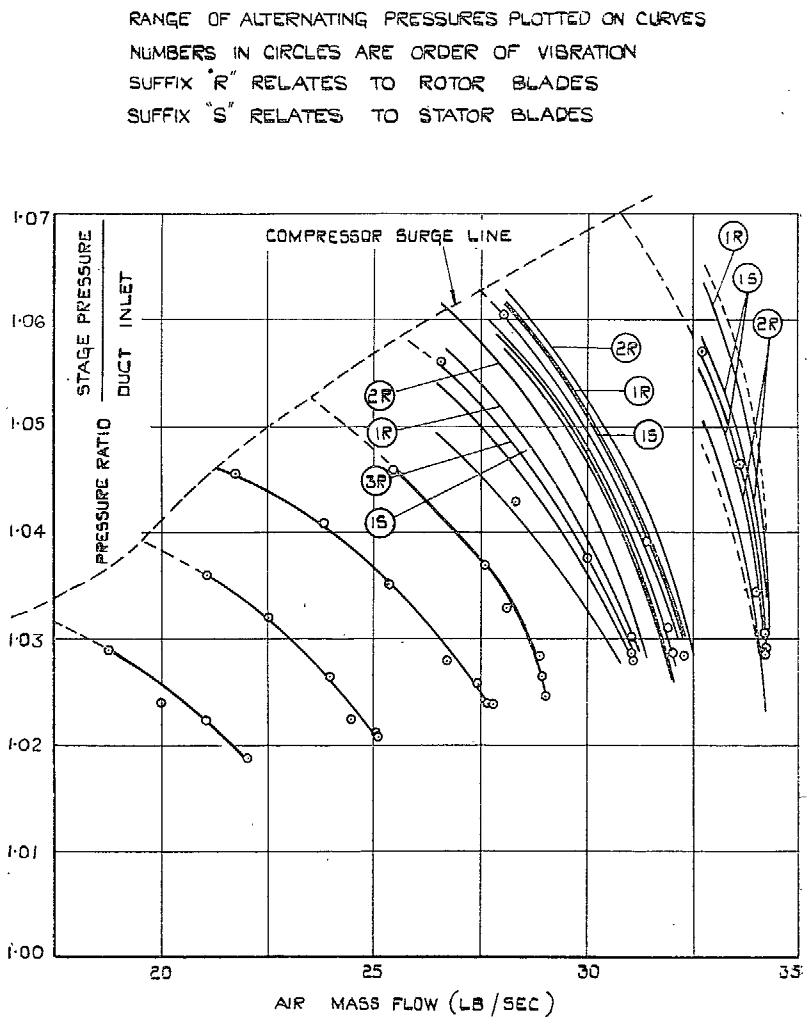



FIG. 16. Alternating pressures in stage 5 plotted on stage characteristics.

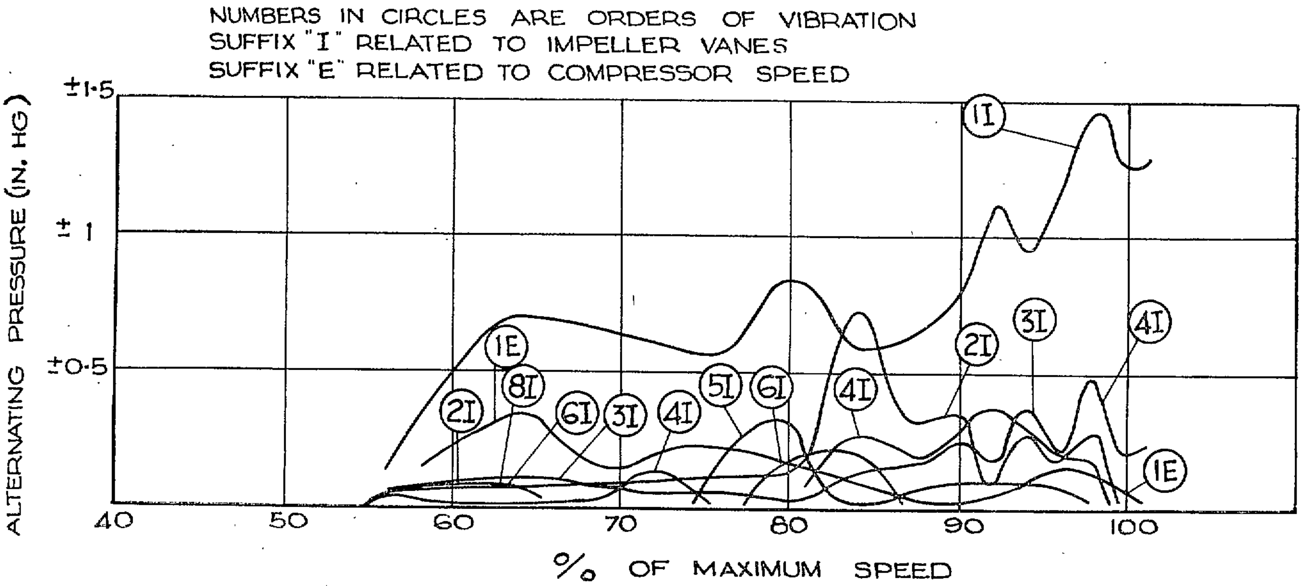



FIG. 17. Alternating pressures at outlet of centrifugal compressor.

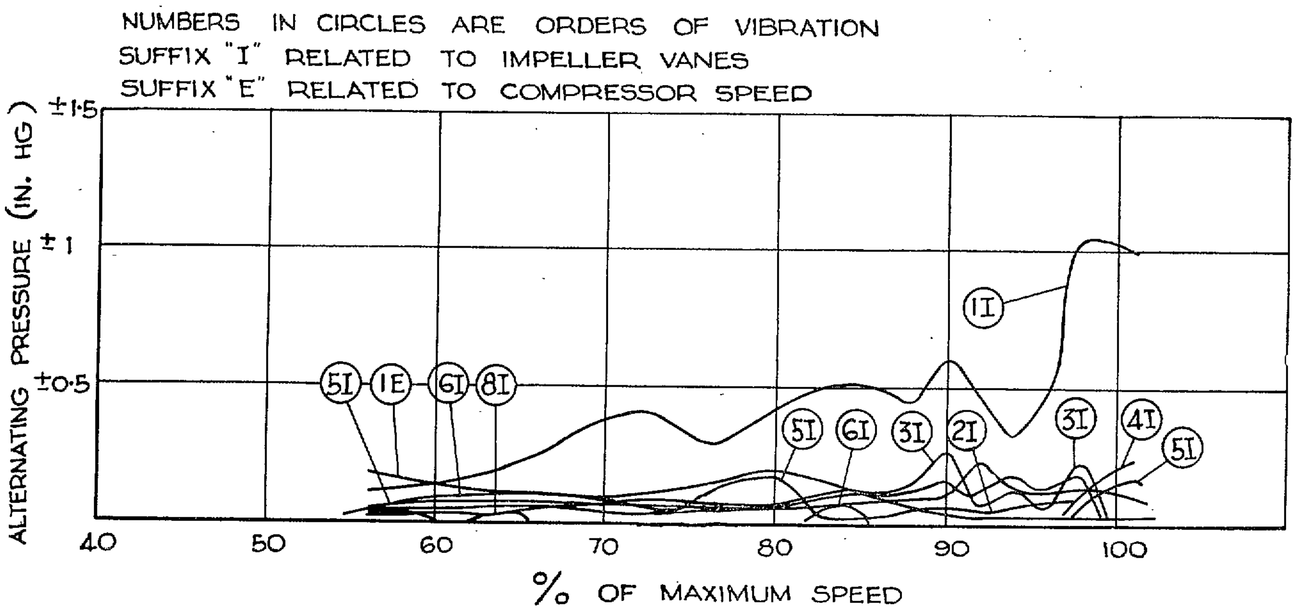



FIG. 18. Alternating pressures in air casing after centrifugal compressor.

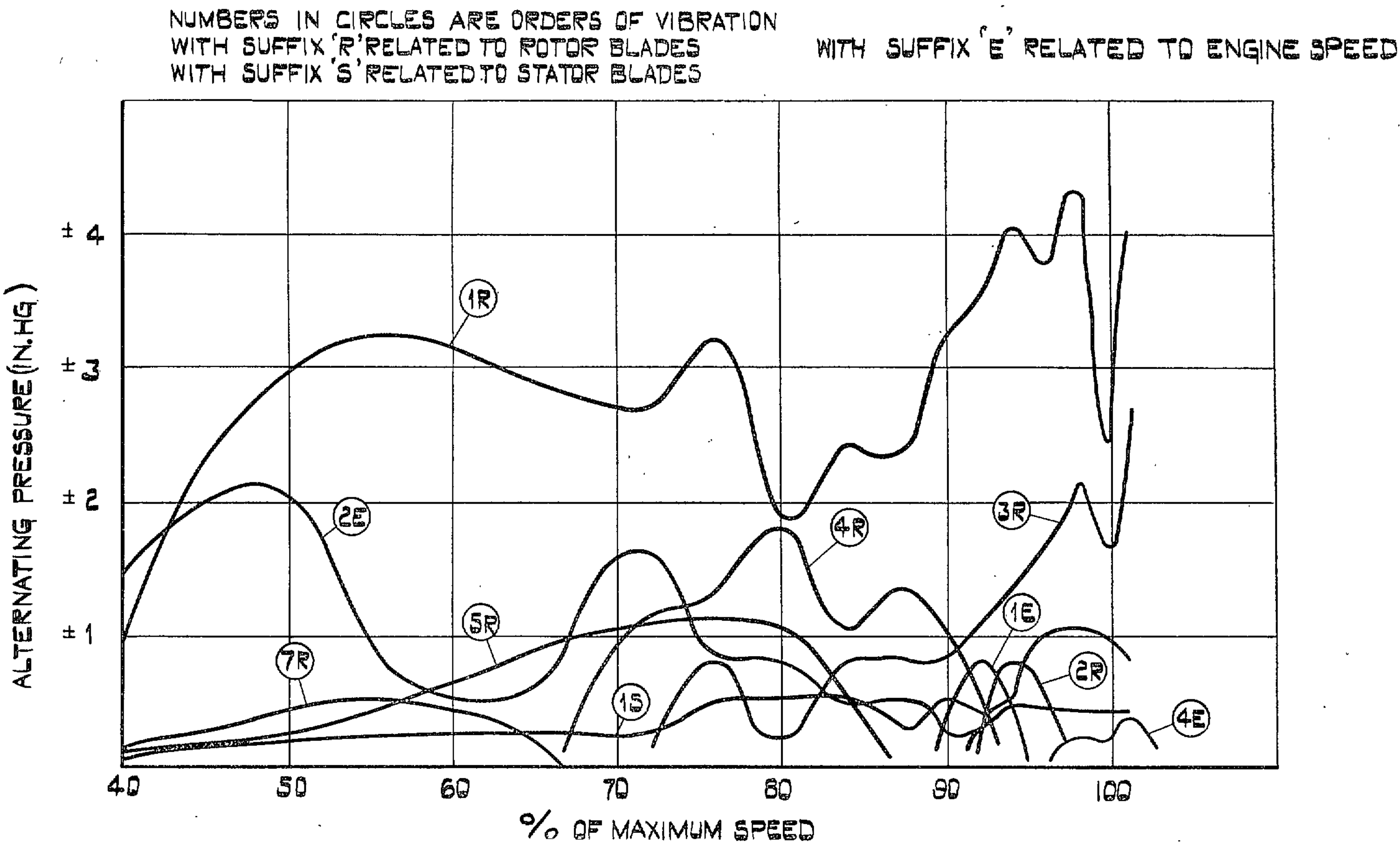



FIG. 19. Harmonic components of alternating pressure in a single-stage turbine.

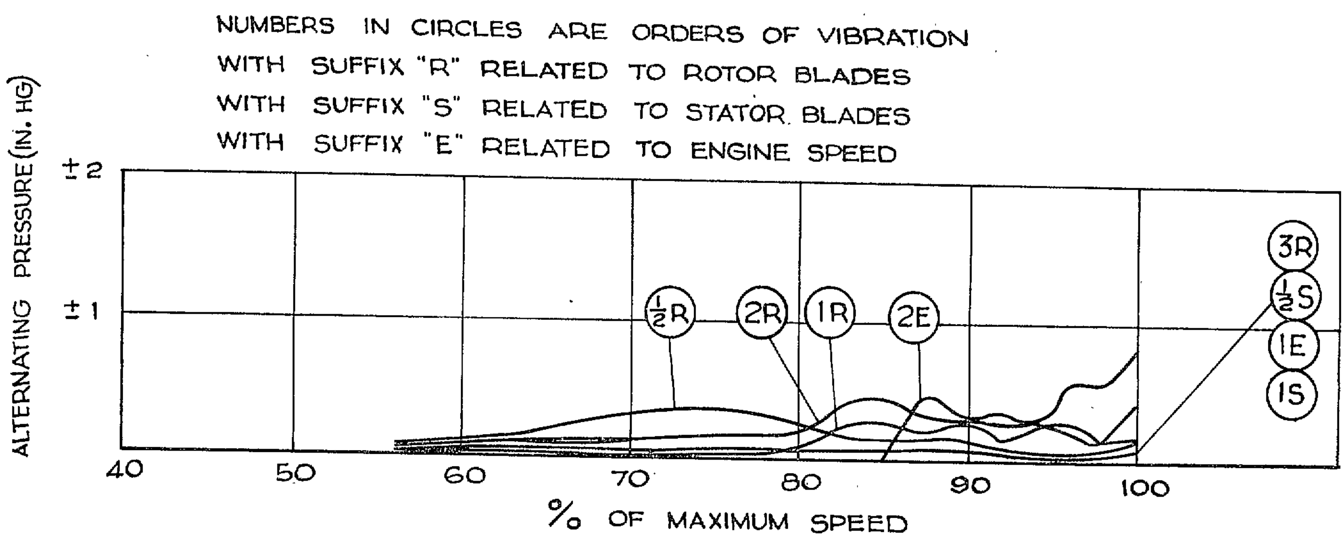



FIG. 20. Alternating pressures in jet pipe after single-stage turbine.

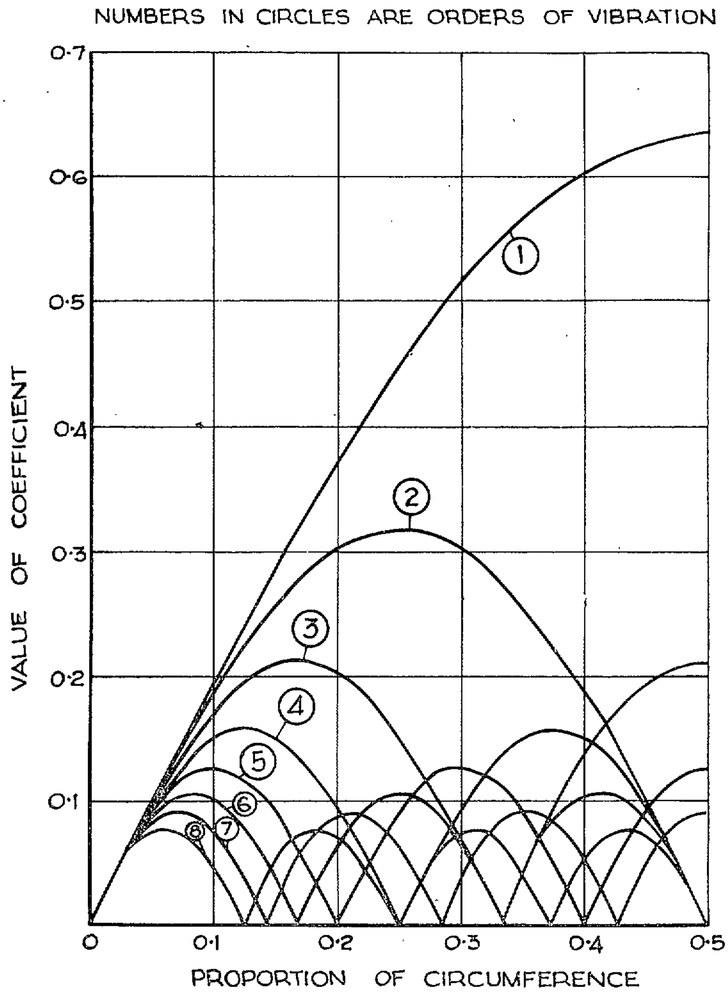



FIG. 21. Harmonic coefficients for various proportions of circumference covered by square wave.

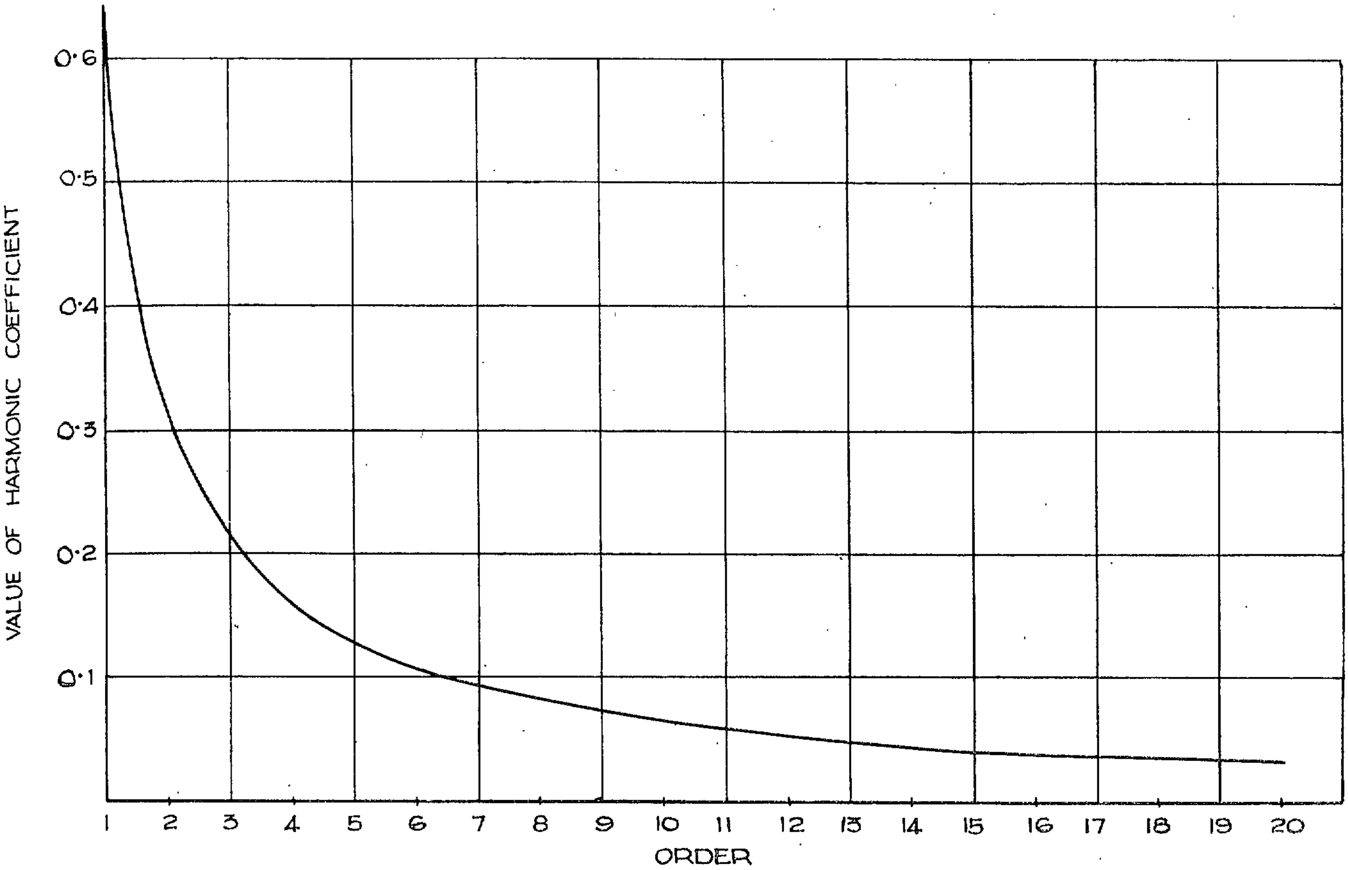



FIG. 22. Maximum values of harmonic coefficients.

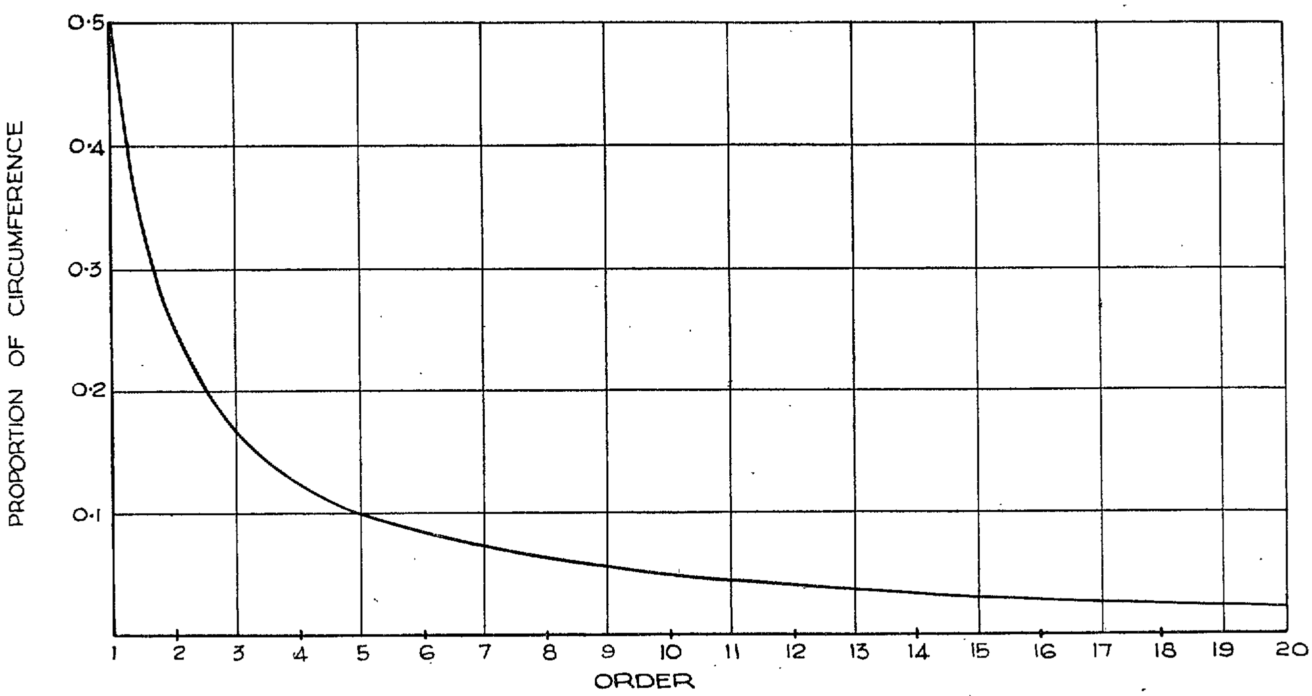



FIG. 23. Proportion of circumference covered by square wave to produce maximum harmonic coefficient.

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