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Some Applications of the Lamé Function Solutions of the Linearised Supersonic Flow Equations

Part I.—Finite Swept-back Wings with Symmetrical Sections and Rounded Leading Edges

Part II.—Cambered and Twisted Wings

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Summary and Introduction.—Some general solutions of the linearised equations of supersonic flow, in terms of Lamé functions, were obtained by G. M. Roper[1,2] (1949, 1950), using the methods of Robinson[3,5] (1946, 1948) and Squire[4] (1947). The results were applied to calculate : (*a*) the pressure distribution over some swept-back wings at zero lift, having symmetrical sections with rounded leading edges[1] ; (*b*) the effect of camber and twist on the pressure distribution and drag on some wings of negligible thickness[2]. The solutions are only valid for surfaces lying wholly within the Mach cone of the apex.

In the present paper, some further special solutions are found. In Part I, some of these solutions are combined with solutions already found[1,4] to give : (*A*) the pressure distribution and wave drag, at zero lift, on some finite unyawed swept-back wings having symmetrical sections with rounded leading edges and wing tips perpendicular to the wind direction ; (*B*) the change in pressure distribution and wave drag at zero lift on the surface of a Squire wing[4], when the local thickness/chord ratio is modified.

The shapes of some curved wings, with swept-back subsonic leading edges were found by Roper[2] (1950), such that the thrust loading on the leading edges, at supersonic speeds, is removed or modified. In Part II of this paper, the effect of a change of Mach number on the aerodynamic characteristics of such a wing, designed for a given Mach number, is calculated.

Some additional solutions of the linearised supersonic flow equations, applicable to cambered and twisted wings, have also been calculated, and the results are given in Appendices III and IV of Part II.

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PART I

Finite Swept-back Wings with Symmetrical Sections and Rounded Leading Edges

1. *Introduction.*—In a previous paper[1], the results of certain general solutions of the linearised differential equations of supersonic flow are applied to find the pressure distribution over some swept-back wings, with rounded leading edges, whose equations are of the form

$$\frac{z}{2t_0} = f(x,y^2)\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2} .$$

x is measured downstream from the apex, y is measured to starboard, and z is measured vertically upwards. c is the chord in the vertical plane of symmetry, $\gamma(= \cot^{-1}k)$ is the apex semi-angle in the horizontal plane of symmetry, and t_0 is a constant proportional to the maximum thickness. The surfaces are symmetrical with respect to the xy and zx-planes, and are set symmetrically to the wind direction, with the apex pointing against the stream. The Mach angle $m(= \mathrm{cosec}^{-1}M)$ is greater than the semi-apex angle γ.

In this paper, the special solutions which give the flow over surfaces of the above form, with $f(x,y^2)$ a function of degree 3, 4 or 5 in x and y, are found. Certain of these solutions are combined with others already found[1,4] to give: (i) the pressure distribution and wave drag, at zero incidence, on some swept-back wings having symmetrical sections, rounded leading edges, a trailing edge parabolic in plan-form, and wing tips perpendicular to the root chord; (ii) the change in pressure distribution and drag on the surface of a Squire wing[4], when the local thickness/chord ratio is modified. Method (ii) could be applied to any surface of a similar type.

By modifying the thickness distribution, in particular by increasing the thickness of the sections near the wing tips, it is believed that the peak suction on the upper surface near the leading edge, at incidence, will be reduced, and that the chance of realising the suction force predicted by theory will be improved. Wind-tunnel tests show only part of the theoretical suction force on the basic Squire wing.

2. *Method of Solution.*—The co-ordinates used are the pseudo-orthogonal co-ordinates introduced by Robinson[2] (1946), where

$$x = \frac{\beta\gamma\mu\nu}{hk}, \quad y = \frac{\gamma(\mu^2 - h^2)^{1/2}(\nu^2 - h^2)^{1/2}}{\beta h}, \quad z = \frac{\gamma(\mu^2 - k^2)^{1/2}(k^2 - \nu^2)^{1/2}}{\beta k}, \qquad (1)$$

$$\left.\begin{aligned} \beta^2 &= M^2 - 1 = \cot^2 m = k^2 - h^2 \\ k^2 &= \cot^2\gamma, \quad h^2 = \cot^2\gamma - \cot^2 m \end{aligned}\right\} . \qquad (2)$$

It is assumed that the surfaces all lie close to the basic plate, whose equation is $\mu = k$, $(z = 0)$, and that the induced velocities on the surface are small and equal to the induced velocities on the plate. Therefore the relation between the shape of the body and its induced velocity potential ϕ is of the form

$$\frac{\partial z}{\partial x} = \frac{1}{V}\left(\frac{\partial\phi}{\partial z}\right)_{\mu=k}, \qquad (3)$$

where V is the free-stream velocity.

For the linearised theory, the pressure coefficient is

$$C_p = -\frac{2}{V}\left(\frac{\partial\phi}{\partial x}\right)_{\mu=k} . \qquad (4)$$

The required solutions of the linearised differential equation for the velocity potential ϕ, in terms of r, μ, ν (equation (5) of R. & M. 2700[1]) are given by combinations of solutions of the form

$$\phi_n^m = C_n r^n F_n^m(\mu) E_n^m(\nu),$$

where $E_n^m(\nu)$ is a standard Lamé function of degree n of the K class, and $F_n^m(\mu)$ is the second Lamé function given by[1,6]

$$F_n^m(\mu) = E_n^m(\mu) \int_\mu^\infty \frac{dt}{[E_n^m(t)]^2 \, [|t^2 - h^2| \, | t^2 - k^2|]^{1/2}} \qquad \ldots \quad (5)$$

$$\equiv E_n^m(\mu) R_n^m(\mu) \ .$$

Solutions for $n = 1, 2$ are given by Squire[4] (1947), and solutions for $n = 3$ by Roper[1] (1949). Solutions for $n = 4, 5, 6$ will now be found.

3. *Solutions for $n = 4$.* For $n = 4$, there are three K functions of the form

$$E_4^m(\mu) = \mu^4 - a_m\mu^2 + b_m \ , \ (m = 1, 2, 3) \qquad \ldots \quad (6)$$

where a_m, b_m are positive constants.

Substituting (6) in the linearised differential equation for ϕ in terms of r, μ, ν, or using relation (1) of Appendix II of Ref. 1, it can be shown that

$$\left.\begin{aligned} &49a_m'^3 - 98(1 + \varkappa^2)a_m'^2 + \{48(1 + \varkappa^2)^2 + 52\varkappa^2\}a_m' - 48\varkappa^2(1 + \varkappa^2) = 0 \ ; \\ &10b_m' = 7a_m'^2 - 6(1 + \varkappa^2)a_m' + 6\varkappa^2, \\ &\text{and hence} \\ &245b_m'^3 - [56(1 + \varkappa^2)^2 + 77\varkappa^2]b_m'^2 + \varkappa^2[24(1 + \varkappa^2)^2 - 37\varkappa^2]b_m' - 3\varkappa^6 = 0 \\ &\text{where } a_m' = a_m/k^2 \ , \ b_m' = b_m/k^4 \ , \ \varkappa^2 = h^2/k^2 \ . \end{aligned}\right\} \quad (7)$$

For a given value of $\varkappa^2$, equations (7) can be solved for a_m', b_m' to any required degree of accuracy. Horner's approximation method has been used to calculate the three values of a_m', b_m' correct to six decimal places, for $\varkappa^2 = 0 \cdot 19$ and $\varkappa^2 = 2/3$. The values are given in Appendix I.

We consider the solution

$$\phi_m = C_4 r^4 F_4^m(\mu) . E_4^m(\nu)$$
$$\equiv C_4 r^4 E_4^m(\mu) . E_4^m(\nu) . R_4^m(\mu) \ . \qquad \ldots \quad (8)$$

At the plate, $\mu \to k$, and

$$r^2 = (x^2 - \beta^2 y^2)/\beta^2, \ r^2\nu^2 = h^2x^2/\beta^2, \qquad \ldots \quad (9)$$

and, using relation (3), it can be shown that (*cf.* equation (20) of R. & M. 2700[1])

$$\frac{\partial z}{\partial x} = \frac{-C_4}{V\beta^4 E_4^m(k)} \left[\frac{(h^4 - a_m h^2 + b_m)x^4 + (a_m h^2 - 2b_m)\beta^2x^2y^2 + b_m\beta^4y^4}{(x^2 - k^2y^2)^{1/2}}\right] \qquad \ldots \quad (10)$$

and therefore

$$z = \frac{-C_4}{V\beta^4 E_4^m(k)} \left[\left\{\tfrac{1}{4}(h^4 - a_m h^2 + b_m)x^3 + [\tfrac{3}{8}k^2(h^4 - a_m h^2 + b_m) + \tfrac{1}{2}\beta^2(a_m h^2 - 2b_m)]xy^2\right\}(x^2 - k^2y^2)^{1/2} + \left\{\tfrac{3}{8}k^4(h^4 - a_m h^2 + b_m) + \tfrac{1}{2}\beta^2k^2(a_m h^2 - 2b_m) + b_m\beta^4\right\}y^4 \int \frac{dx}{(x^2 - k^2y^2)^{1/2}}\right] \qquad \ldots \quad (11)$$

It can be shown that

$$\left(\frac{\partial\phi_m}{\partial x}\right)_{\mu=k} = \frac{C_4}{\beta^4}(k^4 - a_m k^2 + b_m)[4(h^4 - a_m h^2 + b_m)x^3 + 2\beta^2 xy^2(a_m h^2 - 2b_m)]R_4^m(k), \qquad (12)$$

where (*see* Appendix II, R. & M. 2700[1]),

$$R_4(k) = \frac{1}{2k(a_m{}^2 - 4b_m)}\left[\frac{K(\varkappa)}{h^2}\left\{\frac{a_m}{b_m} + \frac{2h^2 - a_m}{h^4 - a_m h^2 + b_m}\right\}\right.$$

$$\left. - k^2E(\varkappa)\left\{\frac{1}{h^2k^2}\frac{a_m}{b_m} + \frac{1}{\beta^2h^2}\left(\frac{2h^2 - a_m}{h^4 - a_m h^2 + b_m}\right) - \frac{1}{\beta^2k^2}\left(\frac{2k^2 - a_m}{k^4 - a_m k^2 + b_m}\right)\right\}\right], \quad \ldots \qquad (13)$$

$K(\varkappa)$, $E(\varkappa)$ being the complete elliptic integrals of the first and second kind respectively, of modulus $\varkappa(= h/k)$.

If we construct a potential

$$\Phi_4 = \sum_{m=1}^{3}(\lambda_m E_4{}^m(k)\phi_m)\,,$$

where the λ_m's are chosen so that the coefficient of

$$y^4\int\frac{dx}{(x^2 - k^2y^2)^{1/2}}$$

in (11) is zero, we obtain the solution for a surface whose equation is of the form

$$z = (c_1x^3 + c_2xy^2)(x^2 - k^2y^2)^{1/2}\,,$$

where c_1, c_2 are constants.

We shall construct solutions for the two surfaces whose equations are of the form

(*a*)
$$\frac{z}{2t_0} = \frac{x^3}{c^3}\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2},$$

(*b*)
$$\frac{z}{2t_0} = \frac{k^2xy^2}{c^3}\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2}.$$

(*a*) *The surface* $\frac{z}{2t_0} = \frac{x^3}{c^3}\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2}$ *at zero incidence.*

If Φ_4 is the induced velocity potential for flow past surface (*a*), it can be shown that

$$\left.\begin{aligned}\sum_{m=1}^{3}(\lambda_m b_m) &= 0\\ \sum_{m=1}^{3}(\lambda_m a_m) &= -3k^2/(\beta^2h^2)\\ \sum_{m=1}^{3}(\lambda_m) &= (k^2 - 4h^2)/(\beta^2h^4)\end{aligned}\right\}, \quad \ldots \qquad (14)$$

if C_4 is chosen so that

$$\frac{-C_4}{V\beta^4} = \frac{2t_0}{c^4}\,. \quad \ldots \qquad (15)$$

Solving (14), and using the notation of equation (7), we obtain

$$k^4\Delta\lambda_3 = \frac{-1}{\varkappa^2(1 - \varkappa^2)}\left[3(b_1' - b_2') - \frac{(1 - 4\varkappa^2)}{\varkappa^2}(a_1'b_2' - a_2'b_1')\right], \quad \ldots \qquad (16)$$

and two similar expressions for λ_1, λ_2, where

$$\Delta \equiv \begin{vmatrix} 1 & 1 & 1 \\ a_1' & a_2' & a_3' \\ b_1' & b_2' & b_3' \end{vmatrix}$$

The pressure coefficient is

$$C_p = \frac{2t_0}{c} k^3 \sum_{m=1}^{3} \lambda_m \frac{(1 - a_m' + b_m')^2}{a_m'^2 - 4b_m'} \Bigg[\frac{K(\varkappa)}{\varkappa^2} \left\{ \frac{a_m'}{b_m'} + \frac{2\varkappa^2 - a_m'}{\varkappa^4 - a_m'\varkappa^2 + b_m'} \right\}$$

$$- E(\varkappa) \left\{ \frac{a_m'}{\varkappa^2 b_m'} + \frac{1}{\varkappa^2(1 - \varkappa^2)} \left(\frac{2\varkappa^2 - a_m'}{\varkappa^4 - a_m'\varkappa^2 + b_m'} \right) \right.$$

$$\left. - \frac{1}{1 - \varkappa^2} \left(\frac{2 - a_m'}{1 - a_m' + b_m'} \right) \right\} \Bigg] \Bigg[4(\varkappa^4 - a_m'\varkappa^2 + b_m') \left(\frac{x}{c} \right)^3$$

$$+ 2(1 - \varkappa^2)(a_m'\varkappa^2 - 2b_m')k^2 \frac{xy^2}{c^3} \Bigg], \quad \ldots \quad (17)$$

where the λ_m's are given by (16).

(*b*) *The surface* $\dfrac{z}{2t_0} = \dfrac{k^2xy^2}{c^3} \left(\dfrac{x^2 - k^2y^2}{c^2} \right)^{1/2}$ *at zero incidence.*

If Φ_4 is the induced velocity potential for flow past surface (*b*), it can be shown that

$$\left. \begin{aligned} \sum_{m=1}^{3} (\lambda_m b_m) &= -\frac{k^4}{\beta^4} \\ \sum_{m=1}^{3} (\lambda_m a_m) &= -\frac{2k^2}{\beta^4} \\ \sum_{m=1}^{3} (\lambda_m) &= \frac{(k^2 - 2h^2)k^2}{h^4\beta^4} \end{aligned} \right\}, \quad \ldots \quad (18)$$

if C_4 is again as chosen in (15).

Hence we obtain

$$k^4 \Delta \lambda_3 = -\frac{1}{(1 - \varkappa^2)^2} \left[2(b_1' - b_2') - (a_1' - a_2') - \frac{1 - 2\varkappa^2}{\varkappa^4} (a_1'b_2' - a_2'b_1') \right], \quad (19)$$

and two similar expressions for λ_1, λ_2. The pressure coefficient is given by (17), where the λ_m's are given by (19).

The values of a_m', b_m' and the corresponding values of λ_m for surfaces (*a*) and (*b*), for $\varkappa^2 = 0{\cdot}19$ and $\varkappa^2 = 2/3$, are given in Appendix I.

4. *Solutions for $n = 5$.*—For $n = 5$, there are three K functions of the form

$$E_5^m(\mu) = \mu^5 - a_m\mu^3 + b_m\mu, \quad m = 1, 2, 3. \quad \ldots \quad (20)$$

It can be shown that : (using notation of (7))

$$27a_m'^3 - 60(1 + \varkappa^2)a_m'^2 + [32(1 + \varkappa^2)^2 + 44\varkappa^2]a_m' - 40\varkappa^2(1 + \varkappa^2) = 0 \quad \ldots \quad (21)$$

and $$14b_m' = 9a_m'^2 - 8(1 + \varkappa^2)a_m' + 10\varkappa^2 \; . \qquad (22)$$

The solution

$$\phi_m = C_5 r^5 F_5^m(\mu) E_5^m(\nu) \equiv C_5 r^5 E_5^m(\mu) E_5^m(\nu) R_5^m(\mu)$$

gives the flow over the surface

$$\begin{aligned} z = \frac{-C_5 h}{V\beta^5 k(k^4 - a_m k^2 + b_m)} \Big[& \tfrac{1}{5}(h^4 - a_m h^2 + b_m)x^4 \\ & + \{\tfrac{4}{15}k^2(h^4 - a_m h^2 + b_m) + \tfrac{1}{3}\beta^2(a_m h^2 - 2b_m)\}\, x^2 y^2 \\ & + \tfrac{8}{15}k^4(h^4 - a_m h^2 + b_m) + \tfrac{2}{3}\beta^2 k^2(a_m h^2 - 2b_m) \\ & + b_m\beta^4\}\, y^4\Big](x^2 - k^2y^2)^{1/2} \; . \end{aligned} \qquad (23)$$

If we construct a potential

$$\Phi_5 = \sum_{m=1}^{3} [\lambda_m k(k^4 - a_m k^2 + b_m)\phi_m] \, ,$$

the λ_m's can be chosen so that Φ_5 gives the flow over any surface of the form

$$z = (c_1 x^4 + c_2 x^2 y^2 + c_3 y^4)(x^2 - k^2 y^2)^{1/2} \, ,$$

where c_1, c_2, c_3 are constants.

For the particular surface

$$\frac{z}{2t^0} = \frac{k^4 y^4}{c^4}\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2} ,$$

it can be shown that

$$\left.\begin{aligned} \sum_{m=1}^{3} (\lambda_m b_m) &= \frac{k^4}{\beta^4} \\ \sum_{m=1}^{3} (\lambda_m a_m) &= \frac{2k^4}{\beta^4 h^2} \\ \sum_{m=1}^{3} (\lambda_m) &= \frac{k^4}{\beta^4 h^4} \end{aligned}\right\} \qquad (24)$$

if C_5 is chosen so that

$$-\frac{C_5 h}{V\beta^5} = \frac{2t_0}{c^5} \qquad (25)$$

and hence we obtain

$$k^4 \Delta (1 - \varkappa^2)^2 \lambda_1 = -\left[(a_2' - a_3') - \frac{2}{\varkappa^2}(b_2' - b_3') - \frac{1}{\varkappa^4}(a_2'b_3' - a_3'b_2')\right] \qquad (26)$$

and two similar formulae for λ_2, λ_3, where

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a_1' & a_2' & a_3' \\ b_1' & b_2' & b_3' \end{vmatrix} . \qquad (27)$$

The pressure coefficient is

$$(C_p)_{k^4y^4} = \frac{4t_0}{kc\Delta(1-\varkappa^2)^2} \sum_{m=1}^{3} \Bigg[\Big\{ k^4\Delta(1-\varkappa^2)^2\lambda_m \Big\} (1 - a_m' + b_m')^2 \, (k^{11}R_5{}^m(k))$$

$$\times \left\{ 5(\varkappa^4 - a_m'\varkappa^2 + b_m') \frac{x^4}{c^4} + 3(1-\varkappa^2)(a_m'\varkappa^2 - 2b_m') \frac{k^2x^2y^2}{c^4} \right.$$

$$\left. + (1-\varkappa^2)^2 b_m' k^4 \frac{y^4}{c^4} \right\} \Bigg] , \qquad (28)$$

where (*see* Appendix II, R. & M. 2700[1])

$$k^{11}R_5{}^m(k) = \frac{1}{2\varkappa^2(a_m'^2 - 4b_m')} \left[\frac{3\{E(\varkappa) - K(\varkappa)\}(2\varkappa^2 - \overline{1+\varkappa^2}a_m' + a_m'^2 - 2b_m')}{(\varkappa^4 - a_m'\varkappa^2 + b_m')(1 - a_m' + b_m')} \right.$$

$$\left. - \frac{\{2(1+\varkappa^2)E(\varkappa) - (2+\varkappa^2)K(\varkappa)\}\{a_m'\varkappa^2 - (1+\varkappa^2)(a_m'^2 - 2b_m') + a_m'(a_m'^2 - 3b_m')\}}{b_m'(\varkappa^4 - a_m'\varkappa^2 + b_m')(1 - a_m' + b_m')} \right] . \qquad (29)$$

The values of a_m', b_m', λ_m for $\varkappa^2 = 0 \cdot 48$ are given in Appendix II.

5. *Solutions for $n = 6$.*—For $n = 6$, there are four K functions of the form

$$E_6{}^m(\mu) = \mu^6 - a_m\mu^4 + b_m\mu^2 - c_m, \quad m = 1, 2, 3, 4 \qquad (30)$$

where (using the notation of (7) and $c_m' = c_m/k^6$)

$$1331a_m'^4 - 5324(1+\varkappa^2)a_m'^3 + [6908(1+\varkappa^2)^2 + 3234\varkappa^2]a_m'^2$$

$$- [2880(1+\varkappa^2)^3 + 7764\varkappa^2(1+\varkappa^2)]a_m' + [4320\varkappa^2(1+\varkappa^2)^2 + 315\varkappa^4] = 0 \qquad (31)$$

$$18b_m' = 11a_m'^2 - 10(1+\varkappa^2)a_m' + 15\varkappa^2, \qquad (32a)$$

$$378c_m' = 121a_m'^3 - 286(1+\varkappa^2)a_m'^2 + \{160(1+\varkappa^2)^2\,\delta + 273\varkappa^2\}a_m'$$

$$- 240\varkappa^2(1+\varkappa^2) . \qquad (32b)$$

The solution

$$\phi_m = C_6 r^6 F_6{}^m(\mu)E_6{}^m(\nu) \equiv C_6 r^6 E_6{}^m(\mu)E_6{}^m(\nu)R_6{}^m(\mu)$$

gives the flow over the surface

$$z = \frac{-C_6}{V\beta^6(k^6 - a_mk^4 + b_mk^2 - c_m)} \Big[\{(h^6 - a_mh^4 + b_mh^2 - c_m)(\tfrac{1}{6}x^5 + \tfrac{5}{24}k^2x^3y^2$$

$$+ \tfrac{5}{16}k^4xy^4) + \beta^2(a_mh^4 - 2b_mh^2 + 3c_m)y^2(\tfrac{1}{4}x^3 + \tfrac{3}{8}k^2xy^2)$$

$$+ \beta^4(b_mh^2 - 3c_m)(\tfrac{1}{2}xy^4)\}\,(x^2 - k^2y^2)^{1/2} + \{\tfrac{5}{16}k^6(h^6 - a_mh^4 + b_mh^2 - c_m)$$

$$+ \tfrac{3}{8}k^4\beta^2(a_mh^4 - 2b_mh^2 + 3c_m) + \tfrac{1}{2}k^2\beta^4(b_mh^2 - 3c_m) + \beta^6c_m\}y^6 \int \frac{dx}{(x^2 - k^2y^2)^{1/2}} \Big] . \qquad (33)$$

If we construct a potential

$$\Phi_6 = \sum_{m=1}^{4} [\lambda_m(k^6 - a_m k^4 + b_m k^2 - c_m)\phi_m],$$

the λ_m's can be chosen so that Φ_6 gives the flow over any surface of the form

$$z = (c_1 x^5 + c_2 x^3 y^2 + c_3 x y^4)(x^2 - k^2 y^2)^{1/2},$$

where c_1, c_2, c_3 are constants.

For the particular surface

$$\frac{z}{2t_0} = k^4 \frac{xy^4}{c^5} \left(\frac{x^2 - k^2 y^2}{c^2}\right)^{1/2},$$

it can be shown that

$$\left.\begin{aligned} \sum_{m=1}^{4} (\lambda_m c_m) &= -\frac{k^6}{\beta^6} \\ \sum_{m=1}^{4} (\lambda_m b_m) &= -\frac{k^4}{\beta^6 h^2}(2h^2 + k^2) \\ \sum_{m=1}^{4} (\lambda_m a_m) &= -\frac{k^4}{\beta^6 h^4}(4h^2 - k^2) \\ \sum_{m=1}^{4} (\lambda_m) &= -\frac{k^4}{\beta^6 h^6}(2h^2 - k^2) \end{aligned}\right\} \quad \ldots \quad (34)$$

if C_6 is chosen so that

$$\frac{-C_6}{V\beta^6} = \frac{2t_0}{c^6} \quad \ldots \quad (35)$$

and hence we obtain

$$k^6 \Delta (1 - \varkappa^2)^3 \lambda_m = \frac{1}{\varkappa^4}(1 - 4\varkappa^2)A_m + \frac{1}{\varkappa^2}(1 + 2\varkappa^2)B_m - C_m + \frac{1}{\varkappa^6}(1 - 2\varkappa^2)D_m, \quad \ldots \quad (36)$$

where

$$\left.\begin{aligned} A_1 &= c_2'(b_3' - b_4') + c_3'(b_4' - b_2') + c_4'(b_2 - b_3') \\ -A_2 &= c_3'(b_4' - b_1') + c_4'(b_1' - b_3') + c_1'(b_3' - b_4') \\ A_3 &= c_4'(b_1' - b_2') + c_1'(b_2' - b_4') + c_2'(b_4' - b_1') \\ -A_4 &= c_1'(b_2' - b_3') + c_2'(b_3' - b_1') + c_3'(b_1' - b_2') \end{aligned}\right\} \quad \ldots \quad (37)$$

The formulae for B_m, C_m are given by (37), with a substituted for b for B_m, and b substituted for c, and a for b for C_m.

$$\left.\begin{aligned} D_1 &= c_2'(a_3'b_4' - a_4'b_3') + c_3'(a_4'b_2' - a_2'b_4') + c_4'(a_2'b_3' - a_3'b_2') \\ -D_2 &= c_3'(a_4'b_1' - a_1'b_4') + c_4'(a_1'b_3' - a_3'b_1') + c_1'(a_3'b_4' - a_4'b_3') \\ D_3 &= c_4'(a_1'b_2' - a_2'b_1') + c_1'(a_2'b_4' - a_4'b_2') + c_2'(a_4'b_1 - a_1'b_4') \\ -D_4 &= c_1'(a_2'b_3' - a_3'b_2') + c_2'(a_3'b_1' - a_1'b_3') + c_3'(a_1'b_2' - a_2'b_1') \end{aligned}\right\} \quad (38)$$

and

$$\Delta = c_1'C_1 + c_2'C_2 + c_3'C_3 + c_4'C_4. \quad \ldots \quad (39)$$

The pressure coefficient is

$$(C_p)_{k^4xy^4} = \frac{4t_0}{kc\Delta(1-\varkappa^2)^3} \sum_{m=1}^{4} \Bigg[\{k^6\Delta(1-\varkappa^2)^3\lambda_m\}(1 - a_m' + b_m' - c_m')^2(k^{13}R_6^m(k))$$

$$\times \Bigg\{ 6(\varkappa^6 - a_m'\varkappa^4 + b_m'\varkappa^2 - c_m')\frac{x^5}{c^5} + 4(1-\varkappa^2)(a_m'\varkappa^4 - 2b_m'\varkappa^2 + 3c_m')k^2\frac{x^3y^2}{c^5}$$

$$+ 2(1-\varkappa^2)^2(b_m'\varkappa^2 - 3c_m')k^4\frac{xy^4}{c^5}\Bigg\}\Bigg], \quad \ldots \quad (40)$$

where

$$k^{13}R_6(k) = \frac{1}{2T_m}\Bigg[\frac{K(\varkappa) - E(\varkappa)}{\varkappa^2}\left(3a_m' + \frac{a_m'^2b_m' - 4b_m'^2}{c_m'}\right)$$

$$+ \frac{1}{\varkappa^2}\left(K(\varkappa) - \frac{1}{1-\varkappa^2}E(\varkappa)\right)H_1 + \frac{E(\varkappa)}{1-\varkappa^2}H_2\Bigg], \quad \ldots \quad (41)$$

$$T_m = 18a_m'b_m'c_m' - 27c_m'^2 + a_m'^2b_m'^2 - 4a_m'^3c_m' - 4b_m'^3 \quad \ldots \quad (42)$$

$$H_1 = [\varkappa^4(2a_m'^2 - 6b_m') - \varkappa^2(2a_m'^3 - 7a_m'b_m' + 9c_m')$$

$$+ (a_m'^2b_m' + 3a_m'c_m' - 4b_m'^2)]/[\varkappa^6 - a_m'\varkappa^4 + b_m'\varkappa^2 - c_m'] \quad \ldots \quad (43)$$

$$H_2 = [(2a_m'^2 - 6b_m') - (2a_m'^3 - 7a_m'b_m' + 9c_m')$$

$$+ (a_m'^2b_m' + 3a_m'c_m' - 4b_m'^2)]/[1 - a_m' + b_m' - c_m']. \quad \ldots \quad (44)$$

The values of a_m', b_m', c_m', λ_m for $\varkappa^2 = 0{\cdot}48$ are given in Appendix II.

Note :

When $\varkappa = 0$, the smallest root of equations (7), (21), (31) for a_m' is, in each case, zero. Therefore, when calculating the smallest roots of the equations for any other value of $\varkappa$, and the corresponding pressure coefficient C_p, we may write a_m', b_m', c_m' as $\varkappa^2a_m''$, $\varkappa^2b_m''$, $\varkappa^2c_m''$. For example, for $n = 6$, (31), (32a), (32b) become :

$$1331\varkappa^6a''^4 - 5324(1+\varkappa^2)\varkappa^4a''^3 + [6908(1+\varkappa^2)^2 + 3234\varkappa^2]\varkappa^2a''^2$$

$$- [2880(1+\varkappa^2)^3 + 7764\varkappa^2(1+\varkappa^2)]a'' + [4320(1+\varkappa^2)^2 + 315\varkappa^2] = 0$$

$$18b'' = 11\varkappa^2a''^2 - 10(1+\varkappa^2)a'' + 15$$

$$378c'' = 121\varkappa^4a''^3 - 286(1+\varkappa^2)\varkappa^2a''^2 + \{160(1+\varkappa^2)^2 + 273\varkappa^2\}a'' - 240(1+\varkappa^2).$$

(40) becomes :

$$(C_p)_{k^4xy^4} = \frac{4t_0}{kc\Delta(1-\varkappa^2)^3} \sum_{m=1}^{4} \Bigg[\{k^6\Delta(1-\varkappa^2)^3\lambda_m\}\{1 - \varkappa^2(a_m'' - b_m'' + c_m'')\}^2(k^{13}R_6'^m(k))$$

$$\times \Bigg\{ 6(\varkappa^4 - \varkappa^4a_m'' + \varkappa^2b_m'' - c_m'')\frac{x^5}{c^5} + 4(1-\varkappa^2)(\varkappa^4a_m'' - 2\varkappa^2b_m''$$

$$+ 3c_m'')k^2\frac{x^3y^2}{c^5} + 2(1-\varkappa^2)^2(\varkappa^2b_m'' - 3c_m'')k^4\frac{xy^4}{c^5}\Bigg\}\Bigg],$$

where

$$k^{13}R_6'(k) = \frac{1}{2T_m'}\left[\frac{K(\varkappa) - E(\varkappa)}{\varkappa^2}\left(3a_m'' + \frac{\varkappa^2 a_m''^2 b_m'' - 4b_m''^2}{c_m''}\right)\right.$$

$$\left. + \frac{1}{\varkappa^2}\left(K(\varkappa) - \frac{1}{1-\varkappa^2}E(\varkappa)\right)H_1' + \frac{E(\varkappa)}{1-\varkappa^2}H_2'\right],$$

$$T_m' = 18\varkappa^2 a_m'' b_m'' c_m'' - 27c_m''^2 + \varkappa^4 a_m''^2 b_m''^2 - 4\varkappa^4 a_m''^3 c_m'' - 4\varkappa^2 b_m''^3$$

$$H_1' = [(2\varkappa^4 a_m''^2 - 6\varkappa^2 b_m'') - (2\varkappa^4 a_m''^3 - 7\varkappa^2 a_m'' b_m'' + 9c_m'')$$

$$+ (\varkappa^2 a_m''^2 b_m'' + 3a_m'' c_m'' - 4b_m''^2)]/[\varkappa^4 - \varkappa^4 a_m'' + \varkappa^2 b_m'' - c_m''],$$

$$H_2' = [(2\varkappa^2 a_m''^2 - 6b_m'') - (2\varkappa^4 a_m''^3 - 7\varkappa^2 a_m'' b_m'' + 9c_m'')$$

$$+ \varkappa^2(\varkappa^2 a_m''^2 b_m'' + 3a_m'' c_m'' - 4b_m''^2)]/[1 - \varkappa^2(a_m'' - b_m'' + c_m'')].$$

For small values of $\varkappa$, the formulae in terms of a_m'' are more convenient for numerical calculations than those in terms of a_m'.

Applications.—(*A*). Pressure distribution and drag, at supersonic speeds and zero lift, on some finite swept-back wings, having symmetrical sections, with rounded leading edges, and wing tips perpendicular to the root chord.

6. *The surface* $\frac{z}{2t_0} = \left(\frac{d}{c} - \frac{x}{c}\right)\left(1 - \frac{x}{c} + \frac{ay^2}{c^2}\right)\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2}$.

By combining the solution (*b*) found in section 3 and those previously given[1,4], a formula can be found for the pressure coefficient for a finite swept-back wing having symmetrical sections, rounded leading edges, and a parabolic trailing edge, except near the wing tips, where the trailing edge is straight and perpendicular to the root chord. For a hyperbolic trailing edge, solution (*a*) would also be used.

The pressure coefficients for the surfaces

$$\frac{z}{2t_0} = \left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2} \text{ and } \frac{z}{2t_0} = \frac{x}{c}\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2}$$

are[4] :

$$(C_p)_0 = \frac{4t_0}{c\beta} f_1 \qquad (45)$$

$$(C_p)_x = \frac{4t_0}{c^2\beta} x f_2 \text{ respectively} \qquad (46)$$

where

$$f_1 = \frac{(1-\varkappa^2)^{1/2}}{\varkappa^2}[K(\varkappa) - E(\varkappa)], \qquad (47)$$

$$f_2 = \frac{(1-\varkappa^2)^{1/2}}{\varkappa^4}[(\varkappa^2 + 2)K(\varkappa) - 2(1 + \varkappa^2)E(\varkappa)]. \qquad (48)$$

The pressure coefficients for the surfaces

$$\frac{z}{2t_0} = \frac{x^2}{c^2}\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2} \text{ and } \frac{z}{2t_0} = \frac{k^2y^2}{c^2}\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2}$$

are[1] :

$$(C_p)_{x^2} = \frac{4t_0}{c^3\beta}(x^2F_1 + k^2y^2F_2), \quad (49)$$

$$(C_p)_{k^2y^2} = \frac{4t_0}{c^3\beta}(x^2F_3 + k^2y^2F_4), \quad (50)$$

respectively, where

$$F_1 = \frac{(1-\varkappa^2)^{1/2}}{2\varkappa^6}[(3\varkappa^4 + \varkappa^2 + 8)K(\varkappa) - (6\varkappa^4 + 5\varkappa^2 + 8)E(\varkappa)], \quad (51)$$

$$F_2 = \frac{(1-\varkappa^2)^{1/2}}{2\varkappa^6}[(\varkappa^4 - 9\varkappa^2 + 8)K(\varkappa) + (\varkappa^4 + 5\varkappa^2 - 8)E(\varkappa)], \quad (52)$$

$$F_3 = \frac{(1-\varkappa^2)^{1/2}}{2\varkappa^6}[(8 - 5\varkappa^2)K(\varkappa) - (8 - \varkappa^2)E(\varkappa)], \quad (53)$$

$$F_4 = \frac{(1-\varkappa^2)^{1/2}}{2\varkappa^6}[(8 - 15\varkappa^2 + 7\varkappa^4)K(\varkappa) - (8 - 11\varkappa^2 + 2\varkappa^4)E(\varkappa)]. \quad (54)$$

Combining these results, the pressure coefficient for the surface

$$\frac{z}{2t_0} = \left(\frac{d}{c} - \frac{x}{c}\right)\left(1 - \frac{x}{c} + \frac{ay^2}{c^2}\right)\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2} \quad (55)$$

is

$$C_p = \frac{d}{c}(C_p)_0 - \left(1 + \frac{d}{c}\right)(C_p)_x + (C_p)_{x^2} + \frac{ad}{ck^2}(C_p)_{k^2y^2} - \frac{a}{k^2}(C_p)_{k^2xy^2} \quad (56)$$

$$= \frac{2t_0}{c}\left[A + B\frac{x}{c} + \tfrac{1}{2}C\frac{x^2}{c^2} + D\frac{y^2}{c^2} + \tfrac{1}{3}E\frac{x^3}{c^3} + F\frac{xy^2}{c^3}\right]. \quad (57)$$

Some numerical examples are given in section 7.

7. *Numerical Examples.*—Some numerical examples, for $\varkappa^2 = 0\cdot19$ and $\varkappa^2 = 2/3$ ($\tan\gamma/\tan m = 0\cdot9$ and $1/\sqrt{3}$ respectively), of the wing described in section 6 are shown in Figs. 1 to 7. The required values of a_m', b_m' given by equation (7), and the corresponding values of λ_m for surfaces (*a*), (*b*) given by equations (14), (18), and also the values of f_1, f_2, F_1, F_2, F_3, F_4 from (47), (48) and (51) to (54) are given in Appendix I. The formulae for the shape of the surface, and the pressure coefficient C_p are given below.

(i)(*a*) $\tan\gamma/\tan m = 0\cdot9$, $\gamma = 45$ deg, $M = 1\cdot345$ (Fig. 1).

The shape of the surface is given by the equation

$$\frac{z}{2t_0} = \left(1\cdot4 - \frac{x}{c}\right)\left(1 - \frac{x}{c} + \frac{1}{2}\frac{y^2}{c^2}\right)\left(\frac{x^2 - y^2}{c^2}\right)^{1/2}.$$

If T_0 is the maximum thickness in the plane of symmetry, $2t_0 = 2\cdot0805T_0$.

The maximum-thickness line is :

$$\frac{3x^3}{c^3} - \frac{x^2}{c^2}\left(4\cdot8 + \frac{y^2}{c^2}\right) + \frac{x}{c}\left(1\cdot4 - 1\cdot3\frac{y^2}{c^2}\right) + \left(2\cdot4\frac{y^2}{c^2} + \frac{1}{2}\frac{y^4}{c^4}\right) = 0 .$$

The pressure coefficient, at zero incidence, is

$$C_p = \frac{T_0}{c}\left[4\cdot9464 - 19\cdot2481\frac{x}{c} + 13\cdot2813\frac{x^2}{c^2} - 0\cdot6426\frac{y^2}{c^2} - 0\cdot0904\frac{x^3}{c^3} - 1\cdot8803\frac{xy^2}{c^3}\right]$$

The trailing edge is supersonic, therefore the solution is valid for the whole surface.

(i)(*b*) $\tan\gamma/\tan m = 0\cdot9, \quad \gamma = 45$ deg, $\quad M = 1\cdot345 \quad$ (Fig. 2).

The shape of the surface is given by the equation

$$\frac{z}{2t_0} = \left(1\cdot9 - \frac{x}{c}\right)\left(1 - \frac{x}{c} + \frac{1}{2}\frac{y^2}{c^2}\right)\left(\frac{x^2 - y^2}{c^2}\right)^{1/2},$$

and

$$2t_0 = 1\cdot3868T_0.$$

The maximum-thickness line is

$$\frac{3x^3}{c^3} - \frac{x^2}{c^2}\left(5\cdot8 + \frac{y^2}{c}\right) + \frac{x}{c}\left(1\cdot9 - 1\cdot05\frac{y^2}{c^2}\right) + \left(2\cdot9\frac{y^2}{c^2} + \frac{1}{2}\frac{y^4}{c^4}\right) = 0.$$

The pressure coefficient, at zero incidence, is

$$C_p = \frac{T_0}{c}\left[4\cdot4748 - 15\cdot5036\frac{x}{c} + 8\cdot9773\frac{x^2}{c^2} - 0\cdot1125\frac{y^2}{c^2} - 0\cdot0602\frac{x^3}{c^3} - 1\cdot2534\frac{xy^2}{c^3}\right].$$

The parobolic trailing edge is subsonic for $x/c > 1\cdot4$. No allowance has been made for the small corrections necessary in the regions near the subsonic portions of the trailing edge.

(ii) $\tan\gamma/\tan m = 0\cdot9, \quad \gamma = 45$ deg, $\quad M = 1\cdot345 \quad$ (Fig. 3).

The shape of the surface is given by the equation

$$\frac{z}{2t_0} = \left(1\cdot35 - \frac{x}{c}\right)\left(1 - \frac{x}{c} + \frac{1}{4}\frac{y^2}{c^2}\right)\left(\frac{x^2 - y^2}{c^2}\right)^{1/2},$$

and

$$2t_0 = 2\cdot1879T_0.$$

The maximum-thickness line is

$$\frac{3x^3}{c^3} - \frac{x^2}{c^2}\left(4\cdot7 + \frac{1}{2}\frac{y^2}{c^2}\right) + \frac{x}{c}\left(1\cdot35 - 1\cdot6625\frac{y^2}{c^2}\right) + \left(2\cdot35\frac{y^2}{c^2} + \frac{1}{4}\frac{y^4}{c^4}\right) = 0.$$

The pressure coefficient at zero incidence is

$$C_p = \frac{T_0}{c}\left[5\cdot0159 - 19\cdot8198\frac{x}{c} + 13\cdot6831\frac{x^2}{c^2} - 1\cdot3984\frac{y^2}{c^2} - 0\cdot0475\frac{x^3}{c^3} - 0\cdot9887\frac{xy^2}{c^3}\right].$$

The trailing edge is supersonic and the solution is valid for the whole surface.

The wave-drag coefficient at zero lift is calculated in section 8.

(iii) $\tan\gamma/\tan m = 0\cdot9, \quad \gamma = 45$ deg, $\quad M = 1\cdot345 \quad$ (Fig. 4).

The shape of the surface is given by the equation

$$\frac{z}{2t_0} = \left(1\cdot6 - \frac{x}{c}\right)\left(1 - \frac{x}{c} + \frac{1}{3}\frac{y^2}{c^2}\right)\left(\frac{x^2 - y^2}{c^2}\right)^{1/2},$$

and

$$2t_0 = 1\cdot7361T_0.$$

The maximum-thickness line is

$$\frac{3x^3}{c^3} - \frac{x^2}{c^2}\left(5\cdot2 + \frac{2}{3}\frac{y^2}{c^2}\right) + \frac{x}{c}\left(1\cdot6 - \frac{4\cdot4}{3}\frac{y^2}{c^2}\right) + \left(2\cdot6\frac{y^2}{c^2} + \frac{1}{3}\frac{y^4}{c^4}\right) = 0.$$

The pressure coefficient at zero incidence is

$$C_p = \frac{T_0}{c}\left[4\cdot7173 - 17\cdot4005\frac{x}{c} + 10\cdot9794\frac{x^2}{c^2} - 0\cdot7999\frac{y^2}{c^2} - 0\cdot0503\frac{x^3}{c^3} - 1\cdot0460\frac{xy^2}{c^3}\right].$$

The trailing edge is supersonic and the solution is valid for the whole surface.

(iv) $\tan\gamma/\tan m = 1/\sqrt{3}$, $\gamma = 30$ deg, $M = 1\cdot414$ (Fig. 5).

The shape of the surface is given by the equation

$$\frac{z}{2t_0} = \left(1\cdot25 - \frac{x}{c}\right)\left(1 - \frac{x}{c} + \frac{y^2}{c^2}\right)\left(\frac{x^2 - 3y^2}{c^2}\right)^{1/2},$$

and

$$2t_0 = 2\cdot4375T_0.$$

The maximum-thickness line is

$$\frac{3x^3}{c^3} - \frac{x^2}{c^2}\left(4\cdot5 + 2\frac{y^2}{c^2}\right) + \frac{x}{c}\left(1\cdot25 - 4\cdot75\frac{y^2}{c^2}\right) + \left(6\cdot75\frac{y^2}{c^2} + 3\frac{y^4}{c^4}\right) = 0.$$

The pressure coefficient, at zero incidence, is

$$C_p = \frac{T_0}{c}\left[4\cdot0556 - 17\cdot2219\frac{x}{c} + 12\cdot9861\frac{x^2}{c^2} - 3\cdot4332\frac{y^2}{c^2} - 0\cdot4246\frac{x^3}{c^3} - 4\cdot0385\frac{xy^2}{c^3}\right].$$

The trailing edge is supersonic and the solution is valid for the whole surface.

(v) $\tan\gamma/\tan m = 1/\sqrt{3}$, $\gamma = 30$ deg, $M = 1\cdot414$ (Fig. 6).

The shape of the surface is given by the equation

$$\frac{z}{2t_0} = \left(1\cdot2 - \frac{x}{c}\right)\left(1 - \frac{x}{c} + 1\cdot274\frac{y^2}{c^2}\right)\left(\frac{x^2 - 3y^2}{c^2}\right)^{1/2},$$

and

$$2t_0 = 2\cdot5834T_0.$$

The maximum-thickness line is

$$\frac{3x^3}{c^3} - \frac{x^2}{c^2}\left(4\cdot4 + 2\cdot548\frac{y^2}{c^2}\right) + \frac{x}{c}\left(1\cdot2 - 4\cdot4712\frac{y^2}{c^2}\right) + \left(6\cdot6\frac{y^2}{c^2} + 3\cdot822\frac{y^4}{c^4}\right) = 0.$$

The pressure coefficient, at zero incidence, is

$$C_p = \left[\frac{T_0}{c}4\cdot1265 - 17\cdot8477\frac{x}{c} + 13\cdot8685\frac{x^2}{c^2} - 3\cdot2269\frac{y^2}{c^2} - 0\cdot5734\frac{x^3}{c^3} - 5\cdot4532\frac{xy^2}{c^3}\right].$$

The trailing edge is supersonic and the solution is valid for the whole surface.

(vi) $\tan\gamma/\tan m = 1/\sqrt{3}$, $\gamma = 30$ deg, $M = 1\cdot414$ (Fig. 7).

The shape of the surface is given by the equation

$$\frac{z}{2t_0} = \left(1\cdot26 - \frac{x}{c}\right)\left(1 - \frac{x}{c} + 0\cdot943\frac{y^2}{c^2}\right)\left(\frac{x^2 - 3y^2}{c^2}\right)^{1/2},$$

and

$$2t_0 = 2\cdot4101T_0.$$

The maximum-thickness line is

$$\frac{3x^3}{c^3} - \frac{x^2}{c^2}\left(4\cdot52 + 1\cdot886\frac{y^2}{c^2}\right) + \frac{x}{c}\left(1\cdot26 - 4\cdot8118\frac{y^2}{c^2}\right) + \left(6\cdot78\frac{y^2}{c^2} + 2\cdot829\frac{y^4}{c^4}\right) = 0.$$

The pressure coefficient, at zero incidence, is

$$C_p = \frac{T_0}{c}\left[4\cdot0421 - 17\cdot1044\frac{x}{c} + 12\cdot8188\frac{x^2}{c^2} - 3\cdot4798\frac{y^2}{c^2} - 0\cdot3959\frac{x^3}{c^3} - 3\cdot7656\frac{xy^2}{c^3}\right].$$

The trailing edge is supersonic and the solution is valid for the whole surface.

8. *Calculation of the Wave Drag at Zero Incidence.*—The wave drag at zero incidence is $D = D_p + D_n$, where D_p is the pressure integral and D_n is the drag due to the high pressure at the rounded leading edges of the wing. The pressure integral is found by integrating the component pressure, along the wind direction, over the plan form, and the corresponding drag coefficient is given by :

$$C_{Dp} \times \text{(area of plan form)} = 2\iint C_p \frac{\partial z}{\partial x}\,dx\,dy,$$

integrated over the plan form,

$$= -2\iint z\frac{\partial C_p}{\partial x}\,dx\,dy, \quad \ldots \quad (58)$$

since z is zero on the leading and trailing edges. R. T. Jones' formula for the force per unit length normal to the leading edge at any point is

$$F_n = \pi R\frac{\rho V^2}{2}\frac{\sin^2\gamma}{(1 - M^2\sin^2\gamma)^{1/2}}$$

where R is the radius of curvature of the leading edge[7]. Hence

$$D_n = 2\tan\gamma\int_0^c F_n\,dx,$$

and the corresponding drag coefficient is

$$C_{Dn} = \frac{2\pi\tan\gamma\ \sin^2\gamma}{S(1 - M^2\sin^2\gamma)^{1/2}}\int_0^c R\,dx, \quad \ldots \quad (59)$$

where S is the area of the plan form.

The total wave drag coefficient at zero lift is

$$C_D = C_{Dp} + C_{Dn}.$$

For a surface given by equation (30), it can be shown that

$$C_{Dp} = -\frac{16t_0^2}{c^3S}\left[\sum_{r=0}^{7}(C_rI_{2r}) + \sum_{r=0}^{4}(C_r'I_{2r}')\right], \quad \ldots \quad (60)$$

where S is the area of the plan form,

$$I_{2r} = \int_0^Y \frac{y^{2r}}{\left\{\left(\frac{a}{c}y^2 + c\right)^2 - k^2y^2\right\}^{1/2}}\,dy, \quad \ldots \quad (61)$$

$$I_{2r}' = \int_Y^{d/k}\frac{y^{2r}}{(d^2 - k^2y^2)^{1/2}}\,dy. \quad \ldots \quad (62)$$

C_r, C_r' are constants (given in Appendix III) and

$$Y^2 = \frac{c}{a}(d - c). \quad \ldots \quad (63)$$

If α_1^2, α_2^2 are the roots of the equation

$$c^4X^2 - (k^2 - 2a)c^2X + a^2 = 0, \qquad (\alpha_1^2 > \alpha_2^2)$$

and $\alpha_1 Y = \text{sn}\, u$, where sn u is a Jacobian elliptic function of modulus $\sigma = |\alpha_2/\alpha_1|$, it can be shown that

$$I_{2r} = \frac{1}{c\alpha_1^{2r+1}} \int_0^u \text{sn}^{2r} u'\, du'$$

$$\equiv \frac{1}{c\alpha_1^{2r+1}} S_{2r} \;. \qquad \qquad (64)$$

The reduction formula for S_{2r} is

$$(2r - 1)\sigma^2 S_{2r} = \text{cn}\, u\ \text{dn}\, u\ \text{sn}^{2r-3} u + (2r - 2)(1 + \sigma^2)S_{2r-2} - (2r - 3)S_{2r-4} \;. \qquad (65)$$

If α_1, α_2 are complex,

$$I_{2r} = \frac{1}{c\alpha^{2r+1}\lambda^{2r+1}} \int_0^v \frac{\text{sn}^{2r} v'\ \text{dn}^{2r} v'}{\text{cn}^{2r} v'}\, dv' \qquad (66)$$

$$\equiv \frac{1}{c\alpha^{2r+1}\lambda^{2r+1}} S_{2r}'',$$

where $(\text{sn}\, v\ \text{dn}\, v)/\text{cn}\, v = \lambda\alpha Y$, α is the real part of α_1 or α_2, and the square of the modulus of the elliptic functions is $1/\lambda^2 = \alpha^2/\alpha_1\alpha_2$. The reduction formula for S_{2r}'' is

$$(2r - 1)S_{2r}'' = \frac{\text{sn}^{2r-3} v\ \text{dn}^{2r-3} v}{\text{cn}^{2r-1} v}\left(\text{dn}^4 v - \frac{1 - \lambda^2}{\lambda^4}\text{sn}^4 v\right)$$

$$+ 4(r - 1)\frac{(2 - \lambda^2)}{\lambda^2} S_{2r-2}'' - (2r - 3)S_{2r-4}'' \;. \qquad (67)$$

The formula for I_{2r}' is

$$I_{2r}' = \frac{d^{2r}}{k^{2r+1}} S_{2r}', \qquad (68)$$

where

$$S_{2r}' = \int_{\sin^{-1} kY/d}^{\pi/2} \sin^{2r} u\, du,$$

and

$$2rS_{2r}' = \cos\theta \sin^{2r-1}\theta + (2r - 1)S_{2r-2}' \qquad (69)$$

where $d \sin\theta = kY$. The drag coefficient due to the leading-edge force is given by

$$C_{D\,n} = \frac{8\pi}{k^2\varkappa} \cdot \frac{t_0^2}{S}\int_0^1 (b - x)^2\left(1 - x + \frac{ax^2}{k^2}\right)^2 x\, dx. \qquad (70)$$

The total wave-drag coefficient at zero lift is

$$C_D = -\frac{16t_0^2}{c^3S}\left[\sum_{r=0}^{7}(C_rI_{2r}) + \sum_{r=0}^{4}(C_r'I_{2r}')\right]$$

$$+ \frac{8\pi}{k^2\varkappa}\frac{t_0^2}{S}\int_0^1 (b - x)^2\left(1 - x + \frac{ax^2}{k^2}\right)^2 x\, dx \qquad (71)$$

where $b = d/c$.

Examples :

For surface (ii), (Fig. 3), $a = \frac{1}{4}$, $b = 1 \cdot 35$, $k = 1$. α_1, α_2 are real and the modulus $\sigma = 1$, amplitude of $u = 36$ deg 16 min, $u = 0 \cdot 680135 = S_0$, $S_2 = 0 \cdot 088527$. Hence, using formulae (64), (65) and (68) to (71), and the formulae given in Appendix III, the drag coefficients for $T_0/c = 0 \cdot 1$, $M = 1 \cdot 345$, are

$$C_{Dp} = 0 \cdot 016, \qquad C_{Dn} = 0 \cdot 051, \qquad C_D = 0 \cdot 067. \qquad (72)$$

For the corresponding complete delta wing with Squire sections[4],

(a) if thickness/chord $= T_0/c = 0 \cdot 1$,

$$C_{Dp} = 0 \cdot 040, \qquad C_{Dn} = 0 \cdot 048, \qquad C_D = 0 \cdot 088. \qquad (73)$$

(b) if thickness/chord $= T_0/1 \cdot 35c = 0 \cdot 074$,

$$C_{Dp} = 0 \cdot 022, \qquad C_{Dn} = 0 \cdot 026, \qquad C_D = 0 \cdot 048. \qquad (74)$$

For surface (v), (Fig. 6), $a = 1 \cdot 274$, $b = 1 \cdot 2$, $k^2 = 3$. α_1, α_2 are complex, $1/\lambda^2 = 0 \cdot 5887$, amplitude of $v = 25$ deg 20 min, $v = 0 \cdot 4510 = S_0''$, $S_2'' = 0 \cdot 03055$. Hence, using formulae (66) to (71) and the formulae given in Appendix III, the drag coefficients for $T_0/c = 0 \cdot 1$, $M = 1 \cdot 414$ are :

$$C_{Dp} = 0 \cdot 066, \qquad C_{Dn} = 0 \cdot 020, \qquad C_D = 0 \cdot 086. \qquad (75)$$

For the corresponding complete delta wing with Squire sections :

(*i*) if thickness/chord $= T_0/c = 0 \cdot 1$,

$$C_{Dp} = 0 \cdot 033, \qquad C_{Dn} = 0 \cdot 015, \qquad C_D = 0 \cdot 048. \qquad (76)$$

(*ii*) if thickness/chord $= T_0/1 \cdot 2c = 0 \cdot 083$,

$$C_{Dp} = 0 \cdot 023, \qquad C_{Dn} = 0 \cdot 010, \qquad C_D = 0 \cdot 033. \qquad (77)$$

(*B*) The change in pressure distribution and drag, at supersonic speeds and zero lift, on a certain swept-back wing having symmetrical sections with rounded leading edges, when the local thickness/chord ratio is modified.

9. *The surface* $\frac{z}{2t_0} = \left(1 + ak^2 \frac{y^2}{c^2} + bk^4 \frac{y^4}{c^4}\right)\left(1 - \frac{x}{c}\right)\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2}$.

By combining the solutions found in sections 3, 4, 5 and those quoted in section 6, a formula is found for the pressure coefficient for a wing whose surface is given by the equation

$$\frac{z}{2t_0} = \left(1 + ak^2 \frac{y^2}{c^2} + bk^4 \frac{y^4}{c^4}\right)\left(1 - \frac{x}{c}\right)\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2}, \qquad (78)$$

where a, b are positive or negative constants. This surface is obtained by multiplying the ordinates of the sections, parallel to the wind direction, of the Squire wing[4] :

$$\frac{z}{2t_0} = \left(1 - \frac{x}{c}\right)\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2} \qquad (79)$$

by the factor

$$1 + ak^2 \frac{y^2}{c^2} + bk^4 \frac{y^4}{c^4}.$$

The root section and the position of the maximum-thickness line of the two wings are the same.

The pressure coefficient for surface (78) is given by

$$C_p = (C_p)_0 - (C_p)_x + a(C_p)_{k^2y^2} - a(C_p)_{k^2xy^2} + b(C_p)_{k^4y^4} - b(C_p)_{k^4xy^4}. \qquad (80)$$

The formulae for the first three terms in (80) are given in section 6, equations (45), (46), (50), and formulae for the last three terms are given by equations (19), (17), (28), (40). The formulae have been computed for $\varkappa^2 = 1 - \tan^2 \gamma/\tan^2 m = 0\cdot48$, (e.g., $M = 1\cdot6$, $\gamma = 30$ deg.,) and the isobars for surface (78) for $k^2 = 3$ and (1) $a = 0\cdot29$, $b = 1\cdot12$, (2) $a = -0\cdot28$, $b = 2\cdot06$, are shown in Figs. 8a and 9a. The variations of local thickness are shown in Figs. 8b and 9b. The pressures on the root chord are the same for surface (78) as for surface (79), but for (79) the pressure coefficient C_p is independent of y, and the isobars are straight lines across the span.

It can be shown that the wave-drag coefficient at zero lift is

$$C_D = - \frac{8t_0 k}{c^3} \int_{y=0}^{c/k} \int_{x=ky}^{c} \left(1 + ak^2 \frac{y^2}{c^2} + bk^4 \frac{y^4}{c^4}\right)\left(1 - \frac{x}{c}\right)(x^2 - k^2y^2)^{1/2} \frac{\partial C_p}{\partial x}\, dx\, dy$$

$$+ \frac{8\pi}{k\varkappa} \frac{t_0^2}{c^2} \int_0^1 (1 - x)^2(1 + ax^2 + bx^4)^2 x\, dx \,. \qquad \ldots \qquad (81)$$

This expression has been integrated and, as an example, C_D has been computed for surface (1), giving $C_D = 0\cdot0142$. For the corresponding Squire wing, $C_D = 0\cdot0113$.

APPENDIX I

Values of a_m', b_m' for $\tan\gamma/\tan m = 0\cdot9$ and $\tan\lambda/\tan m = 1/\sqrt{3}$, and the corresponding values of λ_m for surfaces (a) and (b)

m	$\frac{\tan\gamma}{\tan m}$	$\varkappa^2$	a_m'	b_m'	$k^4\Delta\varkappa^2(1-\varkappa^2)\lambda_m$ for surface (a)	$k^4\Delta(1-\varkappa^2)^2\lambda_m$ for surface (b)
1	0·9	0·19	1·253234	0·318608	−0·17738	0·51289
2			0·938440	0·060422	1·01162	0·50017
3			0·188324	0·004363	−1·05659	−4·03616
1	$1/\sqrt{3}$	2/3	1·672390	0·685432	−0·13761	0·28242
2			1·027598	0·111571	1·02703	−0·02914
3			0·633349	0·047442	−0·42719	−0·11461

Values of f_1, f_2, F_1, F_2, F_3, F_4

$\frac{\tan\gamma}{\tan m}$	f_1	f_2	F_1	F_2	F_3	F_4
0·9	0·7642	1·7347	2·760	−0·4260	0·1610	0·4100
$1/\sqrt{3}$	0·6655	1·5701	2·573	−0·3539	0·2179	0·2859

APPENDIX II

Values of a_m', b_m', c_m' for $n = 4, 5, 6$ and $\varkappa^2 = 0\cdot48$

n	m	$\varkappa^2$	a_m'	b_m'	c_m'
4	1	0·48	1·496680	0·526984	
	2		0·996920	0·098430	
	3		0·466401	0·026107	
5	1	0·48	0·587724	0·067866	
	2		1·167935	0·232021	
	3		1·533230	0·557407	
6	1	0·48	0·708305	0·124208	0·003163
	2		1·328999	0·386635	0·015439
	3		1·648489	0·705280	0·039797
	4		2·234203	1·613449	0·375263

Values of f_1, f_2, F_3, F_4, for $\varkappa^2 = 0\cdot48$

$\varkappa^2$	f_1	f_2	F_3	F_4
0·48	0·7165	1·6576	0·1900	0·3447

Values of λ_m

Surface	m	$k^4\Delta(1-\varkappa^2)^2\lambda_m$
$\frac{z}{2t_0} = k^2\frac{xy^2}{c^3}\left(\frac{x^2-k^2y^2}{c^2}\right)^{1/2}$	1	0·382421
	2	0·007362
	3	−0·422980
		$k^4\Delta(1-\varkappa^2)^2\lambda_m$
$\frac{z}{2t_0} = k^4\frac{y^4}{c^4}\left(\frac{x^2-k^2y^2}{c^2}\right)^{1/2}$	1	0·291089
	2	0·123991
	3	0·144067
		$k^6\Delta(1-\varkappa^2)^3\lambda_m$
$\frac{z}{2t_0} = k^4\frac{xy^4}{c^5}\left(\frac{x^2-k^2y^2}{c^2}\right)^{1/2}$	1	0·090968
	2	+0·012447
	3	−0·014357
	4	−0·078386

APPENDIX III

Formulae for the constants C_r, C_r' in formula (60) for C_{Dp}.
[B, C, E, F are coefficients in the formula (57) for C_p]

$$C_0/c^3 = B(\tfrac{1}{6}b - \tfrac{1}{12}) + C(\tfrac{1}{12}b - \tfrac{1}{20}) + E(\tfrac{1}{20}b - \tfrac{1}{30})$$

$$C_1/c = B[\tfrac{2}{3}ab - (\tfrac{5}{12}a - \tfrac{7}{24}k^2)] + C[b(\tfrac{5}{12}a - \tfrac{7}{24}k^2) - (\tfrac{3}{10}a - \tfrac{13}{120}k^2)]$$
$$+ E[b(\tfrac{3}{10}a - \tfrac{13}{120}k^2) - (\tfrac{7}{30}a - \tfrac{7}{120}k^2)] + F(\tfrac{1}{6}b - \tfrac{1}{12})$$

$$C_2c = B[b(a^2 + \tfrac{2}{5}ak^2 - \tfrac{1}{3}k^4) - (\tfrac{5}{6}a^2 - \tfrac{7}{8}ak^2 + \tfrac{7}{30}k^4)]$$
$$+ C[b(\tfrac{5}{6}a^2 - \tfrac{7}{8}ak^2 + \tfrac{7}{30}k^4) - (\tfrac{3}{4}a^2 - \tfrac{13}{30}ak^2 + \tfrac{1}{6}k^4)]$$
$$+ E[b(\tfrac{3}{4}a^2 - \tfrac{13}{30}ak^2 + \tfrac{1}{6}k^4) - (\tfrac{7}{10}a^2 - \tfrac{7}{24}ak^2 - \tfrac{11}{240}k^4)]$$
$$+ F[b(\tfrac{2}{3}a + \tfrac{1}{15}k^2) - (\tfrac{5}{12}a - \tfrac{7}{24}k^2)]$$

$$C_3c^3 = B[a^2b(\tfrac{2}{3}a + \tfrac{4}{15}k^2) - a(\tfrac{5}{6}a^2 - \tfrac{7}{8}ak^2 + \tfrac{11}{60}k^4)]$$
$$+ C[ab(\tfrac{5}{6}a^2 - \tfrac{7}{8}ak^2 + \tfrac{11}{60}k^4) - (a^3 - \tfrac{13}{20}a^2k^2 + \tfrac{41}{105}ak^4 - \tfrac{2}{15}k^6)]$$
$$+ E[b(a^3 - \tfrac{13}{20}a^2k^2 + \tfrac{41}{105}ak^4 - \tfrac{2}{15}k^6) - (\tfrac{7}{6}a^3 - \tfrac{7}{12}a^2k^2 - \tfrac{11}{80}ak^4 - \tfrac{67}{840}k^6)]$$
$$+ F[b(a^2 + \tfrac{38}{105}ak^2 - \tfrac{1}{3}k^4) - (\tfrac{5}{6}a^2 - \tfrac{7}{8}ak^2 + \tfrac{19}{84}k^4)]$$

$$C_4c^5 = B[\tfrac{1}{6}a^4b - a^3(\tfrac{5}{12}a - \tfrac{7}{24}k^2)]$$
$$+ C[a^3b(\tfrac{5}{12}a - \tfrac{7}{24}k^2) - a^2(\tfrac{3}{4}a^2 - \tfrac{13}{30}ak^2 + \tfrac{22}{105}k^4)]$$
$$+ E[a^2b(\tfrac{3}{4}a^2 - \tfrac{13}{30}ak^2 + \tfrac{22}{105}k^4) - a(\tfrac{7}{6}a^3 - \tfrac{7}{12}a^2k^2 - \tfrac{11}{80}ak^4 + \tfrac{13}{210}k^6)]$$
$$+ F[a^2b(\tfrac{2}{3}a + \tfrac{5}{21}k^2) - a(\tfrac{5}{6}a^2 - \tfrac{7}{8}ak^2 + \tfrac{4}{21}k^4)]$$

$$C_5c^7 = -\tfrac{1}{12}a^5B + C[\tfrac{1}{12}a^5b - a^4(\tfrac{3}{10}a - \tfrac{13}{120}k^2)]$$
$$+ E[a^4b(\tfrac{3}{10}a - \tfrac{13}{120}k^2) - a^3(\tfrac{7}{10}a^2 - \tfrac{7}{24}ak^2 - \tfrac{11}{240}k^4)]$$
$$+ F[\tfrac{1}{6}a^4b - a^3(\tfrac{5}{12}a - \tfrac{7}{24}k^2)]$$

$$C_6c^9 = -\tfrac{1}{20}a^6C + E[\tfrac{1}{20}a^6b - a^5(\tfrac{7}{30}a - \tfrac{7}{120}k^2)] - \tfrac{1}{12}Fa^5$$

$$C_7c^{11} = -\tfrac{1}{30}a^7E$$

$$C_0'/c^3 = Bb^4(\tfrac{1}{6} - \tfrac{1}{12}b) + Cb^5(\tfrac{1}{12} - \tfrac{1}{20}b) + Eb^6(\tfrac{1}{20} - \tfrac{1}{30}b)$$

$$C_1'/c = Bb^2[\tfrac{7}{24}bk^2 + \tfrac{1}{6}ab^2] + Cb^3[-\tfrac{7}{24}k^2 + \tfrac{13}{120}bk^2 + \tfrac{1}{12}ab^2]$$
$$+ Eb^4[-\tfrac{13}{120}k^2 + \tfrac{7}{120}bk^2 + \tfrac{1}{20}ab^2] + Fb^4(\tfrac{1}{6} - \tfrac{1}{12}b)$$

$$C_2'c = Bk^2[\tfrac{1}{15}ab^2 - \tfrac{7}{30}bk^2 - \tfrac{1}{3}k^2] + Cbk^2[-\tfrac{7}{24}ab^2 - \tfrac{1}{6}bk^2 + \tfrac{7}{30}k^2]$$
$$+ Eb^2k^2[-\tfrac{13}{120}ab^2 + \tfrac{11}{240}bk^2 + \tfrac{1}{6}k^2]$$
$$+ Fb^2[\tfrac{1}{6}ab^2 + \tfrac{7}{24}bk^2 + \tfrac{1}{15}k^2]$$

$$C_3'c^3 = -\tfrac{1}{3}k^4aB + Ck^4(\tfrac{19}{84}ab + \tfrac{2}{15}k^2)$$
$$+ Ek^4(\tfrac{73}{420}ab^2 - \tfrac{67}{840}bk^2 - \tfrac{2}{15}k^2) + Fk^2(\tfrac{2}{21}ab^2 - \tfrac{19}{84}bk^2 - \tfrac{1}{3}k^2)$$

$$C_4'c^5 = -\tfrac{2}{15}k^6aE - \tfrac{1}{3}k^4aF.$$

PART II

The Effect of a Change of Mach Number on the Pressure Distribution and Drag at Supersonic Speeds on some Wings having given Camber and Twist

1. *Introduction.*—In R. & M. 2794[2], the effect of camber and twist on the pressure distribution and drag on some wings, of negligible thickness, at supersonic speeds is investigated. The shapes of some curved wings, with swept-back subsonic leading edges, are found, such that the thrust loading on the leading edges is removed or modified, while certain requirements with respect to camber and twist, or aerodynamic properties, are satisfied. The wings are designed for given Mach numbers and are such that, when they are at design incidence, (*a*) there are no leading-edge pressure singularities, and therefore no leading-edge thrust; or (*b*) the leading-edge singularity is modified so that its strength increases along the edge from zero at the apex to a maximum, and then decreases to zero at some point on the edge further downstream. The effect of additional incidence is also calculated.

In the present paper, the effect of a change of Mach number on the aerodynamic characteristics of a wing of type (*b*), designed for a given Mach number, is calculated.

The methods and notation used are those of R. & M. 2794[2]. x is measured downstream from the apex, y is measured to starboard, z is measured vertically upwards. The semi-apex angle γ is less than the Mach angle $\bar{\mu}(= \operatorname{cosec}^{-1} M)$. The surfaces are symmetrical with respect to the zx-plane and are set symmetrically to the wind direction, the apex pointing against the stream.

Some numerical examples showing the effect of a change of Mach number on the lift, drag and moment coefficients of a wing designed for a given Mach number, are given.

2. *Summary of General Results given in R. & M.* 2794[2].—Non-dimensional co-ordinates $x' = x\sigma/c$, $y' = y\sigma/c$, $z' = z\sigma/c$ are used; c is the maximum chord of the wing, and $1/\sigma$ is the distance, in maximum chord lengths (in the free-stream direction) from the apex, of the point of zero pressure on the leading edge. Since these co-ordinates are used throughout the report, the dashes are dropped, and X is written for $(x'^2 - k^2y'^2)^{1/2}$.

The following results are given in Ref. 2. The velocity potential

$$\Omega = A\Phi_2 + B\Phi_3^1 + C\Psi_3 + D\Phi_4^1 + E\Psi_4 \quad \ldots \quad (1)$$

gives the flow over the surface

$$z = ax + bx^2 + d_1x^3 + fx^4 + gk^2xy^2 + h_1k^2x^2y^2 + f(y), \quad \ldots \quad (2)$$

where (*cf.* equations (128), (130) of R. & M. 2794[2])

$$\left.\begin{aligned} a &= -(A + B + D) & f &= \tfrac{1}{4}(f_{12}D - f_{10}E) \\ b &= Af_1 & g &= f_5C - f_7B \\ d_1 &= \tfrac{1}{3}f_6B - f_4C & h_1 &= \tfrac{1}{2}(f_{11}E - f_{13}D) \end{aligned}\right\} \quad \ldots \quad (3)$$

A, B, C, D, E are constants, and $f_1, f_2, \ldots . f_{13}$ are functions of $(\tan\gamma)/(\tan\bar{\mu})$ given in Appendices I, II of R. & M. 2794[2]. (The constant δ which appears in Ref. 2 is here put equal to 1. There is no loss of generality, since this is eventually equivalent to including δ in the constants $A, \ldots . E$.)

The velocity potentials Φ_2, Φ_3^1, Ψ_3, Φ_4^1, Ψ_4, which are combined to give the velocity potential Ω in (1), are the five independent solutions given in R. & M. 2794[2] and are given by:

$$\begin{aligned} &\Phi_2 = \phi_1 - \phi_2, \\ &\Phi_3^1 = \phi_1 - \phi_3^1, && \Psi_3 = \phi_3^1 - k^2\phi_3^2, \\ &\Phi_4^1 = \phi_1 - \phi_4^1, && \Psi_4 = \phi_4^1 - k^2\phi_4^2, \end{aligned} \quad \ldots \quad (4)$$

where $\phi_1, \phi_2, \ldots\ldots$ are the 'basic' solutions given in R. & M. 2794[2] whose values at the plane $z = 0$ are: (putting $\delta = 1$, and writing x for $x' \equiv x\sigma/c$, etc.)

$$\left.\begin{aligned}
(\phi_1)_{z=0} &= \frac{Vc}{\sigma k E(\varkappa)} X, \\
(\phi_2)_{z=0} &= \frac{Vc}{\sigma k E(\varkappa)} xX, \\
(\phi_3{}^1)_{z=0} &= \frac{Vc}{\sigma k E(\varkappa)} x^2X, \\
(\phi_3{}^2)_{z=0} &= \frac{Vc}{\sigma k E(\varkappa)} y^2X, \\
(\phi_4{}^1)_{z=0} &= \frac{Vc}{\sigma k E(\varkappa)} x^3X, \\
(\phi_4{}^2)_{z=0} &= \frac{Vc}{\sigma k E(\varkappa)} xy^2X
\end{aligned}\right\} \qquad (5)$$

k is the cotangent of the semi-apex angle γ, and V is the free-stream velocity.
$\varkappa^2 = 1 - (\tan^2 \gamma)/(\tan^2 \bar{\mu})$ and $E(\varkappa)$ is the complete elliptic integral of the second kind of modulus $\varkappa$.

The pressure coefficient $C_{p\,0}$ and the lift, induced drag and pitching-moment coefficients $C_{L\,0}$, C_{Di}, $C_{M\,0}$, at design incidence, are given by:

$$- C_{p\,0} = \frac{2}{kE(\varkappa)} \left[A \left\{ \frac{x(1-x)}{X} - X \right\} + B \left\{ \frac{x(1-x^2)}{X} - 2xX \right\} \right.$$
$$\left. + C(3xX) + D \left\{ \frac{x(1-x^3)}{X} - 3x^2X \right\} + E \left\{ (4x^2 - k^2y^2)X \right\} \right] \qquad (6)$$

$$C_{L\,0} = \frac{2\pi}{kE(\varkappa)} [A(1-\sigma) + B(1-\sigma^2) + D(1-\sigma^3) + \tfrac{3}{4}(C + E\sigma)\sigma^2]$$
$$= \frac{2\pi}{kE(\varkappa)} [\tfrac{3}{4}(C+E)], \text{ when } \sigma = 1 \qquad (7)$$

$$C_{M\,0} = \frac{2\pi}{kE(\varkappa)} [\tfrac{1}{6}A\sigma + (\tfrac{4}{15}B - \tfrac{1}{5}C)\sigma^2 + (\tfrac{1}{3}D - \tfrac{1}{4}E)\sigma^3] \qquad (8)$$

$$C_{D\,i} = \frac{8\pi}{kE(\varkappa)} [AP_1 + BP_2 + CP_3 + DP_4 + EP_5]$$
$$- \frac{2\pi(k^2 - \beta^2)^{1/2}}{k^2[E(\varkappa)]^2\sigma^2} \int_0^\sigma x[A(1-x) + B(1-x^2) + D(1-x^3)]^2\, dx, \qquad (9)$$

where $P_1, \ldots . P_5$ are given in equations (144) of R. & M. 2794[2].

In R. & M. 2794[2], for a wing of given plan form, the constants $A, \ldots . E$ or $a, \ldots . h_1$, are chosen, and the corresponding coefficients of equation (2) or equation (1) determined for a given Mach number, that is for given values of $f_1, \ldots . f_{13}$. In the following sections, the constants $a, \ldots . h_1$ having been chosen to satisfy certain conditions for a given Mach number, the effect of a varying Mach number on the given wing is calculated. The variable σ is taken equal to 1, that is, at the design Mach number, the points of zero pressure on the leading edges are at the wing tips.

The above results are based on solutions of the linearised supersonic flow equation in terms of Lamé functions of the M class of degree n, for $n = 1, 2, 3, 4$. Since R. & M. 2794[2] was written, the solutions for $n = 5$ have been worked out, and the results are given in Appendix III. These solutions could be used to give three extra terms in expression (1) for Ω, and thus three more arbitrary constants. The equation of the resulting surface (2) would contain three extra terms, *viz.*, constant multiples of x^5, x^3y^2, and xy^4.

3. *The Pressure Coefficient and the Aerodynamic Characteristics of a given Surface at a Varying Mach Number* ($1 < M < \operatorname{cosec}\gamma$).—We first determine the velocity potentials corresponding to the separate terms of the equation of surface (2). Using equations (125), (91), (98), (105) of R. & M. 2794[2], we obtain the following formulae for the velocity potentials in terms of the basic solutions ϕ_1, ϕ_2, $\phi_3^{\ 1}$, $\phi_4^{\ 1}$ and the solutions Ψ_3, Ψ_4, given in Ref. 2. (*cf.* equations (4), (5) of this paper.)

z	*Velocity potential*
x	$-\phi_1$
x^2	$-\dfrac{1}{f_1}\phi_2$
x^3	$\dfrac{3}{f_5f_6 - 3f_4f_7}(f_7\Psi_3 - f_5\phi_3^{\ 1})$
x^4	$\dfrac{4}{f_{11}f_{12} - f_{10}f_{13}}(f_{13}\Psi_4 - f_{11}\phi_4^{\ 1})$
k^2xy^2	$\dfrac{3}{f_5f_6 - 3f_4f_7}(\tfrac{1}{3}f_6\Psi_3 - f_4\phi_3^{\ 1})$
$k^2x^2y^2$	$\dfrac{2}{f_{11}f_{12} - f_{10}f_{13}}(f_{12}\Psi_4 - f_{10}\phi_4^{\ 1})$

. . . . . (10)

The formulae for $f_1, \ldots . f_{13}$ are given in Appendix I, and a table of numerical values in Appendix II. It can be shown that :

$$\tfrac{1}{3}f_5f_6 - f_4f_7 = \frac{1}{4\varkappa^4(E(\varkappa))^2}[(4\varkappa^4 + 11\varkappa^2 - 11)(E(\varkappa))^2 + (1 - \varkappa^2)(16 - 8\varkappa^2)E(\varkappa)K(\varkappa) - 5(1 - \varkappa^2)^2(K(\varkappa))^2], \quad \ldots \quad (11)$$

and

$$f_{11}f_{12} - f_{10}f_{13} = \frac{1}{4\varkappa^8(E(\varkappa))^2}[(32\varkappa^8 - 64\varkappa^6 + 151\varkappa^4 - 119\varkappa^2 + 12)(E(\varkappa))^2 + (1 - \varkappa^2)(32\varkappa^6 - 126\varkappa^4 + 166\varkappa^2 - 24)E(\varkappa)K(\varkappa) + (1 - \varkappa^2)^2(12 - 47\varkappa^2 + 8\varkappa^4)(K(\varkappa))^2]. \quad \ldots \quad (12)$$

Since we are using the linear theory of supersonic flow, the velocity potential for the surface (2) can be obtained by combining the solutions given in (10). Hence it can be shown that the velocity potential giving the flow over the surface

$$z = ax + bx^2 + d_1x^3 + fx^4 + gk^2xy^2 + h_1k^2x^2y^2 + f(y)$$

is

$$\Omega = A_0\phi_1 + A\Phi_2 + B\Phi_3^{\ 1} + C\Psi_3 + D\Phi_4^{\ 1} + E\Psi_4 \quad \ldots \quad (13)$$

where $A_0 = -a - A - B - D$, (at the design Mach number, $A_0 = 0$)

$$A = \frac{b}{f_1}$$

$$\left.\begin{aligned} B &= \frac{3(d_1 f_5 + g f_4)}{f_5 f_6 - 3 f_4 f_7}, & C &= \frac{3 d_1 f_7 + g f_6}{f_5 f_6 - 3 f_4 f_7} \\ D &= \frac{2(2 f f_{11} + h_1 f_{10})}{f_{11} f_{12} - f_{10} f_{13}}, & E &= \frac{2(2 f f_{13} + h_1 f_{12})}{f_{11} f_{12} - f_{10} f_{13}} \end{aligned}\right\} \quad \ldots \quad (14)$$

Hence, since it is assumed that the surface lies close to the plane $z = 0$, the velocity potential on the surface is

$$(\Omega)_{z=0} = \frac{V}{kE(\varkappa)}\Bigg[-aX - \frac{b}{f_1} xX + \frac{3}{f_5 f_6 - 3 f_4 f_7} \{(d_1 f_7 + \tfrac{1}{3} g f_6) X^3 - (d_1 f_5 + g f_4) x^2 X\} + \frac{2}{f_{11} f_{12} - f_{10} f_{13}} \{(2 f f_{13} + h_1 f_{12}) x X^3 - (2 f f_{11} + h_1 f_{10}) x^3 X\}\Bigg] . \quad \ldots \quad (15)$$

The pressure coefficient at design incidence is

$$C_{p\,0} = -\frac{2}{V}\left(\frac{\partial \Omega}{\partial x}\right)_{z=0} = -\frac{2}{kE(\varkappa)}\Bigg[-\frac{ax}{X} - \frac{b}{f_1}\left(\frac{x^2}{X} + X\right) + \frac{3}{f_5 f_6 - 3 f_4 f_7}\left\{(3 d_1 f_7 + g f_6) x X - (d_1 f_5 + g f_4)\left(\frac{x^3}{X} + 2xX\right)\right\} + \frac{2}{f_{11} f_{12} - f_{10} f_{13}}\left\{(2 f f_{13} + h_1 f_{12})(4x^2 - k^2 y^2) X - (2 f f_{11} + h_1 f_{10})\left(\frac{x^4}{X} + 3x^2 X\right)\right\}\Bigg] . \quad \ldots \quad (16)$$

On the leading edges of the wing, $X = 0$, and $C_{p\,0} \to -(2/V)P/(x - k|y|)^{1/2}$, where P is the strength of the singularity in the axial velocity $(\partial \Omega / \partial x)_{z=0}$. P is equal to zero at $x = 0$ and where

$$x^3 + A_1 x^2 + B_1 x + C_1 = 0, \quad \ldots \quad (17)$$

where

$$\left.\begin{aligned} A_1 &= \frac{3(d_1 f_5 + g f_4)(f_{11} f_{12} - f_{10} f_{13})}{2(2 f f_{11} + h_1 f_{10})(f_5 f_6 - 3 f_4 f_7)} \\ B_1 &= \frac{b(f_{11} f_{12} - f_{10} f_{13})}{2(2 f f_{11} + h_1 f_{10})} \\ C_1 &= \frac{a(f_{11} f_{12} - f_{10} f_{13})}{2(2 f f_{11} + h_1 f_{10})} \end{aligned}\right\} \quad \ldots \quad (18)$$

If σ is taken equal to 1, that is the points of zero pressure on the leading edges at the design Mach number are taken at the wing tips, the formulae for the aerodynamic coefficients at design incidence are as follows :

The lift coefficient is

$$C_{L\,0} = \frac{2\pi}{kE(\varkappa)} [\tfrac{3}{4}(C + E) + A_0]. \qquad (19)$$

The total induced-drag coefficient is

$$C_{D\,i} = \frac{8\pi}{kE(\varkappa)} [A_0 P_0 + AP_1 + BP_2 + CP_3 + DP_4 + EP_5]$$
$$- \frac{2\pi(k^2 - \beta^2)^{1/2}}{k^2[E(\varkappa)]^2} \int_0^1 x[A_0 + A(1 - x) + B(1 - x^2) + D(1 - x^3)]^2\, dx, \qquad (20)$$

where

$$\left.\begin{aligned}
P_0 &= -\left(\tfrac{1}{4}a + \tfrac{1}{3}b + \tfrac{3}{8}d_1 + \tfrac{2}{5}f + \tfrac{1}{16}g + \tfrac{1}{10}h_1\right) \\
P_1 &= \tfrac{1}{24}b + \tfrac{3}{40}d_1 + \tfrac{1}{10}f + \tfrac{1}{240}h_1 \\
P_2 &= \tfrac{1}{15}b + \tfrac{1}{8}d_1 + \tfrac{6}{35}f + \tfrac{1}{140}h_1 \\
P_3 &= -3\left(\tfrac{1}{16}a + \tfrac{1}{10}b + \tfrac{1}{8}d_1 + \tfrac{1}{7}f + \tfrac{1}{96}g + \tfrac{1}{56}h_1\right) \\
P_4 &= \tfrac{1}{12}b + \tfrac{9}{56}d_1 + \tfrac{9}{40}f + \tfrac{2}{320}h_1 \\
P_5 &= -\left(\tfrac{3}{16}a + \tfrac{5}{16}b + \tfrac{45}{112}d_1 + \tfrac{15}{32}f + \tfrac{1}{32}g + \tfrac{7}{128}h_1\right)
\end{aligned}\right\} \qquad (21)$$

The pitching-moment coefficient (taken about the chordwise position distant two-thirds of the maximum chord from the vertex) is :

$$C_{M\,0} = \frac{2\pi}{kE(\varkappa)} \left(\tfrac{1}{6}A + \tfrac{4}{15}B - \tfrac{1}{5}C + \tfrac{1}{3}D - \tfrac{1}{4}E\right). \qquad (22)$$

The corresponding formulae for the flat delta wing at (small) incidence α are :

$$\left.\begin{aligned}
C_{L\,0} &= \frac{2\pi\alpha}{kE(\varkappa)} \\
C_{D\,i} &= \frac{2\pi\alpha^2}{kE(\varkappa)} - \frac{\pi\alpha^2(k^2 - \beta^2)^{1/2}}{k^2[E(\varkappa)]^2} \\
C_{M\,0} &= 0
\end{aligned}\right\} \qquad (23)$$

4. *Numerical Examples.*—Some numerical results for two wings, designed to satisfy certain conditions at given Mach numbers, are given. The shapes of the surfaces and their pressure distributions at design Mach number and design incidence, are shown in Figs. 10a and 11a. The variations in lift, drag and moment coefficients, as the Mach number varies, are shown in Figs. 10b, 11b. The results are compared with those for the corresponding flat delta wing having the same lift coefficient at the design Mach number, the incidence remaining unchanged. The formulae giving the shapes of the surfaces, and the numerical values of the lift, induced drag, and moment coefficients at different Mach numbers are given below. The positions of the point of zero pressure on the leading edge are also given.

(i) The first surface chosen is surface (xvi) of R. & M. 2794[2], designed to satisfy the following conditions at $M = 1 \cdot 442$. (σ is taken equal to 1)

(*a*) zero camber at the root,

(*b*) $C_{L\,0} = 0 \cdot 1$,

(*c*) minimum induced drag with conditions (*a*), (*b*) (using solutions given in section 2).

The shape of the surface is given by

$$z = -0\cdot05729x + 0\cdot69610xy^2 - 0\cdot18792x^2y^2 + f(y),$$

the co-ordinates being measured in chord lengths, since $\sigma = 1$.

The semi-apex angle γ is 30 deg.

The numerical values of the aerodynamic coefficients, and the positions of the point of zero pressure, are given in the following table :

M	x co-ordinate on leading edge where $P = 0$	C_{M0}	C_{L0}	C_{Di}	Flat delta wing at incidence 0·035 radians	
					C_{L0}	C_{Di}
1·015	1·099	0·0183	0·111	0·0021	0·126	0·00225
1·058	1·089	0·0175	0·110	0·0022	0·121	0·00228
1·127	1·076	0·0165	0·109	0·0024	0·116	0·00231
1·217	1·056	0·0154	0·107	0·0025	0·111	0·00235
1·323	1·030	0·0144	0·104	0·0026	0·105	0·00238
1·442	1	0·013	0·1	0·0027	0·1	0·00242
1·572	0·969	0·0125	0·095	0·00275	0·095	0·00245
1·709	0·940	0·0118	0·091	0·0028	0·090	0·0025
1·852	0·910	0·0114	0·086	0·00285	0·085	0·0026
2	0·880	0·0109	0·081	0·0030	0·081	0·0028

Finite values are given by the formulae at $M = 1$, but since the linear theory is used, the formulae are not valid when M approaches 1.

(ii) The second surface is chosen to satisfy conditions (a), (c) satisfied by surface (i), (with $\sigma = 1$, $\gamma = 30$ deg), but is designed for $C_{L0} = 0\cdot15$ at $M = 1\cdot6$. The shape of the surface is given by (in the non-dimensional co-ordinates) :

$$z = -0\cdot08882x + 1\cdot06442xy^2 - 0\cdot29578x^2y^2.$$

The numerical values of the aerodynamic coefficients, and the positions of the point of zero pressure, are given in the following table :

M	x co-ordinate on leading edge where $P = 0$	C_{M0}	C_{L0}	C_{Di}	Flat delta wing at incidence 0·056 radians	
					C_{L0}	C_{Di}
1·015	No point	0·0263	0·178	0·0054	0·201	0·00577
1·058	for $M<1\cdot09$	0·0255	0·177	0·0056	0·194	0·0058
1·127	1·184	0·0240	0·174	0·0059	0·186	0·0059
1·217	1·134	0·0223	0·171	0·0062	0·177	0·0060
1·323	1·089	0·0208	0·165	0·0065	0·169	0·0061
1·442	1·047	0·0195	0·159	0·0067	0·160	0·0062
1·572	1·008	0·0184	0·152	0·0068	0·152	0·00627
1·6	1	0·0182	0·15	0·00683	0·15	0·0063
1·709	0·975	0·0175	0·144	0·0069	0·144	0·0064
1·852	0·940	0·0168	0·137	0·0070	0·137	0·0066
2	0·862	0·0162	0·129	0·0074	0·130	0·0073

For both cases considered, it can be seen that the point of zero pressure on a leading edge moves along the edge towards the apex, or downstream of the wing tips, according as the Mach number is greater than or less than the design Mach number.

APPENDIX I

The functions $f_1, \ldots f_{13}$

$$f_1 = f_4 = \frac{1}{2\varkappa^2 E(\varkappa)} \{(2\varkappa^2 - 1)E(\varkappa) + (1 - \varkappa^2)K(\varkappa)\}$$

$$f_5 = \frac{3}{2\varkappa^2 E(\varkappa)} \{(1 + \varkappa^2)E(\varkappa) - (1 - \varkappa^2)K(\varkappa)\}$$

$$f_6 = \frac{1}{2\varkappa^4 E(\varkappa)} \{(1 - \varkappa^2)(2 + 3\varkappa^2)K(\varkappa) - 2(1 + \varkappa^2 - 3\varkappa^4)E(\varkappa)\}$$

$$f_7 = \frac{1}{2\varkappa^4 E(\varkappa)} \{(2 - 3\varkappa^2 + \varkappa^4)E(\varkappa) - 2(1 - \varkappa^2)^2 K(\varkappa)\}$$

$$f_{10} = \frac{1}{2\varkappa^4 E(\varkappa)} \{2(1 - \varkappa^2)(1 + 2\varkappa^2)K(\varkappa) - (2 + 3\varkappa^2 - 8\varkappa^4)E(\varkappa)\}$$

$$f_{11} = \frac{3}{2\varkappa^4 E(\varkappa)} \{2(1 - \varkappa^2 + \varkappa^4)E(\varkappa) - (1 - \varkappa^2)(2 - \varkappa^2)K(\varkappa)\}$$

$$f_{12} = \frac{1}{6\varkappa^6 E(\varkappa)} \{(1 - \varkappa^2)(8 + 7\varkappa^2 + 12\varkappa^4)K(\varkappa) - (8 + 3\varkappa^2 + 7\varkappa^4 - 24\varkappa^6)E(\varkappa)\}$$

$$f_{13} = \frac{1}{2\varkappa_0 E(\varkappa)} \{(8 - 11\varkappa^2 + \varkappa^4 + 2\varkappa^6)E(\varkappa) - (1 - \varkappa^2)(8 - 7\varkappa^2 - \varkappa^4)K(\varkappa)\}$$

APPENDIX II

The functions $f_1, f_4, \ldots f_{13}$. Numerical Values

$\frac{\tan\gamma}{\tan\bar{\mu}}$	$f_1 = f_4$	f_5	f_6	f_7	f_{10}	f_{11}	f_{12}	f_{13}
0	0·5	3	1	0	1·5	3	1	0
0·1	0·5135	2·9600	1·0624	0·0048	1·5751	2·9744	1·1040	0·0142
0·2	0·5390	2·8831	1·1774	0·0176	1·7163	2·9358	1·2929	0·0508
0·3	0·5690	2·7929	1·3093	0·0359	1·8786	2·9002	1·5044	0·1010
0·4	0·6000	2·6999	1·4429	0·0571	2·0430	2·8713	1·7143	0·1578
0·5	0·6300	2·6097	1·5703	0·0799	2·2007	2·8495	1·9123	0·2163
0·6	0·6585	2·5247	1·6894	0·1030	2·3476	2·8338	2·0944	0·2735
0·7	0·6845	2·4463	1·7969	0·1257	2·4817	2·8235	2·2583	0·3284
$\sqrt{52}$	0·6899	2·4304	1·8190	0·1303	2·5088	2·8213	2·2918	0·3386
0·8	0·7085	2·3743	1·8952	0·1473	2·6038	2·8167	2·4066	0·3786
0·9	0·7300	2·3093	1·9820	0·1685	2·7125	2·8146	2·5365	0·4306
1·0	0·7500	2·2500	2·0625	0·1875	2·8125	2·8125	2·6563	0·4688

APPENDIX III

Solutions of the Linearised Supersonic Flow Equation in Terms of the Lamé Functions of the M Class of Degree $n = 5$

For $n = 5$, there are three M Lamé functions of the form

$$E_5{}^m(\mu) = (|\mu^2 - k^2|)^{1/2} P_5{}^m(\mu), \quad m = 1, 2, 3 \qquad \ldots \quad (1)$$

where

$$P_5{}^m(\mu) = \mu^4 - k^2 a_m \mu^2 + k^4 b_m, \qquad \ldots \quad (2)$$

$$27a_m{}^3 - (60\varkappa^2 + 42)a_m{}^2 + (32\varkappa^4 + 68\varkappa^2 + 16)a_m - 2\varkappa^2(12\varkappa^2 + 8) = 0 \qquad \ldots \quad (3)$$

$$b_m = \frac{\varkappa^2 a_m}{(12\varkappa^2 + 8 - 9a_m)}, \qquad \ldots \quad (4)$$

where

$$\varkappa^2 = 1 - \frac{\tan^2 \gamma}{\tan^2 \bar{\mu}}$$

(*cf.* general solutions for $n = 2N + 1$ in Ref. 2).

The roots of equations (3), (4), correct to six decimal places, for different values of $\dfrac{\tan \gamma}{\tan \bar{\mu}}$ are given in Appendix IV.

x, y, z are written for the non-dimensional co-ordinates $x' = x\sigma/c$, $y' = y\sigma/c$, $z' = z\sigma/c$.

The solution for the velocity potential

$$\varphi_m = C_5 r^5 F_5(\mu) E_5(\nu),$$

with

$$C_5 = \frac{\delta V \beta^5 \sigma^4}{c^4 E(\varkappa)},$$

gives

$$\varphi(_m)_{z=0} = \frac{\delta V c(x^2 - k^2 y^2)^{1/2}}{\sigma k E(\varkappa)(1 - a_m + b_m)} [(\varkappa^4 - a_m \varkappa^2 + b_m)x^4$$
$$+ (a_m \varkappa^2 - 2b_m)(1 - \varkappa^2)k^2 x^2 y^2 + b_m(1 - \varkappa^2)^2 y^4] \qquad \ldots \quad (5)$$

and

$$z_m = \frac{\delta(1 - \varkappa^2)}{E(\varkappa)} (1 - a_m + b_m)(k^{11} I_m)[\tfrac{1}{5}(\varkappa^4 - a_m \varkappa^2 + b_m)x^5$$
$$+ \tfrac{1}{3}(1 - \varkappa^2)(a_m \varkappa^2 - 2b_m)k^2 x^3 y^2 + (1 - \varkappa^2)^2 b_m k^4 x y^4] + f(y) \qquad \ldots \quad (6)$$

where

$$k^{11} I_m = k^{11} \int_k^\infty \frac{d}{dt} \left[\frac{1}{t[P_m{}^5(t)]^2(t^2 - h^2)^{1/2}}\right] \frac{dt}{(t^2 - k^2)^{1/2}}$$
$$= \frac{1}{a_m{}^2 - 4b_m} \left[\tfrac{1}{2}\left\{\frac{a_m}{\varkappa^2 b_m} + \frac{2\varkappa^2 - a_m}{(1 - \varkappa^2)^2 \varkappa^2 (\varkappa^4 - a_m \varkappa^2 + b_m)}\right.\right.$$
$$- 3\left(\frac{2 - 2a_m + a_m{}^2 - 2b_m}{(1 - \varkappa^2)(1 - a_m + b_m)^2}\right) + \frac{\varkappa^2 - 2}{(1 - \varkappa^2)^2}\left(\frac{2 - a_m}{1 - a_m + b_m}\right)$$
$$\left. + \frac{4}{(1 - \varkappa^2)(1 - a_m + b_m)}\right\} E(\varkappa) - \left\{2\left(\frac{a_m - 1}{b_m(1 - a_m + b_m)}\right)\right.$$
$$\left.\left. + \frac{\tfrac{1}{2}[a_m{}^2 - 2b_m - 2a_m(a_m{}^2 - b_m) + a_m{}^4 + 4a_m{}^2 b_m - 14b_m{}^2 - 4a_m b_m(a_m{}^2 - 3b_m)]}{b_m{}^2(1 - a_m + b_m)^2}\right\} K(\varkappa)\right]. \quad (7)$$

By constructing potentials of the form

$$\phi_5^s = \sum_{m=1}^{3} [\lambda_m k^4(1 - a_m + b_m)\varphi_m], \quad s = 1, 2, 3 \quad \ldots \quad (8)$$

we obtain the three basic solutions

$$(\phi_5^1)_{z=0} = \frac{V\delta c}{\delta k E(\varkappa)} x^4 X,$$

$$(\phi_5^2)_{z=0} = \frac{V\delta c}{\delta k E(\varkappa)} x^2 y^2 X,$$

$$(\phi_5^3)_{z=0} = \frac{V\delta c}{\delta k E(\varkappa)} y^4 X.$$

The shapes of the corresponding surfaces are given by

$$z_{5,s} = \sum_{m=1}^{3} [k^4 \lambda_m(1 - a_m + b_m) z_m], \quad s = 1, 2, 3. \quad \ldots \quad (11)$$

For the solution ϕ_5^1, we obtain

$$\lambda_1 = \frac{1}{\varkappa^4 k^4 \Delta} (a_2 b_3 - a_3 b_2),$$

where

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

and similar expressions for λ_2, λ_3.

For the solution ϕ_5^2,

$$\lambda_1 = \frac{1}{k^6 \varkappa^2 (1 - \varkappa^2)\Delta} \left[b_2 - b_3 + \frac{1}{\varkappa^2}(a_2 b_3 - a_3 b_2) \right] \quad \ldots \quad (12)$$

and for the solution ϕ_5^3,

$$\lambda_1 = \frac{1}{k^8 (1 - \varkappa^2)^2 \Delta} \left[\frac{1}{\varkappa^4}(a_2 b_3 - a_3 b_2) + \frac{2}{\varkappa^2}(b_2 - b_3) - (a_2 - a_3) \right]. \quad \ldots \quad (13)$$

The values of a_m, b_m are given in Appendix IV, and the values of λ_m for $s = 1, 2, 3$ in Appendix V.

Hence we form three independent solutions of the type given in R. & M. 2794[2].

(i) Using the basic solutions ϕ_5^1, ϕ_5^2, we construct the induced velocity-potential

$$\Psi_5^1 = \phi_5^1 - k^2 \phi_5^2,$$

for which $(X \equiv (x'^2 - k^2 y'^2)^{1/2})$

$$(\Psi_5^1)_{z=0} = \frac{V\delta c}{\sigma k E(\varkappa)} x^2 X^3, \quad \ldots \quad (14)$$

and the pressure coefficient is

$$C_{p\,0} = \frac{-2\delta}{k E(\varkappa)} [(5x^3 - 2k^2 x y^2) X]. \quad \ldots \quad (15)$$

The shape of the surface, at design incidence, is given by

$$z = z_{5,1} - k^2 z_{5,2} \quad \ldots \quad (16)$$

(ii) Using the basic solutions $\phi_5{}^2$, $\phi_5{}^3$, we construct the induced velocity potential

$$\Psi_5{}^2 = \phi_5{}^2 - k^2\phi_5{}^3,$$

for which

$$(\Psi_5{}^2)_{z=0} = \frac{V\delta c}{\sigma k E(\varkappa)} y^2 X^3, \quad \ldots \quad (17)$$

and

$$C_{p\,0} = \frac{-2\delta}{kE(\varkappa)} (3xy^2 X) \quad \ldots \quad (18)$$

The shape of the surface at design incidence is given by

$$z = z_{5,2} - k^2 z_{5,3} \quad \ldots \quad (19)$$

(iii) Using the basic solutions ϕ_1, $\phi_5{}^1$, we construct the induced velocity potential

$$\Phi_5{}^1 = \phi_1 - \phi_5{}^1,$$

for which

$$(\Phi_5{}^1)_{z=0} = \frac{V\delta c}{\sigma k E(\varkappa)} (1 - x^4) X, \quad \ldots \quad (20)$$

and

$$C_{p\,0} = \frac{-2\delta}{kE(\varkappa)} \left[\frac{x(1 - x^4)}{X} - 4x^3 X\right] \quad \ldots \quad (21)$$

The shape of the surface at the design incidence is

$$z = z_1 - z_{5,1} \quad \ldots \quad (22)$$

Example

For $\varkappa^2 = 0 \cdot 48$, the surfaces corresponding to the three basic solutions for $n = 5$ are given by :

$$z_{5,1} = -\delta(0 \cdot 5610x^5 - 0 \cdot 2444k^2x^3y^2 + 0 \cdot 1321k^4xy^4)$$
$$k^2 z_{5,2} = \delta(0 \cdot 0470x^5 - 1 \cdot 0071k^2x^3y^2 + 0 \cdot 3631k^4xy^4)$$
$$k^4 z_{5,3} = -\delta(0 \cdot 0977x^5 - 1 \cdot 4080k^2x^3y^2 + 6 \cdot 8350k^4xy^4).$$

The surfaces corresponding to the basic solutions for $n = 1, 2, 3, 4$ are given by :

$$z_1 = -\delta x$$
$$z_2 = -0 \cdot 6899\delta x^2$$
$$z_{3,1} = -\delta(0 \cdot 6063x^3 - 0 \cdot 1303k^2xy^2)$$
$$k^2 z_{3,2} = \delta(0 \cdot 0835x^3 - 2 \cdot 3001k^2xy^2)$$
$$z_{4,1} = -\delta(0 \cdot 5729x^4 - 0 \cdot 1693k^2x^2y^2)$$
$$k^2 z_{4,2} = \delta(0 \cdot 0542x^4 - 1 \cdot 2414k^2x^2y^2) \, .$$

APPENDIX IV

Numerical Values of a_m, b_m for the Lamé Function $E_5{}^m(\mu) = (\mu^4 - a_m k^2 \mu^2 + b_m k^4)(|\mu^2 - k^2|)^{1/2}$

$\frac{\tan \gamma}{\tan \bar{\mu}}$	a_1	b_1	a_2	b_2	a_3	b_3	$\varkappa^2$
0	0·666667	0·047619	1·111111	0·111111	2	1	1
0·1	0·664970	0·047377	1·103081	0·109729	1·987504	0·987536	0·99
0·2	0·659505	0·046607	1·079327	0·105665	1·950064	0·950563	0·96
0·3	0·648970	0·045153	1·040976	0·099180	1·887832	0·890343	0·91
0·4	0·630543	0·042697	0·990590	0·090794	1·801089	0·808959	0·84
0·5	0·598776	0·038677	0·933138	0·081362	1·690307	0·709324	0·75
0·6	0·544852	0·032358	0·876653	0·072022	1·556272	0·595150	0·64
0·7	0·459468	0·023469	0·829013	0·063494	1·400408	0·471012	0·51
$\sqrt{52}$	0·436934	0·021341	0·820339	0·061748	1·364949	0·444049	0·48
0·8	0·338897	0·013161	0·790825	0·054722	1·225834	0·342759	0·36
0·9	0·184935	0·004078	0·750701	0·040478	1·029338	0·192502	0·19
1·0	0	0	0·666667	0	0·888889	0	0

APPENDIX V

Numerical Values of λ_m in the basic Solutions for $n = 5$

$\frac{\tan \gamma}{\tan \bar{\mu}}$	$k^4\lambda_1$ $s=1$	$k^4\lambda_2$	$k^4\lambda_3$	$k^6\lambda_1$ $s=2$	$k^6\lambda_2$	$k^6\lambda_3$	$k^8\lambda_1$ $s=3$	$k^8\lambda_2$	$k^8\lambda_3$	$\varkappa^2$
0	2·62500	—1·68750	0·06250							1
0·1	2·69840	—1·74222	0·06413	0·68660	114·06225	—12·70684	0·32045	86·20341	10117·588	0·99
0·2	2·93344	—1·91772	0·06934	0·78825	29·67611	—3·33745	0·40779	22·81665	654·9521	0·96
0·3	3·37805	—2·24977	0·07930	0·99635	14·03547	—1·61400	0·53905	11·13325	137·3924	0·91
0·4	4·11647	—2·79575	0·09651	1·39406	8·49012	—1·02646	0·80435	7·10968	47·44718	0·84
0·5	5·24759	—3·59616	0·12636	2·13750	5·74962	—0·93854	1·33448	5·22539	21·88458	0·75
0·6	6·82132	—4·56100	0·18108	3·46218	3·99047	—0·67115	2·36691	4·13520	12·33554	0·64
0·7	9·07704	—5·52492	0·29250	5·75745	2·74579	—0·65701	4·32642	3·53619	8·15027	0·51
$\sqrt{52}$	9·75534	—5·74514	0·33006	6·46478	2·54675	—0·66486	4·95533	3·49116	7·60481	0·48
0·8	14·04160	—6·88558	0·56013	10·78680	2·00364	—0·73407	8·90245	3·75386	6·18168	0·36
0·9	23·96306	—8·93885	0·97539	20·41064	1·77704	—0·85380	17·92285	3·06294	5·45887	0·19
1·0	37·54870	—11·46503	1·61535	33·33941	1·98244	—1·12316	30·68293	5·40664	6·13070	0

Conclusion.—In Part I of this paper, formulae have been found for the pressure distribution and wave drag at zero incidence, at supersonic speeds, for some finite swept-back wings, having symmetrical sections with rounded leading edges and wing tips perpendicular to the root chord. The formulae derived enable a numerical comparison with the drag of a complete delta wing to be made. (*cf.* equations (72) to (77).)

Formulae have also been found for calculating the change in pressure distribution on a Squire wing, when the local thickness/chord ratio, particularly towards the wing tips, is modified. The same method could be applied to any surface of the type of those given in Refs. 4 or 1.

Within the limits of the linearised theory of supersonic flow, a fairly full investigation into the effect of camber and twist on the pressure distribution and drag on a curved wing has now been made. In Ref. 2, wings were designed for given Mach numbers, such that the thrust loading on a leading edge was removed, or decreased to zero at some point on the edge. The effects of varying the position of the point of zero pressure, and of a change of incidence were calculated.

In Part II of the present paper, the effect of a change of Mach number has been calculated. Some additional solutions of the linearised supersonic flow equation are given in Appendices III, IV of Part II. The formulae given in section 3 for any Mach number can easily be extended to include these, or any higher order, solutions.

Acknowledgement.

Acknowledgement is due to Mrs. B. M. Osman for the help she has given with the computation and for preparing most of the drawings.

LIST OF SYMBOLS

Parts I and II :

γ		Apex semi-angle
x		Chordwise co-ordinate (measured downstream from the apex)
y		Spanwise co-ordinate (positive to starboard)
z		Normal co-ordinate (positive upwards)
M		Mach number
β	=	$(M^2 - 1)^{1/2}$
k	=	$\cot \gamma$
V		Free-stream velocity
ρ		Free-stream density
$K(\varkappa)$		Complete elliptic integral of the first kind, modulus $\varkappa$
$E(\varkappa)$		Complete elliptic integral of the second kind, modulus $\varkappa$

Part I :

c		Chord in the vertical plane of symmetry
t_0		Constant determining thickness (in section 6)
	or	Maximum thickness of the wing in the vertical plane of symmetry (in section 9)

T_0		Maximum thickness of the wing in the vertical plane of symmetry (in section 6)
γ, μ, ν		*cf.* equations (1), (2)
m		Mach angle
h	$=$	$(\cot^2 \gamma - \cot^2 m)^{1/2}$
$\varkappa$	$=$	h/k
ϕ, Φ		Induced velocity potential
C_p		Pressure coefficient
$E_n(\mu)$		Standard Lamé function of the K class of degree n
$F_n(\mu)$		Lamé function of the second kind of the K class of degree n
$R_n(\mu)$	$=$	$F_n(\mu)/E_n(\mu)$
a_m b_m, c_m		*cf.* equations (6), (20), (30)
$a_m' = a_m/k^2$		$b_m' = b_m/k^4$ $\quad c_m' = c_m/k^6$
$a_m'' = a_m'/k^2$		$b_m'' = b_m'/k^2$ $\quad c_m'' = c_m'/k^2$
λ_m		*cf.* equations (14, (18), (24), (34)
D		Total drag
D_p		Pressure integral
D_n		Drag due to pressure at rounded leading edge
C_D	$=$	$C_{D\,p} + C_{D\,n}$, total wave-drag coefficient at zero lift
f_1, f_2, F_1, F_2, F_3, F_4		*cf.* equations (45) to (54)
Y	$=$	$\left[\frac{c}{a}(d - c)\right]^{1/2}$ (*cf.* equation (63))
C_r, C_r'		*cf.* formula (60)
a, d		*cf.* equation (55)
b	$=$	d/c
$A, B, C,$ D, E, F		*cf.* equation (57)

Part II :

c	Maximum chord of a triangular wing
δ	Small dimensionless constant, proportional to design lift coefficient $C_{L\,0}$.
$1/\sigma$	Distance in maximum chord lengths, (in free-stream direction) from the apex, of point of zero pressure on a leading edge

$x' = \frac{x\sigma}{c}$, $y' = \frac{y\sigma}{c}$, $z' = \frac{z\sigma}{c}$	Non-dimensional co-ordinates (The dashes are dropped in the text)
X	$= (x'^2 - k^2y'^2)^{1/2}$
γ, μ, ν	*cf.* equations (1), (2) of Ref. 2
$\bar{\mu}$	Mach angle
$\varkappa^2$	$1 - \tan^2 \gamma/\tan^2 \bar{\mu}$
$E_n(\mu)$	Standard Lamé function of the M class of degree n
$F_n(\mu)$	Lamé function of the second kind of the M class of degree n
$P_n(\mu)$	$\{E_n(\mu)\}/(\|\mu^2 - k^2\|)^{1/2}$
φ	Induced velocity potential, *cf.* Appendix III (5)
Ψ, Φ	Induced velocity potential, *cf.* equation (4)
ϕ	Induced velocity potential, *cf.* equation (5)
Ω	Induced velocity potential, *cf.* equation (1)
$C_{p\,0}$	Pressure coefficient at design incidence
P	Strength of singularity in axial velocity on a leading edge
$C_{L\,0}$	Lift coefficient at design incidence
$C_{M\,0}$	Pitching-moment coefficient
$C_{D\,i}$	Total induced-drag coefficient at design incidence
$f_1, \ldots . f_{13}$	Functions of $\tan \gamma/\tan \bar{\mu}$ given in Appendices I, II
$A, B, C,$ D, E, A_0	*cf.* equations (1), (13)
$a, b, d_1,$ f, g, h_1	*cf.* equations (2), (3)
A_1, B_1, C_1	*cf.* equations (17), (18)
$P_0, P_1, P_2,$ P_3, P_4, P_5	*cf.* equations (20), (21)
a_m, b_m	*cf.* Appendix III, (2), (3), (4)

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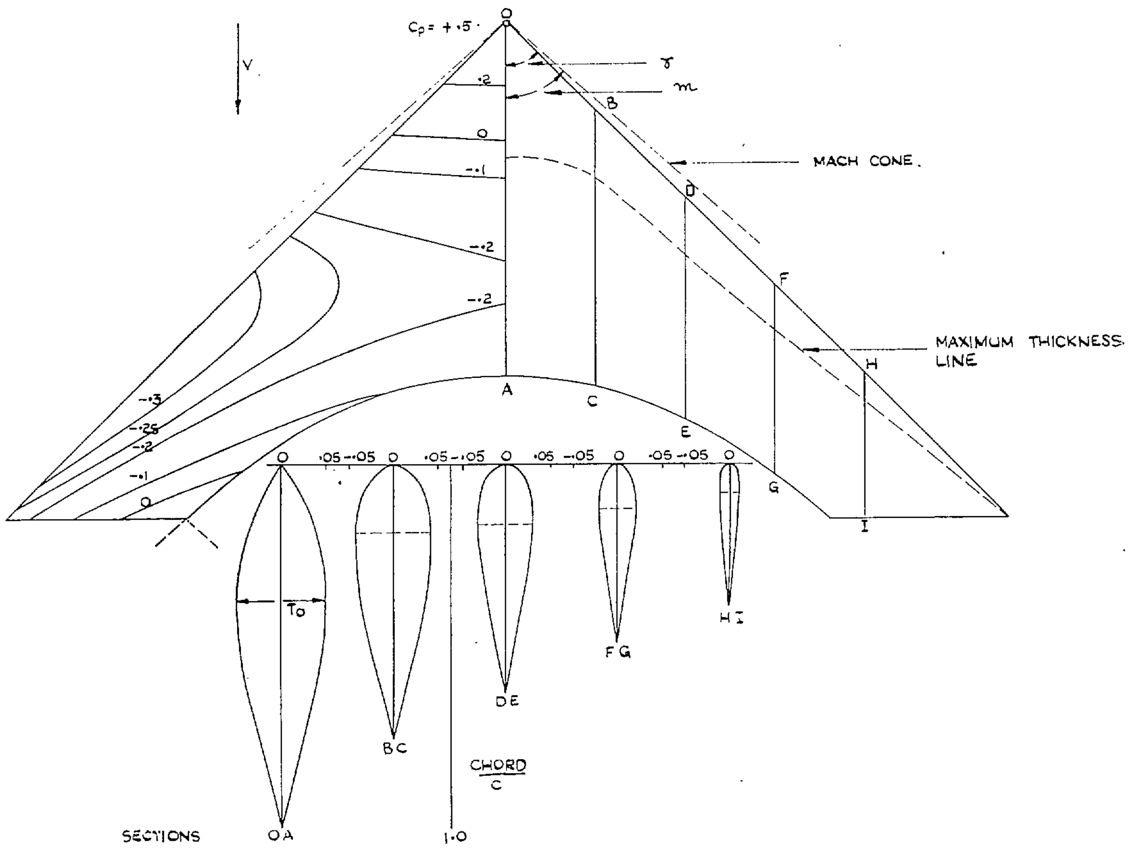

FIG. 1. Surface (i*a*), shape and pressure distribution. $M = 1\cdot345$. $T_0/c = 0\cdot1$.

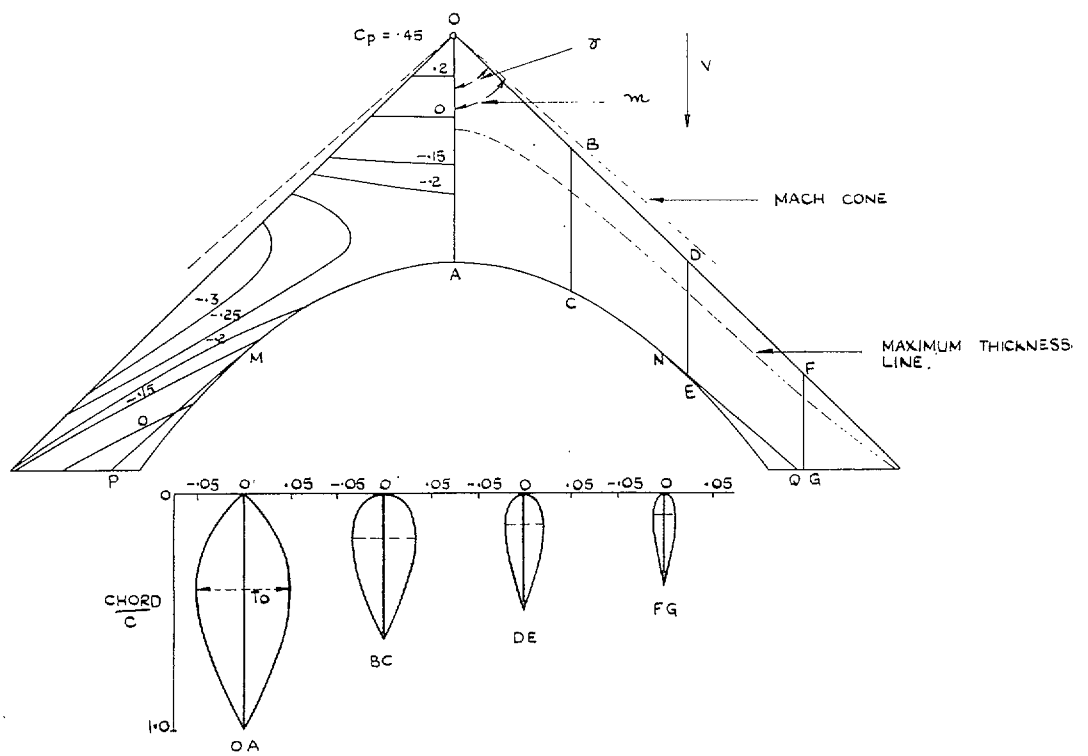

FIG. 2. Surface (i*b*), shape and pressure distribution. $M = 1\cdot345$. $T_0/c = 0\cdot10$.
Solution not valid behind the lines MP, NQ.

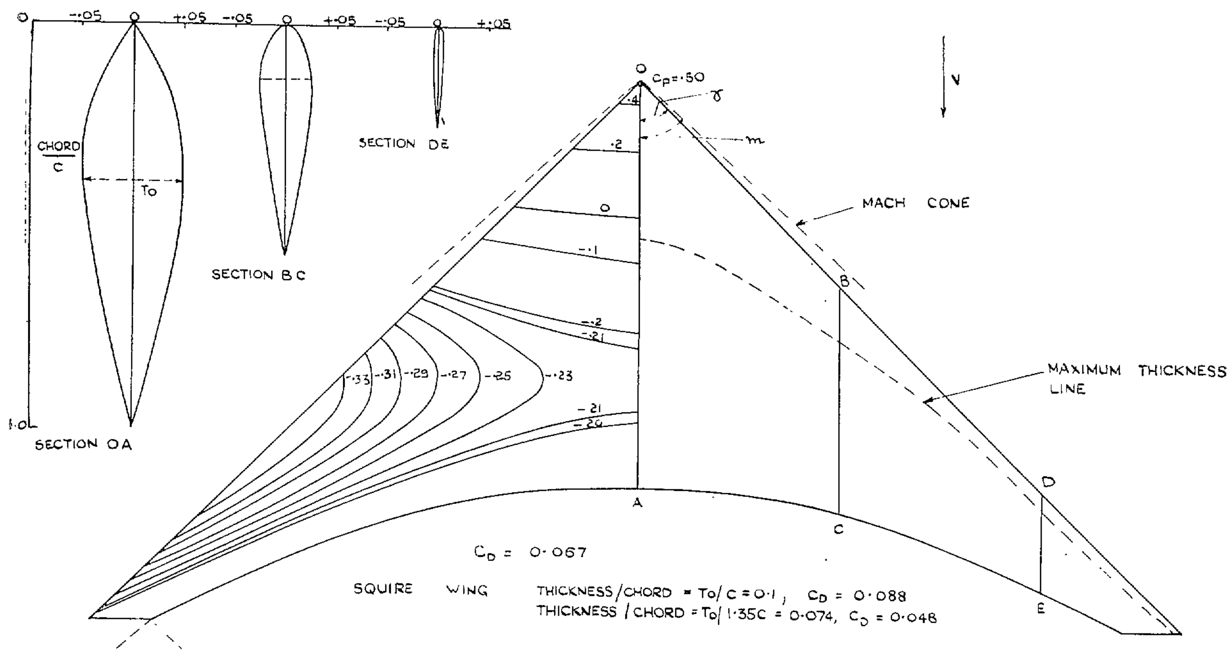

FIG. 3. Surface (ii), shape and pressure distribution. $M = 1 \cdot 345$. $T_0/c = 0 \cdot 1$.

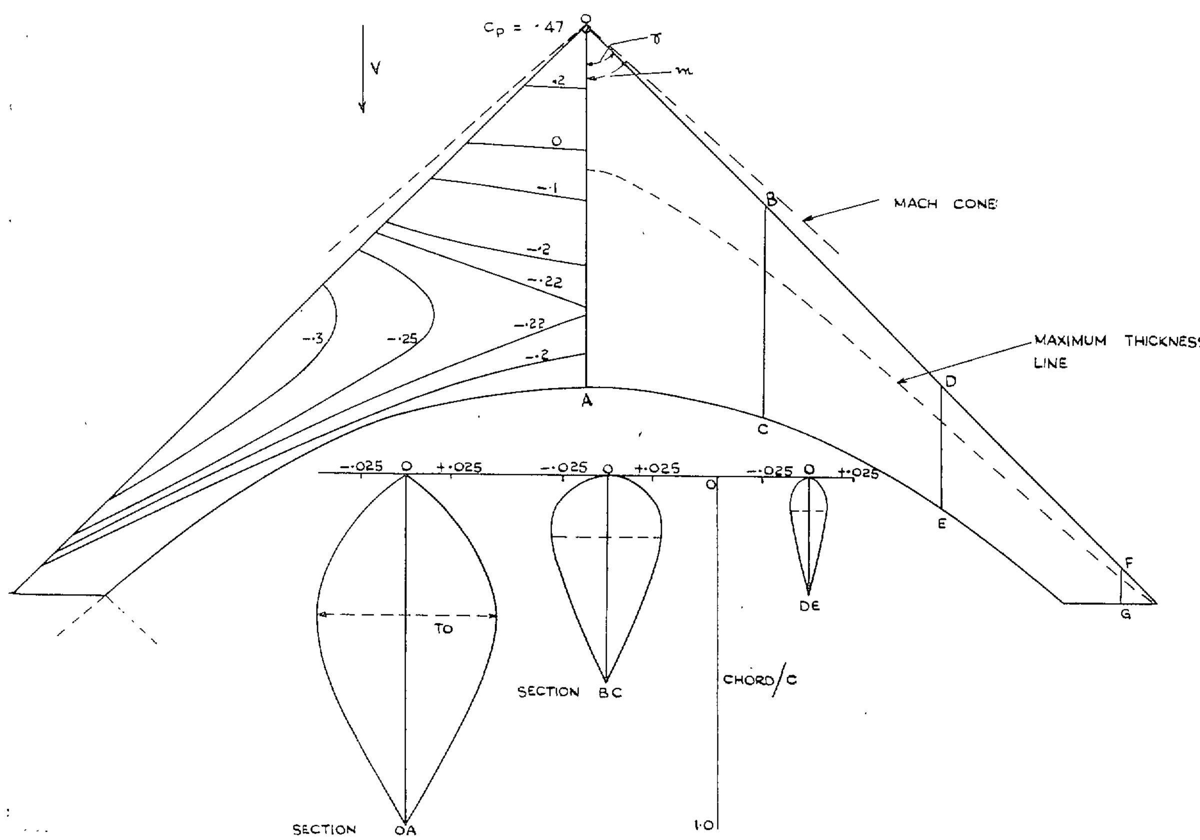

FIG. 4. Surface (iii), shape and pressure distribution. $M = 1 \cdot 345$. $T_0/c = 0 \cdot 1$.

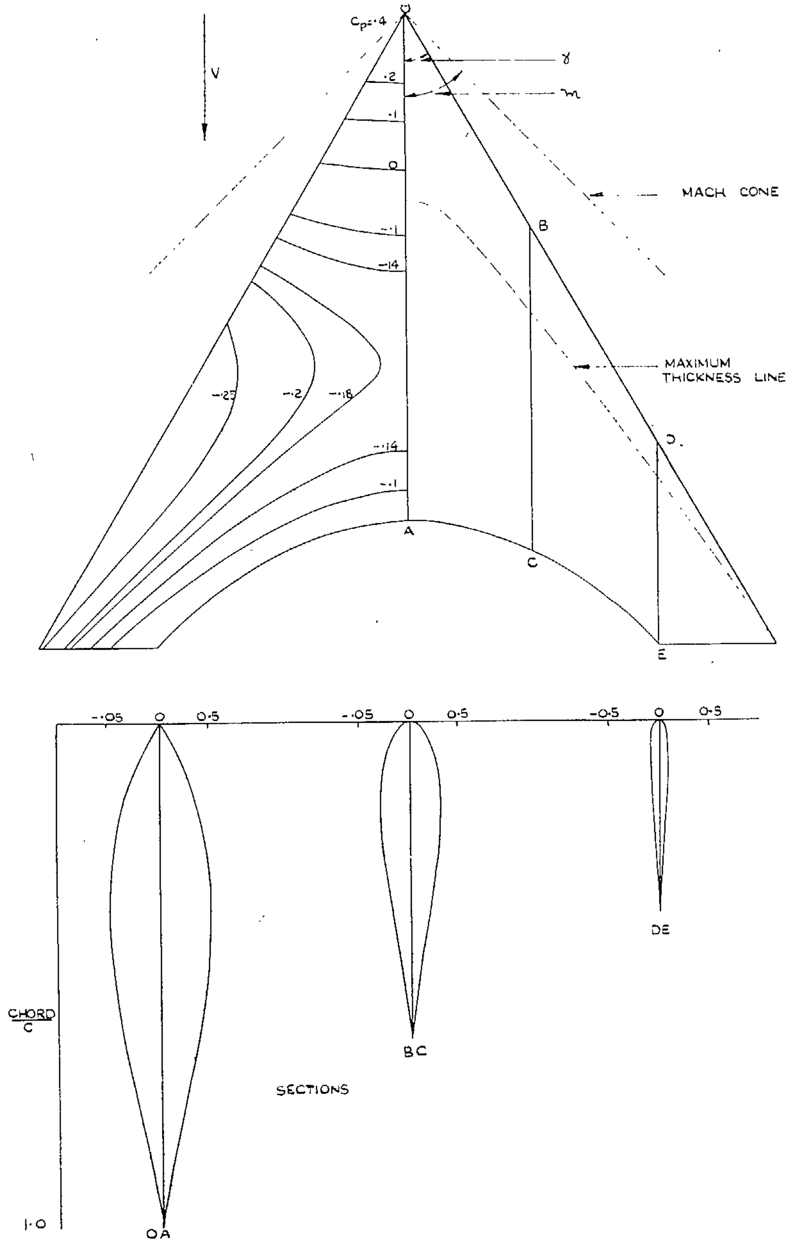

FIG. 5. Surface (iv), shape and pressure distribution. $M = 1 \cdot 414$. $T_0/c = 0 \cdot 1$.

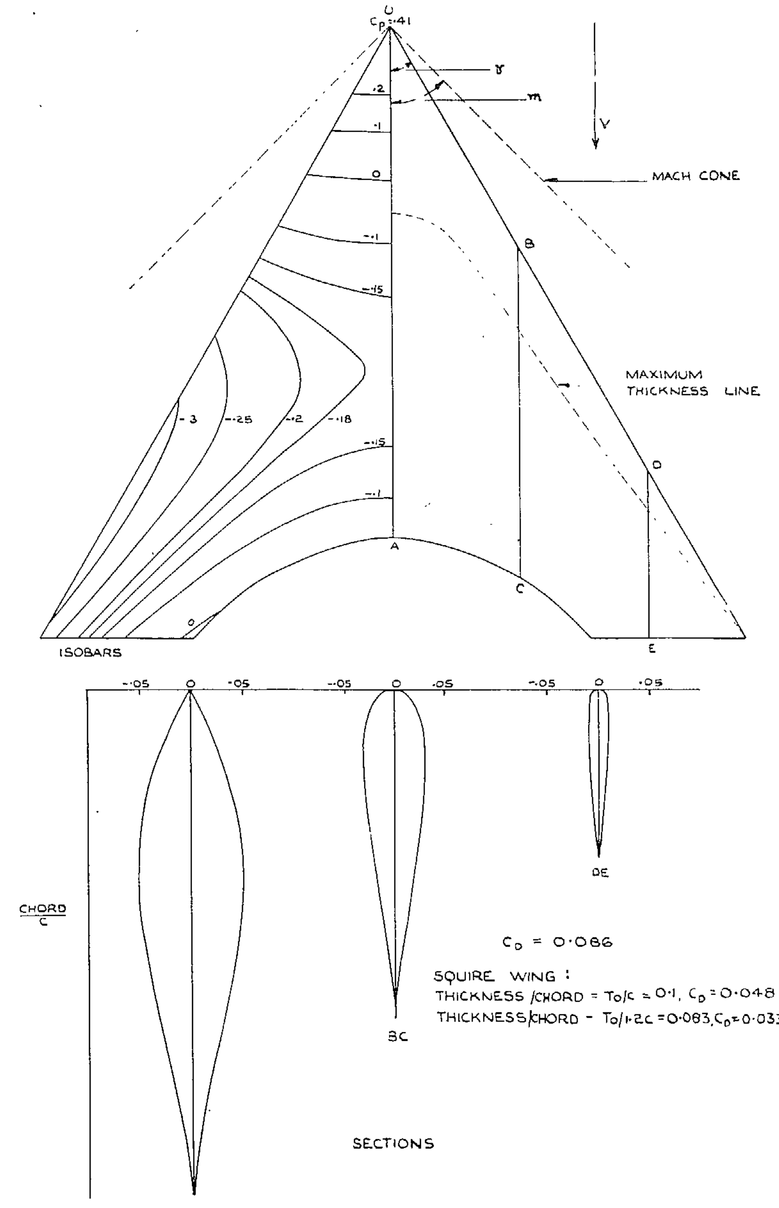

FIG. 6. Surface (v), shape and pressure distribution. $M = 1 \cdot 414$. $T_0/c = 0 \cdot 1$.

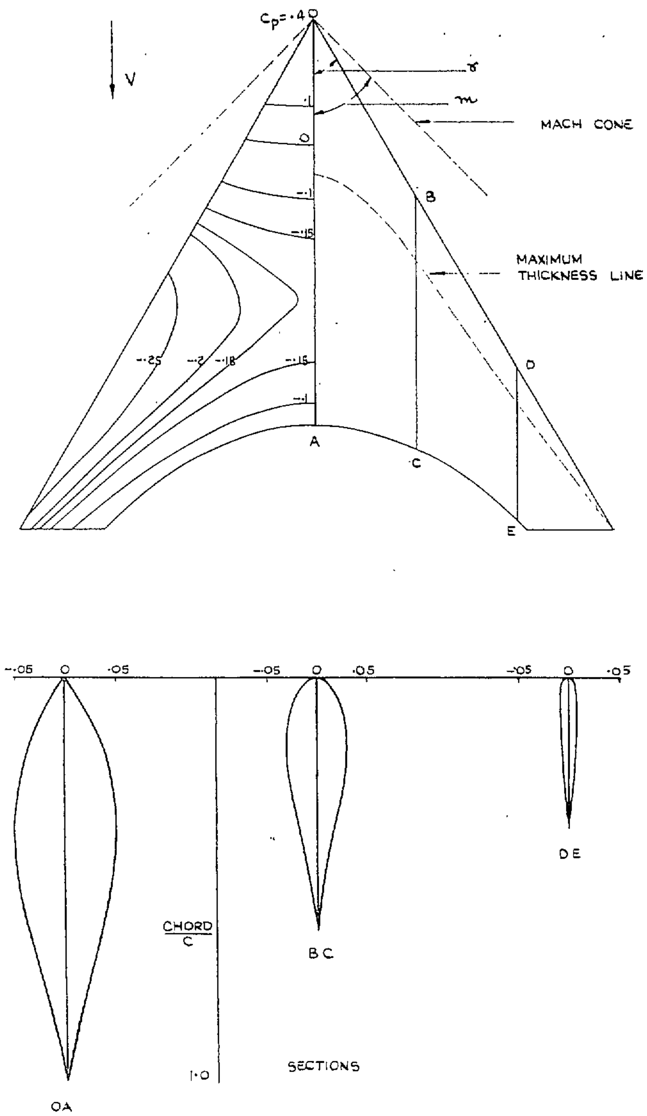

FIG. 7. Surface (vi), shape and pressure distribution. $M = 1 \cdot 414$. $T_0/c = 0 \cdot 1$.

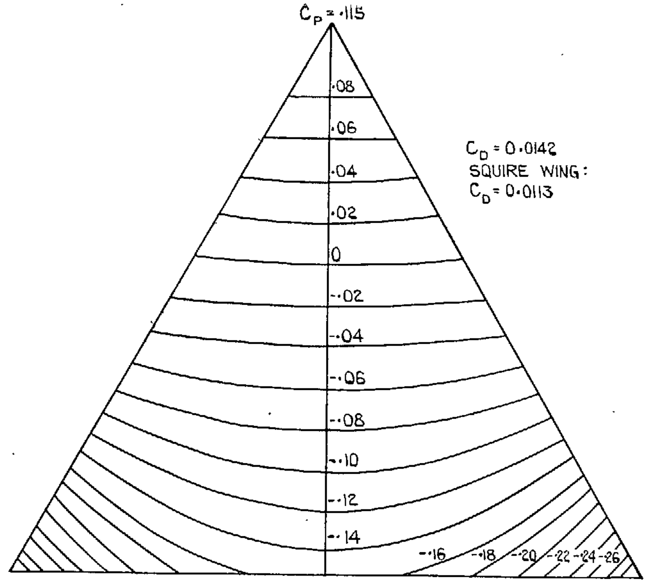

FIG. 8a. Surface (1), isobars at $M = 1 \cdot 6$.

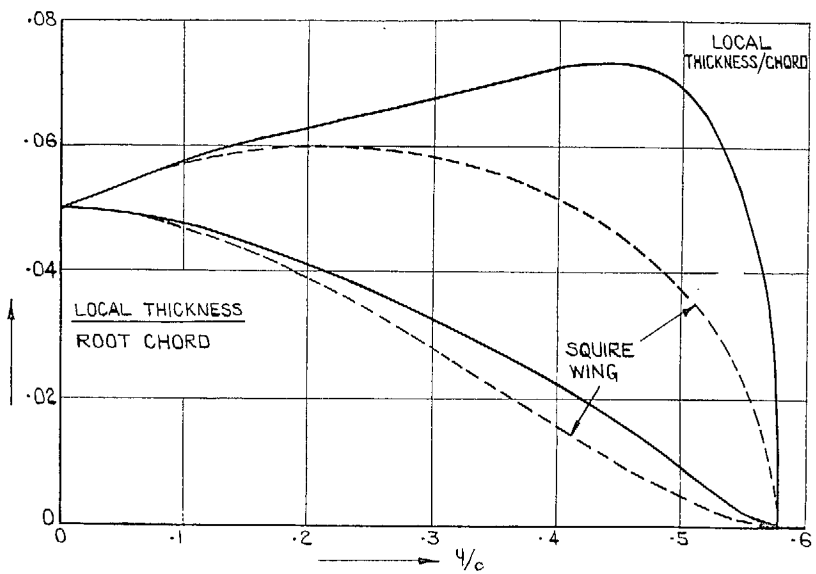

FIG. 8b. Surface (1), variation in local thickness.

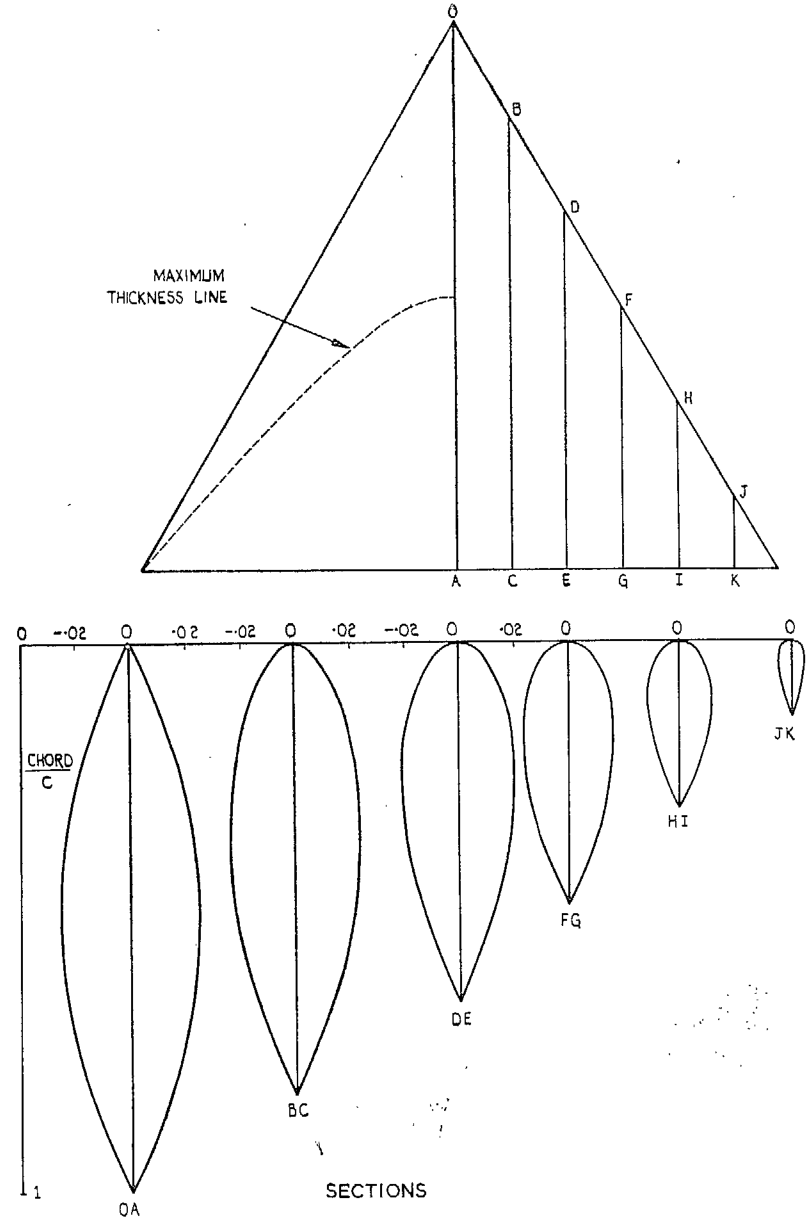

FIG. 8c. Shape of surface (1).

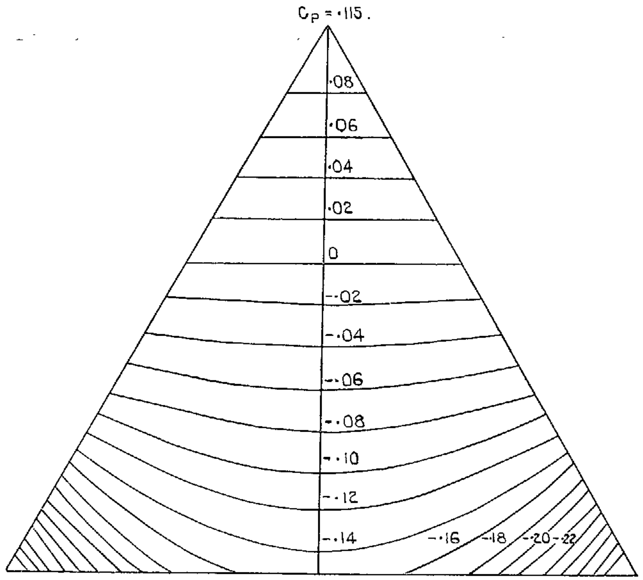

FIG. 9a. Surface (2), isobars at $M = 1 \cdot 6$.

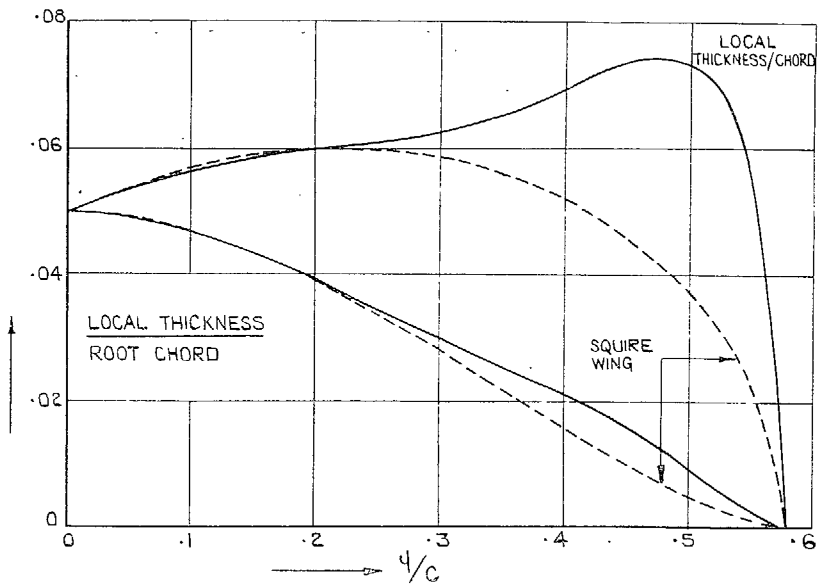

FIG. 9b. Surface (2), variation in local thickness.

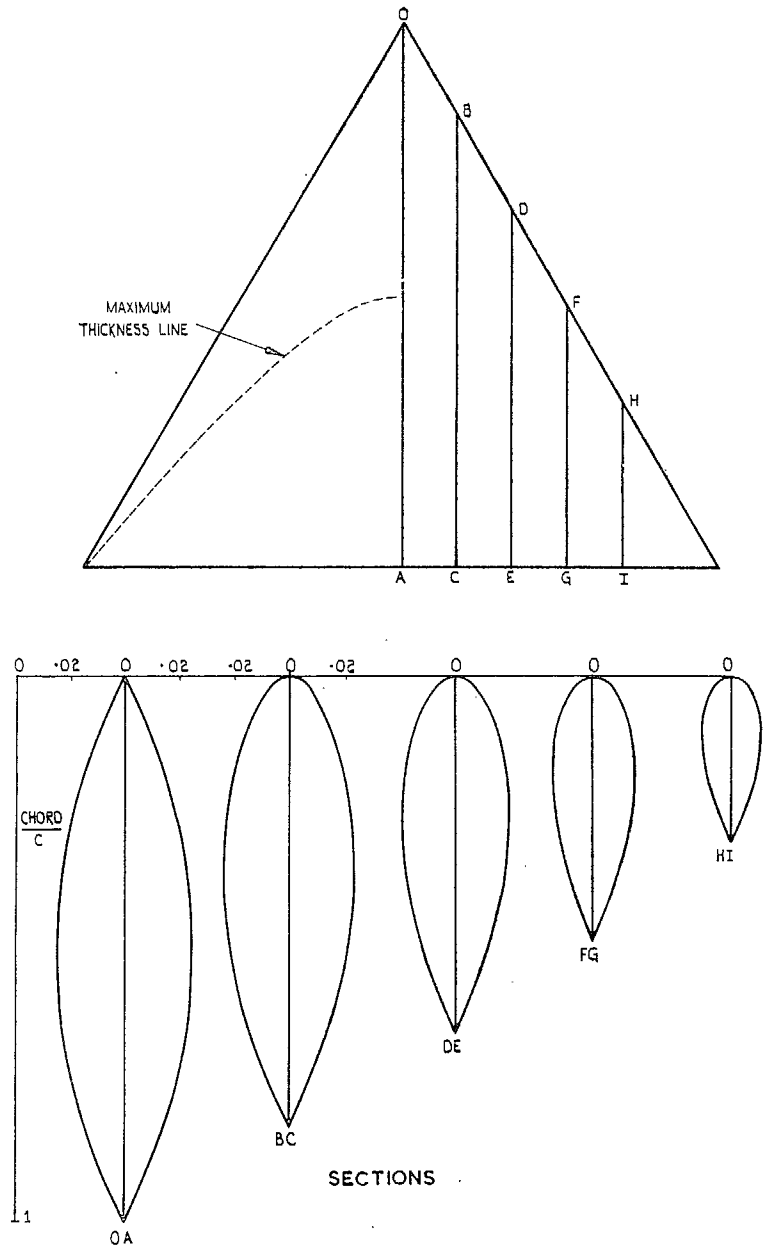

FIG. 9c. Shape of surface (2).

D

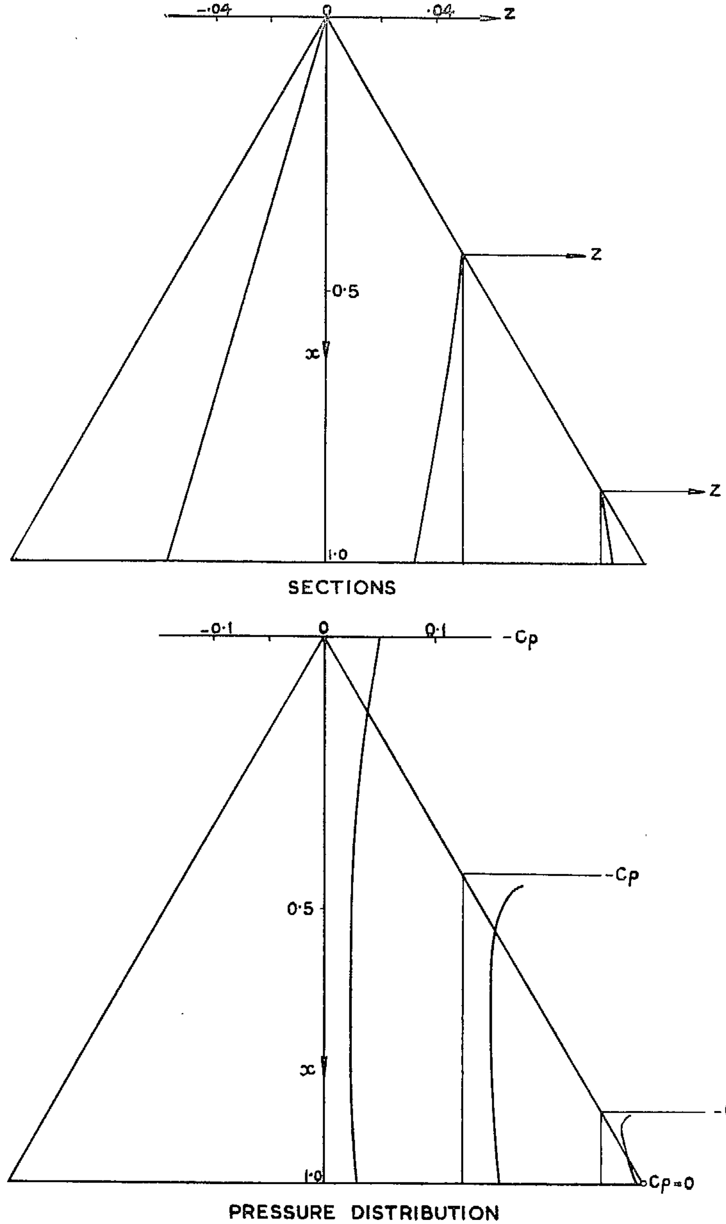

FIG. 10a. Surface (i), shape and pressure distribution. $M = 1 \cdot 442$. $C_{L0} = 0 \cdot 1$.

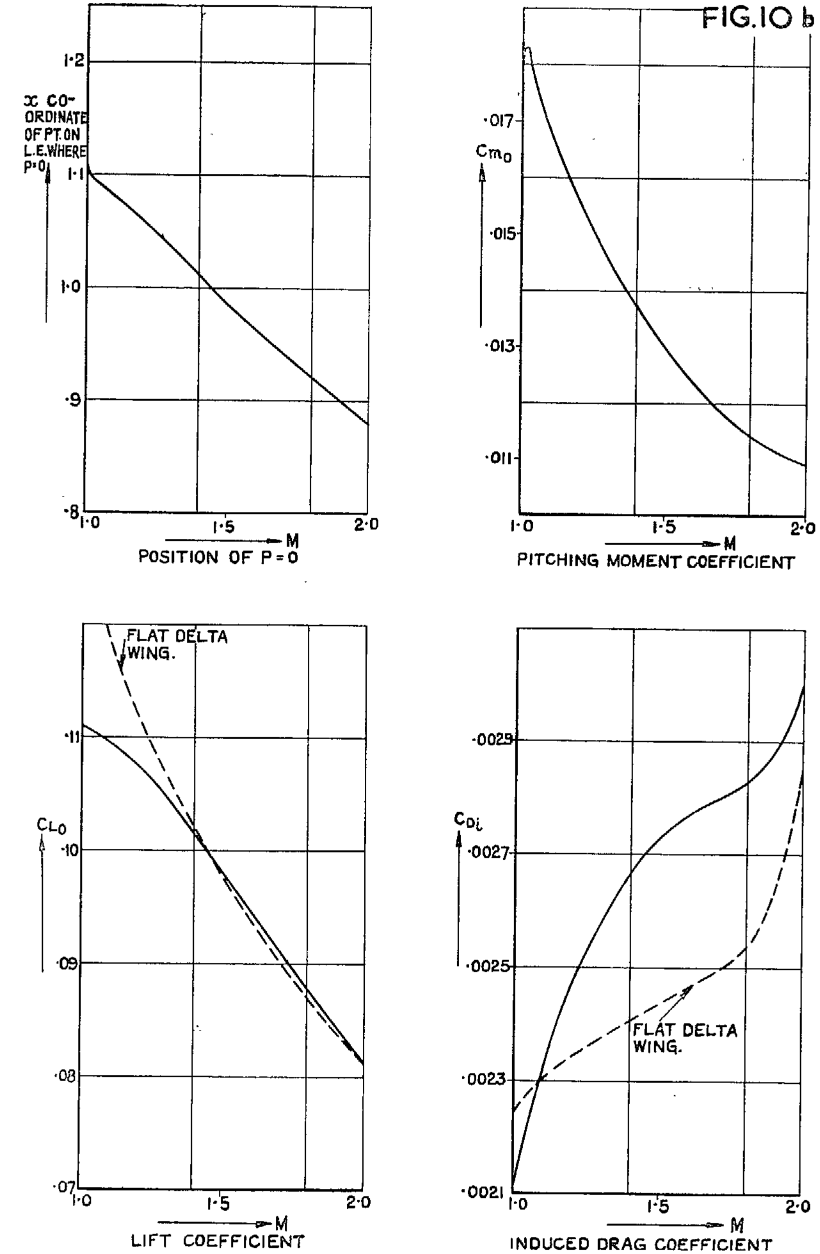

FIG. 10b. Aerodynamic characteristics of surface (i) at varying Mach number ($1 < M < \operatorname{cosec} \gamma$).

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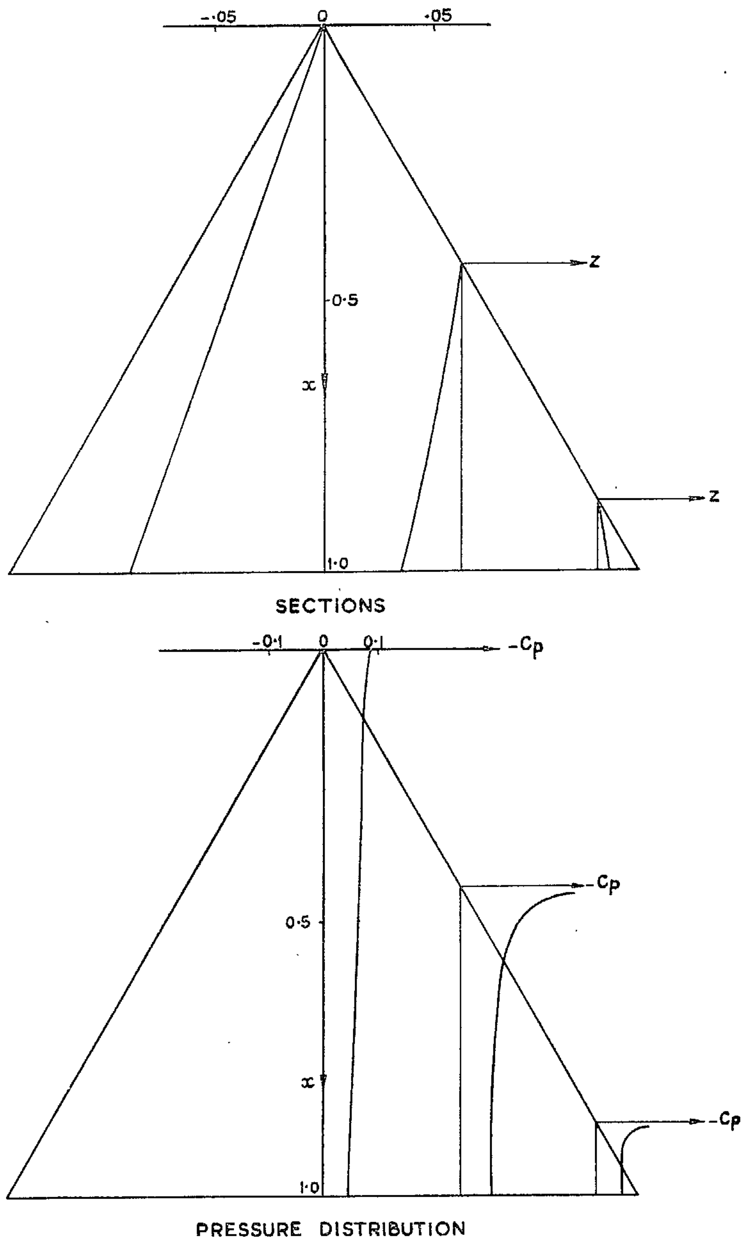

FIG. 11a. Surface (ii), shape and pressure distribution. $M = 1 \cdot 6$. $C_{L0} = 0 \cdot 15$.

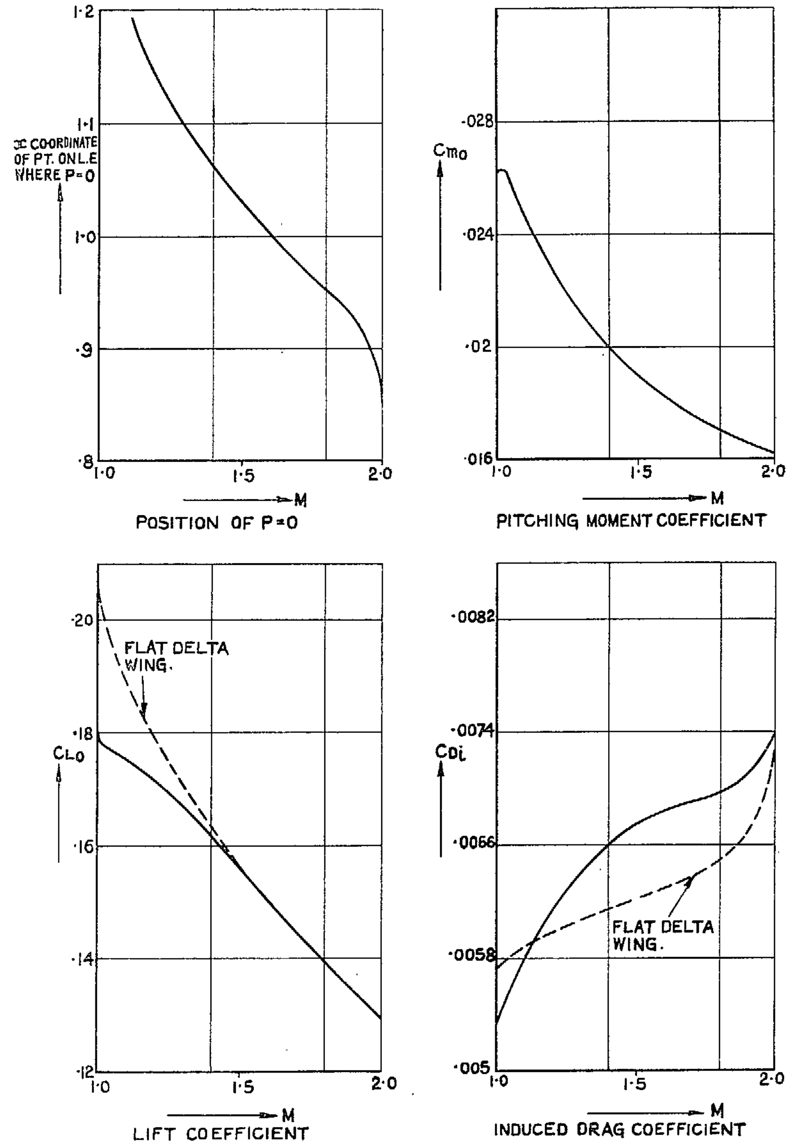

FIG. 11b. Aerodynamic characteristics of surface (ii) at varying Mach number ($1 < M < \operatorname{cosec} \gamma$).

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