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A Simplified Treatment of a Fixed-Root Swept Wing Built on Hill's Isoclinic Principle*

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Summary.—This note shows how the Hill Aero-isoclinic Principle[1] works out in practice for a swept wing, fixed at the root and having straight flexural and inertia axes. The conditions assumed are readily represented in a wind-tunnel model and experiments by Lambourne[2] show good agreement with theory.

The further aft the flexural axis from the quarter-chord position, the smaller is the sacrifice of wing torsional stiffness entailed in making a swept wing isoclinic, and previous work has on that account taken a far-aft position as a reasonable basic assumption, with the result that, to avoid aero-elastic instability, a well-forward position for the inertia axis is arrived at. It is shown here, however, that *by still further sacrifice of torsional stiffness* (whether practicable or not) it is possible to reduce the gap between the two axes very considerably and so simplify one aspect of the constructional problem. It is expected that this conclusion should still hold, qualitatively at least, for the more representative conditions in which body freedoms are included, as in the earlier work referred to above.

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1. *Introduction.*—A swept wing that makes use of Hill's 'aero-isoclinic' principle is one that bends under lift loads, not about an axis normal to the swept-back line, but about an axis parallel to the flight direction. Such a wing is immune from the loss of incidence, and the consequent shift of the neutral point, that inevitably accompanies bending about an axis normal to the swept-back direction, which, in the absence of special measures taken in the design, is the usual way in which a swept wing does bend.

Hill's notion is to arrange for the increased lift, acting at the quarter-chord, to twist the wing by virtue of an aft location of the flexural axis, thus exactly compensating for the loss of incidence otherwise to be expected. This requires a degree of wing torsional flexibility not conventionally acceptable from the point of view of flutter and aerodynamic divergence.

It follows however that, the further aft the flexural axis can be located, the less need be the loss of torsional stiffness, and this is the reason why, in previous discussions of the aero-elastic properties of an aero-isoclinic wing, a far-back position of the flexural axis was considered a reasonable basic assumption to make.

The purpose of this note is to examine the aero-elastic behaviour of an isoclinic wing under very simplified conditions, so as to arrive at correspondingly simple conclusions, capable of being verified in the wind tunnel, and of providing a basis for physical reasoning and hence a pointer to the likely behaviour of an isoclinic wing under more complicated conditions. With this in view a wing fixed at the root and having straight flexural and inertia axes is considered. A further simplification of the analysis is brought about by omitting damping terms from the equations of motion.

So far as verification in the wind tunnel is concerned, the results obtained by Lambourne[2] show remarkable agreement with theory, but how far it is safe to generalise from this it is difficult

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* See the Introduction for a definition of the term 'isoclinic'.

† R.A.E. Report Structures 101, received 24th April, 1951.

to say. The simple solution however, should indicate qualitatively how such factors as the relative disposition of the various axes govern the aero-elastic behaviour, and should therefore provide a useful guide for subsequent quantitative estimates with all the relevant degrees of freedom included. In particular, any such detailed quantitative estimates should include the effect of body freedoms, which have been omitted.

One conclusion from the present simple approach is that although an aero-isoclinic swept wing has, by definition, no divergence speed in the sense ordinarily understood for a straight wing it has instead what may be called a dynamic divergence speed. The divergence of a straight wing is a quasi-static instability, in that it occurs when the upsetting aerodynamic moment about the flexural axis exactly neutralises the restoring elastic moment: it is thus independent of mass and mass distribution. But for a swept wing, if it is isoclinic, the wing incidence is, by definition, the same from root to tip, so that torsional divergence is ruled out. The wing however is only isoclinic under aerodynamic and elastic forces, and, as soon as inertia forces enter, it loses its isoclinic property and thereby becomes liable to divergence—hence the term 'dynamic divergence'*.

The simple case here treated shows that, while there is a range of speeds over which, in the absence of flutter, dynamic divergence would occur, in actual fact flutter always supervenes before the lowest speed necessary for dynamic divergence can be reached. Moreover, if flutter can by any means be avoided, the same means automatically eliminates divergence.

Another conclusion, which at first sight appears to be at variance with previous recommendations, is that, *if a sufficient sacrifice of torsional stiffness* can be tolerated, it is not necessary, in order to avoid flutter, to have a large gap between the flexural and inertia axes. The gap can be reduced to something like $0 \cdot 1c$, thus greatly simplifying the structural difficulties inseparable from a gap of $0 \cdot 3$ or $0 \cdot 4c$. The reason why a large gap was previously recommended has already been stated above; it is that it was then tacitly assumed that the lever arm between the lift force at the quarter-chord position and the effective fulcrum at the flexural axis should be as long as practicable, in order to minimise the loss of torsional stiffness necessary to make the wing isoclinic. Once a position of $0 \cdot 6$ or $0 \cdot 65c$, for example, is assumed for the flexural axis in this way, the inertia axis position must, if flutter is to be avoided, be well forward of this position. What is now pointed out is that by locating the flexural axis much nearer the quarter-chord position (at of course a corresponding loss of torsional stiffness), the gap between the flexural and inertia axes can be substantially reduced, thus overcoming the constructional difficulties inseparably associated with a large (negative) gap between the two axes.

An endeavour is made to present the results in as general a form as possible. The latter aim is greatly facilitated by the fact that in such a wing the bending and torsional stiffnesses are directly related, and by the further fact that the bending stiffness is itself directly related to the bending strength, which, for a given plan form and thickness-chord ratio, is a known quantity. By these inter-relations it is possible to express the results in terms of two governing parameters r and n alone, where

	r		is the ratio of uncoupled bending to uncoupled torsional frequency
and	n	$=$	V/V_d, where
	V		is the flight speed
	$V_d \cos \beta$		is the divergence speed for the same wing unswept
	β		is the angle of sweepback.

The general case is first considered, and then a practical numerical case to see what degree of wing torsional flexibility is required to make a swept wing isoclinic, and to see further how thin the skin has to be to satisfy that requirement.

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* Dynamic divergence is not peculiar to swept wings for it can, under certain conditions, occur for straight wings in addition to ordinary static divergence.

2. *Elementary General Case* (*Product of Inertia Zero*).—We replace the concept of a wing by a simple dynamically equivalent system, taking care however to represent faithfully the essential features of the wing problem.

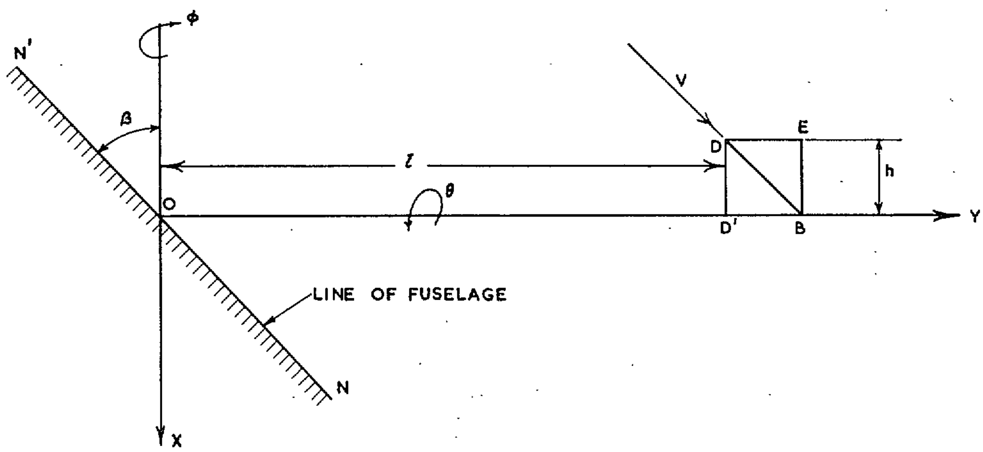

FIG. 1.

Fig. 1 shows in plan-form a strictly equivalent but simplified version of the essential features of the swept-back wing.

O is the point at which the wing flexural axis OB meets the fuselage, which lies along the direction ON parallel to the air stream V. The chordwise section DB in the line of flight (*i.e.*, parallel to V may conveniently be regarded as a reference section where the various equivalent forces act. BE is the wing section at B normal to the flexural axis OB, and the angle DBE (= angle NOX) is thus the angle β of sweepback.

DE lies on the quarter-chord line and we take

l as the distance of D from the axis OX about which the flexural axis bends

h as the distance of D (at quarter-chord) from the flexural axis.

These arbitrary quantities have no significance *per se* for our purpose, as they disappear in the result; they are introduced only to facilitate the analysis.

Physical Characteristics of Simplified Scheme.—A detailed description follows of the physical characteristics of the simplified version of the swept-back wing:

OB is the straight flexural axis hinged at O to enable it to swing in a vertical plane about the axis OX subject to a spring angular constraint.

ϕ is the angular displacement of OB about the axis OX and is positive when B moves down from the plane of the paper.

C_ϕ is the restoring moment of the spring constraint per radian ϕ.

DD′ is a rigid chordwise element which is free to rotate about the axis OY except for an elastic constraining moment.

θ measures the angular displacement of DD′; about OY, positive when D moves up from the plane of the paper.

C_θ is the elastic restoring moment per radian θ.

We assume to begin with that there is no inertial coupling and on this basis let

I_ϕ = moment of inertia of system about axis OX.

I_θ = moment of inertia about axis OY.

The aerodynamic force is determined by the incidence α of the chordwise section DB in the line of flight V, and it is seen that α is defined in terms of the two independent variables θ and ϕ, as

$$\alpha = \theta \cos\beta + \phi \sin\beta, \quad \ldots \quad (1)$$

and if we take $\beta = 45$ deg

$$\alpha = (\theta + \phi)/\sqrt{2}. \quad \ldots \quad (2)$$

For a given forward speed V the upward aerodynamic force F (assumed concentrated at D) may be written as

$$F = K\alpha. \quad \ldots \quad (3)$$

The equations of motion may now be written down in the form

$$\left.\begin{aligned} I_\theta\ddot{\theta} + C_\theta\theta &= K\alpha h = Kh(\theta\cos\beta + \phi\sin\beta) \\ I_\phi\ddot{\phi} + C_\phi\phi + - K\alpha l &= -Kl(\theta\cos\beta + \phi\sin\beta) \end{aligned}\right\}. \quad \ldots \quad (4)$$

We now introduce the isoclinic property of the wing, which requires that a *steady* force F applied at D produces no incidence α of the chord DB. Which means that (from (1))

$$\theta\cos\beta + \phi\sin\beta = 0, \quad \ldots \quad (5)$$

and since

$$\theta = \frac{Fh}{C_\theta} \text{ and } \phi = -\frac{Fl}{C_\phi}, \quad \ldots \quad (6)$$

$$C_\phi = \frac{l}{h}C_\theta \tan\beta \ldots \quad \ldots \quad (7)$$

We also introduce the ratio r, defined, as already mentioned, by

$$r = \frac{\text{uncoupled bending frequency of wing}}{\text{uncoupled torsional frequency of wing}}$$

so that

$$\frac{C_\phi}{I_\phi} = r^2\frac{C_\theta}{I_\theta}. \quad \ldots \quad (8)$$

On substituting (7) and (8) in (4), we now have

$$\left.\begin{aligned} \ddot{\theta} + \frac{C_\theta}{I_\theta}\theta - \frac{Kh}{I_\theta}(\theta\cos\beta + \phi\sin\beta) &= 0 \\ \ddot{\phi} + r^2.\frac{C_\theta}{I_\theta}\phi + \frac{Kl}{I_\phi}(\theta\cos\beta + \phi\sin\beta) &= 0 \end{aligned}\right\} \quad \ldots \quad (9)$$

It is now convenient to relate the aerodynamic force to that required to produce divergence under the condition of a fixed flexural axis (bending prevented—*e.g.*, by infinite inertia). Thus if

$$\frac{V}{V_d} = n,$$

where V_d = divergence speed for swept wing with flexural axis fixed, we have at speed V_d

$$K\alpha h = C_\theta\theta, \quad \ldots \quad (10)$$

and since ϕ is now zero, this gives (by (1))

$$Kh\theta\cos\beta = C_\theta\theta.$$

It follows from the above definition that at speed V

$$Kh\theta\cos\beta = n^2C_\theta\theta$$

$$\text{or } Kh\cos\beta = n^2C_\theta, \quad \ldots \quad (11)$$

so that we can now write (9) in the form

$$\left.\begin{aligned} \ddot{\theta} + p^2\theta - n^2p^2 \sec\beta\,(\theta\cos\beta + \phi\sin\beta) &= 0 \\ \ddot{\phi} + r^2p^2\phi + n^2r^2p^2 \operatorname{cosec}\beta\,(\theta\cos\beta + \phi\sin\beta) &= 0 \end{aligned}\right\} \quad \ldots \quad \ldots \qquad (12)$$

where

$$p^2 = C_\theta/I_\theta = 2\pi f_\theta\,. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (13)$$

Rearranging (12) we get

$$\left.\begin{aligned} \ddot{\theta} + p^2\theta(1 - n^2) - n^2p^2\phi\tan\beta &= 0 \\ \ddot{\phi} + r^2p^2\phi(1 + n^2) + r^2n^2p^2\theta\cot\beta &= 0 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (14)$$

Using the operator D we get

$$\left.\begin{aligned} [D^2 + p^2(1 - n^2)]\theta - n^2p^2\phi\tan\beta &= 0 \\ r^2n^2p^2\theta\cot\beta + [D^2 + r^2p^2(1 + n^2)]\phi &= 0 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \qquad (14a)$$

which, on eliminating ϕ, gives

$$[D^4 + D^2\{r^2p^2(1 + n^2) + p^2(1 - n^2)\} + r^2p^4]\theta = 0\,. \quad \ldots \quad \ldots \quad \ldots \qquad (15)$$

It is of interest to note that this equation is independent of the sweepback angle β, which shows that, *so long as the wing is isoclinic,* and its flexural and inertia axis are coincident, its aero-elastic properties are the same whatever the sweepback angle. It is to be realised however that, in a wing with a very small amount of sweepback and a moderate distance between the quarter-chord and the flexural axis, the problem would be, not how to make the torsional stiffness small enough, but how to make it large enough to render it isoclinic. Under the conditions just described it would be practically impossible to make it large enough.

Putting $\theta = \Theta^{\lambda t}$ in (15) gives the roots of the frequency equation as

$$\lambda^2 = -\frac{p^2}{2}\underbrace{\left\{r^2(1 + n^2) + (1 - n^2)\right\}}_{A} \pm \frac{p^2}{2}\sqrt{\underbrace{\left\{\left[r^2(1 + n^2) + (1 - n^2)\right]^2 - 4r^2\right\}}_{B}} \quad \ldots \qquad (16)$$

and, for convenience of discussion, the quantity inside the first curly bracket is represented by A and the quantity under the radical by B. The salient features of the motion can now be seen.

Divergence.—It is seen at once that since the absolute value of B is necessarily less than that of A, divergence can only take place when A changes sign from the positive value it has at low speeds (*i.e.*, low values of n) to a negative value. In other words the speed (represented by n) at which A becomes zero marks the speed which, *unless flutter has supervened at a still lower speed* produces divergence. Thus we have *potential* divergence as soon as A becomes negative, *i.e.*, when

$$n^2 = \frac{1 + r^2}{1 - r^2}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (17)$$

but, as this value of n must always lie in the flutter speed range of equation (18), the divergence must *remain potential* until the speed is beyond that range. There is therefore no real change in the character of the motion as n^2 passes through the above value.

This confirms the original view that an infinite divergence speed is only realised when bending and torsion keep in step, *i.e.*, when the ratio r of bending to torsional frequency is equal to unity. As soon as the bending frequency falls below the torsional frequency ($r < 1$) the incidence relief due to bending is no longer sufficient to counter the increasing incidence due to wing twist, because the bending action is too slow to keep in step.

An important point to notice, however, is that once the bending frequency becomes greater than the torsional frequency ($r > 1$), the necessary loss of incidence due to bending is always more than punctual enough to offset any increase of incidence due to wing twist, and the wing becomes *immune from divergence at all speeds.*

Flutter.—As it happens, the first instability to occur as the speed (i.e., n) is gradually increased from zero is an oscillating divergence or flutter, because A can never become negative before B, and a change of B from positive to negative marks the onset of flutter irrespective of the sign of A (for we are discussing λ^2 not λ). While B is still negative A next changes sign as n increases, thereby introducing a potential divergence, which only becomes effective however when, with further increase of n, B changes sign a second time to become positive. The range of n values over which B is negative (*i.e.*, the flutter range) is given by

$$\frac{1-r}{1+r} < n^2 < \frac{1+r}{1-r}, \qquad \ldots \qquad (18)$$

which shows that, like divergence, *flutter cannot occur for values of greater than unity.*

Before discussing the significance of this, we may summarise the above remarks concerning the changes in the motion with change of speed n in tabular form.

TABLE 1
(for $r < 1$)

Values of n^2	Values of A	Values of B	Flutter	Divergence
$n^2 < \frac{1-r}{1+r}$	positive	positive	Absent	Absent
$\left(\frac{1-r}{1+r}\right) < n^2 < \left(\frac{1+r^2}{1-r^2}\right)$	positive	negative	Present	Absent
$\left(\frac{1+r^2}{1-r^2}\right) < n^2 < \left(\frac{1+r}{1-r}\right)$	negative	negative	Present	Only potentially present
$n^2 > \frac{1+r}{1-r}$	negative	positive	Absent	Actually present

When $r = 1$ the flutter speed is theoretically zero here, where aerodynamic damping has been omitted. It is known however, that, even with normal damping, the flutter speed is very low usually when the frequencies of the interacting motions—bending and twisting—are equal, so that this result is in accordance with expectations. It is also in accordance with the results of experiments recently carried out by Lambourne[2] at the National Physical Laboratory for the purpose of verifying the simple theoretical approach described in the present report.

As r becomes greater than unity both flutter and divergence speeds suddenly jump to infinity, a phenomenon that is paralleled in Lambourne's experiments by an almost equally abrupt change.

Significance of Case where $r > 1$.—The fact that neither flutter nor divergence can occur when the ratio r is greater than unity opens up a possibility of turning it to good account in an unexpected way. It is known that the principal objection to the isoclinic wing is the low torsional stiffness it must have in order to twist enough under load to offset the loss of incidence due to bending. The objection is not to torsional flexibility *per se*, but to the aero-elastic disabilities it entails. If, however, these disabilities can be made to disappear, the objection vanishes, for even the reduced torsional stiffness is quite adequate for all normal purposes.

Now the shorter the chordwise distance between the flexural axis and the quarter-chord position the greater the torsional flexibilities required to make the wing isoclinic, and hence to a certain extent *the greater the value of r.* Theoretically therefore we can make r anything we please, and if in this way we can increase it beyond unity, no aero-elastic troubles need be feared even under the present highly unfavourable conditions of coincidence of flexural and inertial axes, *i.e.*, of zero mass balance. This matter will be considered again later when numerical values for a concrete case are discussed.

These inferences hold of course only for the case of a wing fixed at the root and zero aerodynamic damping, but they are likely to be a true pointer to the aero-elastic properties under more complex conditions so long as the torsional stiffness is still a major parameter in the motion.

3. *Effect of Inertial Coupling between Bending and Twisting.*—In the above treatment the inertial coupling between torsion and bending was assumed to be zero. As however some degree of mass balance has proved in the past to be a useful deterrent to flutter it is necessary to see how it affects the present problem.

Reverting to Fig. 1, let the inertia axis be located aft of the flexural axis OY so as to make the product of inertia P $(= \Sigma mxy)$ positive.

Equations (9) can now be rewritten with 'product of inertia' terms included as follows:

$$\left.\begin{aligned} \ddot{\theta} + \frac{P}{I_0}\ddot{\phi} + \frac{C_\theta}{I_0}\theta - \frac{Kh}{I}(\theta\cos\beta + \phi\sin\beta) &= 0 \\ \ddot{\phi} + \frac{P}{I_\phi}\ddot{\theta} + \frac{C_\phi}{I_\phi}\phi + \frac{Kl}{I_\phi}(\theta\cos\beta + \phi\sin\beta) &= 0 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

We put $\qquad C_\phi/C_\theta = s$

and $\qquad P = qI_\phi = q_1 I_0$, where, $\quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (20)$

since $\qquad I_\phi = \dfrac{s}{r^2} I_0,\ q_1 = \dfrac{qs}{r^2}$.

Following equations (12), we can now write (19) in the form

$$\left.\begin{aligned} \ddot{\theta} + q_1\ddot{\phi} + p^2\theta - n^2p^2\sec\beta(\theta\cos\beta + \phi\sin\beta) &= 0 \\ \ddot{\phi} + q\ddot{\theta} + r^2p^2\phi + n^2p^2r^2\operatorname{cosec}\beta(\theta\cos\beta + \phi\sin\beta) &= 0 \end{aligned}\right\} \quad \ldots \quad \ldots \quad (19a)$$

or, rearranging and introducing the operator D,

$$\left.\begin{aligned} [D^2 + p^2(1 - n^2)]\theta + [q_1D^2 - (n^2p^2\tan\beta)]\phi &= 0 \\ [qD^2 + r^2n^2p^2\cot\beta]\theta + [D^2 + r^2p^2(1 + n^2)]\phi &= 0 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (19b)$$

which as in (16) leads to the frequency equation

$$\lambda^4(1 - qq_1) + \lambda^2p^2\{(1 - n^2) + r^2(1 + n^2) + n^2(q\tan\beta - q_1r^2\cot\beta)\} + r^2p^4 = 0\,. \qquad (21)$$

The roots are given by

$$\lambda^2 = \frac{-p^2}{2(1 - qq_1)}\overbrace{\left[r^2(1 + n^2) + (1 - n^2) + n^2q(\tan\beta - s\cot\beta)\right]}^{A_1}$$
$$\pm \frac{p^2}{2(1 - qq_1)}\underbrace{\sqrt{\left\{\left[r^2(1 + n^2) + (1 - n^2) + n^2q(\tan\beta - s\cot\beta)\right]^2 - 4r^2(1 - qq_1)\right\}}}_{B_1} \qquad (21a)$$

We note first that the quantity $(1 - qq_1)$ is necessarily positive because

$$(1 - qq_1) = (1 - \frac{s}{r^2}q^2) = \left(1 - \frac{P^2}{I_\theta I_\phi}\right)$$

and $P^2/I_\theta I_\phi$ is necessarily less than unity.

We call

$$P^2/I_\phi I_\theta = \varepsilon^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (22)$$

and use the symbols A_1 and B_1 to represent the quantities shown in (21) marked by horizontal curly brackets.

As in equation (16) it is obvious that B_1 becomes negative before A_1 as n gradually increases, so that flutter again intervenes before dynamic divergence. We also see at once that if the product of inertia qI_ϕ is negative (*i.e.*, inertia axes forward of the flexural axis) a higher value of n than before is required to make B_1 negative. For s ($= C_\phi/C_0$) will be greater than unity (in the range 5 to 10 probably), and therefore the quantity $q(\tan\beta - s\cot\beta)$ will be positive for all practical values of β.

An important point is that in the expression B_1 the effectiveness of a given amount of negative product of inertia (represented by q) is enhanced by the factor $(s\cot\beta - \tan\beta)$ which increases rapidly as the sweepback angle drops to low values. Thus, taking s as 10, we obtain values of 4·1, 9·0 and 27·2 for the above factor, corresponding to β values of 60 deg, 45 deg and 20 deg respectively. A given mass balance therefore becomes very much more effective as the sweep-back angle is reduced. This is no doubt largely due to the fact that the torsional stiffness necessary to render the wing isoclinic is proportional to $\cot\beta$ and becomes very large when β is small.

Reverting to equation (21a), we see that the value of n at which B_1 first becomes zero preparatory to becoming negative is given by the following quadratic equation in n^2.

$$(n^2)^2 - \frac{2n^2(1 + r^2)}{(1 - r^2) + q(s\cot\beta - \tan\beta)} + \frac{(1 - r^2)^2 + 4r^2\varepsilon^2}{\{(1 - r^2) + q(s\cot\beta - \tan\beta)\}^2} = 0 \quad \ldots \qquad (24$$

from which

$$n^2 = \frac{1}{(1 - r^2) + q(s\cot\beta = \tan\beta)}\left[(1 + r^2) \pm 2r\sqrt{(1 - \varepsilon^2)}\right] \quad \ldots \quad \ldots \quad \ldots \qquad (25)$$

a result in agreement with values obtained from the zero-product-of-inertia case if we make $q = 0$.

We are of course interested in the smaller of the two values of n^2 given by (25), *i.e.*,

$$n^2 = \frac{(1 + r^2) - 2r\sqrt{(1 - \varepsilon^2)}}{(1 - r^2) + q(a\cot\beta - \tan\beta)} \cdot \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (26)$$

The net effect of separating the inertia axis from the flexural axis has been to introduce the factor $\sqrt{(1 - \varepsilon^2)}$ in the numerator of (26) and the term $q(s\cot\beta - \tan\beta)$ in the denominator. Both changes, for a negative value of q, make for an increased value of n. The quantitative significance of introducing various amounts of product of inertia is probably best studied from a concrete example of a simplified wing and this is done in section 4.5.

We note however, that since in (26) the numerator is essentially positive, there cannot be a critical speed if the denominator is negative; and as the value of s in practice is not less than about 15, quite a small amount of negative product of inertia q is sufficient to make $q(s\cot\beta - \tan\beta) >$ $(1 - r^2)$ numerically. This point is pursued further in section 4.5.

4. *Practical Numerical Example.*—The following numerical example has been worked out in order to determine, for a swept wing of typical plan-form and thickness/chord ratio, the degree of

torsional flexibility necessary to produce enough aerodynamic incidence by wing twist to offset that lost due to bending of the wing. All the physical quantities involved are kept general and numerical values are considered only in the final formulae obtained.

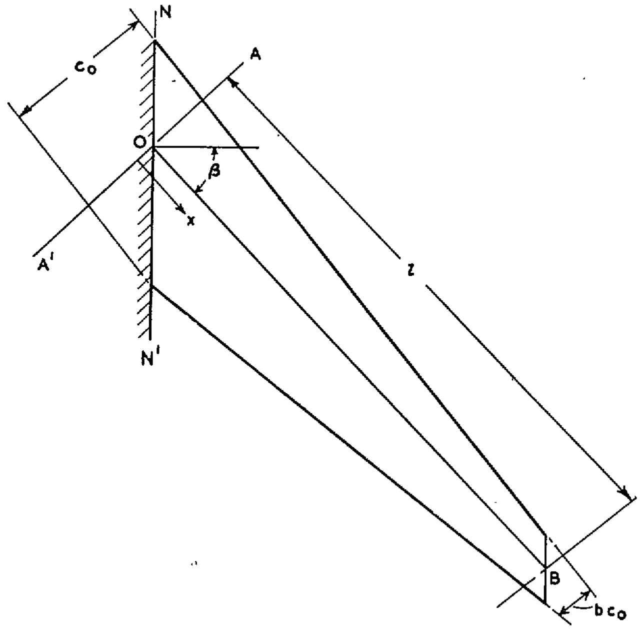

FIG. 2.

Consider a linearly tapered wing swept back through an angle β as shown in Fig. 2, where NN is the fuselage line representing the direction of flight, OB is the flexural axis,

c_0 is the root chord normal to OB

bc_0 is the tip chord normal to OB

2γ is the thickness/chord ratio

l is the length OB of wing along sweepback

x is the distance from wing root along OB

c is the chord at any section x (normal to wing axis).

The simplest approach is to assume that the wing is designed for maximum structural efficiency so that the bending stress produced by the lift forces is uniform from root to tip. Let this stress at full load-factor ($4g$ say) be

$$\sigma = \text{maximum working stress for Dural.} \qquad (27)$$

Under $1g$ level flight conditions (wing loading w_1 lb/ft²) the uniform stress becomes

$$\sigma_1 = \sigma/4 \,. \qquad (28)$$

The curvature of the wing is then given by

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{Mz}{I} \cdot \frac{1}{zE} = \frac{\sigma_1}{zE} \qquad (29)$$

where z = greatest height above the neutral axis

$= \gamma c$.

Now

$$c = c_0\{1 - (1 - b)x/l\} \qquad (30)$$

therefore $$z = \gamma c_0\{1 - (1 - b)x/l\} \,. \qquad (31)$$

From (29) therefore:

$$\text{curvature} \quad \frac{d^2y}{dx^2} = \frac{1}{E\gamma c_0} \Big/ \left\{1 - (1 - b)x/l\right\}, \qquad \qquad (32)$$

$$\text{slope} \quad \frac{dy}{dx} = \frac{\sigma 1}{E\gamma c_0} \int_0^x \frac{1}{1 - (1 - b)x/l} \, dx$$

$$= \frac{\sigma_1}{E\gamma c_0} \cdot \frac{l}{(b - 1)} \left[\log_e\left\{1 - (1 - b)\frac{x}{l}\right\}\right]. \qquad \qquad (33)$$

This represents the variation of wing slope along x if the wing bends under uniform incidence from root to tip, *i.e.*, under isoclinic conditions.

If there is no twist of the wing about its own flexural axis, the slope just found produces a loss of incidence of

$$\alpha_1 = \frac{dy}{dx} \sin \beta .$$

Wing twist produces an increase of incidence of

$$\alpha_2 = \theta \cos \beta ,$$

so that if the net effective change of incidence is to be zero

$$\alpha_1 = \alpha_2$$

or

$$\theta = \frac{dy}{dx} \tan \beta \qquad \qquad (34)$$

and

$$\frac{d\theta}{dx} = \frac{d^2y}{dx^2} \tan \beta = \frac{\sigma_1 \tan \beta}{E\gamma c_0} \Big/ \left\{1 - (1 - b)\frac{x}{l}\right\}. \qquad \qquad (35)$$

Corresponding to this spanwise rate of variation of twist we can now, for any position of the flexural axis relative to the quarter-chord, find the necessary spanwise distribution of torsional stiffness.

4.1. *Torsional Stiffness Required to make Wing Isoclinic.*—Let the flexural axis be situated a distance hc aft of the quarter-chord, so that

$$\left.\begin{array}{l}\text{flexural axis position relative} \\ \text{to leading edge}\end{array}\right\} = (h + \tfrac{1}{4})c . \qquad \qquad (36)$$

Torque dT over element dx of wing span from aerodynamic lift load is given by

$$dT = w_1(c\, dx)hc = w_1 hc^2 \, dx . \qquad \qquad (37)$$

Total torque at x is

$$T_x = \int_x^l w_1 hc^2 \, dx = w_1 hc_0^2 \int_x^l \left(\frac{c}{c_0}\right)^2 dx$$

$$= w_1 hc_0^2 \int_x^l \left\{1 - \frac{x}{l}(1 - b)\right\}^2 dx$$

$$= \frac{l w_1 hc_0^2}{3(b - 1)} \left[b^3 - \left\{1 - \frac{x}{l}(1 - b)\right\}^3\right]. \qquad \qquad (38)$$

The torsional stiffness of the wing at x (*i.e.*, the torque Qx to give one radian twist per unit span) must be such that, under the torque T_x, a rate of change of twist given by (35) is obtained. Thus

$$T_x/Q_x = \frac{d\theta}{dx} = \frac{\sigma_1 \tan \beta}{E\gamma c_0} \left\{\frac{1}{1 - (1 - b)x/l}\right\} \qquad \qquad (39)$$

and therefore the torsional stiffness at any section x is given by

$$Q_x = \frac{Tx}{d\theta/ox} = \frac{\dfrac{lw_1hc_0{}^2}{3(b-1)}\left[b^3 - \left\{1 - \dfrac{x}{l}(1-b)\right\}^3\right]}{\dfrac{\sigma_1 \tan\beta}{E\gamma c_0}\left\{\dfrac{1}{1-(x/l)(1-b)}\right\}} \quad \ldots \quad (40)$$

or

$$Q_x = \frac{E\gamma lw_1hc_0{}^3}{3(b-1)\sigma_1 \tan\beta}\left[b^3 - \left\{1 - \frac{x}{l}(1-b)\right\}^3\right]\left\{1 - \frac{x}{l}(1-b)\right\}. \quad \ldots \quad (40a)$$

4.2. *Corresponding Wing Thickness.*—But, in terms of the skin thickness and the effective area and perimeter of the torsion cell formed by the wing cross-section, we can express the torsional stiffness as

$$Q_x = \frac{4A^2Gt}{\varrho} \quad \ldots \quad (41)$$

where A is the effective area enclosed by torsion cell

$= 0 \cdot 6c \times 0 \cdot 1c$ (say) approximately

$= 0 \cdot 06c^2$,

ϱ is the perimeter of cell

$= 1 \cdot 2c$ (say),

t is the thickness of cell wall

= skin thickness (approx.).

Substituting these approximate values, we get from (41)

$$Q_x = 0 \cdot 012c^3Gt = 0 \cdot 012c_0{}^3Gt\left(\frac{c}{c_0}\right)^3 \quad \ldots \quad (42)$$

which, on being equated to (40a) gives

$$t = 2 \cdot 32\frac{w_1}{\sigma_1} \cdot \frac{E}{G} \cdot \frac{h\gamma l}{\tan\beta}\left[\left\{\frac{1}{(1-b)} - \frac{x}{l}\right\} - \frac{b^3/(1-b)}{\{1-(x/l)(1-b)\}^2}\right] \quad \ldots \quad (43)$$

where t is given in inches when

$w_1 = \text{lb/ft.}^2$

$\sigma_1 = \text{lb/in.}^2$

$l = \text{ft.}$

From the general formula of (43) it is noteworthy that the *skin thickness is independent of aspect ratio.* We see also that the skin thickness varies inversely as $\tan\beta$ which means that, as between sweepbacks of 45-deg and 30-deg the skin thickness required to make the wing isoclinic for a 30-deg sweepback is 75 per cent greater than that for 45-deg sweepback.

We also see that, for a wing untapered in plan, formula (43) gives

$$t = 2 \cdot 32\frac{w_1}{\sigma_1} \cdot \frac{E}{G} \cdot \frac{h\gamma l}{\tan\beta}\left(3 - \frac{x}{l}\right), \quad \ldots \quad (43a)$$

which shows that the variation of skin thickness necessary for attaining the correct distribution of torsional stiffness is very different from that necessary for bending strength.

4.3. *Numerical Values.*—We can now take some numerical values to see the kind of wing that formula (43) leads to.

Reasonable values are

$$\left.\begin{aligned} w_1 &= 50 \text{ lb/ft}^2 \\ &= 7{,}000 \text{ lb/in.}^2 \\ E/G &= 2\cdot5 \\ 2\gamma &= 0\cdot13 \\ l &= 90 \text{ ft} \\ h &= 0\cdot4 \\ b &= 0\cdot2 \end{aligned}\right\} \quad \ldots \qquad (44)$$

Substituting these values in (43) we obtain the curve of Fig. 3, which shows the variation of skin thickness from root to tip. The shape of the curve is dependent only on the plan taper of the wing, so that for a value of $0\cdot2$ for b (which is quite typical) it gives the thickness everywhere along the span in terms the root skin thickness.

$$\begin{aligned} t_{\text{root}} &= 2\cdot32 \frac{w_1}{\sigma_1} \cdot \frac{E}{G} \cdot \frac{h\gamma l}{\tan\beta}(1 + b + b^2) \\ &= 2\cdot88 \frac{w_1}{\sigma_1} \cdot \frac{E}{G} \cdot \frac{h\gamma l}{\tan\beta} \cdot \quad \ldots \qquad (45) \end{aligned}$$

For the numerical values of (44) this gives

$$t_{\text{root}} = 0\cdot12 \text{ in.}$$

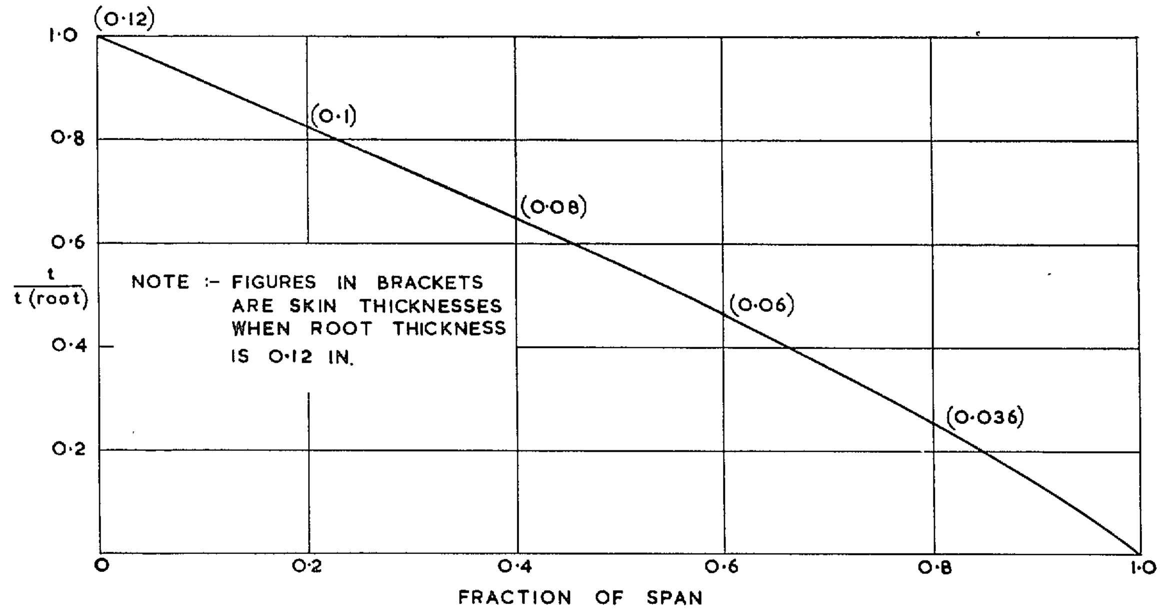

FIG. 3. Variation of skin thickness along span for taper ratio $b = 0\cdot2$ in plan.

Looking at (45) we see that most of the quantities concerned are somewhat rigidly circumscribed; we cannot vary w_1, σ_1, E/G, γ or l much, so that we are left with h and $\tan\beta$, and if further, the sweepback is fixed, only the distance h between the flexural axis and the quarter-chord position is at our disposal. By reducing this distance we can, theoretically make the torsional stiffness as measured by the skin thickness t_1 as small as we like. The consequent forward shift of the flexural axis reduces constructional difficulties but the accompanying very low torsional stiffness increases them.

Again theoretically, even for zero product of inertia, we can make h small enough to reduce the torsional stiffness to a degree that would make r, the ratio of bending to torsional frequency, greater than unity, when the wing becomes immune from both flutter and divergence. The difficulty here is that as r increases from something less than unity to unity the flutter speed progressively drops and only becomes infinite if r is greater than unity. In this particular case therefore, *i.e.*, with zero product of inertia, the choice is either to make h as large as possible or as small as possible.

4.4. *Numerical Value of Frequency Ratio r.*—While on the subject of the scope for varying the frequency ratio r it may be useful to investigate the limits this is likely to have in practice as indicated by the present example. The details of the calculation are not given and only the results are discussed here.

The flexural frequency is found to be 4 c.p.s. and we may reasonably assume that this is independent of skin thickness and dependent on the bending strength alone. The torsional frequency, however, as already discussed above, is determined by the position of the flexural axis relative to the quarter-chord. If we keep the flexural axis at $0 \cdot 65c$, *i.e.*, $0 \cdot 4c$ aft of the quarter-chord, we obtain the skin thickness distribution of Fig. 3. If, further, we take a value of the radius of gyration of chordwise elements of the wing as $0 \cdot 3c$, which is a fairly high figure and not likely to be exceeded, we arrive at a torsional frequency of 23 c.p.s. whence

$$r = 4/23 = 0 \cdot 175 \qquad (52)$$

With the flexural axis in the above position, the wing is at its stiffest torsionally. Suppose therefore the flexural axis is brought forward from $0 \cdot 65c$ to $0 \cdot 35c$ thus reducing its distance from the quarter-chord position to $0 \cdot 1c$ and so reducing the skin thickness necessary for isoclinicism to $\frac{1}{4}$ its previous value and the torsional frequency to $\frac{1}{2}$. This represents the lowest practical value of the torsional frequency since we neglect the accompanying reduction in the radius of gyration consequent upon using a smaller skin thickness with an unchanged spanwise distribution. The reduced value is therefore $23/2 = 11 \cdot 5$ c.p.s. giving a frequency ratio of

$$r = 0 \cdot 35 .$$

This result points strongly to the conclusion that there is no prospect of being able to design for a value of r greater than unity in the search for immunity from aero-elastic troubles. The alternative however, is not, as in the case of zero product of inertia, to go to the other extreme and make r as small as possible, but rather still to make r as large (and hence the torsional stiffness as small) as can be conveniently arranged. This will be clear from the next section in which numerical values are discussed.

4.5. *Inertia Axis Position in Relation to Torsional Stiffness.*—Reverting to equation (26) which gives the critical speeds in terms of the product of inertia q, we are now in a position to assign numerical values to the various quantities. As the numerator is essentially positive, we need only consider the denominator so far as attaining complete immunity is concerned, for, when that changes from positive to negative, n^2 becomes negative also and the flutter speed in consequence infinite. Thus we want to make

$$(1 - r^2) + q(s \cot \beta - \tan \beta) \qquad (47)$$

negative. It has already been shown (equation 46) that, with the flexural axis at $0 \cdot 65c$, or $0 \cdot 4c$ after of quarter-chord

$$r = 0 \cdot 175 \qquad (48)$$

and the value of $s = C_\phi / C_\theta$ is then calculated to be

$$s = 15 . \qquad (49)$$

Taking $\beta = 45$ deg as a first example, we see that expression (47) becomes zero when the absolute value of the negative product of inertia is

$$q = \frac{1 - r^2}{s - 1} = \frac{1 - 0 \cdot 03}{14} = 0.07 \; . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (50)$$

If we now halve the distance between the flexural axis and the quarter-chord, two changes take place, one trivial and the other important. We find that r changes roughly from $0 \cdot 175$ to $0 \cdot 175\sqrt{2}$ ($= 0 \cdot 25$) and so $(1 - r^2)$ changes from $0 \cdot 97$ to $0 \cdot 94$, but the important change is in the value of s, which is doubled.

$$\text{Thus } q = \frac{0 \cdot 94}{30 - 1} = 0 \cdot 032$$

and the distance that the inertia axis has to be placed forward of the flexural axis is about halved. If the flexural axis is moved to $0 \cdot 1c$ of the quarter-chord position, the amount of mass balance is still further reduced to

$$q = \frac{0 \cdot 88}{60 - 1} = 0 \cdot 015,$$

this last value being only about one-fifth of that required when the flexural axis was in the original position of $0 \cdot 65c$. Thus, *if a flexural axis position of* $0 \cdot 65c$ *requires a gap of* $0 \cdot 3c$ *between it and the inertia axis, that gap is reduced to* $0 \cdot 06c$, i.e., *only* 20 *per cent of its previous value if the flexural axis is located at* $0 \cdot 35c$—a fact that can have an important effect on the constructional difficulties.

Again it is to be pointed out that, owing to neglect of aerodynamic damping and the fixing of the wing at the root (thus neglecting rigid body motions of the aircraft), the case treated is a somewhat simplified one, but there is no reason to doubt the broad indications it suggests, particularly as the wind-tunnel results obtained by Lambourne[2] in his experimental check of the theory gave very good agreement in spite of the aerodynamic damping forces that were present.

5. *Conclusions*.—The following are the main conclusions from the above treatment:

(*a*) An isoclinic swept-back wing, although immune from static divergence in virtue of the loss of incidence due to bending having the effect of wiping out any wing twist that a change of lift force might otherwise produce, is subject to dynamic divergence. This follows from the fact that the bending frequency cannot be made equal to, or greater than, the torsional frequency, the consequence of which is that the wing cannot bend quickly enough to relieve the increased incidence due to twisting.

(*b*) The disability remarked in (*a*) remains only a potential one, for flutter invariably intervenes before the divergence can manifest itself, and any device that eliminates flutter *ipso facto* eliminates dynamic divergence.

(*c*) With the flexural and inertia axes coincident, the only way of obtaining immunity from flutter is to make the bending frequency greater than the torsional frequency. An exploration of the possibilities in this direction based on typical data indicates, however, that this is out of the question.

(*d*) When the inertia axis is placed forward of the flexural axis immunity from aero-elastic troubles becomes possible if the gap between the two axes is adequate.

(*e*) The adequacy of the gap referred to in (*d*), for any given angle of sweepback, is dependent on the ratio r of uncoupled bending to uncoupled torsional frequency and on the ratio s of bending to torsional stiffness. To make this clear—for it is an important point—one can do no better than quote the formula involved, namely

$$(1 - r^2) + q(s \cot \beta - \tan \beta) \; ,$$

which has to be negative for safety. The ratios r and s have just been defined, q is the ratio of product of inertia to bending inertia of the wing (*negative* when forward of the flexural axis) and β is the sweepback angle.

The formula shows the interplay of these non-dimensional quantities clearly. What we know is that r cannot be greater than about $\frac{1}{3}$ so that $(1 - r^2)$ is little short of unity and we may call it unity for the purpose of this argument. We also know that s cannot be less than about 10, so that compared with $s \cot \beta$, the value of $\tan \beta$ is negligible for any practical sweepback angle. We are left with

$$1 + qs \cot \beta$$

which incidentally shows the benefit of reducing β. With $\beta = 45$ deg the quantity that must be negative is $(1 + qs)$, which shows at a glance the importance of the ratio s in reducing the absolute value of the negative product of inertia, and hence in reducing the gap between the flexural and inertia axes which represents one of the main structural problems of the isoclinic wing.

As the flexural stiffness may be regarded as constant, the value of s varies inversely with the torsional stiffness, and the torsional stiffness in turn varies directly with the chordwise gap between the flexural axis and the quarter-chord position (to keep the wing isoclinic). By reducing this gap, therefore, from $0 \cdot 4c$ to $0 \cdot 1c$, we make s four times greater, thereby reducing q, and hence the gap between the flexural and inertia axes, to $\frac{1}{4}$ its previous value for the same degree of immunity. The disadvantage is the attendant loss of torsional stiffness, but if aero-elastic troubles are thus by-passed, this loss may be acceptable, particularly as in the Hill scheme rotating wing tips do the work of ailerons.

The possibility of thus reducing the gap between the inertia and flexural axes that flutter avoidance makes necessary is a point not previously brought out. The fact that the possibility can only be realised by further sacrifice of torsional stiffness, already—with the flexural axis at its most aft practicable location—below that conventionally acceptable, explains why this line of approach was not previously investigated. In any event its feasibility in practice remains to be proved; the theoretical possibility alone is indicated here.

(*f*) The degree of skin thickness appropriate to a flexural axis located at $0 \cdot 65c$ is shown in Fig. 3 (section 4.3) and is seen to be not impractical. It is to be noted, however, that a reduced torsional stiffness associated with a more forward position of this axis is attainable by other means than still further reducing the thickness of the wing-cell walls.

(*g*) So long as the flexural and inertia axis are concident the aero-elastic properties of an isoclinic swept-back wing are independent of the sweepback angle (*see* equation (15)). When however the inertia axis is forward of the flexural axis a reduction in the angle of sweepback greatly improves the aero-elastic behaviour (*see* equation (21a)).

(*h*) Other factors being constant (*see* equation (43)), aspect ratio has no effect on the skin thickness required to make a swept-back wing isoclinic.

REFERENCES

No.	*Author*	*Title, etc.*
1	G. T. R. Hill	A proposal for a solution to low-speed stability and high-speed aero-elastic problems of tailless aircraft. Short Bros. & Harland Ltd. Rep. No. S/19. A.R.C. 11,424. Received 12th April, 1948.
2	N. C. Lambourne and A. Chinneck	Flutter properties of a simple aero-isoclinic wing system. R. & M. 2869. January, 1950.

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