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Local Coefficients of Skin Friction and Heat Transfer for Turbulent Boundary Layers in Two-dimensional Diffusers

By

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Local Coefficients of Skin Friction and Heat Transfer for Turbulent Boundary Layers on Two-dimensional Diffusers

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Summary.—This report considers the changes of local coefficients of skin friction and heat transfer for turbulent boundary layers in two-dimensional subsonic air streams under rising pressure gradients.

The values of the local coefficients of skin friction and heat transfer are compared on the basis of Reynolds' Analogy, von Kármán's improvement to Reynolds' Analogy, and on the basis of similar velocity and temperature boundary-layer profiles.

Tests on diffusers at total angles of 4 deg and 7 deg showed that the local coefficients of heat transfer, based on the measured heat transfer values, increased with increase of the stream velocity and diffuser angle relative to the local coefficients of heat transfer, based on Reynolds' analogy and on von Kármán's improvement to Reynolds' analogy. The local coefficients of heat transfer calculated on an assumed similarity of velocity and temperature distributions were unsatisfactory.

1. *Introduction.*—Several complete sets of measurements taken in diffusers at different angles are necessary to obtain a reliable correlation between the local coefficients of skin friction and heat transfer in streams under various rising pressure gradients.

As skin friction and heat transfer results for zero pressure gradient are available, two further series of tests in diffusers at total angles of 4 deg and 7 deg were undertaken. The results are given in this report.

2. *Diffuser and Wind Tunnel.*—A small open-circuit wind tunnel with a test section of large aspect ratio (30 in. × 1 in. at entrance) was constructed with hinged top and bottom walls permitting different pressure gradients to be obtained. The vertical side walls were parallel and readily detachable to allow access to the inside of the diffuser.

The diffuser was preceded by a parallel section of length $1\frac{1}{8}$ in. and a bellmouth entry of parabolic cross-section. The arrangement is shown in Fig. 1.

For heat transfer experiments, a copper steam-heated plate was fitted along 24 in. of the central section of the bottom wall.

After some minor modifications, and by the use of low-loss gauzes in front of the bellmouth entry and downstream of the diffuser exit, the flow in the diffuser became practically two-dimensional.

The laminar boundary layer in the parallel section before the diffuser had a displacement thickness of approximately 0·010 in. No attempt was made to remove this layer at entry to the diffuser. Transition of the laminar boundary layer to a turbulent condition was accelerated by a sharp downward step of about 0·02 in. at the joint between the parallel section and the test diffuser.

3. *Heat Transfer*.—Steam was used for heating as this method offered heat at a constant temperature (the condensation saturation temperature) provided the steam pressure remained constant. With steam initially dry, the amount of heat supplied was represented by the amount of condensate collected.

The difficulty of determining whether the steam was dry was overcome by using steam with a small degree of superheat. The steam pressure was maintained at about 0·25 lb per sq in. above atmospheric pressure.

3.1. *Steam-Heated Plate and Steam Supply*.—The complication and cost of manufacture of a steam-heated plate precluded the use of a plate covering the complete bottom diverging wall of the diffuser (30 in. wide × 24 in. long). In the final design only the central 10 in. of the bottom wall was heated, a central section of $6\frac{1}{2}$ in. wide being used for heat transfer measurements and the remaining $1\frac{3}{4}$ in. on each side acting as 'guard' compartments. With steam supplied to the 'guard' compartments at the same pressure as the steam supplied to other parts of the heated plate, the heat transferred through the air stream passing over the central section of the plate was considered to flow perpendicular to the plate.

The heater plate was designed to have 12 separate sealed compartments, each compartment having a separate steam supply, condensate drain, static tapping, surface-tube tapping, and pitot-tube access hole.

The heat lost from the back of the steam-heated compartments was reduced by using two $\frac{3}{16}$-in. thick Bakelite impregnated fabric covers with a $\frac{5}{16}$-in. air space between them. The air space between the covers was continued down between the webs dividing the guard compartments from those used for testing.

Manganin-constantan thermocouples incorporated in the heater plate at a number of points were employed to determine the plate temperature. The electrical circuit and galvanometer used permitted temperatures to be measured to within 0·25 deg C. Details of the heater plate are given in Fig. 2.

The steam required for the heater plate was supplied by a small gas-heated steam boiler with separate superheater, the feed water required being fed to the boiler by drip feed from an elevated tank. The boiler safety valve was set to 'blow off' continuously during tests at a pressure of 0·25 lb per sq in. gauge.

The size of the compartments in the heater plate was found to be large enough to permit uninterrupted runs of a quarter of an hour and of one hour at the highest and lowest air speeds respectively used during the tests. The heat losses from the back of the various compartments were deduced from the amounts of steam condensed in each compartment over a four-hour period with the top surface of the plate very effectively insulated.

4. *Measurement of Skin Friction*.—Very simple surface tubes were used for skin friction measurement. The surface tubes were formed by soldering small segments of a thin razor blade over tappings in the heater plate. Each was calibrated in laminar flow in a manner similar to that used by A. Fage and V. M. Falkner[2], by using a duct of width 0·030 in. wide and of large aspect ratio. A typical surface tube and the testing duct are shown in Fig. 3.

The pressure reading obtained from a surface tube and its static tapping is a measure of $(u + \bar{u})^2$ and hence an increase in $\bar{u}$ in the laminar sub-layer under test will produce a slight increase in the measured skin friction. However, other methods of measurement of skin friction appear less satisfactory. It is extremely difficult to obtain satisfactory results for skin friction using the von Kármán momentum equation and the degree of turbulence met in diffusing streams appears to affect the result[3]. Hot-wire anemometry is usually tedious and not well suited to the measurement of velocity in steep temperature gradients.

5. *Test Procedure.*—Measurements were taken only in pressure gradients where separation was unlikely to occur. Diffuser total angle settings of 4 deg and 7 deg were adopted.

The highest and lowest air speeds used in tests were dictated by the maximum tunnel speed and by the smallest acceptable surface-tube manometer reading obtained near the diffuser exit. Two intermediate speeds were also used.

Stainless steel hypodermic tubing with the open end flattened was used for velocity measurement, the tubes being traversed through the air stream by means of a micrometer traversing head. The temperature of the air was measured by a thermocouple soldered to the end of a pitot-tube. Electrical contact of the pitot head with the surface of the heater plate was used to zero the movement.

Skin friction values were obtained using a Chattock gauge and eleven surface tubes attached along the surface of the heater plate. The surface tubes were calibrated at ambient temperature, the calibration being later amended to cover heat transfer conditions.

The amounts of water collected in each compartment of the heater plate were used to determine the local rates of heat transfer along the diffuser, due account being taken of the losses. The kinetic temperature was taken as equal to the total or stagnation temperature as the difference between these two temperatures at the velocities met in these tests was small.

The local coefficients of heat transfer (k_H and k_H'), based respectively on the actual heat transfer and von Kármán's improvement to Reynolds' analogy, were computed from the formulae

$$k_H = \frac{Q}{\rho c_p U(t_w - t_s)}$$

$$k_H' = \frac{c_f/2}{1 + \sqrt{\left(\frac{c_f}{2}\right)}\left[5(P_r - 1) + 5 \log_e \frac{5P_r + 1}{6}\right]}$$

or using a mean value of $P_r = 0{\cdot}70$

$$k_H' = \frac{c_f}{2 - 4{\cdot}30\sqrt{c_f}} .$$

The local coefficients of heat transfer have been compared with the local coefficients of skin friction in Figs. 4 to 11. The local coefficients of heat transfer based on the assumption of similar velocity and temperature profiles were calculated but the agreement with the measured values was found to be poor and the values are not included on the graphs.

6. *Analysis.*—The experimental values, collected by Boelter, Martinelli and Jonassen[1] for the local heat-transfer coefficients k_H and k_H' (based respectively on measured heat transfer values and von Kármán's improvement to Reynolds' analogy) are in reasonable agreement for turbulent flow in parallel pipes.

The present experimental results for heat transfer into diffusing flow indicate that there is considerable difference between the values of the local coefficients of heat transfer k_H and k_H'. This difference cannot be due to experimental 'scatter' as repeat measurements give approximately similar results.

The local coefficient of heat transfer (based on an assumed similarity of the boundary-layer velocity and temperature distributions) showed a rapid and unreal increase along the diffuser; this is considered to be primarily due to the increase in the rate of growth of the actual boundary-layer thickness in diffusing flow without a corresponding increase in the thermal boundary-layer thickness. An assumption of similar velocity and temperature distributions therefore gives excessive values of the coefficient of heat transfer. A typical temperature profile is shown in Fig. 13.

From Figs. 4 to 11 it can be seen that the difference between the local coefficients of heat transfer k_H and k_H' increases with increase of wind speed, distance from the diffuser entrance, and with diffuser angle. It therefore appears that the stability and turbulence of the flow is involved.

Other writers have shown that the values of $\tilde{u}/U$, $\tilde{v}/U$ and $\tilde{w}/U$ in turbulent boundary layers increase in rising pressure gradients with distance from the leading edge until separation occurs. At the point of separation the local coefficient of heat transfer can be considerable although the local coefficient of skin friction is zero.

The equation developed from von Kármán's improvement to Reynolds' analogy is based on a universal velocity profile determined for flow under zero pressure gradient. The present results when plotted (*see* Fig. 12) to the same co-ordinates as the universal velocity profile show that the pressure gradient has an effect on the profile and that the differences between the profiles are functions of the increases in static pressure (*see* also Ref. 10).

Except where the boundary layer was thickest, sufficient readings of the velocity were obtained in the buffer layer between the laminar sub-layer and the turbulent layer to enable only approximate equations for the velocity distribution in this layer to be determined. The values of the local coefficients of heat transfer k_H' based on these amended equations for the buffer layer were found to differ by only a minor extent from the values of k_H' found from von Kármán's relationship.

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LIST OF SYMBOLS

x	Distance along diffuser wall parallel to centre-line
y	Distance perpendicular to diffuser wall
z	Distance perpendicular to x and y
u, v, w	Velocity components in boundary layer in directions x, y, z respectively
$\bar{u}, \bar{v}, \bar{w}$	Turbulent velocity components in boundary layer in directions x, y, z respectively
U_0	Velocity of main stream in parallel section before diffuser
U	Velocity at outer edge of boundary layer
ν	Kinematic viscosity
ρ	Density
δ	Boundary-layer thickness
δ_1	Boundary-layer displacement thickness $= \int_0^\delta \left(1 - \frac{u}{U}\right) dy$
θ	Boundary-layer momentum thickness $= \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$
H	Boundary-layer shape parameter $= \frac{\delta_1}{\theta}$
τ_w	Intensity of skin friction at wall
c_f	Local coefficient of skin friction $= \frac{\tau_w}{\frac{1}{2}\rho U^2}$
u^*	Dimensionless shearing stress velocity $= \frac{u}{\sqrt{(\tau_w/\rho)}}$
y^*	Dimensionless perpendicular distance from wall $= \frac{y\sqrt{(\tau_w/\rho)}}{\nu}$
t	Fluid temperature
t_s	Fluid temperature at outer edge of boundary layer
t_w	Fluid temperature at wall
Q	Heat flow per unit area
c_p	Specific heat at constant pressure
P_r	Prandtl number
k_H	Local heat transfer coefficient $= \frac{Q}{\rho C_p U(t_w - t_s)}$
k_H'	Local heat transfer coefficient (von Kármán's improvement to Reynolds' analogy)

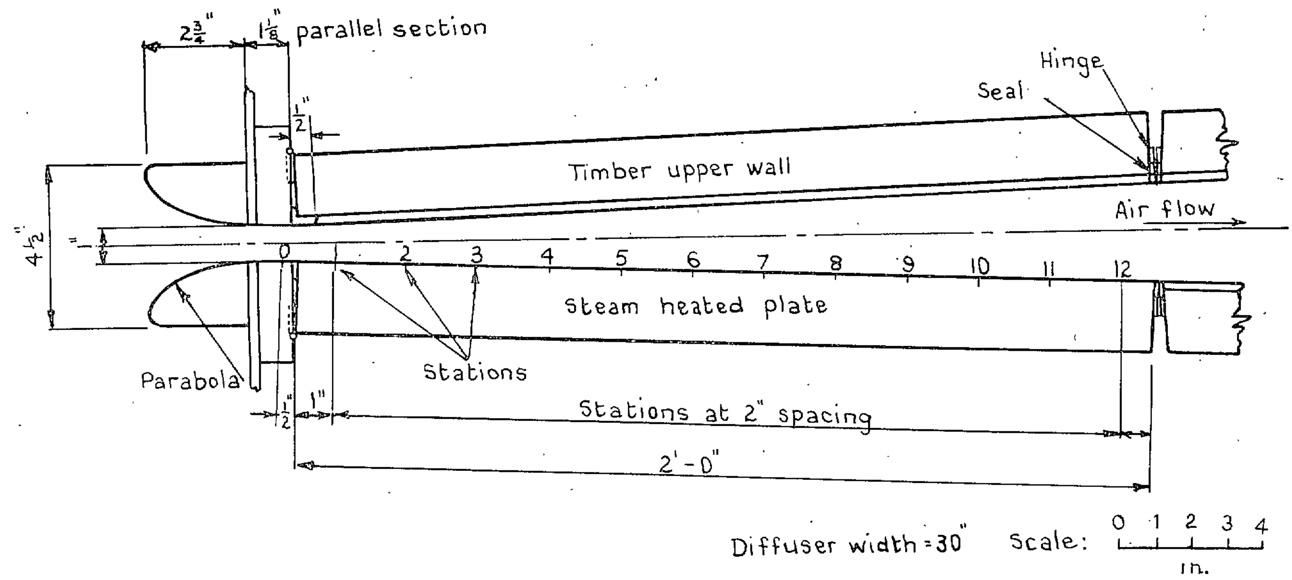

FIG. 1. Details of bellmouth and diffuser.

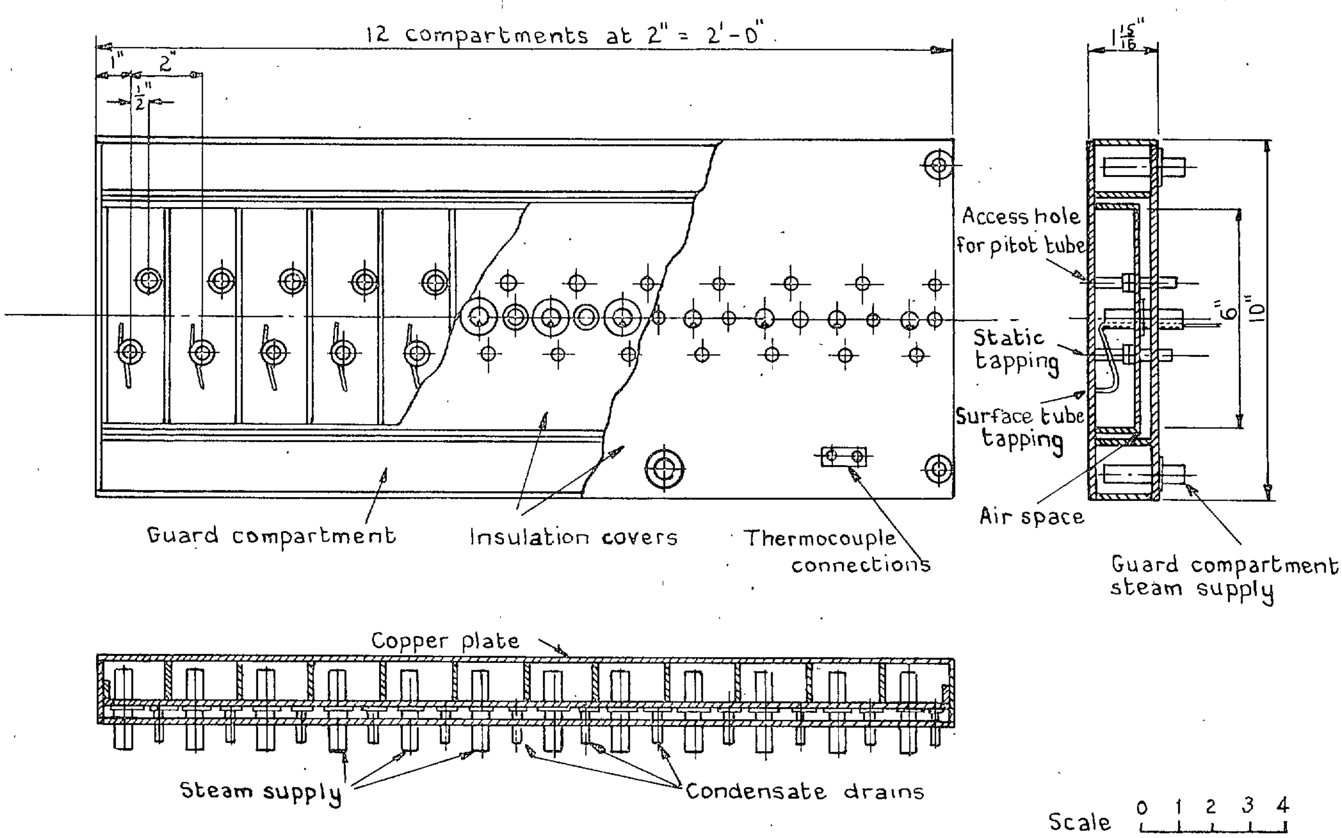

FIG. 2. Details of heated plate.

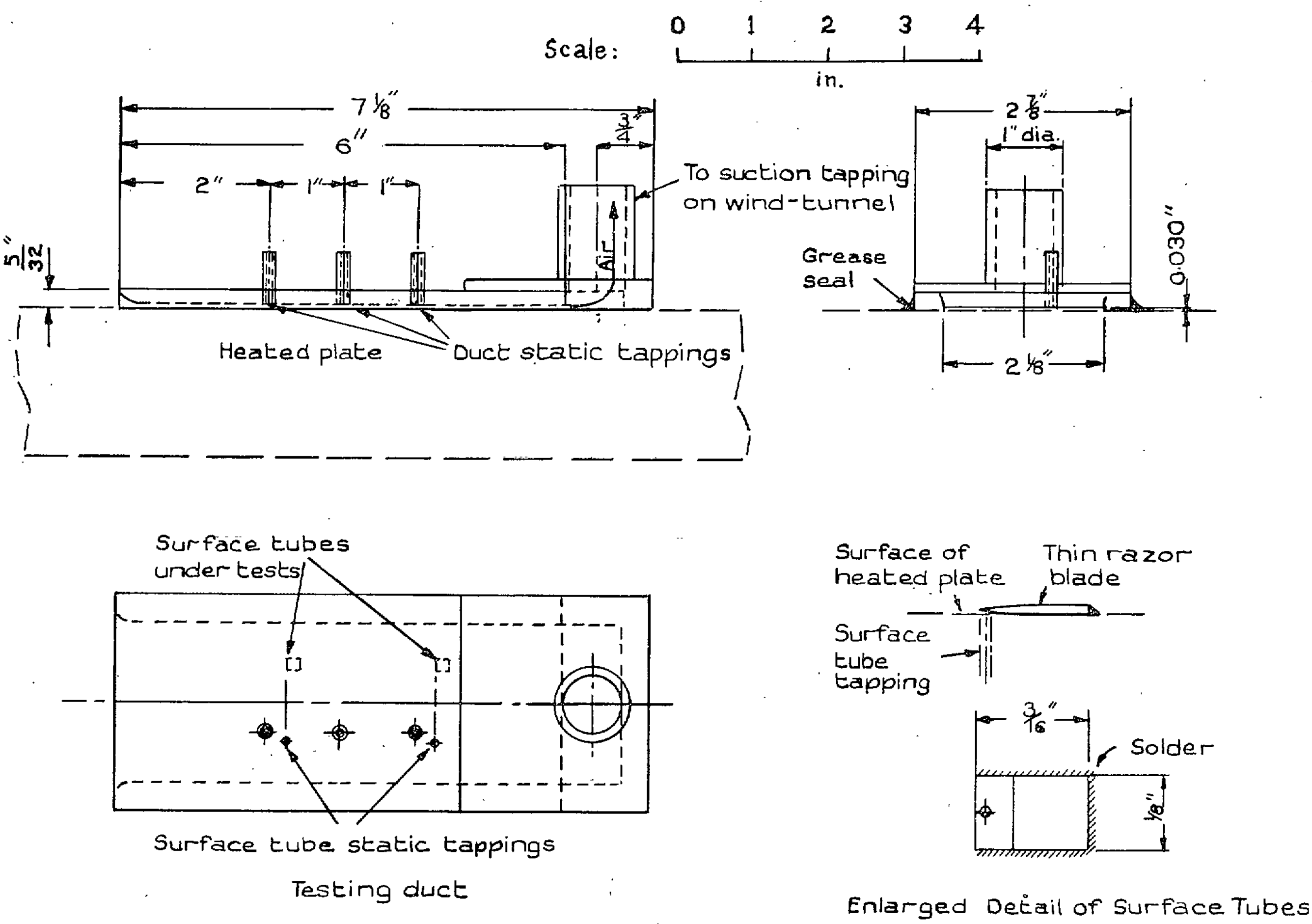

FIG. 3. Details of surface tubes and testing duct.

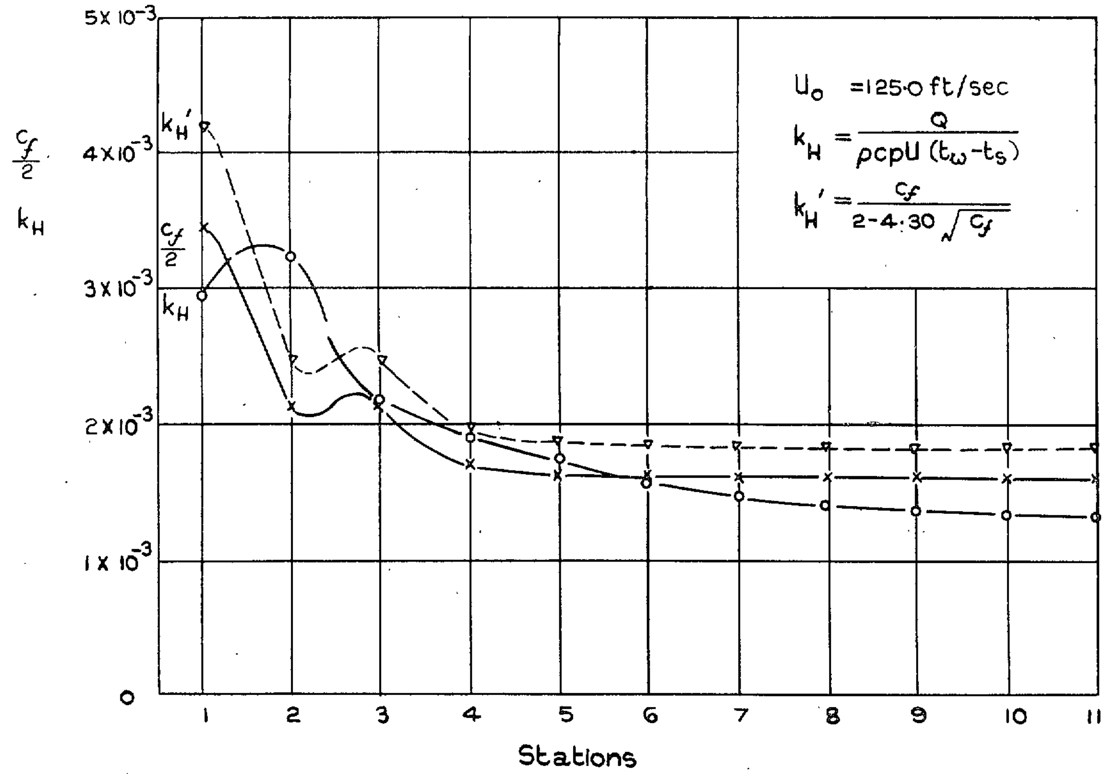

FIG. 4. Variation of c_f (S.T.) and k_H along 4-deg diffuser.

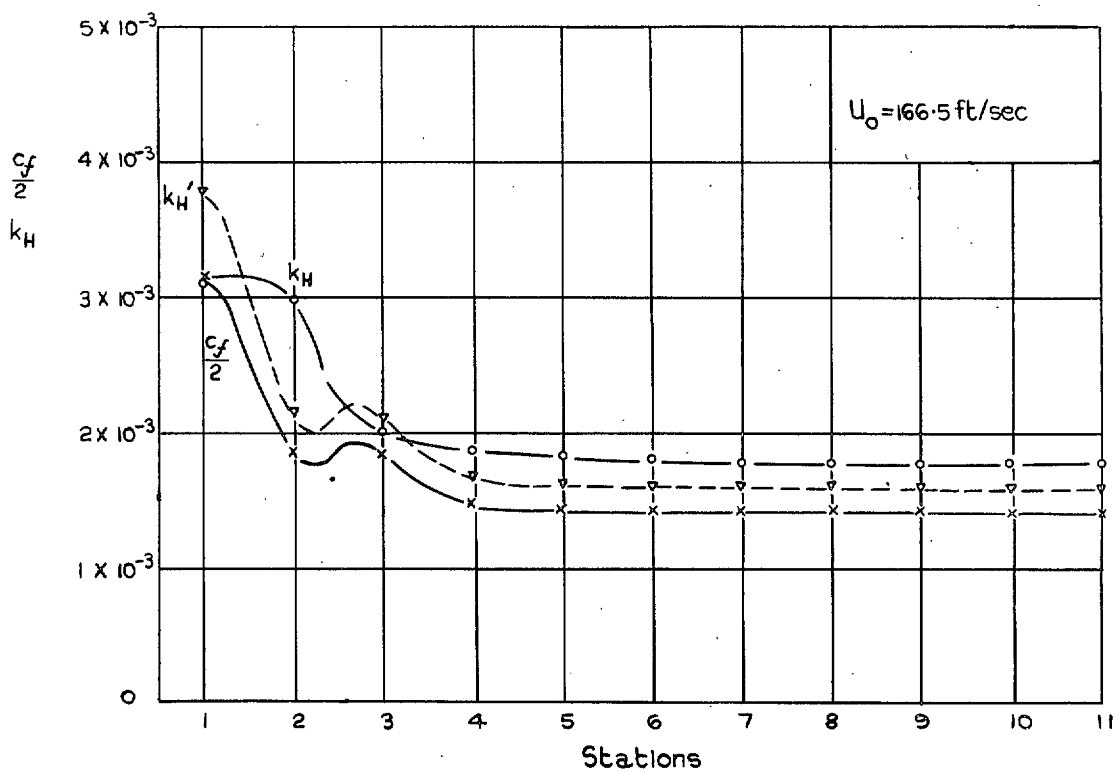

FIG. 5. Variation of c_f (S.T.) and k_H along 4-deg diffuser.

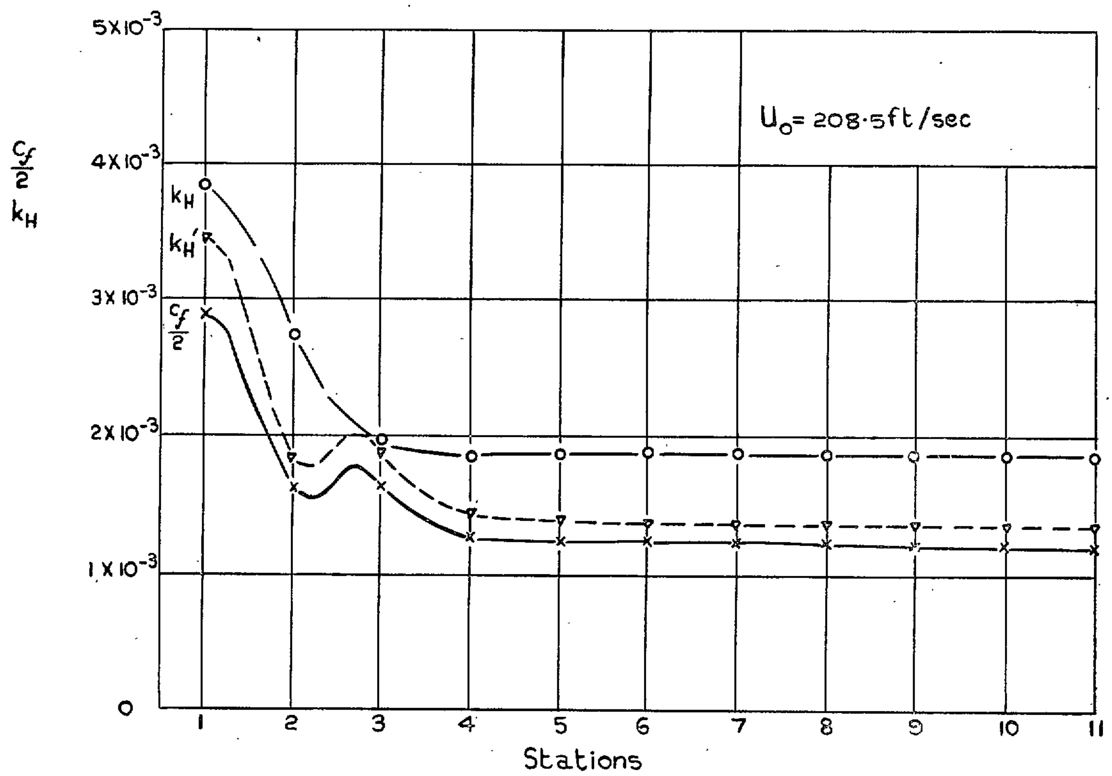

FIG. 6. Variation of c_f (S.T.) and k_H along 4-deg diffuser.

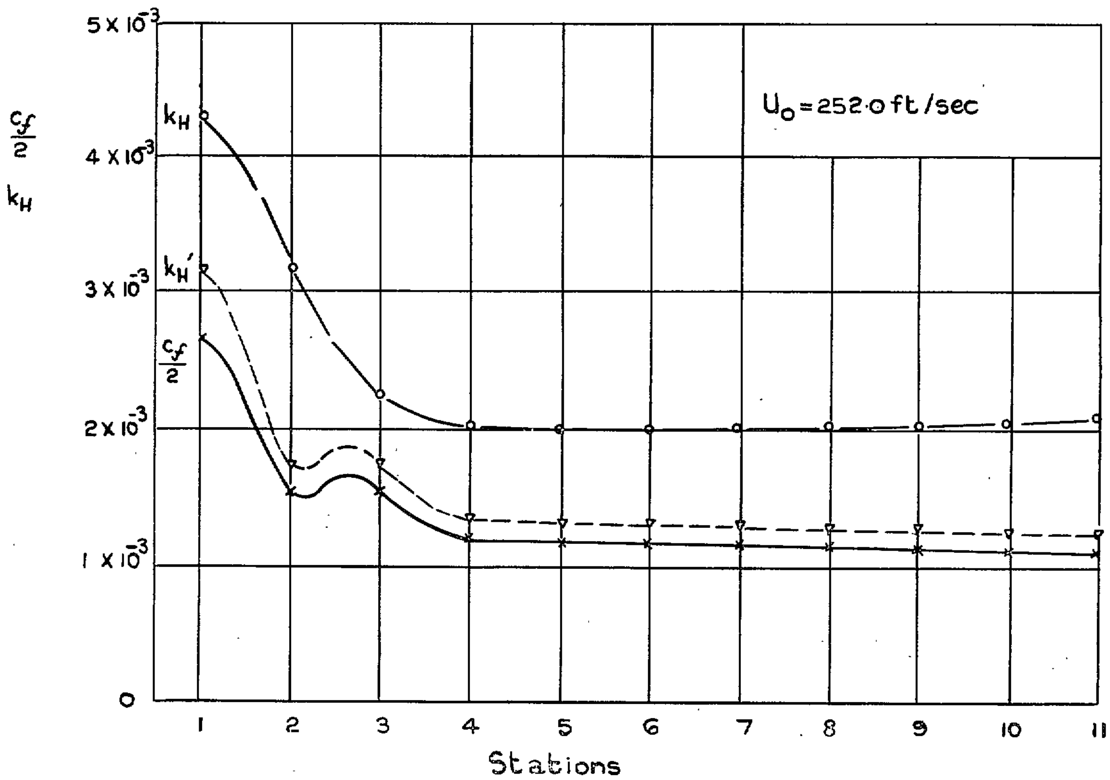

FIG. 7. Variation of c_f (S.T.) and k_H along 4-deg diffuser.

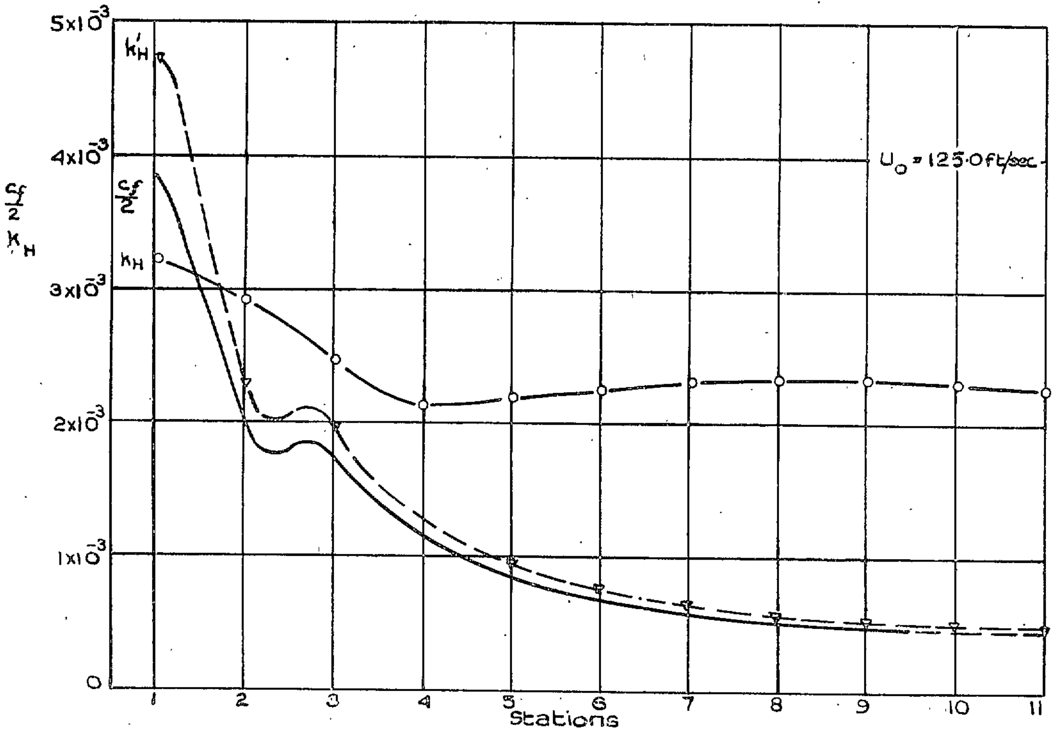

FIG. 8. Comparison of c_f (S.T.) and k_H along 7-deg diffuser.

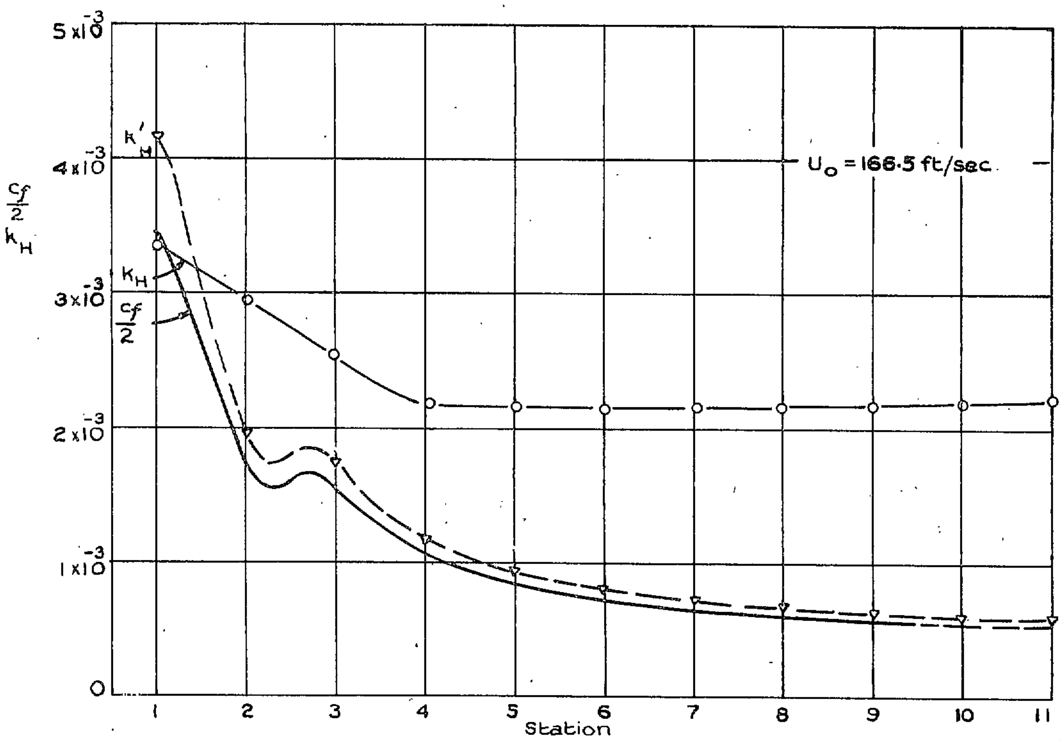

FIG. 9. Comparison of c_f (S.T.) and k_H along 7-deg diffuser.

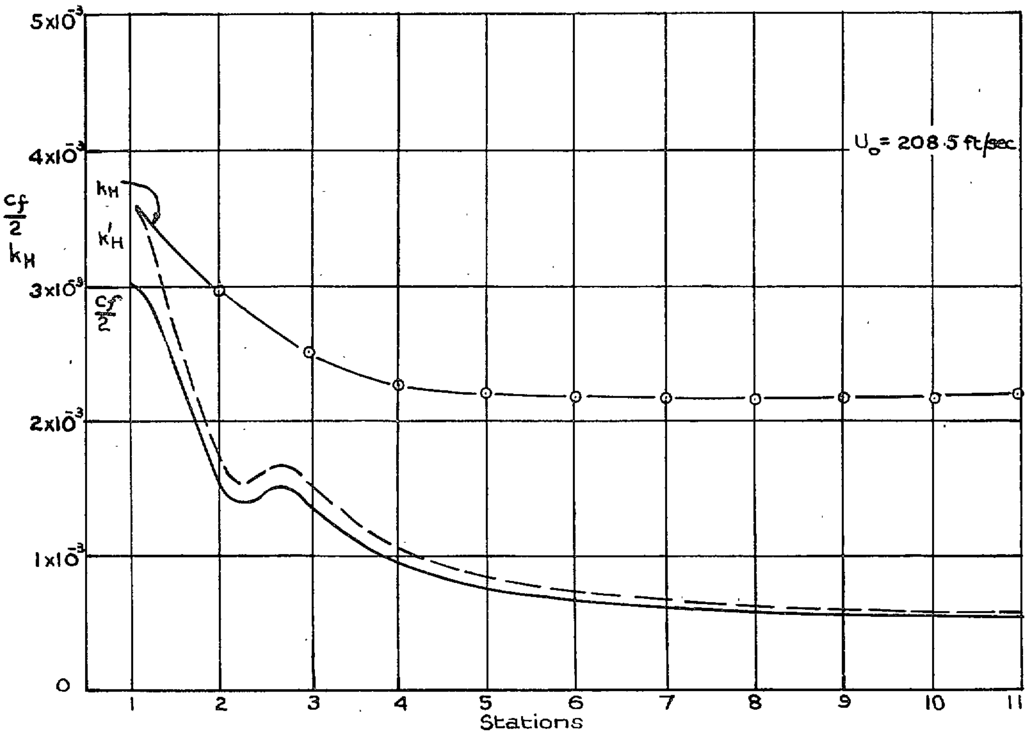

FIG. 10. Comparison of c_f (S.T.) and k_H along 7-deg diffuser.

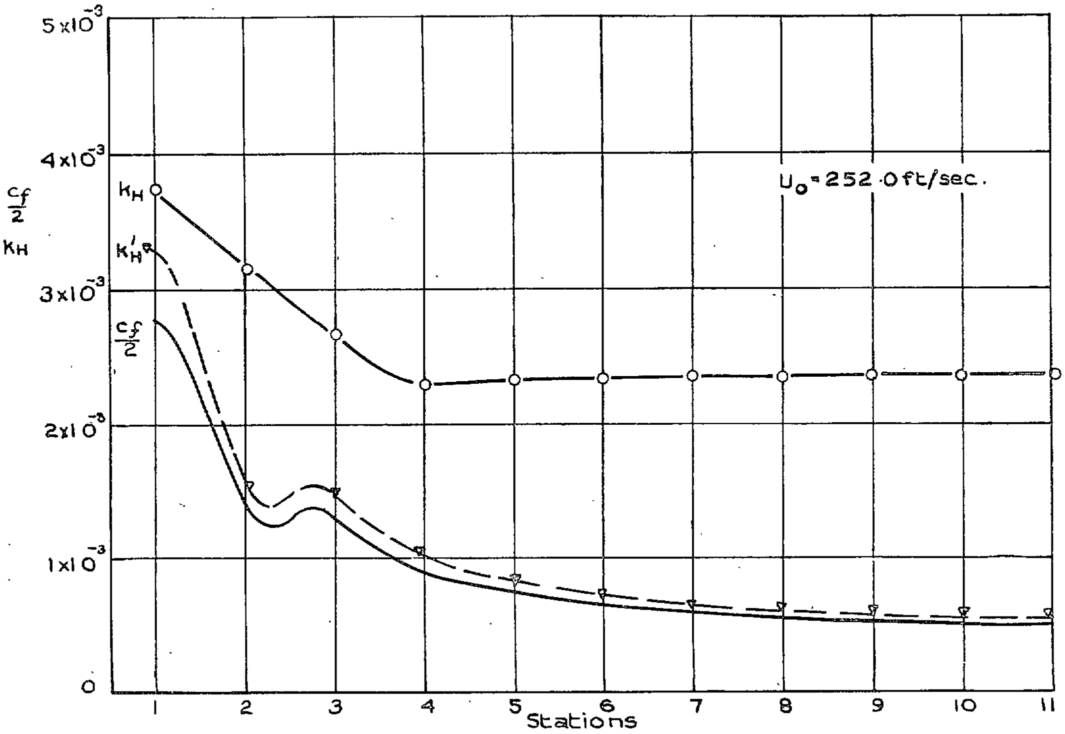

FIG. 11. Comparison of c_f (S.T.) and k_H along 7-deg. diffuser.

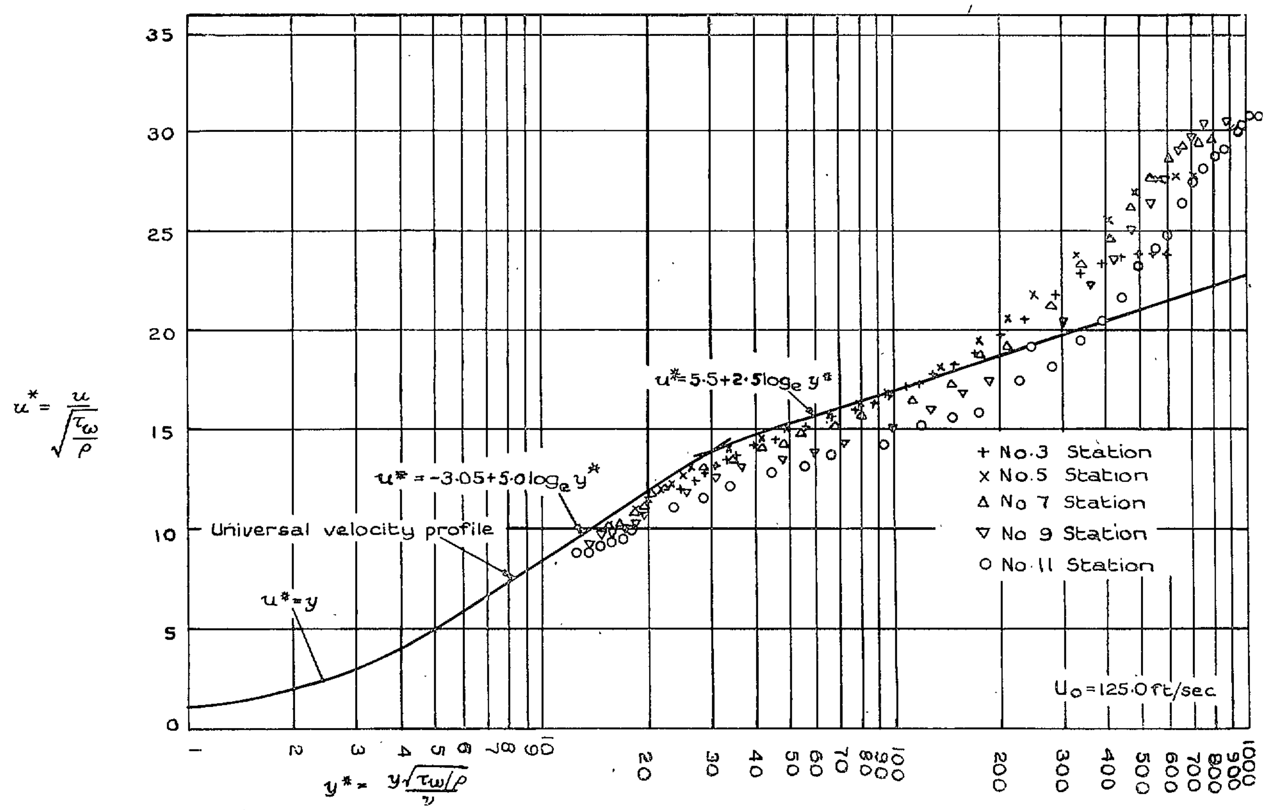

FIG. 12. Non-dimensional velocity profile (4-deg diffuser).

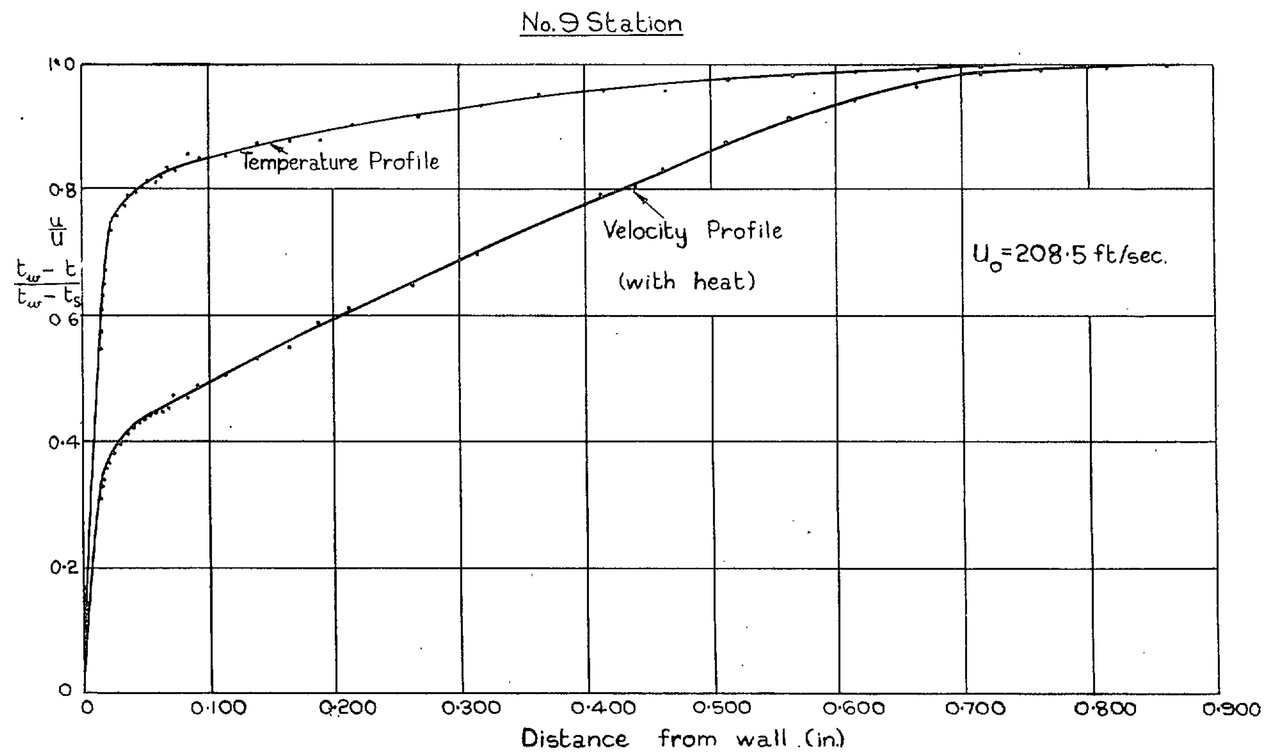

FIG. 13. Typical velocity and temperature profiles (4-deg diffuser).

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